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Abstract

In order to improve the small sample performance of the Generalized Empirical Likelihood Kleibergen type tests (GELK), we propose to re-weight the variance of moments matrix with GEL probabilities. Our modification improves GELK significantly by cutting the size distortion in half. Using simulations, we compare the performance of our modified tests with Kleibergen's K-test and the original GELK tests in a dynamic panel setting. As an empirical application, we use the Arellano and Bond's dynamic panel data for 140 UK firms to estimate labor demand. We compare our results with the traditional Wald test to illustrate the practical importance of using tests which are robust to weak instruments in a dynamic panel setting.

Keywords: GELK, weak instruments, dynamic panel, empirical likelihood.

1 Introduction

The linear instrumental variable (IV) model is widely used in the economic literature. However, when the instruments used in this model are weak, the commonly used two step procedures such as the Generalized Method of Moments (GMM) and Two-Stage Least Squares (2SLS) become problematic. Under weak instruments, the standard asymptotics provide a poor approximation to the sampling distribution, the confidence intervals are poor and the traditional Wald type hypothesis tests for the location of the parameters are size distorted and lead to wrong conclusions. Figure (1) illustrates the size distortion for a simple panel AR(1) model which can be viewed as a linear IV model[1]. In this picture the instruments are weak when the parameter on the x-axis is close to 1. This shows that instead of the perceived size of 5%, the actual size of the Wald test statistic can be over 70%, that is the Wald test mistakenly rejects the null hypothesis of significance over 70% of the time when we think it only does it 5% of the time. Hence, over 70% of the time our conclusions are wrong. To remedy this problem the literature has evolved away from point estimation and towards tests that are robust to weak instruments. Such tests include the tests of Anderson-Rubin (1949), Kleibergen (2002, 2004,2005a, 2005b), Moreira (2002), and the Generalized Empirical Likelihood Kleibergen type tests (GELK) of Guggenberger (2003), Caner (2008) and Guggenberger and Smith (2005)[2]. This paper focuses on improving the small sample performance of the GELK tests and provides an empirical example of labour demand model

[1] The data generating process for this model is described in detail in the simulation section.

[2] The Anderson-Rubin test has its disadvantage in the power properties when the number of instruments is large, since its degrees of freedom depend on the number of instruments. The Moreira test is hard to compute for the Generalized Empirical Likelihood (GEL). Thus, we focus on the Kleibergen type of tests including the Kleibergen test and its GEL alternatives.

to illustrate the importance of using tests that are robust to weak instruments.

The Kleibergen test is based on the Continuous Updating GMM framework developed by Hansen (1982). The alternatives to the Kleibergen test are the Generalized Empirical Likelihood (GEL) counterparts developed by Guggenberger (2003) and Caner (2008), which are based on the GEL framework developed by Qin and Lawless (1994), Hansen, Heaton and Yaron (1996), Smith (1997) and Kitamura and Stutzer (1997). The GEL methodology has been proposed specifically to improve the small sample performance of GMM. The higher order properties of the GEL point estimators were studied by Newey and Smith (2000) who showed that the GEL estimators exhibit better small sample performance. They have also reported that the GEL estimators perform better in the case of outliers and errors from fat tailed distributions. This is true since the estimated GEL probabilities reweigh the moments and place less weights on the outliers. Similarly to point estimators, the GELK test has been proposed as an alternative to the original K test in order to improve K tests's small sample performance. Asymptotically, these tests are equivalent but in small samples it is not clear which test performs better and when. As we show in our paper, all of the original tests are considerably size distorted in the case of a simple dynamic panel model. Moreover, the existing GELK tests have not been able to outperform the K test.

This paper mainly consists of three parts. First, we propose a modification to the existing GELK tests by reweighting the variance matrix. We argue that the estimator for the variance of moments used in the existing GELK tests does not use all of the beneficial information from the GEL framework, namely the estimated GEL probabilities. Alternatively, we propose a modification to the GELK statistics in order to improve their size properties. This is accomplished by reweighting their variance estimator with GEL probabilities. Next, we conduct simulations for a simple panel AR(1) model with fixed effects which can be written as a linear IV model. This model is especially interesting because the quality of instruments depends on the unknown parameter of interest. Hence, it is impossible to know in advance whether the instruments are weak. We find that our modification improves three out of the four GELK tests considered. It cuts the original size distortion almost in half. It also makes one of these tests perform better than Kleibergen's K test in terms of size. Last, for an empirical application, we take the model of employment equations based on the Arellano and Bond (1991) data for 140 UK firms observed over the nine-year period of 1976-1984. We compare the performance of the joint significance tests and the confidence intervals for each coefficient. We consider endogenous and exogenous specifications of the model and find the presence of weak instruments. We show that the conclusions one would make from a GMM procedure would be misleading.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 outlines the construction of the K test. Sections 4 and 5 explain the GEL methodology and GELK test procedures. Section 6 provides the modification of GELK tests. Section 7 presents simulations for a dynamic panel model while Section 8 discusses an application to labor demand. Section 9 concludes.

2 Model

Consider the following linear instrumental variable (IV) model

$$\begin{aligned} Y &= X\phi + \varepsilon \\ X &= Z\pi + v \end{aligned} \tag{1}$$

where Y is an $N \times 1$ vector of the dependent variable, X is an $N \times m$ matrix of the independent variables, and Z is an $N \times k$ matrix of the instruments ($k \geq m$), ε and v are the vectors of error terms and ϕ and π are the unknown parameters. The moment conditions hold at the unique value of $\phi = \phi_h$. To estimate the parameters of interest we use the orthogonality-moment conditions for each i

$$E[Z_i'(Y_i - X_i\phi_h)] = 0 \tag{2}$$

where Z_i' is a $k \times 1$ vector of instruments for individual i.

These instruments are valid if π has a fixed full rank. If $\pi = 0$, the instruments are considered to be invalid. The instruments are called weak if $\pi = \frac{1}{\sqrt{N}}C$ where C is a fixed full rank $k \times m$ matrix. In this paper we are particularly interested in the case where the instruments may be weak.

The most common estimation procedures for this model are the two stage least squares (2SLS) and the Generalized Method of Moments (GMM). However, these procedures become problematic in the cases where the instruments are weak. In these cases, the point estimators are biased, the confidence intervals are inaccurate and the test statistics used for testing the estimated coefficients are size distorted.

In order to know whether our inference is reliable, it would be helpful to know if the instruments we are using are strong. Stock, Wright and Yogo (2002) suggest the rule of thumb: if the joint F-statistic is large enough (10 or greater) then we can suppose the instruments are strong. However, this rule of thumb works for the models with only one endogenous variable. When dealing with a set of endogenous variables instead, which is mostly the case in practice, the procedure for testing the quality of instruments becomes complicated and hard to use.

Next, we outline the procedures to compute the K and GELK tests, and suggest some improvements for the latter.

3 Kleibergen Test Overview

3.1 Testing All Parameters Jointly

The Kleibergen test[3] uses the derivative of the objective function for the continuous updating GMM estimator:

$$J(\phi) = \psi'_N V_{\psi\psi}(\phi)^{-1}\psi_N \tag{3}$$

where

$$V_{\psi\psi}(\phi) = \lim_{N\to\infty} E\left[\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\psi}_i(\phi)\bar{\psi}_j(\phi)'\right] \tag{4}$$

[3] For full derivations see Kleibergen (2002, 2004, 2005a)

is the variance of moments, $\psi_i = Z_i'(Y_i - X_i\phi_0)$ are the moments for individual i, $\psi_N = \sum_{i=1}^N \psi_i(\phi)$, and $\bar{\psi}_i(\phi) = \psi_i(\phi) - E[\psi_i(\phi)]$.

The infeasible K-statistic has the form

$$K(\phi_0) = \frac{1}{N}\psi_N' V_{\psi\psi}(\phi_0)^{-1} D_N(\phi_0) \left[D_N(\phi_0)' V_{\psi\psi}(\phi_0)^{-1} D_N(\phi_0)\right]^{-1} D_N(\phi_0)' V_{\psi\psi}(\phi_0)^{-1}\psi_N \quad (5)$$

where

$$\begin{aligned} D_N(\phi) &= \frac{\partial \psi_N}{\partial \phi} - V_{\phi\psi} V_{\psi\psi}(\phi)^{-1}\psi_N \\ &= -\sum_{i=1}^N Z_i' X_i - V_{\phi\psi} V_{\psi\psi}(\phi)^{-1}\psi_N \end{aligned} \quad (6)$$

and

$$V_{\phi\psi} = \lim_{N\to\infty} E[\frac{1}{N}\sum_i \sum_j (\frac{\partial \bar{\psi}_i(\phi)}{\partial \phi})\bar{\psi}_j(\phi)] \quad (7)$$

This K-statistic is distributed χ^2 with m degrees of freedom (for detailed proof see Kleibergen (2004)).

To make this statistic feasible we replace the unknown variance matrices with their consistent estimators. Here, we consider the most common, easily computable choice of the variance estimators:

$$\hat{V}_{\psi\psi}(\phi) = \frac{1}{N}\sum_{i=1}^N \hat{\psi}_i(\phi)\hat{\psi}_i(\phi)' \quad (8)$$

and

$$\hat{V}_{\phi\psi} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial}{\partial\phi}\hat{\psi}_i(\phi)\hat{\psi}_i(\phi)' \\ = \frac{1}{N}\sum_{i=1}^{N}(-Z_i'X_i\psi_i' + \frac{1}{N}\sum_{i=1}^{N}Z_i'X_i)\hat{\psi}_i(\phi)' \tag{9}$$

where $\hat{\psi}_i(\phi) = \psi_i(\phi) - \frac{1}{N}\sum_{i=1}^{N}\psi_i(\phi)$.

