Game Mining: How to Make Money from those about to Play a Game

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Abstract

It is known that a player in a noncooperative game can benefit by publicly restricting their possible moves before start of play. We show that, more generally, a player may benefit by publicly committing to pay an external party an amount that is contingent on the game's outcome. We explore what happens when external parties – who we call "game miners" – discover this fact and seek to profit from it by entering an outcome-contingent contract with the players. We analyze various bargaining games between miners and players for determining such an outcome-contingent contract. We establish restrictions on the strategic settings in which a game miner can profit, and bounds on the game miner's profit given various structured bargaining games. These bargaining games include playing the players against one another, as well as allowing the players to pay the miner(s) for exclusivity and first-mover advantage. We also establish that when all players can enter contracts with miners, to guarantee the existence of equilibria it is necessary to assume that players can randomize over the contracts they make.

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1 Introduction

1.1 How to Mine a Game

That players can benefit in games by entering contracts that distort payoff functions is well-documented in the economic literature [see Schelling (1956); Sobel (1981); Vickers (1985)]. In this paper we focus on a special case of this phenomenon: A player i may benefit by publicly committing to pay an external party an amount that is contingent on the game's outcome. That benefit to i may or may not be accompanied by a loss to i's opponents. Similarly, i may benefit by publicly paying an external party to make an outcome-contingent payment to i's opponent in the game. In this paper we explore what happens when external parties discover such facts and seek to profit from them.

To ground the discussion, we present two examples.

Example 1. Somali pirates hijack an oil tanker. They can get \$1,000 for it on the black market. To the oil company it is worth \$2,000. The oil company can take the tanker by force at a cost of \$1,700. The pirates have a set business plan, under which they demand a ransom of \$1,500 for tankers of this size. The oil company counters that demand with an offer of \$1,100 and says that if the pirates do not accept it, then rather than pay the \$1,500 they will take the tanker by force. The pirates refuse because it is a non-credible threat (the oil company would lose \$200 by taking the tanker by force, relative to paying the pirates what they ask). The unique Subgame Perfect Equilibrium (SPE) is for the oil company to pay the random. Accordingly, the oil company decides to pay the ransom, and is about to do so.

Game Mining Inc. watches this negotiation. Just before the oil company pays the ransom, they offer the oil company the following contract, making sure the pirates see them do this: "Pay us \$399 up front, and we will keep \$198 no matter what. Make your threat again. If the pirates refuse your threat again and you take the tanker by force, we will give you \$201. Otherwise we will give you nothing."

If the oil company were to accept this contract, then their threat would become credible, and so the pirates would have to yield to it. Accordingly, the oil company accepts this contract because without the contract the unique SPE yields them \$500, while with the contract the unique SPE yields them \$501. The pirates lose \$400 when the oil company makes the contract. Game Mining Inc. makes \$399. ◇

Example 2. There are two cell-phone manufacturers, Anonymous (A) and Brandname (B). They must simultaneously decide how many cell phones to produce. Each firm has two options, high output (H) or low output (L). Anonymous, like its name suggests, is not well known. Therefore, no matter what level of output Brandname produces, Anonymous prefers to produce high output to gain brand recognition. On the other hand, Brandname's choice of output does depend on Anonymous's choice. If Anonymous produces low output, then Brandname prefers to keep prices high by also producing low output. However, if Anonymous produces high output, then Brandname prefers to safeguard its recognized name by also producing high output. The moves and payoffs (in millions of dollars) are summarized by the following matrix.

		B: H	B: L
A	H	$1, .5$	$2, 0$
	L	$0, 0$	$1, 1$

The NE is (H, H), and payoffs are $(1, .5)$.

Game Mining Inc. watches this, and just before Anonymous and Brandname declare their output decisions, Game Mining offers Anonymous the following contract, making sure Brandname sees them do this: "Pay us \$1.5 million right now. Then we will pay you back a certain amount after you and Brandname make your decisions. Here is how much we will pay you, in millions of dollars, for the four possible joint decisions by you and Brandname:"

$$D_A = \begin{bmatrix} 0 & 1.01 \\ 1.5 & 2.0 \end{bmatrix}.$$

So if Anonymous accepts the contract, then the payoff matrix becomes

		B: H	B: L
A	H	$1, .5$	$3.01, 0$
	L	$1.5, 0$	$3, 1$

In the resultant game, Anonymous randomizes, and the unique NE is very close to $(2/3, H)$. For Anonymous, this results in an average payoff of approximately \$1.5 million. This is a \$500,000 improvement over its payoff without the contract. On average, Game Mining Inc. makes approximately \$160,000 for their trouble. Brandname, on average, makes approximately \$333,333. This is a \$166,666 decrease in Brandname's payoff compared to the situation where Anonymous and Game Mining Inc. do not have a contract.

Note that the Coaseian outcome of the game without a game miner would be for Anonymous to pay at least \$500,001 to Brandname for the outcome (H, L) [see Coase (1960)]. So Game Mining Inc. is not merely facilitating the Coaseian outcome. The presence of Game Mining Inc. creates an entirely new strategic setting. $\diamond$

Note that in Ex. 1, GM Inc. could have instead offered a contract that the oil company would accept where the oil company pays only \$100 now, and then pays an outcome-contingent extra amount later. So there may be bargaining over the details of the contract. In that bargaining, in effect GM Inc. is asked by the oil company, "How much do I need to pay you now in exchange for an obligation to pay you more later?" In other words, in situations like in Ex. 1, the only burden on GM Inc. in the bargaining is to maximize the "free money" that the player wants to give it (!). The reason game mining scenarios can be so serendipitous to the game miner(s) is that the player benefits from the obligation to pay the game miner(s). Therefore, the player is willing to pay for that obligation. Similarly a player may be willing to pay the game miner not to sign a contract with the other player for fear of the consequences that the contract would have on the outcome of the game.

In all of these examples, Game Mining Inc. makes considerable profit from recognizing a situation in which one party benefits from an output-contingent contract. But if Game Mining Inc. makes a profit, then other mining firms will surely want to offer the winning party (the oil company or the Anonymous cell-phone company, respectively) their own contract at a lower price. Similarly, the losing party (the pirates or the Brandname cell-phone company, respectively) might want to make contracts with miners.

This raises the issue of what happens if the game miner can offer contracts to both players. As an example, it may be that the miner offers contracts to both players with the following properties. First, both players have a strictly dominant strategy to accept the offered contract. Second, when they both inevitably accept, the outcome is that they are both worse off, and the game miner profits considerably. In fact, the game miner may even promise to pay the players large sums of money if certain outcomes obtain, knowing full well that when all of the players accept the offered contracts, such outcomes will never obtain. (To do this, the game miner effectively creates a prisoner's dilemma among the players for her own benefit.)

There are also considerable timing issues in game mining. When players sequentially sign contracts with game miners, there can be a significant first-mover advantage to the first-signing player. This provides the game miner with yet another opportunity for profit; they can charge the players to move first.

It's also worth noting that in many mining scenarios, *mixed contracts* are needed to guarantee the existence of equilibria. With such contracts, players have uncertainty about their opponent's payoffs. However, unlike in Bayesian Nash equilibrium, in game mining this uncertainty is resolved before the game is played. Its only role is to make the players indifferent among their own contracts. That is, a player will say, "Given that you will select a contract (and therefore an ultimate payoff function) according to that probability distribution, I am indifferent among the following contracts (and therefore ultimate payoff functions). So I will randomize over them with this probability distribution, which in turn makes you indifferent among the contracts in the support of your probability distribution."

1.2 Related Literature

The ideas underlying the game mining concept are implicit in a large body of economic literature. As an illustration, in the model presented in Jackson and Wilkie (2005) (JW), every player specifies outcome-contingent side-payments that they will make after a non-cooperative strategic form game is played and the payoffs are resolved. These side-payments are binding contracts, so the players are *ex ante* determining their preferences over the game's outcomes. In this regard the game that the players actually play is endogenously determined. JW examine whether a mechanism that allows players to make such outcome-contingent side-payments generally results in efficient outcomes and conclude that it does not.

The simplest game-mining scenario, e.g., in Ex.'s 1 and 2, can be viewed as a special case of JW. In this special case, the only outcome-contingent side-payments are between the players and the game miners and the game miners would be indifferent over outcomes of the game if not for the fact that they will be receiving side-payments dependent on those outcomes. Furthermore, the game miners play no part in the game between the players other than to accept contracts for outcome-contingent side-payments and make the contracts public.

In contrast to JW, we do not focus on efficiency issues, and we do not assume that a social planner installs a mechanism for players to make side-payments. Instead, we look at a game without formal mechanisms and ask whether external parties will create contracts for outcome-contingent side-payments in pursuit of profit. In particular, we examine the implications of giving the game miner the power to offer contracts, which will in general increase the game miner's profits. In addition, we relax the assumption in JW that all side-payments are nonnegative. That is, we allow game miners to pay players for certain outcomes. This will be important when examining optimal contracts as well as the extent to which a monopolist game miner can extract profits from players. We also consider how things change when there is more than one miner, when mining contracts are offered in sequence, etc. None of those issues arise in JW.

In another related paper, Renou (2009) analyzes what happens when players are able to embed the original game in a new two-stage game. In the second stage of that new game the players play the original game. However before they do so, in the first stage, the players each simultaneously commit not to play some subset of their possible moves in that game in the second stage. These *commitment games* can be seen as another special case of JW in which (1) player i's side-payments are only contingent on i's action (rather than on the full profile of actions), (2) the side-payments are made to external players and (3) the side-payments are effectively infinite. Renou's analysis does not apply to the full game mining scenario. This is because there are many circumstances in which both the player and the game miner prefer to make contracts that are fully outcome-contingent and that have non-infinite side-payments. One particular example is when the player and game miner find it optimal to agree on a contract that results in an equilibrium where the player uses a mixed strategy with full support (and therefore does not make any commitment in the first stage of Renou's two-stage game).

The idea that there might be pre-game play in which players make choices to affect their own preferences over outcomes is also present in Wolpert et al. (2008). The authors analyze the idea that experimentally observed non-rationality is in fact rational, because by committing to play the game with a non-rational "persona", a player may increase her ultimate payoff. This persona has the same effect as a side-payment or commitment, as it is reflected in a temporary change to the player's utility function. The *persona games* model has been successful in explaining non-rational behavior in non-repeated traveler's dilemma and even in versions of the non-repeated prisoner's dilemma.

There is a subset of the principal-agent literature concerning delegation games that is closely related to game mining. In these models, the principal is able to contract with an agent that will engage in a game with the principal's opponent (or agent of the principal's opponent). One concern of this literature is detailing the optimal contract for a principal [see Vickers (1985); Fershtman (1985); Sklivas (1987)]. Another concern is whether a mechanism that allows specific types of contracts can lead to Pareto efficiency [see Fershtman et al. (1991); Katz (1991)]. Game mining is closely related to a previously unexplored aspect of principal-agent scenarios: the degree to which the agents can profit from delegation contracts.

