SUBJECTIVE EVALUATION OF DELAYED RISKY OUTCOMES: AN EXPERIMENTAL APPROACH

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Subjective Evaluation of Delayed Risky Outcomes:

An Experimental Approach

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This paper uses experimental data to estimate the pure time discount rate for different lengths of times for riskless assets (bonds), and risky assets (delayed lotteries). In moving from the present time (t = 0) to the future, there is a very sharp decline (jump) in the subjective price of the assets for both buy and sell transactions. This jump corresponds to a large increase in the discount rate for the first period and a much lower discount rate for later periods (forward rate). The findings cast doubt on the relevance of the hyperbolic function approach to discounting.

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1. Introduction

Beginning with the model by Strotz (1956) on dynamic inconsistency, and the work by Kahneman and Tversky (1979) on the Certainty Effect, many authors have shown that individual decision-making accounts for risk and time in a non-linear fashion. The recent surge of literature on hyperbolic discounting argues along these lines as well. For example, Choi et al. (2003) develop a model of voluntary opting into 401(k) plans by individuals when utility functions exhibit hyperbolic discounting. As Laibson has shown in earlier papers (1997, 1998, 2001), individuals with hyperbolic utility discounting will choose self-commitment devices to limit time-inconsistency. Other authors have endorsed the view that revealed individual reservation prices are not consistent with constant period-by-period discount rates. For example, Thaler (1981) tested and confirmed three hypotheses concerning the behavior of discount rates inferred from riskless choices. His findings were as follows: (1) the discount rate declines as waiting time increases; (2) the discount rate decreases as the amount of cash flow increases; (3) the discount rates are lower for losses than for gains. Ben-Zion et al. (1989) presented a choice experiment that manipulated four scenarios: postponing a receipt, postponing a payment, expediting a receipt and expediting a payment. The experiment's results were used to test four hypotheses regarding the behavior of discount rates (The Classical Approach, the Market Segmentation Approach, the Implicit Risk Approach, and the Added Compensation Approach). The Classical Approach was flatly rejected; the Market Segmentation Approach was found lacking. The results support an Implicit Risk hypothesis according to which delayed consequences are associated with an implicit risk value; they also support an Added

Compensation hypothesis, asserting that individuals require compensation for a change in their financial positions.

Green et al (1997) present an experiment in which subjects were asked to make a series of choices regarding hypothetical amounts of money, one immediate and one after specified delay. They found that the degree of discounting decreased as the amount of the delayed reward increased.

According to Read (2001), sub-additive time discounting means that "discounting over a delay is greater when the delay is divided into subintervals than when it is left undivided." Read presents three choice experiments that show strong evidence of sub-additive discounting. He found that "when a delay was divided into three, the total discounting over that delay was increased by an average of 40%". This result is inconsistent with the hyperbolic discounting. Read suggests an alternative theory using the "immediacy effect". According to this effect individual requires a relatively high premium for the delay itself and this premium does not decline with time, "but rather a one-time-only charge for delaying consumption".

Rubinstein (2003) argues that the evidence that rejects standard constant discounting can easily reject hyperbolic discounting as well. Rubinstein presents three pairs of experiments that provide evidence against hyperbolic discounting theory. Although the hyperbolic discounting theory predicts the same preference for a pair of amounts and a sequence of amounts, human beings behave differently. He also introduced psychological phenomena that are not exhibited by hyperbolic discount functions, such as "middle aversion".

The present-biased preferences, as labeled by O'Donoghue and Rabin (1999), entail a jump that is apparently realized in the first period. This may entail dynamic

inconsistencies, reversing the preference relationship among alternatives as time goes on.

This paper offers a detailed analysis of the structure of intertemporal valuation in individual choice behavior using an experimental setting. The design allows for replicating and quantifying the first-period jump in implied discount rates, distinguishing between sure bonds and risky lotteries. In our experiments we find significant jumps (or discontinuities) in the term structure of both the pure time and the pure risk discount rates. These latter rates are derived from individual lottery reservation prices, after compounding bond reservation prices.

A significant discontinuity at t=0 was found for the pure time discount, calculated from bond and lottery reservation prices, and for both bid and ask prices, albeit the latter is less pronounced in the case of bonds. The pure risk discount, in contrast, has no significant jump around t=0. It tends to decay steadily over time for bid prices, while it stays flat for ask prices.

To further explore the nature of the discontinuity, we then identify determinants of the size of the 'jump' at t=0, in particular whether it varies with individual risk aversion, or with the size of the investment (i.e. the face value of the bond, and the amount at stake in the lottery). We find no dependence on individual risk aversion, and a negative dependence of the jump on asset size. These results taken together lead us to believe that the discontinuity is not a reflection of the shape of the utility function, as suggested by the hyperbolic and other functional forms put forward in the literature. Rather, we advocate a new interpretation that stresses decision dependence, yielding a time-zero fixed effect and a constant effect afterward. According to this interpretation, choices and evaluations are made in a two-step procedure. First, spot (or time-zero)

alternatives are evaluated. Then, alternatives that entail forward contracting are considered (time greater than zero). In comparison, the other subsequent alternatives entail a fixed cost, which may be thought of as the information costs, or other entry costs associated with evaluating and contracting financial claims that consist of future payoffs. Present time alternatives do not incur these costs.

