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Can the New Keynesian Phillips Curve Explain Inflation Gap Persistence?

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Abstract

Whelan (2007) found that the generalized Calvo-sticky-price model fails to replicate a typical feature of the empirical reduced-form Phillips curve — the positive dependence of inflation on its own lags. In this paper, I show that it is the 4-period-Taylor-contract hazard function he chose that gives rise to this result. In contrast, an empirically-based aggregate price reset hazard function can generate simulated data that are consistent with inflation gap persistence found in US CPI data. I conclude that a non-constant price reset hazard plays a crucial role for generating realistic inflation dynamics.

Contents

1 Introduction

The nature of inflation persistence is a complex phenomenon, because it is influenced by many aspects of the economy. For example, Cogley and Sbordone (2008) argue that it is important to distinguish between the inflation trend persistence and the inflation gap persistence, since they arise from different economic sources. While dynamics of trend inflation results largely from shifts in the long-run target of the monetary policy rule, inflation gap persistence is influenced primarily by the pricing behavior at the firm's level and the price aggregation mechanism.

The focus of this paper is the dynamics of the inflation gap — the difference between the actual inflation and trend inflation. I first document some stylized facts distinguishing inflation gap persistence from inflation level persistence. I find evidence from the U.S. CPI data that the inflation gap constitutes a large part of inflation persistence. Second, I investigate whether the stylized fact can be explained by the theoretical New Keynesian Phillips curve (hereafter: NKPC), and further identify which mechanism of the model is most important for generating inflation gap persistence.

The purely forward-looking NKPC is often criticized for generating too little inflation persistence (See: e.g. Fuhrer and Moore, 1995). To overcome this weakness, various generalizations of the basic NKPC have been developed in the literature, they offer, however, different interpretations on the nature of inflation gap persistence. The hybrid NKPC incorporates lagged inflation into the standard NKPC motivated by the positive backward-dependence of inflation in the empirical reduced-form Phillips curve[1]. According to this line of literature, inflation gap persistence should be interpreted as 'intrinsic' (Fuhrer, 2006) and the dependency between current and lagged inflation should be treated as a fixed primitive relationship, which is independent of monetary policy. By contrast, the more micro-founded general-pricing-hazard models[2] shed new lights on the important role played by inertia of expectations in generating inflation gap persistence. According to this class of models, inflation gap persistence is inherited. It comes from the additional moving-average terms of real driving forces through the lagged expectations. More importantly, since the coefficient on lagged inflation depends on the whole model including the specification of monetary policy, it implies that the hybrid NKPC should be subject to the Lucas critique (Lucas, 1972), and thereby can not be used in the monetary policy analysis.

Despite the theoretical solidity of the general-pricing-hazard model, Whelan (2007) rejected it empirically. He showed that the general-pricing-hazard model fails to replicate the positive backward-dependence of inflation typically found in the empirical reduced-form Phillips curve. In partial equilibrium,Whelan proved that the coefficient on the lagged inflation is always negative, regardless of the form of the price reset hazard function. Furthermore, he used a simple DSGE model to show that, even in general equilibrium, this model still generates negative coefficients on inflation lags.

In this paper, I first replicate his findings and check their robustness to alternative setups of the model. In particular, I test the result using different price reset hazard functions, aggregate demand conditions and monetary policy rules. I find that it is the 4-period-Taylor-contract hazard function used in the Whelan's setup that gave rise to the result. Under an empirically based pricing hazard function estimated by Yao (2010), the simulated data accounts quite well

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[1]See: e.g. Gali and Gertler (1999) and Christiano et al. (2005).
[2]See: e.g. Carvalho (2006), Sheedy (2007), Coenen et al. (2007) and Whelan (2007).

for the inflation gap persistence I find in the U.S. CPI data after the Volcker disinflation period. The reason why the hazard function greatly affects inflation gap persistence is that backward-dependence of inflation in the model is determined by two counteracting channels. The "front-loading channel" always weakens inflation gap persistence, because lagged inflation enters the NKPC with negative coefficients, magnitudes of which are purely determined by the price reset hazard function. By contrast, the second channel works through the expectational terms in the NKPC. In this channel, lagged inflations have positive coefficients when lagged inflations act as leading indicator of other variables. As a result, the magnitude of the "expectation channel" is not only affected by the price reset hazard function, but also by the other general equilibrium forces, such as aggregate demand side of the economy and monetary policy. Overall, inflation gap persistence in this framework results from a more complex propagation mechanism, in which the price reset hazard function exerts crucial effects through various channels.

The general-pricing-hazard models have been studied in the macro literature to understand consequences of different price reset hazard functions for macro dynamics. It is important, because, in recent years, empirical studies using detailed micro-level price data sets[3] generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially across sections. This new evidence issues a serious challenge to the Calvo pricing assumption (Calvo, 1983). In addition, micro empirical evidence largely rejects the constant hazard function, implied by the Calvo model (See, e.g.: Cecchetti, 1986, Alvarez, 2007 and Nakamura and Steinsson, 2008). In response to this challenge, theoretical work by Wolman (1999) raised the issue that inflation dynamics should be sensitive to the hazard function underlying different pricing rules. He showed this result in a partial equilibrium analysis. Kiley (2002) compared the Calvo and Taylor staggered-pricing models and showed the dynamics of output following monetary shocks are both quantitatively and qualitatively different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy. Carvalho (2006) constructed a sticky price model that allows for heterogeneous Calvo-sticky-price sectors. He found that existence of heterogeneity in price stickiness generates large and persistent real effects of monetary policy, which can be replicated by a constant-hazard-pricing model only when it is calibrated with an unrealistic low frequency of price adjustments. Sheedy (2007) derived the generalized NKPC under a recursive formulation of the hazard function and showed that, under this parameterization, the dependence of current and lagged inflation is determined by the slope of the hazard function. This result, however, is not applicable in more general cases. Whelan (2007) derived the NKPC under a general hazard function and showed that backward-dependence of inflation in this structural Phillips curve is mostly negative. Based on this finding he drew the conclusion that this class of models can not explain the observation from the reduced-form Phillips curve regression that inflation is positively dependent on its lags.

It is noteworthy that non-zero trend inflation is also important for the short-run inflation dynamics(See: Ascari, 2004). Furthermore, Cogley and Sbordone (2008) extend the Calvo NKPC by allowing for time-drifting trend inflation and they show that changing trend inflation affects coefficients of the NKPC and hence the short-run inflation dynamics. Even though the general-hazard NKPC does not incorporate this feature, this limitation does not prohibit the

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[3]See: e.g. Bils and Klenow (2004) and Alvarez et al. (2006) among others.

general-price-hazard model from standing as a useful analytical tool for inflation dynamics. Empirical evidence shows that, while non-constant hazard function is a robust feature of the pricing behavior in the data, the time-varying trend inflation is not always equally important all the time. During the oil crises in the 1970's, volatile inflation trend maybe predominated inflation dynamics, but, after early 1980's, U.S. trend inflation became moderate and stable in the data. These two versions of the generalized NKPC complement each other, combining them, however, gives an interesting perspective for future work.

