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First mover advantage in a dynamic duopoly with spillover

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Abstract

We present a dynamic duopoly model of technical innovation where R&D costs decrease exogenously with time, and inter-firm knowledge spillover lowers the second comer's R&D cost. The spillover effect only becomes available after a disclosure lag. These features allow us to identify a new type of equilibrium: the leader delays investment until the R&D cost is low enough that the follower finds it optimal to invest as soon as he can benefit from the spillover. This equilibrium is subgame perfect over a wide range of parameters, and raises several interesting implications. First, in our new equilibrium the time delay between the two R&D investments is realistically short. Second, while the presence of a spillover favors the second mover, this benefit is not enough to rule out a first mover advantage. Indeed, the first mover advantage survives whenever technical progress is sufficiently fast and the disclosure lag is relatively long. Third, in case of a major innovation our equilibrium implies under–investment, which requires a substantial public intervention in favour of the investment activity.

Keywords: R&D, knowledge spillover, dynamic oligopoly
JEL classification: L13, L41, O33.

1   Introduction

The overall performance of an industry can be deeply affected by its innovations. It is therefore important to understand whether firms have a strong incentive to pioneer new technologies. In an oligopoly, the first mover may obtain a competitive advantage such as lower production costs or a higher-quality good. However, being first also involves high R&D costs and the risk of being overtaken by competitors in subsequent improvements. Moreover, the new technology may fail to generate profits or provide unwanted knowledge spillover to other firms.[1] Hoppe (2000) addressed this question by analyzing a dynamic duopoly model similar to the classic Fudenberg and Tirole's (1985) one. The main difference between their models is that Hoppe presumes the profitability of the new technology to be uncertain until the first mover enters the market. Hence, this event produces some informational spillover. Armed with better information, the follower may enjoy higher profits than the leader even when the *ex ante* probability that the new technology will perform poorly is low. Building on the work of Dutta *et al.* (1995), Hoppe and Lehmann-Gruber (2001) considered a vertically differentiated duopoly with sequential entry. In this setting, the second mover chooses a higher quality level in hopes of servicing the richer part of the market and generating larger overall profits. This second mover advantage substantiates in scenarios where the R&D cost rapidly increases over time, so that the first mover is likely to halt investment early and offer a low-quality good.

We consider a process innovation framework in which the first mover's R&D activity generates a technological spillover. The new information becomes available after a time period that we name the "disclosure lag". Empirical papers such as those of Mansfield (1985) and Cohen *et al.* (2002) suggest that disclosure lags vary between one and three years, a figure corroborated by some anecdotal evidence. For example, aircraft engine manufacturing is dominated by three players: General Electric, Rolls Royce and Pratt&Whitney. In this industry, each incremental technical advance introduced by one manufacturer is matched by the others within two years. (The Economist, 10 January 2009, pp. 58-60). In the microprocessor industry, AMD lagged Intel by about four years in bringing a version of the 80486 CPU to market. Hence, disclosure lags represent an important feature of the competitive environment that deserves to be incorporated into a formal model.

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[1]In this vein, since the early 1990's business academics have been questioning whether the pioneer of an innovative activity can expect higher returns than the followers. Lieberman and Montgomery (1988, 1998) review the debate between business scholars.

In the presence of disclosure lags, being a pioneer has two countervailing effects on profitability: the leader enjoys a temporary competitive advantage (for the duration of the disclosure lag) but also pays higher R&D costs. The follower obtains lower profits while working to match the leader's technological advantage, but benefits from new information through the spillover. Notice that this benefit adds to the one granted by the technological externality which drives the dynamics: in fact, in our model – as in Fudenberg and Tirole (1985), Stenbacka and Tombak (1994), Hoppe (2000), and many others – the R&D cost decrease over time.

The presence of inter-firm technological spillovers and of a disclosure lag in the dynamic game allows us to identify a new type of equilibrium: the leader delays investment until the R&D cost is so low that the follower will find it optimal to invest as soon as he benefits from the spillover, that is, immediately after the disclosure lag. This new equilibrium has several interesting implications.

First, in our equilibrium the presence of a spillover effect favoring the second mover is not enough to offset the First Mover Advantage (FMA) in all cases. The FMA survives when technical progress is sufficiently fast and the disclosure lag is relatively long. The follower incurs lower R&D costs than the first mover due to both technical progress and the spillover effect, but the difference between his costs and the leader's ones is limited by the fact that a high technical progress rate encourages the pioneer to wait until R&D costs are low. A FMA is obtained when the extra profits obtained by the leader during the disclosure lag are greater than the difference in R&D costs.[2] This result helps explain the pressure to innovate observed in many high-tech sectors, and some examples of successful pioneers (e.g. Intel and Nintendo).

Second, our equilibrium implies that firms under-invest in major innovations, which therefore require substantial public intervention in favor of the R&D activity. In many previous contributions such as Fudenberg and Tirole (1985), Riordan (1992) and Weeds (2002), the private benefits of a major innovation and the fear of being preempted trigger a socially premature investment, calling for public incentives to *reduce* R&D activity.[3] When we

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[2] When firms are *ex ante* identical the FMA will be dissipated in equilibrium, as demonstrated in many contributions following the work of Fudenberg and Tirole (1985).

[3] Riordan (1992) focuses on the impact of price and entry regulations on the timing of adoption, but only in the early equilibrium. Because regulatory schemes tend to reduce the innovator's rents, they are likely to delay the early adoption, an outcome which can be socially beneficial in his model. Stenbacka and Tombak (1994) analyze the role of experi-

focus on minor innovations – the case in which, according to the literature, the market equilibrium under-invests – our equilibrium implies that policies aimed at stimulating R&D are less sizeable than previously suggested despite the presence of inter-firm spillover.

The new equilibrium exists *in addition to* the early and late equilibria identified in contributions following Fudenberg and Tirole (1985). This literature, which started with Reinganum (1981) and is excellently surveyed by Hoppe (2002), identifies two factors characterizing the equilibria: the length of the follower's strategic delay and the intensity of competitive pressure. We shall argue that in light of empirical evidence, our interpretation of the setting is more realistic than either of these cases.

In the early equilibrium, fear of preemption and lost profits forces the leader to invest as soon as possible. The second mover therefore delays investment for a relatively long period, until the cost of R&D comes down enough to make competition worthwhile. In other words, this decision is driven by a desire to grasp the full benefit of technical progress.

