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# Evaluation of hydropower upgrade projects - a real options approach

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## Abstract

When evaluating whether to refurbish existing hydropower plants or invest in a new power plant, there are two important aspects to take into consideration. These are the capacity chosen for the production facilities and the timing of the investment. This paper presents an investment decision support framework for hydropower producers with production facilities due for restoration. The producer can choose between refurbishing existing power plants and investing in a new production facility. A real options framework is proposed to support the investment decision. Using a case from Norsk Hydro ASA, a Norwegian hydropower producer, we employ the framework to evaluate the investment opportunities. Our main contribution is an approach that combines hydropower scheduling and real options valuation, and the results from our analysis suggest feasible investment strategies for Norsk Hydro ASA.

*Key words:* OR in energy, Real options, Electricity price uncertainty

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## 1. Investment in the Energy Sector

In the IEA study IEO2009 (EIA, 2009), world energy consumption is projected to grow by 34 % in the period from 2010 to 2030 in the reference case. In the same reference case, world net electricity generation is expected to double. The potential for new electricity generation capacity through refurbishing and expanding existing power plants in Norway is estimated to be approximately 15 TWh per year (NVE, 2006). This corresponds to 12 % of the total average Norwegian

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annual generation. The combination of large investment costs, long lifetime of the assets, and uncertainty in future revenues makes investment decisions in the hydropower industry challenging.

Market integration is driven by price differences between regions. It is generally expected that the integration will continue as long as the price differences prevail and lie above the cost of new transmission capacity. The effect of the integration process is uncertain, but studies, such as Statnett (2004), indicate that Nordic electricity prices may change to some extent due to increased transmission capacity between the Nordic and continental European electricity markets.

One important feature of the continental electricity market is the high degree of intraday price variation compared to the Nordic market. This can be illustrated by a peak load contract for 2009 on the German power exchange EEX which is traded for 104 €/MWh (EEX, 2008), while the same contract on the Nordic power exchange Nord Pool is traded for 63 €/MWh (Nord Pool, 2008). The differences in intraday price variations are interesting for hydropower producers in the Nordic market due to the excellent load variation capabilities of hydropower. The short response time and low costs associated with these responses are two of the favorable characteristics of hydropower plants.

Electricity generation and its timing lay the foundation for the cash flows generated by hydropower plants. The cost of water is in principle zero, but since it is a limited resource, it has a value in hydropower production. Producers can therefore calculate a so-called marginal water value (MWV), which can be regarded as an opportunity cost in order to determine a production schedule. The aim is to maximize the profit from the water released, plus the expected value of the water remaining in the reservoir (Wallace and Fleten, 2003). The literature consists of different approaches for estimating the MWV. Yeh (1985) reviews mathematical models developed for reservoir operations where linear programming, dynamic programming, nonlinear programming, and simulations are discussed. Wallace and Fleten (2003), on the other hand, give an overview of optimization models that deal with energy, focusing mainly on stochastic models. Keppo and Näsäkkälä (2008) have developed an approach for production planning based on intuitive MWV calculations. They estimate a production threshold based on information in electricity derivative markets, the reservoir level and on inflow.

Price fluctuations and intraday price differences offer opportunities for hydropower produc-

ers that have storage opportunities for water in connection with their production facilities. It is desirable to decrease the plant's capacity factor, i.e. the relation between annual production and output if it had operated at full capacity throughout the year. A low capacity factor enables the producers to allocate production to peak price load periods, in contrast to producing at a constant rate continuously (Johnsen et al., 1999). It is also desirable to save water from periods with low prices for use in periods with high prices.

Even though capacity expansion projects in the hydropower industry appear attractive, it seems that investors hesitate to realize projects, unless profits are significant. In general, there are three main factors affecting the investment decisions. First, there is uncertainty regarding future cash flows. Second, the investments are irreversible, and third, the investors often have the opportunity to postpone investments. As a result, there is an opportunity cost related to realizing a project (Dixit and Pindyck, 1994). Thus, the reason for the hesitant behavior may be the high option value, i.e. the value of deferring the investment exceeds the value of investing immediately. Bøckman et al. (2008) give an example of how an investment decision in a small hydropower plant will be postponed until a certain electricity price level is reached.

Construction of large power plants often implies long outage times for existing plants, resulting in a considerable profit loss. One of the major costs when refurbishing and maintaining existing production facilities is the production lost due to the generators not being able to produce electricity. Refurbishing existing facilities will thereby to a certain extent imply the same costs as investment in a new production facility. Hence, investment in new production facilities should be considered when the existing facilities are due for restoration and maintenance.

