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CONFLICT, GROWTH AND WELFARE: CAN INCREASING PROPERTY RIGHTS REALLY BE COUNTERPRODUCTIVE?

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Conflict, Growth and Welfare: Can Increasing Property Rights Really be Counterproductive?*

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Abstract

Gonzalez (2007), JET, 137(1), 127-139, sets out a growth model with conflict in which households allocate their resources across consumption, and investment in both productive and unproductive capital. A striking result is obtained: there are circumstances where increasing property rights in society can actually reduce social welfare and hence incremental changes are not necessarily in peoples' interests. This note reassesses this claim in a generalized form of his model with a CRRA utility function (with a risk aversion parameter, $\sigma > 1$ rather than his logarithmic form) and we assume a less than full depreciation of capital. Both these generalizations prove to be critical ones that significantly change the result.

JEL Classification: D23; O10; O40

Keywords: Conflict, growth, property rights, welfare.

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Contents

1 Introduction

In Gonzalez (2007) a growth model with conflict is formulated in which households allocate their resources across consumption, and investment in both productive and unproductive capital, the latter for both offensive and defensive purposes. A striking result is obtained: there are circumstances where increasing property rights in society can actually reduce social welfare and hence incremental changes are not necessarily in peoples' interests.[1] This note reassesses this claim in a generalized form of his model. In our generalization we employ a more general form of utility function (with a risk aversion parameter, $\sigma > 1$ rather than his logarithmic form) and we assume a less than full depreciation of capital.[2] Both generalization prove to be a critical ones that overturn 'counterproductive increasing property rights'.

2 The General Model

The model consists of an intertemporal optimization problem carried out by a representative agent i subject to their resource constraint. The agent at time $t = 0$ with infinite time horizon maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_i(t)) \tag{1}$$

where $c_i(t)$ is consumption at time t and $\beta \in (0, 1)$ is a discount factor. The instantaneous utility function is a concave and increasing function of consumption, c, $u'(c) > 0, u''(c) < 0$. It is assumed that each agent in the economy has similar preferences over the consumption sequence. Generalizing the Gonzales result, we assume a constant relative risk aversion (CRRA) form of the utility function

$$\begin{aligned} u(c) &= \frac{c^{1-\sigma} - 1}{1 - \sigma}; \;\; \sigma > 1 \\ &= \log(c); \;\; \sigma = 1 \end{aligned} \tag{2}$$

[1]For additional discussion of the Gonzalez results see for instance Garfinkel and Skaperdas (2007) and Ray (2007).

[2]The growth, RBC and DSGE model literatures suggest values $\sigma \in [1.5, 3]$ and $\delta \in [0.1, 0.2]$ on an annual basis.

Agent i allocates their output among consumption at time t, $c_i(t)$ and investment in next period's stocks of capital of which $k_i(t+1)$ is productive; $x_i(t+1)$ is unproductive and defensive; and $z_i(t+1)$ is unproductive and offensive. Assume that $k(0) > 0$, $x(0) > 0$, $z(0) > 0$ and that the productive, defensive and offensive capital depreciate at rates δ_k, δ_x and δ_z respectively. The gross investment at time t for each capital is then defined as: $k_i(t+1) - (1-\delta_k)k_i(t)$, $x_i(t+1) - (1-\delta_x)x_i(t)$ and $z_i(t+1) - (1-\delta_z)z_i(t)$.

A stock of productive capital, $k_i(t)$ produces output, $Ak_i(t)$ and economy's average output, $Ak(t)$. With appropriation, agent i retains only $p_i(t)Ak_i(t)$. Agent i can also lay claim to a proportion $q_i(t)$ of society's average output $y(t) = Ak(t)$, proportions $p_i(t)$ and $q_i(t)$ are given below. Under imperfect property rights, the net output of agent i is given by:

$$y_i(t) = A(p_i(t)k_i(t) + q_i(t)k(t))$$

With $A > 0$, households supply a fixed amount of labour, L, and productive capital per head is $k_i(t) \equiv \frac{K(t)}{L}$. Output is only a function of capital and linear in productive capital, $k_i(t)$. The production function has some important characteristics to note: there is no diminishing returns to capital with a constant marginal product of capital equal to A.[3]

