Options, Timing, and Regional Entry of Firms*

Elisabet Berglund

Department of Economics, Umeå University

SE-901 87 Umeå, Sweden

Umeå Economic Studies 505, 1999

Abstract

This paper studies how the possibility to postpone the unrecoverable entry and location cost affects regional entry when post-entry earnings are uncertain. We find that the opportunity cost to enter today is higher when entering a region with high uncertainty relative to a region with low uncertainty.

Key words: Timing, Regional uncertainty, Brownian motion.

JEL Classification: D21, R12, D84, C61.

*I would like to thank Kurt Brännäs for generous and competent guidance during the work on this article. Karl-Gustaf Löfgren is thanked for valuable comments and suggestions and Kaj Nyström for valuable suggestions to improve Section 3 and for help with stochastic differential equations.

1. Introduction

The decision to enter is not a single discrete choice, but may be seen as the outcome of a sequence of conditional choices or the result of a sequential search process in which potential firms maximize future discounted net profits. Entry occurs when the present value of the project's expected cash flow is, at least, as large as its cost. While there has been a number of important theoretical contributions targeting entry decisions to an industry they have almost all neglected the spatial dimension. For further reading about the firm's entry decision see, for example, the survey of Geroski (1991).

An important determinant to the spatial pattern of economic activity, besides the transfer of firms between regions, is firms' entry location. In the framework of Berglund (1999) potential firms are confronted with a collection of disjoint regions from which they choose the one that yields maximal discounted net profits.[1] This allows for regional comparisons of entry locations but sees the entry location decision as a certain and immediate event. The framework therefore ignores future uncertainty in the sense that new information about, for example, prices and costs before commitment of resources may have a value. An example at the national level are the continuously changing environmental standards. At the regional level it may be changing conditions such as labour supply, market demand, possibility to obtain subsidies, public investments, etc. The regional environments are dependent upon regional policies, prices, etc. For this reason we model the entry location decision in an uncertain world and characterize the location decision by irreversibility and the possibility to delay. Since the future is unknown the traditional Marshallian criterion may be incorrect as the option value of preserving the entry location opportunity is

[1] The setup presumes a well-informed entrepreneur or decision maker that knows the values of included variables. The term "Marshallian criterion" is previously used by Dixit (1992). The criterion suggests that we should calculate the net present value of an investment and invest if it is greater than zero. This criterion has also been referred to as simply the net present value criterion.

ignored.[2]

Recent research offers the option value theory criterion, OVTC, see McDonald and Siegel (1986), Pindyck (1991), Dixit (1989, 1992) and Dixit and Pindyck (1994). In contrast to the MTC the OVTC requires that future expected discounted net profits must exceed not only the fixed costs but also the value of keeping the option to entry. The reasons for the existence of the latter value is that future expected cash flows and costs are unknown and that the decision to enter is characterized by irreversibility. If delaying is a viable option, seen over a active period and entry cost, postponed entry may be more favourable than immediate entry.

In this paper, we raise the following question: How does the possibility to postpone entry and location decisions affect regional entry decisions in a model with risky post-entry earnings using the OVTC? Or formulated in another way: How does regional uncertainty with respect to revenues affect the timing of entry into a region? The point is to account for the fact that the opportunity cost to invest today differs between regions due to differences in uncertainty about future earnings.[3]

As far as we know the application of OVTC on regional entry is new. The difference between this and other studies that applies the option value approach to study firm investments is the introduction of region specific uncertainty. Ones the choice of regions is done the entry location cost is sunk, i.e. there is no possibilities to recover it.

Section 2 gives an introduction to how to include uncertainty and the timing dimension for the case of two regions and two time periods. The option value approach with uncertain regional revenue and several periods is developed and discussed in Section 3, while Section 4 concludes.

[2]An option represents the right to buy a security or commodity at a specified price within a specified period of time. By option in this paper we refer to a potential entrant's right, but not its obligation to enter a region within a specific period of time.

[3]The reason that we concentrate on regional earnings is that a major part of new firms serves local markets. Local earnings is thereby an important determinant of new firms profit potential.