The optimal delay in an early equilibrium grants a long period of competitive advantage to the pioneer, implying a large payoff for the leader at the expense of the follower. Hence, to avoid being preempted, the pioneer invests "very soon" and the investment is excessive from a welfarist perspective.[4]

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ence by assuming that the probability of successful innovation is an increasing function of the time since investment. They show that collusive adoption (when pioneer and follower release innovations at the same time) may yield greater welfare than the competitive market equilibrium. This happens when the pace of technical progress is fairly high, so that leaders and followers alike desire to take full advantage of reduced innovation costs. In contrast, the competitive market equilibrium grants a strategic advantage to the pioneer, inducing premature adoption. Weeds (2002) presents a patent race version of the Fudenberg and Tirole (1985) model, in which profits evolve stochastically. She suggests that the early (late) equilibrium over(under)-invests; however, the late equilibrium is closer to the social optimum.

Our under-investing equilibrium in the presence of a major innovation contrasts with previous contributions inspired by Loury (1979) and Lee and Wilde (1980). These authors assumed that a new technique suddenly becomes available to all firms, immediately triggering industry-wide investment in R&D. The competitive pressure induced by the market structure pushes the equilibrium level of R&D investment higher than the social optimum. This result can be partially ascribed to the tournament structure employed in their models. In a non-tournament model, Beath *et al.* (1989) underscored the role of the competitive threat as a major determinant of R&D expenditure: the larger the threat, the more resources firms invest in R&D, so over-investment and more significant innovations are more likely. Delbono and Denicolò (1991), again in a non-tournament framework, find that the equilibrium R&D effort can be lower than the social optimum if the marginal efficiency of R&D expenditure is low (hence each firm invests less and obtains a smaller R&D output).

[4]In a process innovation version of the Fudenberg and Tirole (1985) model based on Cournot competition, the follower's entry delays are typically longer than 15 years. This is so even when exogenous technical progress is fast (7% a year), speeding up the follower's

The possibility of preemption also implies rent equalization, and dissipates the FMA. In contrast, a late equilibrium arises only after technical progress has substantially reduced innovation costs, so that the competitor can immediately copy the pioneer. In this situation, an innovator anticipates that there will be no leadership period so waits until releasing the new product will maximize the joint discounted stream of net profits. The collusive flavor of this equilibrium is immediately apparent. Fudenberg and Tirole suggest that the late market equilibrium under-invests. They also demonstrate that the early equilibrium is subgame perfect when the size of the innovation is large. In this case, in fact, the per-period pioneer profits are considerable, which triggers the preemptive behavior.

We refer to the third case identified in this paper as the "intermediate" equilibrium, since both firms decide to innovate at dates positioned between the early and late equilibria. The pioneer knows that the second mover will attempt to exploit technical spillovers as soon as the relevant information is obtained, i.e., exactly at the end of the disclosure lag. The pioneer's period of competitive advantage is therefore shorter in the intermediate equilibrium than in the early one. This decision reduces the leader's discounted profits but benefits the follower, and may lead to a Second Mover Advantage (SMA). Even when there is a FMA (due to fast technical progress, a relatively long disclosure lag, or a relatively modest spillover), the competitive pressure is weaker than in the early equilibrium. Accordingly, it does not force the pioneer to invest "very soon". However, the competitive pressure is still high enough to prevent a late equilibrium.

By applying the subgame perfection criterion to the overall game, we find that the intermediate equilibrium is particularly relevant to the case of a non-drastic innovation. Indeed, it is the prevailing equilibrium for a wide range of parameters.[5]

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reaction. In the process innovation Cournot competition version of Hoppe (2000), when the probability of a bad technology is in the order of 5-10%, the entry lags are not much shorter. This is because the incentive to be first is still sufficiently strong to unleash preemption. As the probability that the technology performs poorly increases, the incentive to be first disappears, and the pioneer delays her innovation. In case the technology performs well, the rival firm immediately follows suit and we arrive at a second mover advantage equilibrium. Grenadier (1996) applies a stochastic version of the Fudenberg and Tirole (1985) model to the construction sector, arriving at median investment lags ranging from four to eight years as the standard deviation of demand ranges from 0.05% to 0.125%. While these values are adequate for construction, they seem excessive for the manufacturing sector.

[5]In our framework, R&D diffusion and rent equalization do not imply that the R&D investment is excessive from a social planner's perspective, even though these are natural indicators of a highly competitive environment.

The paper proceeds in the standard way. In Section 2 we present our model. Sections 3 and 4 analyze the follower's and the leader's decisions respectively. In Section 5 we discuss the equilibrium concept and compute various market equilibria in which firms compete both in the innovation and in the product stages. The criterion of subgame perfection is invoked to select among market equilibria. In Section 6 we spell out the welfare implications of our analysis. Concluding comments are offered in Section 7.

2 The model

We consider an industry composed of two firms, i and j, involved in a two-stage interaction: first they decide whether or not to innovate, then they compete in the final market. This interaction is repeated over a series of (infinitesimally short) time periods. At the beginning the two firms are symmetric, each enjoying a profit flow of π_0. The subscript indicates the number of firms which have already introduced the innovation. A technical improvement becomes feasible at a date which is set to zero for convenience. From that instant onward, if one firm innovates while the other postpones adopting the technology, the former becomes the leader and obtains a flow profit equal to π_1^L. The latter becomes the follower and obtains an instantaneous profit equal to π_1^F. If both firms innovate, they obtain profits of π_2. As is standard, we assume that

Assumption 1: $\pi_1^L > \pi_2 > \pi_0 > \pi_1^F$.

We also introduce

Assumption 2: $\pi_1^L - \pi_0 > \pi_2 - \pi_1^F$,

which ensures that there is an advantage to adopting first.[6] Assumption 2 is not crucial to our result; it is trivial to adapt the analysis that follows to the case in which it does not hold.

Time is continuous and the firms' horizon is infinite. Firms discount future profits at the common rate r.

In our setup the research project has a fixed size, as assumed by Fudenberg and Tirole (1985), Hoppe (2000), and many other authors. If a firm undertakes the project as soon as the innovation becomes technically feasible, i.e., at time 0, it pays the amount γ. The cost decreases at a constant rate $\rho > 0$, thanks to the advances in basic research and the availability of new results obtained in related areas. Of course, this form of technical progress is exogenous. The R&D cost function is

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[6] As shown in Appendix B, Assumptions 1 and 2 are fulfilled in a linear Cournot duopoly.

$$C_L(t_L) = \gamma e^{-\rho t_L}, \text{ for } t_L \in [0, \infty), \tag{1}$$

where t_L is the time when the leader introduces the technical improvement.