Similarly, we obtain a feasible matrix D by substituting the covariance matrices with their estimators from equations (8) and (9):

$$\hat{D}_N(\phi_0) = -\sum_{i=1}^{N}Z_i'X_i - \hat{V}_{\phi\psi}\hat{V}_{\psi\psi}^{-1} \tag{10}$$

Thus, the feasible K-statistic has the following equation:

$$K(\phi_0) = \frac{1}{N}\psi_N'\hat{V}_{\psi\psi}(\phi_0)^{-1}\hat{D}_N(\phi_0)\left[\hat{D}_N(\phi_0)'\hat{V}_{\psi\psi}(\phi_0)^{-1}\hat{D}_N(\phi_0)\right]^{-1}\hat{D}_N(\phi_0)'\hat{V}_{\psi\psi}(\phi_0)^{-1}\psi_N \tag{11}$$

3.2 Testing the Subset of Parameters

In most models, including the empirical application considered in this paper, we are interested in testing each parameter separately rather than in testing all parameters jointly. In this paper's empirical application we test the subset of parameters using the procedures which were proposed by Kleibergen (2005b). Here, we briefly outline these procedures.

First, we decompose our model in the following way:

$$\begin{aligned} Y &= X_1\beta + X_2\gamma + \varepsilon \\ X_1 &= Z\pi_1 + v_1 \\ X_2 &= Z\pi_2 + v_2 \end{aligned} \tag{12}$$

where X_1 and X_2 are $N \times m_1$ and $N \times m_2$ matrices, π_1 and π_2 are $k \times m_1$ and $k \times m_2$ vectors of unknown parameters and ε, v_1 and v_2 are vectors of error terms, such that $m_1 + m_2 = m$, and β is our parameter of interest. To test the hypothesis $H_0 : \beta = \beta_0$ we use Kleibergen's (2005b) results.

As the first step, we use a maximum likelihood estimator (MLE) of γ, $\tilde{\gamma}$. To find this estimator we use the fact that the likelihood is inverse proportionate to the AR statistic, so we maximize the AR statistic over γ:

$$\max_{\gamma} \frac{1}{AR(\beta_0, \gamma)} \tag{13}$$

This is equivalent to numerically solving equation (14) for $\tilde{\gamma}$

$$\frac{2}{s_{\varepsilon\varepsilon}} \tilde{\pi}_2(\beta_0)' Z'(Y - X\beta_0 - X_2\tilde{\gamma}) = 0 \tag{14}$$

where

$$\begin{aligned}
\tilde{\pi}_2(\beta_0) &= (Z'Z)^{-1}Z'[X_2 - (Y - X_1\beta_0 - X_2\tilde{\gamma})\frac{s_{\varepsilon X_2}(\beta_0)}{s_{\varepsilon\varepsilon}(\beta_0)}] \\
s_{\varepsilon\varepsilon}(\beta_0) &= \frac{(Y - X_1\beta_0 - X_2\tilde{\gamma})'M_Z(Y - X_1\beta_0 - X_2\tilde{\gamma})}{N-k} \\
s_{\varepsilon X_2}(\beta_0) &= \frac{(Y - X_1\beta_0 - X_2\tilde{\gamma})'M_Z X_2}{N-k}
\end{aligned} \tag{15}$$

As the second step, we compute the $K(\beta_0)$ statistics

$$K(\beta_0) = \frac{1}{s_{\varepsilon\varepsilon}(\beta_0)}(Y - X_1\beta_0 - X_2\tilde{\gamma})'P_{M_{Z\pi_2(\beta_0)}Z\tilde{\pi}_1(\beta_0)}(Y - X_1\beta_0 - X_2\tilde{\gamma}) \tag{16}$$

where $\tilde{\pi}_1(\beta_0) = (Z'Z)^{-1}Z'[X - (Y - X_1\beta_0 - X_2\tilde{\gamma})\frac{s_{\varepsilon X_1}(\beta_0)}{s_{\varepsilon\varepsilon}(\beta_0)}]$ and $s_{\varepsilon X_1}(\beta_0) = \frac{1}{N-k}(Y - X_1\beta_0 - X_2\tilde{\gamma})'M_Z X_1$.

This statistic's limiting distribution is bounded from above by $\chi^2(m_1)$ distribution (for proof see Kleibergen (2005b)). Thus, using this distribution provides us with a conservative test.

To obtain the confidence intervals we select the grid of possible values of $\beta : \beta_1, ..., \beta_M$ and plot $1 - pvalue(\beta_j)$ for each of the values of β_j, $j = 1, ..., M$. The confidence region is the area above this curve. For example, if one is interested in a 95% confidence set, we draw a 0.95 line across the graph and look for the points of intersection with the 1-pvalue curve. The interval above the curve and between the intersection points is the confidence interval. In some cases it is possible to have empty sets or a whole real line as a confidence set. When the confidence set is unbounded we do not have enough information to say anything about the value of our parameter.

4 GEL point estimator

In this paper we follow the GEL formulation adopted by Guggenberger (2003) and Guggenberger and Smith (2005). The GEL estimators solve the following problem

$$\min_{\phi \in \Phi} \sup_{\lambda \in \Lambda} \hat{P}_{\rho}(\phi, \lambda) \tag{17}$$

where $\hat{P}_{\rho}(\phi, \lambda) = \frac{2}{N} \sum_{i=1}^{N} \rho(\lambda' \psi_{\iota}) - \rho(0)$, $\rho(\cdot)$ is a function of the corresponding estimator from the GEL class, λ is called the Lagrange Multiplier, and the subscript ρ denotes the corresponding name of the estimator.[4]

We focus on three standard estimators of the GEL class with the corresponding function ρ: the Empirical Likelihood estimator (EL) with $\rho(\lambda'\psi_{\iota}) = \ln(1 - \lambda'\psi_{\iota})$, the Exponential Tilting estimator (ET) with $\rho(\lambda'\psi_{\iota}) = -e^{\lambda'\psi_{\iota}}$, and the Continuous Updating Estimator (CUE) with $\rho(\lambda'\psi_{\iota}) = -(1 + \lambda'\psi_{\iota})^2$.[5] Thus,

$$\hat{P}_{EL}(\phi, \lambda) = 2 \sum_{i=1}^{N} \ln(1 - \lambda'\psi_{\iota}(\phi))/N \tag{18}$$

$$\hat{P}_{ET}(\phi, \lambda) = 2 \sum_{i=1}^{N} \frac{-e^{\lambda'\psi_{\iota}(\phi)}}{N} + 2 \tag{19}$$

$$\hat{P}_{CUE}(\phi, \lambda) = 2 \sum_{i=1}^{N} \frac{-(1 + \lambda'\psi_{\iota}(\phi))^2}{2N} - 1/2 \tag{20}$$

The estimation procedure in equation (17) is conducted in two steps. First, we maximize

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[4] We follow Guggenberg's remapping by subtracting $\rho(0)$ for computational ease. Other papers in the literature might not do so. This does not alter the results in any way.

[5] Note that this is not the same estimator as the Continuous Updating GMM. For the circumstances when they are equivalent see Guggenberger (2003).

over the Lagrange Multipliers λ and denote the solution to this maximization problem

$$\lambda(\phi) = \arg\max_{\lambda \in \Lambda} \hat{P}_\rho(\phi, \lambda) \tag{21}$$

where Λ is the domain of λ.[6] Then, to obtain the GEL point estimators, we plug the maximal multipliers from (21) into the original function in (17) and denote it

$$Q_\rho(\phi) = \hat{P}_\rho(\phi, \lambda(\phi)) \tag{22}$$

The function in (22) is the GEL counterpart of the GMM's objective function J and, hence, it is called the GEL objective function. Analogously to GMM, in order to obtain the GEL point estimator we minimize the corresponding objective function Q_ρ with respect to ϕ. However, the tests discussed in this paper are evaluated at the null parameter and, hence, we forgo the second step (which is used to find the point estimator of ϕ) and only use the first step.

5 GELK test

The GELK test is often referred to as the GEL alternative to K-test. The difference is that it is based on the GEL objective function Q_ρ instead of that of the GMM. The original reason for developing GELK tests was the hope that the GEL procedure can detect the outliers (errors from the fat tails) which is a common occurrence in dynamic panels, and put a smaller weight on such data, so that the tests perform better then the K-test above.

[6] In the case of EL, $\Lambda = \{\lambda : \lambda' \psi_\iota < 1\}$ for all i. In all other cases $\Lambda = R^k$.