Proportions $p_i(t)$ and $q_i(t)$ are given by ratio forms of the contest success functions (CSF):[4]

$$p_i(t) = \frac{\pi(x_i(t))^m}{\pi(x_i(t))^m + (z(t))^m} = \frac{1}{1 + \frac{1}{\pi}\left(\frac{z(t)}{x_i(t)}\right)^m} \tag{3}$$

$$q_i(t) = \frac{(z_i(t))^m}{\pi(x(t))^m + (z_i(t))^m} = \frac{1}{1 + \pi\left(\frac{x(t)}{z_i(t)}\right)^m} \tag{4}$$

where $1 \leq \pi < \infty$ ensures that defence of property claims is more effective than the challenge of other agents' claims. In the context of the model here, the parameter π

[3]It is this assumption that allows technology to be the engine of continuous economic growth.

[4]Two forms of the CSF, discussed in Hirshleifer (2000) at some length, are the *ratio form* and the *difference form*. General forms of the CSF discussed in Skaperdas (1996).

captures *property rights*: $\pi = 1$ would imply that there is no differentiation between defensive and offensive activities. As $\pi \to \infty$ then $p_i(t) \to 1$ and $q_i(t) \to 0$ and with $\pi \to \infty$ then $p_i(t) \to 1$ and $q_i(t) \to 0$. In other words, property rights are guaranteed and we arrive at the standard economic model. Then in the absence of appropriative or defensive activities a stock of productive capital $k_i(t)$ accumulated at time t produces output $y_i(t) = Ak_i(t)$.

The parameter m in (3) and (4) is the *decisiveness parameter* scaling the degree to which a side's greater strength translates into enhanced appropriation success. Following Gonzalez, we assume $0 \leq m \leq 1$ ruling out increasing returns to scale from non-productive activities.

3 The Optimization Problem and Equilibrium

Thus, the resource constraint for agent i is given by

$$\begin{aligned} y_i(t) = A(p_i(t)k_i(t) + q_i(t)k(t)) &\geq c_i(t) + k_i(t+1) - (1-\delta_k)k_i(t) + x_i(t+1) \\ &- (1-\delta_x)x_i(t) + z_i(t+1) - (1-\delta_z)z_i(t) \end{aligned} \tag{5}$$

At time $t = 0$, each agent is endowed with stocks of productive, $k(0) > 0$ and unproductive capital, $x(0) > 0$ and $z(0) > 0$. Taking other agents decision as given, agent i then chooses allocations $\{c_i(t)\}$, $\{k_i(t)\}$, $\{x_i(t)\}$ and $\{z_i(t)\}$ to maximize (1) subject to the resource constraint (5).

To solve the optimization problem of agent i let $\lambda_i(t)$ denote the Lagrangian multiplier and form the Lagrangian:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t [u(c_i(t)) + \lambda_i(t)(A(p_i(t)k_i(t) + q_i(t)k(t)) - c_i(t) \\ &- k_i(t+1) + (1-\delta_k)k_i(t) - x_i(t+1) + (1-\delta_x)x_i(t) - z_i(t+1) + (1-\delta_z)z_i(t))] \end{aligned}$$

The first-order conditions are

$$
\begin{aligned}
c_i(t)^{-\sigma} &= \lambda_i(t) && (6)\\
\beta(Ap_i(t)+1-\delta_k) &= \beta\left(A\frac{\partial p_i(t)}{\partial x_i(t)}k_i(t)+1-\delta_x\right)=\beta\left(A\frac{\partial q_i(t)}{\partial z_i(t)}k(t)+1-\delta_z\right)\\
&= \frac{\lambda_i(t-1)}{\lambda_i(t)} && (7)
\end{aligned}
$$

$$
\begin{aligned}
y_i(t)=A(p_i(t)k_i(t)+q_i(t)k(t)) &= c_i(t)+k_i(t+1)-(1-\delta_k)k_i(t)+x_i(t+1)\\
&- (1-\delta_x)x_i(t)+z_i(t+1)-(1-\delta_z)z_i(t) && (8)
\end{aligned}
$$

From (6), the Lagrangian multiplier, $\lambda_i(t)$ is equal to the marginal utility of consumption. Hence, the extra utility from additional unit of consumption "today" is the opportunity cost of forgone productive or defensive or offensive capital. Equation (7) equates the marginal benefit of appropriative and defensive investment in consumption units with the utility opportunity costs in the previous period. Equation (8) is the binding resource constraint.