In his classic study, Mansfield (1985) reported that in 59% of cases the innovator's rivals need more than twelve months to obtain relevant information about the new technology. More recently, Cohen *et al.* (2002) computed that the average adoption lags for unpatented process innovations in Japan and the US are 2.03 and 3.37 years respectively. Accordingly, we introduce an element in the follower's R&D cost function representing the exogenously determined disclosure lag Δ. An obvious but important consequence of our assumption is that the the leader benefits from a competitive advantage period equal to (at least) Δ years.[7]

Thus, the follower's R&D cost is described by

$$C_F(t_F) = \begin{cases} \gamma e^{-\rho t_F} & \text{for } t_F \in [t_L, t_L + \Delta) \\ (1-\theta)\gamma e^{-\rho t_F} & \text{for } t_F \in [t_L + \Delta, \infty) \end{cases}, \tag{2}$$

where t_F is the adoption time of the follower. The parameter $\theta \in [0, \bar{\theta}]$ represents inter-firm spillover; $\bar{\theta}$ is assumed to be lower than unity. In fact, if θ were unity the follower – bearing no innovation cost – would always invest at the end of the disclosure lag. Hence, this case would deliver results close to those obtained by Katz and Shapiro (1987). When $\theta \neq 0$, the innovation is partially appropriable: the second comer enjoys a reduction in R&D costs by imitating his competitor at $t_F \geq t_L + \Delta$.

This way of introducing spillover and disclosure lag into the model is extremely simple. It would arguably have been preferable to consider a stochastic disclosure lag, where the probability of information diffusion increases with elapsed time since the introduction of the innovation and also depends on the follower's imitation effort. The latter factor should also influence the spillover size.[8] However, even the simplest stochastic formulation – namely one involving a constant probability of information diffusion coupled with a fixed spillover size – precludes the attainment of explicit results. Furthermore, a constant probability of information disclosure does not represent an

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[7] Miyagiwa and Ohno (2002) adopt the same assumption in their R&D model, which is based on that of Lee and Wilde (1980).

[8] For example, we could have followed Jin and Troege (2006), who suggest that firms can raise θ by paying a convex imitation cost. Nevertheless, we preferred not to pursue this development of the model, because our framework is already fairly complex. For the same reason, we do not endogenize the duration of the disclosure lag.

improvement upon our formulation, since the sparse empirical evidence available suggests that the probability of successful imitation increases over time. Hence, our formulation was deemed the optimal compromise between analytical tractability and "realism".

We denote by $V_L(t_L, t_F)$ the stream of future profits, discounted back to time 0, obtained by the firm investing at time t_L while her rival sinks the innovation cost at $t_F \geq t_L$. Hence, we have

$$V_L(t_L, t_F) = \frac{\pi_0}{r} + \frac{\pi_1^L - \pi_0}{r} e^{-rt_L} + \frac{\pi_2 - \pi_1^L}{r} e^{-rt_F} - C_L(t_L) e^{-rt_L}. \quad (3)$$

The second addendum on the right hand side represents the first mover's stand-alone incentive, while the third is the profit reduction imposed by the follower's decision to adopt. The second mover's payoff is

$$V_F(t_L, t_F) = \frac{\pi_0}{r} + \frac{\pi_1^F - \pi_0}{r} e^{-rt_L} + \frac{\pi_2 - \pi_1^F}{r} e^{-rt_F} - C_F(t_F) e^{-rt_F}. \quad (4)$$

The follower's incentive to innovate is summarized by the third addendum, while the profit externality imposed by the leader on the follower is captured by the second one.

Before describing the firms' value functions, we introduce some technical assumptions regarding the admissible values of θ, Δ and γ.

Assumption 3: $1 - \frac{\pi_2 - \pi_1^F}{\pi_1^L - \pi_0} \left[\frac{r(\pi_1^L - \pi_2) + \rho(\pi_1^L - \pi_1^F)}{r(\pi_0 - \pi_1^F) + \rho(\pi_1^L - \pi_1^F)} \right]^{\rho/r} \leq \bar{\theta} < 1.$

As we shall discuss, if the maximum spillover $\bar{\theta}$ were close to zero, the results delivered by our model would be similar to those obtained by Fudenberg and Tirole (1985). Accordingly, by requiring that the maximum spillover is high, we make the discussion more interesting.

Assumption 4: $\Delta \leq \bar{\Delta} = \frac{1}{r} \ln \left(1 + \frac{r}{\rho} \frac{\pi_1^L - \pi_2}{\pi_1^L - \pi_1^F} \right).$

The purpose of Assumption 4 is to limit the number of cases that we need to consider. While Assumption 1 guarantees that $\bar{\Delta} > 0$, in Section 5.1, we shall verify that Assumption 4 does not restrict Δ to values too short to be sensible.

Assumption 5: $\gamma > \bar{\gamma} = \frac{(\pi_2 - \pi_1^F) e^{\rho \bar{\Delta}}}{(r+\rho)(1-\bar{\theta})}.$

According to this hypothesis, when the innovation becomes feasible (i.e. at time 0) the lump-sum cost it bears is sufficiently high that the second

comer wishes to innovate after the completion of the disclosure lag. This ensures that the spillover plays a role in the models considered.

Notice that $\bar{\gamma} > 0$, by Assumption 1.

3 The follower's investment problem

Since the follower reacts optimally to the leader's decisions, it is natural to analyze his behavior first.[9]

After the leader has invested in the early stages of the game, the follower prefers to delay adoption for Δ years or more. In delaying longer than Δ, the follower reaps benefits not only from imitation but also from ongoing progress in pure research. When the R&D cost function is given by (2), maximizing (4) with respect to t_F, we obtain the follower's optimal investment time:

$$T_F^* = -\frac{1}{\rho} \ln \left(\frac{\pi_2 - \pi_1^F}{\gamma(r + \rho)(1 - \theta)} \right). \tag{5}$$

This solution applies when the leader sinks the R&D cost at $t_L \leq T_F^* - \Delta$.[10]

The comparative statics on T_F^* gives sensible results. In particular, the higher the inter-firm spillover, the sooner the second mover will invest. A higher value of $\pi_2 - \pi_1^F$ increases the incentive to innovate and hence advances his decision. An increase in γ or r delays his investment decision, because the innovation is more costly or future profits are more heavily discounted respectively. The role of technical progress (ρ) is ambiguous: on the one hand, a higher value of this parameter implies that the innovation cost is lower at any given date t_F, calling for earlier investment; on the other hand, rapid reduction in innovation costs may induce the follower to wait longer because he knows that the cost will quickly become even smaller. The first (direct) effect prevails over the second (indirect) effect, unless θ is high.