The remainder of the paper is organized as follows: Section 1 documents stylized fact of inflation gap persistence in the U.S. data. In section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows analytical results regarding new insights gained from relaxing the constant hazard function underlying the Calvo assumption and implications for inflation gap persistence is also discussed; in section 4, I simulate the DSGE model with different setups and identify the most important feature in generating inflation gap persistence; section 5 contains some concluding remarks.

2   Inflation Persistence in the Data

Whelan (2007) has documented that U.S. inflation in the post-WWII periods is highly persistent when measured by the sum of autocorrelation coefficients of inflation level and the coefficient of lagged inflation in the reduced-form Phillips curve. Based on this evidence, he rejected the general-pricing hazard model as a valid model for inflation dynamics. However, it is important to distinguish the inflation gap persistence from the inflation trend persistence, because sticky price models are really designed to explain the short-run dynamics of inflation gap which are caused by the collective pricing behavior of firms in the economy, instead of the dynamics of trend inflation which are mainly determined by the central bank's monetary policy targets.

Recently, there are a growing number of studies on inflation persistence controlling a drifting trend inflation. Levin and Piger (2003), Altissimo et al. (2006), Cogley and Sbordone (2008) and Cogley et al. (2008) document using both U.S. and European data that, when correctly accounting for the time-varing trend inflation, various measures of inflation gap persistence fall significantly. Here I present evidence on inflation gap persistence using the U.S. CPI data. In addition, I report results controlling different measures for trend inflation.

I estimate two measures of inflation persistence using the U.S. time series data from 1960 Q1 to 2007 Q4[4]. First, following Andrews and Chen (1992), I calculate the sum of AR coefficients

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[4]I download data from the database FRED maintained by the Federal Reserve Bank of St. Louis. I calculate the inflation rate by using the Consumer Price Index data for all urban consumers: all items and seasonally adjusted (Series: CPIAUCSL). The monthly data is first converted into quarterly frequency by arithmetic averaging and then the annualized Inflation rate is defined as $400 \times \ln(P_t/P_{t-1})$. Furthermore, to measure the real inflationary pressures, I first construct data of real output gap per capita, which is based on the Real GDP (Series: GDPC1). They are in the unit of billions of chained 2005 dollars, quarterly frequency and seasonally adjusted. To calculate real GDP per capita, I use the Civilian Noninstitutional Population (Series: CNP16OV) from the Bureau of Labor Statistics. The monthly data in the unit of thousands is first converted into quarterly frequency by arithmetic averaging. The real GDP per capita is defined as: $\ln(GDP_t \times 1,000,000/POP_t)$. Finally real output gap per capita is obtained by detrending the data by the Hodrick-Prescott filter. In addition, I download the unit labor share for non-farm business sector (Series: PRS85006173) from the U.S. Bureau of Labor Statistics as a measure of real marginal cost.
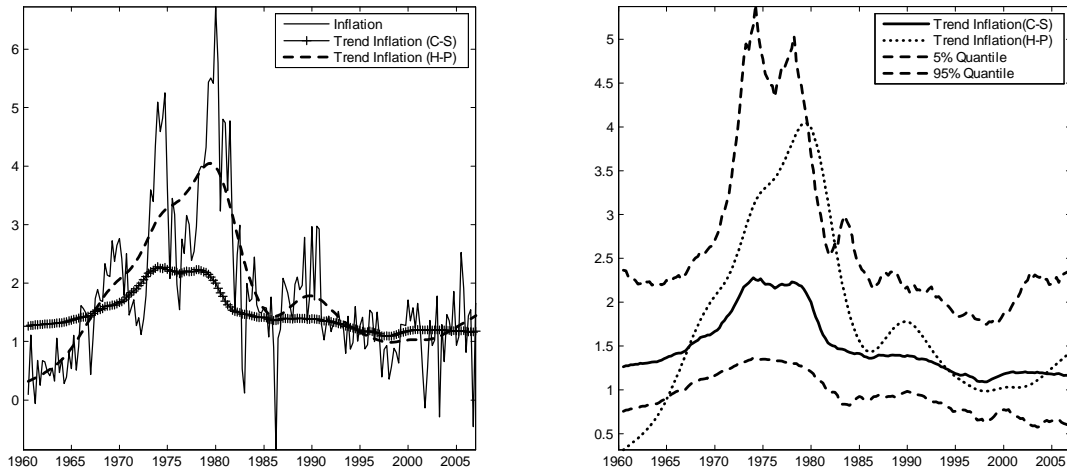
Figure 1: Measures of Trend Inflation

as a measure of overall inflation persistence. Second, following Whelan (2007), I estimate the reduced-form Phillips curve by including real driving forces into the regression. This reduced-form inflation regression distinguishes inflation persistence driven by its own lags[5] from those imparted by persistent real driving forces. The reduced-form inflation regression is specified in the following form and I report the coefficient ρ as the measure of inflation persistence

$$\pi_t = \eta + \rho\pi_{t-1} + \sum_{i=1}^{3} \beta_i \Delta\pi_{t-i} + \sum_{i=0}^{3} \eta_i y_{t-i} + u_t. \tag{1}$$

To construct inflation gap, we need to first calculate measures of the inflation trend. Since there is no standard way to do it in the literature, I first choose a naive method to detrend inflation by the Hodrick-Prescott (H-P) filter. The biggest limitation of this method, however, is that the H-P filter is only based on the univariate process. As argued by Cogley and Sbordone (2008), when the trend inflation is nonzero and drifting over time, it should also depend on other real variables, such as the trend of real marginal cost. To account for this feature of the data, they proposed to estimate a VAR model with drifting parameters and stochastic volatility for four variables - output growth rate, the log of unit labor cost, inflation and the nominal discount factor. After that, they calculate an approximation of trend inflation by defining it as the level to which inflation expectation settles in the long run. Following the same methodology, I construct CPI inflation trend for the periods between 1960 Q1 to 2007 Q4[6].

