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## **Licensing of a lower-cost production process to an asymmetric Cournot duopoly**

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An outside inventor of a new production process seeks to license it to Cournot duopolists which have unequal ex ante costs. Distinguishing “leading-edge” innovations (new cost below both firms’ costs) from “catch-up” innovations (new cost between the two firms’ costs), we compare the equilibria of two license-selling mechanisms: exclusive license auction and non-exclusive price-setting. In contrast to the often-studied case of an innovation that reduces the cost of any licensee by the same amount, we show that licensing of a new process may attenuate the ex ante cost asymmetry, allow the inefficient firm to leapfrog its competitor, and raise the licensee’s net profits.

*JEL classification:* D45, L13, L24, O31

*Keywords:* cost-reducing innovation, technology licensing

### **1. Introduction**

The modern theory of licensing of a cost-reducing innovation uses game-theoretic models to analyze the interaction among an inventor and potential licensees who compete in a downstream market. The externalities that the innovation generates among the downstream competitors makes the upstream trade in technology a much more complex and subtle affair than trade in a typical market for procuring inputs. After more than twenty years of increasingly elaborate models, the theory has examined and clarified many issues that influence the structure, pricing, and allocation of licenses, including royalties vs. fixed-fees, auctions vs. price-setting, inside or outside inventors, and exclusivity. However, the conception of what constitutes “a cost-reducing innovation” remains elementary. Specifically, most licensing theory developed so far has focused on identical firms and assumed that a given cost-reducing technology can bring down unit cost by the same amount at any firm that licenses it. One goal of this paper is to propose alternative conceptions of a cost-reducing innovation that may plausibly arise in a manufacturing industry and engender distinct strategic consequences. A second goal is to explore one of those alternatives in detail and compare the results with previous findings. A third goal is to demonstrate that doing away with the usual assumption that firms are identical leads to substantively richer as well as more realistic models of technology licensing.

We begin in the next section by attempting to ground the conception of a cost-reducing innovation in specific conditions that may arise in a typical

manufacturing industry, without assuming that firms have identical costs. This leads us to interpret the prevalent conception of a cost-reducing innovation as an innovation that eliminates one step in the production process, and to identify other plausible kinds of cost-reducing innovations. Focusing on an innovation which constitutes an entirely new way of producing the downstream good, we proceed to specify a duopoly model in Section 3 and use it to analyze the sale of a new-process license via an auction (Section 4) and via price-setting (Section 5). In contrast to step-eliminating innovations, we show that licensing of a new process can attenuate the ex ante cost asymmetry and allow the inefficient firm to leapfrog its competitor. We also show that an exclusive-license auction does not necessarily earn the inventor more revenue than non-exclusive price-setting.

## ***2. Alternative conceptions of a cost-reducing innovation***

A very general conception of licensing has been put forth in Katz and Shapiro's seminal paper "How to license intellectual property" (1986). Although they conduct much of their discussion in the specific context of an inventor licensing an innovation to an oligopoly, they stress that their model applies to the licensing of any "intangible property" (IP) that satisfies the following conditions

1. there is only one licensor, and it has access to an infinite supply of the IP at zero marginal cost (*i.e.*, development costs have been sunk)
2. there are several identical potential licensees, each of which has use for at most one unit of the IP
3. the profit of the licensee (gross of the license fee) is higher than the profit of a non-licensee
4. the profit of a non-licensee decreases as more of its competitors acquire licenses (*i.e.*, by becoming a licensee, a firm imposes a negative externality on a non-licensee)

Katz and Shapiro stress that these assumptions describe not only the case of licensing of a cost-reducing technology but also the licensing of an industry standard or the sale of access to some central facility. Notably, the authors abstain from specifying a model of how potential licensees compete in the downstream market and how obtaining access to the IP affects a firm's competitive standing. Remaining at a high level of abstraction, Katz and Shapiro (1986) and Shapiro (1985) derive interesting general results including:

- (i) an inventor obtains more licensing revenue via auctioning off a limited number of licenses than by setting a price and letting any interested firm buy, and
- (ii) licensing that maximizes inventor's revenue leaves both licensees and

non-licensees with less net profit than ex ante (*i.e.*, the inventor appropriates some of the licensee's ex ante profit).