In order to capture the value of the expansion option within existing production facilities, real options valuation (ROV) can be employed. The option of investing in an upgrading project can be regarded as an American call option (Dixit and Pindyck, 1994). Chorn and Shokhor (2006) conduct ROV where they combine the Bellman equation and dynamic programming to evaluate petroleum development investments, while Dangl and Wirl (2004) discuss an investment opportunity where a firm has to determine optimal investment timing and capacity at the same time, under conditions of irreversible investment expenditures and uncertainty. Fuss et al. (2008) conduct ROV using dynamic programming on an investment opportunity in a power plant where they take

market and climate policy uncertainty into account, while Kjærland (2007) estimates the value of investment opportunities in the hydropower sector in Norway using ROV. Botterud (2003) includes construction delays when studying investments in new power generation and Tseng and Barz (2002) use ROV to value power plants with unit commitment constraints and propose a solution procedure that integrates forward-moving Monte Carlo simulation with backward-moving dynamic programming.

The purpose of this paper is to analyze a hydropower upgrade project through ROV, and its main contribution is an approach that unites hydropower scheduling and reservoir operations with ROV. Our approach can be used by hydropower plant owners with facilities due for restoration, and the results from our analysis suggest investment strategies for the owner of such facilities.

The structure of this paper is as follows: Section 2 introduces an investment case supplied by Norsk Hydro ASA. Section 3 outlines the models built to describe and evaluate the investment opportunities, while Section 4 presents the empirical results. Section 5 concludes and gives the final remarks.

## **2. Case Presentation**

Norsk Hydro ASA has a sequence of five hydraulically coupled power plants installed in a river system in their production portfolio. These are aged plants that were designed to provide base load electricity production to industries in the surrounding area. The existing configuration suffers from high a response time and low efficiency. The majority of the inflow used for electricity production is accumulated and stored in a large reservoir. This reservoir has a 67.5 % degree of regulation, defined as reservoir size relative to mean yearly inflow (Norsk Hydro, 1987). That is, with no discharge and average inflow, an empty reservoir is refilled in approximately eight months. Due to the degree of regulation opportunities, it is regarded as an multiseasonal reservoir.

During the last two decades, the Nordic electricity market has changed considerably. As a result, the existing hydropower plants are not properly suited to the present market, and maintenance lags lead to large restoration costs (Norsk Hydro, 1987). In order to extract the value found in the increasing volatility in the electricity prices during the day and during the year, Norsk Hydro ASA considers investing in a new power plant in order to improve the generation capacity in the river

system. The company contemplates four new expansion projects in addition to restoration and maintenance of the existing plants. The expansion projects involve replacing three of the existing plants by one large production facility, and expanding the discharge capacity of the two remaining plants.<sup>1</sup> The alternatives differ in geographical location and capacity. In order to keep the number of options manageable, we limit the scope of our analysis to include two of the projects, each with different installed capacities and investment costs.<sup>2</sup>

Each new power plant has a lifetime of 50 years. The existing power plants will also have an extended lifetime of 50 years if the restoration and maintenance requirements is completed. Hence, the problem is reduced to analyzing whether Norsk Hydro ASA should refurbish the existing power plants, or invest in a new power plant. In order to comply with regulatory requirements, the existing power plants must undergo restoration and maintenance today, in eight years and in 18 years if the decision is made to keep them. The costs associated with the restoration and maintenance is given in Table 1:

Table 1: Costs associated with restoration and maintenance of existing facilities.

Year	Cost [MNOK]
Today	250
In 8 years	300
In 18 years	300

If Norsk Hydro ASA chooses to invest in a new power plant, the size and the timing of this investment must be decided. It is important to note that if Norsk Hydro ASA chooses not to build a new power plant today, the option to expand the capacity of the existing power plant at a later stage is still viable. The two investment alternatives considered for a new power plant are mutually exclusive.

Table 2: Overview of the valuation framework.

<b>Production Scheduling</b>	<b>Real Options Valuation</b>	<b>Monte Carlo Simulations</b>
<p><b>Input:</b></p> <ol style="list-style-type: none"> <li>1 Technical parameters for the investment alternatives</li> <li>2 Electricity price dynamics for spot and forward prices</li> </ol>	<p><b>Input:</b></p> <ol style="list-style-type: none"> <li>1 Annual revenues for each investment alternative for different mean annual price levels</li> <li>2 Mean annual price level dynamics</li> </ol>	<p><b>Input:</b></p> <ol style="list-style-type: none"> <li>1 Optimal action for every year and each mean annual electricity price level</li> <li>2 Mean annual price level dynamics</li> </ol>
<p><b>Output:</b></p> <ol style="list-style-type: none"> <li>1 Annual revenues from the investment alternatives for different mean annual price levels</li> </ol>	<p><b>Output:</b></p> <ol style="list-style-type: none"> <li>1 Optimal action for every year and each mean annual price level</li> </ol>	<p><b>Output:</b></p> <ol style="list-style-type: none"> <li>1 Frequencies for how often investment in each investment alternative occurs</li> </ol>

### 3. Valuation Framework

The framework we use for analyzing the investment opportunity consists of the following steps; production scheduling, ROV and Monte Carlo simulation. Table 2 gives an overview of the framework and the inputs and outputs of the different steps. In the production scheduling, electricity price dynamics for spot and forward prices are used to derive annual revenues from each investment alternative for a range of mean annual price levels. These revenues together with the dynamics of the annual price levels are used in the ROV to calculate the optimal action for every year and each mean annual price level. In the third step of the framework, we extract the results by using Monte Carlo simulations<sup>3</sup> and thereby finding how the optimal actions will materialize. When analyzing the investment alternatives, we assume that the electricity price in

Norway contains seasonality, and that the mean annual electricity price level changes every year. These assumptions are supported by the fact that the electricity prices greatly depend on the annual precipitation, and the precipitation varies from year to year. The seasonality is maintained in the production scheduling model, while the characteristic of mean annual price levels is maintained in the ROV. Each part of the framework will be elaborated on in the next sections.