In Figure (1), I plot the two measures of trend inflation. In the left panel, we observe that the two trends differ substantially. While the H-P trend (dashed line) follows closely to actual inflation, the Cogley-Sbordone trend (hereafter: C-S trend) is much more moderate. The median estimate of trend inflation rose by roughly 1% at the annual rate during 1970's and fell back to around 1.3% in the early 80's, then stayed relative stable until 2007. On the right panel, I

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[5]It is denoted as the intrinsic inflation persistence by some authers, e.g.: Sheedy (2007)

[6]For calculating this inflation trend, I implement the MATLAB codes provided by Timothy Cogley and Argia M. Sbordone on their website.

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compare the two trends more closely. As portrayed by the two dash lines, the 90% confident interval of estimated C-S inflation trend is quite wide, especially during the volatile periods in 1970's. It indicates a great deal of uncertainty about trend inflation associating with the C-S method. Even through the H-P trend is substantially different to the C-S trend, it lies within the confident interval for the most of sample periods. Due to this reason, in Table (1), I report measures of inflation gap persistence for both H-P and C-S trend inflation.

Sample	Inflation level			Inflation Gap (H-P)			Inflation Gap (C-S)		
	AR	$\rho(\hat{y})$	$\rho(LS)$	AR	$\rho(\hat{y})$	$\rho(LS)$	AR	$\rho(\hat{y})$	$\rho(LS)$
$1960-2007$	0.887 (0.041)	0.897 (0.041)	0.882 (0.046)	0.559 (0.082)	0.479 (0.095)	0.548 (0.084)	0.825 (0.051)	0.849 (0.053)	0.807 (0.055)
$1960-1985$	0.902 (0.048)	0.895 (0.047)	0.906 (0.051)	0.659 (0.094)	0.574 (0.109)	0.642 (0.103)	0.858 (0.056)	0.873 (0.058)	0.850 (0.063)
$1986-2007$	0.491 (0.145)	0.494 (0.155)	0.475 (0.153)	0.064 (0.185)	0.013 (0.200)	0.062 (0.187)	0.376 (0.16)	0.364 (0.172)	0.378 (0.165)

Note: Numbers in the parenthesis are the standard deviations.

Table 1: Empirical Results based on the Inflation Data

The first row of the table indicates which definition of inflation is used to calculate the measures of persistence. I report results for inflation level, inflation gap detrended by the H-P filter and inflation gap detrended by the Cogley and Sbordone method. Under each label, three measures of inflation persistence are presented, i.e. the sum of autocorrelation coefficients AR, the coefficient of lagged inflation in the reduced-form Phillips curve when the real driving force is measured by H-P detrended real output per capita $\rho(\hat{y})$, and the coefficient of lagged inflation in the reduced-form Phillips curve when the real driving force is measured by the unit labor share $\rho(LS)$. The first noteworthy result from the table is that the CPI inflation was indeed highly persistent over the subsample from 1960 to 1985. It fell dramatically, however, after the Volcker disinflation of 1980's. This finding is consistent with what is found in the literature. Second, the magnitude of inflation gap persistence crucially depends on the measure of trend inflation. When the H-P trend is used, inflation gap persistence is significantly lower than that in the inflation level. It becomes even insignificant from zero during the second subsample. By contrast, when the C-S trend is used, inflation gap persistence is lower, but much closer to the measured inflation level persistence. It is instructive to compare the C-S trend with two extreme cases of inflation detrending, namely the linear detrending and the detrending by the H-P filter. While the mean detrending does not change the inflation persistence at all, the H-P detrending reduces it to the greatest extent. The multivariable-based C-S method gives values between these two extreme cases. Even through it is not very accurate, one can still draw conclusion from this evidence that the true inflation gap persistence is significant and positive and inflation gap persistence constitutes a large part of inflation persistence. In the later section, I will use the C-S measure of inflation gap persistence as the benchmark for evaluating the performance of the theoretical model.

In the light of these results, we can sum up some stylized facts of inflation gap persistence.
1. Inflation gap persistence constitutes a large part of inflation persistence in the U.S. CPI data.
2. CPI inflation gap is highly persistent during periods between 1960 to 1985. The sum of coefficients on lagged inflation lies in the range around 0.85 with the standard deviation of 0.06.

3. inflation gap persistence reduces significantly after the Volcker disinflation period. The sum of coefficients on lagged inflation reduces to around 0.37 with the standard deviation of 0.16.

3   The Model

In this section, I use a DSGE model to analyze the persistence of inflation gap found in the U.S. data. The main feature of the model is the incorporation of a general price reset hazard function into an otherwise standard New Keynesian model. A hazard function of price setting is defined as the probabilities of price adjustment conditional on the spell of time elapsed since the last price change. In this model, the hazard function is a discrete function taking values between zero and one on its time domain.

3.1   Representative Household

A representative, infinitely-lived household derives utility from the composite consumption good C_t, and its labor supply L_t, and it maximizes a discounted sum of utility of the form:

$$\max_{\{C_t, L_t, B_t\}} \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\delta}}{1-\delta} - \chi_H \frac{L_t^{1+\phi}}{1+\phi} \right) \right].$$

Here C_t denotes an index of the household's consumption of each of the individual goods, $C_t(i)$, following a constant-elasticity-of-substitution aggregator (Dixit and Stiglitz, 1977).

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\eta > 1$, and it follows that the corresponding cost-minimizing demand for $C_t(i)$ and the welfare-based price index, P_t, are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t \tag{3}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{4}$$

For simplicity, I assume that households supply homogeneous labor units (L_t) in an economy-wide competitive labor market.

The flow budget constraint of the household at the beginning of period t is

$$P_t C_t + \frac{B_t}{R_t} \leq W_t L_t + B_{t-1} + \int_0^1 \pi_t(i) di. \tag{5}$$

Where B_t is a one-period nominal bond and R_t denotes the gross nominal return on the bond. $\pi_t(i)$ represents the nominal profits of a firm that sells good i. I assume that each household owns an equal share of all firms. Finally this sequence of period budget constraints is supplemented with a transversality condition of the form $\lim_{T \to \infty} E_t \left[\frac{B_T}{\Pi_{s=1}^T R_s} \right] \geqslant 0$.

The solution to the household's optimization problem can be expressed in two first order necessary conditions. First, optimal labor supply is related to the real wage:

$$\chi_H L_t^\phi C_t^\delta = \frac{W_t}{P_t}. \tag{6}$$

Second, the Euler equation gives the relationship between the optimal consumption path and asset prices:

$$1 = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\delta \frac{R_t P_t}{P_{t+1}} \right]. \tag{7}$$

3.2 Firms in the Economy

3.2.1 Real Marginal Cost

The production side of the economy is composed of a continuum of monopolistic competitive firms, each producing one variety of product i by using labor. Each firm maximizes real profits, subject to the production function

$$Y_t(i) = Z_t L_t(i) \tag{8}$$

where Z_t denotes an aggregate productivity shock. Log deviations of the shock, $\hat{z}_t$, follow an exogenous AR(1) process $\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$, and $\varepsilon_{z,t}$ is white noise with $\rho_z \in [0,1)$. $L_t(i)$ is the demand of labor by firm i.

