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PRICE REGULATION, INVESTMENT AND THE  
COMMITMENT PROBLEM

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# Price Regulation, Investment and the Commitment Problem\*

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## Abstract

We consider a dynamic model of price regulation with asymmetric information where strategic delegation is available to the regulator. Firms can sink non-contractible, cost-reducing investment but regulators cannot commit to future price levels. We fully characterise the perfect Bayesian equilibrium and show that, with incentive contracts and no delegation, under-investment occurs. We then show that delegation to a suitable regulator can both improve investment incentives and ameliorate the ratchet effect by credibly offering the firm future rent. Simulations indicate significant welfare gains from these two effects and that a wide range of regulatory preferences can achieve this result.

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# 1 Introduction

In many settings, an investment ‘hold-up’ problem can arise because one party is unable to commit to appropriate action once the other has sunk the investment costs. A familiar example occurs in the context of industry price regulation because regulators face a conflict between their desire to encourage investment and their obligations towards consumers. The weak incentives implicit in such arrangements have been blamed for poor investment performance in several regulatory environments (e.g. Levy and Spiller (1994); Lyon (1995); Newbery (1999)).

The purpose of this paper is to analyse the under-investment problem in the context of a dynamic non-commitment relationship between a regulator and a regulated monopoly. We assume that the regulator observes neither the firm’s productivity nor whether investment has taken place, but can observe the firm’s total costs (though not the individual components of cost). The presence of asymmetric information means that the welfare costs of suboptimal investment are compounded by those of the ratchet effect that typically afflicts such dynamic non-commitment problems. The paper makes two contributions. First, it proposes (and demonstrates) a solution to the under-investment problem based on strategic delegation to an independent regulator with suitable preferences. We suggest that commitment to such ‘types’ often occurs in practice. Second, this takes place in a model that extends the theoretical literature on regulation in two ways: (i) situations where the regulator observes neither investment nor the other components of cost have received little attention yet, for some types of activity, are clearly appropriate; (ii) we examine the problem in the context of optimal (subject to asymmetric information) price regulation, where the regulator is prevented from making lump-sum transfers to the firm. Again, this reflects much regulatory practice.

A number of authors have considered the under-investment problem. The literature can be usefully divided into papers that develop ‘reputational’ trigger-strategy equilibria where strategies are history-dependent, and those that focus on sequential or perfect Bayesian or subgame-perfect equilibria (depending on the information assumptions)

without history-dependence.<sup>1</sup> Considering the former first, in a complete information dynamic game between the utility and a benevolent regulator, Salant and Woroch (1992) and Gilbert and Newbery (1994) show that first-best levels of investment can be sustained as a subgame-perfect trigger-strategy equilibrium. Unfortunately, there are well-known problems with this approach concerning the arbitrary length of the punishment phase (usually infinity) and the infinite number of such equilibria. Even if coordination is possible, the equilibrium is not ‘renegotiation-proof’. This questions the credibility of trigger-strategy equilibria, even though they are subgame perfect.<sup>2</sup>

Turning to the second group of papers, Goodwin and Patrick (1992) focus on the speed with which regulators should allow sunk cost recovery. Alternatively, Lyon (1995) uses a full information model to show how allowing the regulator to engage in hindsight review can prevent investment with uncertain costs. Lewis and Sappington (1991) consider the effects on investment of changing regulators when investment cost is uncertain but is guaranteed to be reimbursed to the firm. On average, under-investment occurs. Besanko and Spulber (1992) assume that the regulator cannot observe the firm’s cost and cannot commit to a particular price level: she must offer a transfer and price once investment has been observed. In their sequential equilibrium, the firm can signal its type through this observable investment and the under-investment problem is ameliorated; see also Urbiztondo (1994). Dalen (1995) looks at a two-period model in which investment takes place in period 1. The regulator cannot observe firms’ costs and again provides transfers to the firm. When investment is contractible, it reduces the ratchet effect by inducing more first-period separation. When investment is non-contractible,

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<sup>1</sup>There is a literature that restricts the set of allowable equilibria to those for which both parties (in our context, the regulator and the firm) agree in each period. These ‘renegotiation-proof’ contracts are binding unless both players agree to replace the original contract. They assume an intermediate degree of commitment between the ‘full’ and the ‘no’ intertemporal commitment to policy scenarios studied in this paper. Our delegation game, set out in section 4, assumes that the regulator cannot commit, either fully or partially, to pricing policy, but in appointing the regulator the government can commit to a particular type. See Laffont and Tirole (1993), chapter 10, for details of two-period renegotiation-proof price contracts, but without investment.

<sup>2</sup>See al Nowaihi and Levine (1994) who, in the context of a monetary policy game, argue for a refinement they term ‘chisel-proofness’ to resolve this difficulty. It should be noted that the renegotiation-proof equilibria used in repeated games differ from the concept used in the contract literature discussed in the previous footnote. They do not necessarily involve contracts or even negotiation, but should be interpreted as allowing players to re-coordinate their expectations of strategies. For this reason the term ‘recoordination-proof’ equilibria is often used instead. For further discussion see Fudenberg and Tirole (1993), section 5.4.

under-investment occurs.<sup>3</sup>

As explained above, we amend this literature by considering a long-term regulatory relationship with non-contractible investment, asymmetric information about costs and no lump-sum transfers. Specifically, we build on Laffont and Tirole (1993)'s framework. Firms can be efficient or inefficient and can take (costly) action to reduce their costs in each of two periods. In the first, they can also undertake costly activity to lower their costs in future ('investment'). The regulator observes none of these three actions but observes total costs and must design prices to encourage cost-reducing effort and investment, as well as generating consumer surplus. Being unable to commit to future prices means that sub-optimal investment and the ratchet effect occur. We characterise the full range of perfect Bayesian equilibria for this setting, which can involve complete separation as well as partial and complete pooling.

Our set-up explores a particular solution to the sub-optimal investment problem, and the ratchet effect: strategic delegation to an independent regulator with pro-industry preferences.<sup>4</sup> The intuition is that a regulator's preference for industry profits dilutes the commitment problem she faces. In the current paper, the effects of this idea are particularly strong because the firm's awareness of the regulator's pro-industry tastes makes it more confident of retaining profits from cost-reducing investment and avoiding the ratchet effect of early information revelation. Interestingly, delegation can produce, *over*-investment as well as under-investment. However, delegation to particular (identifiable) types of regulator generates *optimal* investment. We present simulations to illustrate potentially significant welfare gains from such delegation.<sup>5</sup>

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<sup>3</sup>A related literature, beginning with Averch and Johnson (1967) compares investment incentives under alternative forms of price regulation, typically rate-of-return and price-capping. In a recent contribution, allowing for complete information and technical progress, Biglaiser and Riordan (2000) demonstrate that sub-optimal investment may be generated by both these schemes: with rate-of-return regulation this happens because of the treatment of depreciation in the face of technical progress; with price-capping it results from the commitment problems associated with regulatory reviews.

<sup>4</sup>Baron and Besanko (1987) consider an alternative mechanism for avoiding the ratchet effect. They assume that the regulator is committed (by some means) to choose from 'fair' mechanisms (those which respect the firm's zero profit constraint) and, in exchange, the firm by choosing to participate in the first period, waives its right not to participate in the second period. Their set-up therefore assumes some degree of commitment by both the regulator and the firm, whereas we assume commitment only by the government (to a regulatory 'type'). In addition, the problem they address differs fundamentally from ours by excluding investment and moral hazard.

<sup>5</sup>The literature identifies several kinds of strategic delegation. In particular, apart from delegation to a (publicly observable) 'type' of regulator (Baron (1988), Spulber and Besanko (1992), Currie

We argue that this solution is appealing because of its correspondence with practice in several economies: for example, utilities regulation in the UK and certain features of environmental regulation in the US. One of the arguments for regulation over public ownership has been the independence this injects into the oversight of the industries concerned (Armstrong *et al.* (1994)) and it is certainly the case that British regulators are contractually independent from considerable government interference.<sup>6</sup> Further, it is clear that different ‘types’ of regulator are available to a government and that these types are observable before appointment (for example, from the candidate’s track-record).<sup>7</sup> Thus, it seems that a government can commit to a particular type of regulator more easily than to a particular policy. This point has also been made by Spulber and Besanko (1992) in the context of US environmental regulation and by Rogoff (1985) in the context of monetary policy.

The paper is structured as follows. In the next section, we set out the basic model and derive a full information benchmark: the Ramsey-optimum prices and investment levels. Section 3 then looks at price regulation with asymmetric information and commitment to two-period contracts, in order to explain the ‘time-inconsistency’ problem caused by non-commitment. In section 4, we relax the commitment assumption and, instead, introduce the idea of strategic delegation to a regulator whose ‘type’ is observable *ex ante*. We characterise the full set of equilibria for this model and illustrate them by simulation. Of particular interest here is the potential increases in investment and Pareto improvements in welfare that come about through delegation and the wide range of regulator types capable of achieving this outcome. The final section offers

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*et al.* (1999)) one might consider delegation to a given regulator whose actions are then governed by an incentive contract or set of instructions which may (or may not) be publicly observable—see Fershtman *et al.* (1991), Fershtman and Kalai (1997) and Fershtman and Gneezy (2001). Analogously, literature on central banking also distinguishes these two kinds of strategic delegation: *c.f.* Rogoff (1985) and Walsh (1995). The key ingredient is that observable commitment (to a type, a contract, an instruction or to the use of delegation) can improve welfare from the delegator’s perspective.

<sup>6</sup>For example: “. . . the [BT regulator] is officially independent of ministerial control and . . . is not due for reappointment for another three years” (Financial Times, 22/9/00, p. 1). See Graham (2000) for an account of the constitutional status of utility regulators in the UK.

<sup>7</sup>Again, examples are available in the UK. Thus, Tom Winsor’s appointment as UK rail regulator in 1999 was regarded as a “hawkish” move amongst commentators because of his strong track record in consumer law (e.g. Daily Telegraph, 24/3/99; 28/5/99). We do not consider the mechanisms available for choosing such regulators (see Baron (1988), Spulber and Besanko (1992), for examples of how the political process might do this). Instead, our aim is to illustrate the gains available from such delegation in the current context.

conclusions and suggestions for further research.

