The University of Adelaide
School of Economics

Research Paper No. 2008-01

Wage Posting Without Full Commitment

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This version : September, 2008

Abstract

Wage posting models of job search typically assume that firms can commit to paying workers the posted wage. This paper investigates the consequences of relaxing this assumption. Under "downward" commitment, firms can commit only to paying at least their advertised wage. We show that wage posting is always an equilibrium, although in special cases other equilibria can exist. Surprisingly, the wage posting equilibrium in our economy is identical to the equilibrium when firms can commit to paying exactly their posted wage. When firms cannot even commit to paying at least their advertised wage, equilibrium exhibits job auctions with wage dispersion which generally is not constrained efficient. JEL CLASSIFICATION: E24, J64

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Much recent research in the job search literature has adopted the wage posting model of directed search. Typically in these models, each firm posts a wage, workers look across all the wage postings and then each worker applies to a single vacancy.[1] The successful applicants are then paid the posted wage. Directed search models have multiple appealing properties. First, the structure of the job application game usually allows for a structural matching function to be derived as an equilibrium outcome. These matching functions are policy invariant thus sidestepping a criticism of standard black-box matching functions often used in other labour search models. Second, the baseline wage posting equilibrium is constrained efficient so that no inefficiencies are injected into the model due to lack of microfoundations in the matching process.

However, wage posting models almost always assume that firms commit to paying workers the posted wage. If workers were allowed to ask for a wage that differed from the posted wage, would firms submit to such wage demands? How would this affect the outcomes of standard wage posting models? In this paper we relax the assumption of full commitment to posted wages by allowing workers to ask for more than the posted wage. Specifically, this paper addresses the question of how such a lack of commitment alters the outcomes of standard directed search models. We examine two types of commitment issues : (i) a "downward commitment" case, in which firms can only commit to paying *no less* than the advertised wage, and (ii) a "no commitment" case, in which firms cannot commit to either paying more or less than the advertised wage.

Under standard wage posting models as illustrated in Burdett, Shi, and Wright (2001) and Peters (2000), the unique equilibrium is such that all firms post the same wage, workers randomize across all firms with equal probability and equilibrium is constrained efficient. At first pass it is not clear whether these results carry over to the case where workers can ask firms for more than their posted wage. Without properly articulating the process by which workers compete for jobs through wage demands, it is very difficult to know when workers stop asking for more than the posted wage. Also, without being explicit about the procedures followed by job auctions, it is unclear what the efficiency properties of job auctions will be. Combining these two problems it is then not clear what circumstances will give rise to wage posting equilibria or job auction equilibria along with their corresponding efficiency properties.

[1] Galenianos and Kircher (2006) is an example where workers can apply to multiple job vacancies.

In order to answer these questions our model builds upon the work of Burdett, Shi, and Wright (2001) and Peters (2000). Firms open job vacancies and advertise a wage that they will pay a worker. After looking at all the advertised wages, each worker applies to a single vacancy. In our economy, however, firms cannot commit to paying exactly their advertised wage. Consequently each worker submits a wage demand along with his or her job application. As firms cannot commit to the posted wage, workers have an incentive to demand higher wages. The inability of workers to coordinate their application strategies means that some firms will receive applications from only a few workers, some firms will receive only a single application and other firms will receive no applications. Firms with few applicants may have no choice but to concede to a worker's wage demand. Importantly, workers are unable to commit to not work for less than their demanded wage so if a firm chooses an applicant but rejects the initial wage demand, then the firm and worker bargain over the wage to be paid to the worker. Knowing this, optimal application strategies result in each firm choosing the applicant offering the lowest wage demand and paying this wage to the successful applicant. This allows for the possibility of job auctions, in which firms advertise wages, but the wage bids of competing applicants determines the wage paid to the successful applicant.[2]

Perhaps surprisingly, in the case where firms possess only limited, downward commitment, wage posting is, typically, the only equilibrium outcome. In particular, despite the fact that workers can ask for more than the posted wage, in equilibrium, they choose not to do so. This result obtains because competition between firms causes firms to post wages so high that workers prefer not to risk losing out on a job by asking for higher wages. Furthermore, when firms can commit to paying at least their advertised wage, equilibrium is always constrained efficient in the sense that a social planner that can open job vacancies but cannot direct job applications will choose to open the same number of vacancies as does the competitive economy. This result arises because in submitting their wage bids, workers internalize the effects that they have on the job finding probabilities of other workers. In turn, when firms advertise their guaranteed wages, by understanding the optimal reservation wage bidding strategies of workers, firms internalize the congestion effect that their vacancy imposes on other firms.

In a special case, namely where workers are credibly able to ask for the full amount of

[2]Shimer (1999) and Peters and Severinov (1997) are two examples of auction papers that have a similar flavour to our model.

output created by a match, both wage posting and job auctions exist in equilibrium. In other words, there can be a continuum of equilibria, with different mixes of wage posting and job auctions. All equilibria that consist of job auctions exhibit wage dispersion. In this special case, equilibria with job auctions are also constrained efficient.

When there is no commitment, such that firms cannot commit to anything except having a vacancy, the equilibrium outcome is job auctions and equilibrium exhibits wage dispersion. Typically job auction equilibria are not constrained efficient with the exception, again, of the case where workers have all the bargaining power in the bargaining phase of wage determination.

The results of this paper highlight the robustness of the results from the standard wage posting model and also emphasize conditions under which job auctions and wage posting are equivalent. Kultti (1999), and more recently Eeckhout and Kircher (2008), show that auctions and price posting can coexist in equilibrium which resembles the real indeterminacy result in our paper. However, our paper shows that if workers cannot credibly ask for the full amount of output created by a match, then equilibrium exhibits degenerate wage distributions as firms choose to advertise wages that are so high that workers do not choose to ask for more.[3] Therefore, by pointing out a crucial assumption driving the equilibrium real indeterminacy outcome, our results complement the previous work that finds equivalence between wage posting and job auctions.

The paper proceeds as follows: Section 1 lays out the structure of the job search game that is played between workers and firms and characterizes the equilibrium of a labour market in which firms possess downward commitment. Section 2 examines the efficiency properties of the labour market with downward commitment. Next, Section 3 extends the model to a labour market in which firms possess no commitment to advertised wages. A discussion about the results is carried out in Section 4, and Section 5 provides some concluding remarks.

1 The Economy

The economy consists of U homogeneous, risk-neutral, unemployed workers, each of whom can apply to only one job vacancy, and N homogeneous firms, each of which has the potential to open one job vacancy. We assume that N is a multiple of U, such that

[3]This is the case if, for example, there is bargaining over the wage after the firm and worker meet.

$N = \tau U$ with $\tau >> 1$ so that the number of firms is much greater than the number of workers.[4] The basic production unit of the economy, which consists of one worker and one firm, produces y units of output.

The timing of events in the job search game is as follows. All firms simultaneously decide whether to incur a cost, z, to create a job vacancy, and if a vacancy is created a firm advertises a wage, $\underline{w}$, in an attempt to attract job applicants. An implication of this structure is that a firm does not observe the actions of other firms when making its decision. Workers observe the advertised wages and choose which firm to apply to. When applying to a firm, a worker also submits a wage demand to the firm. All workers submit job applications simultaneously, so no worker observes the application decisions of other workers. Firms look at their pool of applicants and choose a worker to whom its job vacancy is offered. If the firm is indifferent across multiple job applicants, we assume that the firm randomly selects from amongst these worker with equal probability. Once a firm selects an applicant to offer its job, it is unable to recall any other of its applicants. Should a firm not find any of the wage demands that its applicants submit palatable then it chooses an applicant and bargains over the wage to be paid to the chosen applicant. For simplicity of exposition, we take the outcome of the bargaining process to be $\hat{w}$, with $\hat{w} \leq y$. This pins down the maximum wage that a worker can credibly demand during the application phase.[5] Unsuccessful applicants remain unemployed and enjoy consumption of b, which can represent a combination of home production, leisure and unemployment benefits.