Given the CSFs, the partial derivative with respect to $x_i(t)$ and $z_i(t)$ are

$$
\begin{aligned}
\frac{\partial p_i}{\partial x_i} &= \frac{m}{x_i}\left(\frac{\pi x_i^m}{\pi x_i^m+z^m}\right)\left(\frac{z^m}{\pi x_i^m+z^m}\right)=\frac{m}{x_i}p_i(1-p_i) && (9)\\
\frac{\partial q_i}{\partial z_i} &= \frac{m}{z_i}\left(\frac{z_i^m}{\pi x^m+z_i^m}\right)\left(\frac{\pi x^m}{\pi x^m+z_i^m}\right)=\frac{m}{z_i}q_i(1-q_i) && (10)
\end{aligned}
$$

which now completely characterizes the equilibrium.

We now seek a symmetric balanced growth steady state with $\delta_k=\delta_x=\delta_z=\delta$, $k_i(t)=k(t)$, $x_i(t)=x(t)$ and $z_i(t)=z(t)$ and $\frac{c(t+1)}{c(t)}=\frac{k(t+1)}{k(t)}=\frac{x(t+1)}{x(t)}=\frac{z(t+1)}{z(t)}=1+\gamma$ where γ is the continuous growth rate yet to be determined. Imposing these properties on the first order conditions we arrive at the equilibrium

$$
\begin{aligned}
\frac{c(t+1)}{c(t)} &= \frac{k(t+1)}{k(t)}=\frac{y(t+1)}{y(t)}=(1+\gamma)=[\beta(Ap(\pi)+1-\delta)]^{\frac{1}{\sigma}} && (11)\\
x(t) &= z(t)=m(1-p(\pi))k(t) && (12)\\
p(\pi) &= \frac{\pi}{1+\pi} && (13)\\
c(t) &= [A-(1+2m(1-p(\pi)))(\gamma+\delta)]k(t) && (14)
\end{aligned}
$$

given the initial condition $k(t) = k(0)$ at $t = 0$. It is easily verified that if $\sigma = 1$ and $\delta = 1$ this equilibrium reduces to that of Gonzalez.

For use later are savings rates

$$s^k = \frac{1}{A}(\gamma + \delta) \tag{15}$$

$$s^x = s^z = \frac{m(1 - p(\pi))}{A}(\gamma + \delta) \tag{16}$$

where $s^k \equiv \frac{k_i(t+1)-(1-\delta)k_i(t)}{Ak_i(t)}$, s^x and s^z, defined analogously are the savings rates for productive, unproductive defensive and unproductive offensive capital respectively.

If $\delta = 1$ and $\sigma = 1$, the growth rate becomes, $\beta p(\pi)A - 1$, which is the growth rate in Gonzalez. Since $p \geq \frac{1}{2}$ for $\pi \geq 1$, the sufficient condition for positive growth is that $\beta A > 2$. Further, if $\pi \to \infty$ and $p(\pi) \to 1$ we arrive at the standard endogenous growth model growth rate outcome under perfect property rights. If $\infty > \pi > 1$ and thus $\frac{1}{2} < p(\pi) < 1$, the growth rate under imperfect property rights is always less than the growth rate under perfect property rights.

4 Property Rights and Social Welfare

For $\sigma = 1$ and $\delta = 1$, Gonzalez asserts that a gradual piecemeal increment in property rights (π) does not always improve welfare. Formally he shows:

Proposition 1

If $\sigma = \delta = 1$, for relatively very high values of m and β there exists a sequence (a, b), and $\pi \in (a, b) \subset [1, \infty)$ such that even though property rights are improving and positive growth is assured, the utility of agent i declines for $\pi \in (a, b)$.

Proof. See Appendix A.

As Gonzalez states , this proposition defines a sequence in which there exist an interior equilibrium, for all $\pi \in [1, \infty)$ but for some π utility is decreasing. The graphs labelled $\sigma = 1$ in figure 1 illustrates the Gonzalez proposition as m increases from $m = 0.1$ to $m = 0.97$ and parameter values are otherwise as in his paper.