Following the notation in Guggenberger (2003), we consider two types of his GELK test statistics: Wald and Lagrange Multiplier, denoted as $GELK^{W}$ and $GELK^{LM}$ respectively. Both of these tests are computed by evaluating the first order condition of the GEL objective function in (22) at the null.[7]

The first order derivative is denoted

$$dQ_{\rho} = \frac{\partial \hat{P}_{\rho}(\phi_0, \lambda(\phi_0))}{\partial \phi} \equiv 2\lambda'(\phi)D_{\rho}(\phi)$$

where $D_{\rho}(\phi)$ denotes the derivative of the corresponding GEL function $\rho(\cdot)$. This derivative takes the following expressions for different GEL class estimators:

$$D_{EL}(\phi) = \frac{1}{N}\sum_{i} \frac{1}{1-\lambda'\psi_i(\phi)}(Z_i'X_i) \tag{23}$$

$$D_{ET}(\phi) = \frac{1}{N}\sum_{i} e^{\lambda'\psi_{\iota}} Z_i'X_i \tag{24}$$

and

$$D_{CUE} = \frac{1}{N}\sum_{i}(1+\lambda'\psi_{\iota})Z_i'X_i \tag{25}$$

Thus, Guggenberger's infeasible $GELK_{\rho}^{LM}$ has the following expression

$$GELK_{\rho}^{LM} = N\lambda(\phi_0)'D_{\rho}(\phi_0)\left[D_{\rho}(\phi_0)'V_{\psi\psi}^{-1}(\phi_0)D_{\rho}(\phi_0)\right]^{-1}D_{\rho}(\phi_0)'\lambda(\phi_0) \tag{26}$$

[7] For full derivations see Guggenberger (2003)

Similarly to the Kleibergen's K-statistic,

$$GELK_{\rho}^{LM} \to^{d} \chi^{2}(m) \tag{27}$$

at the null.

Since the expression in equation (26) is based on the Lagrange Multipliers $\lambda(\phi_0)$, this test is referred to as the LM type GELK test. In the simulations section we also denote it as GELKlm.

The Wald-type GELK test is constructed using the following result

$$\lambda(\phi_0) = V_{\psi\psi}^{-1}(\phi_0)\frac{1}{N}\psi_N \tag{28}$$

Plugging the above equation into the expression in (26), we obtain the Wald-type GELK test based on the moments

$$GELK_{\rho}^{W} = \frac{1}{N}\psi_N'(\phi_0)V_{\psi\psi}^{-1}(\phi_0)D_{\rho}(\phi_0)\left[D_{\rho}(\phi_0)'V_{\psi\psi}^{-1}(\phi_0)D_{\rho}(\phi_0)\right]^{-1}D_{\rho}(\phi_0)'V_{\psi\psi}^{-1}(\phi_0)\psi_N(\phi_0) \tag{29}$$

This test is also distributed $\chi^2(m)$ under the null by analogous arguments (for more detailed proofs and derivations see Guggenberger (2003)).

6 GELK Modification

Both of the tests above are infeasible because the covariance matrix is unknown. In order to make them feasible, the previous papers estimate this matrix in the same fashion as in

Section 3 for the K-test. While their approach is theoretically correct, we argue that it does not take advantage of all the benefits of the GEL procedure. In particular, the GEL procedure allows us to compute the variance estimator which is third order efficient (see Newey and Smith (2004)). This variance has been traditionally used in the context of the GEL point estimators, however, it has been overlooked in the context of the robust GELK tests. Thus, we propose to modify the feasible GELK statistic by using the GEL estimated probabilities to reweigh the moments in the variance matrix estimator. To do this, let

$$\hat{p}_i = \frac{\rho_1(\lambda(\phi_0)'\psi_i(\phi_0))}{\sum_{j=1}^{N} \rho_1(\lambda(\phi_0)'\psi_j(\phi_0))} \tag{30}$$

where $\rho_1(\bullet)$ is the first order derivative of the GEL function ρ. Then, the GEL variance estimator is

$$\hat{V}_{\psi\psi}^{GEL}(\phi_0) = \sum_{i=1}^{N} \hat{p}_i(\psi_i(\phi_0) - \bar{\psi})(\psi_i(\phi_0) - \bar{\psi})' \tag{31}$$

where $\psi_i(\phi_0)$ are the moments evaluated at the null, $\bar{\psi} = \sum_{i=1}^{N} \psi_i(\phi_0)$ and $\lambda(\phi_0)$ is the vector of optimal Lagrange multipliers for the null parameter ϕ_0.

Thus, our modified feasible Wald and LM type GELK statistics are:

$$GELK_\rho^W = \frac{1}{N}\psi'_N(\phi_0)\hat{V}_{\psi\psi}^{GEL\,-1}(\phi_0)D_\rho(\phi_0)\left[D_\rho(\phi_0)'\hat{V}_{\psi\psi}^{GEL\,-1}(\phi_0)D_\rho(\phi_0)\right]^{-1}D_\rho(\phi_0)'\hat{V}_{\psi\psi}^{GEL\,-1}(\phi_0)\psi_N(\phi_0) \tag{32}$$

and

$$GELK_{\rho}^{LM} = N\lambda(\phi_0)'D_{\rho}(\phi_0)\left[D_{\rho}(\phi_0)'\hat{V}_{\psi\psi}^{GEL-1}(\phi_0)D_{\rho}(\phi_0)\right]^{-1}D_{\rho}(\phi_0)'\lambda(\phi_0) \tag{33}$$

where

$$D_{\rho}(\phi_0) = \frac{1}{N}\sum_{i=1}^{N}\rho_1(\lambda(\phi_0)'\psi_i(\phi_0))\frac{\partial\psi_i(\phi)}{\partial\phi} \tag{34}$$

7 Monte Carlo Experiment

7.1 Model and Data Generating Process

To demonstrate the impact of reweighting the variance in the GELK tests we use a simple panel AR(1) model which can be viewed as an instrumental variable type model described in equation (1)

$$y_{it} = \alpha_i + \phi y_{it-1} + \varepsilon_{it} \tag{35}$$

where T is small and N is large ($t = 1, ..., T$, $i = 1, ..., N$), $\phi \in [0, 1]$, $\alpha_i \sim N(0, \sigma_{\alpha}^2)$ and ε_{it} are independent across T and N with mean 0 and variance σ_{ε}^2. To cancel out the fixed effects, we take the usual first order difference and obtain the new equation

$$\Delta y_{it} = \phi\Delta y_{it-1} + \Delta\varepsilon_{it} \tag{36}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$ for $t = 2, ...T$ and $i = 1, ..., N$.

Following Arellano and Bond (1991), we take the instruments to be all values of y_{it} with a lag of two periods and more, i.e. $(y_{it-2}, y_{it-3}, ..., y_{i1})$. Thus, the instruments matrix for each individual i is denoted

$$Z_i' = \begin{bmatrix} y_{i,1} & 0 & 0 & 0 & 0 \\ 0 & y_{i,1} & 0 & 0 & 0 \\ 0 & y_{i,2} & 0 & 0 & 0 \\ 0 & 0 & y_{i,1} & 0 & 0 \\ 0 & 0 & y_{i,2} & 0 & 0 \\ 0 & 0 & y_{i,3} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & y_{i,1} \\ 0 & 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & 0 & y_{i,T-2} \end{bmatrix} \tag{37}$$

This model[8] is a particularly interesting case of the linear IV models because the strength of the instruments here depends on the unknown parameter ϕ. When ϕ is close to 1, the instruments are weak and the standard Wald test based on the two stage least squares (2SLS) over-rejects almost 70-80% of the time.

For our simulations we test the hypothesis $H_0 : \phi = \phi_0$ at 5% significance level for different values of $\phi_0 = 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 0.9, 1$ and compare the actual size curves of different test statistics.

[8] To see that this model can be represented as the model in equation (1) let $f_{it} = \Delta y_{it} - \phi \Delta y_{i,t-1}$. The moment conditions for this model are $E[\psi_i(\phi)] = 0$, where $\psi_i = Z_i' f_i$, and $f_i = (f_{i2}', ..., f_{iT}')'$ for $i = 1, ..., N$. We stack all of the time period observations in a vector for each individual and denote $\Delta y_i = (\Delta y_{i3}', ..., \Delta y_{iT}')'$, $\Delta y_{i,-1} = (\Delta y_{i2,...,}' \Delta y_{iT-1}')'$ and $\Delta \varepsilon_i = (\Delta \varepsilon_{i3}, ..., \Delta \varepsilon_{iT})'$. Stacking the observations for all individuals in a vector form, we get $Y = (\Delta y_1', ..., \Delta y_N')'$, $X = (\Delta y_{1,-1}', ..., \Delta y_{N,-1}')'$, $\varepsilon = (\Delta \varepsilon_1', ..., \Delta \varepsilon_N')'$ and $Z = (Z_1', ..., Z_N')'$ which correspond to the same name variables in the instrumental variable model.

For the data generating process we take ε_{it} from a t-distribution with 10 degrees of freedom[9]. We take $\alpha_i = (1-\phi)\mu_i$ and $\mu_i \sim N(0,2)$. To make sure the initial condition is not zero we set $y_{i0} = \alpha_i + \varepsilon_{i0}$ where $\varepsilon_{i0} \sim N(0,1)$. We compute size and power curves for $N = 50$, 100 and 250, and $T = 6$. The number of replications in each experiment is $M = 100000$.

7.2 Simulation Results

First, we present the improvement results for the GELK test statistics. Figure (2) shows the size curves for each test separately when $N = 50$ and $T = 6$. All statistics show a stable size improvement except for the GELKw-ET. On average, the size was improved by over 5% for GELKw-EL, 4% for GELKlm-ET, and over 2% for GELKlm-EL. This improvement tends to cut the original size distortion in half. For example, for the original GELKw-EL the size was 14% and the new GELKw-EL size is only 9% on average.