Finally, our work is related to the general literature on commitment in games because, at its core, game mining is about what happens when players benefit by strategically restricting themselves. One well-studied aspect of commitment is the role of timing. Papers such as Hamilton and Slutsky (1990), van Damme and Hurkens (1996) and Romano and Yildirim (2005) concern endogenous timing and Stackelberg-like commitments. Another area of study is the role of commitment in repeated games. In their study of finitely repeated games, Garcia-Jurado and Gonzalez-Diaz (2006) introduce a weakening of SPE called virtual subgame perfect equilibrium. Kalai et al. (2007) also study commitment in finitely repeated games, but do so in a manner similar to JW. That is, they are concerned with the role of commitment in bringing about efficiency.

1.3 Overview

We start by introducing the game-mining model and notation. We then analyze the ways in which games can be mined.

Next, in Sec. 2, we assume that only one player interacts with one game miner. We derive bounds on the aggregate payoffs that the game miner and player can earn together. We show that they can select a contract to divide these payoffs in any way between them. We also show that outcome-contingent contracts cannot be profitable for both the game miner and contracting player if the other player has a strictly dominant strategy.

In the rest of the paper we consider various market structures, i.e., various structured bargaining games involving the players of the underlying noncooperative game and one or more external firms trying to mine that the players of that underlying game. First, in the next section we analyze a structured bargaining game between two players and a single game miner (i.e. a monopolist firm that handles all outcome-contingent contracts). We begin with the assumption that players offer contracts to the game miner and the game miner must choose either one of the offered contracts or neither contract. We show that the game miner can profit by more than the maximal payoff to either player in the game without contracts. (This is because players can suffer a loss if their opponent outbids them for the right to contract with the game miner.)

Next we relax some of these assumptions. First we allow the game miner to accept both offers if she so chooses. This reduces the game miner's bargaining power, and we find that the game miner can always do at least as well by restricting herself to accept only one contract. Next we discuss the role of timing and first-mover advantage, establishing that the players may be willing to pay for the right to contract first with the game miner, even when their opponent has a strictly dominant strategy.

We end this section by analyzing the case where the miner has the bargaining power, i.e., she is the one making the offers. We show that this allows the game miner to "play the players against one another" and thereby increase her profit. We also derive an upper bound on this profit.

In section five we look at perfect competition and duopoly miner market structures. We develop the notion of best contract response correspondences and mixed contracts to discuss the existence of equilibria when players simultaneously choose contracts. We also detail the way in which a duopolist game miner's profits depend on the game that arises as a result of contracts.

In section six we briefly discuss several new research areas opened by game-mining. These areas include games of more than two players, and risk aversion on the part of the game miner. We also briefly consider unstructured bargaining among the players and the miner to determine the contract. We also touch on an "inverted" version of this topic, where the underlying game is itself unstructured, while the miner(s) negotiate with the player(s) via structured bargaining to determine a contract for that underlying game. We also discuss the idea that one player signs a contract that obligates him to pay the other player outcome-contingent amounts. This obligation may actually help the payer and hurt the payee.

2 Notation

We study a two-player, one-stage simultaneous-move game of complete information. However, we relax the usual assumption that the two players cannot make outcome-contingent contracts (or simply contracts) with players external to the game.

Specify the two-player *pre-contract* game as $\Gamma = (\{A, B\}\{X_A, X_B\}, \{U_A, U_B\})$. U_i is an $|X_A|$-by-$|X_B|$ matrix for which the (m, n) entry gives the payoff to i when A chooses his m'th pure strategy and B chooses his n'th pure strategy. Player i's set of mixed strategies is Δ_i, $i = A, B$, and the set of mixed strategy profiles is $\Delta = \Delta_A \times \Delta_B$. We write all of i's pure and mixed strategies as $|X_i|$-by-one vectors σ_i for which the m'th entry gives the probability that σ_i assigns to playing i's m'th pure strategy. Therefore, we can write player i's expected payoff from $\sigma = (\sigma_A, \sigma_B)$ as

$$\mathbb{E}_\sigma(U_i) = \sigma_A^T U_i \sigma_B$$

where superscript$-T$ indicates matrix transpose.

Player i's best response correspondence is given by $R_i^\Gamma(\cdot) : \Delta_j \to 2^{\Delta_i}$, so that

$$R_A^\Gamma(\sigma_B) = \{\sigma_A \in \Delta_A : \quad \sigma_A^T U_A \sigma_B \geq \sigma_A'^T U_A \sigma_B \quad \forall \quad \sigma_A' \in \Delta_A\}. \tag{1}$$

Therefore, the set of Nash equilibria of game Γ is given by

$$NE(\Gamma) = \{(\sigma_A, \sigma_B) : \sigma_A \in R_A^\Gamma(\sigma_B) \text{ and } \sigma_B \in R_B^\Gamma(\sigma_A)\}.$$

An outcome-contingent contract between player A and an external player C is a matrix D_A that specifies a (possibly negative) transfer from A to C for every outcome of Γ. We refer to player C as the *game miner*, and if players use strategies (σ_A, σ_B), then under contract D_A player A expects to pay $\sigma_A^T D_A \sigma_B$ to player C. Hence, defining $U_A^{D_A} \equiv U_A - D_A$, player A's expected payoff is $\sigma_A^T U_A^{D_A} \sigma_B$. Therefore, we can view D_A as a transformation of Γ. We write the *post-contract* game as $\Gamma(D_A) = \{\{A, B\}, \{X_A, X_B\}, \{U_A^{D_A}, U_B\}\}$. We write the set of possible contracts as $\mathcal{D} = \mathbb{R}^{|X_A|} \times \mathbb{R}^{|X_B|}$. The notation D_0 denotes the null contract, where all entries are zero.

3 Maximal Mining

Before introducing a formal strategic setting for game mining in the next section, we first explore the way that a player A and game miner C can work together to extract gains from Γ. We will first need to know the *aggregate payoffs* from a contract. These are the amounts that the contracting parties can earn in equilibrium and divide among themselves. Suppose A and C are the contracting parties and D_A is their contract. Then the aggregate payoff that is apportioned between A and C is given by the payoff that A gets at a NE in $\Gamma(D_A)$ *before* A pays to C the amount specified in D_A.

Definition 1. The *aggregate payoff set* for A and C from D_A is:

$$M_A(D_A) = \{\sigma_A^T U_A \sigma_B : (\sigma_A, \sigma_B) \in NE(\Gamma(D_A))\}$$

We denote by $M_A^*(D_A)$ the maximum of the aggregate payoff set from D_A. The maximum over all aggregate payoff sets is $\mathcal{M}_A \equiv \max_{D_A \in \mathcal{D}}\{M_A^*(D_A)\}$. It is the maximum that A and C can possibly have to divide among themselves in any NE of any game in which they sign a contract. We refer to this quantity as the *maxagg* (maximum aggregate payoff). The maxagg is the subject of our first result.

Theorem 1.

$$\mathcal{M}_A = \max_{\sigma_A, \sigma_B}\{\sigma_A^T U_A \sigma_B : \sigma_B \in R_B^\Gamma(\sigma_A)\}. \tag{2}$$

Proof. From the definition of maxagg we have:

$$\mathcal{M}_A = \max_{D_A}\{\max\{\sigma_A^T U_A \sigma_B : (\sigma_A, \sigma_B) \in NE(\Gamma(D_A))\}\} \tag{3}$$

Recall that the contract D_A does not affect B's payoffs U_B. This means that $NE(\Gamma(D_A)) = \{\sigma \in \Delta : \sigma_A \in R_A^{\Gamma(D_A)}(\sigma_B) \quad \text{and} \quad \sigma_B \in R_B^\Gamma(\sigma_A)\}$. The trouble is to choose D_A so that A's best response correspondence meets B's best response function at the maximizers that correspond to $\mathcal{M}_A$, (σ_A, σ_B). This problem is solved trivially by choosing D_A such that A is indifferent among all strategy pairs. Then every action of A is a best response to every action of B, including σ_B, which, by assumption is in $R_B^\Gamma(\sigma_A)$.

So equation 3 becomes

$$\mathcal{M}_A = \max_{D_A}\{\max\{\sigma_A^T U_A \sigma_B : \sigma_B \in R_B^\Gamma(\sigma_A)\}\},$$

which is the same as equation 2 because the set $\{\sigma_A^T U_A \sigma_B : \sigma_B \in R_B^\Gamma(\sigma_A)\}$ is independent of D_A. □

So to find the maximum aggregate payoffs for A and C, we simply search B's best response correspondences to all of A's moves for the one giving maximum payoff to A. This allows us to restrict our analysis to the values of U_A along B's best response correspondence. Note how this differs from the NE concept: Here A has the freedom not to be forced to make his best response to B. Only B is being forced to make a best-response.

We do not mean to imply that maxagg is some reasonable refinement of a NE. By definition of maxagg, such a claim would imply that the players coordinate on the NE that most benefits A. Instead, maxagg is nothing more than an upper bound on what is possible for A and C to obtain by making a contract.

In the real world, a game miner would be concerned with downside risk of any given contract. That is, the game miner would be reluctant to sign a contract D_A if the game $\Gamma(D_A)$ has NE in which C loses money. Now consider the Subgame Perfect Nash Equilibrium (SPE) concept applied to the extensive form game in which C first decides whether to accept a given contract D_A, and then the associated underlying game $\Gamma(D_A)$ is played if C accepts the contract. Under that equilibrium concept, when deciding whether to accept D_A, C knows what NE of $\Gamma(D_A)$ would be played if C accepts. Hence, under that concept, C is only concerned with her payoff as prescribed by the strategies of A and B in some single associated NE of $\Gamma(D_A)$ (see analysis below of SPE of game mining). In the real world though, if $\Gamma(D_A)$ contains multiple NE, the a game miner does *not* know with certainty which NE of $\Gamma(D_A)$ would be played if C accepted the contact D_A. Due to this, in the real world, a "conservative" game miner C might choose a contract that maximizes the *minimum* aggregate payoff to be divided between A and C, to minimize how bad the situation for C could be a "worst case" NE of $\Gamma(D_A)$.

We write this minimum of the aggregate payoff set $M_A(D_A)$ as $\underline{M}_A(D_A)$. Maximizing $\underline{M}_A(D_A)$ over all contracts D_A, we get the maximum minimum aggregate payoff $\underline{\mathcal{M}}_A$, called the *maxminagg*:

$$\underline{\mathcal{M}}_A = \max_{D_A} \min_{\sigma} \{\sigma_A^T U_A \sigma_B : \sigma \in NE(\Gamma(D_A))\}.$$

Trivially, $\underline{\mathcal{M}}_A \geq \underline{M}_A(D_0)$. Comparing maxminagg with maxagg, we also know that $\underline{\mathcal{M}}_A \leq \mathcal{M}_A$. And when there exists a contract D_A such that $M_A(D_A) = \{\mathcal{M}_A\}$, we have that $\underline{\mathcal{M}}_A = \mathcal{M}_A$. That is, if there exists a contract D_A such that the only NE of $\Gamma(D_A)$ yields the maxagg to A and C, then the maxagg and maxminagg are the same.