The two-step procedure suggests that hyperbolic discounting is relevant only for small stakes, because for large stakes the first period jump vanishes. Experimental and other evidence has not sufficiently addressed the issue of size-dependence. The current paper represents a first step in this direction. Note that for small stakes, the fixed cost hypothesis and the hyperbolic discounting hypothesis make the same prediction, while this is no longer true for (relatively) larger stakes. The important difference to be stressed is that with the fixed costs explanation, the jump in the implicit discount function cannot be attributed to the shape of the utility function, but rather is caused by features of the decision situation. Thus, the fixed costs explanation leads to different policy conclusions from those derived under the hyperbolic discounting hypothesis, as discussed in the concluding section. Note that our explanation is also related to a decision model recently suggested by Rubinstein (2003).

The rest of the paper is organized as follows: Section 2 presents the definitions and hypotheses. Section 3 introduces the experimental design. Section 4 presents the main results. Finally, Section 5 discusses the results and points to further research questions related to the study.

2. Definitions and Hypotheses

The process of lottery discounting implicit in a bidder's reservations price is assumed to consist of two interwoven rates, temporal discounting (time discounting) and probability discounting. The notation used in the discussion is as follows:

R_L^t is the pure time discount rate, up to period t, for lottery L (effective interest rate, or yield to maturity).

R_B^t is the pure time discount rate, up to period t, for a riskless bond B.

CP_B^t is the current price of the future bond's face value F_t realized in t periods. $CP_B^0 = F_t$ is an assumption we make, i.e. a bond maturing in t=0 has a reservation price equal to its face value.

CP_L^0 is the current price of the current lottery.

CP_L^t is the current price of the future lottery played over t periods.

The implied time discount rate for a zero coupon bond with face value F and maturity t is defined as the ratio of two bond values (or reservation prices) (e.g; Williams (1938):

$$R_B^t = (\frac{CP_B^0}{CP_B^t})^{1/t} - 1$$

Similarly, the per-period time discount rate for a risky asset with maturity t is defined by (e.g., Anderhub et al, 2001):

$$R_L^t = (\frac{CP_L^0}{CP_L^t})^{1/t} - 1$$

There has been extensive discussion in the literature concerning the shape of the discount function. The current consensus assumes that the intertemporal discount rate is time dependent, i.e. varies with the distance from the present (t=0). An influential formulation of the time-dependent discount function Z(t) was proposed by Prelec and Loewenstein (1993), who suggested a generalized hyperbolic functional form, where Z(t) is the discounted present value of one € (seeAhlbrecht/Weber 1995).

$$Z(t) = (1+R^t)^{-f(t)} \qquad (1)$$

Note that for f(t)=t, Z(t) is the classical discount function.

Prelec and Loewenstein suggest $f(t) = (1+\alpha t)^{-\beta/\alpha}$, of which special cases have also been used in the literature. Setting α=1 leads to Harvey's (1986) formulation $f(t)= (1+t)^{-\beta}$. β/α=1 yields $f(t)= (1+\alpha t)^{-1}$ Others have suggested a quasi-hyperbolic discount function of the form:

$$Z(t) = \begin{cases} 1 & \text{if } t=0 \\ \beta\delta^t & \text{if } t\in\{1,2,...\} \end{cases} \qquad (2)$$

In all these cases, the shape of the discount function diverges from the classical form in that the discount factor 'jumps' at t=0. Afterwards, it gradually decreases, similar to the exponential discount function typically used in finance (Thaler 1981).

In this study, first we examine the relative price of a bond in terms of an otherwise identical asset with different maturity. This examination is applied to a set of lotteries as well. The forward rate is calculated between two dates, k and w (k<w), and defined as: R_L^{w-k} for the lottery and R_B^{w-k} for the bond.

For example, the forward rate between period 1 and period 4 for the lottery is

$R_L{}^{4\text{-}1} = (\dfrac{CP_L^1}{CP_L^4})^{1/3} - 1$, and for fixed bond payoffs it is $R_B{}^{4\text{-}1} = (\dfrac{CP_B^1}{CP_B^4})^{1/3} - 1$.

One formula that yields a hyperbolic discount function for a given lottery is

$$(\frac{CP_L^t}{CP_L^0}) = \frac{1}{(1+\beta t)} \tag{3}$$

where β, $0<\text{\ss}<1$, is the constant rate for one period.