The intuition for this result can be seen by considering the two effects of better property rights and therefore higher growth on welfare. First the higher savings entailed means that current consumption $c(0)$ falls as can be seen from (A.3). This negative effect on welfare is offset by the positive from the increase in growth. The condition (A.2) is for the latter growth effect to outweigh the former current consumption effect. For the perfect property rights case as $\pi \to \infty$ and $p(\pi) \to 1$ we have that

$$\frac{dU}{dp} = [(1+\gamma)^{\sigma} - \beta(1+\gamma)]\, 2m(\gamma+\delta) \tag{17}$$

Since $\beta < 1$, for $\sigma \geq 1$ (17) is positive. Thus for the perfect property rights model the welfare effect of higher growth is (not unsurprisingly) positive.

As property rights are eroded the unambiguously positive effect of growth on welfare breaks down as shown in proposition 1 for the restrictive case of $\sigma = \delta = 1$. The reason is that higher growth increases inefficient non-productive investment and may no longer offset the current consumption effect. However this effect is crucially dependent on the choice of the risk-aversion parameter σ and depreciation rate δ. First we can show:

Proposition 2

For δ=1, there exists a $\sigma = \sigma^* > 1$ such that for any $p \in [\frac{1}{2}, 1]$ we have $\frac{dU}{dp} > 0$.

Proof. See Appendix A.

The graphs labelled $\sigma = 1.01$ and $\sigma = 1.1$ show how sensitive the welfare-p relationship is to small increases in σ and that σ^* need only to be slightly over unity for a monotone relationship to emerge.

Next we put $\sigma = 1$ and examine $\delta < 1$. Then we have the proposition:

Proposition 3

For σ=1, there exists a $\delta = \delta^* < 1$ such that for any $p \in [\frac{1}{2}, 1]$ we have $\frac{dU}{dp} > 0$.

Proof. See Appendix A.

Figure 2 shows the case where $\delta \in (0, 1)$ and $\sigma = 1$ and again illustrates how sensitive the Gonzalez result is to small deviations from $\delta = 1$.

The intuition behind propositions 2 and 3 is somewhat the same as the proposition 1. In what follows, the rates of savings for each capital increase in δ and γ but γ decreases in σ and δ. Therefore, in proposition 2, the increase in σ causes the growth rate, γ to fall, this means the future productive savings also falls but the most important implication is that the returns to defensive and offensive activities are also falling. There is, in fact, a further effect of fall in savings rates that is the current consumption rises. Whereas, in proposition 3, even though the effect of the fall in δ increases growth rate, γ, and hence this increases saving rates, there is a direct and greater effect of δ on savings rates. Formally, taking the partial derivatives of (15) and (16) with respect to δ it is easy to verify that;

$$\frac{\partial s^k}{\partial \delta} = \frac{1}{A}(1-\beta) \qquad \text{and} \qquad \frac{\partial s^x}{\partial \delta} = \frac{\partial s^z}{\partial \delta} = \frac{m(1-p(\pi))}{A}(1-\beta)$$

Hence since $\beta < 1$, the net effect of fall in δ is that it reduces the savings rates and hence the welfare effects of non-productive investment.

5 Conclusions

Our extension of the Gonzalez model shows that the Gonzalez findings do not hold for $\forall\ \sigma > 1$ and for $\forall\ \delta \in [0,1]$. With $\delta = 1$, for as low a value as $\sigma = 1.1$, figure 1 indicates that a small increment in property rights has a positive effect on utility throughout. Similarly with $\sigma = 1$, for as high a value as $\delta = 0.8$ in figure 2 the monotone welfare-property rights relationship emerges. The Gonzalez result in other words is crucially dependent on the assumptions of a logarithmic utility function in consumption and complete depreciation of capital.

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A Proofs of Propositions

Proposition 1

In order to analyze this claim under the extended model the utility function is re-examined. Putting $c(t) = (1+\gamma)c(t-1) = (1+\gamma)^t c(0)$ we obtain

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \left[\frac{c(t)^{1-\sigma}-1}{1-\sigma}\right] = \sum_{t=0}^{\infty} \beta^t \left[\frac{((1+\gamma)^t c(0))^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma}\right] \\ &= \frac{1}{(1-\sigma)}\left[\frac{c(0)^{1-\sigma}}{1-\beta(1+\gamma)^{1-\sigma}} - \frac{1}{1-\beta}\right] \end{aligned} \tag{A.1}$$

which is valid if and only if $\beta(1+\gamma)^{1-\sigma} < 1$. For $\sigma \geq 1$ this clearly holds for $\gamma > 0$ and $\beta < 1$.