2. Regional Entry

We start by introducing the notation and the intuition behind the OVTC on the regional system by revising the two region framework of Berglund (1999). There, a profit maximizing firm that has decided to enter an industry faces a system of disjoint regions, and uses present value maximization for the choice of location. The firm also faces a set of regional variables, factor prices and a final price of the product, p_{it}, from the time of entry $t = 0$ until a known time of exit ϵ. With two regions, the firm chooses region one, if the discounted net profit in that region is higher than in region two, i.e. if

$$\pi_1 - \pi_2 = F_2 - F_1 + \int_0^{\epsilon} [(p_{1t}q_{1t} - p_{2t}q_{2t}) - (D_{1t} - D_{2t})] \exp(-rt)dt > 0, \qquad (1)$$

where $p_{it}q_{it}$ represents revenues and D_{it} operating costs in region i at time t, with q_{it} denoting the level of output. Each region's demand curve is downward-sloping. Regional supply is driven by the own price, production costs, input prices and regional market organization. The sunk and irretrievable fixed costs is represented by F_i, while $\exp(-rt)$ is a common discount factor of future revenues and costs, where $r > 0$ is the risk free nominal interest rate.[4] We include uncertainty by letting future regional revenues be uncertain.

In this section we give an example with only two time periods within which variables are constant. In the next section some of these constants would be realizations of stochastic processes. The fixed costs difference $\bar{F} = F_2 - F_1$, is known with certainty. Once a firms has entered the market, it is assumed to produce one unit per period at zero unit cost of production. In period one the regional revenues are certain, but there are two different and mutually exclusive outcomes of regional revenues in both regions in the second period. The potential entrant values the revenues in period two, contingent on the probability that the price will be high or low. We denote high revenues in region i by $R^+_{it_2}$, and by $R^-_{it_2}$ if revenues are

[4]Since we use the risk free interest rate we must assume that the risk over future revenues is unrelated to what happens with the overall economy.

low. Revenues are high with probability α and low with probability $1-\alpha$. Exit is random in the sense that the entrant survives with probability β. Exit and revenues are independent. If we consider the above facts and take expectations we can write equation (1) as the difference in expected discounted net profit between the regions

$$\begin{aligned} E[\pi_1]-E[\pi_2] &= F_2 - F_1 + (R_{1t_1} - R_{2t_1})\exp(-rt_1) \\ &\quad +\beta[\alpha(R^+_{1t_2} - R^+_{2t_2}) + (1-\alpha)(R^-_{1t_2} - R^-_{2t_2})]\exp(-rt_2)]. \end{aligned} \quad (2)$$

According to MTC the entrant chooses region one if this difference exceeds zero. The firm is indifferent between the two regions if equation (2) is equal to zero. If equation (2) is less then zero, the entrant chooses region two.

An application of the OVTC does not necessarily yield the same outcome. Assume that the firm also has the possibility to postpone entry and let $\bar{R}_{t_1} = R_{1t_1} - R_{2t_1}$ denote the revenue difference in period one. We denote the period two difference in high revenues by $\bar{R}^+_{t_2}$, while a low revenue difference is denoted by $\bar{R}^-_{t_2}$. Equation (2) can then be rewritten as

$$E[\bar{\pi}] = E[\pi_1] - E[\pi_2] = \bar{F} + \bar{R}_{t_1}\exp(-rt_1) + \beta[\alpha\bar{R}^+_{t_2} + (1-\alpha)\bar{R}^-_{t_2}]\exp(-rt_2). \quad (3)$$

Given the possibility to postpone entry or to enter selectively in period two, we should compare (3) to the best available alternative in period two. The potential entrant then faces the following decision rule: Enter into region one in the first period if the following is true

$$\bar{F} + \bar{R}_{t_1}\exp(-rt_1) + \beta\left[\alpha\bar{R}^+_{t_2} + (1-\alpha)\bar{R}^-_{t_2}\right]\exp(-rt_2) \geq \alpha(\bar{R}^+_{t_2} - \bar{F})\exp(-rt_2), \quad (4)$$

if not, wait. In the second period the firm knows the outcome of revenues in both regions and can thereby choose the most profitable region. The option value, i.e. the value of the possibility to wait can be derived from (4)

$$\begin{aligned} O(\bar{R}_{t_1}) &\equiv \alpha\bar{R}^+_{t_2}\exp(-rt_2) - \bar{F}(1+\alpha\exp(-rt_2)) - \bar{R}_{t_1}\exp(-rt_1) \\ &\quad -\beta\alpha\bar{R}^+_{t_2}\exp(-rt_2) - \beta(1-\alpha)\bar{R}^-_{t_2}\exp(-rt_2). \end{aligned} \quad (5)$$

Table 1: Entry location choices.