The current analysis uses a slightly more general formulation that allows determining the extent of the jump in t=0 separate from the slope of the discount function after t=0. This "jump discount" function is described by equation (4):

$$(\frac{CP_L^t}{CP_L^0}) = \frac{1}{(1+\alpha)(1+\beta)^{(t-1)}} \tag{4}$$

where α is the rate for the first period, β is the constant rate for one period except for the first period, and $\alpha > \beta$. The case of $\alpha = \beta$ yields the simple financial function. In the case of bonds, the latter is given by $(\dfrac{CP_B^t}{CP_B^0})$. We are now in a position to formulate the first hypothesis, which replicates results in the literature.

> *Hypothesis 1: The difference in prices between period zero and period 1 exhibits a significant jump, which can be reflected in a very high spot discount rate at time zero (Jump discount).*

This jump compensation for the delayed current amount or lottery from time zero to time 1 can be associated with two known behavioral effects. The first is the status-quo bias (Samuelson and Zeckhauser, 1988), meaning that subjects ask for high compensation for changing their state from current action to future action. This

explanation is also consistent with Read (2001) who suggests that individual requires a relatively high premium for the delay itself and this premium does not decline with time. Getting the payment in the future period instead of today may involves other cost such as planning travel cost and the need to remember to collect the future payment (Green et al (1997)). The second is the Kahneman and Tversky (1979) 'certainty effect'. According to this effect, delaying a reward is risky because it increases the possibility that something will prevent payment (e.g., Stevenson 1986, Green and Myerson 1996, and Myerson et al. 2003). We test the hypothesis using both bid and ask prices for both bonds and lotteries. Based upon the literature, we test whether delayed rewards are seen by investors as risky, possibly because delaying a reward may increase the expectation that some unforeseen event will prevent the payment from being realized.

The next hypothesis relates to individual risk aversion. We want to test whether the size of the jump in the discount function is related to risk aversion on the part of the investor. A positive relationship between risk aversion and the size of the jump will indicate that the latter involves a compensation for switching payoffs from the present to an uncertain future.

> *Hypothesis 2: The initial jump in the discount function is positively related to the investor's risk aversion.*

Next we test the dependency of the jump in the discount function on the size of the asset's face value. A negative relationship between jump size and asset size suggests the existence of a fixed cost responsible for the jump in the discount function. Such a fixed cost may be rationalized by information costs or by monitoring costs that are positive but independent of the properties of the claim. This leads to Hypothesis 3.

Hypothesis 3: In the cross section, the jump of the discount function is inversely related to the face value of the asset.

A possible formulation of such a model is as follows:

Let X be the payoff of a time-zero asset, and let F(X) be a fixed compensation which depends on X, and incurred if the asset pays off at a future date T. Let i be the market interest rate of asset. Then we can approximate the required yield of the asset (i_{req}) as

$$i_{req} \; = \; i + \frac{F(X)}{XT} \tag{5}$$

We can easily see that the required yield of the asset rises in F, and it decreases at the payment date T. For example, if F(X) is a constant (not dependent on X), subjective bid reservation prices will yield a decreasing term structure of effective interest rates, giving the appearance of a hyperbolic functional run. However, by construction, the term structure of reservation prices in this example is correctly described by including a jump at t=0.

If F(X) is linear with X the required yield of the asset is independent of X and there is no magnitude effect on the discount factor. In other cases where $\frac{F(X)}{X}$ is a decreasing function of X we will have a magnitude effect where the discount function will decline with the magnitude (Loewenstein & Prelec (1992) , Green et al (1997), Chapman & Winquist (1998)).

For a given sum one could consider the jump as an hyperbolic function with magnitude effect, however the hyperbolic approach assume (Mazur formula) that the process continue for later periods, where the jump does not specify any functional form. There is no need to add to the jump any specific discounting form such as hyperbolic. And in fact there was no further discounting between the latter

periods.

The next step is to determine the pure risk premium, and to estimate its term structure. Of particular interest is whether the pure risk discount rate also exhibits a jump in its term structure. When the payoff is certain, as in the case of bonds, their reservation prices *CP* can be estimated from observed bid prices, yielding an effective rate $R_B{}^t$:

$$CP_B^t = \frac{F_t}{1+Z_t} = \frac{F_t}{(1+R_B^t)^t} \qquad (6)$$

where Z_t is the discount rate for a sure payoff in period t, and $R_B{}^t$ is the one period risk-free effective discount rate (or yield to maturity). If $F^*{}_t$ is a random lottery payoff with expectation $E(F^*{}_t) = F_t$, the current price CP_L^t can again be estimated from observed reservation (bid) prices. In order to estimate the functional form of the discount function, we express the ratio of the current price of the bond and the lottery as the product of the present risk premium λ and the intertemporal risk discount rate δ. Of course, if the risk of the lottery is taken into consideration only once, $\delta=0$. In this case the lottery reservation price is set as a certainty equivalent:

$$CP_L^t = \frac{F_t}{(1+Z_t)(1+\lambda)} = \frac{F_t}{(1+R_B^t)^t(1+\lambda)}. \qquad (7)$$

Equation (7) assumes that the substitution rate between present values of the bond payoff and the lottery payoff is constant, yielding

$$\frac{CP_B^t}{CP_L^t} = (1+\lambda) \qquad (8)$$

In contrast, if the reservation price is described by a constant per-period price of risk, we get

$$\frac{CP_B^t}{CP_L^t} = (1+\lambda)*(1+\delta)^{at} \qquad (9)$$

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The critical question is, which of the two expressions applies, and whether $\lambda=\delta$, i.e., whether the reservation price adjustment for a lottery played in t=0 is of the same order of magnitude as a lottery played at a later date. This brings us to Hypothesis 4.

