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Hysteresis*

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Abstract

Three competing conceptualizations of hysteresis in economics are identified: the unit/zero root approach, "true" hysteresis, and hysteresis conceived as a product of historical time. The properties of these conceptualizations are discussed and their pros and cons considered.

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Hysteresis

Hysteresis is a form of path dependence – that is, it describes a process whereby the past has a lasting influence on the present and future. The concept of hysteresis provides an "organizing concept" (similar, for example, to equilibrium) around which dynamical models of specific economic processes can be built, in a way that is sympathetic to the notion that "history matters" in the determination of economic outcomes.

Competing conceptualizations of hysteresis in economics

In path dependent systems, outcomes – including anything that can be construed as a long run or final outcome – are affected by the path (the prior sequence of adjustments and associated outcomes) that led up to them. The term hysteresis is sometimes used as a synonym for path dependence, but properly conceived it describes a specific set of mechanisms or processes that give rise to path dependence.

The concept of hysteresis originated in the natural sciences, in studies of the magnetic properties of ferric metals (Cross and Allen, 1988). Despite the fact that the importance of historical contingency has long been recognized in economics, appeal to hysteresis is more recent. The term was popularized in critiques of the natural rate hypothesis during the 1980s (for the earliest example, see Hargreaves Heap, 1980), although it appears earlier in Georgescu-Roegen's (1966) theory of consumer behaviour.

In economics, several competing conceptualizations of hysteresis have arisen, although their properties are by no means mutually exclusive (see, for example, Setterfield, 2009). These are: the unit/zero root approach; models of "true" hysteresis; and hysteresis conceived as a product of historical time.

The unit/zero root approach

This approach is strongly associated with critiques of the natural rate of unemployment (see, for example, Wyplosz, 1987; Franz, 1990; Layard, Nickell and Jackman, 1991). It involves assuming the existence of a unit (zero) root in systems of linear difference (differential) equations. Suppose, for example, that the variable x can be described by the first-order linear difference equation:

$$x_t = \alpha x_{t-1} + \beta + \varepsilon_t \qquad [1]$$

where β is a constant and ε is a stochastic term. Equation [1] can be re-written as:

$$x_t = \alpha^t x_0 + \sum_{i=1}^{t} \alpha^{t-i}(\beta + \varepsilon_i) \qquad [2]$$

If $\alpha = 1$ – that is, if the difference equation in [1] has a unit root – then the solution to [2] is:

$$x_t = x_0 + t\beta + \sum_{i=1}^{t} \varepsilon_i \qquad [3]$$

This implies that, in any period, the current value of x depends on the past – specifically, initial conditions and the entire history of shocks to the system. In the unit/zero root approach, this result is called hysteresis. Note, however, that if $\alpha < 1$ (and assuming that $\varepsilon = 0$ in the stationary state), the solution to [2] becomes:

$$x = \frac{\beta}{1-\alpha} \qquad [4]$$

Equation [4] is recognizable as a conventional (ahistorical) equilibrium outcome, in which the value of x depends only on the time-invariant data α and β. For small values of t, the past can be said to exert an influence on the current value of x, as is evident from equation [2]. In the unit/zero root approach, this result is called *persistence*. But in the limit persistence disappears, as x converges to the value in [4] that is defined independently of events in the past. In the unit/zero root approach, then, hysteresis is a special case, arising only if a unit/zero root is observed.

The unit/zero root approach has been widely criticized for misrepresenting and over-simplifying the concept of hysteresis – and in particular, for its neglect of the importance of non-linearities and structural change in the genesis of hysteresis effects (Amable et al, 1993, 1995; Cross, 1993, 1995, Setterfield, 1993, 1998a). Nevertheless, models with unit/zero roots are attractively simple to formulate, and bear easy comparison with traditional (ahistorical) equilibrium systems, to which they default in the absence of a unit/zero root. Moreover, they do capture at least one key property of hysteresis, namely, the propensity for even transitory causes to have permanent effects. (Note that $\varepsilon_i \neq 0$ will forever affect the value of x_t in equation [3]). For these reasons, the unit/zero root approach to hysteresis has proved popular in the radical political economy tradition among authors studying short- and long-term macrodynamic phenomena such as the impact of monetary policy (Lavoie 2006) and accumulation and growth (Dutt, 1997).