	MTC=OVTC, $O(\bar{R}_{t_1}) \leqslant 0$	MTC≠OVTC, $O(\bar{R}_{t_1}) > 0$
$E[\bar{\pi}] < 0$	Region two, Entry	Wait
$E[\bar{\pi}] = 0$	Indifferent, Entry	Wait
$E[\bar{\pi}] > 0$	Region one, Entry	Wait

If $O(\bar{R}_{t_1})$ is positive, it is optimal to wait. We have summarized the decision rules in Table 1.

We conclude that in the two period case, an increased difference in the first period revenues decreases the option value. Also, if the difference between the fixed costs increases the regional entry option decreases.

This exercise tells us that considerations of regional uncertainty and the relaxation of the now or never assumption matter for the entry location choice, even in this simple single firm, two region, two period framework.

3. Uncertain Regional Revenue and Entry - Several Periods

In this section assume that there are more than two periods and that $\beta = 1$, i.e. the survival probability equals one. The risk neutral profit maximizing potential entrant, with monopoly right to invest, faces an instant, continuous and random revenue, R_{it}, in region i for all t. The entrant does not know the future values of regional revenues, but only their probability distributions. We assume revenues to arise according to the following stochastic differential equation (a geometric Brownian motion process)

$$dR_{it} = \gamma_i R_{it} dt + \sigma_i R_{it} dz_t,$$

with $R_{i0} > 0$ the revenue at time zero. Here, dz_t is the increment of a Wiener process, $dz_t = \varepsilon_t \sqrt{dt}$ with $\varepsilon_t \sim N(0, 1)$. The drift parameter, γ_i, is the rate of change of R_{it}. If it is negative it means that revenues in region i decrease with time. If $\gamma_i \geq 0$ it means that the revenues are constant or increasing in t. The σ_i is the diffusion coefficient, measuring the stochastic fluctuations around the mean. Using Ito's Lemma (e.g., Ito and Mckean, 1965) one can show that $R_{it} = R_{i0}\exp(\gamma_i - \frac{1}{2}\sigma_i^2)t + \sigma_i z_t$. The expectation of the revenue process in region i at time $t \geq 0$, starting at R_{i0}, is given by

$$E_0(R_{it}) = R_{i0}\exp(\gamma_i t). \tag{6}$$

To study the MTC we define the expected discounted profit to entry in region i at time $t = 0$ and survival to infinity as

$$E_0[\pi_i] = E_0\left[\int_0^\infty (R_{it} - F_i)\exp(-rt)dt\right], \tag{7}$$

where $r > 0$ is the risk free nominal interest rate. For both regions we assume that $\gamma_i < r$ because otherwise waiting longer would always be a better policy. Since future values of R_{it} are unknown and investments are irreversible in the sense that fixed costs cannot be recovered, there exists an opportunity cost to invest today in regions one and two.

To study the OVTC we define the expected discounted profit of entry and survival to infinity in region i starting from the unknown time $t = \tau_i$ as

$$E_0[\Omega_i] = \sup_{\tau_i} E_0\left[\int_{\tau_i}^\infty (R_{it} - F_i)\exp(-rt)dt\right]. \tag{8}$$

The supremum is taken over all stopping times τ_i. With τ_i^* we denote the optimal stopping time for the problem in (8). At each time t the potential entrant then has an option to enter region one but also an option to enter region two, and as a result the optimal entry time may differ between the two regions. We want to show that the expected value of the optimal entry time in region i, $E(\tau_i^*)$ depends on the uncertainty in region i, σ_i, in the sense that if $\sigma_1 > \sigma_2$ (ceteribus paribus), then $E(\tau_1^*) > E(\tau_2^*)$. This will indicate that the opportunity cost to enter today in region one, exceeds the one of region two.

Equation (8) can be rewritten as the expectation of the, known, "now or never starting time" to infinity, minus the opportunity cost of exercise the regional entry option in region i:

$$E_0\left[\int_0^{\infty}(R_{it}-F_i)\exp(-rt)dt\right]+\sup_{\tau_i}E_0\left[\int_0^{\tau_i}-(R_{it}-F_i)\exp(-rt)dt\right]. \qquad (9)$$

Substitution of (6) into the Marshallian term of (9), i.e. into the first term, and performing the integration we may conclude that the value of the first term in (9) equals

$$\frac{R_{i0}}{r-\gamma_i}-\frac{F_i}{r}.$$

To study in what way $E(\tau_i^*)$ depends on σ_i we have to solve,

$$\sup_{\tau_i}E_0\left[-\int_0^{\tau_i}(R_{it}-F_i)\exp(-rt)dt\right]. \qquad (10)$$