The next example uses maxagg and maxminagg to illustrate a distinction between general game mining and commitment games.[1] In this example, maxagg is associated with a NE σ_A that is a mixed strategy with full support. However in this example, a commitment by A to play (or not play) certain pure strategies will never allow A and C to achieve the maxagg; communication games are a restricted subclass of game mining.

On the other hand, contracts that achieve the maxagg also give rise to a NE with an aggregate payoff lower than maxagg. This mean that we would not expect a conservative miner to choose that maxagg. To address this. we show that there are contracts that yield a unique NE for which the aggregate payoff is arbitrarily close to the maxagg. Because the NE is unique, this aggregate payoff is the maxminagg of concern to a conservative miner. Hence even a conservative miner would want a contract that causes σ_A to be fully mixed, so that A would not make any commitment.

Example 3. Consider again the game Γ presented in Ex. 2 above:

	l	r
t	$1, .5$	$2, 0$
b	$0, 0$	$1, 1$

where A is the row player. Write $p \equiv \sigma_A(t)$. Then B's best response correspondence is

$$R_B^\Gamma(p) = \begin{cases} l & \text{if } p > \frac{2}{3} \\ r & \text{if } p < \frac{2}{3} \\ q \in [0,1] & \text{if } p = \frac{2}{3}. \end{cases}$$

If A chooses $p < \frac{2}{3}$, then B will choose r, and the payoff to A will be $2p + 1(1-p) = 1+p$. Likewise, if A chooses $p > \frac{2}{3}$, then B will choose l, and the payoff will be p. If A chooses $p = \frac{2}{3}$ then B chooses any combination of l and r yielding payoffs to A between $\frac{2}{3}$ and $\frac{5}{3}$. Therefore, the maximum payoff for A along R_B^Γ is when $\sigma_A = (p, 1-p) = (\frac{2}{3}, \frac{1}{3})$ and

[1] Commitment games, in which a player commits before the game not to play certain pure strategies, are a special case of game mining, where only contracts that result in one or more strictly dominated moves by A are allowed.

$\sigma_B = (0,1)$:

$$\mathcal{M}_A = \sigma_A^T U_A \sigma_B = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{5}{3}.$$

In other words, the maxagg payoff for A and C is achieved by a mixed strategy. The problem is that there is no contract D_A such that $\Gamma(D_A)$ has a unique NE with maxagg the payoff to A. Moreover, for those D_A's such there is a NE of $\Gamma(D_A)$ with A's payoff equaling 5/3, there are other NE with A's payoff less than 1. 1 is also A's payoff in every NE of Γ, so it would appear that A has no incentive to form a contract with Game Mining. However, there are contracts that produce a unique NE under which the aggregate payoff is arbitrarily close to the maxagg. An example of a contract that gets arbitrarily close to the maxagg is

$$D_A = \begin{bmatrix} 1.5 & .5-\varepsilon \\ 0 & -.5 \end{bmatrix}.$$

With this contract A's payoffs are now given by

$$U_A^{D_A} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1.5 & .5-\varepsilon \\ 0 & -.5 \end{bmatrix} = \begin{bmatrix} -.5 & 1.5+\varepsilon \\ 0 & 1.5 \end{bmatrix}$$

There is a unique NE for all $\varepsilon > 0$. As ε approaches zero, that NE approaches $(2/3, r)$, so the maxminagg approaches 5/3. Finally, we note that the Coaseian outcome is for A to pay B to play r for a price of $.5+\varepsilon'$ for some small ε'. This differs from the outcome under game mining. This illustrates that game mining is not just a way to facilitate Coaseian outcomes when players cannot directly cooperate. Rather the presence of a game miner transforms the strategic setting in a way that cooperation cannot.

means that this Coaseian transaction cannot take place. Therefore, in this example, the game miner provides a service that the players could not provide themselves. $\diamond$

The maxminagg is a reasonable concept, especially when the game miner cannot know for certain which of multiple NE will be played by the players. However, in this paper we will rely exclusively on the SPE concept applied to extensive form games where the contracts are chosen before the underlying game is played. This concept requires that when choosing a contract the game miner knows which of multiple NE will be adopted by the players in the following, underlying game. Accordingly, SPE says that the miner can perfectly forecast which NE of the underlying game gets played. Whether or not such perfect forecasting is realistic — and it arguably is not —- it is demanded by the SPE concept. Therefore, from now on, with few exceptions we depart from the maxminagg concept, leaving for future work an analysis of game mining that incorporates maxminagg more fully.

The preceding results consider what A and C can achieve together. However, throughout the remainder of the paper, we will be interested in the profits that the game miner is able to extract from the strategic situation given by Γ. Therefore, we now address the way in which A and C are able to divide aggregate payoffs. The following result says that they are able to incorporate any division of the aggregate payoffs directly into the contract without affecting the best response correspondence of A or B. (Whether A and C would accept such a division is a different issue.)

Theorem 2. *For any $a \in \mathbb{R}$ and $\sigma^* = (\sigma_A^*, \sigma_B^*)$ such that $\sigma_B^* \in R_B^\Gamma(\sigma_A^*)$, there exists a contract D_A^* such that:*

1. $(\sigma_A^*, \sigma_B^*) \in NE(\Gamma(D_A^*))$, *and*
2. $\sigma_A^{*T} U_A^{D_A^*} \sigma_B^* = a$.

Proof. By theorem 1 there exists a contract D_A such that $(\sigma_A^*, \sigma_B^*) \in NE(\Gamma(D_A))$. Let $\underline{1}$ stand for a matrix all of whose entries are 1. So $(\sigma_A^*)^T \underline{1} \sigma_B^* = 1$. Therefore there is a scalar $x \in \mathbb{R}$ such that

$$\left(\sigma_A^{*T}\left(U_A - D_A - x\underline{1}\right)\sigma_B^*\right)\underline{1} = a\underline{1}.$$

This gives us $D_A^* \equiv D_A + x\underline{1}$. Since $U_A^{D_A^*} = U_A^{D_A} - x\underline{1}$, $(\sigma_A^*, \sigma_B^*) \in NE(\Gamma(D_A^*))$. □

Theorem 2 says that the aggregate payoffs that A and C get by mining are not affected by a restriction on the way in which A and C divide those payoffs. So how $\mathcal{M}_A$ is divided between A and C in equilibrium will be determined by strategic rather than technical considerations. This will be convenient when we introduce a formal strategic environment in the next section.

In some settings, there is no contract such that A and C can both benefit in any NE of the post-contract game. One example where this is always the case is when B has a strictly dominant strategy. The intuition is that A's contract with C will never change B's payoffs. Therefore, B will always play his dominant strategy, no matter what the contract. Therefore there is nothing that a contract can do to help A. This intuition is formalized in the following result.

Theorem 3. *If B has a strictly dominant strategy, then there is no contract D_A such that A and C both strictly benefit in any NE of $\Gamma(D_A)$.*

Proof. Suppose $\tilde{x}_B$ is a strictly dominant strategy for player B. Then $\tilde{x}_B = R_B^{\Gamma(D_A)}(\sigma_A) = R_B^{\Gamma}(\sigma_A)$ for all $\sigma_A \in \Delta_A$. Hence the set of NE in Γ is

$$\{\sigma^* \in \Delta : \sigma_A^* = \underset{\sigma_A}{\operatorname{argmax}}\, \sigma_A^T U_A \tilde{x}_B \text{ and } \sigma_B^* = \tilde{x}_B\}$$

and the set of NE in $\Gamma(D_A)$ is

$$\{\tilde{\sigma} \in \Delta : \tilde{\sigma}_A = \underset{\sigma_A}{\operatorname{argmax}}\, \sigma_A^T U_A^{D_A} \tilde{x}_B \text{ and } \tilde{\sigma}_B = \tilde{x}_B\}.$$

If C benefits by entering contract D_A, then $\tilde{\sigma}_A^T D_A \tilde{x}_B > 0$.

But this means that $\tilde{\sigma}_A^T U_A \tilde{x}_B > \tilde{\sigma}_A^T U_A^{D_A} \tilde{x}_B$. Since $\sigma_A^{*T} U_A \tilde{x}_B \geq \tilde{\sigma}_A^T U_A \tilde{x}_B$, by combining we have $\sigma_A^{*T} U_A \tilde{x}_B > \tilde{\sigma}_A^T U_A^{D_A} \tilde{x}_B$ for all $\sigma^*, \tilde{\sigma}$. Accordingly, A will not benefit by signing D_A. □

Theorem 3 puts a restriction on the set of games Γ in which A will benefit from the services of a game miner. However, as is shown in later sections, there are ample opportunities for a game miner to profit from situations in which player B has a strictly dominant strategy. In general, such a situation requires that B also has an opportunity to make a payoff-contingent contract with the miner.

The next result establishes limits on game mining of interest to a "conservative" miner, when one player has a weakly dominant strategy.

Theorem 4. *If B has a weakly dominant strategy, then there is no contract D_A such that both A and C strictly benefit in every NE of $\Gamma(D_A)$ compared to not signing any contract.*

Proof. By contradiction. Suppose x_B^* is weakly dominant and D_A is a contract such that both A and C benefit in every NE of $\Gamma(D_A)$. There is a NE (x_A^*, x_B^*) of Γ. For every $x_A \in X_A$ we have that

$$x_A^T U_A x_B^* \leq x_A^* U_A x_B^*.$$

If A is better off in every NE of $\Gamma(D_A)$, then for $x_A \in R_A^{\Gamma(D_A)}(x_B^*)$

$$x_A U_A^{D_A} x_B^* > x_A^* U_A x_B^*$$

which implies

$$x_A D_A x_B^* < 0$$

which in turn contradicts the fact that C is better off. □

Theorem 4 tells us that A and C cannot eliminate the risk of loss in a NE of $\Gamma(D_A)$ if B has a weakly dominant strategy in such a game. If A and C are both conservative and require that they gain in every NE of $\Gamma(D_A)$, then no contract will be made between them.

4 Monopolist Miner

4.1 C Accepts One Contract

Consider a situation in which players A and B encounter each other in a simultaneous move game with perfect information, $\Gamma = \langle\{A, B\}\{X_A, X_B\}, \{U_A, U_B\}\rangle$. There is only one external party, C, that is willing to accept publicly observable outcome contingent contracts. Before A and B play Γ, they simultaneously offer contracts to C. These contracts are called D_A and D_B respectively.