> *Hypothesis 4: The risk premium, i.e., the rate of substitution between bond reservation value and lottery reservation value, is independent of when the lottery is played.*

We use the following equation to estimate the dependency of the pure risk premium on the maturity of the lottery.

$$Ln(\frac{CP_B^t}{CP_L^t}) = Ln(1+\lambda) + a*t*Ln(1+\delta) \tag{10}$$

Next, we compare the two sets of rates (bond and lottery) with one another.

> *Hypothesis 5: The time discount rate for a given lottery is lower than the time discount rate for a bond of equal maturity.*

According to Hypothesis 5, $R_L^t = R_B^t$, or that $(\frac{CP_B^0}{CP_B^t}) > (\frac{CP_L^0}{CP_L^t})$.

Furthermore, let α_B and α_L be the alphas from equation (4) for bonds and lotteries respectively, and let β_B and β_L similarly be the betas from equation (4). Then, Hypothesis (5) implies that $\alpha_B > \alpha_L$ and $\beta_B > \beta_L$.

We suggest three alternative explanations to hypothesis (5), as follows:

(1) Consistent with Keren and Roelofsma (1995) and Ahlbrecht and Weber (1997), implicit discounting for uncertainty overlaps with delay discounting, and therefore reduces the final level of the delay discount rate. We assume that this effect holds not only for choice tasks (as tested by Keren and Roelofsma, and by Ahlbrecht and Weber), but also for evaluation tasks.

(2) An alternative explanation is the "certainty effect", which was presented in the prospect theory (Kahneman and Tversky, 1979). According to this effect, when an outcome is certain and becomes less probable, it has a greater impact than when the outcome was merely probable before the probability was reduced by the same amount. Delaying rewards is risky because delaying a reward increases the possibility that something will prevent payment (e.g., Stevenson 1986, Green and Myerson 1997, and Myerson et al. 2003). Since a delay is similar to reducing probability, it affects bonds more than lotteries, because of the "certainty effect", implying a larger time discount rate.

(3) Another explanation is the anticipation effect. This effect (Loewenstein 1987) describes additional utility (disutility) associated with delayed consumption of desirable (undesirable) goods. The notion of anticipatory emotions has been studied by Elster and Loewenstein (1992), who define savoring as the positive utility derived from anticipating a desirable outcome, and dread as the disutility derived from anticipating an undesirable outcome. In the gain domain, anticipation may be a source of positive utility, causing individuals to experience the hedonic utility of the future prospect in the current period. These emotions were also studied by Caplin and Leahy (2001, 2004), who extend expected utility theory to situations in which agents experience feelings of anticipation prior to resolution of uncertainty. They demonstrate that these anticipatory feelings may result in time inconsistency and provide an example from portfolio theory to illustrate the potential impact of anticipation on asset prices.

Delaying a lottery can be an example of anticipation of playing or winning a gamble. An example from real life situations can be seen in the number of subjects who

participate in national lotteries even though they pay more than the lottery's expected outcome. The potential feeling of being rich until the lottery is completed (anticipation effect) leads to a lower time discount rate for buying the lottery.

3. Experimental design and data

Two experiments were conducted.

The first experiment.

The participants in the experiment were 64 undergraduate students of economics[1]. The computerized experiment took place in a computer laboratory at Ben-Gurion University, and lasted approximately one hour.

First, the subjects read the instructions[2], including examples, and then the experimenter answered questions. In the instructions, subjects were told that the assets were to be sold and bought from them using a second-price auction. In the case of a buying auction, the subject with the *highest* bidding price will win the auction, but will pay the *second highest* bidding price in the group participating in the auction. In the case of a selling auction, the subject with the *lowest* asking price will win the auction, but will receive the *second lowest* asking price in the group participating in the auction.

The auctions were presented in a random order to avoid any order effect (with the exception of auction 1, which was used for risk attitude measurement). We asked the subjects: (a) to bid the maximum price they are willing to pay (WTP) today for a

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[1] The students were from Ben-Gurion University, Beer Sheba, Israel. In the discussion in Section 5 of the paper, we also refer to experimental sessions run at Frankfurt (Germany)'s Goethe University; see Benzion and Krahnen (1997: Hyperbolic discounting: an alternative explanation, mimeo).

lottery and a sure amount at different times, and (b) to ask the minimum price they are willing to accept today for a lottery and a sure amount they own at different times (WTA). In each auction, subjects received an initial balance. In the case of selling problems, the subjects owned the initial balance and the lottery or the sure amount[3]. Each problem was presented separately, and subjects answered one problem at a time. This procedure was carried out to avoid the possibility of subjects returning to their previous answers while answering the current problem.