### *3.1. Production Scheduling*

In order to assess the water in the reservoir, we calculate a MWV. This is done for the purpose of comparing the value of using the water today with the value of saving the water for later. If the MWV is higher than the spot price, we save the water for later use, and if the spot price is higher than the MWV, we use the water for production. The value of the MWV will be different for each of the investment alternatives. When scheduling the production of the investment alternatives, we have chosen to use the framework suggested by Keppo and Näsäkkälä (2008). Our model differs from theirs in the following ways;

1. The MWV is a function of the deviation from the median reservoir level. This intuition is supported by Tipping and Read (2010) for hydropower-dominated electricity markets. We express the MWV as a function of the storage level, in terms of deviation from a median reservoir level. This intuition is also supported in the work of Keppo and Näsäkkälä (2008), but then in terms of spillage probability.
2. The production strategy is executed as a "bang-bang" strategy. That is, the power plant shifts instantaneously between zero and full capacity. Consequently, we do not take minimum discharge and ramping constraints into account. This is a simplification, since the capacity used can be chosen to be between zero and the maximum capacity of the facility, as illustrated in Zhao (2009).
3. The minimum reservoir level requirement converges to 90 % of maximum reservoir level in the end of the planning horizon.<sup>4</sup> This approach is supported by Fleten et al. (2002) who proposes alternatives for avoiding end effects in the reservoir level, one of them being to choose the time such that it makes sense to constrain the reservoir to be either full or empty



in the end of the planning horizon. Keppo and Näsäkkälä (2008) let the MWV be a function of time<sup>5</sup> and let the MWV converge to zero towards the end of the planning period, implying that the reservoir is full and there is a risk of spillage.

In our production scheduling model, all reservoirs are aggregated into one large reservoir, and the existing power plants are aggregated into one large power plant. The output is considered to be independent of head variation effects. In general, these are common assumptions used in long-term generation planning (Wallace and Fleten, 2003).

In the production scheduling, the set  $J$  contains all the mean annual price levels in the range  $[Q_{min}, Q_{max}]$ , while  $T$  is the set of all days in the planning horizon and  $H$  is the set of all hours in a day. The set  $I$  contains the three investment alternatives.<sup>6</sup> The parameterized mathematical form of the MWV is given by  $K(\cdot)$  in Eq. (1), while Eq. (2) gives the average future forward price,  $\bar{F}_t^j$ :

$$K_{\bar{F}_t^j, x_t, v_t}^j = \alpha_{\bar{F}} \cdot \bar{F}_t^j \cdot e^{\alpha_x \cdot \left(\frac{\bar{x}_t - x_t}{\bar{x}_t}\right) + \alpha_v \cdot \left(\frac{\bar{v}_t - v_t}{\bar{v}_t}\right)}, \quad t \in T, j \in J \quad (1)$$

$$\bar{F}_t^j = \frac{\sum_{i=1}^q F_{t,t+i}^j}{q}, \quad t \in T, j \in J \quad (2)$$

The MWV at day  $t$  and price level  $j$  is a function of the reservoir level,  $x_t$ , the inflow,  $v_t$ , and the average future forward price,  $\bar{F}_t^j$ . In order to find the average future forward price, we estimate the forward curve on day  $t$  with maturity in  $q$  days, and use the average future forward price in our calculations, as given in Eq. (2). The median reservoir level is given by  $\bar{x}_t$ <sup>7</sup> and the median inflow is given by  $\bar{v}_t$ <sup>8</sup>. The parameters  $\alpha_{\bar{F}}$ ,  $\alpha_x$  and  $\alpha_v$  denote the rate of decrease in the MWV as a function of the average future forward price, the reservoir level and the future inflow estimate, respectively. In order to find the value of the parameters, we estimate the expected NPV for the investment alternatives for a set of parameter combinations, and the parameter set chosen is the one maximizing the expected NPV.