After observing D_A and D_B, C chooses either D_A, D_B or D_0 (the null contract). Players A and B observe this contract and recognize its legally binding nature. A and B then engage in the simultaneous move game $\Gamma(D_i)$. $\Gamma(D_i)$ is the *post-contract subgame.*

Formally, this is a perfect information extensive form game with three stages:

Stage One: Players A and B simultaneously offer contracts D_A and D_B to C.

Stage Two: C chooses D_A, D_B or the null contract D_0.

Stage Three: Players A and B play $\Gamma(D_j)$.

A strategy S_i for $i = A, B$ in the extensive form game is a pair $S_i = (D_i, s_i)$. The first component, $D_i \in \mathcal{D}$, is the offer that i makes to C in the first stage. The second component is a function from the space of all possible contracts, $\mathcal{D}$, to the space of probability distributions over actions x and y, i.e. $s_i : \mathcal{D} \mapsto \Delta_i$. In other words, s_i gives i's strategy for every possible post-contract subgame. The profile of strategies of player A and player B are written as S_{-C} where $s_{-C} = (s_A, s_B)$.

In stage two, C selects an element of the choice set $\mathcal{D}_C = \{D_A, D_B, D_0\}$. S_C is the function that takes as input the history (D_A, D_B) and returns an element of $\mathcal{D}_C$ as C's choice. Note that $\mathcal{D}_C$ is specified by $s_{-C} = (s_A, s_B)$.

Given a full strategy profile (S_A, S_B, S_C), C's payoffs are

$$U_C(S_A, S_B, S_C) = s_A(S_C(D_A, D_B))^T S_C(D_A, D_B) s_B(S_C(D_A, D_B)).$$

$S_C(D_A, D_B)$ is C's stage two choice given the stage one actions (D_A, D_B), and $s_A(S_C(D_A, D_B))$ is A's stage three reaction to that choice. For $i = A, B$, the payoffs are

$$U_i(S_A, S_B, S_C) = s_A(S_C(D_A, D_B))^T U_i^{S_C(D_A,D_B)} s_B(S_C(D_A, D_B)),$$

where $U_i^{S_C(D_A,D_B)}$ gives i's payoffs in the post-contract game $\Gamma(S_C(D_A, D_B))$. As shorthand, we represent this extensive form game as

$$\Gamma_C = \langle \{A, B, C\}, \Gamma, \{\mathcal{S}_i\}_{i=A}^B, \mathcal{S}_C, U_C \rangle.$$

Definition 2. A *subgame perfect equilibrium (SPE)* of Γ_C is a strategy profile $S = (S_A, S_B, S_C)$ such that:

1. $(s_A(D), s_B(D)) \in NE(\Gamma(D))$ for all contracts $D \in \mathbb{R}^2 \times \mathbb{R}^2$.

2. S_C is optimal given s_{-C} for all pairs (D_A, D_B), i.e.

$$s_A(S_C(D_A, D_B))^T U_C(S_C(D_A, D_B)) s_B(S_C(D_A, D_B)) \geq \\ s_A(S'_C(D_A, D_B))^T U_C(S'_C(D_A, D_B)) s_B(S'_C(D_A, D_B))$$

   for all S'_C.

3. D_A is optimal given S_C, s_A and S_B, i.e.

$$s_A(S_C(D_A, D_B))^T U_A(S_C(D_A, D_B)) s_B(S_C(D_A, D_B)) \geq ... \\ s_A(S_C(D'_A, D_B))^T U_A(S_C(D'_A, D_B)) s_B(S_C(D'_A, D_B))$$

   for all D'_A (*mutatis mutandi* for B).

We turn our attention to finding the maximum amount that can be mined from Γ. To do so, we introduce a concept that is related to the aggregate payoff set from definition 1:

Definition 3. The *aggregate payoff function* for A and C from D_A is:

$$m_A(D_A|s_{-C}) = s_A(D_A)U_A(D_A)s_B(D_A).$$

The aggregate payoff function differs from the aggregate payoff set. Whereas the aggregate payoff set includes payoffs for all NE of $\Gamma(D_A)$, the aggregate payoff function simply returns the sum of A and C's payoffs when s_{-C} is played in $\Gamma(D_A)$. For example, if s_{-C} selects a NE of the post-contract subgame $\Gamma(D_A)$, then the aggregate payoff function $m_A(D_A|s_{-C})$ selects one element from the aggregate payoff set $M_A(D_A)$. We denote by $\hat{D}_i$ a contract that maximizes A's aggregate payoff function. We denote by $\mathcal{D}_A$ the set of all such maximizers.

In an SPE, C will choose whichever contract yields her the highest payoff as determined by (s_A, s_B). Given that, i's contract should offer more to C than is offered by j's contract only if j's contract offers less than $m_i(\hat{D}_i|s_{-C}) - s_A(D_j)^T U_i s_B(D_j)$. The most that i will ever be willing to offer C is therefore determined by finding the contract of j that results in the smallest payoff for i, called $\underline{D}_j$. Following this logic reveals that, loosely speaking, C will contract with the player that has the greatest willingness to pay. In other words, there will not be an SPE in which C accepts a contract from one player while the other has a greater willingness to pay. From the players' willingness to pay, we get the maximum SPE payment to C in the following theorem.

Theorem 5. *The maximum SPE payment to C is*

$$\bar{U}_C = \max_i \{\mathcal{M}_i - \min_{\sigma_A,\sigma_B} \{\sigma_A^T U_i \sigma_B : \sigma_i \in R_i^\Gamma(\sigma_{-i})\}\}$$

Proof. First, let $\underline{D}_j(s_{-C}) = \text{argmin}_{D_j}\, s_A(D_j)^T U_i s_B(D_j)$. That is, $\underline{D}_j$ is the contract that minimizes i's payoff given s_{-C}. The proof follows from the strategic considerations of the players. Either (1) neither player pays C, or (2) player i pays C. In the case of (2), i will offer C no more than necessary, which is the minimum increment above what C would get by accepting j's offer, D_j. Player i will only be willing to pay this amount if it is less than the amount that she gains by changing the game from $\Gamma(D_j)$ to $\Gamma(D_i)$,

$$\delta_i(D_i, D_j|s_{-C}) = m_i(D_i|s_{-C}) - m_i(D_j|s_{-C}).$$

This is the difference between i's payoffs in $\Gamma(D_j)$ and $\Gamma(D_i)$. This difference is maximized by choosing D_j to minimize i's payoff in $\Gamma(D_j)$ and choosing D_i to maximize i's payoff in $\Gamma(D_i)$. Given s_{-C}, these arguments are $\underline{D}_j$ and $\hat{D}_i$ respectively. So we have that the maximum i will pay in an SPE of Γ_C given s_{-C} is $\delta_i(\hat{D}_i, \underline{D}_j | s_{-C})$.

Maximizing i's payoff over all functions s_{-C} and contracts D_i we get the maxagg $\mathcal{M}_i$. Minimizing i's payoff over all functions s_{-C} we get $\min_{s_{-C}} \underline{D}_j(s_{-C})$ where the minimizer $s^i_{-C} = \operatorname{argmin}_{s_{-C}} s_A(\underline{D}_j(s_{-C}))^T U_i s_B(\underline{D}_j(s_{-C}))$ yields $\underline{D}_j(s^i_{-C})$. However, we know that since s^i_{-C} is part of an SPE, that $s^i_{-C}(\underline{D}_j)$ is a NE of $\Gamma(\underline{D}_j)$. Therefore, $s^i_i(\underline{D}_j) \in R^\Gamma_i(s^i_j(\underline{D}_j))$. In other words, the only requirement in constructing s^i_{-C} is that i is always playing a best response. This is because D_j can be such that $s^i_j(\underline{D}_j)$ is a best response to i. Therefore,

$$s^i_A(\underline{D}_j)^T U_i s^i_B(\underline{D}_j) = \min_{\sigma_i, \sigma_j} \{\sigma_A^T U_i \sigma_B : \sigma_i \in R^\Gamma_i(\sigma_j)\}.$$

Putting this together with i's maxagg and choosing i we get the result. □

Theorem 5 gives an upper bound on the amount that the monopolist game miner can extract from the game. This amount is bounded by the players' payoff functions. So a monopolist game miner cannot, in this situation, extract arbitrary profits. However, the SPE concept here allows for some behavior that is unreasonable from a trembling hand perspective. For example, in order for C to achieve her maximum payment, it may be necessary for A to offer just under B's maximum willingness to pay, $\mathcal{M}_B - \min_{\sigma_A, \sigma_B} \{\sigma_A^T U_B \sigma_B : \sigma_B \in R^\Gamma_B(\sigma_A)\}$, to C for the contract $\underline{D}_A$. This is despite the fact that A might *prefer* the outcome under $\hat{D}_B$ to the outcome under $\underline{D}_A$. That is, A offers quite a bit of money to C for a deal she wants not to take effect. A's offer is only a best response to B's slightly greater offer because C will choose B's contract, so that this unreasonable offer by A, $\underline{D}_A$, will never be accepted by C. But if C trembled and chose $\underline{D}_A$, the outcome could be disastrous for A. In short, for some Γ_C, there exist SPE in which C achieves her maximum payoff only if one of the players acts in a manner that seems unreasonable.

This suggests that a more reasonable set of outcomes is one in which players will only offer contracts $\hat{D}_i$ $(i = A, B)$ such that $m_i(D_i | s_{-C})$ is maximized. That is, players will maximize the aggregate payoff function regardless of the way in which that money is divided. They would do this because at least one $\hat{D}_i \in \hat{\mathcal{D}}_i$ is a best response to every

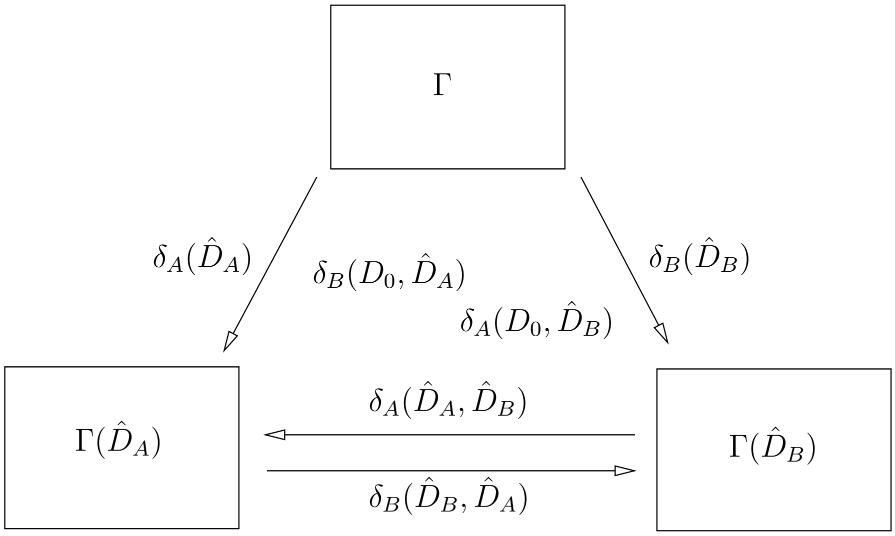
D_j and s_{-C}. The following flow diagram illustrates how these strategies would translate into the monopolist miner's payoffs. Let

$$\delta_i(D_i|s_{-C}) = m_i(D_i|s_{-C}) - m_i(D_0|s_{-C})$$

be the change in i's payoff by going from Γ to $\Gamma(D_i)$. Similarly define

$$\delta_i(D_i, D_j|s_{-C}) = m_i(D_i|s_{-C}) - m_i(D_j|s_{-C})$$

as the change in i's payoff by going from $\Gamma(D_j)$ to $\Gamma(D_i)$. Positive quantities represent proportional movements in the direction of the associated arrow.