The above solution is not optimal if the leader invested at a time later than $T_F^* - \Delta$, i.e., when $t_L > T_F^* - \Delta$. In this case the fixed cost at T_F^* is unacceptably high because the disclosure lag has not yet elapsed. The follower will choose either to wait exactly Δ periods before investing (just long enough to grasp the inter-firm spillover), to wait fewer than Δ periods, or to copy immediately.

We now analyze the above strategies, discussing separately the case of

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[9] For ease of exposition, hereafter we refer to the follower as if it were headed by a male CEO and to the leader as if it were run by a female CEO.

[10] Assumption 5 guarantees that $T_F^* - \Delta \geq 0$ for any $\Delta \in [0, \bar{\Delta}]$, $\theta \in [0, \bar{\theta}]$.

high, and of low spillovers. The boundary is given by

$$\theta'(\Delta) = 1 - \frac{r + \rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta}.$$

Notice that $\theta'(0) = 0$, that $\partial\theta'(\Delta)/\partial\Delta > 0$, and that $\partial^2\theta'(\Delta)/(\partial\Delta)^2 > 0$. Suppose first that $\theta > \theta'(\Delta)$, i.e. that the spillover is high. In this case, the choice of waiting less than Δ is never optimal. When the spillover is sizable, and the innovation cost is still high, waiting Δ years implies an R&D cost saving that is large enough to compensate for the efficiency disadvantage during the disclosure lag. Hence the follower invests at the end of the disclosure lag, i.e. at $t_L + \Delta$. Instead, when the R&D cost is low, because the innovation leader has decided to invest "late", it is optimal for the second firm to enter immediately without exploiting the inter-firm spillover. We define $\bar{T}$ as the first date on which the payoff of the "wait Δ years before following" strategy matches that of the "immediately follow " strategy. Solving the equation $V_F(t_L, t_L) = V_F(t_L, t_L + \Delta)$, in which the follower's value function is given by (4), we immediately obtain

$$\bar{T} = -\frac{1}{\rho}\ln\left[\frac{\pi_2 - \pi_1^F}{\gamma r}\frac{1 - e^{-r\Delta}}{1 - (1-\theta)e^{-(r+\rho)\Delta}}\right]. \tag{6}$$

Notice that an increase in the spillover parameter raises $\bar{T}$. That is, firms are encouraged to postpone innovation when the benefit of imitation is high.[11]

[Figure 1 about here]

The above arguments are summarized in Figure 1, and formally presented in:

Proposition 1 *When $\theta \in [\theta'(\Delta), \bar{\theta}]$, the follower's optimal strategy is to invest at time*
(a) T_F^*, if $t_L \in [0, T_F^* - \Delta]$
(b) $t_L + \Delta$, if $t_L \in (T_F^* - \Delta, \bar{T}]$
(c) t_L, if $t_L \in (\bar{T}, \infty)$.

Proof: refer to Appendix A.

When the spillover is low $(\theta < \theta'(\Delta))$, the above analysis must be partly modified for $t_L > T_F^* - \Delta$. In this case, waiting Δ periods is less rewarding

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[11]Apart from the role of θ just described, the effects of various parameters on $\bar{T}$ are quite similar to those on T_F^*.

for the follower. Accordingly, we presume that waiting fewer than Δ periods becomes the optimal decision for some $t_L > T_F^* - \Delta$.

Defining

$$T_F' = -\frac{1}{\rho} \ln \left(\frac{\pi_2 - \pi_1^F}{\gamma(r + \rho)} \right) \tag{7}$$

as the follower's optimal investment date in the absence of spillover, we find

Proposition 2 *When $\theta \in [0, \theta'(\Delta))$, the follower's optimal strategy is to invest at time*
(a) T_F^* if $t_L \in [0, T_F^* - \Delta]$;
(b) $t_L + \Delta$ if $t_L \in (T_F^* - \Delta, \breve{T}_L]$, where $\breve{T}_L \in (T_F^* - \Delta, T_F']$ is the time instant such that $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$;
(c) T_F' if $t_L \in (\breve{T}_L, T_F']$;
(d) t_L if $t_L \in (T_F', \infty)$.

Proof: refer to Appendix A.

Proposition 2 has an interesting consequence:

Corollary 3 $\lim_{\theta \to 0} \breve{T}_L = T_F^* - \Delta$

Proof: refer to Appendix A.

Accordingly, when $\theta = 0$, the time interval sub (b) in Proposition 2 collapses to $\emptyset$. Because $\lim_{\theta \to 0} T_F^* = T_F'$, the follower's optimal strategy is to invest at $T_F'(= T_F^*)$ if $t_L \in [0, T_F']$ and immediately follow the leader if $t_L \in (T_F', \infty)$. This result comes as no surprise: if there is no spillover, the disclosure lag can have no effect on the follower's optimal decision. Hence, we recover the optimal strategy identified by Fudenberg and Tirole (1985).

4 The leader's investment decision and the behavior of the value functions

We now solve the leader's optimal decision problem, determining her payoff.

In a model without spillover, if the leader opts for early adoption, she is aware that her competitor will postpone investment for quite a long time. This situation generates an inverted-U payoff function determined by two opposing forces.

An increase in the leader's adoption time induces a reduction in her innovation cost, which increases her value function, but implies also a shortening in

her efficiency advantage period, which reduces her discounted profits. When t_L is relatively low, the former effect dominates the latter because the cost reduction induced by the technological externality is quantitatively relevant. From the value function (3), it is clear that the leader's optimal entry date is obtained balancing the later attainment of the stand-alone incentive, with the decrease in the innovation cost. Maximizing (3) with respect to t_L, one immediately obtains

$$T_L^* = -\frac{1}{\rho} \ln \left(\frac{\pi_1^L - \pi_0}{\gamma(r + \rho)} \right). \tag{8}$$

Although they do not appear in the above result, spillover and disclosure lag do bring about several effects relevant to the leader. First of all, because θ reduces T_F^*, spillover shortens the leader's cost advantage period and reduces her value function.[12] Second, a longer Δ reduces the time interval during which the leader knows that T_F^* represents the follower's optimal response. T_F^* is computed assuming that the follower exploits the spillover, and hence completes the disclosure lag. The joint effect of Δ and θ (which reduces T_F^*) may imply that $T_L^* > T_F^* - \Delta$, in which case the leader's value function is increasing in $[0, T_F^* - \Delta]$.

In Figure 2, the dashed line represents the leader's value function. Note that for $t_L \in [0, T_F^* - \Delta]$, this function has the usual inverted-U shape. Figure 3 considers cases in which $T_L^* > T_F^* - \Delta$ (so T_L^* is not shown).