Since all these tests are asymptotically equivalent, as the sample size increases, the actual size of the tests converges to the nominal size of 5% and the test statistics converge to the same asymptotic value although at a different rate. Figure (3) demonstrates this with the size curves for $N = 100$ and $T = 6$. Here the improvement remains stable for the three tests: GELKw-EL, GELKlm-ET and GELKlm-EL. Since they all converge to the same statistic at the infinite sample size, the improvement for this sample size is slightly smaller than their counterpart when $N = 50$. Figure (4) shows similar results for $N = 250$ and $T = 6$.

[9] We have also performed the same simulations for the errors coming from normal and $\chi^2(2)$ distributions. The results are almost identical to the above ones. Hence we omit their report here. However, it is interesting to note the interpretation of these results. The presence of outliers or errors coming from a thick tailed distribution does not change the results and the ranking remains the same.

Now we compare the performance of all the Kleibergen type of tests: K, GELKw-EL (new), GELKw-ET, GELKw-CUE, GELKlm-EL (new), GELKlm-ET (new), and GELKlm-CUE. Notice that we only include the improved version of the GELK tests since we already know that they perform better than the original ones except for GELKw-ET. Figure (5) shows the size curves for $N = 50$ and $T = 6$. We can see that all of these tests are robust to weak instruments. Their size remains stable across all values of the parameter. Although all of the statistics are still size distorted, they are less so than the original non-improved ones. The GELKlm-CUE shows the smallest size distortion (its size is the closest to the desired 5%). However, this test tends to underreject the null. The rest of the tests still overreject the null. In terms of the size, the GELKw-EL (new) is the best with the average size of about 9%, followed by the K (12.5%), GELKw-CUE and GELKw-ET (13%), and GELKlm-ET (new) (18%), with the GELKlm-EL (new) being the last (22%). Figures (6) and (7) show the same size curves for $N = 100$ and $N = 250$ with $T = 6$ respectively. Here the size decreases for all tests at slightly different rates, converging to the desired 5%, but the ranking remains the same.

As for power, the results exhibit the usual power-size trade off (the smaller the size the smaller the power) for most of the tests. Figure (8) shows the results for $N = 50$ and $T = 6$ while Figures (9) and (10) show the power curves for $N = 100$ and $N = 250$ with $T = 6$ respectively. The GELKlm- EL and GELKlm-ET show smaller power and higher size than K when the null values of $\phi = 0.7$, 0.98, i.e. when the instruments are weak. So the K test is superior to GELKlm-ET and GELKlm-EL in terms of both size and power. When the sample size is increased the K test power becomes visibly higher than the power of the rest of the tests. This power curve has much more curvature.

Given these size and power properties we recommend using the best three tests: GELKlm-CUE, GELKw-EL (new) and K.

8 Application: Labor Demand

For our empirical illustration we use the model of employment demand based on Arellano and Bond (1991) dataset of 140 UK companies observed over the nine-year period of 1976-1984. This is an unbalanced panel data in the sense that some of the firms do not have observations for all of the years in this time period. To deal with this problem, we delete the missing equations and the rows in the instruments matrix that correspond to these equations, and we replace the missing level entries in the IV matrix with zeros.

The model is a more complicated extension of the simple AR(1) panel model we used for our simulations in equation (35). Here, we take two lags of the dependent variable, an AR(2) process, and add three current and lagged independent variables. This particular economic model of employment is taken from Layard and Nickell (1986), and consequently has been tested by Arellano and Bond (1991) using GMM. The dynamic employment equations are of the form:

$$\begin{aligned} n_{it} &= \alpha_i + \gamma_t + \beta_1 n_{i(t-1)} + \beta_2 n_{i(t-2)} + \\ &\quad + \beta_3 w_{it} + \beta_4 w_{i(t-1)} + \beta_5 k_{it} + \beta_6 k_{i(t-1)} + \beta_7 k_{i(t-2)} + \beta_8 ys_{it} + \beta_9 ys_{i(t-1)} + \beta_{10} ys_{i(t-2)} + v_{it} \end{aligned} \tag{38}$$

where n_{it} is the natural log of employment, w_{it} is log of the real product wage, k_{it} is the log of gross capital and ys_{it} is the log of industry output in company i at time period t. The equation also includes a time specific effect γ_t that is the same for all firms, an unobservable firm-specific effect α_i which is permanent across time and an error term v_{it}. The lag structure represents firms' sluggish adjustments and the industry output captures the industry demand shocks.

8.1 Exogenous wages and capital

First, we consider the case where the firms are price takers. Hence, the wages, capital and industry output variables are treated as exogenous.

To estimate this model, we take the first difference

$$\Delta n_{it} = \beta_1 \Delta n_{i(t-1)} + \beta_2 \Delta n_{i(t-2)} + \Delta x_{it}\delta + \Delta v_{it} \tag{39}$$

where $\Delta n_{it} = n_{it} - n_{i(t-1)}$, Δx_{it} is the first difference of the lagged exogenous variables including the time dummies, $\delta = (\beta_3, ..., \beta_{10})'$and Δv_{it} is the first difference of the error terms for $t = 4, ...9$. We lose the three first waves because of the dynamic nature of the model, differencing to avoid the incidental parameter problem and instrumenting.

To instrument for the lagged unemployment variables we take the instruments to be all values of n_{it} with a lag of two periods or more. The matrix of instruments takes the following form for each i :

$$Z_i = \begin{bmatrix} n_{i1} & n_{i2} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \vdots & \Delta x_{i4} \\ 0 & 0 & n_{i1} & n_{i2} & n_{i3} & & 0 & & 0 & \vdots & \Delta x_{i5} \\ \vdots & & & & & \ddots & & & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & n_{i1} & \cdots & n_{i7} & . & \Delta x_{i9} \end{bmatrix} \tag{40}$$

Here, if the exogeneity assumption is valid, the strength of the instruments only depends on the sum of the unknown parameters $\beta_1 + \beta_2$, that is for some values of these parameters the instruments are going to be weak. In such cases the GMM and 2SLS point estimators are biased and the t-statistics are distorted. Without looking at the robust results and comparing them to the results obtained from the two step procedures, we can't determine which case we are facing. Therefore, the point estimators have to be interpreted with caution.

To test the hypothesis that all the coefficients are jointly insignificant we compute the Wald statistic based on the GMM estimator and the best three robust K-type statistics. All of these test statistics have a limiting distribution of $\chi^2(m)$ where $m = 16$ is the number of independent variables including the time dummies, so the 5% critical level here is 26.296. The results are provided in Table (1).

All of the above tests show that the coefficients are jointly significant except for GELKlm-CUE. The latter test tends to underreject and be very conservative. The second best test, GELKw-EL, overrejects slightly. Due to the fact that our tests do not yield uniform results, in this situation the results can be interpreted at our discretion, depending on how conservative we choose to be. The difference between the Wald test and the GELKlm-CUE could be caused by either the fact that the instruments are weak or the fact that the rest of the tests are simply overrejecting. Since we are dealing with many variables, the joint test graph would be multidimensional and hard to interpret. Hence the best way to visualize the results is to look at the p-values for each parameter separately in order to determine if there are weak instruments or if there are other complications.

Table (2) reports the optimal two-step GMM estimators for all the right hand side variables except time dummies, standard errors are shown in parenthesis.

Since robust tests do not yield point estimators, we compare their conclusions with those based on the GMM procedure by investigating confidence intervals. We provide 1-pvalue curves for each of the coefficients, except the time dummies, for the GMM t-statistic and the robust K statistic. Figure (11) shows the $1 - pvalue$ curves[10]. The 95% confidence interval is the area above the curve and between the intersection points of the curve with the 0.95 line.

Figure (11) shows that the GMM and K tests result in different confidence intervals. While the GMM intervals are always well defined and closed, the K-test intervals are not. The first picture on the left shows $1 - pvalue$ curves for the n_{it-1} coefficient. Here, the K test's $1 - pvalue$ curves do not asymptote to 1, indicating that the confidence interval is the whole real line. This means that there is weak identification in the model: either the AR(1) coefficient or at least one of the parameters we projected out using the MLE estimator, or both, are poorly identified. There could be two sources for this problem: (1) the exogeneity assumption is false and we have an endogeneity problem, (2) the GMM instruments are weak. If the exogeneity assumption is valid, the identification in this equation only depends on $\beta_1 + \beta_2$ value. When this value is close to 1 in absolute value, the GMM instruments are weak. But if the exogeneity assumption is false, instrumenting for these variables as if they were exogenous would also introduce weak identification and can influence the $1 - pvalue$ curves of the AR(1) coefficient, that is there is a spill over effect from weak instruments for one of the coefficients onto the identification of the rest of the coefficients. Figure (11) shows that k_{it-1} and k_{it-2} coefficients are also not identified.

To understand better the source of identification problem we look at the long run coefficients by slightly rearranging the original equation (39):

[10] The x-axis label shows the name of the variable for which the coefficient is estimated. The numbers on the x-axis actually represent the coefficient values.

$$\Delta n_{it} = (\beta_1 + \beta_2)\Delta n_{i(t-1)} - \beta_2(\Delta n_{i(t-1)} - \Delta n_{i(t-2)}) + \Delta x_{it}\delta + \Delta v_{it} \quad (41)$$

Table (3) shows the results for GMM estimator with the standard errors in parenthesis. The 1-pvalue curves for this equation are shown in Figure (12).