To evaluate at $\sigma = 1$ (the Gonzalez Case) we apply L'Hopital's rule to obtain

$$\lim_{\sigma \to 1} \sum_{t=0}^{\infty} \beta^t \left[\frac{(1+\gamma)^t c(0)^{1-\sigma}}{1-\sigma} - 1\right] = \frac{1}{1-\beta}\log(c(0)) + \frac{\beta}{(1-\beta)^2}\log(1+\gamma)$$

We now examine the conditions for welfare to increase monotonically or not as property rights captured by the parameter π increase; i.e, the conditions under which $\frac{dU}{d\pi} = \frac{dU}{dp(\pi)} \geq 0$. First note that

$$\frac{dU}{d\pi} = \frac{dU}{dp(\pi)}\frac{dp(\pi)}{d\pi}$$

where $\frac{dp(\pi)}{d\pi} > 0 \; \forall \; \pi \in [1, \infty)$. Hence, considering the condition $\frac{dU}{dp(\pi)} \geq 0$ will suffice. We then have that

$$\frac{dU}{dp(\pi)} = \frac{d\left(\frac{1}{(1-\sigma)}\left(\frac{c(0)^{1-\sigma}}{1-\beta(1+\gamma)^{1-\sigma}} - \frac{1}{1-\beta}\right)\right)}{dp(\pi)} \geq 0$$

if and only if

$$[(1+\gamma)^{\sigma} - \beta(1+\gamma)]\frac{dc(0)}{dp(\pi)} + \beta c(0)\frac{d\gamma}{dp(\pi)} \geq 0 \tag{A.2}$$

noting from the equilibrium in section 3 above that

$$\begin{aligned}
\frac{dc(0)}{dp(\pi)} &= \left[2m(\gamma+\delta) - \frac{1}{\sigma}[1+2m(1-p(\pi))][\beta(p(\pi)A+1-\delta)]^{\frac{1-\sigma}{\sigma}}\beta A\right]k(0) \\
\frac{d\gamma}{dp(\pi)} &= \beta A\frac{1}{\sigma}[\beta(p(\pi)A+1-\delta)]^{\frac{1-\sigma}{\sigma}} \\
c(0) &= [A-(1+2m(1-p(\pi)))(\gamma+\delta)]k(0) && \text{(A.3)}
\end{aligned}$$

Next we evaluate the condition at $\sigma = 1$ and $\delta = 1$ to give

$$\begin{aligned}
\left.\frac{dc(0)}{dp(\pi)}\right|_{\sigma=1,\delta=1} &= [2m(\beta Ap(\pi)) - [1+2m(1-p(\pi))]\beta A]\,k(0) \\
\left.\frac{d\gamma}{dp(\pi)}\right|_{\sigma=1,\delta=1} &= \beta A
\end{aligned}$$

Substituting these into (A.2) and assuming $\sigma = 1$ and $\delta = 1$, the Gonzalez proposition (1) is verified by solving the following condition for (A.2) to hold:

$$p(\pi)[1+2m(1-(2-\beta)p(\pi))] \leq 1$$

The left hand side is quadratic in $p(\pi)$ and the condition fails at some $p(\pi) < 1$ if

$$m > \frac{1}{2} \qquad \text{and} \qquad \frac{(1+2m)^2}{8m} > 2-\beta$$

which completes the proof.[5]

Proposition 2

Rearranging (A.2) for $\delta = 1$ we now have

$$\begin{aligned}
\frac{dU}{dp} &= [(1+\gamma)^{\sigma} - \beta(1+\gamma)]\,2m(\gamma+1) + \frac{(\beta A)^2}{\sigma}(1+\gamma)^{1-\sigma}[1-p(1+2m(1-p))] \\
&\equiv F(\sigma,p) + G(\sigma,p) && \text{(A.4)}
\end{aligned}$$

Clearly $F(\sigma,p) \equiv [(1+\gamma)^{\sigma} - \beta(1+\gamma)]\,2m(\gamma+\delta) > 0$ for all $\sigma \geq 1$. Furthermore putting $(1+\gamma)^{\sigma} = \beta Ap$:

$$G(\sigma,p) \equiv \frac{(\beta A)^2}{\sigma}(\beta Ap)^{\frac{1-\sigma}{\sigma}}(1-p(1+2m(1-p)))$$

[5]For $m = 1$ the condition for $\frac{dU}{dp} < 0$ at some $p(\pi) < 1$ becomes $\beta > \frac{7}{8}$.

which is negative if $(1 - p(1 + 2m(1 - p))) < 0$. For low levels of m this does not occur for any p and $\frac{dU}{dp} > 0$ for all σ and p. But for $m = 1$ and $p = \frac{3}{4}$, $G(\sigma, p) < 0$. For the latter case it is clear that $-G(\sigma, p)$ is downward sloping in σ and tends to zero as $\sigma \to \infty$ as illustrated in figure 3. This proves the proposition.