In this paper we study the properties of the large economy where N and U are pushed towards infinity. Given the relationship between N and U, this amounts to taking the limit as U tends to infinity. Importantly, as τ is constant, the ratio of workers to firms is retained in the large economy. This allows us to derive the limiting payoff functions for the players and also to obtain explicit matching functions. This is standard in the directed search literature (see Burdett, Shi, and Wright (2001) and Peters (2000)).[6] We

[4]This avoids potential corner equilibria, in which all firms choose to create a vacancy.

[5]In section 4 we examine how $\hat{w}$ might be endogenously determined as a result of ex-post wage bargaining by matched firms and workers. Particularly, suppose that a firm rejects all its applicants' wage demands. Then when the firm selects one of its applicants and bargains over the wage paid to the applicant, the outcome of the bargaining process is a wage, $\hat{w} \leq y$.

[6]The papers referenced above use this technique as a way of showing that the equilibrium in the large economy can be derived as the limit of the finite economy. In this paper we restrict our attention to the properties of the equilibrium of the large economy.

consider only *symmetric* equilibrium in which all firms use the same strategy and all workers use the same strategy. This is justified by the restriction that firms and workers are homogeneous and cannot communicate with each other in order to coordinate their actions, which, given the size of the economy, seems to be a plausible assumption. Thus we solve for the symmetric subgame perfect equilibrium of the job search game of the large economy.

Each firm's wage advertising problem is to choose a minimum guaranteed wage to maximize expected profits taking the strategies of all other players as given. Mixed strategies in wage advertising can be allowed by letting $G(\underline{w})$ be the probability that a firm will advertise a wage less than or equal to $\underline{w}$. The entry decision for the firm is then to choose whether to create a vacancy at a fixed cost, $z \in (0, y - b)$. Let p_v be the probability that a firm chooses to create a vacancy.

A worker's application strategy consists of a function $p(\underline{w})$ and, for each possible advertised wage $\underline{w}$, a distribution $F_{\underline{w}}(w)$. The function $p(\underline{w})$ denotes the probability that a worker will apply to a particular vacancy advertising a minimum guaranteed wage of $\underline{w}$ and the distribution, $F_{\underline{w}}(w)$, denotes the probability that a worker will submit a wage demand weakly less than w when applying to a vacancy with an advertised wage of $\underline{w}$.

In solving for the subgame perfect equilibrium of the job search game and showing its efficiency properties, the following proposition, which is the main result of this paper, will be proven.

Proposition 1 *When firms possess downward commitment the equilibrium of the job search game is always constrained efficient. Furthermore,*

1. *when $\hat{w} < y$, the equilibrium allocations and payoffs for workers and firms are identical to those under a wage posting model where firms can commit to paying their workers exactly the posted wage, and*

2. *when $\hat{w} = y$, there may exist many equilibria. Particularly, when $\hat{w} = y$, equilibrium payoffs to workers and firms are the same at any advertised wage in the interval $[b, w_p]$ for an endogenous threshold, w_p; that is, real indeterminacy exists.*

1.1 Worker Bidding Strategies

We begin by putting some structure on the application strategies of the workers. First note that if at least one vacancy advertises a wage $\underline{w}$, then $p(\underline{w}) > 0$. In order to see this, suppose equilibrium is such that $p(\underline{w}) = 0$ if at least one firm advertises $\underline{w}$. Consider a deviating worker that applies to such a vacancy. Knowing that no other worker will apply to this vacancy, the deviating worker's best strategy is to submit a reservation wage of $\hat{w}$ which is the highest wage a firm will accept. This is the best possible wage that a worker can receive and the deviating worker is guaranteed this payoff. At any other vacancy, there is a probability that another worker will apply, so the expected payoff for a worker from any other vacancy is strictly less than $\hat{w}$.

Now, some structure can be put on workers' wage demand strategies.

Lemma 1 *For any wage $\underline{w}$ in the support of $G(\underline{w})$:*

1. *if $\exists$ an atom in $F_{\underline{w}}(w)$ it can only be at $\underline{w}$,*

2. *if $\exists$ a gap in supp $F_{\underline{w}}(w)$ then it must be that there is an atom at $\underline{w}$ and for a $w^* > \underline{w}$, $F_{\underline{w}}(w)$ is continuous on $[w^*, \hat{w}]$ so that supp $F_{\underline{w}}(w) = \{\underline{w}\} \cup [w^*, \hat{w}]$,*

3. *otherwise, for a $w_m > \underline{w}$, $F_{\underline{w}}(w)$ is continuous and atomless on the interval $[w_m, \hat{w}]$.*

Proof : *Construct this proof in steps :*

1. *First we show that $F_{\underline{w}}(w)$ cannot be degenerate for any wage in $(\underline{w}, \hat{w}]$ for $\underline{w}$ with $p(\underline{w}) > 0$. If $F_{\underline{w}}(w)$ is degenerate at $w' \in (\underline{w}, \hat{w}]$ then suppose a worker considers deviating. By offering a deviating wage $w'' = w' - \epsilon$ for some arbitrarily small ϵ the worker is guaranteed to obtain a job at any firm advertising $\underline{w}$. This results in a discontinuous increase in the deviating worker's expected job finding probability for an arbitrarily small decrease in expected wages so there is a profitable deviation. Taking the limit as $\epsilon \to 0$ produces a profitable deviation. The same logic can be applied to show that there cannot be an atom in $(\underline{w}, \hat{w}]$.*

2. *Suppose that for $\underline{w}$ with $p(\underline{w}) > 0$, supp $F_{\underline{w}}(w)$ contains an open interval so that $F_{\underline{w}}(w)$ has support $[\underline{w}, w'] \cup [w'', \hat{w}]$. In order for a worker to be indifferent between w' and w'' it must be that announcing w' or w'' yields the same expected payoff. However, as neither w' nor w'' can be a mass point, the job finding probabilities are*

identical. Thus as $w'' > w'$, this cannot be an equilibrium mixed strategy. Notice that this argument does not carry over to the case in which there is an open interval in $F_{\underline{w}}(w)$ such that its support is $\{\underline{w}\} \cup [w^*, \hat{w}]$ as there may be an atom at $\underline{w}$.

3. *If there is no atom at $\underline{w}$ and no gaps in the distribution $F_{\underline{w}}(w)$ with $p(\underline{w}) > 0$ then it must be that $F_{\underline{w}}(w)$ is atomless and continuous on $[\underline{w}, \hat{w}]$.* $\square$

Thus the distribution of wage demands takes on one of three forms. First, it can take the form of a standard job auction, in which all workers ask for more than the advertised wage, and the distribution of bids is continuous with no atoms. In this case, workers ignore the advertised wage, and it is as if the firm never advertised a wage at all but rather only announced an open vacancy. Workers will choose to follow this type of bidding strategy at firms that advertise a sufficiently low wage.