[Figure 2 about here]

[Figure 3 about here]

For $t_L \in (T_F^* - \Delta, \bar{T}]$, the leader's value function again tends to assume an inverted-U shape. This behavior can be easily understood with reference to the case $\theta > \theta'(\Delta)$, in which the leader is aware that the follower will grant her a constant efficiency period equal to the disclosure lag (refer to Proposition 1). Therefore, if t_L is close to $T_F^* - \Delta$, the reduction in the fixed cost due to the technological externality outweighs the effects of postponing high post-innovation profits (in terms of current value, the latter do not change over time). As t_L increases, however, the disadvantages of postponing innovation outweigh the benefits.[13]

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[12]Exploiting equations (3) and (5), it is easy to show that $\frac{\partial V_1(t_L, T_F^*)}{\partial \theta} < 0$.

[13]When θ gets smaller, $\bar{T}$ is reduced (as implied by Eq. (6)). Hence, the negative effect may not have time to become strong enough to induce an inverted-U shape in the leader's value function.

We now characterize in more detail the maximum value functions for various values of θ and Δ, focusing first on $t_L \in (0, \bar{T}]$ and later on $t_L \in (\bar{T}, \infty)$.

Accordingly, for $t_L \in (0, \bar{T}]$, we first analyze the case in which θ is small with respect to Δ. We make this concept precise by defining

$$\theta''(\Delta) = 1 - \frac{r(\pi_2 - \pi_1^F)e^{(r+\rho)\Delta}}{(r+\rho)(\pi_1^L - \pi_1^F)(e^{r\Delta} - 1) + r(\pi_2 - \pi_1^F)},$$

and assuming that $\theta \leq \theta''(\Delta)$. Note that $T_F^* - \Delta$ lies inside the interval $(0, \bar{T}]$, providing another natural boundary for discussing the behavior of the value function. Figure 4 is helpful to locate the portion of space we are considering.[14]

[Figure 4 about here]

In the earlier portion of the interval, $[0, T_F^* - \Delta]$, the leader's payoff is higher than the follower's for some t_L (refer to Figure 2, panels (A1) and (A2)). Moreover, a relatively long disclosure lag implies that the interval $[0, T_F^* - \Delta]$ is short. Thus, the leader's value function is higher than the follower's at the end of this interval (i.e., for $\theta \leq \theta''(\Delta)$, we have that $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$). A relatively long disclosure lag has another important implication: the leader's value function is higher than the follower's throughout the later portion of the interval ($t_L \in (T_F^* - \Delta, \bar{T}]$). Thus, the follower must let the leader enjoy a long cost advantage period.

The leader's value functions for various values of θ are depicted in Figure 2, and formally described below:

Proposition 4 (a) *When* $\theta \in [0, \theta''(\Delta)]$, *for some* $t_L \in [0, T_F^* - \Delta]$, $V_L(t_L, T_F^*) \geq V_F(t_L, T_F^*)$;
(b) *when* $\theta \in (\theta'(\Delta), \theta''(\Delta)]$, *for* $t_L \in (T_F^* - \Delta, \bar{T}]$, $V_L(t_L, t_L + \Delta) > V_F(t_L, t_L + \Delta)$;
(c) *when* $\theta \in [0, \theta'(\Delta)]$, *for* $t_L \in (T_F^* - \Delta, T_F']$, $V_L(t_L, T_F') \geq V_F(t_L, T_F')$, *with the equality applying at* $t_L = T_F'$.

Proof: refer to Appendix A.

We now consider – again for $t_L \in (0, \bar{T}]$ – the effects of a relatively large spillover ($\theta > \theta''(\Delta)$).

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[14]Notice that (a) $\theta''(0) = 0$, and (b) for $\Delta < \bar{\Delta}$, $\partial\theta''(\Delta)/\partial\Delta > 0$ while $\theta''(\Delta) > \theta'(\Delta)$.

In this case, $V_L(T_F^* - \Delta, T_F^*) \leq V_F(T_F^* - \Delta, T_F^*)$ (refer to Figure 3). As will become clear in Section 5, it is important to verify that the leader's maximum value function is increasing in $t_L \in (0, T_F^* - \Delta]$. Simple calculations show that $V_L(t_L, T_F^* - \Delta)$ is increasing for $t_L \in (0, T_F^* - \Delta]$, because $T_L^* \geq T_F^* - \Delta$, for values of $\theta \geq \theta^*(\Delta)$, where

$$\theta^*(\Delta) = 1 - \frac{\pi_2 - \pi_1^F}{\pi_1^L - \pi_0} e^{\rho\Delta}.$$

In Figure 4, $\theta^*(\Delta)$ is the downward sloping bold curve, portrayed only for values such that $\theta^*(\Delta) > \theta''(\Delta)$.[15]

We now restrict our attention to the sub-case $\theta > \max\{\theta^*(\Delta), \theta''(\Delta)\}$, so that the innovation leader's value function is increasing in the interval $[0, T_F^* - \Delta]$, and $V_L(t_L, T_F^*)$ and $V_F(t_L, T_F^*)$ can be drawn for $t_L \in [0, T_F^* - \Delta]$ as in Figure 3.

In comparison to the previous case (i.e., $\theta \leq \theta''(\Delta)$), the higher spillover reduces the leader's payoff in the early interval. This happens because an increase in θ advances the follower's optimal reply date and reduces the leader's efficiency advantage period.[16]

A relatively high spillover also has important consequences for the intermediate interval. By delaying entry for Δ periods, the follower obtains a large reduction in fixed costs, shifting his value function upward. Accordingly, the follower may enjoy a payoff larger than that of the first mover. This is possible only in the initial part of the interval $(T_F^* - \Delta, \bar{T}]$. In fact, increasing the spillover parameter also involves a second effect. Because the benefit of postponing imitation is larger (and the follower will have no desire to renounce the benefit), $\bar{T}$ increases. Thus, by entering the market in the late part of $(T_F^* - \Delta, \bar{T}]$, the leader is able to grasp higher payoffs because the R&D costs are low. These behaviors for the value functions imply the existence of a unique intersection point, denoted by T^{ip}, which is the solution of $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$.

An increase in θ – benefiting the follower – shifts $V_F(t_L, t_L + \Delta)$ upward and therefore postpones T^{ip}. When the spillover is relatively modest, T^{ip} is earlier than $\hat{T}_L$, the date at which $V_L(t_L, t_L + \Delta)$ is maximum. This case is portrayed in Figure 3, Panel (B_1). When the spillover is substantial, $V_L(t_L, t_L + \Delta)$

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[15]While it is obvious that $\partial\theta^*(\Delta)/\partial\Delta < 0$, the condition $\theta^*(0) > 0$ only holds when Assumption 2 is satisfied. It is trivial to adapt the analysis that follows to the case in which such an Assumption does not hold.