Following equation (3), demand for intermediate goods is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t. \tag{9}$$

In each period, firms choose optimal demands for labor inputs to maximize their real profits given nominal wage, market demand (9) and the production technology (8):

$$\max_{L_t(i)} \Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) \tag{10}$$

And real marginal cost can be derived from this maximization problem

$$mc_t = \frac{W_t/P_t}{Z_t}.$$

Furthermore, using the production function (8), output demand equation (9), the labor supply condition (6) and the fact that at the equilibrium $C_t = Y_t$, I can express real marginal cost only in terms of aggregate output and technology shock.

$$mc_t = Y_t^{\phi+\delta} Z_t^{-(1+\phi)}. \tag{11}$$

3.2.2 Pricing Decisions under Nominal Rigidity

In this section, I introduce a general form of nominal rigidity, which is characterized by a set of hazard rates depending on the spell of the time since last price adjustment. I assume that monopolistic competitive firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing prices are dictated by the hazard rates, h_j, where j denotes the time-since-last-adjustment and $j \in \{0, J\}$. J is the maximum number of periods in which a firm's price can be fixed.

Dynamics of the Price-duration Distribution In the economy, firms' prices are heterogeneous with respect to the time since their last price adjustment. Table 2 summarizes key notations concerning the dynamics of the price-duration distribution.

Duration	Hazard Rate	Non-adj. Rate	Survival Rate	Distribution
j	h_j	α_j	S_j	$\theta(j)$
0	0	1	1	$\theta(0)$
1	h_1	$\alpha_1 = 1 - h_1$	$S_1 = \alpha_1$	$\theta(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	h_j	$\alpha_j = 1 - h_j$	$S_j = \prod_{i=0}^{j} \alpha_i$	$\theta(j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	$h_J = 1$	$\alpha_J = 0$	$S_J = 0$	$\theta(J)$

Table 2: Notations of the Dynamics of Price-vintage-distribution.

Using the notation defined in Table 2, and also denoting the distribution of price durations at the beginning of each period by $\Theta_t = \{\theta_t(0), \theta_t(1) \cdots \theta_t(J)\}$, we can derive the ex-post distribution of firms after price adjustments ($\tilde{\Theta}_t$) as

$$\tilde{\theta}_t(j) = \begin{cases} \sum_{i=1}^{J} h_i \theta_t(i) \text{ , when } j = 0 \\ \alpha_j \theta_t(j) \text{ , when } j = 1 \cdots J. \end{cases} \tag{12}$$

Firms reoptimizing their prices in period t are labeled with 'Duration 0', and the proportion of those firms is given by hazard rates of all duration groups multiplied by their corresponding densities. The firms left in each duration group are the firms that do not adjust their prices. When the period t is over, this ex-post distribution, $\tilde{\Theta}_t$, becomes the ex-ante distribution for the new period, Θ_{t+1}. All price duration groups move to the next one, because all prices age by one period.

As long as the hazard rates lie between zero and one, dynamics of the price-duration distribution can be viewed as a Markov process with an invariant distribution, Θ, and is obtained by solving $\theta_t(j) = \theta_{t+1}(j)$. It yields the stationary price-duration distribution $\theta(j)$:

$$\theta(j) = \frac{S_j}{\sum_{j=0}^{J-1} S_j}, \text{ for } j = 0, 1 \cdots J - 1. \tag{13}$$

In a simple example, when $J = 3$, the stationary price-duration distribution

$$\Theta = \left\{ \frac{1}{1 + \alpha_1 + \alpha_1 \alpha_2}, \frac{\alpha_1}{1 + \alpha_1 + \alpha_1 \alpha_2}, \frac{\alpha_1 \alpha_2}{1 + \alpha_1 + \alpha_1 \alpha_2} \right\}.$$

I assume the economy converges to this invariant distribution fairly quickly, so that regardless of the initial price-duration distribution, I only consider the economy with the invariant distribution of price durations. This assumption makes aggregation problem of the economy tractable.

The Optimal Pricing under the General Form of Nominal Rigidity Given the general form of nominal rigidity introduced above, the only heterogeneity among firms is the time when they last reset their prices, j. Firms in price duration group j share the same probability of adjusting their prices, h_j, and the distribution of firms across durations is given by $\theta(j)$.

In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon in which the new price is expected to be fixed. The probability that a new price will be fixed at least for j periods is given by the survival function, S_j, defined in Table 2.

I setup the maximization problem of an adjuster as follows:

$$\max_{P_t^*} E_t \sum_{j=0}^{J-1} S_j Q_{t,t+j} \left[Y_{t+j|t}^d \frac{P_t^*}{P_{t+j}} - \frac{TC_{t+j}}{P_{t+j}} \right].$$

Where E_t denotes the conditional expectation based on the information set in period t, and $Q_{t,t+j}$ is the stochastic discount factor appropriate for discounting real profits from t to $t+j$. An adjusting firm maximizes the profits subject to the demand for its intermediate good in period $t+j$ given that the firm resets the price in period t and can be expressed as.

$$Y_{t+j|t}^d = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\eta} Y_{t+j}.$$

This yields the following first order necessary condition for the optimal price:

$$P_t^* = \frac{\eta}{\eta - 1} \frac{\sum\limits_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1} MC_{t+j}]}{\sum\limits_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]}, \tag{14}$$

where MC_t denotes the nominal marginal cost. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, whose weights depend on the survival rates. In the Calvo case, where $S_j = \alpha^j$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the stationary distribution, $\theta(j)$, aggregate price can be written as a distributed sum of all optimal prices. I define the optimal price which was set j periods ago as P_{t-j}^*. Following the aggregate price index from equation (4), the aggregate price is then obtained by:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P_{t-j}^{*1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{15}$$

4 New Keynesian Phillips Curve

In this section, I derive the New Keynesian Phillips curve for this generalized sticky price model. To do that, I first log-linearize equation (14) around the flexible price steady state. The log-

linearized optimal price equations are obtained by

$$\hat{p}_t^* = E_t \left[\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Omega} \left(\widehat{mc}_{t+j} + \hat{p}_{t+j} \right) \right], \tag{16}$$

$$where \quad :$$

$$\Omega = \sum_{j=0}^{J-1} \beta^j S(j) \quad and \quad \widehat{mc}_t = (\delta + \phi)\hat{y}_t - (1 + \phi)\hat{z}_t.$$

In a similar fashion, I derive the log deviation of the aggregate price by log linearizing equation (15).