The second possibility is that some workers only ask for the advertised wage, but others ask for more. The distribution has an atom at $\underline{w}$, with the rest of the mass distributed continuously across an interval of wages exceeding the advertised wage. In this case, there is a range of wages between the atom and the continuous part of the distribution which are not demanded. This is because workers bidding more than the advertised wage always ask for discretely more in order to offset the discrete reduction in the probability of obtaining the job that goes along with bidding a wage above the advertised wage. We call this an atomic job auction and workers will end up choosing this type of bidding strategy at vacancies with intermediate levels of the advertised wage.

Finally, the distribution can be degenerate, with all of the mass at the advertised wage. This occurs when the minimum wage is so high that no workers submit reservation wage bids that exceed it, as the reduction in the probability of winning the job is too great to be compensated for by any wage that a deviating worker could credibly ask for. This is wage posting, and it will be shown to arise at firms which advertise a sufficiently high wage.

The next three subsections examine the three possible job application subgames and we calculate the expected payoffs to workers and firms *conditional* on choosing to participate in each of these subgames. As vacancies are created prior to the application stage, we fix the number of vacancies to be equal to v. Later, in accounting for entry of vacancies into the labour market, the expected number of vacancies will be $v = p_v N$ where p_v is the probability that a firm chooses to open a vacancy. At the moment though, what is of relevance is ratio $q(\underline{w}) = p(\underline{w})U$ so we will take p_v and v as fixed for now.

1.1.1 Standard Job Auctions

In this subsection we derive the optimal wage bidding strategy for workers, as well as the expected payoffs for workers and firms, for a vacancy where the firm advertises a minimum guaranteed wage in the interval $[b, w_m]$.

Conditional on approaching a given firm and demanding a wage w, the expected payoff to a worker in a standard job auction with a minimum guaranteed wage of $\underline{w} \in [b, w_m]$ is equal to the probability that no other worker shows up at the chosen firm offering a wage lower than w times the worker's bid plus the probability that the worker is undercut by another worker times the payoff from unemployment.[7] In the large economy where we look at the limit as $U \to \infty$, while retaining the $v - U$ ratio, this expected payoff is given by

$$e^{-q(\underline{w})F_{\underline{w}}(w)}w + (1 - e^{-q(\underline{w})F_{\underline{w}}(w)})b.$$

As workers mix across wages in the interval $[w_m, \hat{w}]$, each worker must be indifferent in equilibrium across offering any of these wages. Thus for any wage $w \in [w_m, \hat{w}]$,

$$e^{-q(\underline{w})F_{\underline{w}}(w)}w + (1 - e^{-q(\underline{w})F_{\underline{w}}(w)})b = e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b$$

which can be solved to yield the distribution

$$F_{\underline{w}}(w) = 1 - \frac{1}{q(\underline{w})} \ln\left(\frac{\hat{w} - b}{w - b}\right) \tag{1}$$

and the density

$$f_{\underline{w}}(w) = \frac{1}{q(\underline{w})}\left(\frac{1}{w - b}\right). \tag{2}$$

In order to pin down the bottom end of the support, w_m, it must be that $F_{\underline{w}}(w_m) = 0$ so

$$w_m = e^{-q(\underline{w})}\hat{w} - (1 - e^{-q(\underline{w})})b.$$

The expected utility of participating in a standard job auction and applying to a firm advertising $\underline{w}$ can be calculated as

$$V_s(\underline{w}) = e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b.$$

[7] In a standard auction, the wage demand strategies of the workers will be continuous and atomless, so that the probability that multiple workers submit the same reservation wage is zero.

It is useful to notice that as workers are indifferent over all wage demands in $[w_m, \hat{w}]$ then the expected payoff from partaking in a standard job auction is equal to the expected payoff from asking for the highest possible wage, $\hat{w}$.

It has already been shown that workers mix across all job vacancies in a symmetric equilibrium. Letting R be the expected return from mixing across all job vacancies under the optimal application strategy, the expected return to applying to a job advertising $\underline{w} \in [b, w_m]$ is given by

$$e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b = R.$$

This reveals that each worker's indifference condition requires that the expected queue length obtained by a firm advertising a wage $w \in [b, w_m]$ is $q = \ln\left(\frac{\hat{w}-b}{R-b}\right)$. Notice that the expected queue length at any standard auction is independent of the advertised wage.[8]

The expected profit for a firm participating in a standard job auction and advertising wage $\underline{w}$ can be found to be

$$\begin{aligned}\pi_s(\underline{w}) &= \int_{w_m}^{\hat{w}} q(\underline{w}) f_{\underline{w}}(w) e^{-q(\underline{w})F_{\underline{w}}(w)}(y - w)dw \\ &= (1 - e^{-q(\underline{w})})(y - b) - q(\underline{w})e^{-q(\underline{w})}(\hat{w} - b).\end{aligned}$$

In order to understand this profit function, notice that the first term is equal to the output created by a match less the participation wage necessary to be paid to the worker for employment. The second term is equal to the job finding bonus paid to the worker. Consider the expected wage paid to a worker bidding $\hat{w}$. This is paid out by the firm when the worker demanding $\hat{w}$ is the only applicant. Such an event occurs with probability $q(\underline{w})e^{-q(\underline{w})}$. As workers are indifferent over all wages in the support of $F_{\underline{w}}(\underline{w})$ then the firm is essentially faced with the situation of paying its worker an expected wage bonus of $\hat{w} - b$. Substitution of the firm's expected queue length from conducting a standard auction results in an expected profit function of

$$\pi_s = \left(\frac{\hat{w} - R}{\hat{w} - b}\right)(y - b) - \ln\left(\frac{\hat{w} - b}{R - b}\right)(R - b).$$

Therefore if a firm advertises a minimum guaranteed wage that results in a standard auction, the firm will be unable to affect its expected queue length.

[8] The reason that the expected queue length is independent of the advertised wage under standard job auctions is discussed in detail at the end of section 1.1.2.

1.1.2 Atomic Job Auctions

In this subsection we derive the optimal wage bidding strategy as well as the expected payoffs for workers and firms, for a vacancy where the firm sets a minimum guaranteed wage, $\underline{w}$, such that $\underline{w} \in [w_m, w_p]$. When the advertised wage falls in this interval, the result is an atomic job auction.

Remark 1 *In an atomic job auction the wage strategy of the workers, $F_{\underline{w}}(w)$, admits an atom at the firm's minimum guaranteed wage and for an endogenously determined threshold, $w^*(\underline{w})$, a continuous support on an interval $[w^*(\underline{w}), \hat{w}]$.*

Conditional on approaching a given firm and offering a wage $w > [w^*(\underline{w}), \hat{w}]$, the expected payoff to a worker in an atomic job auction with a minimum guaranteed wage of $\underline{w}$ is equal to the probability that no other worker shows up at the chosen firm offering a wage lower than w times the worker's bid plus the probability that the worker is undercut by another worker times the payoff from unemployment. For bids $w \geq w^*$ this expected payoff is

$$e^{-q(\underline{w})F_{\underline{w}}(w)}w + (1 - e^{-q(\underline{w})F_{\underline{w}}(w)})b$$

and for bids equal to the advertised minimum wage, this expected payoff is

$$\frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})}\underline{w} + \left[1 - \frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})}\right]b.$$

As the workers mix across the minimum guaranteed wage, $\underline{w}$, as well as wages in the interval $[w^*(\underline{w}), \hat{w}]$, each worker must be indifferent in equilibrium across offering any of these wages. Thus for any wage $w \in [w^*(\underline{w}), \hat{w}]$,

$$e^{-q(\underline{w})F_{\underline{w}}(w)}w + (1 - e^{-q(\underline{w})F_{\underline{w}}(w)})b = e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b \tag{3}$$

with

$$e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b = R.$$

The indifference condition in equation (3) can be solved to yield the wage demand distribution and density functions, which take the same form as in a standard job auction (equations (1) and (2), respectively).