These results have been confirmed and elaborated in the context of specific models of competition among potential licensees in a downstream market. Following the seminal papers of Kamien and Tauman (1984, 1986), most models assume identical firms with constant marginal cost  $c > 0$  engaged in Cournot competition in a market with linear demand. (Kamien, 1992; Wang, 1998; Sen and Tauman, 2007) The innovation is defined as a reduction in cost from  $c$  to  $c - \varepsilon$ , assuming  $0 < \varepsilon \leq c$ . All firms are assumed identical ex ante and it is also assumed that any firm can realize the same amount of cost-reduction  $\varepsilon$  if it puts the innovation to use.

The case of ex ante cost asymmetries has been largely ignored. Although the asymmetric case is much less tractable, there are at least three important reasons why it deserves attention. First, perfect cost symmetry is unlikely in any real industry. Second, the assumption of perfect cost symmetry leads to auction equilibria in which potential licensees make identical bids, which then requires a random draw to allocate the license. Some properties of such equilibria may arise from the tie-breaking rather than from substantive aspects of downstream competition. In particular, as our analysis will show, findings (i) and (ii) above may be violated when firms are not identical. Thirdly, including ex ante cost asymmetries in a model forces us to clarify the conception of a cost-reducing innovation by identifying and distinguishing specific ways in which a new technology may reduce production costs. This is so because, if firms are not identical in terms of their ex ante costs, it is likely that they may also differ in the extent to which the innovation can reduce their costs.

We propose that the following three types of cost-reducing innovation may plausibly arise in a manufacturing industry and engender distinct strategic consequences for the allocation and pricing of licenses.

*Step-eliminating innovation.* Ex ante, the unit production cost is  $c_i > 0$  for firm  $i$ . The production processes used by the firms may differ, but each process includes a step that is common to all firms. The cost of this common step is  $\varepsilon < \min_i(c_i)$  per unit.

The innovation eliminates this step, thereby reducing the production cost of a licensee to  $c_i - \varepsilon$ . This type of innovation is congruent to the innovation assumed in most models of the strategic licensing literature cited above.

*New process innovation.* Ex ante, the unit production cost is  $c_i > 0$  for firm  $i$ . The

innovation is a whole new process to produce the same output good at a per-unit cost  $e < \max_i(c_i)$ . To use the innovation, a licensee must abandon its previous process and replace it with the new process. We will take up this case in detail in the following sections and show that licensing of a new process innovation is substantively different from licensing a step-eliminating innovation.

*General cost-reducing innovation.* Ex ante, the unit production cost is  $c_i > 0$  for firm  $i$ . By adopting the innovation, firm  $i$  can achieve a unit cost  $e_i \leq c_i$ . Such firm-specific cost-reductions may arise if a particular input (e.g., electricity) is used by firms in different amounts and the innovation reduces the cost of procuring that input (e.g., new electricity generator). Since cost-reductions are firm-specific, the gains to becoming a licensee and the losses from not becoming a licensee are also firm-specific. An auction to allocate a limited number of licenses to such an innovation is a special case of an auction with identity-dependent externalities, a difficult problem that has received some attention from auction theorists. (see Aseff and Chade, 2008; Das Varma, 2002; Funk, 1996) Applying the theory of auctions with interdependent valuations to the specific case of licensing appears to be a promising new direction.

In the remainder of the paper we analyze the licensing of a new process innovation and compare the results to the case of a step-eliminating innovation.

### 3. Model

A Cournot duopoly producing undifferentiated goods faces inverse demand given by  $p(q_1 + q_2) = a - q_1 - q_2$ , where  $q_i$  is the quantity produced by firm  $i \in \{1, 2\}$ ,  $a > 0$  is a demand parameter, and  $p$  is the market-clearing price. We assume that firms have positive costs and that firm 1 is more efficient.<sup>1</sup> To focus attention on non-drastic innovations, we further assume that the inefficient firm would produce a positive amount even if the efficient firm were to succeed in reducing its unit cost to zero. The following condition incorporates these assumptions about costs:

$$0 < c_1 < c_2 < \frac{a}{2} \quad (1)$$

We will use the following notation to denote the Cournot equilibrium profit of firm  $i$  as a function of its and rival's unit costs:

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<sup>1</sup> To avoid repeating "more" and "less," we will simply refer to firm 1 as "efficient" and firm 2 as "inefficient."

$$\pi(c_i | c_j) \equiv \frac{1}{9}(a - 2c_i + c_j)^2$$

where  $i, j \in \{1, 2\}$  and  $i \neq j$ . Ex-ante profits of the duopolists will be denoted by  $A_i \equiv \pi(c_i | c_j)$  and the ex ante cost difference by  $\Delta c \equiv c_2 - c_1$ .