## 2 Full Information and the Ramsey Optimum

### 2.1 The Model and Payoffs

First, we set out the basic elements of the model. There are two periods. In period  $t = 1, 2$ , the firm produces a quantity  $q_t$  of a homogeneous good at cost

$$C_t = \beta_t - e_t + cq_t; \quad \beta_1 = \beta + i; \quad \beta_2 = \beta - f(i) \quad (1)$$

where  $e_t$  is total cost-reducing effort of which an amount  $i$ , ‘investment’, is devoted to reducing fixed costs in the second period by an amount  $f(i)$ .<sup>8</sup> Marginal costs are fixed and given by  $c$ . We make the standard assumptions  $f' > 0, f'' < 0, f(0) = 0, f'(0) = -\infty$ . We also assume that the efficiency parameter is sufficiently large to ensure that fixed costs are never negative; i.e.,  $\beta_t - e_t \geq 0$ . The good is sold at a price  $p_t = \phi(q_t)$  where  $\phi(\cdot)$  is the inverse demand curve.

Both the firm and regulator maximize a two-period welfare function with the same discount factor  $\delta$  and with single-period payoffs given respectively by

$$\begin{aligned} U(q_t, e_t, \beta_t) &= R(q_t) - C_t - \psi(e_t) \\ &= R(q_t) - cq_t - \beta_t + e_t - \psi(e_t) \end{aligned} \quad (2)$$

$$\begin{aligned} W(q_t, e_t, \beta_t, \alpha) &= S(q_t) - R(q_t) + \alpha U_t \\ &= S(q_t) - (1 - \alpha)R(q_t) - \alpha[cq_t + (\beta_t - e_t + \psi(e_t))] \end{aligned} \quad (3)$$

In (2),  $\psi(e_t)$  is the disutility of effort and again we make standard assumptions:  $\psi', \psi'' > 0$  for  $e_t > 0$ ,  $\psi(e_t) = 0$  otherwise. In (3),  $S(q_t)$  is the gross consumer surplus of the industry,  $R(q_t) = p_t q_t$  is the revenue,  $S(q_t) - R(q_t)$  is the net consumer surplus and the weight  $\alpha$  is placed on the firm’s profit by the regulator. A utilitarian

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<sup>8</sup>The assumption that effort only reduces fixed and not variable costs can be relaxed but a considerable cost in terms of tractability. For example, we could assume two types of imperfectly substitutable effort with managers dividing their total effort in each period between reducing fixed and variable costs. Laffont and Tirole (1993) consider situations where all effort is devoted to reducing variable costs.



regulator would have  $\alpha = 1$ , but in this paper we examine the effect of delegating to a regulator chosen to have different preferences. Suppose that the government has preferences defined by  $\alpha = \alpha_s \leq 1$ . Then a choice  $\alpha > \alpha_s$  signifies a more ‘pro-industry’ (pro-rent) regulator type than the government, while  $\alpha < \alpha_s$  would signify a more ‘pro-consumer’ type.

## 2.2 The Ramsey Optimum (RO)

We first solve for the ‘Ramsey Optimum’ (RO); that is the social optimum subject to a two-period individual rationality constraint for the firm.<sup>9</sup> This provides a full information benchmark for later results. Suppose that the social planner adopts the single-period social welfare function (3) with weight  $\alpha = \alpha_s$ . Then the RO is found by the maximization of the intertemporal social welfare function  $\Omega = W_1 + \delta W_2$  with respect to  $(q_t, e_t)$ ,  $t = 1, 2$  and  $i$ , where  $W_t$  is given by (3) with weight  $\alpha = \alpha_s$ , subject to a two-period individual rationality constraint

$$IR : U_1 + \delta U_2 \geq 0$$

To solve this maximization problem define a Lagrangian  $\mathcal{L} = \Omega + \mu(U_1 + \delta U_2)$  where  $\mu$  is a Lagrangian multiplier. The Kuhn-Tucker first-order and complementary slackness (CS) conditions are

$$e_t : \quad \psi'(e_t) = 1; \quad t = 1, 2 \quad (4)$$

$$i : \quad \delta f'(i) = 1 \quad (5)$$

$$q_t : \quad S'(q_t) + (\alpha_s - 1 + \mu)R'(q_t) = (\alpha_s + \mu)c; \quad t = 1, 2 \quad (6)$$

$$CS : \quad \mu(U_1 + \delta U_2) = 0$$

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<sup>9</sup>We use the term ‘Ramsey-Optimal’ because the pricing formula involves a (Ramsey) inverse elasticity mark-up to cover fixed costs. Notice that the *unconstrained* social optimum would have  $p_t = c$  and would require investment to be subsidized.

Using the standard result  $S'(q_t) = p_t$ , (6) can be written

$$L_t = \frac{p_t - c}{p_t} = \frac{\mu + \alpha_s - 1}{(\mu + \alpha_s)\eta(q_t)} \quad (7)$$

where  $L_t$  is the Lerner index and  $\eta(q_t) = -p_t q'_t / q_t$  is the elasticity of demand. It follows from (7) that the price in each period is the same. Furthermore, since fixed costs can never be negative by assumption, this common price must exceed the marginal cost, otherwise the IR constraint cannot be satisfied; thus  $L_t > 0$ . It follows from (7) that  $\mu > 0$ , and the IR condition therefore binds, iff

$$L_t = \frac{p_t - c}{p_t} \geq \frac{\alpha_s - 1}{\alpha_s \eta(q_t)}; \quad t = 1, 2$$

Clearly this condition holds if  $\alpha_s \leq 1$ , which we presently assume.

From (7) Ramsey prices  $p_1 = p_2 = p^{RO}$  and hence output  $q_1 = q_2 = q^{RO}$  are equal in the two periods, but not yet determined. Denote by  $e^{RO}$  and  $i^{RO}$  the Ramsey-optimal levels of  $e$  and  $i$  given by (4) and (5) respectively. Substituting back into the binding IR constraint then determines the Ramsey-optimal output  $q^{RO}$  and hence the price  $p^{RO} = \phi(q^{RO})$ , completing the social planner's problem.<sup>10</sup>

### 3 Asymmetric Information with Commitment

We now seek to establish the nature of the commitment problem in our model. First, we present results for the case where commitment is feasible, then we explain how these break down when the regulator cannot commit to a contract with the firm.

In contrast with the previous section, suppose that neither effort nor the productivity parameter  $\beta$  are observed by the regulator so she faces both an adverse selection and moral hazard problem. The regulator observes total cost and knows that  $\beta$  belongs to a two-point support:  $\beta = \underline{\beta}$  and  $\beta = \bar{\beta}$  ( $\bar{\beta} > \underline{\beta} > 0$ ), over which she holds priors  $\nu_1$

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<sup>10</sup>With commitment plus full information about total costs and demand, the RO can be implemented if the regulator faces only a moral hazard ( $\beta$  but not  $e$  or  $i$  observable) or an adverse selection ( $e$  and  $i$  but not  $\beta$  observable). In the former case, she commits to a two-period contract specifying only  $p^{RO}$  and rent maximizing managers choose  $e^{RO}$  and  $i^{RO}$ . In the latter case, the regulator can calculate  $\beta$  from observable cost, demand, effort and investment.

and  $1 - \nu_1$  respectively at the beginning of period 1. Investment does not need to be contractible, nor indeed observable for our results to hold.

Following Laffont and Tirole (1993), the regulator must now design contracts  $(\underline{p}_t, \underline{C}_t)$ ,  $(\bar{p}_t, \bar{C}_t)$ ,  $t = 1, 2$  for the efficient and inefficient firms respectively. In doing so, she must recognise the incentive compatibility constraints introduced by asymmetric information: each firm can mimic the other's costs by suitable choice of unobservable effort. Letting  $\underline{p}_1^C = \underline{p}_2^C = \underline{p}^C$ ,  $\bar{p}_1^C = \bar{p}_2^C = \bar{p}^C$ , etc., denote the solution to this problem is<sup>11</sup>

**Proposition 1 (Commitment Equilibrium).** *Assume  $\alpha = \alpha_s \leq 1$  and fixed costs are always positive. Then for the two-period contract under commitment we have that:*

(i)  $\underline{e}^C = e^{RO}$ ;  $\bar{e}^C < e^{RO}$ .

(ii)  $\underline{i}^C = \bar{i}^C = i^{RO}$ .

(iii) If the elasticity  $\eta(q_t)$  is non-increasing in  $q_t$ ,  $\bar{p}^C > \underline{p}^C$ .

(iv) For both types of firm, rent is less in the first period than the second. For the inefficient firm, rent is negative in the first period and positive in the second.

Parts (i) and (iii) of this proposition reflect the single-period trade-off between effort and rent that typifies such incentive contracts (see Laffont and Tirole (1993)). However, (ii) tells us that the regulator's ability to commit assures the firm of sufficient second-period rent (see (iv)) to encourage Ramsey-optimal investment.

Having examined the nature of the commitment solution in the presence of asymmetric information, now suppose that such commitment is *not feasible*. In this case, the contracts described in Proposition 1 are *time-inconsistent*: although they are optimal *ex ante*, *ex post* in period 2 they cease to be optimal and there exists a temptation for the regulator to re-optimize. This temptation exists for two reasons. First, the contract is a revelation mechanism that reveals the type of firm. In the second period an optimizing regulator will offer a new contract at a lower price that removes any information rent to the efficient firm. This is the familiar 'ratchet effect' which, when anticipated by the efficient firm, requires higher information rent in the first period to satisfy the first-period incentive-compatibility constraint. Second, the first-period investment is a

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<sup>11</sup>See Levine and Rickman (2001) for a proof of the case where investment is contractible. Proof of the non-contractible and non-observable result is available from the authors.

sunk-cost. The *ex ante* contract sees negative rent in the first period and positive rent in the second period for both types. However, in the absence of a binding commitment, *ex post* an optimizing regulator will renege on the promise of positive rent and offer a new contract at a lower price just sufficient to satisfy the second-period individual rationality constraint. Anticipating this opportunistic behaviour, in the absence of commitment both firms will under-invest in the first period. We now move to a formal analysis of the non-commitment case in order to show how the extent, or indeed the existence, of both these problems can be influenced by the choice of regulator.

## 4 Asymmetric Information without Commitment

### 4.1 The Delegation Game

Consider a two-period, two-type delegation game with the same structure and information assumptions as section 3, but now with the assumption that the regulator cannot commit to a two-period price contract. The government however can commit to a particular regulator over this interval.<sup>12</sup> Asymmetric information introduces dynamics through the process of learning about the firm's type. At the beginning of the game the firm knows its type  $\beta$ . The government and all types of regulators have the prior  $\nu_1$  that  $\beta = \underline{\beta}$ . Then the sequence of events for the delegation game is given by:

1. The government has preferences as for the regulator, except that rent carries a weight  $\alpha_s$  (reflecting social welfare), and delegates to an independent regulator of type  $\alpha \neq \alpha_s$  for the two periods. In the absence of delegation, the regulator is government-dependent and adopts a weight  $\alpha = \alpha_s$ .
2. The regulator offers a choice of two first-period price contracts from which the firm chooses one or neither. First-period effort  $e_1$  and investment  $i$  are applied by the firm, the cost  $C_1$  is realized and observed by regulator.
3. The regulator updates her prior  $\nu_1$  to  $\nu_2$ .