This shows how the players' willingness to pay affects the outcome as long as the players are not acting in the unreasonable manner described above (i.e. offering more than their willingness to pay). So if $\delta_B(\hat{D}_B) > 0$, we know that B is willing to pay to change the game from Γ to $\Gamma(\hat{D}_B)$. Hence, Γ will not be the post-contract subgame in an SPE. Next, if $\delta_B(\hat{D}_B, \hat{D}_A) > \delta_A(\hat{D}_A, \hat{D}_B) > 0$, then B is willing to pay more to change the game from $\Gamma(\hat{D}_A)$ to $\Gamma(\hat{D}_B)$ than A is willing to pay to change it from $\Gamma(\hat{D}_B)$ to $\Gamma(\hat{D}_A)$. B will offer C a contract such that C's payment is just greater than A's is willing to pay to change the game from $\Gamma(\hat{D}_B)$ to $\Gamma(\hat{D}_A)$. In some games, the restriction that players choose only contracts from $\mathcal{D}_i$ $(i = A, B)$ will decrease C's maximium SPE payoff.

Another implication of theorem 5 is that C's payoff can be greater than $\max\{\mathcal{M}_A, \mathcal{M}_B\}$. In other words, the winning contract may pay C more than the maxagg for either player. The following example illustrates how a monopolist miner can make both players worse off than they were without the opportunity to mine. We demonstrate that this is the case even when players are restricted to choosing contracts that maximize aggregate payoff functions.

Example 4. Consider the game Γ:

	x	y	z
x	$-1, 2$	$-1, 3$	$0, 0$
y	$-1, -1$	$0, 0$	$3, -1$
z	$-1, -1$	$-1, -1$	$2, -1$

where A is the row player. There is one pure NE (y, y) of Γ. Calculating the A's aggregate payoff function for $\hat{D}_A$, $\hat{D}_B$ and $\hat{D}_0$ as well as A's willingness to pay, we get:

$$m_A(\hat{D}_A|s_{-C}) = 2, \quad m_A(\hat{D}_B|s_{-C}) = -1 \quad \text{and} \quad m_A(D_0|s_{-C}) = 0$$
$$\Rightarrow \quad \delta_A(\hat{D}_A, \hat{D}_B) = 3 \quad \text{and} \quad \delta_A(\hat{D}_A) = 2$$

By symmetry the quantities for B are the same as the corresponding quantities of A.

The fact that $\delta_i(\hat{D}_i) = 2 > 0$ for $i = A, B$ means that both players are willing to pay to change the game from Γ to $\Gamma(\hat{D}_i)$, so D_0 will not be the outcome. Next, because $\delta_i(\hat{D}_i, \hat{D}_j) = 3 > 0$ for $i = A, B$, we know that C will get a payoff of $\delta_i(\hat{D}_i, \hat{D}_j) = 3$ in equilibrium. This payoff is greater than $\mathcal{M}_i - m_A(D_0|s_{-C}) = 2$. In other words, if i's contract is accepted, then the contract between i and C pays C more than the increase in aggregate payoffs $\mathcal{M}_i - m_A(D_0|s_{-C}) = 2$. The reason is that i is paying to avoid having $\Gamma(\hat{D}_j)$ become the equilibrium game.

We also observe that $s_A(\hat{D}_i)^T U_i^{D_i} s_B(\hat{D}_i) = -1 < 0 = m_A(D_0|s_{-C})$. This says that i gets less by having the equilibrium contract with C than i would get if neither player had the opportunity to offer contracts. For j, the player that does not win the equilibrium contract, the SPE payoff is also -1. Therefore, the winner and the loser are both made worse off by the opportunity to mine $\diamond$

4.2 C Accepts Both Contracts

We now relax the assumption that C must choose between D_A and D_B. After all, if C is a true monopolist game miner and there are gains to be made by simultaneously contracting with both parties, then C will certainly want to do this.

The strategies in Γ_C must be modified to accommodate this new possibility. First, a strategy S_C for C selects an element of C's choice set $\mathcal{D}_C$ after the history $(D_A, D_B) \in \mathcal{D}^2$. Since C can now choose to accept both contracts if she wishes, the choice set $\mathcal{D}_C$ is given by:

$$\mathcal{D}_C = \{(D_A, D_B), (D_A, D_0), (D_0, D_B), (D_0, D_0)\}.$$

This induces the game $\Gamma(D_i, D_j)$ where D_i $(i = A, 0)$ is the contract between A and C and D_j $(j = B, 0)$ is the contract between B and C. Therefore, the game $\Gamma(D_i, D_j)$ is one in which A's preferences are $U_A^{D_i}$ and B's preferences are $U_B^{D_j}$. This means that the stage three strategy profile $s_{-C} = (s_A, s_B)$ is defined on $\mathcal{D}^2$ so that $s_i : (\mathcal{D})^2 \mapsto \Delta_i$ $(i = A, B)$. In other words, players select a strategy for every possible post-contract subgame of the form $\Gamma(D_i, D_j)$.

We refer to the stage three game that is played in equilibrium of Γ_C as the *equilibrium game*. If C accepts only D_A, then the equilibrium game is $\Gamma(D_A)$. If C accepts D_A and D_B, then the equilibrium game is $\Gamma(D_A, D_B)$ and so on. The game $\Gamma(D_A, D_B)$ was not possible when C could only accept a single contract. However, when C can accept both contracts it is possible.

This raises the issue of determining how A chooses D_A given that B is choosing D_B. Given a function $s_{-C} = (s_A, s_B)$, B's contract D_B and C's decision S_C, A chooses a contract in order to maximize his payoff.

$$\max_{D_A} \quad s_A(S_C(D_A, D_B))^T U_A^{D_A} s_B(S_C(D_A, D_B)) \tag{4}$$

This gives rise to a best response correspondence for A.

Definition 4. Player A's *best contract response correspondence given* s_{-C} is a set valued function $\Phi_A(^T|s_{-C}) : \mathcal{D} \mapsto 2^{\mathcal{D}}$ that gives all of the contracts D_A that maximize 4 when B makes contract D_B given s_{-C}.

By requiring that s_{-C} selects a NE of every post-contract subgame, we guarantee that $s_A(S_C(D_A, D_B))$ is a best response to $s_A(S_C(D_A, D_B))$ and vice versa. When s_{-C} meets this requirement, the best contract response correspondence amounts to a best response correspondence for the extensive form game. The following result uses the concept of a best contract response correspondence to categorize a monopolist game miner's payoffs when able to accept both contracts.

Theorem 6. *The monopolist game miner's equilibrium payoffs under the restriction that C can only accept one contract are always as good and sometimes better than her payoffs without that restriction.*

Proof. Suppose $\delta_A(\hat{D}_A) \geq 0$ and/or $\delta_B(\hat{D}_B) \geq 0$ and $\delta_A(\hat{D}_A, \hat{D}_B) \geq \delta_B(\hat{D}_B, \hat{D}_A) \geq 0$, then with the restriction, C gets $\delta_B(\hat{D}_B, \hat{D}_A)$. However, without the restriction, there is the possibility that for some D_A and D_B, A and B both prefer $\Gamma(D_A, D_B)$ to $\Gamma(\hat{D}_A)$ and $\Gamma(\hat{D}_B)$. If $D_A \in \Phi_A(D_B)$ and $D_B \in \Phi_B(D_A)$ given s_{-C}, then this will be an equilibrium. When the equilibrium game is $\Gamma(D_A, D_B)$, neither player is paying for exclusivity, so C's payoff is zero instead of $\delta_B(D_B, D_A)$.

Further, the threat of an outcome $\Gamma(D_A, D_B)$ can never induce i to pay more than $\delta_j(\hat{D}_j, \hat{D}_i)$ for exclusivity. This is because $\delta_j(\hat{D}_j, \hat{D}_i)$ is the value for j of going from $\Gamma(\hat{D}_A)$ to $\Gamma(\hat{D}_B)$. Given that i pays for exclusivity, there is no payment that j can make to change the game from $\Gamma(\hat{D}_A)$ to $\Gamma(D_A, D_B)$ because i's contract is contingent on exclusivity. □

Theorem 6 says that a monopolist game miner cannot be made worse off by restricting herself to accept a single contract. The reason is that when C does not restrict herself, then she does not give up D_A in order to accept D_B. Therefore, if C accepts D_B, then her best response is to accept any contract D_A for which her payoff under $\Gamma(D_A, D_B)$ is at least her payoff under $\Gamma(D_B)$. Knowing this, A will choose D_A such that C's payoff under $\Gamma(D_A, D_B)$ is exactly what it is under $\Gamma(D_B)$. The same holds for B. Therefore, C is made worse-off by the ability to make contracts with both players. Put differently, the threat of an equilibrium game $\Gamma(D_A, D_B)$ never induces the players to pay more, and it is sometimes better for the players.

The above suggests that the one-contract restriction might be the result of payoff maximizing behavior. That is, C's payoff in equilibrium of the one-contract game might be equivalent to a payment not to contract with the other player. Hence, the restricted game is equivalent to a game in which A and B submit two-element stage-one offers, (D_i, z_i), where D_i is the matrix of strategy-contingent transfers and z_i is a payment not to make a contract with j. If $z_i = 0$, then i places no exclusivity restriction on C's acceptance of D_i. Therefore, C's payoff from accepting A's contract is $z_A + s_A(D_A, D_0)^T D_A s_B(D_A, D_0)$. If $z_A = z_B = 0$ then C's payoff from accepting both contracts is $s_A(D_A, D_B)^T (D_A + D_B) s_B(D_A, D_B)$.

4.3 Sequential Contracts

We now examine the role of timing on game mining outcomes. The game is exactly as previously described, except that A first selects a contract to be observed by B before B selects a contract. In this setting we find that A may have a first-mover advantage and also that contracts are not equivalent to the pre-commitments of Renou (2009). Both points are demonstrated in the following example.