An independent inventor patents a new production process that can be used to produce the same goods more efficiently than the inefficient firm. Specifically, the new process has a unit cost  $e < c_2$ . We distinguish innovations that are a breakthrough for the industry from innovations that can help the inefficient firm catch up, as follows.

**Definition.** A *leading-edge* innovation is a new production process with unit cost  $e$  that satisfies  $0 \leq e < c_1$ .

**Definition.** A *catch-up* innovation is a new production process with unit cost  $e$  that satisfies  $c_1 \leq e < c_2$ .

As the following analysis will show, the strategic consequences of licensing leading-edge and catch-up innovations are quite distinct.

#### 4. *Exclusive licensing via auction*

The inventor may choose to hold an auction to allocate an exclusive zero-royalty license to the innovation.<sup>2</sup> Each firm's bid in the auction represents a per-period fixed fee the firm is willing to pay to become the exclusive licensee. The auction is conducted as a sealed-bid second-price auction. Let  $W_i$  represent the profit of firm  $i$  if it wins at auction, gross of the license fee it will have to pay. Let  $L_i$  represent the profit of firm  $i$  if it loses the auction. Since the auction is second-price, it is optimal for each firm to enter a bid equal to its willingness-to-pay. The firms thus bid  $b_i = W_i - L_i$ , the highest bidder becomes the licensee, and thereafter pays a license fee  $F = \min(b_1, b_2)$  per period. We can decompose the willingness to pay for the license into "use value" and "loss-prevention value." The use value of the license to firm  $i$  is  $W_i - A_i$ , since this is how much it can gain from using the new technology. The loss-prevention value is  $A_i - L_i$ , which represents the loss a firm will suffer if its rival gets the license.

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<sup>2</sup> We limit attention to zero-royalty fixed-fee licenses since such licenses have been shown to maximize outside inventor revenue under basic assumptions. (Kamien, 1992)

#### 4.1 Exclusive licensing of a catch-up innovation

The efficient firm has zero use value for a catch-up innovation, since if it obtains the license, it would be most profitable to shelve the new technology and keep producing with the old process. However, the efficient firm has loss-prevention value for a catch-up innovation, because by obtaining a license it can prevent its rival from becoming more efficient. Specifically, the gross payoffs to the efficient firm from winning or losing the auction are:

$$\begin{aligned} W_1^{CU} &= A_1 \\ L_1^{CU} &= \pi(c_1 | e) \end{aligned}$$

The inefficient firm has zero loss-prevention value but positive use value for a catch-up innovation, since the inefficient firm loses nothing if the efficient firm licenses the innovation and then shelves it. Specifically, after the auction the inefficient firm stands to earn one of the following gross profit levels:

$$\begin{aligned} W_2^{CU} &= \pi(e | c_1) \\ L_2^{CU} &= A_2 \end{aligned}$$

In the auction, the firms bid  $b_i^{CU} = W_i^{CU} - L_i^{CU}$ . Regardless of which firm wins the license, the inefficient firm will not suffer a loss in net profit but the efficient firm will. The next proposition makes this precise.

*Proposition 1.* Allocation of an exclusive license to a catch-up innovation via an auction weakly increases the net profit of the inefficient firm and strictly decreases the net profit of the efficient firm.

*Proof.* The inefficient firm can retain its ex ante profit by bidding zero in the auction, letting the efficient firm win the license and shelve the new process. Thus, if the inefficient firm chooses to make a positive bid, it must be for the purpose of obtaining a higher net profit in the case of winning the auction. If the efficient firm wins the license, it shelves the technology. Its profit from production remains unchanged but net profit falls by the amount of the license fee. ■

Which firm wins the license leads to different consequences not only for the firms' profits but also for consumer surplus and the source of inventor's compensation. If the inefficient firm gets the license, the inventor appropriates part of the new profit the innovation brings to the licensee, but not any of its ex ante profit. The lower cost

enjoyed by the licensee will lead to more output, lower price, greater consumer surplus, higher net profit for the inefficient firm, but lower profit for the efficient firm. Thus, by licensing a catch-up innovation to the inefficient firm, the inventor ends up hurting the efficient firm indirectly, via competition in the output market, and ends up helping its direct client – the licensee – as well as downstream consumers. However, if the efficient firm wins the license, there will be no change in quantities, prices, consumer surplus, or the inefficient firm's profit. The inventor will have simply appropriated part of its client's ex ante profit – a pure transfer of an incumbent's rent with no efficiency consequences.