According to Theorem 10.18 of Oksendal (1995) (variational inequalities for optimal stopping) an essential step in solving the problem in (10) is to solve an equation of the form $L\phi=\exp(-rs)R_{i0}$ where ϕ is a function of (s,R_{i0}) and $L\phi$ is defined as

$$L\phi=\frac{\partial\phi}{\partial s}+\gamma_iR_{i0}\frac{\partial\phi}{\partial R_{i0}}+\frac{1}{2}\sigma_i^2R_{i0}^2\frac{\partial^2\phi}{\partial R_{i0}^2}. \qquad (11)$$

Equation (11) may be solved by separation of variables, assuming $\phi(s,R_{i0})=\exp(-rs)\theta(R_{i0})$. Then θ solves

$$\frac{1}{2}\theta^2R_{i0}^2\frac{\partial\theta^2}{\partial R_{i0}^2}+\gamma_iR_{i0}\frac{\partial\theta}{\partial R_{i0}}-r\theta=R_{i0}. \qquad (12)$$

The general solution to equation (12) is then given by

$$\theta(R_{i0})=AR_{i0}^{\beta_{i1}}+BR_{i0}^{\beta_{i2}}+\frac{R_{i0}}{\gamma_i-r}, \qquad (13)$$

where A and B are constants and β_{i1} and β_{i2} are the negative and positive roots of the quadratic equation $\frac{1}{2}\sigma_i^2\beta_i(\beta_i-1)+\gamma_i\beta_i-r=0$, i.e.

$$\beta_i=\frac{1}{\sigma_i^2}\left[\frac{\sigma_i^2}{2}-\gamma_i\pm\sqrt{\left[\gamma_i-\frac{\sigma_i^2}{2}\right]^2+2r\sigma_i^2}\right]$$

Obviously the expression under the root sign is positive and we denote the negative root by β_{i1} and the positive root by β_{i2}, i.e. $\beta_{i1}<0$ and $\beta_{i2}>0$.

In order to have ϕ finite at $R_{i0} = 0$ we assume that $A = 0$ and rewrite the equation for ϕ as

$$\phi(s, R_{i0}) = \exp(-rs) \left[B R_{i0}^{\beta_{i2}} + R_{i0}/(\gamma_i - r) \right].$$

We now redefine this function in the following way (the parameter $\tilde{R}_{i0}$ is defined below)

$$\phi(s, R_{i0}) = \exp(-rs) \left[B R_{i0}^{\beta_{i2}} + R_{i0}/(\gamma_i - r) \right], \text{ if } 0 < R_{i0} < \tilde{R}_{i0}$$

and

$$\phi(s, R_{i0}) = -\exp(-rs)\ F_i/r, \text{ if } \tilde{R}_{i0} \leq R_{i0} < \infty.$$

The B and $\tilde{R}_{i0}$ are determined using the conditions that ϕ should be once continuously differentiable at $(s, \tilde{R}_{i0})$. First $\phi(s, R_{i0})$ should be continuous at $(s, \tilde{R}_{i0})$, which implies the condition

$$B \tilde{R}_{i0}^{\beta_{i2}} + \tilde{R}_{i0}/(\gamma_i - r) = -F_i/r.$$

Second, $\partial\phi(s, R_{i0})/\partial R_{i0}$ should be continuous at $(s, \tilde{R}_{i0})$, which is equivalent to

$$1/(\gamma_i - r) + \beta_{i2} B \tilde{R}_{i0}^{\beta_{i2}-1} = 0.$$

If we solve these two equations for B and $\tilde{R}_{i0}$ we get

$$B = -\left[\frac{F_i}{r} + \frac{\tilde{R}_{i0}}{\gamma_i - r} \right] \tilde{R}_{i0}^{-\beta_{i2}}$$

$$\tilde{R}_{i0} = \frac{\beta_{i2} F_i}{r} \frac{\gamma_i - r}{1 - \beta_{i2}} \tag{14}$$

Now the conclusion of Theorem 10.18 of Oksendal (1995) states that the optimal entry time τ_i^* in our original problem, i.e. the problem in (10), is given by $\tau_i^* = \tau_{iD}$, where

$$\tau_{iD} = \inf \left\{ t > 0 : R_{it} \notin (0, \tilde{R}_{i0}) \right\}.$$

Hence, τ_i^* is the first time at which revenues exit the interval $(0, \tilde{R}_{i0})$, with $\tilde{R}_{i0}$ calculated as above. Furthermore, the maximal profit in (10) is given by the function $F_i/r + \phi(0, R_{i0})$. If $R_{i0} \geq \tilde{R}_{i0}$, then $E(\tau_{iD}) = 0$, immediate entry into region i is the optimal behavior.