Example 5. Consider the game Γ where A is the row player. The unique NE of this

	x	y	z
x	2,5	0,0	5,4
y	1,3	1,2	2,0
z	0,3	0,1	2,0

game is (x, x). Note that x is a strictly dominant strategy for B. By theorem 3, there is no contract D_A such that A gets a better payoff in a NE of $\Gamma(D_A)$ than in a NE of Γ. Despite this fact, there is a contract D_A such that $s_A(D_A, D_B)^T U_A^{D_A} s_B(D_A, D_B) > s_A(D_0)^T U_A s_B(D_0)$ where $D_B \in \Phi_B(D_A)$. In other words, there is a contract D_A such that when B chooses his best contract response to D_A, A gets a higher payoff in $\Gamma(D_A, \Phi_B(D_A))$ than in any NE of Γ. For example, if A signs a contract with C to pay C 2 whenever the outcome is (x, x), then the unique NE of $\Gamma(D_A)$ is (y, x). The resultant game, $\Gamma(D_A)$ is given by:

	x	y	z
x	0,5	0,0	5,4
y	1,3	1,2	2,0
z	0,3	0,1	2,0

Then B's best response is a contract D_B that promises to pay C 4 if the outcome is (y, x) and 3 if the outcome is (y, y). This will make (x, z) the unique NE of $\Gamma(D_A, D_B)$.

	x	y	z
x	0,5	0,0	5,4
y	1,-1	1,-1	2,0
z	0,3	0,1	2,0

The final outcome is best for A. Note that if B was the first to select a contract, then B would choose D_0 to which A's best response is D_0.

This example draws a sharp distinction between game mining and pre-commitments to play or not play certain strategies. Suppose A instead selected a contract that made x a never-best-response. Then B's best response is D_0, and the outcome is (y, x), which is worse for A. A does not want to commit to not playing x because (x, z) is the ultimate goal. He rather wants to commit to (x, x) not being the outcome, so that B will commit to (y, x) and (y, y) not being the outcome $\diamond$

Exploiting contract timing is yet another way that game miners game miners can extract profits from players even when players are making the offers. Since A has a first-mover advantage, and B has a second-mover disadvantage, both are willing to pay to move first. Suppose A recognizes this advantage before B and approaches C with his desired contract D_A. C could potentially put A on hold and notify B to start a bidding war over the first-mover advantage. The first-mover advantage is worth more to A than it is to B, five versus one, so A would end up paying B's maximum willingness to pay. This is despite the fact that players are offering the contracts and C is free to accept both.

4.4 C Makes Offers

Until this point we have assumed a particular bargaining structure in which A and B make take-it-or-leave-it offers to C. This implies that C's only bargaining power is in rejecting contracts that result in negative payoffs. Suppose now that we change the game so that C makes publicly observable offers to A and B, and then A and B simultaneously accept or reject the offers C has made. So A and B will now accept any contract that does not make them worse off, given the other's choice. This clearly places more power in the hands of C.

To accommodate the new structure of the game, we alter the definition of strategies. Now C's stage one strategy is $s_C \in \mathcal{D}^2$. A and B have binary stage two strategies $s_i^2 : \mathcal{D}^2 \mapsto \{\text{accept}, \text{reject}\}$ and stage-three strategies $s_i^3 : \mathcal{D}^2 \mapsto \Delta_i$ $(i = A, B)$, which we sometimes shorten to be s_{-C}^2 and s_{-C}^3. So C selects a contract for each player, s_C. Then each player chooses to accept or reject the contract they are offered, s_i^2 for $i = A, B$. Then, the players play the resultant game, s_i^3 for $i = A, B$.

We want to characterize C's payoffs in an SPE. To do so, consider the following devious plan where C can sometimes create a high-order Prisoner's Dilemma (PD) between A and B. This is illustrated in the example below.

Example 6. Consider the game Γ where A is the row player.

	w	x	y	z
w	4,4	0,0	0,0	0,5
x	0,0	1,2	0,0	0,0
y	0,0	0,0	2,1	0,0
z	5,0	0,0	0,0	3,3

The NE of Γ is (z, z) at which both players get a payoff of 3. C's plan is the following: choose D_A and D_B so that both players have a strictly dominant strategy to accept, given s^3_{-C} (in this example the caveat "given s^3_{-C}" won't come into play because we make sure that Γ, $\Gamma(D_A)$, $\Gamma(D_B)$, and $\Gamma(D_A, D_B)$ all have unique NE) Suppose C sets D_A so that $\Gamma(D_A)$ is:

	w	x	y	z
w	1.01,4	1,0	1,0	0,5
x	0,0	0,2	0,0	.5,0
y	0,0	0,0	5,1	0,0
z	0,0	0,0	0,0	0,3

The NE of $\Gamma(D_A)$ is (y, y) where payoffs are $(5, 1)$. So when $D_B = D_0$ (i.e. B rejects the contract offered to him), A has the incentive to accept D_A because A's payoff will increase from 3 under Γ to 5 under $\Gamma(D_A)$. Note that A's payoff for (y, y) in Γ was only 2, so this means that D_A stipulates that C *pays* A when (y, y) occurs. Then C sets D_B so that $\Gamma(D_B)$ is:

	w	x	y	z
w	4,1.01	0,0	0,0	0,0
x	0,1	1,5	0,0	0,0
y	0,1	0,0	2,0	0,0
z	5,0	0,0	0,.5	3,0

The NE of $\Gamma(D_B)$ is (x, x) where payoffs are $(1, 5)$. So B has the incentive to accept D_B given that $D_A = D_0$ (i.e. A rejects the contract offered to him) because B's payoff will increase from 3 to 5. Note that B's payoff for (x, x) in Γ was only 2, so this means that D_B stipulates that C *pays* B when (x, x) occurs.

If both of the players accept their respective contracgts, we get $\Gamma(D_A, D_B)$:

The NE of $\Gamma(D_A, D_B)$ is (w, w) where payoffs are $(1.01, 1.01)$. So A has the incentive to accept D_A given that B accepts because A's payoff will increase from 1 under $\Gamma(D_B)$ to

	w	x	y	z
w	1.01,1.01	1,0	1,0	0,0
x	0,1	0,5	0,0	.5,0
y	0,1	0,0	5,0	0,0
z	5,0	0,0	0,.5	0,0

1.01 under $\Gamma(D_A, D_B)$. Similarly B has the incentive to accept D_B given that A accepts because B's payoff will increase from 1 under $\Gamma(D_A)$ to 1.01 under $\Gamma(D_A, D_B)$[2].

This is very similar to a PD game because both players have a strictly dominant strategy to accept the contract that C offers. This moves them from a situation where the only NE gives them $(3, 3)$ to a situation where the only NE gives them $(1.01, 1.01)$. By playing A against B the game miner C gets $2(4 - 1.01) = 5.98$ in the unique SPE of Γ_C. The situation can be visualized alternatively as the following PD game where A is the row player and B is the column player:

	accept	*reject*
accept	$1.01, 1.01$	$5, 1$
reject	$1, 5$	$3, 3$

◇

The example above shows that C can potentially do better for herself by selecting contracts that both A and B will accept than by contracting with one player exclusively. The intuition for why this is possible is that C relies on the fact that $\Gamma(D_A)$ and $\Gamma(D_B)$ will never obtain in equilibrium. Therefore, C is free to offer contracts D_A and D_B such that she loses money in the NE of $\Gamma(D_A)$ and $\Gamma(D_B)$. This allows her the flexibility to make sure the NE of $\Gamma(D_A, D_B)$ is in her favor. Note that this is true even in many games where players have strictly dominant strategies in Γ as well as constant-sum games. The following example depicts a game where both players have dominant strategies.

Example 7. Consider the game Γ where A is the row player and both players have a

[2]Note that the NE of $\Gamma(D_A, D_B)$, (w, w), is not in $R_i^\Gamma(\sigma_j)$ for any $\sigma_j \in \Delta_j$ for $i = A, B$, $j = A, B$ and $j \neq i$

strictly dominant strategy to choose z.

	x	y	z
x	$5,5$	$1,5$	$1,6$
y	$5,1$	$1,1$	$2,2$
z	$6,1$	$2,2$	$3,3$

.

The NE is (z, z), and both players get a payoff of 3. Then if C offers the following contract to A

$$D_A = \begin{bmatrix} 2.99 & -2 & 0 \\ 3 & 0 & -3 \\ 4 & 0 & 0 \end{bmatrix}$$

the NE of $\Gamma(D_A)$ is (y, z). A's payoff in $\Gamma(D_A)$ is $2 - (-3) = 5$, so A would accept D_A, getting 5 rather than 3. B's payoff in $\Gamma(D_A)$ is 2. If C offers $D_B = D_A^T$ to B, then the equilibrium of $\Gamma(D_A, D_B)$ is (x, x). The payoff to B is $5 - 2.99 = 2.01$. Therefore, B would accept D_B given that A accepts D_A because he will get 2.01 rather than 2. By the symmetry of Γ, D_A and D_B, we know that each player i has a dominant strategy to accept D_i regardless of whether $j \neq i$ accepts or rejects his offered contract $\diamond$

The following theorem provides an upper bound on the game miner's payoff when she extracts profits according to this scheme.

Theorem 7. *The maximum that a monopolist game miner can profit by offering contracts* (D_A, D_B) *such that A and B have weakly dominant strategies to accept is*

$$\max_{x_A, x_B} x_A^T(U_A + U_B)x_B - \min_{x'_A, x'_B} \{x'^T_A U_B x'_B : x'_B \in R_B^\Gamma(x'_A)\} - \min_{x''_A, x''_B} \{x''^T_A U_B x''_B : x''_A \in R_A^\Gamma(x''_B)\}$$

Proof. The first term is the maximum amount that A and B can earn in any outcome of Γ. The second term is the minimum amount that C can force B to get by designing D_A such that x'_A is a best response to x'_B. This is because C is constrained so that x'_B must lie on B's best response correspondence for Γ. The third term is the equivalent of the second term for A rather than B.