The first figure on the left shows the confidence intervals for the long-run coefficient $\beta_1 + \beta_2$. If the exogeneity assumption is correct, this model is identified when this coefficient is between -1 and 1 and small. However, here, the K-statistic shows that the confidence interval is empty in the sensible parameter region. Hence, there is no solution in the sensible parameter region, this model is misspecified. The misspecification most likely comes from the fact that the assumption that capital and wages are exogenous is false. For many coefficients here, the K and GMM procedures do not agree on the significance verdict either.

We also have obtained the results for the specification which omits the insignificant dynamics (the insignificance conclusion is derived from K test). Doing so had little change on the long-run properties of the model and did not purge the identification problem. Hence, we do not report these results.

8.2 Endogenous wages and capital

The results in Figures (11) and (12) suggest that exogeneity assumption may not be sensible. Now, we relax this assumption by allowing the wages and capital to be endogenous. To deal with this endogeneity we change the instruments variable matrix to the following:

$$Z_i = \begin{bmatrix} n_{i1} & n_{i2} & w_{i1} & w_{i2} & k_{i1} & k_{i2} & \cdots & 0 & \cdots & 0 & & & & & \vdots & \Delta x_{i4} \\ \vdots & & & & & & \ddots & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & n_{i1} & \cdots & n_{i7} & w_{i6} & w_{i7} & k_{i6} & k_{i7} & . & \Delta x_{i9} \end{bmatrix} \tag{42}$$

where Δx_{it} is a vector that now contains all the exogenous variables excluding wages and capital.

Table (4) shows the results for the joint significance hypothesis. The critical value is 26.296.

Table (5) shows the results for GMM estimators with standard errors in parenthesis.

Figure (13) shows the p-values for equation (39) where wages and capital are now endogenous.

Again, while the GMM confidence intervals are always nice and bounded, the K tests confidence intervals for n_{it-1}, $w_{it}, w_{it-1}, k_{it-2}$ and ys_{it-2} are unbounded, showing a problem with identification.

To get a better idea of the source of identification we look at the long run equation with endogenous wages and capital. Figure (14) shows the $1 - pvalue$ curves while Table (6) shows the GMM estimators.

The long run coefficient $\beta_1+\beta_2$ is not identified along with the coefficients for w_{it}, k_{it}, k_{it-2}. Omitting insignificant variables designated by K-test does not solve the identification problem. So we do not report the results.

This shows that we may be dealing with weak GMM instruments. The time series are too persistent, so the instruments are weak and do not allow us to identify the coefficients. This is a very different conclusion from the one that would result from GMM procedures alone.

9 Conclusion

In this paper we have suggested an improvement for the Generalized Empirical Likelihood Kleibergen type (GELK) tests. This improvement is based on reweighting the variance of moments matrix with GEL probabilities. We have shown that the GELK tests that use the GEL probabilities as weights in the variance of moments matrix perform better in small samples than those that use conventional $1/N$ equal weights.

To evaluate small sample performance of GELK and K tests we performed simulations for a simple dynamic panel AR(1) model with fixed effects. This model is especially interesting because the quality of instruments depends on the unknown parameter of interest. In this setting we found that the GELK's GELKlm-CUE, the improved GELK's GELKw-EL and the Kleibergen's K statistics are the best in terms of their size distortion performance, in that order.

For an empirical application, we have used employment equations to estimate the elasticity of labor demand. We have used data from Arellano and Bond (1991) for 140 UK firms observed over the period of 1976-1984. The empirical case demonstrates the importance of using the tests that are robust to weak instruments, especially when dealing with dynamic panel models. This is because the strength of the instruments depends on the value of the unknown parameters. The Arellano and Bond (1991) data for modeling labor demand of a price-taking firm shows that the optimal GMM instruments are weak and cause the identification problems. Hence, the conclusions based on GMM procedures would be misleading. None of the specifications of Arellano and Bond that we have tested using the Kleibergen test are identifiable. Some of the models are actually misspecified within the sensible parameter region.

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Tables and Figures

Table 1: Test statistics for testing joint significance: Exogenous wages and capital. Short run.

	Test statistic	**Reject**
GELKlm-CUE	20.193	NO
GELKw-EL*	305.85	YES
K	292.59	YES
W-GMM	1771.9	YES

Notes:
(a)* indicates the modified GELK test
(b) The critical 5% value: 26.296
(c) Degrees of freedom: 16

Table 2: GMM Estimates: Exogenous wages and capital. Short run.

Independent Variable	**GMM**
$\Delta n_{i(t-1)}$	0.494 (0.105)
$\Delta n_{i(t-2)}$	−0.083 (0.027)
Δw_{it}	−0.480 (0.077)
$\Delta w_{i(t-1)}$	0.171 (0.096)
Δk_{it}	0.320 (0.048)
$\Delta k_{i(t-1)}$	0.037 (0.050)
$\Delta k_{i(t-2)}$	−0.012 (0.024)
Δys_{it}	0.499 (0.113)
$\Delta ys_{i(t-1)}$	−0.290 (0.150)
$\Delta ys_{i(t-2)}$	−0.012 (0.108)

Notes:
(a) All variables are in logs
(b) Standard errors in parenthesis
(c) Based on 2-step GMM estimator
(d) Exogenous wages and capital, short run

Table 3: GMM Estimates: Exogenous wages and capital. Long run.

Independent Variable	**GMM**
$\Delta n_{i(t-1)}$	0.411 (0.099)
$\Delta n_{i(t-1)} - \Delta n_{i(t-2)}$	0.083 (0.027)
Δw_{it}	−0.480 (0.077)
$\Delta w_{i(t-1)}$	0.171 (0.096)
Δk_{it}	0.319 (0.048)
$\Delta k_{i(t-1)}$	0.037 (0.050)
$\Delta k_{i(t-2)}$	−0.012 (0.024)
Δys_{it}	0.499 (0.113)
$\Delta ys_{i(t-1)}$	−0.290 (0.150)
$\Delta ys_{i(t-2)}$	−0.012 (0.108)

Notes:
(a) All variables are in logs
(b) Standard errors in parenthesis
(c) Based on 2-step GMM estimator
(d) Exogenous wages and capital, long run

Table 4: Test statistics for testing joint significance: Endogenous wages and capital. Short run.

	Test statistic	**Reject**
GELKlm-CUE	17.859	NO
GELKw-EL*	537.03	YES
K	310.84	YES
W-GMM	2850.1	YES

Notes:
(a)* indicates the modified GELK test
(b) The 5% critical value: 26.296
(c) Degrees of freedom: 16

Table 5: GMM Estimates: Endogenous wages and capital. Short run.

Independent Variable	**GMM**
$\Delta n_{i(t-1)}$	0.865 (0.053)
$\Delta n_{i(t-2)}$	−0.106 (0.025)
Δw_{it}	−0.450 (0.110)
$\Delta w_{i(t-1)}$	0.667 (0.106)
Δk_{it}	0.338 (0.070)
$\Delta k_{i(t-1)}$	−0.266 (0.049)
$\Delta k_{i(t-2)}$	−0.037 (0.019)
Δys_{it}	0.687 (0.135)
$\Delta ys_{i(t-1)}$	−0.792 (0.177)
$\Delta ys_{i(t-2)}$	0.214 (0.121)

Notes:
(a) All variables are in logs
(b) Standard errors in parenthesis
(c) Based on 2-step GMM estimator
(d) Endogenous wages and capital, short run

Table 6: GMM Estimates: Endogenous wages and capital. Long run.

Independent Variable	**GMM**
$\Delta n_{i(t-1)}$	0.758 (0.048)
$\Delta n_{i(t-1)} - \Delta n_{i(t-2)}$	0.106 (0.025)
Δw_{it}	−0.450 (0.110)
$\Delta w_{i(t-1)}$	0.667 (0.106)
Δk_{it}	0.338 (0.070)
$\Delta k_{i(t-1)}$	−0.266 (0.049)
$\Delta k_{i(t-2)}$	−0.037 (0.019)
Δys_{it}	0.687 (0.135)
$\Delta ys_{i(t-1)}$	−0.792 (0.177)
$\Delta ys_{i(t-2)}$	0.214 (0.121)

Notes:
(a) All variables are in logs
(b) Standard errors in parenthesis
(c) Based on 2-step GMM estimator
(d) Endogenous wages and capital, long run

Figure 1: Size distortion example: Size curves for Wald Homosckedastic (solid), Wald Heteroskedastic (dotted) and robust K (dashed) tests.