$$\hat{p}_t = \sum_{k=0}^{J-1} \theta(k) \, \hat{p}_{t-k}^*. \tag{17}$$

After some algebraic manipulations on equations (16) and (17), I obtain the New Keynesian Phillips curve as follows[7]

$$\hat{\pi}_t = \sum_{k=0}^{J-1} \frac{\theta(k)}{1 - \theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right)$$

$$- \sum_{k=2}^{J-1} \Phi(k)\hat{\pi}_{t-k+1}, \quad where \ \Phi(k) = \frac{\displaystyle\sum_{j=k}^{J-1} S(j)}{\displaystyle\sum_{j=1}^{J-1} S(j)}, \quad \Psi = \sum_{k=0}^{J-1} \beta^j S(j). \tag{18}$$

4.1 Economic Intuition

The general-hazard NKPC differs from the standard NKPC in two aspects. First, the general-hazard NKPC has not only current and forward-looking terms but also lagged variables and lagged expectations. In addition, all coefficients in the new NKPC are nonlinear functions of price reset hazard rates ($\alpha_j = 1 - h_j$) and the subjective discount factor β. Thereby, short-run dynamics of inflation gap are affected by both the shape and magnitude of the price reset hazard function. To see the dynamic structure more clearly, I write down a simple example of the NKPC with $J = 3$.

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[7]The detailed derivation of the NKPC can be found in the technical Appendix (A).

$$\hat{\pi}_t = \frac{1}{(\alpha_1 + \alpha_1\alpha_2)\,\Psi}\widehat{mc}_t + \frac{\alpha_1}{(\alpha_1 + \alpha_1\alpha_2)\,\Psi}\widehat{mc}_{t-1} + \frac{\alpha_1\alpha_2}{(\alpha_1 + \alpha_1\alpha_2)\,\Psi}\widehat{mc}_{t-2}$$
$$+ \frac{1}{\alpha_1 + \alpha_1\alpha_2}E_t\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+2} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+2}\right)$$
$$+ \frac{\alpha_1}{\alpha_1 + \alpha_1\alpha_2}E_{t-1}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+1} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1}\right)$$
$$+ \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2}E_{t-2}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t}\right)$$
$$- \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2}\hat{\pi}_{t-1}, \tag{19}$$

$$where \quad : \quad \Psi = 1 + \beta\alpha_1 + \beta^2\alpha_1\alpha_2.$$

Even though, from the first glance, the general-hazard NKPC differs substantially from the Calvo NKPC, they share the same economic intuition. In fact, should the hazard function be constant over the infinite horizon, the general-hazard NKPC (18) reduces to the standard Calvo NKPC[8]:

$$\hat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}mc_t + \beta E_t\hat{\pi}_{t+1} \tag{20}$$

The general-hazard NKPC nests the Calvo NKPC in the sense that, under a constant hazard function, lagged inflation terms exactly cancel lagged expectations, leaving only current variables and forward-looking expectations of inflation in the expression.

To understand the economic intuition of the general-hazard NKPC, we need to categorize its dynamic components and exam the effect of each component on inflation. The general-hazard NKPC can be decomposed into three parts: 1) all forward-looking and current terms, 2) Lagged expectations and 3) lagged inflations. In the following analysis, I represent these three parts with short-hand symbols $E_t(.)$, $E_{t-j}(.)$ and $\hat{\pi}_{t-k}$ respectively and $W_x(h_j)$ denotes coefficients of those terms. Furthermore, by definition, inflation is equal to the log difference between two consecutive aggregate prices and the aggregate price in the period t can be further written as the distributed sum of current and past optimal reset prices. As illustrated in the following expressions (21), these three dynamic components of the general-hazard NKPC affect inflation through current reset price, past reset prices and past aggregate price respectively.

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$$

$$\hat{\pi}_t = \overbrace{\theta(0)\hat{p}_t^*} + \overbrace{\theta(1)\hat{p}_{t-1}^* + \cdots + \theta(J-1)\hat{p}_{t-J-1}^*} - \hat{p}_{t-1} \tag{21}$$
$$\Uparrow \qquad\qquad \Uparrow \qquad\qquad \Uparrow$$
$$\hat{\pi}_t = W_1(h_j)E_t(.) + W_2(h_j)E_{t-j}(.) - W_3(h_j)\hat{\pi}_{t-k}$$

---

[8] Proof : see Appendix (B).

The economic reasons why those three components should show up in the general-hazard NKPC is that: first, the current and forward-looking terms - $E_t(.)$ - enter the Phillips curve through their influence on the current reset price. As same as in the Calvo sticky price model, the price setting in this model is forward-looking. The optimal price decision is based on the sum of current and future real marginal costs over the time span the reset price is fixed. The only difference now is that the time horizon of the pricing decision is not infinite, but depends on the hazard function. Second, due to price stickiness, some fraction of past reset prices continue to affect the current aggregate price. Lagged expectational terms -$E_{t-j}(.)$- represent influences of past reset prices on current inflation. Last, past inflations enter the NKPC, because they affect the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price would be, and thereby it deters current inflation to be high.

The new insights gained from this analysis is that the two new dynamic components have opposing effects on inflation through $\hat{p}_t$ and $\hat{p}_{t-1}$ respectively. The magnitudes of these effects depend on the price reset hazard function. In the general case, they should be different to each other. Conversely, in the Calvo case, the constant hazard function leads reset prices to exert the exactly same amount of impact on both $\hat{p}_t$ and $\hat{p}_{t-1}$, and thereby causes lagged expectations and lagged inflation to be cancelled out.

This cancellation can be also seen in the derivation of the Calvo NKPC:

$$
\begin{aligned}
\hat{p}_t &= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\hat{p}_{t-j}^* \\
&= (1-\alpha)\left[\hat{p}_t^* + \alpha\hat{p}_{t-1}^* + \alpha^2\hat{p}_{t-2}^* + \cdots\right] \\
&= (1-\alpha)\hat{p}_t^* + \underbrace{(1-\alpha)\left[\alpha\hat{p}_{t-1}^* + \alpha^2\hat{p}_{t-2}^* + \cdots\right]}_{=\alpha\hat{p}_{t-1}} \\
\hat{p}_t &= (1-\alpha)\hat{p}_t^* + \alpha\hat{p}_{t-1} \\
&\ \ \vdots \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t + \beta E_t(\hat{\pi}_{t+1}).
\end{aligned}
$$

The crucial substitution from line (3) to line (4) is only possible, when the distribution of price durations takes the form of a power function. In conclusion, we learn that, lagged inflation and lagged expectations are not extrinsic to the time-dependent sticky price model. They are missing in the Calvo setup only because of the restrictive constant-hazard assumption.