The production planning depends on the spot price and the MWV, but also on the reservoir level. The reservoir level is given in the following equation:

$$x_{t+1} = x_t + v_t - \sum_{h=1}^H u_{h,t} - s_t, \quad t \in T \quad (3)$$

Eq. (3) states that the reservoir level the next day,  $x_{t+1}$ , is equal to the current reservoir level,  $x_t$ , plus inflow,  $v_t$ , minus discharge throughout the current day,  $\sum_{h=1}^H u_{h,t}$ , minus spillage,  $s_t$ . That is;

all inflow to the reservoir is either used for production or spilled. The water reservoir level is constrained by upper and lower bounds,  $x_{max,t}$  and  $x_{min,t}$ , respectively, and the bounds depends on the season. We assume that inflow occurs at the beginning of the day.

The spot price at day  $t$  for mean annual price level  $j$  is given by  $S_t^j$ . This spot price is adjusted for the deterministic intraday price differences before it is used in the production scheduling, and we thereby get an hourly resolution of the spot price,  $S_{h,t}^j$ . The lower bound for the discharge is set to zero, while the upper bound is the plant's maximum discharge capacity,  $u_{max}$ . Since we assume a bang-bang strategy, the discharge will either be zero or  $u_{max}$ , given that the restrictions for the reservoir level are maintained. The production strategy is to discharge water whenever the spot price is higher than the MWV. Depending on the spot price, the MWV and the reservoir level, there are four possible outcomes for the discharge,  $u_{h,t}$ , as given in Eq. (4) and Eq. (5):

$$S_{h,t}^j > K_{\bar{F}_t^j, x_t, v_t}^j \begin{cases} \text{if } x_t + v_t + 24 \cdot u_{max} \geq x_{min,t} & \rightarrow u_{h,t} = u_{max} \\ \text{if } x_t + v_t + 24 \cdot u_{max} < x_{min,t} & \rightarrow u_{h,t} = 0 \end{cases} \quad (4)$$

$$S_{h,t}^j \leq K_{\bar{F}_t^j, x_t, v_t}^j \begin{cases} \text{if } x_t + v_t \leq x_{max,t} & \rightarrow u_{h,t} = 0 \\ \text{if } x_t + v_t > x_{max,t} & \rightarrow u_{h,t} = u_{max} \end{cases} \quad (5)$$

The first part of Eq. (4) states that if the spot price is higher than the MWV, and the reservoir level and the inflow plus the water required for one day of full discharge are higher than the minimum reservoir level, the discharge is  $u_{max}$ . If, on the other hand, the reservoir level, adjusted for the inflow and the required water for one day of full production, is beneath its minimum, the last part of Eq. (4) ensures that the discharge is zero. If the spot price is lower than the MWV, the first part of Eq. (5) states that the discharge is zero, if there is no risk of spillage. If there is a risk of spillage, the last part of Eq. (5) ensures that the discharge is  $u_{max}$ .

The value of the investment alternatives are maximized by finding the MWV, and producing according to the strategy outlined in Eq. (4) and Eq. (5). The value of alternative  $i$  depends on the plant capacity,  $\bar{\omega}_i$ , the plant efficiency,  $\eta_i$ , the discharge,  $u_{h,t}$ , the spot price,  $S_{h,t}^j$ , the maintenance cost,  $c_i$ ,<sup>9</sup> and the discount rate,  $\mu$ . The annual present value of an investment alternative can be

summarized as in Eq. (6):

$$NP_{\bar{\omega}_i, \eta_i, S_{h,t}, x_t, u_{h,t}}^{j,i} = \mathbb{E}[\bar{\omega}_i \cdot \eta_i \cdot \sum_{t=1}^T (e^{-\mu \cdot t} \cdot \sum_{h=1}^H (u_{h,t} \cdot (S_{h,t}^j - c_i))) | \bar{F}_t^j], \quad i \in I, j \in J \quad (6)$$

### 3.2. Electricity Price Model

The production scheduling described in the previous section requires specifications of spot and forward prices. Lucia and Schwartz (2002) have developed a framework for electricity prices suited for the Nordic electricity market, and we represent spot and forward price dynamics similar to their one-factor model. Our electricity spot price dynamics consist of three parts; a mean annual price level, a deterministic seasonal variation, and a stochastic part. In the production scheduling, we change this parameter in order to find the present value of the annual revenues for each investment alternative in a range of mean annual price levels. The deterministic part reflects the seasonal changes in the price over the year, while the stochastic part represents the unforeseeable changes in prices due to e.g. weather (temperature and precipitation). While the deterministic part follows a cosine function, the stochastic part is a mean-reverting process. The electricity price dynamics are summarized in Eq. (7), the deterministic part is described in Eq. (8) while the stochastic part is described in Eq. (9).

$$S_t^j = j + d_t + \chi_t, \quad t \in T, j \in J \quad (7)$$

$$d_t = \gamma \cdot \cos((t + \phi) \cdot \frac{2\pi}{365}), \quad t \in T \quad (8)$$

$$\Delta \chi_t = -\kappa \cdot \Delta t + \sigma \cdot \Delta Y_t, \quad t \in T \quad (9)$$

where  $S_t^j$  represents the spot price on day  $t$  for mean annual price level  $j$ . In the price dynamics,  $j$  represents the mean annual price level, and the deterministic part of is represented by  $d_t$  where  $\gamma$  and  $\phi$  are constants. The stochastic part  $\chi_t$  has a mean of zero,  $\kappa$  denotes the rate of short term mean reversion,  $\sigma$  is the volatility of the process and  $\Delta Y_t$  is the increment of a Wiener process. The time increment is given by  $\Delta t$ .