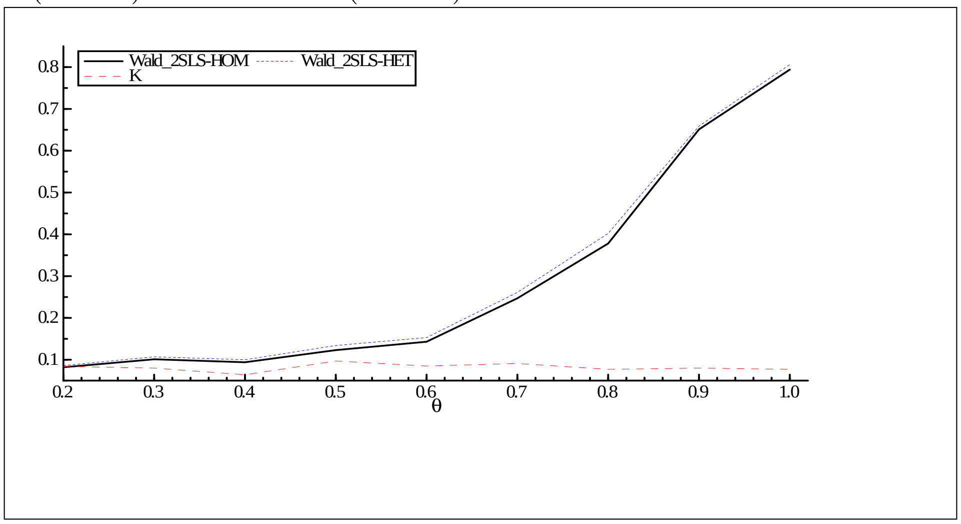

Figure 2: Modified GELK: Size curves for N=50, T=6. Old GELK tests (dashed), New GELK tests (solid)

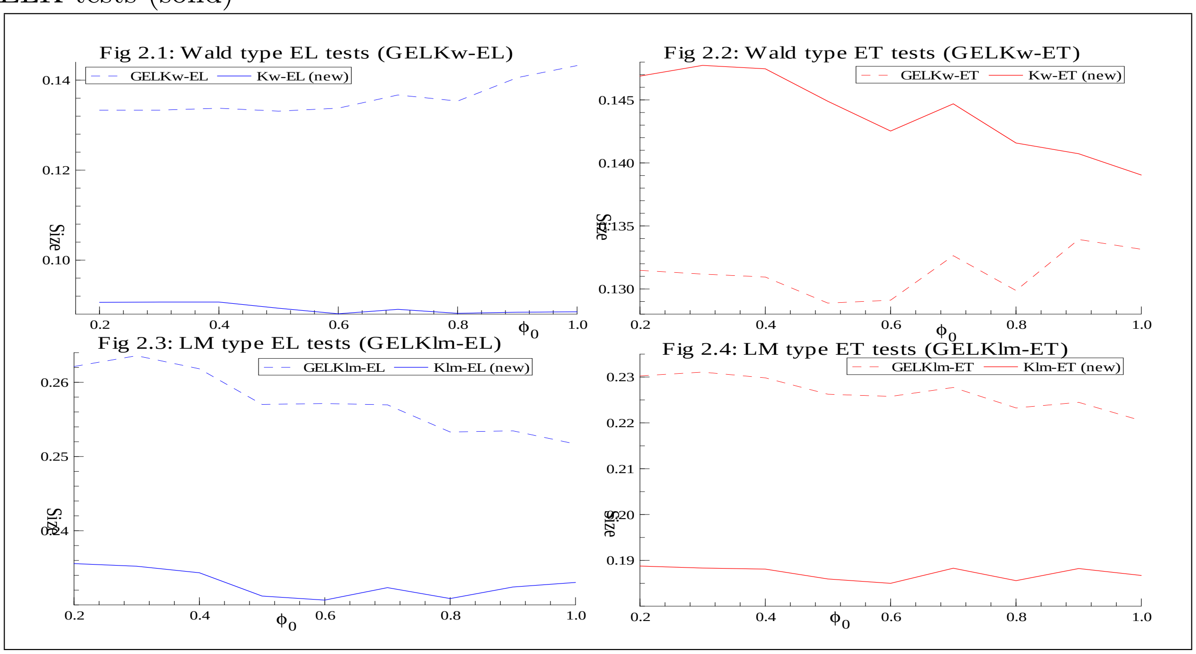

Figure 3: Modified GELK: Size curves for N=100, T=6. Old GELK tests (dashed), New GELK tests (solid)

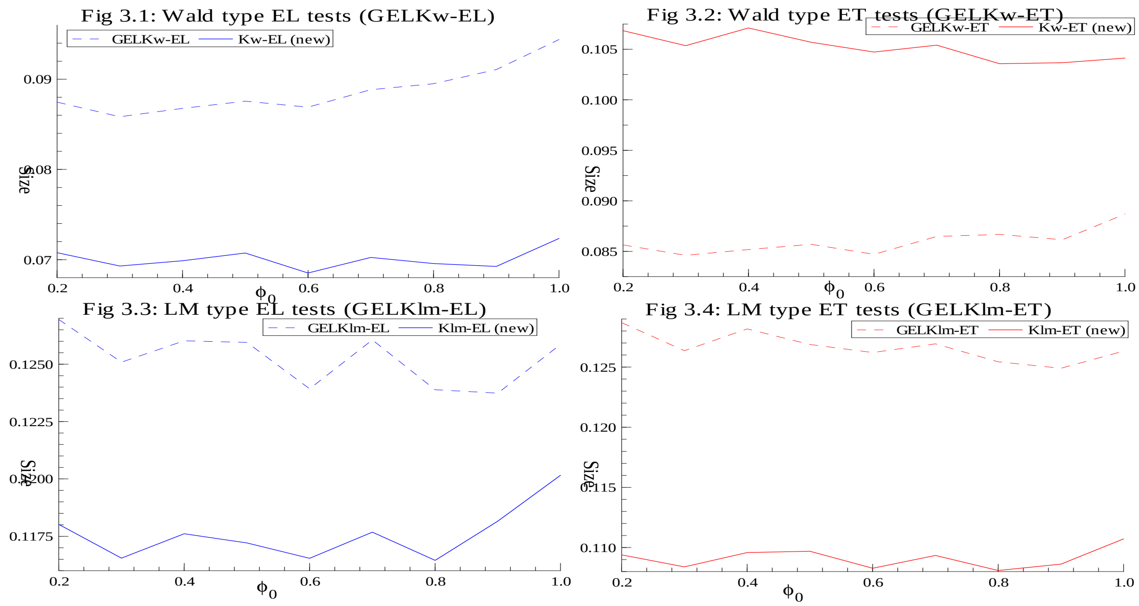

Figure 4: Modified GELK: Size curves for N=250, T=6. Old GELK tests (dashed), New GELK tests (solid)

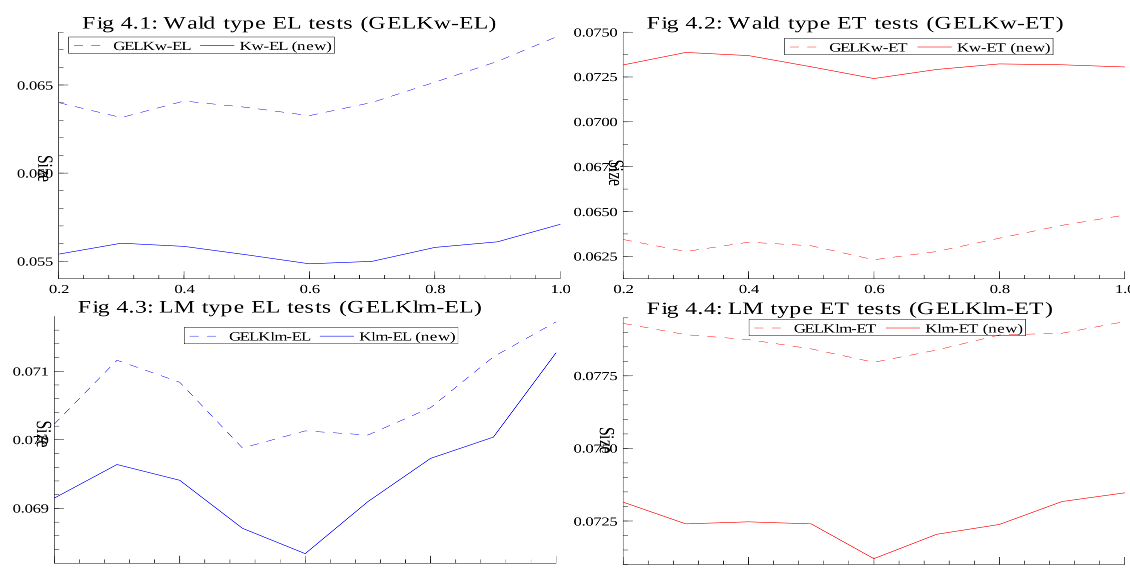

Figure 5: GELK tests comparison: Size curves for N=50, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

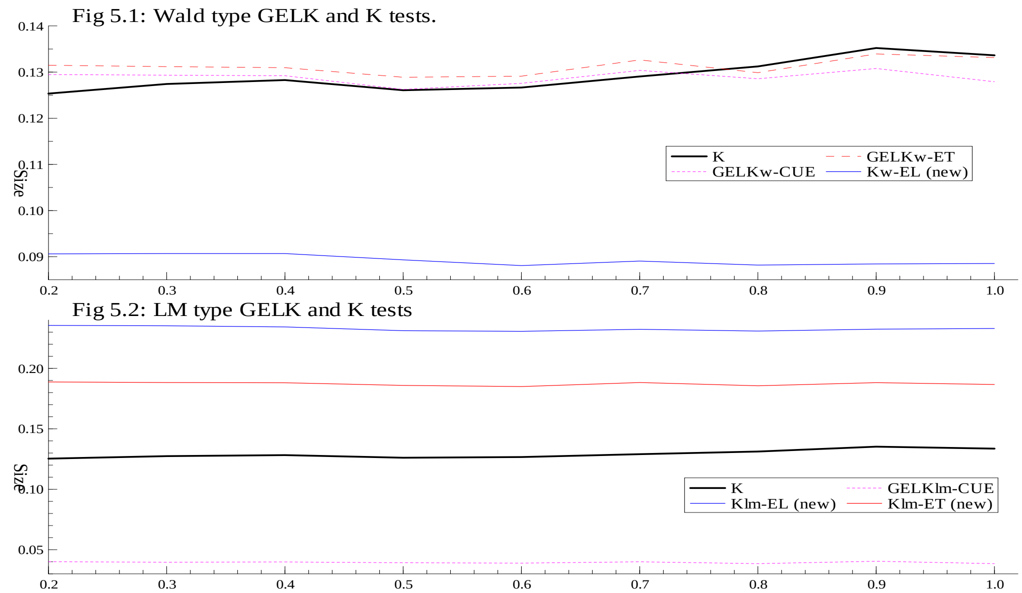

Figure 6: GELK tests comparison: Size curves for N=100, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