In order to pin down the bottom end of the support, $w^*(\underline{w})$, it must be that $F_{\underline{w}}(w^*) = F_{\underline{w}}(\underline{w})$ so by indifference

$$\frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})}(\underline{w} - b) = e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}(w^*(\underline{w}) - b).$$

Similarly, to pin down the probability with which workers offer to work for the minimum guaranteed wage, $F_{\underline{w}}(\underline{w})$, use the indifference condition between $\underline{w}$ and $\hat{w}$. Then

$$\frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})}(\underline{w} - b) = e^{-q(\underline{w})}(\hat{w}(\underline{w}) - b)$$

which implicitly defines $F_{\underline{w}}(\underline{w})$.

Taking the limit as $F_{\underline{w}}(w)$ approaches one, the upper threshold for advertised wages that bring about atomic auctions, w_p, can be found to be

$$w_p \equiv \frac{q_a e^{-q_a}}{1 - e^{-q_a}}(\hat{w} - b) + b. \tag{4}$$

When firms advertise wages above w_p workers all submit wage demands equal to w_p because the expected gains from demanding any wage above w_p are outweighed by the costs of not obtaining the job.

The expected payoff to the worker from an atomic job auction with minimum guaranteed wage $\underline{w}$ is denoted by $V_a(\underline{w})$. Note that optimal wage demand strategies require mixing across $\underline{w}$ and the interval $[w^*(\underline{w}), \hat{w}]$. Thus the expected payoff from demanding any of these wages must equal the payoff from demanding the highest wage, $\hat{w}$, and so

$$V_a(\underline{w}) = e^{-q(\underline{w})}\hat{w} + (1 - e^{-q(\underline{w})})b.$$

As application strategies call for workers to mix across all job vacancies, in any equilibrium it must be that $V_a(\underline{w}) = R$ for all $\underline{w} \in [w_m, w_p]$. This pins down the expected queue length at any atomic auction to be $q_a = \ln\left(\frac{\hat{w}-b}{R-b}\right)$ which is independent of the advertised wage.

The expected profit for a firm in an atomic job auction with minimum guaranteed wage of $\underline{w}$ is

$$\begin{aligned}
\pi_a(\underline{w}) &= \int_{w^*(\underline{w})}^{\hat{w}} q(\underline{w}) f_{\underline{w}}(w) e^{-q(\underline{w})F_{\underline{w}}(w)}(y - w)dw + \left(1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}\right)(y - \underline{w}) \\
&= (1 - e^{-q_a})(y - b) - q_a e^{-q_a}(\hat{w} - b).
\end{aligned}$$

Thus the expected profit from conducting an atomic auction is independent of the advertised wage for $\underline{w} \in [w_m, w_p]$, so $\pi_a(\underline{w}) = \pi_a$. Substituting the expected queue length, q_a, into the expected profit function π_a it is easily shown that $\pi_a = \pi_s$.

Observe that as $q_a = q_s$, standard and atomic job auctions are identical in terms of payoffs delivered to firms and workers.[9] The reason is that when firms advertise wages in this interval, workers have the incentive to submit reservation wages above the advertised minimum guaranteed wage. This arises because the probability of not obtaining a job by asking for a higher wage is not sufficiently large enough to deter the worker from asking for more. Particularly, workers are willing to demand a wage of $\hat{w}$. For all auctions with a minimum guaranteed wage below w_p, a worker who asks for $\hat{w}$ only obtains the job if no other worker applies to the given vacancy. As the upper end of the wage demand action space is the same at all such auctions, and is in the support of each worker's mixed strategy, this results in all auctions offering workers the same expected payoff. It follows that workers and firms are indifferent across all vacancies offering any minimum guaranteed wage in the interval $[b, w_p]$.

1.1.3 Wage Posting

Consider a firm that is considering advertising a wage in the interval $[w_p, y - z]$. In this case all workers that apply to the deviator choose to submit a reservation wage equal to the posted minimum guaranteed wage. Then the expected payoff to the worker from applying to a firm advertising $\underline{w} \in [w_p, y - z]$ and asking for the advertised wage is

$$V_p(\underline{w}) = \left(\frac{1 - e^{-q(\underline{w})}}{q(\underline{w})}\right)\underline{w} + \left(1 - \frac{1 - e^{-q(\underline{w})}}{q(\underline{w})}\right)b.$$

Again, as workers mix across all vacancies, in any equilibrium it must be that $V_p(\underline{w}) = R$.

1.2 Equilibrium

In solving for the equilibrium when $\hat{w} < y$, it will next be shown that all firms will choose to post a wage sufficiently high so that workers don't ask for more than the advertised wage. Specifically, consider a firm choosing the optimal wage to advertise in the interval $[w_p, y - z]$. This firm solves the problem

$$\begin{aligned} \pi_p^* &= \max_{q_p, \underline{w}_p} \left(1 - e^{-q_p}\right)(y - \underline{w}_p) \\ &\text{s.t.} \\ &\left(\frac{1 - e^{-q_p}}{q_p}\right)\underline{w}_p + \left(1 - \frac{1 - e^{-q_p}}{q_p}\right)b = R. \end{aligned}$$

[9] Since the payoff functions are identical, symmetry implies that all workers apply to all job vacancy advertising a wage in the interval $[b, w_p]$ with equal probability. As a result, the expected queue lengths, $q(\underline{w})$, are identical across all such vacancies.

This problem resembles those faced by firms in standard wage posting models as illustrated by Burdett, Shi, and Wright (2001) and Peters (2000) with the exception that the endogenous lower bound on wages necessary to bring about a wage posting subgame, w_p, potentially constrains the wage choice of firms. As is standard across wage posting models, the firm faces a trade-off between paying a higher wage and obtaining a higher expected queue length.

At an interior solution, the optimal queue length and wage for this firm are given by

$$q_p = \ln\left(\frac{y-b}{R-b}\right) \tag{5}$$

$$\underline{w}_p = \frac{y-b}{y-R}\ln\left(\frac{y-b}{R-b}\right)(R-b)+b \tag{6}$$

with an associated expected profit of

$$\begin{aligned}\pi_p &= (1-e^{-q_p})(y-b) - q_p e^{-q_p}(y-b) \\ &= y-R-\ln\left(\frac{y-b}{R-b}\right)(R-b).\end{aligned}$$

At the corner $\underline{w} = y - z$, it is easy to show that the expected profit from opening a vacancy is negative so this is never an optimal strategy. At the other corner $\underline{w} = w_p$, and optimality requires that the expected queue length at this corner is $q_a = \ln\left(\frac{\hat{w}-b}{R-b}\right)$. The associated expected profit at this corner is then

$$\pi_a = \left(\frac{\hat{w}-R}{\hat{w}-b}\right)(y-b) - \ln\left(\frac{\hat{w}-b}{R-b}\right)(R-b).$$

In order to show that the interior solution is the most profitable option for a firm advertising a wage in $[w_p, y-z]$, consider the derivative of the profit function

$$\pi(\hat{w}) = \left(\frac{\hat{w}-R}{\hat{w}-b}\right)(y-b) - \ln\left(\frac{\hat{w}-b}{R-b}\right)(R-b).$$

As $\pi'(\hat{w}) > 0$ and $\pi''(\hat{w}) < 0$, when $\hat{w} < y$ the interior solution, $\underline{w}_p$, is strictly more profitable than the corner solution, w_p. Furthermore, the expected profit at the corner is identical to the expected profit from any other minimum guaranteed wage in $[b, w_p]$. Therefore, when $\hat{w} < y$, $\underline{w}_p$ is the optimal wage in $[b, y-z]$ for a firm. It is also transparent that when $\hat{w} = y$, the expected profits from offering any minimum guaranteed wage in the interval $[b, w_p]$ are the same as the under wage posting so any advertised wage maximizes expected profits.