We now assume that $R_{i0} \in (0, \tilde{R}_{i0})$, i.e. the option to wait has a positive value and we want to calculate $E(\tau_{iD})$ and understand how this quantity depends on σ_i. Under the assumption that $\gamma_i - \sigma_i^2/2 > 0$ one can use Ito's Lemma to show that

$$E_0(\tau_{iD}) = \ln\left[\frac{\tilde{R}_{i0}}{R_{i0}}\right] / \left[\gamma_i - \frac{1}{2}\sigma_i^2\right] = q(\sigma_i). \tag{15}$$

We want to study $q'(\sigma_i)$ assuming all other parameters fixed. In the Appendix we show that

$$q'(\sigma_i) = \frac{\tilde{R}'_{i0}(\cdot)\left[\gamma_i - \frac{1}{2}\sigma_i^2\right]/\tilde{R}_{i0} + \left[\ln \tilde{R}_{i0} - \ln R_{i0}\right]\sigma_i}{\left[\gamma_i - \frac{1}{2}\sigma_i^2\right]^2} > 0.$$

We may therefore conclude that the optimal entry time into region i increases with an increased σ_i, i.e. we have shown that if $\sigma_1 > \sigma_2$, then $E_0(\tau_1^*) > E_0(\tau_2^*)$.

4. Conclusions

The model sketched in this paper applied the option value approach to model the entry location choice within a two period and uncertainty framework. In Section 2 the value of a flexible entry location decision is compared to an immediate one. An increased difference in the first period revenues decreases the option value. Also, if the difference between the fixed costs increases the regional entry option decreases. In Section 3 the value of the option to postpone can explicitly be calculated as

$$O_i = \frac{F_i}{r} - \frac{R_{i0}}{r - \gamma_i}[1 - \beta_{i2}^{-1}\left[\frac{R_{i0}}{\tilde{R}_{i0}}\right]^{\beta_{i2}-1}]. \tag{16}$$

Equation (16) is only relevant when R_{i0} is less or equal to the threshold $\tilde{R}_{i0} > 0$ explicitly given in Section 3. If $R_{i0} \geqslant \tilde{R}_{i0}$ the value of the option to postpone entry is zero. This value can be visualized, as a function of R_{i0}, as in Figure 1.

O_i is a strictly decreasing function of R_{i0} and $R_{i0} = F_i(r - \gamma_i)/r$ is the unique initial revenue giving the Marshallian profit equal to zero. If $R_{i0} \in (F_i(r - \gamma_i)/r$, $\tilde{R}_{i0})$ then the difference between the OVTC and MTC profit (as a function of the initial revenue) is equal to the opportunity cost of exercise the regional entry option, which is positive.

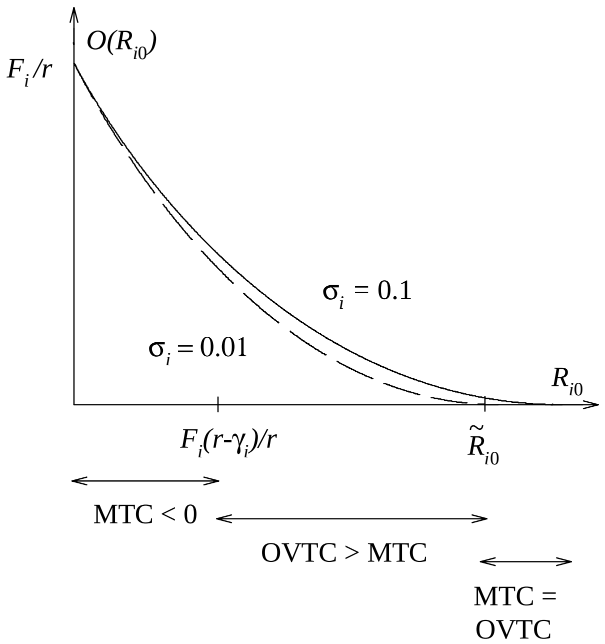

Figure 1: The option value $O(R_{i0})$ versus R_{i0} evaluated at two values of σ_i. In addition, $F_i = 1000, \gamma_i = 0.03$ and $r = 0.05$.