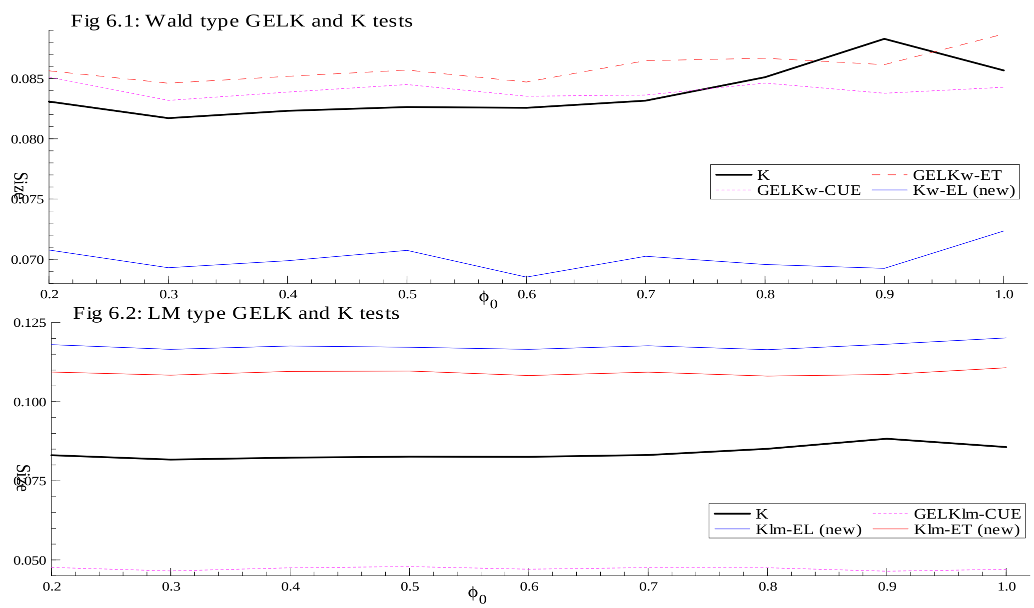

Figure 7: GELK tests comparison: Size curves for N=250, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

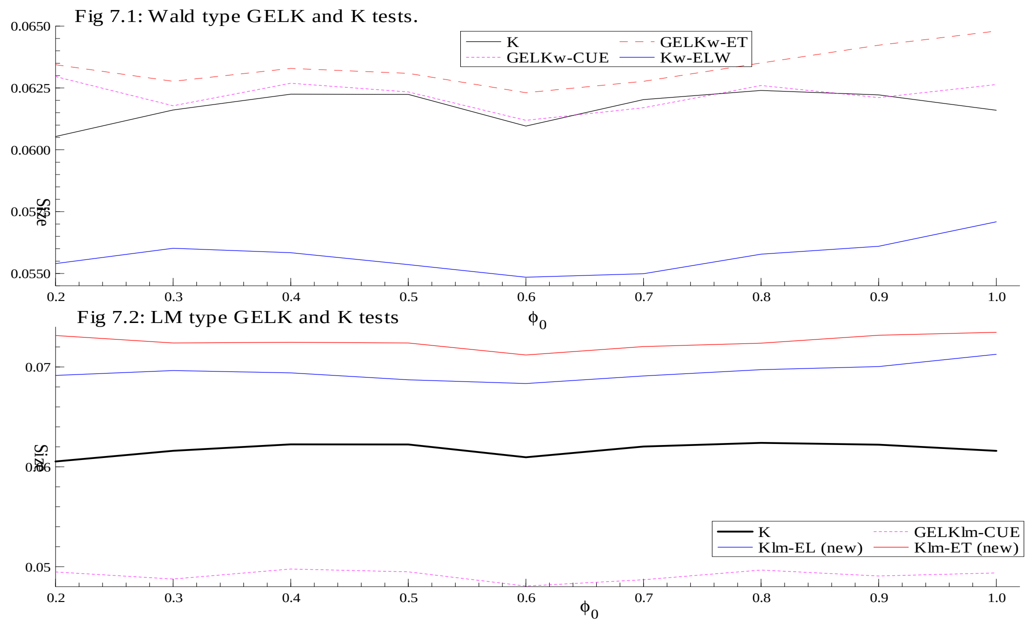

Figure 8: GELK tests comparison: Power curves for N=50, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

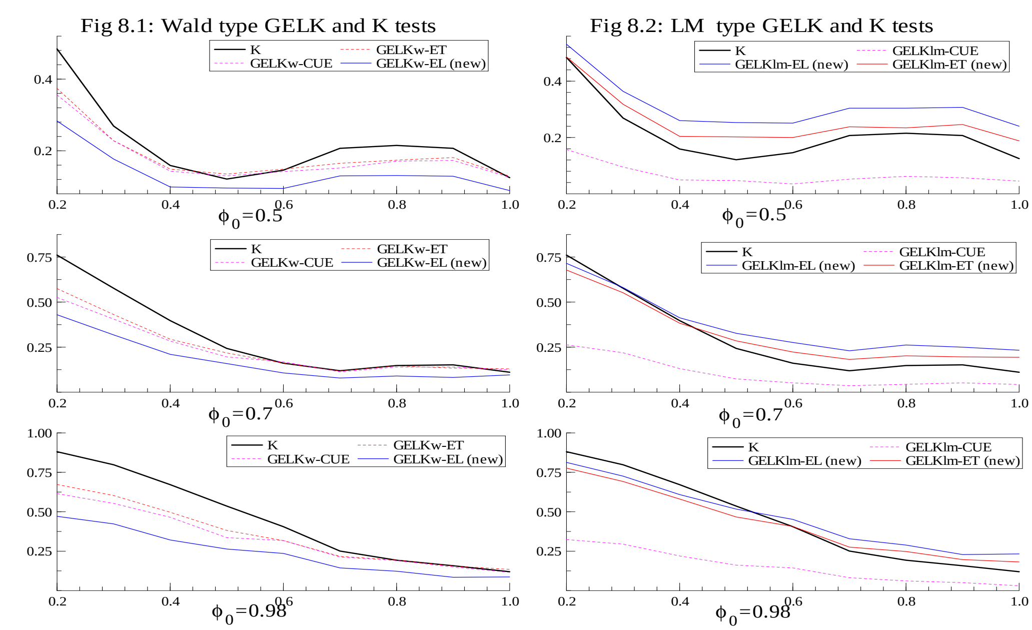

Figure 9: GELK tests comparison: Power curves for N=100, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

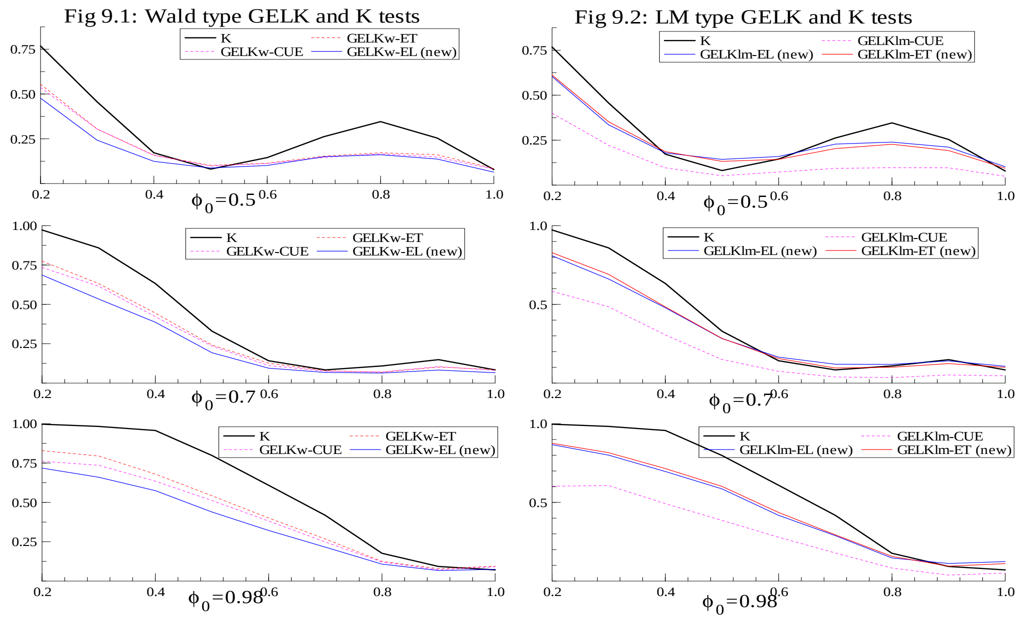

Figure 10: GELK tests comparison: Power curves for N=250, T=6. Old GELK tests (dashed), New GELK tests (solid), K test (black bold)