The production scheduling requires predictions of forward prices. Given the spot price in Eq. (7) and the future mean annual electricity price level presented in Section 3.3, an approximation of the forward price is given in Eq. (10):

$$F_{t,q}^j = j + d_q + (S_t^j - (j + d_t)) \cdot e^{-\kappa \cdot q} + \alpha^* (1 - e^{-\kappa \cdot q}) - (1 - e^{-\lambda \cdot q}) \cdot (j - \bar{Q}), \quad t \in T, j \in J \quad (10)$$

where  $F_{t,q}^j$  is the forward price for electricity delivered in  $q$  days for each mean annual price level  $j$ . In the production scheduling, we assume that the mean annual spot price level is the same throughout the year. The forward curve includes the expectation of the mean annual price level for the next year. The last part of Eq. (10) ensures that this relationship is maintained, where  $\bar{Q}$  is the long term mean annual electricity price level, and  $\lambda$  is the long term mean reversion rate.

The parameters are found using regression analysis, and the volatility in Eq. (9),  $\sigma$ , and the risk premium in Eq. (10), represented through  $\alpha^*$ , is calculated as outlined in Lucia and Schwartz (2002).

In our electricity spot price model, the price does not only change every day, it also changes each hour within the day. We let intraday price variation be deterministic, and vary depending on the day of the week and season.

The parameters for the price models are estimated based on price information from the Nordic electricity exchange Nord Pool.<sup>10</sup> The estimation of the parameters has been conducted using the nonlinear least square method. The parameter values and the corresponding  $t$ -statistics are presented in Table 6 in Appendix 2.

The input to the ROV is the present value of the annual revenues from each of the investment alternatives for a range of mean annual price levels. When calculating the present value of the annual revenues, the price level, presented by  $j$  in Eq. (8), is taken from the range  $[Q_{min}, Q_{max}]$ .

The parameters required for the ROV are summarized in Table 7 in Appendix 2, while the parameters required for the production scheduling are given Table 5 in Appendix 1.

### 3.3. Real Options Valuation

In this analysis, we study three investment alternatives for the aging hydropower facilities. The first alternative is to refurbish the existing plants. The second alternative is to invest in a new

medium size power plant, and the third alternative is to invest in a new large power plant. If the producer chooses to refurbish the existing facilities, the investment opportunity in a new facility is still present. If the producer chooses to construct a new power plant, the power plant will be operating throughout its lifetime.

We analyze this investment opportunity using dynamic programming. The input to the ROV is the present value of annual revenues from the investment alternatives, and the price dynamics of the mean annual electricity price. In the production scheduling, we found the annual revenues for a range of annual mean price levels. We now need to determine at which level we are each year throughout the lifetime,  $L$ , of the investment alternatives. After deciding the mean annual price level, we can find the annual revenues by using the results from the production scheduling.

The mean annual electricity price level, used in the ROV, is determined by the mean-reverting price process in Eq. (11):

$$\Delta Q_y = \xi \cdot (\bar{Q} - Q_y) \cdot \Delta y + \psi \cdot \Delta Z_y, \quad y \in L \quad (11)$$

where  $Q_y$  is the annual electricity price level in year  $y$ ,  $\bar{Q}$  is the level to which the process reverts, and  $\xi$  is the mean-reversion factor. The volatility of the process is given by  $\psi$ , and  $\Delta Z_y$  is the increment of a Wiener process. The time increment is given by  $\Delta y$ . We assume no correlation between the Wiener processes of the  $S_t^j$  price process (Eq. (9)) and the  $Q_y$  price process (Eq. (11)).

The parameters for the mean annual price level model are found by using an AR(1) process as suggested by Dixit and Pindyck (1994). The AR(1) process is presented in Eq. (12):

$$\Delta Q_y = \theta + \omega \cdot Q_{y-1} + e_y, \quad y \in L \quad (12)$$

The parameters for the AR(1) process and their corresponding  $t$ -statistics are summarized in Table 8 in Appendix 2,<sup>11</sup> and are statistically significant at the 94 % confidence level. The parameters for the mean-reverting process are found as suggested by Dixit and Pindyck (1994) and are summarized Table 9 in Appendix 2. The level to which the process reverts,  $\bar{Q}$ , however, is set to an estimated average of long term forward prices at Nord Pool.

In order to compare the investment alternatives, we annualize the construction costs for the two new projects. By doing this, we can compare investment today with investment in the future,

without expanding the planning horizon. The restoration and maintenance costs of the existing facilities stay the same regardless. Capital is not a limiting factor in our model. This implies that the decision made is not restricted by the amount of capital available. If the producer chooses to invest in a new power plant, the new power plant can be taken into use immediately. Also, restoration of the existing plant does not affect the cash flows from the plant, other than the costs included in the maintenance costs.