Which firm will win the license to a catch-up innovation depends on the ex ante cost asymmetry and the size of the innovation. The next three propositions specify the relevant conditions.

*Proposition 2.* If the ex ante cost asymmetry is larger than  $\Delta c > \frac{2}{5}(a - c_1)$ , then any catch-up innovation will be licensed by the inefficient firm.

*Proposition 3.* If the ex ante cost asymmetry is smaller than  $\Delta c < \frac{1}{5}(a - c_1)$ , then any catch-up innovation will be licensed by the efficient firm.

*Proposition 4.* If the ex ante cost asymmetry falls in the range  $\frac{1}{5}(a - c_1) < \Delta c < \frac{2}{5}(a - c_1)$ , then there exists a critical size of a catch-up innovation  $\bar{e} \in (c_1, c_2)$  such that any more significant catch-up innovation  $e \in (c_1, \bar{e})$  will be licensed by the inefficient firm and any less significant catch-up innovation  $e \in (\bar{e}, c_2)$  will be licensed by the efficient firm. The critical size of a catch-up innovation is  $\bar{e} = \frac{1}{5}(2a + 8c_1 - 5c_2)$ . The bigger (smaller) the ex ante cost asymmetry, the broader (narrower) the range of catch-up innovations licensed by the efficient firm.

*Proof.* The bids of the efficient and inefficient firms for an exclusive license to a catch-up innovation are, respectively,  $b_1 = A_1 - \pi(c_1 | e)$  and  $b_2 = \pi(e | c_1) - A_2$ . From this it follows that  $b_1 > b_2$  if and only if  $e > \frac{1}{5}(2a + 8c_1 - 5c_2)$ . Imposing the restrictions  $e > c_1$  and  $e < c_2$ , and using the definition of ex ante cost asymmetry  $\Delta c = c_2 - c_1$  leads to Propositions 2 through 4. ■

The overall picture that emerges from these results is that a catch-up innovation



will end up being put to use by the inefficient firm only when the ex ante cost asymmetry is large enough and if the innovation promises a large enough cost reduction. Thus, only significant catch-up innovations in significantly asymmetric duopolies can be expected to end up in the hands of the inefficient firm and thereby reduce the cost asymmetry and increase consumer surplus. Conversely, a sufficiently small ex ante cost asymmetry will be perpetuated in spite of any catch-up innovations offered for licensing by outside inventors. Such inventions will be kept out of use by preemptive licensing by the efficient firm. This implies that it is possible for a slightly more efficient firm to be driven to progressively lower profit levels by a series of outside inventors auctioning off catch-up innovations. Considered in isolation, each such auction for a catch-up innovation makes it rational for the efficient firm to outbid the inefficient firm and then shelve the innovation. However, by accumulating such licenses the efficient firm will progressively pay out more of its gross profit to the inventors, eventually ending up with zero net profit.<sup>3</sup> We caution that the rationality of such preemptive licensing may be questioned in a model that allows firms to anticipate future innovations.

#### *4.2 Exclusive licensing of a leading-edge innovation*

A leading-edge innovation has both use value and loss-prevention value for both firms. Specifically, the gross payoffs to firm  $i$  from winning or losing the auction, respectively, are:

$$\begin{aligned} W_i^{LE} &= \pi(e | c_j) \\ L_i^{LE} &= \pi(c_i | e) \end{aligned}$$

For each firm, winning a license auction increases gross profit ( $W_i^{LE} > A_i$ ) whereas

losing decreases it ( $L_i^{LE} < A_i$ ). This win-or-lose situation is structurally similar to the

case of a step-eliminating innovation that is the focus of most strategic licensing models cited earlier. However, because the licensing of a new process to an asymmetric duopoly engenders different amounts of cost-savings for the two firms, there are equilibria different from those identified in the literature on step-eliminating innovations. Specifically, whereas in the case of a step-eliminating innovation the efficient firm always outbids the inefficient firm and thereby increases the cost asymmetry via licensing, the opposite outcome is possible in the case of a leading-edge new process

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<sup>3</sup> The same problem plagues most strategic licensing models that predict transfer of ex ante incumbent profits to the outside inventor via licensing auctions.

innovation. The next three propositions make this precise.