It is easy to show that the expected return to the worker is under wage posting is given by

$$V_p = e^{-q_p}y + (1 - e^{-q_p})b.$$

The following Lemma summarizes the findings so far.

Lemma 2 *For cases $\hat{w} < y$, a firm's expected profit is maximized by advertising the minimum guaranteed wage, $\underline{w}_p$. This results in all workers who approach the deviating firm submitting a reservation wage of $\underline{w}_p$. For the case $\hat{w} = y$, firms are indifferent between advertising any minimum guaranteed wage in the interval $[b, w_p]$ and all advertisements in this interval maximize expected profits.*

Given the results of Lemma 2, in a symmetric equilibrium where $\hat{w} < y$, the expected queue length for each firm is $q = q_p$ and the unique minimum guaranteed wage is $\underline{w} = \underline{w}_p$. The equilibrium condition for the expected queue length in equation (5) pins down entry probability, p_v. Zero expected profits from opening a vacancy requires that in equilibrium

$$\pi_p - z = 0.$$

This implicitly defines the expected return offered by the labour market to the job searcher, R, so that R satisfies the entry condition

$$y - R - \ln\left(\frac{y-b}{R-b}\right)(R-b) - z = 0.$$

It has been shown in Lemma 2 that firms want to advertise the wage $\underline{w}_p \in [w_p, y - z)$ if possible. It remains to be shown that it is possible for $\underline{w}_p$ to be offered in equilibrium.

Lemma 3 *When $\hat{w} < y$ it feasible for firms to post a wage $\underline{w}_p$.*

Proof : *It has already been shown that when $\hat{w} < y$ firms will choose a wage $\underline{w}_p \in (w_p, y - z)$. Now it is shown that, in equilibrium, $\underline{w}_p < y - z$ so that $\underline{w}_p$ is always feasible. Suppose this is not the case. Then*

$$\frac{q_p e^{-q_p}}{1 - e^{-q_p}}(y - b) + b \geq y - z$$

and regrouping terms

$$(1 - e^{-q_p})z \geq (1 - e^{-q_p} - q_p e^{-q_p})(y - b).$$

In equilibrium $\pi_p = (1 - e^{-q_p} - q_p e^{-q_p})(y - b)$ *and zero profits requires* $\pi_p = z$ *so for* $\underline{w}_p \geq y - z$, *it must be that*

$$e^{-q_p} z \leq 0$$

which is not possible as this would require $q_p \to \infty$ *reducing the job finding probability of a worker to zero. No worker would then apply to such a vacancy, contradicting* $q_p \to \infty$ *as being part of an equilibrium.* $\square$

Given Lemma 3, wage posting is always a feasible option for firms in equilibrium. Furthermore, in a symmetric equilibrium, the queue length adjusts such that the wage posting interior is always feasible. Thus in the symmetric equilibrium, for $\hat{w} < y$, wage posting is always the outcome and this symmetric equilibrium exists.

Importantly, solving the wage posting problem and ignoring the constraint that $\underline{w} \geq w_p$ yields the same equilibrium outcome. This establishes that the equilibrium allocations in our economy are identical to the equilibrium allocations from the standard wage posting model in which firms can commit to paying exactly their posted wages.

By Lemma 2, when $\hat{w} = y$, the expected profits from offering any minimum guaranteed wage in the interval $[b, w_p]$ are the same. Hence all advertisements in this interval maximize expected profits, and there is equilibrium indeterminacy. In order to understand the indeterminacy result, it is important to note the double layer of competition in the economy. In the first tier of competition, workers compete to obtain jobs by offering reservation wages while in the second tier of competition firms compete to attract workers by advertising minimum guaranteed wages. When $\hat{w} = y$, competition amongst workers leads to optimal queue lengths for the firm irrespective of the advertised wage, resulting in firms being indifferent across minimum guaranteed wages. However, when $\hat{w} < y$ competition amongst workers fails to deliver the optimal queue length for the individual firm. In response, each firm adjusts its advertised wage in order to affect its expected queue length resulting in a unique, determinate equilibrium. In the present environment, this means that firms adjust their wages until the expected job finding bonus paid to the worker exactly equals the expected surplus generated by a match.

Figure 1 illustrates the isoprofit and indifference curves in an equilibrium where $\hat{w} < y$. It can be seen that the point of tangency between a firm's isoprofit curve and a worker's indifference curve occurs at a wage that is strictly greater than the highest minimum

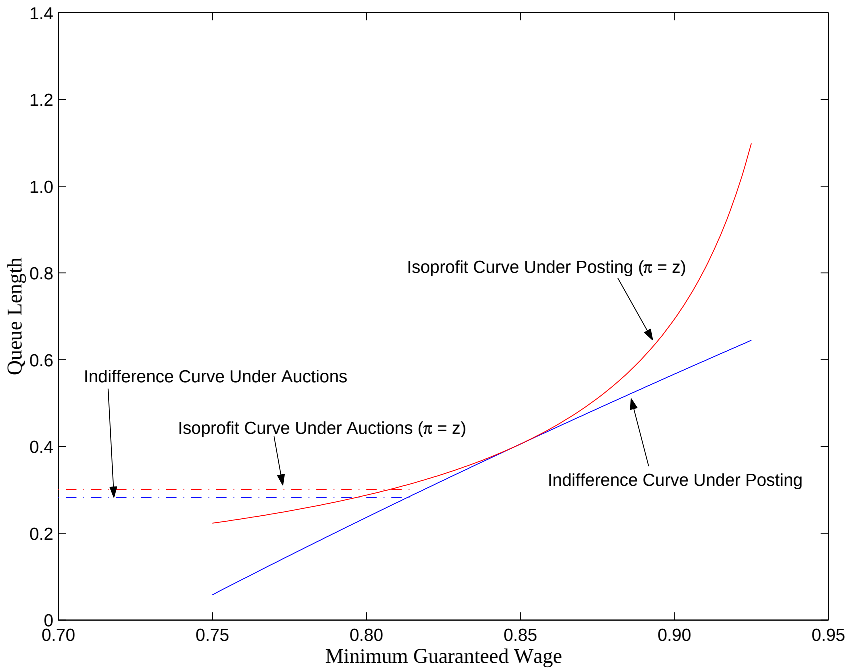

Figure 1: $\hat{w} < y$

guaranteed wage that results in workers being willing to ask for more than the advertised minimum guaranteed wage, w_p. When minimum guaranteed wages are below w_p the relevant portion of the isoprofit curves is the horizontal dashed line. This is because in this interval, workers are willing to submit reservation wages that exceed the minimum guaranteed wage and so the subgame behave like an auction. For minimum guaranteed wages equal to or greater than w_p, workers do not expect to gain from submitting a reservation wage above the advertised wage and so the subgame behaves as in the standard wage posting models. Even though firms cannot commit not paying more than their advertised wage, workers choose not to ask for more. When wages are above w_p then the relevant portion of the isoprofit curves is the segment of the solid isoprofit curve to the right of w_p which lies at the end of the dashed isoprofit curve. By the same logic, when wages are below w_p, the relevant portion of the indifference curves is the horizontal dashed indifference curve. When wages are above w_p, the relevant portion of the indifference curves is segment of the curved solid indifference curve to the right of w_p.