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<sup>12</sup>In common with much of the strategic delegation literature, we do not examine the reasons why a government may find it easier to commit to a type of regulator than (say) to a pricing policy: our intention is to demonstrate the effects that such delegation can have on investment and welfare.

4. The regulator offers a choice of two second-period contracts from which the firm chooses one or neither. Second-period effort  $e_2$  is applied by the firm, the cost  $C_2$  is realized and observed by regulator.

In the first period, given  $\nu_1$ , the regulator designs contracts  $(\underline{p}_1, \underline{C}_1)$  and  $(\bar{p}_1, \bar{C}_1)$ . In general we must consider equilibria in which the efficient firm may mimic the inefficient and *vice versa*. When the efficient firm chooses the low cost contract it chooses output  $q_1 = \phi^{-1}(\underline{p}_1)$  and effort  $(\underline{e}_1, \underline{i})$  such that observed cost  $\underline{C}_1 = \underline{\beta} - \underline{e}_1 + \underline{i} + cq_1$ . Similarly when the inefficient firm chooses the high cost contract it chooses output  $\bar{q}_1 = \phi^{-1}(\bar{p}_1)$  and effort  $(\bar{e}_1, \bar{i})$  such that observed cost  $\bar{C}_1 = \bar{\beta} - \bar{e}_1 + \bar{i} + c\bar{q}_1$ . Denote mimicking effort for the efficient and inefficient firms by  $(\tilde{\underline{e}}_1, \tilde{\underline{i}})$  and  $(\tilde{\bar{e}}_1, \tilde{\bar{i}})$  and  $\Delta\beta \equiv \bar{\beta} - \underline{\beta}$ .<sup>13</sup> In order to realize the appropriate observed costs, these mimicking efforts must satisfy

$$\tilde{\underline{e}}_1 = \bar{e}_1 - \Delta\beta + \tilde{\underline{i}} - \bar{i}; \quad \tilde{\bar{e}}_1 = \underline{e}_1 + \Delta\beta + \tilde{\bar{i}} - \underline{i} \quad (8)$$

Suppose that the efficient firm chooses the low cost contract with probability  $x$  and the high cost contract with probability  $1 - x$ . Similarly suppose that the inefficient firm chooses the high cost contract with probability  $y$  and the low cost contract with probability  $1 - y$ . The appropriate equilibrium concept for this game is a perfect Bayesian equilibrium (PBE) found by backward induction starting at event 4. We define the regulator's information sets at this point as follows: **H** (resp. **L**) if  $(\bar{p}_1, \bar{C}_1)$  (resp.  $(\underline{p}_1, \underline{C}_1)$ ) was accepted in period 1.

## 4.2 The Second-Period Contract

At **L** and **H**, the regulator designs contracts  $(\underline{p}_2, \underline{C}_2)$ , and  $(\bar{p}_2, \bar{C}_2)$  for low and high cost types respectively, given the (updated) probabilities  $\nu_2(\mathbf{L})$  and  $\nu_2(\mathbf{H})$  that the firm is efficient. At **L** we have that  $\underline{\beta}_2 = \underline{\beta} - f(\underline{i})$  and  $\bar{\beta}_2 = \bar{\beta} - f(\tilde{\underline{i}})$ . Similarly at **H**,  $\underline{\beta}_2 = \underline{\beta} - f(\tilde{\bar{i}})$  and  $\bar{\beta}_2 = \bar{\beta} - f(\bar{i})$ . Contracts must be designed to satisfy the following

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<sup>13</sup>We adopt the following notation:  $\tilde{\underline{z}}$  is some outcome for the efficient firm who mimics the inefficient firm and  $\tilde{\bar{z}}$  is the corresponding outcome for the inefficient firm who mimics the efficient firm.

incentive compatibility (IC) and individual rationality (IR) constraints for each firm:

$$\begin{aligned}
\underline{IC}_2 : \quad & \underline{U}_2 \geq \tilde{U}_2 = \bar{U}_2 + \Phi(\bar{e}_2) \\
\overline{IC}_2 : \quad & \bar{U}_2 \geq \tilde{U}_2 = \underline{U}_2 - \Phi(\underline{e}_2 + \Delta\beta_2) \\
\overline{IR}_2 : \quad & \bar{U}_2 \geq 0 \\
\underline{IR}_2 : \quad & \underline{U}_2 \geq 0
\end{aligned}$$

where  $\Phi(\bar{e}_2) = \psi(\bar{e}_2) - \psi(\bar{e}_2 - \Delta\beta_2)$  and  $\Phi(\underline{e}_2 + \Delta\beta_2) = \psi(\underline{e}_2 + \Delta\beta_2) - \psi(\underline{e}_2)$  are the firms' information rents. Because  $\underline{IC}_2 + \overline{IR}_2 \Rightarrow \underline{IR}_2$ , we can drop the latter constraint.

It is convenient to formulate the regulator's problem in terms of the choice of output and effort levels bearing in mind that contracts are designed as prices, contingent on observed total costs. The regulator's problem, to be carried out at each information set characterized by the state variables given by the vector  $s = [\nu_2, \underline{\beta}_2, \bar{\beta}_2]$ , is now:

**Given  $s = [\nu_2, \underline{\beta}_2, \bar{\beta}_2]$ , choose  $(\bar{q}_2, \bar{e}_2)$  and  $(\underline{q}_2, \underline{e}_2)$  to maximize the expected welfare**

$$E[W_2] = \Omega_2 = \nu_2 W(\underline{q}_2, \underline{e}_2, \underline{\beta}_2, \alpha) + (1 - \nu_2) W(\bar{q}_2, \bar{e}_2, \bar{\beta}_2, \alpha) \quad (9)$$

**subject to  $\underline{IC}_2$ ,  $\overline{IC}_2$  and  $\overline{IR}_2$ .**

To solve this optimization problem, let  $\mu_2 \geq 0$ ,  $\zeta_2 \geq 0$  and  $\xi_2 \geq 0$  be the Lagrangian multipliers associated with the  $\underline{IC}_2$ ,  $\overline{IC}_2$  and  $\overline{IR}_2$  constraints respectively. Then defining the Lagrangian

$$\mathcal{L}_2 = \Omega_2 + \mu_2(\underline{U}_2 - \bar{U}_2 - \Phi(\bar{e}_2)) + \zeta_2(\bar{U}_2 - \underline{U}_2 + \Phi(\underline{e}_2 + \Delta\beta_2)) + \xi_2 \bar{U}_2$$

the first-order conditions are:

$$\underline{L}_2 = \frac{p_2 - c}{\underline{p}_2} = \frac{\mu_2 - \zeta_2 + \nu_2(\alpha - 1)}{(\mu_2 - \zeta_2 + \nu_2\alpha)\eta(\underline{q}_2)} \quad (10)$$

$$\bar{L}_2 = \frac{\bar{p}_2 - c}{\bar{p}_2} = \frac{\xi_2 - \mu_2 + \zeta_2 + (1 - \nu_2)(\alpha - 1)}{(\xi_2 - \mu_2 + \zeta_2 + (1 - \nu_2)\alpha)\eta(\bar{q}_2)} \quad (11)$$

$$(\nu_2\alpha + \mu_2 - \zeta_2)(1 - \psi'(\underline{e}_2)) = -\zeta_2\Phi'(\underline{e}_2 + \Delta\beta_2) \quad (12)$$

$$((1 - \nu_2)\alpha + \xi_2 - \mu_2 + \zeta_2)(1 - \psi'(\bar{e}_2)) = \mu_2\Phi'(\bar{e}_2) \quad (13)$$

$$\mu_2(\underline{U}_2 - \bar{U}_2 - \Phi(\bar{e}_2)) = 0 \quad (14)$$

$$\zeta_2(\bar{U}_2 - \underline{U}_2 + \Phi(\bar{e}_2 + \Delta\beta_2)) = 0 \quad (15)$$

$$\xi_2\bar{U}_2 = 0 \quad (16)$$

To characterise the period 2 equilibrium and how it is affected by the type of regulator we need to examine the behaviour of the constraints as  $\alpha$  increases. In Appendix A we characterize three second period equilibrium categories<sup>14</sup>, depending on the value of  $\alpha \geq 1$ . In particular, there are threshold values  $\bar{\alpha}_2 > \underline{\alpha}_2 > 1$  such that:

- $\alpha \in (1, \underline{\alpha}_2]$ :  $\underline{IC}_2$  and  $\bar{IR}_2$  both bind. We call this *second-period equilibrium category b*.
- $\alpha \in (\underline{\alpha}_2, \bar{\alpha}_2]$ :  $\bar{IR}_2$  binds. We call this *second-period equilibrium category c*.
- $\alpha > \bar{\alpha}_2$ : unconstrained. We call this *second-period equilibrium category d*.

Notice that, in principle, we could have a second-period equilibrium, category *a* say, in which all three constraints  $\underline{IC}_2$ ,  $\bar{IC}_2$  and  $\bar{IR}_2$  bind. A familiar one-period result for a utilitarian regulator ( $\alpha = 1$ ) is that  $\bar{IC}_2$  does not bind (see Laffont and Tirole (1993)) and therefore  $\xi_2 = 0$ . Since the effect of increasing  $\alpha$  is to relax constraints, this means that equilibrium category *a* does not exist in the second period for  $\alpha > 1$  either. Equilibrium category *b* is then the familiar result for a single-period model. As  $\alpha$  increases (i.e. as the regulator becomes increasingly pro-industry), first  $\underline{IC}_2$  ceases to bind ( $\mu_2 = 0$ ) at  $\alpha = \underline{\alpha}_2$  and then  $\bar{IR}_2$  ceases to bind too ( $\xi_2 = 0$ ), at  $\alpha = \bar{\alpha}_2$ .<sup>15</sup>

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<sup>14</sup>Our equilibrium ‘categories’ are equivalent to the equilibrium ‘types’ in Laffont and Tirole (1993). We introduce this new terminology having reserved ‘type’ to describe a regulator of given preference for rent and a firm of ‘type  $\beta$ ’. Note also that although different equilibrium categories exist, for a given set of parameter values (including  $\alpha$ ) our numerical results always converge to the same equilibrium, regardless of initial values. This strongly suggests that the equilibrium is unique (given parameter values) and that problems of *multiple equilibria* do not arise.

<sup>15</sup>As can be confirmed from (13) to (16) this order for relaxing the constraints assumes  $\eta'(q_2) \leq 0$  and  $e^{RO} > \Delta\beta_2$ . The first of these implies  $\underline{p}_2 < \bar{p}_2$  (See Levine and Rickman (2001) for further details.)