*Proposition 5.* If the ex ante cost asymmetry is larger than  $\Delta c > \frac{2}{5}(a - c_1)$ , then any leading-edge innovation will be licensed by the efficient firm.

*Proposition 6.* If the ex ante cost asymmetry is smaller than  $\Delta c < \frac{2}{5}(a - 5c_1)$ , then any leading-edge innovation will be licensed by the inefficient firm.

*Proposition 7.* If the ex ante cost asymmetry falls in the range  $\frac{2}{5}(a - 5c_1) < \Delta c < \frac{2}{5}(a - c_1)$ , then there exists a critical size of a leading-edge innovation  $e^* \in (0, c_1)$  such that any more significant innovation  $e \in (0, e^*)$  will be licensed by the efficient firm and any less significant leading-edge innovation  $e \in (e^*, c_1)$  will be licensed by the inefficient firm. The critical size of a leading-edge innovation is  $e^* = \frac{1}{8}(5(c_1 + c_2) - 2a)$ . The bigger the sum of the ex ante costs, the larger the range of leading-edge innovations licensed by the efficient firm.

*Proof.* Firm  $i$  bids for an exclusive license to a leading-edge innovation  $e \in (0, c_1)$  in the amount of  $b_i = \pi(e | c_j) - \pi(c_i | e)$ . From this it follows that  $b_1 > b_2$  if and only if  $e < \frac{1}{8}(5(c_1 + c_2) - 2a)$ . Imposing the restrictions  $e < c_1$  and  $e > 0$ , and using the definition of ex ante cost asymmetry  $\Delta c = c_2 - c_1$  leads to Propositions 5 through 7. ■

The overall picture that emerges from these results is that a sufficiently large ex ante cost gap will be widened by a leading-edge innovation. However, if the initial cost asymmetry is not too large and the leading-edge innovation is not too significant, then the inefficient firm will license the innovation and leapfrog the efficient firm. Such leapfrogging is not possible when licensing a step-eliminating innovation, as the next proposition shows.

*Proposition 8.* Any step-eliminating innovation  $\varepsilon \in (0, c_1)$  is licensed by the efficient firm.

*Proof.* Firm  $i$  bids for an exclusive license to the step-eliminating innovation in the amount of  $b_i = \pi(c_i - \varepsilon | c_j) - \pi(c_i | c_j - \varepsilon)$ . From this it follows that  $b_1 - b_2 = 2\varepsilon\Delta c > 0$ , which implies firm 1 wins the auction. ■

In particular, if a step-eliminating innovation satisfies  $\varepsilon > \Delta c$ , then potentially the inefficient firm could leapfrog its competitor if only it could obtain the exclusive license. However, according to Proposition 8, the efficient firm will outbid the inefficient firm in the auction and thereby widen the cost asymmetry.

## **5. Non-exclusive licensing of a new production process via price-setting**

Instead of holding an auction to allocate an exclusive license, the inventor can set a price and offer a non-exclusive license to any firm willing to pay the price. In this section, we consider the allocation of licenses and inventor revenue under such price-setting, and identify conditions under which the inventor prefers price-setting to auctioning.

### *5.1 Licensing of a catch-up innovation via price-setting*

An inventor choosing how to sell license(s) to a catch-up innovation would choose to hold an auction only for those innovations that would be won and shelved by the efficient firm. If the auction would lead to licensing by the inefficient firm, the inventor could earn more by non-exclusive price-setting. The next proposition makes this precise.

*Proposition 9.* If the inventor offers non-exclusive licenses to a catch-up innovation, then (i) the efficient firm will not buy a license at any positive price, and (ii) the inventor will earn higher licensing fees than via exclusive auctioning if the innovation satisfies conditions in Propositions 2 and 4 under which the inefficient firm wins the auction.