4.2 Implications for inflation gap persistence

The purely forward-looking NKPC is often criticized for generating too little inflation gap persistence(See: e.g. Fuhrer and Moore, 1995). In response to this challenge, the hybrid NKPC has been developed to capture the positive dependence of inflation on its lags (See:e.g. Gali and Gertler, 1999 and Christiano et al., 2005). According to this strand of the literature, inflation persistence should be interpreted as 'intrinsic' and the dependency between current and lagged inflation is mechanically modeled as a fixed primitive relationship, which is independent of changes in monetary policy. By contrast, the generalized Calvo sticky price model, such as the one introduced in the previous section, captures this backward-dependency of inflation in a

more micro-founded way. Unlike the hybrid models, inflation gap persistence in this framework is the result of two counteracting channels. The first channel gives lagged inflation a direct role, which works through the past aggregate price. I call it the "front-loading channel" because it weakens inflation gap persistence, and its magnitude is purely determined by the price reset hazard function. By contrast, the second channel is an indirect one, where lagged inflation affects current inflation only through the expectational terms in the NKPC, I name it the "expectation channel". In this channel, lagged inflations have positive coefficients when lagged inflations act as the leading indicator of other variables. Because, in the general equilibrium, the expectation formulation is determined by the whole setup of the model, the magnitude of the "expectation channel" is not only affected by the price reset hazard function, but also by the other general equilibrium forces, such as aggregate demand side of the economy and monetary policy.

$$
\hat{\pi}_t = \underbrace{W_1(h_j)E_t(.) + W_2(h_j)E_{t-j}(.)}_{Expectation\ Channel} - \underbrace{W_3(h_j)\hat{\pi}_{t-k}}_{Front-loading\ Channel}
$$

$$
\pi_t = \sum_{i=0}^{I} \gamma_i mc_{t-i} + \sum_{i=1}^{I} \rho_i \pi_{t-i} + \epsilon_t.
$$

In the light of these results, the general-hazard NKPC preserves the implication of the standard Calvo NKPC for inflation gap persistence, which is in stark contrast to those from the hybrid NKPC. First of all, inflation gap persistence can not be interpreted as 'intrinsic'. Instead, more persistence come from the additional moving-average terms of real driving forces introduced by the expectations. The positive coefficient on lagged inflation in the reduced-form Phillips curve results from the correlation between lagged inflation and other variables in the general equilibrium, and therefore it is not a real economic behavioral relation, but a "statistical illusion". More importantly, since the coefficient on lagged inflation depends on the whole model, changes in any part of the general equilibrium setup ultimately affects its value. Consequently, hybrid sticky price models are subject to the Lucas critique, and thereby can not be used in the monetary policy analysis.

Overall, inflation gap persistence in this framework is the result of these two counteracting channels. Whelan (2007) has proved that, in the partial equilibrium setting, the net effect of these two opposing forces is always negative, regardless of the form of the hazard function. He further showed that, even in the general equilibrium, the general-hazard sticky price model fails to replicate the positive backward-dependence of inflation. My numerical analysis reveals, however, that it is the 4-period-Taylor-contract hazard function that gave rise to this result. When I use an empirically based hazard function, the simulated data can account well for the inflation gap persistence I find in the U.S. aggregate data after the Volcker disinflation period.

4.3 The General Equilibrium Analysis

In this section, I study the behavior of inflation dynamics in the general equilibrium setup. For this purpose, I close the model by adding the aggregate demand side of the economy and a monetary policy rule. The log-linearized equilibrium equations are summarized in the following table:

Aggregate Supply:

$$\hat{\pi}_t = \sum_{k=0}^{J-1} W_1(k) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k) \hat{\pi}_{t-k+1}$$

$$\widehat{mc}_t = (\phi + \delta)\,\hat{y}_t - (1 + \phi)\,\hat{z}_t$$

$$\hat{z}_t = \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \quad \epsilon_t \backsim N(0, \sigma_z^2)$$

Aggregate Demand:

$$E_t\left[\hat{y}_{t+1}\right] = \hat{y}_t + \frac{1}{\delta}\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right)$$
or:
$$\hat{y}_t = \hat{m}_t - \hat{p}_t \qquad and \qquad \hat{m}_t = \delta \hat{y}_t - \frac{\beta}{1-\beta} \hat{\imath}_t$$

Monetary Policy:

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + q_t, \quad q_t \backsim N(0, \sigma_q^2)$$
or:
$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + g_t \quad where \quad g_t \backsim N(0, \sigma_g^2)$$

Where all variable are expressed in terms of log deviations from the non-stochastic steady state. The weights (W_1, W_2, W_3, W_4) in the general-hazard NKPC are defined in the equation (18). $\hat{m}_t$ is the real money balance, and g_t denotes the growth rate of the nominal money stock. The aggregate demand block is motivated either by the standard household intertemporal optimization problem outlined in the model section or by the quantity theory of money[9]. The monetary policy is specified in terms of either a nominal money growth rule or a simple Taylor rule.

4.3.1 Calibration

In the calibration of the general equilibrium model, I choose some common values for the standard structural parameters. For the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\delta = 1$, implying log utility of consumption. The Frisch elasticity of the labor supply is set to be 0.5, a value that is motivated by using balanced-growth-path considerations in the macro literature. In addition, I choose the elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%.

Since the main purpose of the paper is to study the impact of the hazard function on inflation gap persistence, I calibrate the hazard function as follows: My first hazard function takes the

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[9] In this case, model has not enough structure to pin down the relationship between real marginal cost and output gap. To make the results quantitatively comparable, I assume, in this case, that real marginal cost holds the same relationship to output gap as in the complete model $\widehat{mc}_t = (\phi + \delta)\,\hat{y}_t - (1 + \phi)\,\hat{z}_t$.
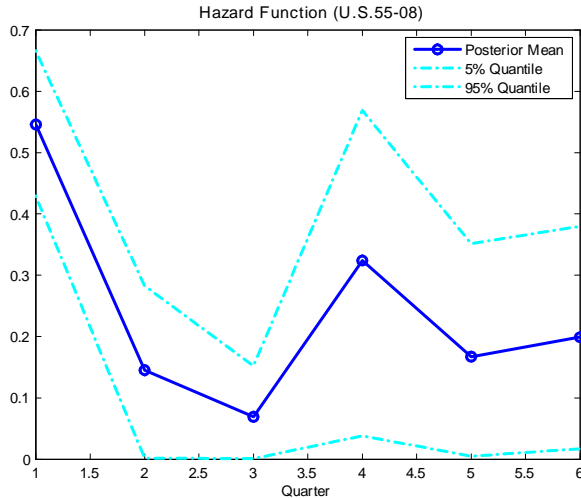
Figure 2: Empirical Hazard Function

form of $\{0, 0, 0, 1\}$, which is motivated by the 4-period-Taylor-contract theory. This hazard function is used in the general equilibrium analysis of Whelan (2007). Alternatively, I refer to the empirical finding by Yao (2010), who estimates the aggregate hazard function using the same framework and the same aggregate data set applied in this paper. As seen in the table (3) and the figure (2), the empirical hazard function differs sharply to the theoretical hazard function. Overall, the aggregate hazard function is first decreasing and then increases slowly with the age of the price. In comparison to the Taylor hazard function, where firms only adjust their prices after 4 quarters, the empirical hazard function highlights two important frequencies of the price adjustment. Additional to the yearly frequency, it is also evidence of a large flexible price setting sector in the economy.