We thus move from equilibrium category  $b$  to  $c$ , then  $d$  as the regulator becomes more pro-industry.

The intuition is as follows. Since  $\bar{p}_2 > \underline{p}_2$  there is no incentive for the inefficient type to mimic the efficient type. Therefore constraint  $\overline{IC}_2$  does not bind. The following possibilities remain:  $\underline{IC}_2$  and  $\overline{IR}_2$  bind (i.e., equilibrium  $b$ ), only  $\underline{IC}_2$  binds, only  $\overline{IR}_2$  binds, and no constraints bind. Of these, an equilibrium with only  $\underline{IC}_2$  binding must be sub-optimal because it implies rent for the inefficient type which must also be passed on to the efficient type. As  $\alpha$  increases, the progression between each equilibrium tells us that the increasingly generous regulator eventually supplies enough rent to the efficient firm to remove its incentive to mimic, and then allows the inefficient firm positive rent.

By setting the appropriate multipliers to zero in (10)–(16), and eliminating the rest, we can determine the nature of the second period contracts offered by different types of regulator; see Appendix A. Thus, regulators of type  $\alpha \in (0, \bar{\alpha}_2]$  offer a high-powered contract to the efficient firm ( $\psi'(\underline{e}_2) = 1$ ) and one involving a measure of cost-sharing for the inefficient one ( $\psi'(\bar{e}_2) < 1$ ). More pro-industry regulators ( $\alpha > \bar{\alpha}_2$ ) offer high-powered contracts to both firms and secure Ramsey-optimal effort in either case. At the same time, the fact that more rent is available to both firms as  $\alpha$  increases will provide investment incentives in period 1. We now turn to this investment decision.

### 4.3 The First-Period Investment Decision

Our analysis now moves to the first period where there are two decisions: the firm's investment decision and the regulator's contract offers. Beginning with the former, consider a firm of either type who has accepted a first-period contract specifying price and cost,  $(p_1, C_1)$ , and faces the prospect of a rent  $U_2 = U(q_2, e_2, \beta_2)$  corresponding to one of the second-period equilibria  $b, c$  or  $d$  at **L** or **H**. From the second-period optimization we know that  $(q_2, e_2)$  is a function of the state vector  $s = [\nu_2, \underline{\beta}_2, \bar{\beta}_2]$  at the relevant information set. Thus we can write  $U_2 = U_2(s)$ . Then given  $(p_1, C_1)$  and therefore  $q_1 = \phi^{-1}(p_1)$ , the firm chooses  $i$  to maximize

$$U_1 + \delta U_2 = p_1 q_1 - C_1 - \psi(\beta + i + c q_1 - C_1) + \delta U_2(\beta_2(i)) \quad (17)$$



The first-order condition for a *local* maximum (we consider whether this is also global below) is

$$\psi'(\beta + i + cq_1 - C_1) = \psi'(e_1) = -\delta \frac{\partial U_2}{\partial \beta_2} f'(i) \quad (18)$$

using  $\beta_2 = \beta - f(i)$ , from which  $\beta_2'(i) = -f'(i)$ . This is the familiar condition that the marginal cost of investment ( $MC(e_1) = \psi'(e_1)$ ) must equal its marginal benefit ( $MB(i)$ ). It is immediately apparent that the firm's investment decision depends on its first-period effort and anticipated second-period rent: the former offsets the effects of  $i$  on costs; the latter funds the investment. Accordingly, the regulator can influence investment behaviour through the power of the first-period contract and the credibility of offers of future rent (i.e., prices). In particular, the position of the MB curve is determined by the regulator's type since different second-period equilibrium categories ( $b, c, d$ ) generate different  $U_2$  and, thus, different  $\frac{\partial U_2}{\partial \beta_2}$ . Writing the solution to (18) as  $i = i(e_1)$  and differentiating gives

$$\psi''(e_1) = -\delta \left[ \frac{\partial U_2}{\partial \beta_2} f''(i) - \frac{\partial^2 U_2}{\partial \beta_2^2} (f'(i))^2 \right] \frac{di}{de_1} \quad (19)$$

$$\Rightarrow \frac{di}{de_1} < 0 \text{ provided that } \left[ \frac{\partial U_2}{\partial \beta_2} f''(i) - \frac{\partial^2 U_2}{\partial \beta_2^2} (f'(i))^2 \right] > 0 \quad (20)$$

Stated differently, the condition in (20) is that  $MB(i) = -\delta \frac{\partial U_2}{\partial \beta_2} f'(i)$  is decreasing in  $i$ .<sup>16</sup> Recalling (5), (18) tells us that the firm's choice of investment is optimal ( $i = i^{RO}$ ) when  $\psi'(e_1) = \left| \frac{\partial U_2}{\partial \beta_2} \right| = 1$ ; i.e., the firm must get a one-for-one return on its investment in period 2. Equation (4) then tells us that optimal investment also requires  $e = e^{RO}$ .

Figure 1 illustrates our results; both parts show optimal MB and MC curves, along with a pair relating to a low-powered contract and a regulator who generates  $\left| \frac{\partial U_2}{\partial \beta_2} \right| < 1$  (so that the firm's rent does not fully benefit from its investment). Here, the second-period prospects for lower rent and the low power of the first-period contract (which reduces the marginal cost of investment) work in opposite directions: the former lowering and the latter raising investment. Depending on which effect dominates we can

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<sup>16</sup>For small changes in  $\underline{\beta}_2$  and  $\bar{\beta}_2$  we can linearise  $U_2(s)$  around  $\underline{\beta}$  and  $\bar{\beta}$ , the second term in this condition can be ignored and the condition becomes  $\frac{\partial U_2}{\partial \beta_2} f''(i) > 0$ . Since  $f'' < 0$  and  $\frac{\partial U_2}{\partial \beta_2} < 0$  is necessary for any investment, the condition then holds. We are not able to show that the condition holds more generally, but numerical results indicate that this may be the case.

have under- or over-investment (Figures 2a and b respectively). Thus the value of  $|\frac{\partial U_2}{\partial \beta_2}| < 1$  is crucial for the investment decision and Appendix B provides details of this expression for the second-period equilibrium categories  $b$ ,  $c$  and  $d$ .

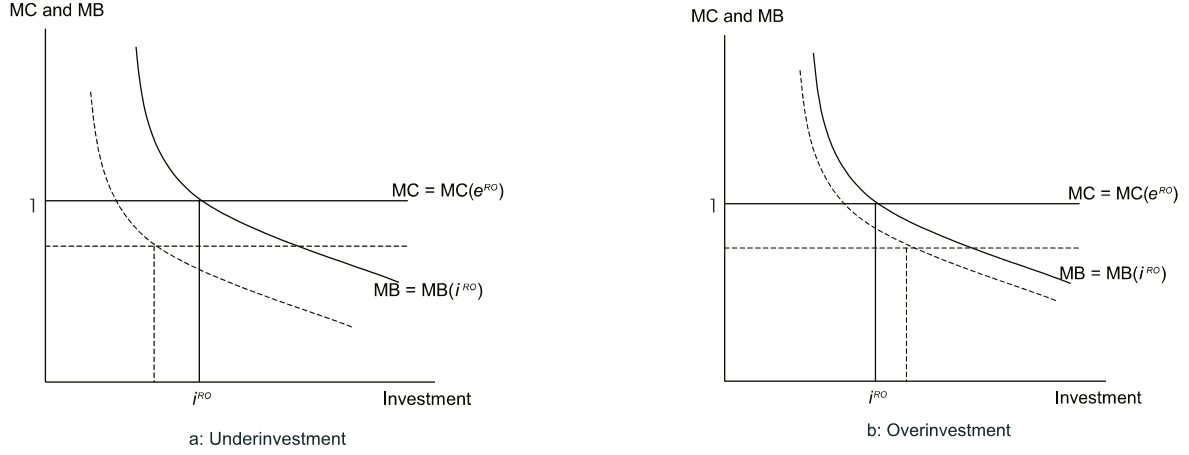
As we have stated, (18) defines a local optimum. If the firm chooses to invest at all it will choose  $i = i(e_1)$ . However the firm may choose not to invest. Given the anticipated second-period regulated price (which depends on  $\alpha$ ),  $i = i(e_1)$  is preferable to no investment,  $i = 0$ , only if  $-\psi(e_1) + \delta U_2(\beta_2(i)) > -\psi(e_1 - i) + \delta U_2(\beta_2(0))$ ; i.e.,

$$\delta[U_2(\beta_2(i)) - U_2(\beta_2(0))] > \psi(e_1) - \psi(e_1 - i) \quad (21)$$

This *investment condition* states that the second-period price must be sufficient for the future gain in rent to outweigh the current marginal cost of investing. Notice that if, in the second period, the constraint  $\overline{TR}_2$  binds then  $U_2(\beta_2(i)) = U_2(\beta_2(0)) = 0$  and (21) cannot hold for  $i > 0$ . Only when the regulator's type is sufficiently pro-rent that  $\alpha > \bar{\alpha}_2$  and we have a second-period equilibrium category  $d$ , can this condition hold for both the efficient and inefficient firm. However, the efficient firm may optimally invest, or over-invest, in second-period equilibrium categories  $b$ ,  $c$  and  $d$  because of the existence of information rent. We summarize our results on the firm's investment decision in the following proposition:

**Proposition 2 (The firm's investment decision).** *There is an investment-effort trade-off in the first period and more investment can only be secured at the expense of lower effort (i.e., a lower power contract) in the first period, provided (21) and the condition in (20) are satisfied. Over-investment or under-investment can occur.*

It is interesting that, in principle, the regulator's commitment problem can generate *over-investment* as well as *under-investment*. We now examine the regulator's first-period contract offer and confirm that both forms of investment behaviour can arise in equilibrium. We also examine how the type of regulator may achieve Ramsey-optimal investment.