Appendix

We want to show that

$$q'(\sigma_i) = \frac{\tilde{R}_{i0}'(\sigma_i)\left[\gamma_i - \frac{1}{2}\sigma_i^2\right]/\tilde{R}_{i0} + \left[\ln \tilde{R}_{i0} - \ln R_{i0}\right]\sigma_i}{\left[\gamma_i - \frac{1}{2}\sigma_i^2\right]^2} > 0.$$

To prove this inequality we only have to prove that $\tilde{R}_{i0}'(\sigma_i) > 0$, since $\gamma_i - \frac{1}{2}\sigma_i^2 > 0$ and $R_{i0} \leq \tilde{R}_{i0}$. Using equation (14) we have $\tilde{R}_{i0} = C_0\beta_{i2}/(\beta_{i2}-1)$, where $C_0 = F_i(r-\gamma_i)/r$ is a positive constant. Hence, $\tilde{R}_{i0}' = -C_0\beta_{i2}'/(\beta_{i2}-1)^2$, i.e. the sign of $\tilde{R}_{i0}'(\sigma_i)$ is determined by $\beta_{i2}'(\sigma_i)$. To derive $\beta_{i2}'(\sigma_i)$ we rewrite β_{i2} as

$$\beta_{i2} = \frac{1}{2} - \frac{\gamma_i}{\sigma_i^2} + \sqrt{\left[\frac{1}{2} - \frac{\gamma_i}{\sigma_i^2}\right]^2 + \frac{2r}{\sigma_i^2}}$$

and introduce $\delta = 1/2 - \gamma_i/\sigma_i^2$. Then $\beta_{i2} = \delta + (\delta^2 + 2r/\sigma_i^2)^{1/2}$ and

$$\beta_{i2}' = \delta' + \frac{(2\delta\delta' - 4r\sigma_i^{-3})}{2\sqrt{\eta}} = \frac{\delta' 2\sqrt{\eta} + \left[2\delta\delta' - 4r\sigma_i^{-3}\right]}{2\sqrt{\eta}},$$

where $\delta' = 2\gamma_i/\sigma_i^3$ and $\eta = \delta^2 + 2r/\sigma_i^2$. We may therefore conclude that $\beta_{i2}' > 0$, if and only if

$$2\delta'\sqrt{\eta} > -2\delta\delta' + \frac{4r}{\sigma_i^3}.$$

We can rewrite the inequality above as

$$\frac{4\gamma_i}{\sigma_i^3}\sqrt{\eta} > \frac{4r}{\sigma_i^3} - 2\delta\left[\frac{2\gamma_i}{\sigma_i^3}\right].$$

Equivalently

$$\gamma_i\sqrt{\eta} > r - \delta\gamma_i > 0$$

Taking squares gives $\gamma_i^2\eta > (r - \delta\gamma_i)^2$. Expanding we see that $\beta_{i2}' > 0$, if and only if $\gamma_i > r$. But since we assume that $\gamma_i < r$ we therefore have $\beta_{i2}' < 0$. Hence, $\tilde{R}_{i0}'(\sigma_i) > 0$ which implies that $q'(\sigma_i) > 0$.

References

Berglund, E. and Brännäs, K. (1999). Plant's Entry and Exit in Swedish Municipalities. Umeå Economic Studies 497, Umeå University.

Berglund, E. (1999). Human Capital and Regional Entry of Firms. Umeå Economic Studies 500, Umeå University.

Dixit, A. K. (1989). Entry and Exit Decisions under Uncertainty. *Journal of Political Economy* **97**, 620-638.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment Under Uncertainty.* Princeton University Press, Princeton.

Geroski, P. (1991). *Market Dynamics and Entry.* Basil Blackwell, Oxford.

Ito, K. and Mckean, H. P. (1965). *Diffusion Processes and Their Sample Paths.* Springer-Verlag, Berlin.

McDonald, R. and Siegel, D. (1986). The Value of Waiting to Invest. *Quarterly Journal of Economics* **101**, 707-728.

Oksendal, B. (1995). *Stochastic Differential Equations: An Introduction with Applications.* Fourth edition, Springer-Verlag, Berlin.

Pindyck, R. (1991). Irreversibility, Uncertainty and Investment. *Journal of Economic Literature* **29**, 1110-1148.