Hazard function	h_1	h_2	h_3	h_4	h_5	h_6
4-period-Taylor-contract	0	0	0	1	0	0
Yao (2010)	0.55	0.15	0.07	0.33	0.17	0.20

Table 3: Hazard Function Calibration

Proceeding with monetary policy parameters, the responses of nominal interest rate to inflation and output gap (ϕ_π and ϕ_y) are chosen at the values commonly associated with the simple Taylor rule. Following Taylor (1993), I set ϕ_π to be 1.5, and the response coefficient to output gap ϕ_y to be 0.5. Finally, I set the standard deviation of the innovation to monetary policy shock to be 25 basic points per quarter.

4.3.2 Numerical Results

To evaluate the quantitative implications of the hazard function for inflation gap persistence, I simulate different setups of the general-pricing-hazard model, then estimate the reduced-form Phillips curve using the artificial data. The reduced-form Phillips curve is specified in the

following form

$$\pi_t = \eta + \rho\pi_{t-1} + \sum_{i=1}^{3} \beta_i \Delta\pi_{t-i} + \sum_{i=0}^{3} \gamma_i mc_{t-i} + \sum_{i=0}^{3} \eta_i y_{t-i} + \epsilon_t.$$

I include both output gap and real marginal cost into the reduced-form Phillips curve, because in the theoretical model real marginal cost is the driving force of inflation and output gap also affect the inflation dynamics through the monetary feedback rules.

	Model Setup			**inflation gap persistence**
Model	Hazard Function	Monetary Police	Agg. Demand	ρ
1	4-period-contract	Money growth rule	$\hat{y} = \hat{m} - \hat{p}$	-0.538
2	4-period-contract	Money growth rule	IS curve	-1.068
3	4-period-contract	Taylor rule (1.5,0.5)	IS curve	-0.805
4	Yao (2010)	Money growth rule	$\hat{y} = \hat{m} - \hat{p}$	0.286
5	Yao (2010)	Money growth rule	IS curve	0.242
6	Yao (2010)	Taylor rule (1.5,0.5)	IS curve	0.308
7	Yao (2010)	Taylor rule (2,0)	IS curve	0.217

Table 4: Simulation based Empirical Results

In Table (4), I report the sum of AR coefficients of lagged inflations (ρ) generated by the simulated data of different theoretical setups. The first three rows are models applying the 4-period-Taylor-contract hazard function. All these models produce negative coefficient on inflation lag, implying no inflation persistence. The benchmark case (Model 1) has the same setup as in Whelan (2007), combining 4-period-Taylor-contract hazard function with the nominal money growth rule and simple aggregate demand equation. In this model, the reduced-form lagged inflation coefficient is negative (-0.538). Model 2 replaces the simple aggregate demand equation with the intertemporal IS curve derived from the household problem. This setup generates a even more negative coefficient on inflation lag than Model 1. In Model 3, I replace the money growth rule with the simple Taylor rule for monetary policy. inflation gap persistence in this case becomes a little stronger than that in Model 2. By contrast, setups using the empirical hazard function (Model 4 to 7) generate realistic inflation gap persistence as we observe in the data from 1986 to 2007. This comparison reveals that it is the unrealistic hazard function that drives the result that leads Whelan to reject the general-pricing-hazard model. From the analysis in the previous section, we know that the hazard function has direct influence on both propagation channels in the general-hazard NKPC. When the magnitude of the second channel is large enough to compensate the negative coefficients introduced by the first channel, the reduced-form Phillips curve reveals a positive backward-dependence of inflation. From the numerical results, it turns out that the hazard function is the most important factor in the complex propagation mechanism of inflation dynamics.

Moreover, other parts of the general equilibrium model plays also a role in determining the magnitude of inflation gap persistence. In contrast to the hazard function, this general equilibrium influence mainly occurs through the expectation channel. Similar to the pattern revealed by the model 1 2 and 3, Model 4, 5, 6 conduct the same numerical experiments under the

empirically based hazard function. In the model 4, the reduced-form lagged inflation coefficient is positive (0.286). Model 5 replaces the simple demand equation with the IS curve and generates a slightly less inflation gap persistence than Model 4. The reason why inflation becomes even less persistent is that, with the intertemporal optimizing IS curve, demand shocks are not propagated completely to output gap and inflation dynamics, but they are partially dampened by the rise of real interest rate. So that expectational channel becomes less powerful than the previous case. In Model 6, I replace the money growth rule with the simple Taylor rule. inflation gap persistence in this case becomes a little stronger than that in Model 4. The Taylor rule changes inflation gap persistence, because it introduces an extra channel, through which inflation and real forces feedback to the economy, so that the expectation channel is strengthened. In addition, in Model 7, I apply another Taylor rule with a stronger inflation response parameter and a zero response parameter to output gap. Shutting down the feedback of output gap to the interest rate rule makes the Taylor rule less powerful, so that it performs similar to the money growth rule.

In conclusion, both monetary policy rule and demand side of economy are important in propagating inflation dynamics, but the fundamentally important factor in this mechanism is the hazard function. Using the empirically based hazard function along with the Taylor rule and IS curve (Model 6), the general-pricing-hazard model preforms best in replicating the stylized fact of inflation gap persistence found in the U.S. CPI data from 1986 to 2007. It is not a surprising result, because most macroeconomists agree that monetary policy is well approximated by the simple Taylor rule with coefficients conforming to the Taylor principle during this period of time. In addition, this time span is also characterized by low and stable trend inflation. This character of data validates the use of the general-pricing-hazard model.