The subjects were told that at the end of the experiment, the computer program would randomly divide them into groups of five subjects. Using the second-price auction, the five subjects in each group compete on buying and selling the assets.

To provide concrete incentives, we told all subjects that one of the problems will be randomly selected (at the end of the experiment) and that we will pay them according to their final balance at the due date, i.e., at the actual date when the bond matures. This date may be several weeks after the experimental session. Subjects were asked to return to the experimenter on or after the due date to recoup their money. For example, if the subjects won a lottery to be played out in four weeks, the pay day was four weeks after the experiment.

The extra effort to collect the money is one possible factor, which affect the compensation required by individual and the jump. However, in our experiment all the subjects were student in the university during the regular academic year and the extra cost of time is rather small. The effect of forgetting suggested by Green et al (1997) is also a factor. However, in our experiment this effect is not very significant as we

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[2] Translated instructions to be provided upon request.

[3] In order to avoid an income effect in the selling problems, the subjects' initial balance was lower than in the buying problems. We reduced the initial balance in these problems by 70, the lottery's

promised to call (send a SMS or mail) the student on the payment date.

Description of the assets

The assets presented in the first experiment are described in Table 1.

The lottery pays 100 with a probability of 60%, and 25 otherwise (the expected payoff

is 70), and the bond face value is 70, the same as the expected value of the lottery.

The auctions included current bid and ask prices for present and future assets.

<div align="center">**<Insert Table 1 about here>**</div>

Subjects' risk attitude.

Risk aversion was measured by using the ratio between WTP and the expected value

of a lottery, in accordance with Kachelmeier and Shehata (1992), Anderhub et al.

(2001), and Holt and Laury (2002).

The lottery used to measure risk aversion pays 80 with a probability 70% and 30

otherwise; it thus differs from the lotteries in the other sessions. This lottery's

expected value E(L) equals 65.

The risk aversion Index RA is defined as follows (see Anderhub et al. 2001, and

Krahnen et al. 1997 for a discussion of this measure of risk aversion):

$$RA = \frac{E(L) - WTP_1}{E(L)}$$

The second experiment.

The second set of experiments was carried out at Frankfurt University. Subjects were

asked to specify their reservation price ("quotes") for either buying or selling a zero

coupon bond maturing at a specified date in the future. Each decision ("round") is

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expected value.

treated separately, and there is no carry-over from one round to the next. If a task

consists of purchasing the security, the subject is endowed initially with a sufficient

amount of cash; if the task requires selling the security, the subject is endowed with

one unit of the bond. Subjects had to specify their reservation values for a series of 19

bond transactions, involving purchase and sale transactions. Subjects also performed

some additional tasks related to the purchase of securities with uncertain payoffs

(lotteries), but this paper refers to the bond purchase transactions only.

The face value and the term of the bonds were varied randomly across sessions and

subjects. Terms were between 15 days (minimum) and 360 days (maximum). Face

values ranged from a minimum of 200 monetary units [MU] up to a maximum of 920

units. The tasks were presented on a computer screen, and subjects had to type in their

respective reservations values.

No interaction between participants was allowed. The final payoff was determined by

randomly selecting one round. For inducing revelation of individual reservation

prices, we relied on Becker et al. (1964). A random number was drawn from a

uniform distribution spanning the interval between bond face value minus 50% and

bond face value. This number is the relevant transaction price of that round. If the

subject's task was to purchase [sell] the specified bond, the exchange of money

against the bond became effective if and only if the quote entered by the subject, i.e.,

the reservation price, turned out to be above [below] the transaction price. Each

subject received the monetary equivalent of the bond in his or her possession, plus the

remaining cash. The exchange rate of Monetary Units into Deutschmarks was 20:1.

Payoff in cash was effective immediately after the end of the session, while bonds

were paid out only at the actual maturity of the asset.

We conducted four experimental sessions with a total of 47 participants, yielding 47

independent observations of 19 variables. Subjects were always offered the opportunity "to opt out", i.e. to refuse to answer any single question. The opportunity cost of not complying with a specific task was a 30-second waiting period before the next task was presented to the subject. To limit the impact of typing errors or misunderstandings on the results, we eliminated all values that differed by more than 1.5 standard deviations from the mean of each variable[4]. There were 59 such observations in the data, equaling 8.6 % of our data points. Table A-1 (Appendix A) summarizes all bond valuation tasks in the two sessions of the second experiment.

The session for experiment 1 were carried out in May 2002 at Ben-Gurion University, Israel, while the sessions for experiment 2 were carried in October 1996 at Frankfurt University, Germany.

4. Main Results.

We start with Hypothesis 1. Table 2 presents the forward time discounting for the bond and the lotteries in experiment 1.