"True" hysteresis

Introduced into economics by Amable et al (1993, 1994, 1995) and Cross (1993, 1994, 1995), "true" hysteresis is based on models of hysteretic processes developed in theoretical physics. The two key components of "true" hysteresis are the non-ideal relay (Krasnosel'skii and Pokrovskii, 1989) and the aggregation effects that result from heterogeneity at the micro level (Mayergoyz, 1986). The workings of both of these components can be demonstrated by means of reference to Figure 1.

[FIGURE 1 GOES HERE]

Consider first panel (a) of Figure 1, which depicts the relationship between the dependent variable *x* and the independent variable *y* for the i^{th} agent. To make this relationship more concrete, we might think of *y* as the size of a regional market, and *x* as a binary measure of the i^{th} firm's activity in this market, where x_{i0} denotes absence and x_{i1} denotes presence. Suppose we begin at point A with $y = y_1$ and $x_i = x_{i0}$. Now suppose that a shock increases the size of the market to $y = y_2$. According to Figure 1(a) we will now arrive at point B, with $x_i = x_{i1}$. This is because the size of the market has crossed a critical threshold ($y = y_{iu}$) sufficient to induce firm *i* to enter. Suppose, however, that the shock that triggered market entry is temporary, and that size of the market subsequently declines to $y = y_3 = y_1$. As indicated in Figure 1(a), we will nevertheless find ourselves at point C where it is still the case that $x_i = x_{i1}$. Technically, this is because of the non-linearity of the upper and lower "arms" (denoted by the thick solid lines) of the non-ideal relay depicted in Figure 1(a), which govern the response of x_i to variations in *y*. In terms of the example of firm entry and exit used above to motivate Figure 1(a), this might be explained by sunk costs, which result in firm *i*'s continued participation in the regional market even after the factors that induced its initial entry have disappeared. The upshot of all this, as depicted in Figure 1(a), is that a temporary shock to *y* can have a permanent effect on x_i. More generally, we will observe that variations in *y* – even if transitory – that cross the upper or lower bounds y_{iu} and y_{il} will permanently alter x_i, while variations in *y* within these bounds will leave x_i unchanged. Note, then, that not *all* shocks change the outcomes of the system, which is thus said to have a *selective* memory (in contrast to the complete memory of the unit root system in equation [3]).

In order to understand the importance of aggregation effects in systems of this type, consider now both panels (a) and (b) of Figure 1, which together illustrate the non-ideal relays characterizing two different agents. The same transitory shock contemplated above (where y rises to y_2 before falling back to y_1) will result in the aggregate outcome $X = x_{i1} + x_{j1}$. Notice, however, that a subsequent transitory shock that sets $y = y_4$ before reverting to $y = y_1$ will result in the aggregate outcome $X = x_{i1} + x_{j0}$ (because $y_{il} < y_4 < y_{jl}$), whereas the result of this same shock would have been $X = x_{i1} + x_{j1}$ had it been the case that agents i and j were identical to one another (specifically, if we observed $y_{jl} = y_{il} < y_4$). The *aggregate* effect of a symmetric, transitory shock is therefore sensitive to the *composition* of the system – specifically, the way in which responses to shocks vary among heterogenous agents.

Finally, and starting again from $y = y_1$ in Figure 1, a transitory shock that sets $y = y_5$ before reverting to $y = y_1$ will result in the aggregate outcome $X = x_{i0} + x_{j0}$ (because $y_5 < y_{il} < y_{jl}$) *regardless* of the preceding sequence of events. This illustrates the importance of *non-dominated extrema* (such as $y = y_5$) in models of "true" hysteresis. A transitory shock that sets $y = y_5$ will erase the effects of what are now the dominated extrema y_2 and y_4, rendering the latter irrelevant in the determination of current outcomes. This example also illustrates the capacity of certain types of shocks to "wipe" the memory of systems displaying "true" hysteresis.

"True" hysteresis provides a well-specified formal model of the processes responsible for hysteresis effects. As yet, however, this formal model has found few applications in economics beyond a literature (exemplified by Cross, 1995) that focuses on the Phillips curve (but see Lang and de Peretti (2009) for an important exception).