**Figure 1: Determinants of under/over-investment**

#### 4.4 First-Period Contract

Now consider the design of contracts  $(\underline{p}_1, \underline{C}_1)$  and  $(\bar{p}_1, \bar{C}_1)$ , given  $\nu_1$ . Since the efficient firm may mimic the inefficient firm with probability  $1 - x$ , and the inefficient may mimic the efficient firm with probability  $1 - y$ , the probabilities of arriving at **L** and **H** are  $\Pr(\mathbf{L}) = \nu_1 x + (1 - \nu_1)(1 - y)$  and  $\Pr(\mathbf{H}) = \nu_1(1 - x) + (1 - \nu_1)y$ . Then by Bayes' Rule we have

$$\begin{aligned} \nu_2(\mathbf{L}) &= \Pr(\text{firm is efficient} \mid \text{low cost contract has been accepted}) \\ &= \frac{\nu_1 x}{\Pr(\mathbf{L})} = \frac{\nu_1 x}{\nu_1 x + (1 - \nu_1)(1 - y)} \end{aligned} \quad (22)$$

$$\begin{aligned} \nu_2(\mathbf{H}) &= \Pr(\text{firm is efficient} \mid \text{high cost contract has been accepted}) \\ &= \frac{\nu_1(1 - x)}{\Pr(\mathbf{H})} = \frac{\nu_1(1 - x)}{\nu_1(1 - x) + (1 - \nu_1)y} \end{aligned} \quad (23)$$

It is convenient to formulate the regulator's problem in terms of the choice of output and effort levels and the probabilities  $x$  and  $y$ . With  $E[W_2] = \Pr(\mathbf{L})E[W_2 \mid \mathbf{L}] + \Pr(\mathbf{H})E[W_2 \mid \mathbf{H}]$ , the first-period optimization problem for the regulator of type  $\alpha$  is:

Given  $\nu_1$ , choose  $x, y, (\bar{q}_1, \bar{e}_1)$  and  $(\underline{q}_1, \underline{e}_1)$  to maximize

$$\begin{aligned}\Omega = E[W_1 + \delta W_2] &= \nu_1[xW(\underline{q}_1, \underline{e}_1, \underline{\beta} + i(\underline{e}_1), \alpha) + (1-x)W(\bar{q}_1, \tilde{e}_1, \underline{\beta} + i(\tilde{e}_1), \alpha)] \\ &+ (1-\nu_1)[yW(\bar{q}_1, \bar{e}_1, \bar{\beta} + i(\bar{e}_1), \alpha) + (1-y)W(\underline{q}_1, \tilde{e}_1, \bar{\beta} + i(\tilde{e}_1), \alpha)] \\ &+ \delta E[W_2]\end{aligned}\tag{24}$$

subject to  $\underline{IC}_1, \overline{IC}_1, \underline{IR}_1$  and  $\overline{IR}_1$ .

Let the rent obtained when each firm mimics the other be given by

$$\tilde{U}_1 = \bar{U}_1 + \psi(\bar{e}_1) - \psi(\tilde{e}_1); \quad \tilde{\bar{U}}_1 = \underline{U}_1 + \psi(\underline{e}_1) - \psi(\tilde{e}_1)\tag{25}$$

where from (8) and (18) we have that  $\tilde{e}_1 = \bar{e}_1 - \Delta\beta + i(\tilde{e}_1) - i(\bar{e}_1)$  and  $\tilde{\bar{e}}_1 = \underline{e}_1 + \Delta\beta + i(\tilde{\bar{e}}_1) - i(\underline{e}_1)$ . Hence  $\tilde{e}_1 = \tilde{e}_1(\bar{e}_1)$  and  $\tilde{\bar{e}}_1 = \tilde{\bar{e}}_1(\underline{e}_1)$  and (25) can be written

$$\tilde{U}_1 = \bar{U}_1 + \Theta(\bar{e}_1); \quad \tilde{\bar{U}}_1 = \underline{U}_1 - \Gamma(\underline{e}_1)$$

Also, let  $s(\mathbf{L})$  and  $s(\mathbf{H})$  denote the state vectors at  $\mathbf{L}$  and  $\mathbf{H}$  respectively. Then the first-period incentive compatibility and individual rationality constraints are given by:

$$\begin{aligned}\underline{IC}_1 : \underline{U}_1 + \delta \underline{U}_2(s(\mathbf{L})) &\geq \tilde{U}_1 + \delta \underline{U}_2(s(\mathbf{H})) \\ \overline{IC}_1 : \bar{U}_1 + \delta \bar{U}_2(s(\mathbf{H})) &\geq \tilde{\bar{U}}_1 + \delta \bar{U}_2(s(\mathbf{L})) \\ \underline{IR}_1 : \underline{U}_1 + \delta \underline{U}_2(s(\mathbf{L})) &\geq 0 \\ \overline{IR}_1 : \bar{U}_1 + \delta \bar{U}_2(s(\mathbf{H})) &\geq 0\end{aligned}$$

Once again, it is clear that  $\underline{IC}_1 + \overline{IR}_1 \Rightarrow \underline{IR}_1$  so that we can ignore the latter. Also, since  $\bar{U}_2 = 0$  in second-period equilibrium  $b$  and  $c$ , and  $\bar{U}_2$  is independent of  $\mathbf{L}$  and  $\mathbf{H}$  in equilibrium  $d$ , we must have that  $\bar{U}_2(s(\mathbf{H})) = \bar{U}_2(s(\mathbf{L}))$ . The  $\overline{IC}_1$  constraint therefore simplifies to  $\bar{U}_1 \geq \tilde{\bar{U}}_1$ .

As before, to solve this optimization problem, we let  $\mu_1 \geq 0, \zeta_1 \geq 0$  and  $\xi_1 \geq 0$  be the Lagrangian multipliers associated with the  $\underline{IC}_1, \overline{IC}_1$  and  $\overline{IR}_1$  constraints respec-

tively. Then the Lagrangian and first-order conditions are given by:

$$\mathcal{L}_1 = \Omega + \mu_1[\underline{U}_1 - \tilde{\underline{U}}_1 + \delta(\underline{U}_2(s(\mathbf{L})) - \underline{U}_2(s(\mathbf{H})))] + \zeta_1[\bar{U}_1 - \tilde{\bar{U}}_1] + \xi_1[\bar{U}_1 + \delta\bar{U}_2(s(\mathbf{H}))]$$

$$\underline{L}_1 = \frac{p_1 - c}{\underline{p}_1} = \frac{\mu_1 - \zeta_1 + (\nu_1 x + (1 - \nu_1)(1 - y))(\alpha - 1)}{[\mu_1 - \zeta_1 + (\nu_1 x + (1 - \nu_1)(1 - y))\alpha]\eta(\underline{q}_1)} \quad (26)$$

$$\bar{L}_1 = \frac{\bar{p}_1 - c}{\bar{p}_1} = \frac{\xi_1 - \mu_1 + \zeta_1 + (\nu_1(1 - x) + (1 - \nu_1)y)(\alpha - 1)}{[\xi_1 - \mu_1 + \zeta_1 + (\nu_1(1 - x) + (1 - \nu_1)y)\alpha]\eta(\bar{q}_1)} \quad (27)$$

$$\begin{aligned} & (\alpha\nu_1 x + \mu_1 - \zeta_1)(1 - \psi'(\underline{e}_1)) + \zeta_1\Gamma'(\underline{e}_1) \\ & - [\alpha\nu_1 x(1 - \delta f'(\underline{i})) + \mu_1(1 - \psi'(\underline{e}_1)) - \zeta_1]i'(\underline{e}_1) \\ & - \alpha(1 - \nu_1)(1 - y)(1 - \delta f'(\tilde{i}))i'(\tilde{e}_1)\tilde{e}'_1(\underline{e}_1) = 0 \end{aligned} \quad (28)$$

$$\begin{aligned} & (\alpha(1 - \nu_1)y + \xi_1 - \mu_1 + \zeta_1)(1 - \psi'(\bar{e}_1)) - \mu_1\Theta'(\bar{e}_1) \\ & - [\alpha(1 - \nu_1)y(1 - \delta f'(\tilde{i})) + \xi_1(1 - \psi'(\bar{e}_1)) - \mu_1 + \zeta_1]i'(\bar{e}_1) \\ & - [\alpha\nu_1(1 - x)(1 - \delta f'(\tilde{i})) + \mu_1\psi'(\tilde{e}_1)]i'(\tilde{e}_1)\tilde{e}'_1(\bar{e}_1) = 0 \end{aligned} \quad (29)$$

$$\mu_1(\underline{U}_1 - \tilde{\underline{U}}_1 - \delta(\underline{U}_2(s(\mathbf{H})) - \underline{U}_2(s(\mathbf{L})))) = 0 \quad (30)$$

$$\zeta_1(\bar{U}_1 - \tilde{\bar{U}}_1) = 0 \quad (31)$$

$$\xi_1(\bar{U}_1 + \delta\bar{U}_2(s(\mathbf{H}))) = 0 \quad (32)$$

In period 1, unlike period 2 the  $\bar{IC}_1$  constraint can bind. The reason for this is the ratchet effect: the higher rent required by the efficient type to prevent it from mimicking and thus enjoying information rent in the second period is also attractive to the inefficient firm. The ratchet effect increases with the discount factor  $\delta$  (and disappears as  $\delta \rightarrow 0$  where the set-up in effect is static). As the weight  $\alpha$  increases in period 2, second-period equilibrium categories  $c$  then  $d$  emerge, offering the  $\underline{\beta}$ -firm second-period rent even when it reveals its type in period 1. This in turn reduces the ratchet effect and constraints  $\bar{IC}_1$ ,  $\underline{IC}_1$  and  $\bar{IR}_1$  cease to bind in that order giving four

first-period equilibrium categories: ‘category  $a$ ’ where all bind, ‘category  $b$ ’ where  $\underline{IC}_1$  and  $\overline{IR}_1$  bind, ‘category  $c$ ’ where only  $\overline{IR}_1$  binds and ‘category  $d$ ’ the unconstrained case. The intuition is the same as that set out for the second period.

## 4.5 The Two-Period Equilibrium

Taking the second and first-period contracts together, we now have a number of possible outcomes, depending on the cost and demand conditions and, in particular, the type of regulator ( $\alpha$ ). Each configuration of parameters determines which  $IC$  and  $IR$  constraints bind in each period. Table 1 sets out the possibilities. Each row describes a particular combination of first-period constraints. The columns describe second-period constraints and depend on whether a low cost (**L**) or high cost (**H**) first-period contract has been observed.<sup>17</sup> The delegation decision on the type of regulator, captured by  $\alpha$ , is particularly crucial for determining which equilibrium category applies. As with the second-period contract, each of these outcomes can be characterised by setting the relevant multipliers to zero in (26)–(31) and solving the resulting simplified first-order conditions: see Appendix B.