<center>**<Insert Table 2 about here>**</center>

Using the Wilcoxon Signed Ranks Test, we find that in both assets and in both positions (i.e. buy and sell), the jump rate is extremely high (the rate between current time to week 1) and significantly different (p-value < 0.05) from the forward rates of the next periods (1 to 4 weeks and 4 to 8 weeks). The forward rates do not different significantly from each other. These results are consistent with hypothesis (1).

We next run a regression analysis for each asset in each position, estimating equation (11):

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[4] All observations are in the interval of 90% under a standard normal distribution (μ - 1.645σ,

$$R^t = a_0 + a_1 * t_1 + a_2 * t_2 + a_3 * RA \tag{11}$$

where,

t_1, t_2– are dummy variables for a duration of one week and four weeks, respectively.

Since t_1, t_2, t_3, and the dummy variable for 4 weeks to 8 weeks are linearly related, variable t_3 is excluded from the regression.

a_0 – is the discount rate for 8 weeks.

RA – is the risk aversion index.

The results are presented in Table 3.

\<Insert Table 3 about here\>

The regression confirms the jump discount for the first weeks (a_1) and the lower discounts for the following weeks, which do not differ from zero.

Column 5 in Table 3 shows that the coefficient of risk aversion (a3) is insignificant in the case of lotteries, and mildly significant for bonds. For the case of bonds we find the same result as Anderhub et al. (2001), who found higher discounts for subjects with higher risk aversion, in line with Hypothesis 2.

We now turn to Hypothesis 3, the effect of size on the discount rate. This question is important for our analysis because it is an indirect test of the fixed cost hypothesis. To conduct the analysis, a minimum of variation across asset nominal values and asset terms is needed. For this reason we include the second experiment, as described in section 3. Given the variety of face values and maturities present in the data from the second experiment (see Table A-1 in the appendix), we can use a multivariate

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$\mu + 1.645\sigma$).

regression to isolate the effect of asset size, i.e. face values of bonds, on individual reservation prices.

Table 4 shows implicit interest rates according to equation (12), where T is a vector of maturity dummies, ranging from two weeks up to one year (see appendix), and Beta (β) is the size coefficient. We are interested in the shape of the term structure and the existence of any size dependence. The results are summarized in Table 4.

$$\ln\left(\frac{Facevalue}{Bid}\right) = \alpha + \beta \ln(size) + \gamma T \qquad (12)$$

<Insert Table 4 about here>

Table 4 shows two interesting results. First, the size coefficient is negative and highly significant (p<0.01), suggesting that the time discount rate, after controlling for term differences between bonds, is negatively related to the nominal value of the claim. The higher the face value of the bond, the smaller is the discount *rate* (not necessarily the absolute discount). Second, after controlling for the asset size, the discount rate for different periods is effectively flat, consistent with the idea that the hyperbolic shape of the discount function is due to a fixed effect at time zero for time periods above 30 days. Above 60 days, the implicit discount rate fluctuates moderately around 10%. The coefficients for period 4 to 7 are highly significant. The coefficient for period 2 (16-30 days) is 21%, but is insignificant due to large variability. The estimated coefficients for periods 2 and 3 are neither significantly different from each other nor from zero.

Finally, the fixed term α, which is intended to capture the impact of a fixed forward charge, differs significantly from zero with a mean value of 0.16.

We now turn to Hypothesis 4. The risk premium is independent of when the lottery is

played. Table 5 presents the average prices and the substitution rates: $\dfrac{CP_B^t}{CP_L^t}$ -1.

For the valuation of the bond at t=0, we assume a valuation discount of zero, and we thus take the hypothetical buy price to equal its face value.

<Insert Table 5 about here>

Once again, we use regression (see equation 9) to test whether the substitution rate

$\dfrac{CP_B^t}{CP_L^t}$ -1 is dependent on the asset realization time, where t=0,1,4,8.

We run the regression separately for buying and selling.

The results are:

For purchase decisions (see equation (10)):

$$Ln(\dfrac{CP_B^t}{CP_L^t}) = 0.253 - 0.012 * t \qquad \{\text{R-square} = 0.01, \text{ p=0.16}) \qquad (13a)$$

$$(0.00) \quad (0.159)$$

For sell decisions ((see equation (10)):

$$Ln(\dfrac{CP_B^t}{CP_L^t}) = 0.353 - 0.013 * t \qquad \{\text{R-square} = 0.01, \text{ p=0.14}) \qquad (13b)$$

$$(0.00) \quad (0.14)$$

In both regressions, the coefficient of the realization time does not differ from zero, implying that adjustment for risk happens only once and is independent of the overall term of the security. This is consistent with Hypothesis 4.

We now turn to Hypothesis 5, which claims that the time discount rate for a given

lottery is lower than the time discount rate for a bond of equal maturity.

Table 6 summarizes the weekly discount rates for both assets and both trade

directions, buys and sells. The comparison relies on the Wilcoxon Signed Ranks test.