*Proof.*

- (i) The efficient firm would not pay for a non-exclusive license since the use value is zero and loss-prevention value cannot be realized without exclusivity.
- (ii) If an auction for an exclusive license is won by the inefficient firm, then the firms' bids must have satisfied  $b_2 = W_2 - A_2 > A_1 - L_1 = b_1$  and the license fee must be the smaller bid  $F^E = A_1 - L_1$ . If instead of holding an auction, the inventor were to set the price slightly below  $F^{NE} = W_2 - A_2$ , then the inefficient firm would have bought the license, since doing so would raise its profit. Since  $F^{NE} > F^E$ , the inventor would have earned more revenue. ■

Thus, it makes sense for the inventor to hold an exclusive auction for a

catch-up innovation only if the loss-prevention value for the efficient firm exceeds the use value for the inefficient firm. Otherwise, price-setting is a more profitable way for the inventor to sell the license to the inefficient firm. Even though price-setting potentially makes the innovation available to both firms on a non-exclusive basis, in effect the outcome is a single license to the inefficient firm.

### 5.2 Licensing of a leading-edge innovation via price-setting

If both firms obtain a license to a leading-edge innovation, then each firm will earn a gross profit which we will denote by  $B \equiv \pi(e|e)$ . Figure 1 shows the firms' payoff matrix in the price-setting licensing game in which the inventor first announces a license fee  $F$  per period and then each firm decides whether to buy a license or not. As the following propositions establish, there are two candidate prices that the inventor must choose from when setting a non-exclusive license fee to a leading-edge innovation: either  $F_{12} = B - L_1$  or  $F_2 = W_2 - A_2$ .

		Firm 2	
		Don't buy	Buy
Firm 1	Don't buy	$A_1 \quad A_2$	$L_1 \quad W_2 - F$
	Buy	$W_1 - F \quad L_2$	$B - F \quad B - F$

Figure 1. Payoff matrix of the game in which each firm decides whether to buy a license at the price  $F$  per period set by the inventor.

*Lemma 1.* For any leading-edge innovation  $e \in [0, c_1)$  and any cost and demand parameters satisfying (1), Cournot profits gross of the license fee satisfy all of the following inequalities:

- (i)  $L_2 < L_1 < A_2 < A_1 < W_2 < W_1$
- (ii)  $A_2 < B < W_2$
- (iii)  $W_2 - A_2 > W_1 - A_1$
- (iv)  $W_2 - A_2 > B - L_1$

*Proof.* Inequalities (i), (ii) and (iii) follow from the definitions of the Cournot profit functions. To prove (iv), we note that the equation  $W_2 - A_2 = B - L_1$  reduces to  $e^2 + c_1(c_2 - 2e) + (a - c_1 - c_2)(c_2 - c_1) = 0$ , which has no real roots in  $e$ . Thus  $W_2 - A_2 - (B - L_1)$  as a function of  $e$  is a parabola with no zero-crossings. Therefore, for all  $e$ , either  $W_2 - A_2 > B - L_1$  or  $W_2 - A_2 < B - L_1$ . When  $e = c_1$ ,

$W_2 - A_2 > B - L_1$ ; therefore this must hold for all  $e$ . ■

*Proposition 10.* The highest price an inventor can set such that both firms will buy a license to a leading-edge innovation is  $F_{12} = B - L_1$ . Both firms will suffer a decrease in net profit as a result of buying a license at this price.

*Proof.* From the payoff matrix in Figure 1, it can be seen that (Buy, Buy) is a Nash equilibrium of the price-setting licensing game if and only if  $F < B - L_1$  and  $F < B - L_2$ . According to Lemma 1(i),  $L_2 < L_1$ . Therefore,  $F < B - L_1$  is the binding upper limit on the license fee. After the firms purchase licenses at this price, each firm's net profit will be  $B - (B - L_1) = L_1$ , which is less than  $A_1$  and  $A_2$  by Lemma 1(i). ■

*Proposition 11.* The highest price an inventor can set such that exactly one firm will buy a license to a leading-edge innovation is  $F_2 = W_2 - A_2$ . This price will induce only the inefficient firm to become a licensee. The licensee's net profit will be the same as ex ante; non-licensee's profit will be lower than ex ante.