Figure 2 illustrates the isoprofit and indifference curves in an equilibrium where $\hat{w} = y$. In this case point of tangency between a firm's isoprofit curve and a worker's indifference curve occurs at any wage that is less than or equal to the highest minimum guaranteed

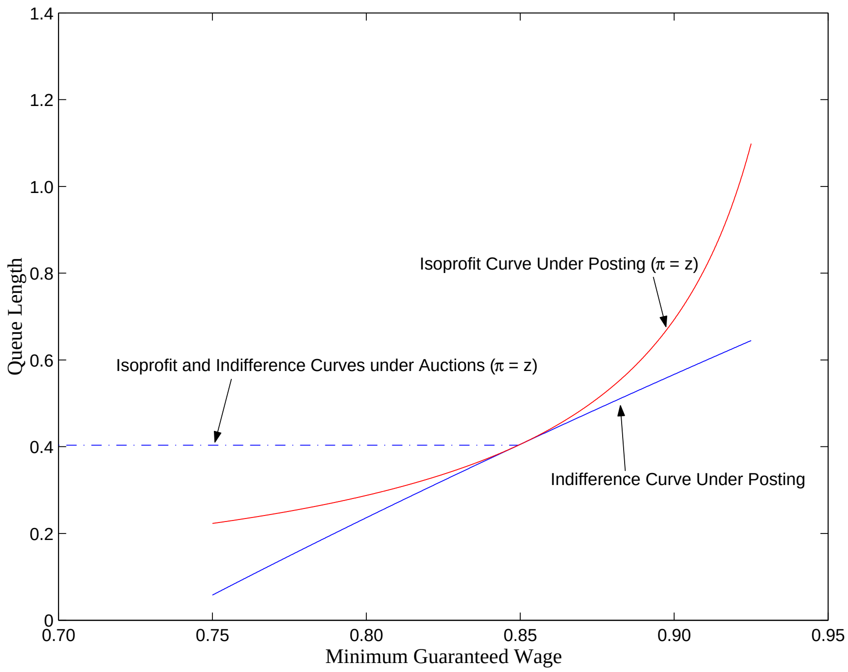

Figure 2: $\hat{w} = y$

wage that results in workers being willing to ask for more than the advertised minimum guaranteed wage, w_p.

2 Efficiency

This section illustrates the efficiency properties of the model. The main result is that equilibria are always constrained efficient when firms possess downward commitment.

2.1 The Social Planner

The social planner chooses vacancies to maximize expected aggregate output. Under the centrally planned economy workers apply to each vacancy with equal probability which minimizes congestion at all vacancies. Thus the job filling probability is given by $1 - e^{-q}$ and the job finding probability is $\frac{1-e^{-q}}{q}$. The planner's problem is given as

$$\max_{v}(1 - e^{-q})yv - vz + \left(1 - \frac{1 - e^{-q}}{q}\right) bu$$

where $q = \frac{u}{v}$. The optimality condition for the social planner is such that

$$(1 - e^{-q^*})(y - b) - q^* e^{-q^*}(y - b) = z$$

where the first term is the expected increase in output from the marginal vacancy and the second term is the expected loss in output from increased congestion caused by the the marginal vacancy on the existing vacancies. Importantly, the congestion externality is such that the social planner only cares about congestion if it means that the marginal vacancy "steals" its worker from another firm which only has a single applicant. If the worker obtained by the marginal firm is taken from another firm that has at least two applicants then no output is destroyed by the creation of this additional vacancy. This interpretation is easily seen as $1 - e^{-q^*}$ is the probability that a vacancy receives at least one worker and $q^* e^{-q^*}$ is the probability that a vacancy receives only one applicant.

2.2 The Decentralized Economy with Wage Posting

In the decentralized economy firms must choose the wage that they post and workers choose their application probabilities based on the vector of posted wages. In equilibrium workers adjust their application probabilities across firms such that all firms offer the same equilibrium return. Let R denote the "market return" that the labour market offers to each worker under optimal application strategies. A firm's problem is to choose its wage (and thus its expected queue length) to solve

$$\begin{array}{ll} \max_{q^d, w^d} & (1 - e^{-q^d})(y - w^d) \\ \text{s.t.} & \left(\dfrac{1 - e^{-q^d}}{q^d}\right) w^d + \left(1 - \dfrac{1 - e^{-q^d}}{q^d}\right) b = R \end{array}$$

where $1 - e^{q^d}$ is the probability that the deviating firm receives at least one job applicant and $\frac{1-e^{q^d}}{q^d}$ is the probability that a worker applying to the deviating firm expects to obtain the job. The optimality conditions for the deviating firm are such that

$$\left(\frac{1 - e^{-q^d}}{q^d}\right) (\underline{w}^d - b) = e^{-q^d}(y - b)$$

along with the worker's indifference condition from the optimal application strategy, which is the firm's constraint. By applying to the deviating firm, a worker is guaranteed b and earns a job finding "bonus" of $\left(\frac{1-e^{-q^d}}{q^d}\right)(\underline{w}^d - b)$. The optimal queue length for the deviating firm requires that the bonus paid to the successful applicant is equal to a fraction of the surplus created by the match, $y - b$, with the weight equal to the probability that no other worker applies to the deviating firm, e^{-q^d}. If we consider $e^{-q^d}(y - b)$ as the marginal increase in surplus created by a small increase in the queue length at the deviating firm, holding constant the queue lengths at other firms, then the condition for optimal queue length requires that the expected job finding bonus paid by the deviating firm equal this gain in surplus.

Rearranging this optimality condition yields a condition on the expected wage payment by the deviating firm

$$(1-e^{-q^d})\underline{w}^d = q^d e^{-q^d}(y-b) + (1-e^{-q^d})b.$$

It can be seen that the expected wage payment by the deviating firm compensates its applicants for the expected increase in congestion that it creates, which is the first term on the righthand side, and covers the successful applicant's loss output from home production, which is given by the second term on the righthand side. Relating expected wages to the congestion externality, $q^d e^{-q^d}$ is the probability that only one worker shows up at the deviating firm. Therefore, the expected wage pays the worker a bonus equal to a share of the additional output created by the match weighted by the probability that the successful applicant is the only worker that applies to the deviating firm. Note that the marginal value of having more than one applicant at the deviating firm is zero. Therefore in considering the appropriate compensation for a worker taking market conditions as given, the wage paid by firms in the symmetric equilibrium is exactly equal to the congestion externality created by a firm by opening its vacancy.

Using the workers' indifference condition over job applications, the expected return from applying to the deviating firm is

$$e^{-q^d}(y-b) + b = R.$$

Given concavity of the profit function, in a symmetric equilibrium $q = q^d$, so

$$\pi_p = (1-e^{-q} - qe^{-q})(y-b)$$

and by free entry

$$(1-e^{-q} - qe^{-q})(y-b) = z$$

which is the same as the social planner's optimality condition. The constrained efficiency of the wage posting economy is a well known property.[10] The reason the decentralized

[10] See Moen (1997) and Peters (2000). Julien, Kennes, and King (2000) illustrate the efficiency properties of a "reservation wage" posting game in which workers post reservation wages and firms direct their search for workers. Their model exhibits ex post worker auctions such that workers that receive multiple job offers force the firms to compete in Bertrand auctions.

economy is constrained efficient is because wage competition between firms in the labour search market internalizes the congestion effect that firms impose on one another. When a firm considers the value of attracting a single worker and the compensation required to get workers to direct their search towards its vacancy, firms essentially decide to part with a portion of the surplus created by a match that equals the value to the firm of obtaining a single applicant. In a symmetric equilibrium, the value to a firm of obtaining a single applicant is equal to the value of a worker to a firm that losses its only applicant. In equilibrium this means that all firms consider the effects that their wage policy has on their queue length which is the equilibrium queue length. Hence in considering the wage that it will have to pay a worker when it makes its vacancy creation decision, each firm internalizes the congestion externality.