<p align="center">**<Insert Table 6 about here>**</p>

The table shows that for purchases, the lottery's weekly time discount is lower than

for bonds. However, for the selling position we were not able to reject the hypothesis

that the lottery's time discount equals the bond's time discount. Thus, Hypothesis 4 is

confirmed for sell but not for purchase decisions. A possible interpretation is that the

three effects mentioned when discussing Hypothesis 5, i.e. implicit discounting

overlapping with delay discounting, the "certainty effect" and the "anticipation

effect", are only relevant for the buying position and not for the selling position.

5. Summary and conclusions.

The discounting rate for uncertain delayed rewards combines time discounting and

probability discounting, with probability discounting corresponding to the risk

premium. To estimate the risk premium from our data, we use the bid and ask prices

for a current and future lottery, and compare them to the price of a current and future

bond with the same maturity. Bid and ask prices, in turn, are elicited from a second

price auction in a computerized experiment, using the Becker-DeGroot-Marschak

mechanism. It is important to mention that all data in this paper refer to preference

elicitation, and are not market prices or equilibrium outcomes.

We show that the substitution rate between the bond present values and the lottery

present values is constant, implying a one-time adjustment for risk, independent of the

maturity of the security. Comparing the bids for bonds and lotteries, we observe a significant jump between the current price (time zero) and the next period (period 1), and a relatively small change between prices in later periods. This observation is true both for bid and for ask positions. The jumps in prices translate into high first period (implicit) discount rates, and into relatively low and stable (implicit) forward rates. Our findings to this point are consistent with earlier observations in the literature referring to hyperbolic discounting. Using additional data from a second experiment, we find that the jump discount is negatively related to asset size. This finding, however, is clearly not consistent with the concept of hyperbolic discounting. Thus, if discounting were to obey the idea of a hyperbolic discount function, the jump observed in reservation prices between period zero and period one should be independent of the face value of the bond, and independent of the expected payoff from the lottery. This, however, is not what we found. What, then, is a possible explanation for the jump? We base our interpretation on the observed negative size dependence. The pattern of bid and ask prices observed in our experimental sessions is consistent with the presence of a fixed decision cost effect, i.e. a lump sum cost incurred by the decision-maker at the moment when future claims are evaluated. A lump sum cost is due to a general characteristic of a decision situation involving future choices, e.g. the fact that evaluation of alternatives implies a relatively more complex analysis. The increased complexity may have many dimensions concerning the effort of labor needed to prepare and undertake the decision, including the difficulty of assessing possible outcomes, the identification of a suitable benchmark, the consideration by the decision-maker concerning the chances of survival until maturity, and so on.

Nevertheless, to be consistent with a fixed cost interpretation, the cost incurred by the decision-maker must be related to the choice between present versus future claim, and it must be unrelated to the amount at stake (the size of the asset).

When holding the amount of the asset constant, a fixed cost of switching from a decision setting with current choices to another setting with future choices yields a per-period discount rate that is inversely related to the maturity of the asset. Laibson (1997) has argued that it will always be possible to fit a hyperbolic or a quasi-hyperbolic discount function to the implicit discount rates, provided they are downward sloping (see also Angeletos et al. (2001) on this).

The presence of decision costs has been discussed in a work by Smith & Walker (1993), who give several examples of experimental studies where the omission of generic decision costs leads to biased results. The bias is found to disappear if stakes get high enough to outweigh decision costs. This is consistent with our finding of a diminishing first period jump in the implied discount rate if payouts rise.

The difference between our interpretation of the functional form of the implied discount rates and the prevailing view in the literature is not only a matter of taste. Rather, it has relevant economic implications. First, in the fixed cost interpretation that we favor, the jump disappears if asset size gets large. In economic terms, if we focus on institutional investors who manage large funds, and whose individual transactions are commensurate to their holdings, the difference between i_{nom} and i_{eff} may be negligible (see equation (5)). Second, and with respect to financial markets, most if not all financial transactions relate to cash flow streams that are entirely within the realm of future payments. Thus, all calculations and comparisons refer to points in

time that are no longer affected by the fixed cost argument, i.e. in periods where the implicit discount function has its usual shape.

Third, our fixed cost interpretation still leaves open the question of where the fixed cost comes from. There are several observations in the literature that fit our interpretation, including Tversky's "certainty effect" (1977), and the explanation in Keren and Roloefsma (1999), or in Rubinstein (2003).

Fourth, finding out more about the determinants of the fixed cost effect is an important research agenda, the results of which will also shed light on the role of the institutional environment in fixing the level of decision costs. We conjecture that the emphasis on dynamic inconsistency in much recent work on hyperbolic preferences may be augmented by an emphasis on lowering decision costs for individuals.