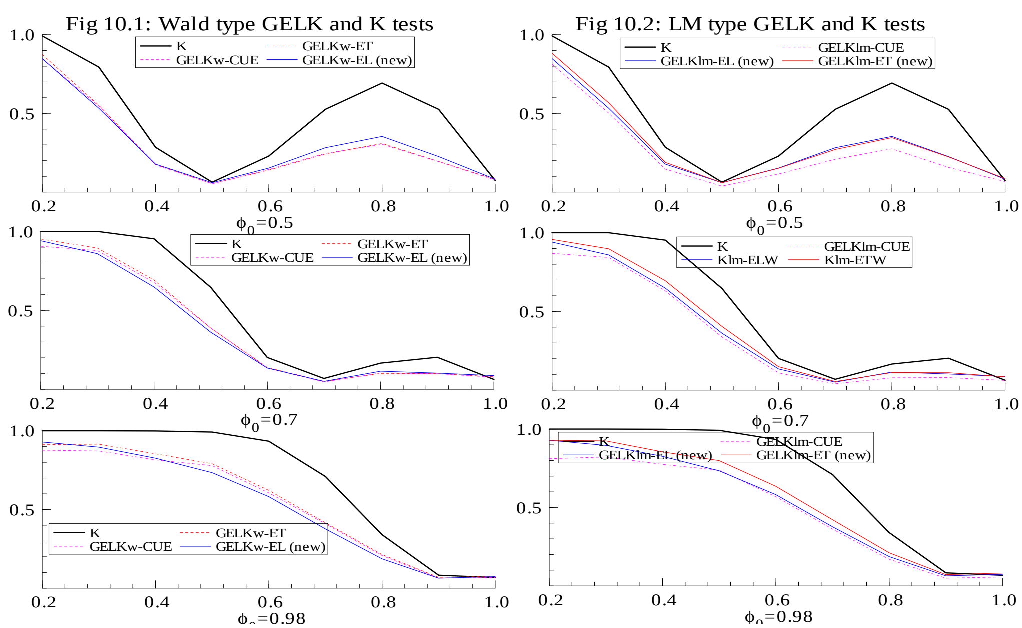

Figure 11: 1-pvalue curves: Exogenous wages and capital. Short run. GMM (solid), K(dashed).

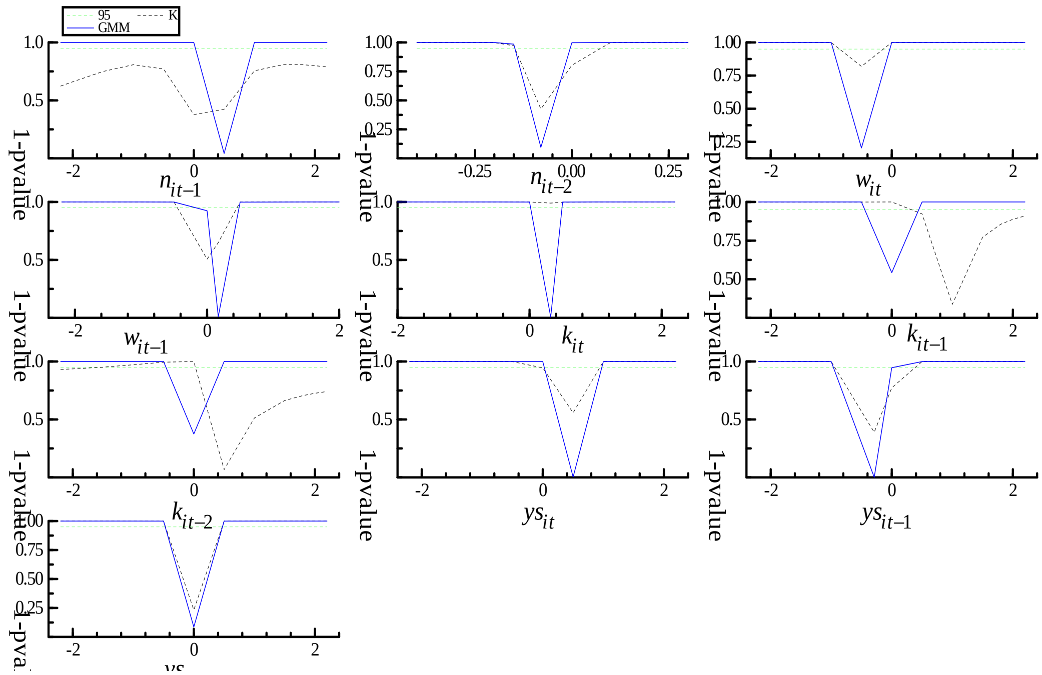

Figure 12: 1-pvalue curves: Exogenous wages and capital. Long run. GMM(solid), K(dashed)

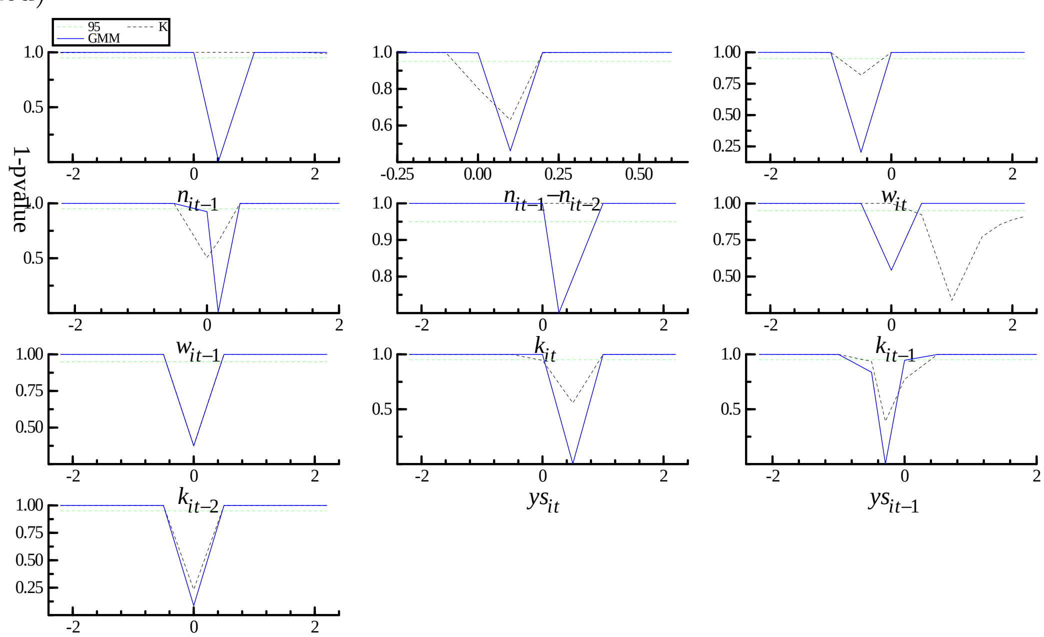

Figure 13: 1-pvalue curves: Endogenous wages and capital. Short run. GMM (solid), K (dotted).

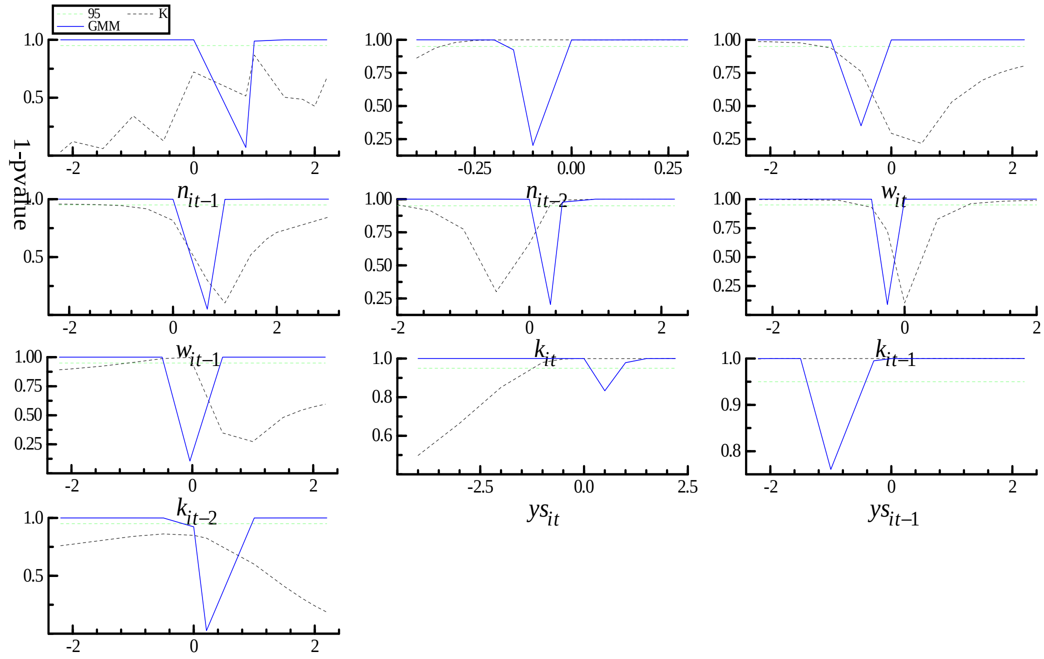

Figure 14: 1-pvalue curves: Endogenous wages and capital. Long run.GMM (solid), K (dotted).