Proposition 3

Rearranging (A.2) for $\sigma = 1$

$$\begin{aligned}\frac{dU}{dp} &= 2m(1-\beta)(1+\gamma)(\gamma+\delta) + (\beta A)^2 - (\beta A)^2 p(\pi)(1 + 2m(1 - p(\pi))) \\ &\equiv H(\delta, p) + K(p) \qquad \text{(A.5)}\end{aligned}$$

where $H(\delta, p) \equiv 2m(1-\beta)(1+\gamma)(\gamma+\delta) + (\beta A)^2 > 0 \ \forall \delta$ and downward sloping in δ, and $K(p) \equiv -(\beta A)^2 p(\pi)(1 + 2m(1 - p(\pi)) < 0$. Consider a value of $p(\pi)$ for which at $\delta = 1$ the Gonzalez result $\frac{dU}{dp} < 0$ is obtained, as in figure 4. The proposition now follows.

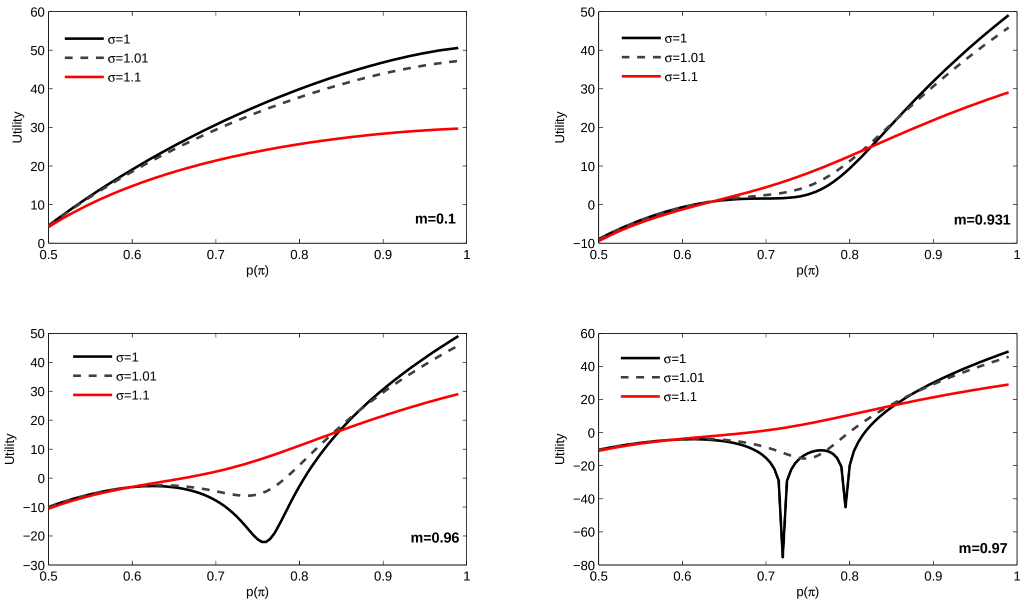

Figure 1: Utility and Property Rights with $\delta = 1$: $A = 2.3$, $\beta = 0.9$ and $k(0) = 1$.