Suppose C earns more than this maximum, then

$$s_A(D_A, D_B)^T(D_A + D_B)s_B(D_A, D_B) > \max_{x_A, x_B} x_A^T(U_A + U_B)x_B ... \tag{5}$$
$$- \min_{x'_A, x'_B} \{x'^T_A U_B x'_B : x'_B \in R^\Gamma_B(x'_A)\} - \min_{x''_A, x''_B} \{x''^T_A U_B x''_B : x''_A \in R^\Gamma_A(x''_B)\}$$

but because A and B each have a weakly dominant strategy to accept, we know that

$$s_A(D_A, D_B)^T(U_A - D_A)s_B(D_A, D_B) \geq s_A(D_0, D_B)^T U_A s_B(D_0, D_B)$$

and

$$s_A(D_A, D_B)^T(U_B - D_B)s_B(D_A, D_B) \geq s_A(D_A, D_0)^T U_B s_B(D_A, D_0).$$

We can rewrite these as

$$s_A(D_A, D_B)^T U_A s_B(D_A, D_B) - s_A(D_0, D_B)^T U_A s_B(D_0, D_B) \geq s_A(D_A, D_B)^T D_A s_B(D_A, D_B)$$

and

$$s_A(D_A, D_B)^T U_B s_B(D_A, D_B) - s_A(D_A, D_0)^T U_B s_B(D_A, D_0) \geq s_A(D_A, D_B)^T D_B s_B(D_A, D_B)$$

which imply that

$$s_A(D_A, D_B)^T(D_A + D_B)s_B(D_A, D_B) \leq s_A(D_A, D_B)^T(U_A + U_B)s_B(D_A, D_B) ...$$
$$- s_A(D_0, D_B)^T U_A s_B(D_0, D_B) - s_A(D_A, D_0)^T U_B s_B(D_A, D_0)$$

The left-hand-side is C's profit, and the maximum of the right-hand-side is given by the right-hand-side of inequality 5. Therefore, we have a contradiction. □

This result is important because it says that the game miner cannot make an arbitrary profit from the players by giving each a dominant strategy to accept her offer. Therefore, the monopolist can always do only limited damage.

5 Multiple Miners

Since a monopolist game miner can extract profits from the interaction between A and B, it is reasonable to think that other game miners will enter this market. In addition, the opportunity to sign game mining contracts can benefit players. So if one player finds a game miner to contract with, then the other player is likely to seek out his own game miner. For these reasons, we introduce multiple game miners to examine the role of competition on outcomes. We call this extensive game Γ_N.

5.1 Perfect Competition

We begin with the assumption that there are a very large number of game miners available for contracting. This assumption means that i will never pay a game miner not to contract with j, because j can readily find another external party to contract with if a contract is desirable. In this section we also adhere to the assumption that players make contract offers to game miners, and that these contracts are made simultaneously.

Naturally, in this competitive environment, game miners will earn the marginal cost of their service, which is assumed to be zero. Therefore, we worry primarily about characterizing the *equilibrium game* $\Gamma(D_A, D_B)$ (i.e. the post-contract subgame that is played in equilibrium). To do so, we first consider the problem that A faces when choosing a contract D_A given that B chooses D_B. It is similar to the problem from equation 4.

$$\begin{aligned} &\max_{D_A} s_A(D_A, D_B)^T U_A^{D_A} s_B(D_A, D_B) \\ &\text{s.t.} \quad s_A(D_A, D_B)^T D_A s_B(D_A, D_B) \geq 0 \end{aligned} \tag{6}$$

B has an analogous problem. So A and B are simultaneously choosing payoffs $U_A^{D_A}$ and $U_B^{D_B}$, and each pair of utilities is mapped to strategies by s_{-C}.

Before our next result, we introduce some notation. Let $M_i(D_i|D_j)$ be the aggregate payoff set for i given that j's contract is D_j. Let $M_i^*(D_i|D_j)$ be the maximum element of $M_i(D_i|D_j)$. And let $\mathcal{M}_i(D_j) = \max_{D_i}\{M_i^*(D_i|D_j)\}$ be the maximum of all the aggregate payoff sets for all contracts $D_i \in \mathcal{D}$ given D_j.

Theorem 8. *If Γ' is an equilibrium game of Γ_N for all s_{-C}, then i gets $\mathcal{M}_i(D_j)$ in all NE of Γ'.*

Proof. We can add a conditional argument to i's aggregate payoff function to indicate that j's contract is given. The equilibrium payoffs $\Gamma(D_A, D_B)$ are then characterized by

$$m_A(D_A|D_B, s_{-C}) = m_A(\hat{D}_A|D_B, s_{-C})$$
$$m_B(D_B|D_A, s_{-C}) = m_B(\hat{D}_B|D_A, s_{-C})$$

Otherwise $m_i(D_i|D_j, s_{-C}) < \mathcal{M}_i(D_j)$, and for some $s'_{-C} \neq s_{-C}$, there is an alternative D'_i such that $m_i(D'_i|D_j, s'_{-C}) = \mathcal{M}_i(D_j)$. □

One example of a game Γ' that is an equilibrium game for all s_{-C} is one in which players have strictly dominant strategies. Theorem 3 says that if B has a strictly dominant strategy, then A cannot benefit from a game mining contract without hurting the game miner. So if $U_A^{D_A}$ and $U_B^{D_B}$ both exhibit a strictly dominant strategy, then neither player will have the opportunity to make a beneficial contract with an outside party. In other words, such a D_A and D_B would be an equilibrium game $\Gamma(D_A, D_B)$ because neither player will want to deviate. Of course not all games with strictly dominant strategies are equilibrium games. It must be that D_A does not require A to pay the game miner when choosing his best response to B's strictly dominant strategy under D_B (likewise for D_B). In other words, if (x_A, x_B) is the NE under $\Gamma(D_A, D_B)$ where $U_A^{D_A}$ and $U_B^{D_B}$ exhibit strictly dominant strategies, then we must have that $x_A^T D_A x_B = 0$. Otherwise, A could do better against D_B by choosing a contract where $x_A^T D_A x_B = 0$ holds.

However, if U_A and U_B in Γ both exhibit strictly dominant strategies it still is entirely possible to have an equilibrium game $\Gamma(D_A, D_B)$ where players sign something other than the null contract (i.e. $D_A \neq D_0 \neq D_B$). That is, if B selects D_B so that $U_B^{D_B}$ does not exhibit a strictly dominant strategy, then A might have a best response $D_A \in \Phi_A(D_B|s_{-C})$ so that $U_A^{D_A}$ does not exhibit a strictly dominant strategy (or exhibits a different strictly dominant strategy). If $D_B \in \Phi_B(D_A|s_{-C})$, then we have an equilibrium game $\Gamma(D_A, D_B)$ that is different from Γ. This is despite the fact that both U_A and U_B exhibit strictly dominant strategies.

Another important issue that arises is whether A and B must be allowed to randomly select contracts to offer their respective game miners in order to guarantee the existence of an equilibrium.

Definition 5. A *mixed contract* for player i is a mapping $\Sigma_A : \mathcal{D} \mapsto [0, 1]$ such that $\int_{\mathcal{D}} \Sigma_i(D_i) dD_i = 1$.

In an SPE where players' strategies employ (proper) mixed contracts, each player will be indifferent among the contracts that he plays with positive probability given the other player's mixed contract strategy and s_{-C}. In particular, if A is randomizing between contracts D_A and D'_A and B's mixed contract is Σ_B, then

$$\int_{\mathcal{D}} \Sigma_B(D_B)\left[s_A(D_A, D_B)^T U_A^{D_A} s_B(D_A, D_B)\right] dD_B = ...$$
$$\int_{\mathcal{D}} \Sigma_B(D_B)\left[s_A(D'_A, D_B)^T U_A^{D'_A} s_B(D'_A, D_B)\right] dD_B$$

Using this description of mixed contract equilibria, we explore the existence of SPE of Γ_N.

Theorem 9. *There exist games Γ_N and functions $s_{-C} = (s_A, s_B)$, such that s_{-C} is part of an SPE if and only if A and B use mixed contracts.*

Proof. Let $\Phi = (\Phi_A, \Phi_B)$. By Kakutani's fixed point theorem, we know that if $\mathcal{D}^2$ non-empty, compact and convex, and $\Phi : \mathcal{D}^2 \mapsto 2^{\mathcal{D}^2}$ is a set valued function on $\mathcal{D}^2$ with a closed graph and the property that $\Phi(D_A, D_B)$ is nonempty and convex for all $(D_A, D_B) \in \mathcal{D}^2$, then Φ has a fixed point. However, if Φ only includes pure best responses, then we lose convexity, i.e. for distinct D_A and $D'_A \in \Phi_I(D_B)$, we don't generally have that $D''_A = \alpha D_A + (1-\alpha)D'_A \in \Phi_I(D_B)$. This is true because $s_A(D_A, D_B)^T D_A s_B(D_A, D_B) = 0$ and $s_A(D'_A, D_B)^T D'_A s_B(D'_A, d_B) = 0$ implies $s_A(D''_A, D_B)^T D''_A s_B(D''_A, d_B) = 0$ only if $s_i(D_A, D_B) = s_i(D'_A, D_B) = s_i(D''_A, D_A)$, which does not generally hold.

The fact that s_{-C} must be a NE of $\Gamma(D_A, D_B)$ restricts the set of fixed points (D_A, D_B) rather than restricting the set of correspondences Φ for Γ_N. Further, there are no restrictions on U_A or U_B. Therefore, for every D_A and D_B, we can find matrices U_A and U_B such that $D_A \in \Phi_A(D_B)$. Hence, we invoke Kakutani's theorem to say that the convexity of Φ is necessary for the general existence of fixed points of $\Phi(^T|s_{-C})$. Hence mixed strategies are necessary for fixed points of $\Phi(^T|s_{-C})$, for some Γ_N and s_{-C}. □

When A and B use mixed contracts, each has uncertainty about which game $\Gamma(D_A, D_B)$ will ultimately be played. This is reminiscent of what occurs in a Bayesian Nash equilibrium. Yet there is an important difference. The difference is that A and B's realized contracts are announced publicly after the game miners accept them in stage two. So when they play $\Gamma(D_A, D_B)$, they both know which game they are playing. Hence, there is uncertainty about contracts, but only in stage one when they are selecting contracts, not when they are playing the post-contract subgame in stage three. So game mining introduces the possibility for a new kind of uncertainty about other players' payoffs. This uncertainty affects how one chooses one's own payoffs but not how one plays the game once payoffs are chosen.

5.2 Duopoly

We now alter the assumption that there are many game miners. Instead suppose that there are only two. This makes it possible for i to pay for exclusive contracts with both game miners in order to keep j from obtaining a contract of his own. Like before, we assume players A and B make simultaneous take-it-or-leave-it contract offers. However, this time they are making offers to game miners C and E. Each player makes one contract offer to each of the game miners, i.e. $D_i = (D_i^C, D_i^E) \in \mathcal{D}^2$ $(i = A, B)$. Each game miner cannot accept more than one contract offer.

Here, because there are only two game miners it might be reasonable for one player to pay for exclusivity with both game miners. In other words, even though A only cares about $D_A = D_A^C + D_A^E$, or his "aggregate contract," he may be willing to split the contract between C and E so that C and E both prefer A's offered contract over any contract B is willing to offer. Whether or not A is willing to pay each duopolist enough to exclude B will depend on A's relative gains to exclusivity versus A's outcome when he does not exclude B. We will use the construction, i *blocks* j *from* Γ', to refer to a situation in which i pays for exclusivity in a way that prevents j from getting a contract that brings about Γ'.