The main idea of the dynamic programming approach for evaluating real options is to estimate the optimal decisions for each possible price in each time period. We assume that the the producer can only make investment decisions once each year. The annual profits from the investment depend on the power plant in use:

$$NP_{g_y, a_y, Q_y} = I_{g_y, Q_y} - c_{a_y}, \quad y \in L \quad (13)$$

$NP_{g_y, a_y, Q_y}$  are the annual profits and depend on the power plant in use, the action undertaken and the annual price level in year  $y$ . The action undertaken in year  $y$  is denoted by  $a_y$ , and  $c_{a_y}$  are the costs associated with action  $a_y$ .  $I_{g_y, Q_y}$  are the annual revenues from each power plant, given the price level  $Q_y$  and the power plant running,  $g_y$ . The power plant running in year  $y$ ,  $g_y$ , is 0 for the existing power plant, 1 for the medium size power plant and 2 for the large power plant. The power plant running in year  $(y + 1)$  depends on the state of the system and its action in year  $y$ . The set of possible actions when the state is  $g_y$  is denoted by  $A(g_y)$ . In the first year, the set of possible actions contain all the alternatives,  $A(g_1) = \{0, 1, 2\}$ , where 0 means no action (keep existing facilities), 1 stands for investing in the medium size power plant and 2 for investing in the large power plant. The state of the system in year  $(y + 1)$  depends only on the state in year  $y$ , and on the action taken in the same year. As long as no investment in a new power plant has been undertaken, the possible actions remain the same. If an investment in a new power plant is undertaken in year  $y'$ , the possible action changes to  $A(g_y) = 0$  for  $y > y'$ . This is due to the following assumptions: (1) once a new power plant is installed, we cannot go back to use the old facilities, and (2) the medium and large power plant alternatives are mutually exclusive. In order

to maximize total revenues, we solve Eq. (14):

$$\max_{a_y \in A_{g_y}} \left\{ \sum_{y=1}^L e^{-\mu \cdot y} \cdot \mathbb{E}[NP_{g_y, a_y, Q_y}] \right\} \quad (14)$$

where  $NP$  are the annual profits and the discount factor is given by  $\mu$ .<sup>12</sup> The lifetime of the investment alternatives are  $L$  years.

The model is thus formulated as an optimal control problem that can be solved by dynamic programming. Let  $\pi_{y, g_y, Q_y}$  denote the value function. The terminal condition is given in Eq. (15):

$$\pi_{L, g_L, Q_L} = 0 \quad (15)$$

This equation states that the salvage value of each investment alternative is zero. The main problem is then to determine the optimal investment strategies,  $g_y$ . The optimal decision for each year can be found recursively by solving the Bellman equation, which is stated in Eq. (16):

$$\pi_{y, g_y, Q_y} = \max_{a_y \in A_{g_y}} \left\{ NP_{g_y, a_y, Q_y} + e^{-\mu} \cdot \mathbb{E}[\pi_{y+1, g_{y+1}, Q_{y+1}} | Q_y] \right\}, \quad y \in L \quad (16)$$

where  $NP_{g_y, a_y, Q_y}$  is the immediate profit the investor receives the first year after the investment decision, and  $(e^{-\mu} \mathbb{E}[\cdot])$  is the discounted continuation value. The discounted continuation value is obtained by using Monte Carlo simulations, similar to Fuss et al. (2008). The mean annual price level for the next year,  $Q_{y+1}$ , depends on the mean annual price level in year  $y$ . The expected value of  $\pi_{y+1, g_{y+1}, Q_{y+1}}$  can then be estimated as an average over  $\pi_{y+1, g_{y+1}, Q_{y+1}}$ . Each year, the optimal action is the action that maximizes Eq. (16). This procedure yields a matrix containing the optimal action (investment) for each year and each mean annual price level possible.

The results, that is, how the actions will materialize in a certain price path, are extracted by conducting a Monte Carlo simulation "on top of" the matrix generated by the Bellman equation. We do this by simulating the annual price levels using Eq. (11) and match the mean annual price level with the matrix for optimal actions. This procedure yields the frequencies, that is, the number of times, investment in each of the investment alternatives will occur for each year throughout  $L$ .

Summarizing, the analysis is conducted in three steps. First, we obtain the annual revenues from each power plant for a range of mean annual price levels in the production scheduling. Second, we find the optimal action for each year and mean annual electricity price level. The third

and final step is to use Monte Carlo simulations where we simulate price paths that we match with the optimal action for this price level in the given year. We do this a number of times and extract the frequencies with which each investment alternative is chosen.

#### 4. Numerical Results

The alternatives presented to the power producer, who owns power plants that are due for restoration and maintenance, includes keeping the existing facilities, or investing in either a medium size power plant or a large power plant, each with greater discharge capacity and improved efficiency compared to the existing facilities. If the power producer decides to keep the existing facilities, restoration costs have to be paid today, in eight years and in 18 years, as given in Table 1.