	$\underline{IC}_{2\mathbf{L}}, \overline{IR}_{2\mathbf{L}}$	$\overline{IR}_{2\mathbf{L}}$	None	$\underline{IC}_{2\mathbf{H}}, \overline{IR}_{2\mathbf{H}}$	$\overline{IR}_{2\mathbf{H}}$	None
$\underline{IC}_1, \overline{IC}_1, \overline{IR}_1$	$(a, b_{\mathbf{L}})$	$(a, c_{\mathbf{L}})$	$(a, d_{\mathbf{L}})$	$(a, b_{\mathbf{H}})$	$(a, c_{\mathbf{H}})$	$(a, d_{\mathbf{H}})$
$\underline{IC}_1, \overline{IR}_1$	$(b, b_{\mathbf{L}})$	$(b, c_{\mathbf{L}})$	$(b, d_{\mathbf{L}})$	$(b, b_{\mathbf{H}})$	$(b, c_{\mathbf{H}})$	$(b, d_{\mathbf{H}})$
$\overline{IR}_1$	$(c, b_{\mathbf{L}})$	$(c, c_{\mathbf{L}})$	$(c, d_{\mathbf{L}})$	$(c, b_{\mathbf{H}})$	$(c, c_{\mathbf{H}})$	$(c, d_{\mathbf{H}})$
None	$(d, b_{\mathbf{L}})$	$(d, c_{\mathbf{L}})$	$(d, d_{\mathbf{L}})$	$(d, b_{\mathbf{H}})$	$(d, c_{\mathbf{H}})$	$(d, d_{\mathbf{H}})$

**Table 1. The Two-Period Equilibrium**

In fact we can rule out some of the outcomes in Table 1. The ratchet effect means that first-period constraints  $\overline{IC}_1$  and  $\underline{IC}_1$  must bind before their second-period counterparts. Similarly  $\overline{IR}_1$  must bind before  $\overline{IR}_2$ ; otherwise the contracts offer rent to the inefficient type in the first period, but not the second; yet the only reasons for offering

<sup>17</sup>Laffont and Tirole (1993), chapter 9, derive a non-commitment PBE equilibrium for a procurement problem where contracts are transfers conditional on cost, there is no delegation ( $\alpha = \alpha_s = 1$ ), and no investment. What they call types III and I equilibria correspond to our equilibrium categories  $(a, b)$  and  $(b, b)$  respectively.

the inefficient type rent would be a pro-industry regulator who sufficiently likes rent, in which case she would offer it in both periods (equilibrium  $(d, d)$ ), or a regulator who wishes to encourage investment, in which case rent is offered in the second-period only. These considerations imply that as  $\alpha$  increases above unity, second-period constraints cease before their first-period counterparts, ruling out the lower-diagonal equilibrium categories  $(c, b_{\mathbf{L}})$ ,  $(d, b_{\mathbf{L}})$ ,  $(d, c_{\mathbf{L}})$  and  $(c, b_{\mathbf{H}})$ ,  $(d, b_{\mathbf{H}})$ ,  $(d, c_{\mathbf{H}})$ .

Table 1 provides the main insights into the effects of selecting a particular type of regulator; once the government has selected  $\alpha$ , the category of equilibrium follows immediately. It is clear that only equilibrium categories  $(*, d)$  can generate investment by the inefficient firm since  $\bar{U}_2 > 0$  only when  $\bar{TR}_2$  slackens. Similarly, as we move from  $(b, *)$  to  $(c, *)$ , increasingly credible promises of future rent gradually overcome the ratchet effect ( $\underline{IC}$  ceases to bind) and  $\underline{e}_1$  and  $\bar{e}_1$  can both equal  $e^{RO}$  (see Appendix B)—a necessary condition for  $i = i^{RO}$ . Of course, because removing the ratchet effect reduces rents, prices can fall when this happens.

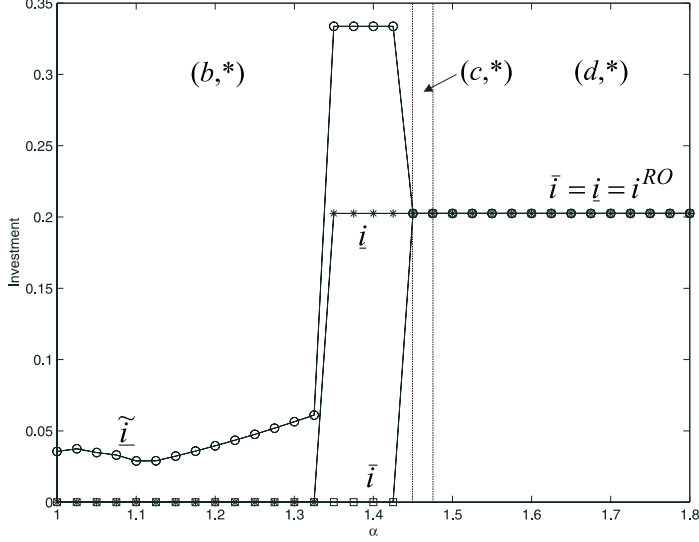
Focusing more closely on investment behaviour, and first period effort consider Figures 2 and 3. These provide a numerical example of how investment and first period effort respectively are affected by the choice of regulator and can be explained using Table 1.<sup>18</sup>Note that Figure 3 excludes  $\underline{e}_1 (= e^{RO} = 1)$  for simplicity. For our choices of functional forms and parameter values  $(a, *)$  equilibrium categories do not occur, but if they did we find in Appendix C the possibility of all efforts and investment being greater or less than the Ramsey optimum.<sup>19</sup>

To begin with, the chosen regulator is of the type to produce equilibrium category  $(b, b)$ . Using Appendices B and C and Figure 2, we can characterise investment for this category as follows. First, since  $\bar{TR}_2$  binds,  $\bar{i} = 0$ . Next, suppose the efficient firm does

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<sup>18</sup>We choose functional forms:  $\psi(e) = \frac{\gamma}{2}(\max(0, e))^2$ ,  $q = \phi(p) = Ap^{-\eta}$ ,  $\eta > 1$  and  $f(i) = Bi^\theta$ ;  $\theta \in (0, 1)$ , and parameters:  $\underline{\beta} = 2$ ,  $\bar{\beta} = 2.5$ ,  $c = \gamma = B = 1$ ,  $A = 10$ ,  $\eta = 1.5$ ,  $\nu_1 = \theta = 0.5$ ,  $\delta = 0.9$  and  $\alpha = \alpha_s = 1$  (no delegation). With these choices we have  $e^{RO} = 1/\gamma = 1$  and  $i^{RO} = (\delta\theta B)^{\frac{1}{1-\theta}} = 0.2025$

<sup>19</sup>For the  $(b, *)$  type equilibrium categories, which do occur, the optimal incentive mechanism is found by maximizing the social welfare function over  $x \in [0, 1]$ , where, we recall,  $x$  is the probability that the efficient firm mimics the inefficient firm in period 1. However here we avoid the complications arising from  $x$  changing with every parameter combination and present results for an exogenously chosen  $x = 0.5$ . Thus, we actually underestimate the welfare gains from delegation reported in section 5. All numerical results are obtained using programs written in MATLAB. These are available to the reader on request.

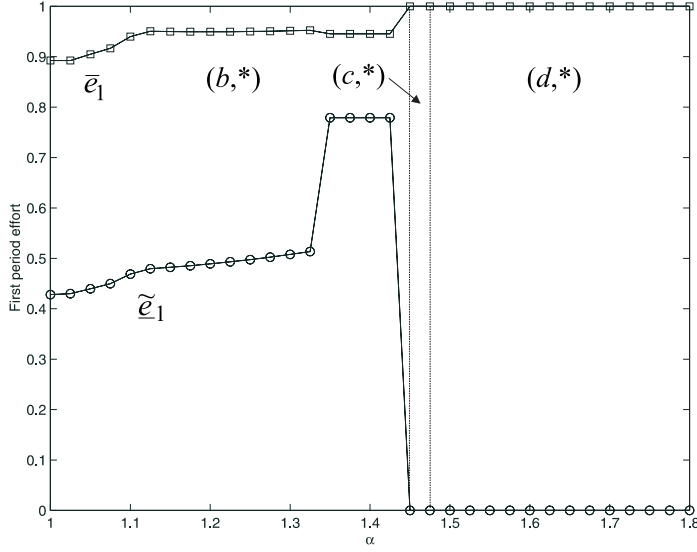


**Figure 2: Delegation and Investment**

not mimic the inefficient one (i.e.  $(b, b_L)$ ). From Appendix B,  $\underline{e}_1 = e^{RO}$  (since  $\overline{IC}_1$  does not bind); from Appendix B, we have  $|\frac{\partial U_2}{\partial \beta_2}| < 1$  and therefore from (18)  $MB(i) = \delta |\frac{\partial U_2}{\partial \beta_2}| f'(i) < \delta f'(i)$ . Referring back to Figure 1, we thus have  $0 < \underline{i} < i^{RO}$  — assuming (21) holds (otherwise  $\underline{i} = 0$ ). Thus, under-investment or, as in Figure 3, no investment occurs. Now suppose that the efficient firm mimics (i.e.  $(b, b_H)$ ). We now have  $\tilde{e}_1 < e^{RO}$  (See Figure 3 and Appendix C) along with  $MB(i) < \delta f'(i)$ . From Figure 1 (and assuming (21) holds) the lower marginal cost and marginal benefit of investment lead to  $\tilde{i} \begin{matrix} \geq \\ < \end{matrix} i^{RO}$ ; in our example the net effect is under-investment.

Selecting a more pro-industry regulator (higher  $\alpha$ ) moves us through the various  $(b, *)$  equilibrium categories and at around  $\alpha = 1.32$  the regulator is of the type to generate equilibrium category  $(b, c)$  and then, as  $\alpha$  increases,  $(b, d)$ . When the latter is reached, we know that both the efficient and inefficient firm may now invest since  $\overline{IR}_2$  slackens, and indeed, the inefficient firm can *over-invest* if  $\bar{e}_1 < e^{RO}$ . However the investment condition (21) must also be satisfied. Since the inefficient firm receives no information rent in the second period this condition is only satisfied at higher values of  $\alpha$  than for the efficient firm. In Figure 2 this does not happen and in equilibrium categories  $(b, c)$  and  $(b, d)$  the inefficient firm does not invest at all.





**Figure 3: Delegation and first-period effort**

For the efficient firm, when  $(b, c)$  is reached, non-mimicking investment is Ramsey-optimal as can be confirmed from Appendix C (no mimicking so  $\bar{e}_1 = e^{RO}$ ) and Appendix B ( $MB(i) = -1$ ). However its mimicking investment involves over-investment; see Figure 2. This is because its marginal cost of investment is low ( $\tilde{e}_1 < e^{RO}$ ) while its  $MB(i)$  is optimal. Thus, as noted in Proposition 2, we have the interesting prospect of the regulator's commitment problem creating *over*-investment.

Still more pro-industry regulators move us towards the bottom righthand corner of the table (through  $(c, *)$  equilibrium categories for  $\alpha \in [1.45, 1.47]$ ), then  $(d, *)$ , as  $\underline{IC}_1$ , and  $\overline{TR}_1$  cease to bind in turn. Now  $\underline{e}_1 = \bar{e}_1 = e^{RO}$  and Ramsey-optimal investment by both efficient and inefficient firms can take place if the investment condition (21) holds, as is the case in Figure 2. Then the marginal cost of investment is Ramsey-optimal and the regulator is sufficiently pro-industry that the marginal benefit of investment is similarly optimal (Appendices B and C).