We examine this blocking behavior below for each possible equilibrium game Γ, $\Gamma(D_i)$ for $i = A, B$ and $\Gamma(D_A, D_B)$. First assume the following notation:

$$\delta_A((\Phi_A, D_B), D_B|s_{-C}) = \Phi_A(D_B|s_{-C}) - m_A(D_B|s_{-C}).$$

So $\delta_A((\Phi_A, D_B), D_B|s_{-C})$ is the amount that A will pay in order to change the game from $\Gamma(D_B)$ to $\Gamma(\Phi_A(D_B), D_B)$. Like before, we omit the conditional argument s_{-C} to simplify the notation.

Given that the equilibrium game is Γ, there are two possibilities

1. neither player blocks the other

2. i blocks j from $\Gamma(D_j)$

The first situation results in duopolist payoffs of $(0, 0)$ because neither player is paying for an exclusive contract. This might occur if there is no opportunity for game mining (i.e. $\hat{D}_i = D_0$ for $i = A, B$). The second situation results in duopolist payoffs of $(\delta_j(\hat{D}_j), \delta_j(\hat{D}_j))$. That is, i pays each duopolist a quantity that just exceeds j's willingness to pay to change the game from Γ to $\Gamma(\hat{D}_j)$. This might occur if i prefers Γ to $\Gamma(\hat{D}_j)$ at least twice as much as j prefers $\Gamma(\hat{D}_j)$ to Γ.

Given that the equilibrium game is $\Gamma(D_i)$, there are three possibilities

1. neither player blocks the other
2. i blocks j from $\Gamma(\hat{D}_j)$
3. i blocks j from $\Gamma(D_A^k, D_B^m)$ for $k, m \in \{C, E\}$, $k \neq m$.

The first situation again results in duopolist payoffs of $(0, 0)$. The second situation results in duopolist payoffs of $(\alpha\delta_j(\hat{D}_j), (1-\alpha)\delta_j(\hat{D}_j))$ where $\alpha \in [0, 1]$. The reason is that j will have to get exclusive contracts with both game miners if he is to induce $\Gamma(\hat{D}_j)$. Therefore, if i wants to block j, i needs the sum of his payments to C and E to be greater than j's willingness to pay to change the game from $\Gamma(D_i)$ to $\Gamma(\hat{D}_j)$. The third situation is a bit more complicated. It is a setting where i has contracted for exclusivity with both game miners, offering contracts D_i^C and D_i^E. If j was to pay one of the game miners, say E, more than i offered E, then the resultant game would be $\Gamma(D_i^C, D_j^E)$. If i is to block j from doing this, then player i has to offer E at least $\delta_B((D_i^C, \phi_j), D_i)$. This quantity is j's willingness to pay in order to change the game from $\Gamma(D_i)$ to $\Gamma(D_i^C, \phi_j(D_i^C))$. Naturally, i must pay C the quantity $\delta_B((D_i^E, \phi_j), D_i)$ to block j from changing the game to $\Gamma(D_i^E, \phi_j(D_i^E))$.

Finally we explore what happens when the equilibrium game is $\Gamma(D_A, D_B)$. Here there are two possibilities

1. neither player blocks the other
2. i blocks j from $\Gamma(\hat{D}_j)$
3. i blocks j from $\Gamma(\hat{D}_j)$ and j blocks i from $\Gamma(\hat{D}_i)$.

The first situation again results in duopolist payoffs of $(0, 0)$. The second situation results in duopolist payoffs of $(\delta_j(\hat{D}_j), 0)$. Here i pays C the amount of j's willingness to pay, $\delta_j(\hat{D}_j)$, in order to keep j from getting an exclusive contract with C. However, j pays nothing to block i from getting an exclusive contract with E. This might be because j prefers $\Gamma(\hat{D}_j)$ to $\Gamma(D_A, D_B)$ while i prefers $\Gamma(D_A, D_B)$ to $\Gamma(\hat{D}_i)$. The third situation is an extension of the second. Here both players are blocking the other from getting an exclusive contract. Player i will make offers to both C and E. These offers will be such that, given j's offers, C accepts and E rejects. E instead accepts j's offer. Player i pays C enough in $\Gamma(D_A, D_B)$ so that it is not worthwhile for j to offer C a greater amount, causing C to accept and inducing the game $\Gamma(\hat{D}_j)$. The amount that j pays E in $\Gamma(D_A, D_B)$ is enough so that it is not worthwhile for i to offer E a greater amount, causing E to accept and inducing the game $\Gamma(\hat{D}_i)$.

Of course there are many more SPE of the game mining duopoly than are characterized by the above blocking behavior. In fact, many of the SPE can involve combinations of the above blocking behavior, where A and B are both blocking each other from various games. Other SPE may rely on the type of "unreasonable" behavior discussed at the end of section 4.1. For example, i offers contracts that pay more than i's maximum willingness to pay, only because j is willing to offer yet more.

6 Discussion

As mentioned in the introduction, the game mining analysis opens up several new research areas. One natural extension is to consider game mining situations in which there are more than two players. With multiple players simultaneously choosing contracts, the blocking and exclusivity concerns we address above are likely to become much more complicated. An open question is whether the game miner has more or less opportunity to profit as the number of players grows.

Another research area concerns the difficulty of modeling the game miner's uncertainty over which outcome will obtain in the game $\Gamma(D_A)$ after signing the contract D_A with player A. In an SPE analysis, the game miner's decision to sign or not sign D_A occurs through the process of backward induction. That is, the SPE approach assumes the game miner somehow knows the outcome of $\Gamma(D_A)$, $s_{-C}(D_A)$, before she signs the contract with A. In the real world, the game miner would likely not be so certain about future events. In fact, the game miner's beliefs would likely assign nonzero probability to the occurrence of non-equilibrium outcomes of $\Gamma(D_A)$. Hence, rather than an equilibrium-approach, it may be valuable to adopt a statistical approach to game mining, such as the Predictive

Game Theory (PGT) models described in Wolpert (2008).

Yet another research question is whether replacing the structured bargaining between players and game miners with unstructured bargaining will change the profitability of game mining. One might also consider the reverse a situation in which the stage game Γ is a game of unstructured bargaining between A and B, but the negotiations between players and game miners follow structured bargaining. Here, we would have that players sign contracts through structured bargaining in an attempt to gain an advantage in the unstructured bargaining that follows. The question is how would players design contracts to distort their utility possibilities set in such a way that benefits them in the ensuing unstructured bargaining. An interesting technical issue arises in deciding whether to allow players to sign contracts that lead to utility possibility sets that are nonconvex. If so, then a solution concept other than the Nash Bargaining Solution is needed [see Nash (1950); Kalai and Smorodinsky (1975)].

The following phenomenon, which is closely related to game mining, also deserves a rigorous analysis. That is, player A may want to form a contract with the game miner, C, to pay player B an outcome-contingent amount. This is much like the setup of JW, where players make outcome-contingent side-payments to each other.

However, there are some situations in which a contract between A and C to pay B outcome-contingent amounts can actually hurt B. In such instances, C will be faced with this peculiar question from B: "How much do I have to pay you never to give me money?" Here A benefits from an obligation to pay B, while B would be hurt by such a payment. Hence, unlike JW, where the question is whether side-payments can bring about the efficient outcome, we ask if this is an opportunity for the game miner to profit. That is, can the game miner leverage A's desire to have the contract against B's desire not to have the contract in order to extract profit?

To gain intuition for why such a strange arrangement might be beneficial for A and detrimental for B, consider the following two-stage game of complete and perfect information. A, as the row player, moves first, and then B moves (i.e., a Stackelberg game). The payoff bimatrix is

	l	r
t	$-1, 1$	$10, 0$
b	$0, 3$	$0, 3$

(7)

If A were to move T(op), then B would move L(eft), and A would get -1. If instead A

were to move D(own), then A would get 0. So A moves D, gets 0, and B gets 3.

If instead the bimatrix were

	l	r
t	$-1, 1$	$10, 2$
b	$0, 3$	$0, 3$

(8)

then if A were to move T, B would move R(ight), and A would get 10. So now the equilibrium is (T, R), with A getting 10.

So if we start with the bimatrix 7, but A can get the game miner, C, to pay B an outcome-contingent amount given by a contract D_B,

	l	r
t	$0, 0$	$0, 2$
b	$0, 0$	$0, 0$

then we wind up with the bimatrix 8, where A benefits by $10 - 0 = 10$, and B loses by $2 - 3 = -1$.

So for example, if C is a bank to which B owes a lot of money, and A can pay the bank so that B's debt is reduced by D_B, it benefits A. As an example, a payment to C of 3 would work.

As an example, say that B owes a loan shark a lot of money, and this loan shark would have no qualms about who pays him in B's name. Then the miner can pay off (output-contingent amounts of) B's debt. More generally, even if we simply had Game Mining Inc. send a bar of gold to B, wouldn't it be the case that B cannot pretend that he will throw the gold away? After all, that would be a non-credible threat; the original game is over, and B gets a bar of gold in the mail, so he cannot credibly claim that he would throw it away. Even if B could somehow protect himself pro-actively against gold delivered by the Post Office, Game Mining Inc. could simply commit to sending *something* to B, at some unspecified time within the next year, with the needed value.

Naturally, whenever B is hurt by such a contract between A and C, B might want to form a contract with another external party, saying "if I get a gift from Game Mining Inc., you take the exact same amount from my bank account." But this is exactly the idea behind our arguments above. The "external party" that B uses can potentially demand a profit for their services.

All of this raises a crucial question: Why aren't real game mining firms wreaking havoc on real markets? Game mining appears to be very possible according to basic game theory, so if it is not generally possible in the real world, what assumptions are being violated?

There are many potential answers to this question. One tempting explanation is that the payoff structure of most real world games makes them unable to be mined. This seems a strange assertion because, as shown, even games in which both players have strictly dominant strategies can be mined for profit depending on the market structure. Other potential answers are that the calculations are too difficult in practice, that the time frame in real world games is too short, that game mining could be considered illegal, that imperfect information limits game mining opportunity, or that some kind of strategic uncertainty makes game mining impractical. These explanations should be explored in future work because they might shed light on the way game theory applies to real world strategic settings.

There are other questions to explore. For instance, does game mining imply that certain games should never exist because the minute they appear they will be mined into an alternate game? In some sense this gives rise to a meta-game whereby a player that finds himself involved in an easily mineable game might assume that the game will be mined and therefore conclude that he is actually playing a different game. Or, in a game with multiple equilibria, one equilibrium might make the game susceptible to mining by an outside party that ultimately makes both parties worse off (like what happens when a monopolist makes offers that give players strictly dominant strategies to accept). Therefore, that susceptible equilibrium might become less likely than an equilibrium that is more robust. In this way game mining introduces an equilibrium refinement: choose the equilibrium that makes game mining least profitable.

These questions and others are not only interesting for their ability to shed light on game mining concepts, but also more generally for their ability to shed light on the noncooperative theory.

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