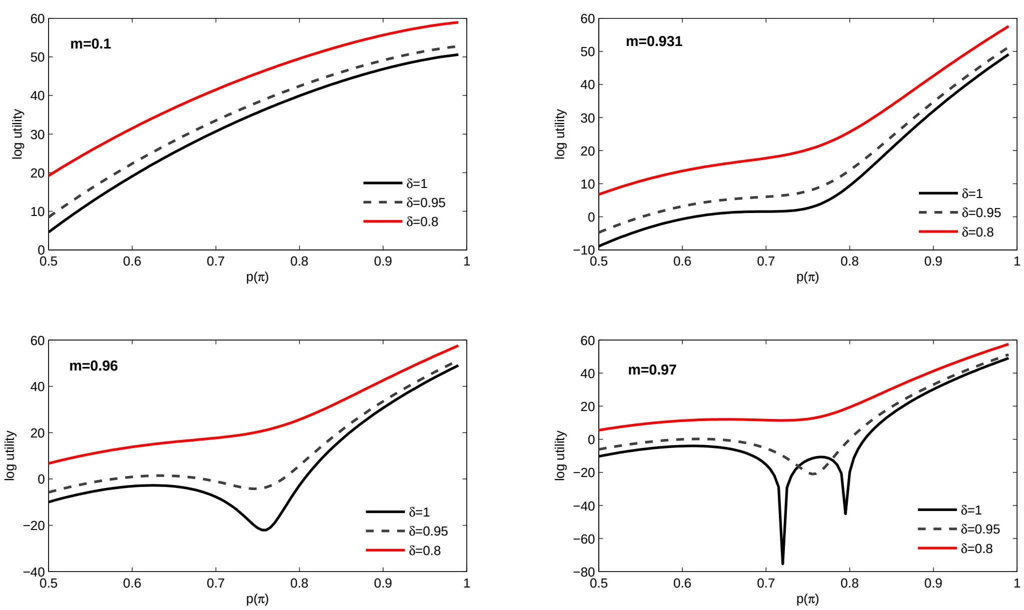

Figure 2: Utility and Property Rights with $\sigma = 1$: $A = 2.3$, $\beta = 0.9$ and $k(0) = 1$.

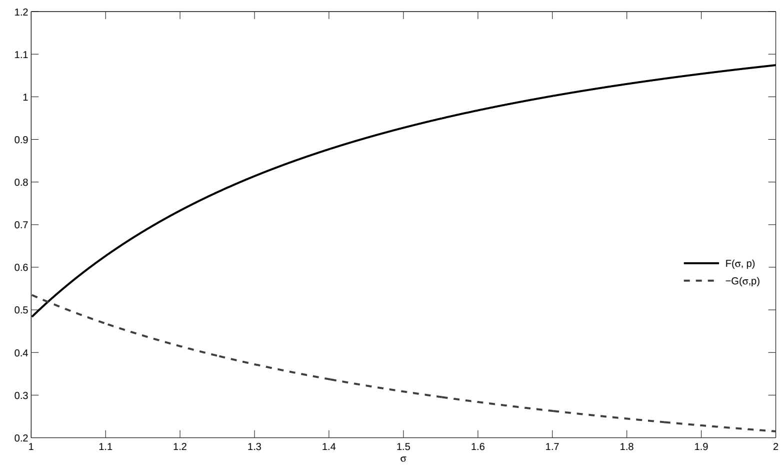

Figure 3: $F(\sigma, p)$ and $-G(\sigma, p)$: $A = 2.3$, $\beta = 0.9$, $m = 1$ and $p = 3/4$.

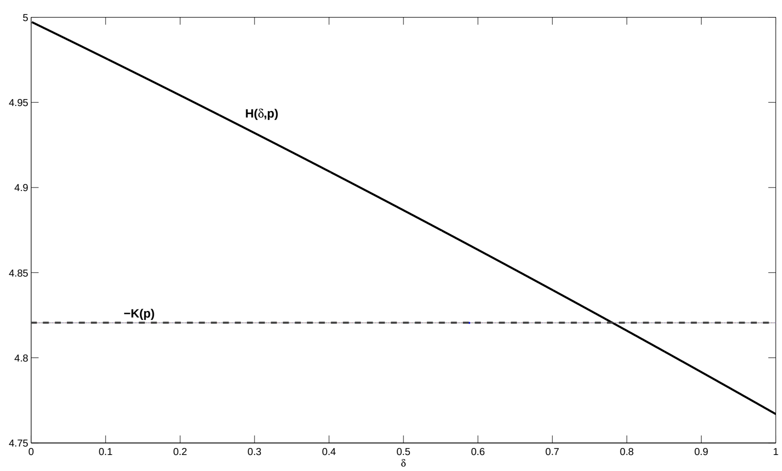

Figure 4: $H(\delta, p)$ and $-K(p)$: $A = 2.3$, $\beta = 0.9$, $m = 1$ and $p = 3/4$.