It is also possible to confirm (see Levine and Rickman (2001)) that as the  $(b, d)$  equilibrium category is entered, the regulator is offering sufficient second-period rent to prevent the ratchet effect from taking place. Thus, at this point, regulated prices fall as they no longer take account of the extra information rent required by the efficient firm.

Working through Table 1 in the above fashion gives us:

**Lemma 1.** *Any positive investment requires (21) to hold, otherwise investment is zero. Then the equilibrium categories exhibit the following first-period effort and investment behaviour:*

$$\begin{aligned}
& \left. \begin{aligned} (a, b), (a, c) : \quad \tilde{\underline{i}}, \underline{i} \begin{cases} \geq \\ \leq \end{cases} i^{RO}, \bar{i} = 0 \\ (a, d) : \quad \tilde{\underline{i}}, \underline{i}, \tilde{\bar{i}}, \bar{i} \begin{cases} \geq \\ \leq \end{cases} i^{RO} \end{aligned} \right\} \underline{e}_1, \tilde{\underline{e}}_1, \bar{e}_1, \tilde{\bar{e}}_1 \begin{cases} \geq \\ \leq \end{cases} e^{RO} \\ \\
& \left. \begin{aligned} (b, b) : \quad \tilde{\underline{i}} \begin{cases} \geq \\ \leq \end{cases} i^{RO}, \underline{i} < i^{RO}, \bar{i} = 0 \\ (b, c) : \quad \tilde{\underline{i}} > i^{RO}, \underline{i} = i^{RO}, \bar{i} = 0 \\ (b, d) : \quad \tilde{\underline{i}}, \bar{i} > i^{RO}, \underline{i} = i^{RO} \end{aligned} \right\} \underline{e}_1 = e^{RO}; \bar{e}_1, \tilde{\underline{e}}_1 < e^{RO} \\ \\
& \left. \begin{aligned} (c, c) : \quad \underline{i} = i^{RO}, \bar{i} = 0 \\ (c, d), (d, d) : \quad \underline{i} = \bar{i} = i^{RO} \end{aligned} \right\} \underline{e}_1 = \bar{e}_1 = e^{RO}
\end{aligned}$$

Bringing Lemma 1 together with the effect of  $\alpha$  on the constraints yields the following result:

**Proposition 3 (Delegation and investment).** *Unlike relatively utilitarian regulators, relatively pro-industry ones are able to guarantee Ramsey-optimal investment (if sufficiently pro-industry, by both firms). A necessary condition for Ramsey-optimal investment is  $\alpha > \bar{\alpha}_2$  where  $\bar{\alpha}_2$  is the regulator's weight on rent at which all second period IC and IR constraints cease to bind. The sufficient condition is that  $\alpha$  must rise further to insure Ramsey-optimal investment is preferable to no investment and (21) is satisfied.*

## 5 Delegation and Welfare

We have seen that delegation to a pro-industry regulator with a carefully chosen preference parameter  $\alpha$  can increase investment, reduce the ratchet effect and result in both lower prices, benefiting consumers, and higher rent: it can, in other words, be Pareto improving. This section investigates these welfare gains further, compares them with the welfare gain from full commitment and examines the scope for a wrong choice of

$\alpha$  that leads to counterproductive delegation. First consider the single-period social welfare (3):

$$W_t = S(p_t) - R(p_t) + \alpha_s U_t = W(p_t, U_t, \alpha_s)$$

Then having obtained prices and rents in a Perfect Bayesian Equilibrium with a regulator of type  $\alpha$ , we can write the two-period social welfare as

$$\begin{aligned} \Omega(\alpha, \alpha_s) &= \nu_1[xW(\underline{p}_1(\alpha), \underline{U}_1(\alpha), \alpha_s) + (1-x)W(\bar{p}_1(\alpha), \tilde{U}_1(\alpha), \alpha_s)] \\ &+ (1-\nu_1)[yW(\bar{p}_1(\alpha), \bar{U}_1(\alpha), \alpha_s) + (1-y)W(\underline{p}_1(\alpha), \tilde{U}_1(\alpha), \alpha_s)] + E[W_2(\alpha, \alpha_s)] \end{aligned}$$

where

$$\begin{aligned} E[W_2(\alpha, \alpha_s)] &= (\nu_1 x + (1-\nu_1)(1-y))E[W_2|\mathbf{L}] + (\nu_1(1-x) + (1-\nu_1)y)E[W_2|\mathbf{H}] \\ E[W_2|\mathbf{L}] &= \nu_{2\mathbf{L}}W(\underline{p}_{2\mathbf{L}}(\alpha), \underline{U}_{2\mathbf{L}}(\alpha), \alpha_s) + (1-\nu_{2\mathbf{L}})W(\bar{p}_{2\mathbf{L}}(\alpha), \bar{U}_{2\mathbf{L}}(\alpha), \alpha_s) \\ E[W_2|\mathbf{H}] &= \nu_{2\mathbf{H}}W(\underline{p}_{2\mathbf{H}}(\alpha), \underline{U}_{2\mathbf{H}}(\alpha), \alpha_s) + (1-\nu_{2\mathbf{H}})W(\bar{p}_{2\mathbf{H}}(\alpha), \bar{U}_{2\mathbf{H}}(\alpha), \alpha_s) \end{aligned}$$

We measure the welfare gain from delegation,  $G(\alpha)$  as follows. Let  $\Omega^C$  be the optimal two-period social welfare under commitment. Then

$$G(\alpha) = \frac{\Omega(\alpha, \alpha_s) - \Omega(\alpha_s, \alpha_s)}{\Omega^C - \Omega(\alpha_s, \alpha_s)} \times 100$$

Thus  $G(\alpha) \leq 100\%$  and measures the extent to which delegation can substitute for full commitment.

Figure 4 plots  $G(\alpha)$  against  $\alpha$  for  $B = \{0, 1, 1.5\}$ . The case of  $B = 0$  shows the ability of delegation to mitigate the ratchet effect on its own, without investment considerations. These results demonstrate the possibility of significant welfare gains from delegation with the appropriate choice of  $\alpha$ .<sup>20</sup> However without investment considerations a regulator who is only slightly too pro-industry leads to a welfare loss: the negative welfare effects of increasing rent (i.e., prices) cut in quickly. Delegation is far

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<sup>20</sup>It is clear from our numerical example that, given our choice of parameter values, there exists an ‘optimal  $\alpha$ ’. It would be desirable to produce an analytical existence result, but this is precluded by the complexities of the set-up that includes two-period dynamics, moral hazard and adverse selection—all essential ingredients in the regulation game with investment.

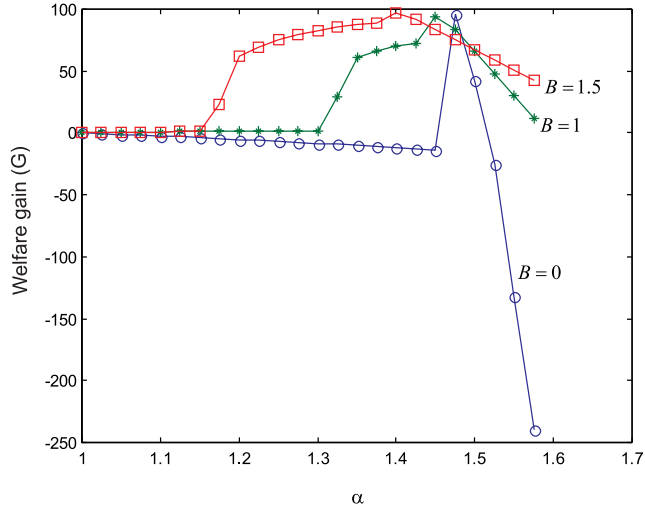


Figure 4: Welfare gains from delegation

more robust (and the range of ‘beneficial’ regulators is considerably wider) if investment is introduced, especially if its impact on costs is at the higher level of  $B = 1.5$ .

**Proposition 4 (Delegation and Welfare).** *Numerical Results show that welfare can be increased by delegation to a range of pro-industry regulators. As investment becomes more effective, a wider range of regulators achieves this result.*

## 6 Conclusions and Future Research

The question of how to encourage investment by regulated industries is a central one for regulators. Problems arise because despite the benefits of investment (lower costs), regulators ex post have an incentive to lower prices, which firms anticipate. A number of authors have identified the resulting ‘under-investment’ in a variety of regulatory settings. The present paper considers a dynamic non-commitment problem and makes several contributions to the analysis of the under-investment problem. First, we show how strategic delegation to a suitable type of regulator can overcome the under-investment problem (as well as the ratchet effect that also arises in the model). Second, we focus on non-contractible investment in the presence of asymmetric information about other cost-reducing effort by the firm. Third, the regulator is not permitted to use

transfers in order to reimburse the firm. The full set of Perfect Bayesian Equilibria is characterised. We suggest that each of these contributions accords with features of the regulatory environment, for example in the UK.

Our results throw some light on how a regulatory regime might achieve effective regulation. This must achieve: (i) the freedom to respond to the latest information regarding the industry; i.e., it must involve discretion; (ii) socially optimal investment and effort, ruling out direct controls or ‘rate-of-return’ regulation and (iii) consumer benefits from higher investment through lower prices. Our paper shows that, with discretion, delegation of price regulation to an independent regulator of the appropriate type will achieve these objectives.

This, in a sense, is a positive rather than normative result. If we observe good regulation it could be coming about through this mechanism. To derive normative conclusions we note that, in common with much of the strategic delegation literature, we have relocated the problem as one of choosing the correct  $\alpha$ , but we have not addressed directly how an appropriate regulator can be found. While it is reasonable to suppose that track records can play a valuable role here, it may still be sensible to consider safeguards against ‘mistakes’. In this respect, Spulber and Besanko (1992)’s suggestion that legal rules can be helpful for implementing simple (but clear) policy objectives is relevant. Thus, one could imagine statutory limits on the maximum prices that regulators could set, so as to curtail excessively pro-industry behaviour. Furthermore new regulators without a clear track-record should be aware of the problem posed in our model and be prepared to build up a reputation for achieving the ‘right balance between the needs of consumers and the firm’ (i.e., a reputation for having the right  $\alpha$ ). Formal modelling of this process would be worthwhile in future work.

Our analysis makes predictions about the effects of regulatory independence on investment, costs and prices (see also Currie *et al.* (1999)). An important requirement for testing these predictions would be a suitable index of regulatory independence in various countries/industries in order to compare different regulatory regimes. Naturally, such an index would be complex to produce. However, to the extent that regulatory independence can be shown to have benefits in theory, such empirical work would provide important insights for policy makers in this area.