2.3 Efficiency of Standard and Atomic Auctions

The equilibrium expected profit from opening a vacancy in either a standard or an atomic job auction is

$$\pi_s = \pi_a = (1 - e^{-q_s})(y - b) - q_s e^{-q_s}(\hat{w} - b). \tag{7}$$

The firm's expected profit is comprised of two components. The expected surplus left after paying its worker the participation cost of b and the job finding bonus paid to the successful applicant. This bonus is equal to the maximum surplus that the worker can extract via wage demand competition and is paid by the firm if only one worker applies to the job vacancy.

Free entry requires that

$$(1 - e^{-q_s})(y - b) - q_s e^{-q_s}(\hat{w} - b) = z. \tag{8}$$

Comparison of the social planner's benchmark with the case of standard and atomic auctions reveals that such auctions are constrained efficient with $\hat{w} = y$.[11]

At first pass, this result is curious as firms cannot offer wages and thus firms do not have access to an instrument which will allow them to internalize the congestion externality. Note, however, that workers are indifferent across all equilibrium bids, which

[11] Auctions are not constrained efficient when $\hat{w} < y$. With downward commitment, firms' advertised wages only lead to job auctions when $\hat{w} = y$. In the next section, we examine the no commitment case, where job auctions occur in equilibrium even when $\hat{w} < y$.

are all equivalent in ex ante payoffs to the bid of $\hat{w}$. By bidding the highest feasible wage, $\hat{w}$, a worker will only secure a job if no other applicant applies to the same vacancy. Therefore, in contemplating its highest possible bid, a worker internalizes the maximum possible surplus it can extract from a firm.

Of course when $\hat{w} = y$ this is the expected value to a firm from having only one applicant. In other words, as long as workers can secure the expected surplus from possibly being the only worker to apply to a firm, wages will reflect the expected value to a firm of having only one applicant. Comparison with the social planner's outcome shows that this equals the congestion externality created by an additional vacancy.

3 No Commitment

When firms cannot commit to paying more or less than their advertised wage, then equilibrium is such that standard auctions arise at all vacancies and advertised wages play no role. The idea is quite simple given the description of equilibrium with downward commitment. We know that with downward commitment no worker asks for more than $\underline{w}_p$ which is the equilibrium wage and is downward binding. Now get rid of downward commitment. Consider a firm that advertises a wage $\underline{w} \in (w_p, \hat{w})$. Suppose equilibrium is such that all workers that apply to this firm demand wages no less than $\underline{w}$. Now consider a single deviating worker that asks for a wage less than $\underline{w}$. In this case the firm is best off hiring this deviating applicant and the applicant obtains the job; a profitable deviation has been constructed. Now it is easy to show that equilibrium is such that all firms can only offer applicants standard job auctions.

As noted in the previous section, auctions are not efficient when $\hat{w} < y$. In such cases, workers are essentially leaving some surplus on the table for the firm whenever a match is formed. Workers then compete over the remaining match surplus and wage competition across workers do not result in wages reflecting the full value of a match to a firm. Due to this, competition to secure jobs amongst workers does not fully internalize the congestion externality so that job auctions are not constrained efficient.

Proposition 2 *In the absence of commitment, the equilibrium only exhibits standard auctions. The equilibrium is constrained efficient only in the special case where $\hat{w} = y$.*

4 Discussion

4.1 Determining the Maximum Wage Demand

In previous sections, we took the maximum wage demand that a worker could credibly obtain, $\hat{w}$, as a parameter. In this section we endogenize $\hat{w}$ as the outcome of ex-post bargaining over wages.

We start by modeling bargaining as an alternating offers game (Rubinstein (1982)). Consider a worker and a firm that play the alternating offers game on the interval $w \in [\underline{w}, y]$. The worker makes the first offer which is denoted $w_W(1)$. If the firm rejects this "wage demand" then it makes a counteroffer, $w_F(2)$, in the second stage. If the worker rejects this offer then it makes its first counteroffer, $w_W(3) \in [\underline{\mathrm{w}}, y]$. Note that once the worker's first offer is rejected, the worker's subsequent counteroffers are not constrained by its original wage demand. Assume that the worker and firm share the same discount factor, $\beta \in [0, 1)$. Once a worker is selected from a firm's job queue the firm cannot recall a different applicant from its queue.

The worker's initial wage demand, which is determined in the cross-worker competition to secure a job, serves as the worker's first offer in the alternating offers game. Here we determine the maximum wage that a worker can ask for as a credible wage demand in the job application stage of the game.

Aside from the worker's initial reservation wage bid, this is the setting considered by Rubinstein (1982), and we can solve for the subgame perfect equilibrium using the approach therein. The solution for the maximum reservation wage is then

$$\hat{w} = \frac{1}{1+\beta} y.$$

Knowing that ex-post bargaining is feasible as a fall-back position, once workers have submitted their wage demands, the firm will reject any wage demand above $\hat{w}$. Therefore, in the first stage of the game, workers will only make offers in the interval $w \in [\underline{w}, \hat{w}]$. Notice that only in the special case where the worker and firm are extremely impatient, $\beta = 0$, will $\hat{w} = y$.

An alternative way to model the alternating offer game is to allow for both parties to incur a fixed cost each time an offer is made. For example, consider the case where the worker incurs a cost of c_w each time an offer is made and a firm incurs an analogous fixed cost of c_f. The first offer is made by the worker in submitting a wage demand during the job application stage of the job search game. It is well known that in such a set-up, the party with the lower cost will extract the entire surplus. In this model, this amounts to $\hat{w} = y$ when the worker's cost is lower than the firm's, $\hat{w}$ equal to the firm's advertised minimum guaranteed wage when the firm's cost is lower than the worker's and $\hat{w}$ being indeterminate but in the interval $[\underline{w}, y]$ when the worker and firm have equal fixed costs. Thus, closing the model with Nash Bargaining in lieu of the alternating offer game will result in $\hat{w} = y$ only if workers have all the bargaining power.

These results suggest that equilibrium indeterminacy, which obtains only in the case where $\hat{w} = y$, is a special case that arises only under extreme assumptions about the nature of either impatience or the distribution of bargaining power in ex-post wage negotiations.

4.2 Ex Post Auctions

Another alteration that could be made to the job search environment is to allow firms to announce that wages will be determined by an ex post Bertrand auction. In this event, if only one worker applied to a firm then the worker would obtain $\hat{w}$ and if a firm obtained multiple applicants then a worker would be randomly selected with equal probability and paid a wage equal to the worker's outside option, b. The expected profits from such an advertisement would be

$$\pi_B = (1 - e^{-q_B})(y - b) - q_B e^{-q_B}(\hat{w} - b) \tag{9}$$

with $q_B = q_s$, which is identical to the expected profits from running a standard auction or an atomic auction. This means that the results carry over to allowing for ex post auctions. The equivalence between ex post auctions and wage posting was brought to light in Kultti (1999) with the difference in his paper being that firms can commit to not paying more than their posted wage. Thus in his set-up firms will always be indifferent between auctions and wage posting and it cannot be the case that firms strictly prefer wage posting.[12]

[12] Recently, Eeckhout and Kircher (2008) have generalized this indeterminacy result between auctions and wage posting in the homogeneous worker and homogeneous firm economy when firms can commit to their posted wage. **?** illustrate that indeterminacy can arise in directed search games if firms can post price schedules that allocate the good depending on the number of buyers that show up ex post.