[16]Formally, we have $\partial V_L(t_L, T_F^*)/\partial T_F^* > 0$.

reaches $\hat{T}_L$ at a date earlier than T^{ip}, as in Figure 3, panel (B$_2$). As we shall argue in the next section, this may lead to SMA games.

We now define

$$\theta'''(\Delta) = 1 - \frac{[\rho(\pi_1^L - \pi_1^F) + r(\pi_2 - \pi_1^F)](e^{-r\Delta} - 1) + r(\pi_2 - \pi_0)}{r[\pi_1^L - \pi_0 - (\pi_1^L - \pi_2)e^{-r\Delta}]}e^{(r+\rho)\Delta},$$

as the spillover value for which $T^{ip} = \hat{T}_L$. The line $\theta'''(\Delta)$ is indicated in Figure 4. We formally present the above arguments below:[17]

Proposition 5 *(a) When $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, the maximum of the leader's value function within the interval $[0, \bar{T}]$ lies in the sub-interval $(T_F^* - \Delta, \bar{T}]$;*
(b) when $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$, at T^{ip} the leader's value function has not reached its maximum;
(c) when $\theta \in (\theta'''(\Delta), \bar{\theta}]$, at T^{ip} the leader's value function is non-increasing.

Proof: refer to Appendix A.

To complete this discussion of the parameter space, we need to consider the small interval $\theta \in [\theta''(\Delta), \theta^*(\Delta))$ (refer to Figure 4). Such spillovers are sufficiently low to allow $V_L(t_L, T_F^*) \geq V_F(t_L, T_F^*)$ for some $t_L \in [0, T_F^* - \Delta]$. On the other hand, these spillovers are also high enough to induce a SMA for some $t_L \in (T_F^* - \Delta, \bar{T}]$. In this case, the firms' value functions are depicted in Figure 5.

[Figure 5 about here]

Up to this point, we have only considered values of t_L in the range $[0, \bar{T}]$. We now consider the cases where the leader innovates "late", that is in the interval $(\bar{T}, \infty)$.

In this case, the R&D cost is so low that it is optimal for the second firm to renounce the spillover benefit and immediately imitate his rival's investment. Naturally, the first firm is well aware of this fact and makes her decision anticipating such behavior. This leads to an equilibrium where the two firms maximize their joint payoff. In this context the two firms are symmetric, so maximizing a single firm's payoff yields the joint maximum as well. Accordingly, the payoff for the first firm is:

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[17]It is possible to show that $\theta'''(0) = 0$, and that when Assumption 2 is satisfied, $\partial\theta'''(\Delta)/\partial\Delta|_{\Delta=0} > 0$.

$$V_S(t_S) = \frac{\pi_0}{r} + \frac{\pi_2 - \pi_0}{r} e^{-rt_S} - \gamma x e^{-(r+\rho)t_S}, \qquad (9)$$

where S stands for 'symmetric'.

Maximizing (9) with respect to t_S under the constraint $t_S \geq \bar{T}$ yields the optimal timing:

$$T^{le} = \begin{cases} \bar{T} & \text{if } \theta > 1 - e^{(r+\rho)\Delta}\left[1 - (r+\rho)\frac{1-e^{-r\Delta}}{r}\frac{\pi_2-\pi_1^F}{\pi_2-\pi_0}\right] \\ -\frac{1}{\rho}\ln\left(\frac{\pi_2-\pi_0}{\gamma(r+\rho)}\right) & \text{if } \theta \leq 1 - e^{(r+\rho)\Delta}\left[1 - (r+\rho)\frac{1-e^{-r\Delta}}{r}\frac{\pi_2-\pi_1^F}{\pi_2-\pi_0}\right] \end{cases}.$$

The superscript stands for 'late'.

If $\bar{T}$ is small, which is the case when θ is modest, $V_S(t_S)$ displays an inverted-U shape. When t_S is close to $\bar{T}$, decreasing R&D costs prevail over losses due to the delay of higher post-innovation profits. When $\bar{T}$ is high, which is the case if θ is large, the second effect already prevails at $\bar{T}$, and the value function is decreasing. In Figures 2, 3 and 5 we have depicted the case in which $V_S(t_S)$ takes its standard inverted-U shape.

5 The market equilibrium

In this section we discuss the equilibrium in the non-cooperative R&D game. Subgame perfection is the natural criterion to apply in this context. As in many dynamic games, we wish to restrict our attention to pure strategies. Accordingly, we begin by introducing certain assumptions that allow us to disregard mixed strategies.

In our setup, only one research project is available to the firms and the choice to innovate is an irreversible stopping decision. Our model therefore belongs to the class of symmetric timing games, which can be divided into two sub-classes depending upon which firm (the first mover or the second mover) obtains the higher payoff.

To clarify this point, assume for the moment that we have exogenously assigned the task of moving first to one of the two firms. Then there is a first mover advantage (FMA) if this firm obtains the higher payoff, and a second mover advantage (SMA) if it obtains the lower payoff. The first mover is assumed to choose the innovation time that maximizes its payoff, given the second mover's optimal choice.

To deal with FMA games, we drop the hypothesis of exogenously assigned roles and follow Hoppe and Lehman-Gruber (2005) by assuming the following.

Assumption 6: If at some time t the two firms have no preference between the roles of first mover and second mover, then each firm will try to become the leader. The firm that actually moves first is randomly selected with probability 1/2. The other firm may then choose to immediately follow the leader or postpone adopting the innovation.[18] If the leader is indifferent between adopting at time t or later, then it chooses t.

Assumption 6 is commonly used in the literature to rule out the possibility of coordination failure as an equilibrium outcome. In other words, firms do not choose to move at the same instant if they know that they will regret this choice afterwards.[19]

In dealing with SMA games, we assume that the equilibrium is driven by expectations and make the following assumption:

Assumption 7: When the innovation leader maximum payoff is lower than the corresponding second mover's value, firm h, $h \in \{i, j\}$ – randomly selected with probability 1/2 – believes that the other never enters first, and becomes the leader.

The above hypothesis (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out equilibria due to mixed strategies, which in this context are often referred to as wars of attrition (Fudenberg and Tirole, 1991). However, if we reject Assumption 7, the alternative would be to let our firms to start to randomize their entry decision at $\hat{T}_L$. At every instant of time, the firms would then obtain an expected payoffs identical to the leader's one. Hence, rejection of Assumption 7 in the SMA case leads to later adoption dates but the same expected payoff for the leader than in the case we study. Fudenberg and Tirole (1985) argued that the Pareto dominant equilibrium is most reasonable. In our case, Pareto ranking implies that all firms prefer the pure strategy equilibrium involving an advantage for the follower.