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## A Details of Second-Period Equilibrium Categories

**Second-Period Equilibrium b:**  $\alpha \in [1, \underline{\alpha}_2]$ . Only  $\underline{IC}_2$  and  $\overline{IR}_2$  constraints bind.

Putting  $\zeta_2 = 0$  and eliminating  $\mu_2$  and  $\xi_2$  the first order conditions (foc) for this equilibrium gives the following four equations in  $\underline{q}_2, \overline{q}_2, \underline{e}_2$  and  $\overline{e}_2$ :

$$\underline{U}_2 = U(\underline{q}_2, \underline{e}_2, \beta_2) = \Phi(\overline{e}_2) \quad (\text{A.1})$$

$$\overline{U}_2 = U(\overline{q}_2, \overline{e}_2, \beta_2) = 0$$

$$\psi'(\underline{e}_2) = 1; \quad \text{i.e., } \underline{e}_2 = e^{RO}$$

$$\frac{(1 - \nu_2)}{(1 - \eta(\overline{q}_2)\overline{L}(\overline{q}_2))} (1 - \psi'(\overline{e}_2)) = \nu_2 \left[ \frac{1}{(1 - \eta(\underline{q}_2)\underline{L}(\underline{q}_2))} - \alpha \right] \Phi'(\overline{e}_2) \quad (\text{A.2})$$

**Second-Period Equilibrium c:**  $\alpha \in (\underline{\alpha}_2, \overline{\alpha}_2]$ . Only  $\overline{IR}_2$  binds. The foc are:

$$\underline{L}_2 = \frac{\underline{p}_2 - c}{\underline{p}_2} = \frac{\alpha - 1}{\alpha \eta(\underline{q}_2)} \quad (\text{A.3})$$

$$\overline{U}_2 = U(\overline{q}_2, \overline{e}_2, \beta_2) = 0 \quad (\text{A.4})$$

$$\psi'(\overline{e}_2) = \psi'(\underline{e}_2) = 1; \quad \text{i.e., } \overline{e}_2 = \underline{e}_2 = e^{RO} \quad (\text{A.5})$$

$$\underline{U}_2 > \overline{U}_2 + \Phi(\overline{e}_2) \quad (\text{A.6})$$

**Second-Period Equilibrium Category d:**  $\alpha > \overline{\alpha}_2$ . For this unconstrained case the



foc are:

$$\underline{L}_2 = \frac{p_2 - c}{\underline{p}_2} = \frac{\alpha - 1}{\alpha \eta(\underline{q}_2)} \quad (\text{A.7})$$

$$\bar{L}_2 = \frac{\bar{p}_2 - c}{\bar{p}_2} = \frac{\alpha - 1}{\alpha \eta(\bar{q}_2)} \quad (\text{A.8})$$

$$\bar{U}_2 > 0 \quad (\text{A.9})$$

$$\psi'(\bar{e}_2) = \psi'(\underline{e}_2) = 1; \text{ i.e., } \bar{e}_2 = \underline{e}_2 = e^{RO} \quad (\text{A.10})$$

$$\underline{U}_2 > \bar{U}_2 + \Phi(\bar{e}_2) \quad (\text{A.11})$$

## B Details of The Investment Decision

Differentiating the foc in Appendix A we can evaluate the derivatives  $|\frac{\partial U_2}{\partial \beta_2}|$ :

**Second-Period Equilibrium Category b:**  $\frac{\partial U_2(\mathbf{H})}{\partial \beta_2} > -1$ ;  $\frac{\partial U_2(\mathbf{L})}{\partial \beta_2} > -1$ ;  $\frac{\partial \bar{U}_2(\mathbf{H})}{\partial \beta_2} = \frac{\partial \bar{U}_2(\mathbf{L})}{\partial \beta_2} = 0$ . To prove this result, first note that the second-period information rent  $\Phi = \psi(\bar{e}_2) - \psi(\tilde{e}_2)$  is a function of  $\bar{e}_2$  and  $\Delta\beta_2$ , the latter depending on investment in the first period. Write  $\Phi = \Phi(\bar{e}_2, \Delta\beta_2)$ . Then differentiating (A.1)-(A.2) we have that

$$\frac{\partial \underline{U}_2}{\partial \beta_2} = \frac{\partial \Phi}{\partial \bar{e}_2} \frac{\partial \bar{e}_2}{\partial \beta_2} - \psi'(\tilde{e}_2) = -\frac{\partial \Phi}{\partial \bar{e}_2} \frac{a_2}{a_0 + a_1} - \psi'(\tilde{e}_2) \quad (\text{B.1})$$

Therefore the result holds iff  $\frac{a_2}{(a_0 + a_1)} \frac{\partial \Phi}{\partial \bar{e}_2} < (1 - \psi'(\tilde{e}_2))$  where we have defined

$$\begin{aligned} a_0 &= \frac{(1 - \nu_2)}{(1 - \eta \bar{L}_2)} \left[ \frac{\eta \bar{L}_2' (1 - \psi'(\bar{e}_2))^2}{(1 - \eta \bar{L}_2)(\bar{p}_2(1 - \frac{1}{\eta}) - c)} + \psi''(\bar{e}_2) \right] \\ a_1 &= \nu_2 \left[ \mu_2(\psi''(\bar{e}_2) - \psi''(\tilde{e}_2)) + \frac{\eta \bar{L}_2' \left( \frac{\partial \Phi}{\partial \bar{e}_2} \right)^2}{(1 - \eta \bar{L}_2)^2 (\underline{p}_2(1 - \frac{1}{\eta}) - c)} \right] \\ a_2 &= \nu_2 \left[ -\mu_2 \psi''(\tilde{e}_2) + \frac{\eta \bar{L}_2' \frac{\partial \Phi}{\partial \bar{e}_2} (1 - \psi'(\tilde{e}_2))}{(1 - \eta \bar{L}_2)^2 (\underline{p}_2(1 - \frac{1}{\eta}) - c)} \right] \end{aligned}$$

From the definitions of  $a_2$  and  $a_1$  and the fact that  $a_0 > 0$  we have that  $\frac{a_2}{(a_0 + a_1)} \frac{\partial \Phi}{\partial \bar{e}_2} < \frac{a_1}{a_0 + a_1} (1 - \psi'(\tilde{e}_2)) < (1 - \psi'(\tilde{e}_2))$ , which proves the result.

Hence, providing (21) holds,  $\underline{i} \geq 0$ , and mimicking investment  $\tilde{\underline{i}} \geq 0$  (where  $\underline{i} = \tilde{\underline{i}} = 0$  if  $\frac{\partial U_2}{\partial \beta_2} \leq 0$ ) for the efficient firm, but  $\bar{i} = \tilde{\bar{i}} = 0$  for the inefficient firm. For second-period equilibrium categories c and d it is straightforward to obtain the following results:

**Second-Period Equilibrium Category c:**  $\frac{\partial U_2}{\partial \beta_2} = -1$ ;  $\frac{\partial \bar{U}_2}{\partial \beta_2} = 0$ . Hence, as before if (21) holds,  $\underline{i} \geq 0$ , and mimicking investment  $\tilde{\underline{i}} \geq 0$  for the efficient firm, but  $\bar{i} = \tilde{\bar{i}} = 0$  for the inefficient firm.

**Second-Period Equilibrium Category d:**  $\frac{\partial U_2}{\partial \beta_2} = \frac{\partial \bar{U}_2}{\partial \beta_2} = -1$ . Now, as a result of the extra rent offered by a regulator of type  $\alpha > \bar{\alpha}_2$ ,  $\underline{i}$ ,  $\bar{i}$ ,  $\tilde{\underline{i}}$  and  $\tilde{\bar{i}}$  can all be positive.

## C Details of First Period Equilibrium Categories

Let us now consider each row of this table in turn:

**Equilibria**  $(a, *)$ :  $\overline{IC}_1, \underline{IC}_1, \overline{IR}_1$  bind ( $\zeta_1, \mu_1, \xi_1 > 0$ ).

Then given  $x$  and  $y$ ,  $\underline{q}_1, \bar{q}_1, \underline{e}_1$ , and  $\bar{e}_1$ , are given by (26), (27), (28), (29), (30) and (31), given the functions  $i = i(e_1)$  and  $i'(e_1)$  obtained in section 4.3. This system of equations allows the possibility of all efforts being greater or less than the Ramsey optimum. The optimal mechanism for a regulator of type  $\alpha$  is then found by maximizing the intertemporal utility (24) with respect to  $x$  and  $y$ .

**Equilibria**  $(b, *)$ :  $\underline{IC}_1, \overline{IR}_1$  bind ( $\zeta_1 = 0$ ;  $\mu_1, \xi_1 > 0$ ).

The inefficient firm now does not mimic, so the solution is found by putting  $y = 1$ , solving (26), (27), (28), (29), (30) and (32), for  $\mu_1, \xi_1 > 0$ ,  $\underline{q}_1, \bar{q}_1, \underline{e}_1$ , and  $\bar{e}_1$ , for a given  $x$ , and then maximizing (24) with respect to  $x$ . Now we have that  $\underline{e}_1 = e^{RO}$ .

**Equilibria**  $(c, *)$ :  $\overline{IR}_1$  binds ( $\zeta_1 = \mu_1 = 0$ ;  $\xi_1 > 0$ ).

There is now no mimicking by either type of firm and it is now easy to characterize the equilibrium. Putting  $x = y = 1$ , information sets L and H become singletons and we have that  $\nu_2(L) = 1$ ,  $\nu_2(H) = 0$ ,  $Pr(L) = \nu_1$  and  $Pr(H) = 1 - \nu_1$ . Then:

$$\underline{L}_1 = \frac{\underline{p}_1 - c}{\underline{p}_1} = \frac{\alpha - 1}{\alpha \eta(\underline{q}_1)} \quad (C.1)$$

$$\bar{U}(\bar{q}_1, \bar{e}_1, \bar{\beta}_1) + \delta \bar{U}_2 = 0 \quad (C.2)$$

$$\bar{e}_1 = \underline{e}_1 = e^{RO} \quad (C.3)$$

**Equilibria** ( $d, *$ ): Unconstrained. ( $\zeta_1 = \mu_1 = \xi_1 = 0$ )

This is the simplest case to characterise. Equations (C.1) and (C.3) apply as before and (C.2) now becomes

$$\bar{L}_1 = \frac{\bar{p}_1 - c}{\bar{p}_1} = \frac{\alpha - 1}{\alpha\eta(\bar{q}_1)} \quad (\text{C.4})$$