*Proof.* From the payoff matrix in Figure 1, it can be seen that (Don't buy, Buy) is a Nash equilibrium of the price-setting licensing game if and only if  $F < W_2 - A_2$  and  $F > B - L_1$ . Such a fee  $F$  exists by Lemma 1(iv). Next, we need to confirm that the inventor cannot obtain a higher license fee in the (Buy, Don't buy) equilibrium. (Buy, Don't buy) is a Nash equilibrium if and only if  $F < W_1 - A_1$  and  $F > B - L_2$ . According to Lemma 1(iii), the upper bound on  $F$  in this equilibrium is below that of the (Don't buy, Buy) equilibrium. Finally, ex post profits in the (Don't buy, Buy) equilibrium with the price  $F_2 = W_2 - A_2$  are  $L_1 < A_1$  for firm 1 and  $W_2 - (W_2 - A_2) = A_2$  for firm 2. ■

The next proposition identifies conditions under which the inventor prefers to set the license fee aiming to license both firms, and conditions under which the inventor prefers to set the fee so as to license only the inefficient firm.

*Proposition 12.* An inventor who uses price-setting to sell licenses to a leading-edge innovation maximizes licensing revenue by setting the fee to  $F_{12} = B - L_1$ , and thereby licensing both firms, if  $c_2 < 2c_1$  and  $e \in [0, 2c_1 - c_2]$ . Otherwise (i.e., if  $c_2 > 2c_1$ , or  $c_2 < 2c_1$  and  $e \in (2c_1 - c_2, c_1)$ ), the inventor maximizes licensing revenue by setting the fee to  $F_2 = W_2 - A_2$  and thereby licensing only the inefficient firm.

*Proof.* The inventor can set the price to attract either one or both firms. Propositions 10 and 11 establish the highest price possible for each case. Inventor's maximum revenue from licensing both firms is  $2F_{12}$ , which exceeds the maximum revenue from licensing one firm  $F_2$  iff  $2(B - L_1) > W_2 - A_2$ . This inequality reduces to  $c_1 + c_2 - a < e < 2c_1 - c_2$ . Applying assumption (1) constrains  $e$  to the range  $0 < e < 2c_1 - c_2$ . ■

Finally, we consider conditions under which an inventor of a leading-edge innovation earns more via exclusive auctioning than via non-exclusive price-setting. As the next proposition establishes, if price-setting results in licensing only a single firm, the inventor can earn more revenue by instead holding an auction for an exclusive license.

*Proposition 13.* If licensing a leading-edge innovation via price-setting would result in licensing only one firm, the inventor can earn more revenue by instead auctioning off an exclusive license.

*Proof.* If  $c_2 > 2c_1$  or if  $c_2 < 2c_1$  and  $e \in (2c_1 - c_2, c_1)$ , then, according to Proposition 12, the profit-maximizing price is  $F_2 = W_2 - A_2$ , only the inefficient firm buys, and the licensing revenue is  $F_2$ . An auction for an exclusive license would yield  $\min_{i=1,2}(W_i - L_i)$  in licensing revenue, which exceeds  $F_2$  according to Lemma 1(i). ■

As can be easily verified with a numerical counter-example, the converse of Proposition 13 does not hold. Thus, in cases when price-setting would result in both firms buying a license, the inventor may or may not find it more profitable to instead auction off an exclusive license. Unlike in models with identical firms licensing a step-eliminating innovation, the inventor in our model does not necessarily earn more via exclusive-license auctioning than via non-exclusive price-setting.

## 6. Conclusion

We began by inquiring into how the usual theoretical definition of a cost-reducing innovation may be interpreted in the context of a typical manufacturing industry, and whether there are other plausible kinds of cost-reducing innovations. We have outlined a brief typology of cost-reducing innovations that distinguishes step-eliminating, new-process, and general cost-reducing innovations. We then

focused on the licensing of a new process innovation, keeping track of its two sub-types: leading-edge and catch-up innovations. The licensing equilibria that we have derived are substantively different from those in previous studies focused on the case of symmetric firms licensing a step-eliminating innovation. We have shown that when firms with different ex ante costs engage in a game to allocate a license to a new process via auction or price-setting, outcomes that have been ruled out in many strategic licensing models become possible. In particular, the relatively inefficient firm may catch-up to and even leapfrog its rival, price-setting without quantity restrictions may yield higher licensing revenue to the inventor than auctioning off a restricted number of licenses, and the licensee does not necessarily end up with less net profit than ex ante.

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