Subgame perfection requires that the equilibrium must survive all the possible off-equilibrium deviations. Accordingly, to test a candidate equilibrium, we need to compare the leader's payoff at the proposed equilibrium with her payoff at all earlier instants. If we can find some instant when the leader's payoff is higher than the discounted value of her payoff at the candidate equilibrium, then the leader would prefer to invest at that date rather than wait

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[18] Adoption by one firm may result in instantaneous adoption by the other. That is, the two firms adopt 'consecutively but at the same instant of time', obtaining the same payoff.

[19] As Hoppe and Lehman-Gruber (2005) remark, an equilibrium involving coordination failures cannot be obtained in a continuous-time game without a grid, as in this case equilibria are defined as the limits of discrete-time mixed strategies (Fudenberg and Tirole, 1985; 1991).

for the proposed equilibrium. When the leader's payoff is higher than the follower's, we also need to take into account the possibility of preemption by the follower. (In the following discussion, we presume that the roles of the leader and follower are *not* pre-assigned. If the follower's payoff is lower than the leader's, the former has an incentive to preempt the latter. For each game, we will then describe the effect of the simplifying assumptions (6 and 7) given above.)

The logic to obtain the unique subgame perfect equilibrium in FMA games can be described by exploiting Panel (A_1) in Figure 2. When both firms invest simultaneously at T^{le}, they obtain $V_S(T^{le})$. However, each firm would prefer to adopt first at T_L^*, the date that maximizes her discounted payoff. Consider that the roles of innovation leader and follower are not pre-assigned. If one firm knows that the other will adopt at time T_L^*, it is in his interest to preempt at time $T_L^* - dt$. By backward induction, we conclude that the only equilibrium strategy is to invest as soon as the leader's payoff is equal to the follower's (i.e., at $\underline{T}_L$). Assumption 6 then grants each firm a 50% chance of being the first innovator, and ensures that only one firm invests at $\underline{T}_L$. The preemption argument yields equal payoffs to the two firms in the subgame perfect equilibrium. Hence, in this case the equilibrium involves rent dissipation.

To conclude that in the SPNE of Figure 2 the leader will invest at time $\underline{T}_L$, and the follower at T_F^*, we do not need to show that $V_L(T_L^*, T_F^*) > V_S(T^{le})$. It is sufficient to find a $V_L(t_L, t_L + \Delta) > V_S(T^{le})$. It is then in the first mover's interest to deviate from $\{T^{le}, T^{le}\}$, and backward induction leads to the rent dissipation equilibrium $\{\underline{T}_L, T_F^*\}$.

As an example of a SMA game, consider panel (B_2) of Figure 3. Investing simultaneously at T^{le}, both firms obtain $V_S(T^{le})$. However, $V_L(\hat{T}_L, \hat{T}_L + \Delta) > V_S(T^{le})$. Hence, by Assumption 7, the firm that believes that the other one never enters first chooses $t_L = \hat{T}_L$. The other firm has no incentive to preempt its rival before date t_L.

Having clarified the equilibrium concept, we now exploit the results obtained in the previous sections to determine the SPNE.

When $\theta \in [\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, Propositions 5 and 1 imply that the SPNE is either T^{le} or in the intermediate interval $(T_F^* - \Delta, \bar{T}]$. The equilibrium is at T^{le} (i.e., it is "late") if $V_S(T^{le}) > V_L(\hat{T}_L, \hat{T}_L + \Delta)$. If $V_S(T^{le}) \leq V_L(\hat{T}_L, \hat{T}_L + \Delta)$, on the other hand, the entry date belongs to the interval $(T_F^* - \Delta, \bar{T}]$) and the equilibrium is "intermediate". Moreover, from Proposi-

tion 5(b), if $\theta \in [\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$ we have $\hat{T}_L \geq T^{ip}$. In this case the first firm invests at T^{ip} and the second at $T^{ip} + \Delta$. Notice that this SPNE is of the FMA type, and implies preemption (refer to Figure 3, panel (B$_1$)). When $\theta \in [\max\{\theta^*(\Delta), \theta'''(\Delta)\}, \bar{\theta}]$, Proposition 5(c) implies that $\hat{T}_L < T^{ip}$ and the SPNE is of the SMA type (refer to panel (B$_2$) in Figure 3). As intuition suggests, the candidate equilibrium can shift from a first mover advantage to a second mover advantage if θ increases for a given disclosure lag. The higher the spillover, the lower the fixed costs for the follower, and the more probable a SMA.

If $\theta \in [0, \theta''(\Delta)]$, the leader's adoption time in the SPNE is either T^{le} or in the interval $[0, T_F^* - \Delta]$. The equilibrium is late (at T^{le}) when $V_S(T^{le})$ is larger than the leader's maximum deviation payoff in $[0, \bar{T}]$. Otherwise, Propositions 4, 1, and 2 guarantee that the SPNE is a preemptive equilibrium where the leader adopts at $\max\{0, \underline{T}_L\}$ and the follower adopts at T_F^*. Both panels in Figure 2 provide examples of this equilibrium.

Finally, if $\theta \in [\theta''(\Delta), \theta^*(\Delta))$, we have three candidate equilibria. Not surprisingly, there is (as usual) a simultaneous entry date that maximizes the joint payoff. The spillover is sufficiently low to allow for a candidate equilibrium in $[0, T_F^* - \Delta]$, when the advantage for the leader is still high. Nonetheless, θ is large enough to induce an higher payoff for the follower for some $t_L \in (T_F^* - \Delta, \bar{T}]$, which implies the existence of a candidate equilibrium in the intermediate interval. Accordingly, we also have a candidate SPNE at $(T_F^* - \Delta, \bar{T}]$ (refer to Figure 5). As before, the candidate equilibrium in the intermediate interval $(T_F^* - \Delta, \bar{T}]$ is of the FMA type if $\hat{T}_L \geq T^{ip}$, and of the SMA type when $\hat{T}_L < T^{ip}$.

To sum up, we have up to three candidate equilibrium for SPNE: an "early" one when the leader's entry date belongs to the interval $[0, T_F^* - \Delta]$, an "intermediate" one when the leader's entry date belongs to $[T_F^* - \Delta, \bar{T}]$, and a "late" one such that the two firms simultaneously invest in $(\bar{T}, \infty]$.