The ROV suggests that the optimal strategy for Norsk Hydro ASA is to invest in the medium size power plant today. Figure 1 gives the relation between the change in expected value when investing in a new production facility, compared to keeping the existing facilities, and the number of simulations in the Monte Carlo procedure. As can be seen from the figure, the results converge to the value of MNOK 225 for the medium size power plant and MNOK 215 for the large size power plant. That is, investing in a medium size power plant today has a NPV that is MNOK 225 higher compared to keeping the existing facilities.

— Figure 1 here —

Figure 2 presents the optimal action for all mean annual price levels for the next 19 years. In the dark grey area, the optimal action is to invest in the large size power plant. In the light grey area, the optimal action is to invest in the medium size power plant, while the white area indicates that keeping the existing facilities is the optimal action. The dotted lines represent three different simulated paths for the annual price level.

— Figure 2 here —



As can be seen from the figure, the optimal action in year zero (today) is to invest in either the medium size power plant or the large size power plant, depending on the annual price level today. The existing power plants are due for restoration and maintenance today, in eight years and in eighteen years, and it is optimal to invest in a new power plant at, or before, these times. If the power producer for some reason<sup>13</sup> chooses not to invest in the medium size power plant today, the ROV gives decision support for the upcoming years. As can be seen from Figure 2, the optimal action depends on the development of the annual price level. Table 3 summarizes how often investment in a new facility will occur, given that there is no investment in a new power plant before the given year. As can be seen both from Table 3 and Figure 2, if no investment

Table 3: Number of times when investment in a new facility will occur, given that there is no investment in a new power plant before the given year. The numbers are based on 15 000 simulations.

Year	Optimal action		
	Medium power plant	Large power plant	Existing power plants
1	8.7 %	8.5 %	82.8 %
2	8.8 %	9.3 %	81.9 %
3	10.8 %	8.9 %	80.3 %
4	12.6 %	8.9 %	78.8 %
5	18.6 %	8.9 %	72.5 %
6	27.5 %	8.5 %	64 %
7	49.6 %	9.4 %	41 %
8	90.8 %	9.2 %	0 %

has been undertaken before the time when the next restoration costs occur, investment in a new facility will be undertaken at this time. The investment alternative chosen depends on the annual price level, and at high prices, investment in the large power plant alternative is preferred.

In order to test how sensitive our results are, we investigate how much each of the parameters investment cost, maintenance cost, discount rate and the present value of the annual revenues must change before the frequency of investing in a medium size power plant is less than 50 % during

the next 50 years. The result of this exercise is summarized in Table 4. The changes required

Table 4: Changes required for the frequency of optimal action to invest in a new production facility is less than 50 % of the time.

Parameter	Change required
Discount rate	Increased to 9.9 %
Investment cost of medium and large power plant	Increased by 31 %
Present value of annual revenues	Reduced by 30 %
Long term annual price level	Reduced by 41 %

for the optimal action to change are quite extensive, as can be seen from Table 4. In our results, the medium size power plant dominates the large power plant alternative. Considering the annual revenues from the large power plant however, they need to increase by only 1 % to dominate the medium size power plant. If the long term annual price level is 337 NOK/MWh, the large size power plant alternative dominates the medium size alternative.

If the Nordic electricity market is more closely integrated with the European electricity market, intraday price variation in the Nordic market might increase. Increased intraday price variation gives incentives for increased power production during the day when prices are high, and reduced power production during the night when prices are low. A power plant with higher discharge capacity is better suited for exploiting the new opportunities in the market given by the increased intraday price variation. Next, we therefore study the effect of increased intraday price variation of 30 %.

When increasing the intraday price variation by 30 % in our model, the ROV still suggests investment in a medium size power plant today as the optimal action. The added value from investing in a new facility, compared to keeping the existing facilities, increases to MNOK 242. Considering the annual revenues from the large power plant again, they need to increase by only 0.5 % to dominate the medium size power plant. Investment in a new power plant is more attractive when intraday price variation increases due to the opportunity of using the water when the electricity price is high.

## 5. Conclusion and Further Work

We study the investment decision of a power producer owning facilities that are due for restoration and maintenance. The investment alternatives presented for the power plant owner are restoration of the existing facilities, investment in a medium size power plant and investment in a large power plant. The analysis is conducted by ROV, where a production scheduling model is built to find the annual revenues of the investment alternatives. These serve as input to the ROV.