For example, Angeletos et al. (2001) argue that utility functions that give rise to hyperbolic discounting will lead to (too) high indebtedness via expensive credit card accounts. In contrast, our alternative interpretation stresses the way financial products influence individual decision costs. Thus, while credit card accounts are specifically designed to keep individual decision costs low, thereby allowing households to increase indebtedness with no discernable increase in (decision) costs, the opposite is true for a classical collateralized bank loan. Hence, we expect households with credit card access to accumulate a disproportionate amount of credit card debt.

The findings of this paper need further analysis, both experimental and empirical. On the experimental side, a detailed investigation of the decision cost function is warranted to enhance our understanding of cost determinants. On the empirical side, the relationship between the type of decision situation, or its framing, and the derived

form of the decision cost function may be analyzed with data from financial product markets.

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Tables

Table 1: <u>The assets in the experiment</u>

Asset	Payment	Realization Time	Expected Value
Lottery	Today	Today	70
	Today	1 Week	70
	Today	4 Week	70
	Today	8 Week	70
Fixed sum	Today	1 Week	70
	Today	4 Week	70
	Today	8 Week	70

Table 2: <u>Forward weekly time discount of lottery and bond</u>.

Position	Period	Bond (R_B^{w-k})	Lottery (R_L^{w-k})
Buy (WTP)	Current To 1 Week	0.38	0.15
	1 Week To 4 Weeks	0.02	0.01
	4 Week To 8 Weeks	0.02	0.02
Sell (WTA)	Current To 1 Week	0.20	0.19
	1 Week To 4 Weeks	0.01	0.01
	4 Week To 8 Weeks	0.02	0.02

Table 3: <u>Regression analysis of forward discount rate for different time periods and risk aversion.</u>

Position/asset	a_0	a_1	a_2	a_3	R Square
Buy Lottery	*0.001* (0.97)	*0.13* (0.00)	*-0.006* (0.872)	*0.11* (0.16)	*0.08* (0.001)
Sell Lottery	*0.06* (0.24)	*0.159* (0.02)	*-0.008* (0.89)	*-0.187* (0.14)	*0.05* (0.02)
Buy Bond	*-0.013* (0.85)	*0.336* (0.00)	*0.034* (0.71)	*0.315* (0.08)	*0.09* (0.00)
Sell Bond	*0.011* (0.73)	*0.157* (0.00)	*-0.015* (0.71)	*0.214* (0.01)	*0.14* (0.00)

[a] Significance levels (p-values) are in brackets.

Table 4: <u>The effect of size and time on the jump function.</u>

Variable	Coefficient	p(Coefficient)

Constant		.16	<.001
ln(size)		-.02	.007
Time dummy variable	16 - 30	.21	.25
	31 - 60	.10	.18
	61 - 120	.12	.002
	121 - 180	.09	.0002
	181 - 270	.11	.000
	271 - 360	.09	.000

[a] Reservation prices from bond purchase quotes are used to estimate average implicit discount rates over different maturities according to equation (12).

Table 5: <u>Bidding prices and substitution rate between bond and lottery prices</u>.

Position	Realization Time	Lottery (CP_L^t)	Bond (CP_B^t)	$\dfrac{CP_B^t}{CP_L^t} - 1$
Buy (WTP)	Spot	53.9	70	0.42
	1 Weeks	48.7	58.6	0.35
	4 Weeks	48.7	55.9	0.31
	8 Weeks	46.1	53.4	0.36
Sell (WTA)	Spot	51.7	70	0.54
	1 Weeks	48.2	62.15	0.54
	4 Weeks	49.2	61.18	0.54
	8 Weeks	46.7	57.56	0.43

Table 6: <u>Bond and lottery time discount.</u>

Position	Period	R_B^t	R_L^t	Wilcoxon Signed Ranks Test. Z (significance)

Buy (WTP)	1 Week	0.38	0.15	1.96 (0.03)
	4 Weeks	0.08	0.04	2.95 (0.00)
	8 Weeks	0.04	0.03	2.22 (0.01)
Sell (WTA)	1 Week	0.2	0.19	0.94 (0.17)
	4 Weeks	0.05	0.03	1.34 (0.09)
	8 Weeks	0.03	0.02	1.12 (0.13)

Appendix A (data sets from second experiment (1997, Frankfurt-Germany)

Maturities	Face values of bonds purchase tasks									Observations
	200	290	410	500	530	650	730	810	920	
15	4	7	8	47	-	3	6	4	9	88
30	3	11	-	-	6	-	-	20	-	40
60	6	-	10	43	3	8	-	9	-	79
90	11	6	-	-	-	3	-	10	8	38
120	-	4	10	-	14	-	10	15	3	56
150	7	4	4	-	-	4	4	-	-	23
180	80	12	-	44	-	13	5	4	-	158
210	3	-	3	-	6	10	2	2	-	26
240	12	6	5	-	-	10	7	-	-	40
270	-	-	7	-	12	2	8	-	4	33
300	2	-	3	-	4	-	-	-	7	16
330	-	10	-	-	-	-	7	-	4	21

360	-	4	8	43	11	-	-	-	-	66
Observations	128	64	58	177	56	53	49	64	35	684
Outliers										