5 Conclusion

In this paper, I investigate whether the general-hazard NKPC is capable of accounting for the inflation gap persistence. In the empirical part, I find that, after detrending inflation by the Cogley-Sbordone method, inflation gap persistence is still significant and large in the U.S. CPI data. In the theoretical part, I redo the general equilibrium analysis by Whelan (2007), and check robustness of the result to different setups of the model. I find that the general-pricing-hazard model with empirically based price reset hazard function can account quite will for inflation gap persistence found in the data of post Volcker's disinflation periods. The key mechanism at work in this model is the expectational channel in the generalized NKPC, which depends on the setup of the whole model, therefore inflation gap persistence is also not independent of monetary policy. This result directly implies that the hybrid sticky price model should be subject to the Lucas critique, and thereby can not be used in the monetary policy analysis.

However, one should also be aware of the limitation of the model. It can not account for time-varing trend inflation, which affects also the coefficients in the NKPC (Cogley and Sbordone, 2008). As a result, the general-pricing-hazard model is only suitable to model a economy with a stable monetary policy regime.

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A Deviation of the New Keynesian Phillips Curve

Starting from 16

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right] \tag{22}$$

$$= E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j}\right] \tag{23}$$

The last term can be further expressed in terms of future rates of inflation

$$\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j} = \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1}$$

$$= \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\left(\hat{p}_{t+1}-\hat{p}_t\right) + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1}$$

$$= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}\right)\hat{p}_t + \sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \frac{\beta^2 S_2}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2}$$

$$= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\frac{\beta^2 S_2}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1}$$

$$+\frac{\beta^3 S_3}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2}$$

$$\vdots$$

$$= \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j}$$

$$+\sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1} + ... + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{\pi}_{t+1}$$

$$= \hat{p}_t + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}$$

The optimal price can be expressed in terms of inflation rates, real marginal cost and aggregate prices.

$$\hat{p}_t^* = \hat{p}_t + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right] \tag{24}$$

Next, I derive the aggregate price equation as the sum of past optimal prices. I lag equation

24 and substitute it for each $\hat{p}^*_{t-j}$ into equation 17

$$
\begin{aligned}
\hat{p}_t &= \theta(0)\ \hat{p}^*_t + \theta(1)\ \hat{p}^*_{t-1}\ + \cdots + \theta(J-1)\hat{p}^*_{t-J+1} \\
&= \theta(0)\left[\hat{p}_t + E_t\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right) + E_t\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right)\right] \\
&\quad + \theta(1)\left[\hat{p}_{t-1} + E_{t-1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-1}\right) + E_{t-1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-1}\right)\right] \\
&\quad \vdots \\
&\quad + \theta(J-1)\left[\hat{p}_{t-J+1} + E_{t-J+1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-J+1}\right) + E_{t-J+1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-J+1}\right)\right] \\
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\left[\hat{p}_{t-k} + \underbrace{E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-k}\right)}_{F_{t-k}}\right] \qquad (25)
\end{aligned}
$$

Where F_t summarizes all current and lagged expectations formed at period t.
Finally, we derive the New Keynesian Phillips curve from equation 25.

$$
\begin{aligned}
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} + \underbrace{\sum_{k=0}^{J-1}\theta(k)F_{t-k}}_{Q_t} \\
\hat{\pi}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + \theta(0)\hat{p}_{t-1} + \theta(1)\hat{p}_{t-1} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + (\theta(0)+\theta(1))\ \hat{p}_{t-1} + \theta(2)\hat{p}_{t-2} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \underbrace{\theta(0)}_{W(0)}\hat{\pi}_t + \underbrace{(\theta(0)+\theta(1))}_{W(1)}\hat{\pi}_{t-1} + (\theta(0)+\theta(1)+\theta(2))\ \hat{p}_{t-2}\cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&\quad \vdots \\
&= W(0)\ \hat{\pi}_t + W(1)\hat{\pi}_{t-1} + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{W(J-1)}_{=1}\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= W(0)\ \hat{\pi}_t + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{\hat{p}_{t-J+1} - \hat{p}_{t-J+2}}_{-\hat{\pi}_{t-J+2}} + \hat{p}_{t-J+2} - \cdots + \underbrace{\hat{p}_{t-2} - \hat{p}_{t-1}}_{-\hat{\pi}_{t-1}} + Q_t \\
(1-W(0))\hat{\pi}_t &= -(1-W(2))\hat{\pi}_{t-1} - \cdots - (1-W(J-1))\hat{\pi}_{t-J+2} + Q_t \\
\hat{\pi}_t &= -\sum_{k=2}^{J-1}\frac{1-W(k)}{1-\theta(0)}\hat{\pi}_{t-k+1} + \sum_{k=0}^{J-1}\frac{\theta(k)}{1-\theta(0)}F_{t-k}
\end{aligned}
$$

The general-hazard New Keynesian Phillips curve is:

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$$\hat{\pi}_t \;=\; \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi} \hat{\pi}_{t+i-k} \right)$$

$$-\sum_{k=2}^{J-1} \Phi(k)\hat{\pi}_{t-k+1}, \quad where \; \Phi(k) = \frac{\sum\limits_{j=k}^{J-1} S_j}{\sum\limits_{j=1}^{J-1} S_j}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S_j \tag{26}$$

B Proof

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Denote the constant hazard as $h = 1 - \alpha$, and substitute it into the NKPC (18):

$$
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} = (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left((1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-k} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right)
$$

$$
\begin{aligned}
\hat{\pi}_t + \alpha\hat{\pi}_{t-1} + \alpha^2\hat{\pi}_{t-2} + \cdots &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+ \alpha^2 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots
\end{aligned}
\tag{27}
$$

Iterate this equation one period forward, I obtain

$$
\begin{aligned}
\hat{\pi}_{t+1} + \alpha\hat{\pi}_t + \alpha^2\hat{\pi}_{t-1} + \alpha^3\hat{\pi}_{t-2}\cdots &= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+ \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&\vdots
\end{aligned}
$$

Use equation (27) to substitute terms in the left hand side of the equation $(\hat{\pi}_t, \hat{\pi}_{t-1}, \hat{\pi}_{t-2}\cdots)$, I get

$$\hat{\pi}_{t+1} + \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)$$

$$+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)$$

$$+\alpha^3 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right)$$

$$\vdots$$

$$= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)$$

$$+\alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)$$

$$+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right).$$

$$\vdots$$

After canceling out equal terms from both sides of the equation, It yields the following equation:

$$\hat{\pi}_{t+1} = E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)$$

Lag this equation and rearrange it, I obtain the NKPC of the Calvo model.

$$
\begin{aligned}
\hat{\pi}_t &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \quad (28) \\
\hat{\pi}_t &= (1-\alpha)(1-\alpha\beta)mc_t + (1-\alpha)\hat{\pi}_t + \alpha\beta E_t \left(\hat{\pi}_{t+1} \right) \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \left(\hat{\pi}_{t+1} \right)
\end{aligned}
$$

Proof done

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