The ROV suggests that constructing a medium size power plant today is the optimal action. The parameters for discount rate, investment cost, annual revenues, and/or maintenance costs must change substantially in order for the real option model to suggest anything but investment in a medium size power plant as the optimal action. The sensitivity of the result to the revenues from a large power plant, on the other hand, is quite substantial. These only need to increase by 1 % for the large power plant to dominate the medium size power plant. If the investor chooses to refurbish and maintain the existing power plants today, the ROV suggests optimal actions in the upcoming years. For the years between restorations, the optimal action depends on the mean annual electricity price level. For the years when restoration costs occur, the optimal action is to construct a medium size power plant, regardless of the mean annual electricity price level. Hence, investment in a new power plant should be considered in the years before, or in the same year, as the existing facilities are due for restoration and maintenance. If intraday price variation increases, the real option analysis still suggests investment in a medium size power plant as the optimal action, but the sensitivity regarding the annual revenues from the large power plant alternative increases.

Future work will be concerned with the following points: The electricity price process chosen is crucial for estimating future cash flows and present values of the investment alternatives. The price models in this paper can be extended to include more sophisticated multi-factor models to better represent future prices and the uncertainties in the electricity price. Also, the effect of increased discharge capacity, in combination with storage capacity, is the ability to produce more electricity when prices are high and save water when prices are low. Therefore, the electricity price models should also allow for spikes to value the increased capacity of a new power plant

with greater discharge capacity. In addition, the results depend on the investment alternatives chosen in the study. Including more of the investment alternatives will yield a better platform for decision making.

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## Notes

<sup>1</sup>The discharge capacity decides how much water per time unit the power plant can use. Discharge of water results in power production.

<sup>2</sup>Throughout the paper, we will denote these alternatives as the medium and the large power plant alternative.

<sup>3</sup>The main idea of Monte Carlo simulation is to generate possible price paths and hence the development of the investment project's value.

<sup>4</sup>This approximately equals the historical median reservoir level.

<sup>5</sup>That is, time to maturity. The valuation procedure is developed for option pricing.

<sup>6</sup>The investment alternatives are keeping the existing power plants, invest in a medium size power plant and invest in a large power plant.

<sup>7</sup>That is, the historical median reservoir level for day  $t$  of the year.

<sup>8</sup>That is, the historical median inflow for day  $t$  of the year.

<sup>9</sup>The variable costs of each investment alternative depends on the production. This is a simplification made for modeling purposes.

<sup>10</sup>We use spot price data from the period 01.01.1993 to 31.12.2008.

<sup>11</sup>The parameters are estimated based on price information from the Nordic electricity exchange Nord Pool and the data used in the regression analysis spans the period 01.01.1993 to 31.12.2008.

<sup>12</sup>Note that this is the same discount factor as used in the production planning.

<sup>13</sup>This may be due to priorities of the owners and investment strategies.

## Appendix 1

### *Parameters Required for Production Scheduling*

We are not able to disclose neither the discharge capacities considered nor the costs associated with the investment alternatives. Table 5 provides the parameters we can share.

Table 5: Parameters required for the hydropower scheduling.

Parameter	Description	Value
$T$	Length of period under consideration	365 days
$H$	Hours in a day	24
$q$	Length of forward curve considered	180 days <sup>a</sup>
$\Delta t$	Time increment in price process in production scheduling model	1 day

<sup>a</sup>The length of the forward curve is set by the authors.

## Appendix 2

Table 6: Estimated parameters for the electricity price model.

Parameter	$\alpha^*$	$\sigma$	$\lambda^a$	$\kappa$	$\phi$	$\gamma$
Value	8.26	19.2	0.0023	0.05	756	36.6
t-statistic	-	-	-	221	74.9	5.75

<sup>a</sup> $\lambda$  is the daily mean-reversion level equivalent to the annual  $\xi$  in Table 9.

Table 7: Parameters required for the real options valuation.

Parameter	Description	Value
$\Delta y$	Time increment in mean annual price level process	1 year
$\rho$	Discount rate	7 % <sup>a</sup>
$K$	Length of future forward curve	180 days
$T$	Length of period under consideration	365 days
$J$	Mean annual price level, in the range $[Q_{min}, Q_{max}]$	140
$Q_{min}$	Lower bound on price range under consideration	5 NOK/MWh <sup>b</sup>
$Q_{max}$	Upper bound on price range under consideration	700 NOK/MWh
$L$	Period spanned by the lifetime of the power plant alternatives	50 years

<sup>a</sup>In order to be consistent with common practice in electricity companies, we set the discount rate to 7 %.

<sup>b</sup>Note that in constructing the policy of optimal actions, we use a price resolution of 5 NOK/MWh. That is, the annual present value is found at a mean annual electricity price level of 320 NOK/MWh, 325 NOK/MWh, 330 NOK/MWh etc.

Table 8: Estimated parameters for AR(1) process, which serves as the basis for finding the rate of mean-reversion and the volatility of the mean annual price level dynamics.

	$\theta$	$\omega$
Parameter value	128.5	-0.57
t-statistic	-2.26	2.08

Table 9: Estimated parameters for the mean-reverting price process.

$\bar{Q}$	$\xi$	$\psi$
320 <sup>a</sup>	0.728	83.33

<sup>a</sup>This is approximately the average price of yearly forward contracts with delivery in Norway (system price) in 2009.