5 Conclusion

We have presented a simple model in which we examined the role of incomplete commitment in wage posting models. Surprisingly, when firms can only commit to not paying less than their advertised wage, the equilibrium outcome is identical to the wage posting outcome in which firms *can* commit to paying exactly their posted wage. Specifically, all firms offer the same wage, workers all demand the posted wage and equilibrium is constrained efficient. Under a special case, where workers can credibly demand that their wages be equal to all the output created by a match, other constrained efficient equilibria can exist which exhibit wage dispersion.

Importantly, the paper offers a theory of when wage posting will dominate job auctions in equilibrium and hence emphasizes the robustness of the results typically obtained in directed search models. Lastly, when firms lack any kind of commitment, equilibrium only exhibits job auctions that feature wage dispersion and typically are not constrained efficient.

Appendix

A Derivations for Standard and Atomic Auctions

A.1 Standard Auctions

For now, we fix the number of vacancies to be equal to $v = p_v N$ where p_v is the probability that a firm chooses to open a vacancy. At the moment what is of relevance is ratio $q = p(\underline{w})U$ so we will take p_v as fixed for now. Let a worker's wage strategy be given by the distribution $F_{\underline{w}}(w)$ which yields the probability with which a worker will offer a reservation wage below w to a firm advertising a wage $\underline{w}$.

Conditional on approaching a given firm and offering a wage w, the expected payoff from wages of a worker in a standard job auction with a minimum guaranteed wage of $\underline{w} \in [b, w_m(\underline{w})]$ is equal to the probability that no other worker shows up at the chosen firm offering a wage lower than w,

$$\gamma_S(w; U, v, \underline{w})w - (1 - \gamma_S(w; U, v, \underline{w}))b$$

where $\gamma_S(w; U, v, \underline{w})$ is defined as

$$\gamma_S(w; U, v, \underline{w}) = 1 - \sum_{i=1}^{U-1} \binom{U-1}{i} \left(p(\underline{w})F_{\underline{w}}(w)\right)^i \left(1 - p(\underline{w})F_{\underline{w}}(w)\right)^{U-1-i}.$$

Taking the limit as $U \to \infty$ holding fixed the ratio $q(\underline{w}) = p(\underline{w})U$,

$$\gamma_S(w; U, v, \underline{w}) \quad = \quad e^{-q(\underline{w})F_{\underline{w}}(w)}.$$

The probability that a particular worker finds a job is given by

$$\begin{aligned} \int_{w_m}^{\hat{w}} e^{-q(\underline{w})F_{\underline{w}}(w)} dF_{\underline{w}}(w) &= -\frac{1}{q(\underline{w})} e^{-q(\underline{w})F_{\underline{w}}(w)} \mid_{w_m}^{\hat{w}} \\ &= \frac{1 - e^{-q(\underline{w})}}{q(\underline{w})}. \end{aligned}$$

The expected profits of a firm participating in a standard job auction can be found to be

$$\begin{aligned} \pi_s(\underline{w}) &= \int_{w_m}^{\hat{w}} q(\underline{w}) f_{\underline{w}}(w) e^{-q(\underline{w})F_{\underline{w}}(w)} (y - w) dw \\ &= -y \left[e^{-q(\underline{w})F_{\underline{w}}(w)}\right] \mid_{w_m}^{\hat{w}} - \left[we^{-q(\underline{w})F_{\underline{w}}(w)} - (\hat{w} - b)e^{-q(\underline{w})} \ln(w - b)\right] \mid_{w_m}^{\hat{w}}. \end{aligned}$$

Substitute the indifference condition of the worker's bidding strategy, $w_m - b = e^{-q(\underline{w})}(\hat{w} - b)$, to obtain

$$\pi_s(\underline{w}) \quad = \quad (1 - e^{-q(\underline{w})})y - q(\underline{w})e^{-q(\underline{w})}\hat{w} - (1 - e^{-q(\underline{w})} - q(\underline{w})e^{-q(\underline{w})})b.$$

A.2 Atomic Auctions

Conditional on approaching a given firm and offering a wage w, the expected payoff for a worker in an atomic job auction with a minimum guaranteed wage of $\underline{w}$ is equal to the probability that no other worker shows up at the chosen firm offering a wage lower than w,

$$\gamma_a(w; U, v, \underline{w})w - (1 - \gamma_a(w; U, v, \underline{w}))b$$

where

$$\gamma_a(w; U, v, \underline{w}) = 1 - \sum_{i=1}^{U-1} \binom{U-1}{i} \left(p(\underline{w})F_{\underline{w}}(w)\right)^i \left(1 - p(\underline{w})F_{\underline{w}}(w)\right)^{U-1-i}.$$

Taking the limit as $U \to \infty$,

$$\gamma_a(w; U, v, \underline{w}) = e^{-q(\underline{w})F_{\underline{w}}(w)}$$

where $q(\underline{w}) = p(\underline{w})U$. However, at the minimum guaranteed wage, the expected payoff from wages of a worker is

$$\gamma_a(\underline{w}; U, v, \underline{w})\underline{w} - (1 - \gamma_a(\underline{w}; U, v, \underline{w}))b$$

where

$$\gamma_a(\underline{w}; U, v, \underline{w}) = \sum_{i=0}^{U-1} \binom{U-1}{i} (p(\underline{w})F(\underline{w}))^i (1 - p(\underline{w})F(\underline{w}))^{U-1-i} \frac{1}{i+1}$$

which sums over the events in which $i = 0, 1, 2, ...$ other workers show up at the same firm offering the minimum guaranteed wage, $\underline{w}$. In such a case the firm randomizes across this set of workers with equal probability. Taking the limit $U \to \infty$

$$\gamma_a(\underline{w}; U, v, \underline{w}) = \frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})}.$$

The expected profits of a firm in an atomic job auction with minimum guaranteed wage of $\underline{w}$ is

$$\begin{aligned}
\pi_a(\underline{w}) &= \int_{w^*(\underline{w})}^{\hat{w}} q(\underline{w})f_{\underline{w}}(w)e^{-q(\underline{w})F_{\underline{w}}(w)}(y - w)dw + \left(1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}\right)(y - \underline{w}) \\
&= -y\left[e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}\right]\Big|_{w^*(\underline{w})}^{\hat{w}} + \left[we^{-q(\underline{w})F_{\underline{w}}(w)} - \hat{w}e^{-q(\underline{w})}\ln(w)\right]\Big|_{w^*(\underline{w})}^{\hat{w}} \\
&\quad + \left(1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}\right)(y - \underline{w}).
\end{aligned}$$

Then using the indifference conditions that $e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}(w^*(\underline{w}) - b) = e^{-q(\underline{w})}(\hat{w} - b)$ and $\hat{w} - b = \frac{1-e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})e^{-q(\underline{w})}}(\underline{w} - b)$ it is found that

$$\pi_a(\underline{w}) \quad = \quad (1 - e^{-q(\underline{w})})(y - b) - q(\underline{w}) \left(\frac{1 - e^{-q(\underline{w})F_{\underline{w}}(\underline{w})}}{q(\underline{w})F_{\underline{w}}(\underline{w})} \right) (\underline{w} - b).$$

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