Hysteresis as a product of historical time

Some radical economists have taken a more behavioural approach to conceptualizing hysteresis. This approach – associated with Setterfield (1997, 1998a) and Katzner (1998, 1999) – can be thought of as "re-tooling" hysteresis for the social sciences, by grounding the concept in what are understood to be the dynamical properties of specifically *social* systems. Hence Setterfield (1993) associates this project with the absence of Lucasian "deep parameters" from social systems, as a result of which "the only truly exogenous factor is whatever exists at a given moment of time, as a heritage of the past" (Kaldor, 1985, p.61). On this view, social systems are necessarily open systems and are therefore subject to radical or fundamental uncertainty.

In the approach taken by Setterfield (1997, 1998a), the possibility of hysteresis is demonstrated by considering a sequence of "cumulatively neutral" changes in some dependent variable of interest – in other words, an adjustment path that, starting from some initial value (such as an equilibrium), leads the variable back to its initial value through a sequence of changes that sums to zero. Hysteresis exists if: (a) the "data" (the alleged exogenous parameters) responsible for determining the value of the variable of interest are, in fact, sensitive to changes in this variable; and (b) changes in the "data" as the variable of interest traverses its cumulatively neutral adjustment path are *not*, themselves, cumulatively neutral (i.e., do not sum to zero). The second condition means that the system will have undergone structural change in the course of the series of cumulatively neutral changes in the variable of interest. This will subsequently alter its outcomes – i.e., the value of the variable of interest – despite the fact that the changes to this variable initially contemplated were, by design, cumulatively neutral. The processes described here can be summarized in terms of the reduced form equation employed by Katzner (1999):

$$x_t = f_t(x_{t-1}, \varepsilon_t) \quad [5]$$

Assuming that $\varepsilon_t = 0$ for all t for simplicity, if both $f_t^{'} \neq 0$ and $f_t^{'} \neq f_{t-1}^{'}$ for some t, then a hypothetical sequence of cumulatively neutral changes in x between periods t – 1 and t + n (so that $x_{t+n} = x_{t-1}$) will nevertheless imply that $x_{t+n+1} \neq x_t$.

On the basis of this analysis, Setterfield (1998) identifies condition (b) above as a sufficient condition for hysteresis, associating the adjustment asymmetries it describes with "threshold effects". Specifically, extreme experiences that propel a system sufficiently far from its current state are thought to result in structural change and hence permanently different system outcomes. Note, then, that not all history matters on this view – only extreme experiences that trigger threshold effects. For example, numerous relatively uneventful journeys away from and back to one's home may leave preferences for travel (and hence, *ceteris paribus*, future travel decisions) unaffected. But a single traumatic experience (involving, for example, a bad accident) may "scar" the decision maker, altering his/her preferences and hence future travel decisions. In this approach to conceptualizing hysteresis, then, historically contingent systems are again expected to display selective memories. Note, also, that if there are no "deep parameters" in the social realm defining a (deterministic or stochastic) "true" model governing the evolution of preferences (or, more generally, the function f_t in equation [5]), then predicting future outcomes in systems that are hysteretic in the sense described here will be subject to radical or fundamental uncertainty.

One of the main advantages of this approach to conceptualizing hysteresis is its sensitivity to perceived characteristics of the specifically *social* material that is the economist's object of analysis. One of the main drawbacks is that its formal model of hysteresis is under-developed. Despite this, the approach has found applications in growth theory (Setterfield, 1998b, 2002). See also Harris (2005) for an implicit application (drawing on equation [5]) to the analysis of Joan Robinson's thinking on "history versus equilibrium".

Summary and Conclusions

Properly conceived, hysteresis is a type of (rather than a synonym for) path dependence, that arises from properties of the adjustment dynamics of a system associated with non-linearities and structural change. Minimally, and in tandem with various other concepts of path dependence, hysteresis implies that earlier states of the world affect later ones and that even transitory causes can have permanent effects. Closer consideration reveals that hysteretic systems also display other, distinguishing features, such as selective rather than complete memories.

As discussed at length in recent surveys of the concept (see, for example, Göcke, 2002; Setterfield, 2009), there are various competing conceptualizations of hysteresis in economics. Among these, the unit/zero root conceptualization has been criticized for the paucity of its characterization of hysteresis. Nevertheless, even this approach has its uses. Between them, the various conceptualizations of hysteresis discussed in this entry provide potential alternatives to equilibrium as an organizing concept in formal analyses of dynamical economic systems.

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Figure 1: The Non-Ideal Relay and "True" Hysteresis in Systems featuring Multiple, Heterogeneous Agents