5.1 Numerical results

The SPNE cannot be determined analytically as a function of the parameters, due to the high degree of nonlinearity in our model. Hence, we now present some numerical results for the Cournot competition framework with linear demand detailed in Appendix B.[20]

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[20] Our routine has been written in Matlab, and it is based on a discretization of the space $[\theta \times \Delta]$ for $\theta \in [10^{(-10)}, 0.8]$ and $\Delta \in [10^{(-10)}, 3]$. We have used 300,000 gridpoints, but our results do not change significantly for any number of points larger than 20,000. The Matlab routine is available upon request from the authors.

In our simulations, we normalize the market dimension parameter A to unity and fix the discount rate r to 0.03. The latter is consistent with our time unit, which is set to one year. The parameter γ does not play a substantial role, provided that $\gamma \geq \bar{\gamma}$; the effect of higher γ (i.e., less efficient R&D) is to postpone all equilibria without changing their relative convenience. Hence, we choose $\gamma = 50$ with no loss of generality. As for ρ, we refer to the prior literature on industry-specific innovation costs. Cummins and Violante (2002), for example, estimated the rate of technical change in several sector-specific capital goods. Unsurprisingly, the sector whose productivity has grown at the fastest pace (more than 20% per year in the US over the entire postwar period) is "computers and office equipment". Apart from this outlier, the greatest rates of technical change occurred in communications equipment (9% per year), aircraft (8%), and instruments (6%). We then have a 5% annual change in the production costs of "service industry machinery". Productivity growth rates in all other sectors range between 0.1% and 3.8% per year. Because a non-negligible share of the productivity increase is retained by the producer, we simulate the model for: $\rho \in \{0.01; 0.04; 0.07\}$. The first value of ρ characterizes technologically mature sectors, which benefit from minor technical progress in the industries producing their required machinery. We label this scenario Industry I. We set $\rho = 0.04$ to represent a fairly dynamic sector (Industry II). The case of $\rho = 0.07$, denoted Industry III, is a frontier sector.

As detailed in Appendix B, to preserve the duopolistic structure of our market we consider only non-drastic innovations. Hence, the size of the R&D output, x, must be lower than A ($x < 1$). We investigate two types of output: a minor innovation where $x = 0.05A (= 0.05)$, and a major innovation where $x = 0.5A (= 0.5)$.[21]

We know that $\bar{\Delta}$ decreases with x. To see this, take the definition of $\bar{\Delta}$ given in Assumption 4 and then substitute in the values for π_0, π_1^L, π_1^F, and π_2 obtained in Appendix B. We can compute $\lim_{x \to 0} \bar{\Delta}$ for $\rho \in \{0.01; 0.04; 0.07\}$, obtaining $\{23.105, 7.438, 4.451\}$. Hence, the restriction implied by Assumption 4 is realistic in most contexts.

Figure 6 portrays the equilibria arising in case of a minor innovation. In panel (a), we see that a low spillover implies a late equilibrium in Industry I. As the spillover increases, however, an intermediate equilibrium prevails. For instance, when $\Delta = 2.5$, the late equilibrium prevails for $\theta \leq 0.058$ and the

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[21]Reasonable perturbations in r, ρ, and x do not significantly affect our results.

intermediate equilibrium prevails for $\theta > 0.058$.

[Figure 6 about here]

An intuitive explanation of this result is as follows. As underscored by Fudenberg and Tirole (1985), the smaller the cost reduction, the weaker the incentive to innovate first.[22] Hence, a small value for x means that even the highest deviation payoff for the first mover is low, so that the early equilibrium never prevails over the late one. Moreover, a low spillover gives rise to a late equilibrium because it shrinks the intermediate region; the second firm has little incentive to wait Δ years (refer to the definition for $\bar{T}$ and to Figure 3). Hence, the late equilibrium prevails over any possible deviation occurring in the intermediate period.

Industry	Innovation			
	minor		major	
I	$\theta \leq 0.058$	late	$\theta \leq 0.114$	early
	$\theta > 0.058$	intermediate	$\theta > 0.114$	intermediate
II	$\theta \leq 0.061$	late	$\theta \leq 0.121$	early
	$0.061 < \theta \leq 0.069$	early		
	$\theta > 0.069$	intermediate	$\theta > 0.121$	intermediate
III	$\theta \leq 0.061$	late	$\theta \leq 0.138$	early
	$0.061 < \theta \leq 0.082$	early		
	$\theta > 0.082$	intermediate	$\theta > 0.138$	intermediate

Table 1: R&D equilibria ($\Delta = 2.5$)

As θ increases the intermediate region expands, leading to a situation in which the first mover's deviation payoff becomes greater than her late equilibrium payoff. This leads to dominance of the intermediate equilibrium.

Panel (b) in Figure 6 shows the equilibria arising in Industry II. Again, if the spillover is very low for a given value of Δ, the late equilibrium prevails in the R&D stage for reasons explained before. However, as θ increases (while remaining below $\theta''(\Delta)$), the balance shifts to an early equilibrium. This shift happens in the small area contained between the two curves exiting from the origin in Figure 6 (refer also to Table 1). To understand this result, bear in mind that an increase in ρ raises payoffs in the intermediate region because R&D costs are lower.[23] Increasing the deviation payoff in the intermediate

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[22]This happens because the first mover's profit function, π_1^L, is more convex in x than the second mover's profit function, π_2 (see Appendix B).

[23]The effect of ρ on the late equilibrium payoff is of course similar, but less significant since at that time the R&D cost is already very low.

region destroys the late equilibrium, and moves the firms to adopt the early equilibrium as in the case portrayed in Figure 2, panel (A_1) (refer also to our discussion in Section 5).

Finally, further increasing θ (above $\theta''(\Delta)$) reduces the first mover's payoff in the early stage, making the intermediate equilibrium dominant as shown in Figure 3.

Figure 6(c) shows the equilibrium selection in Industry III ($\rho = 0.07$). The pattern is similar to that observed for Industry II, the only difference being that the θ threshold separating the early and intermediate equilibria is higher. This happens because the payoffs in the early region benefit more from a rapid technical progress than those in the intermediate region, since the R&D costs are higher.

Having computed that portion of the parameter space in which the intermediate equilibrium is subgame perfect, we now distinguish the FMA cases from the SMA cases (refer again to Figure 5). Our simulations show several areas in which the intermediate equilibrium is of the FMA type. This result stands in remarkable contrast to Hoppe's (2000) contribution. If one inserts the same specifications for demand and costs that we have adopted for our simulations into her framework, all equilibria imply a SMA for any strictly positive probability that the technology will perform poorly. A minor innovation induces the late equilibrium, where the pioneer delays innovation and her rival (if the technology performs well) immediately follows suit. Accordingly, the second mover obtains higher expected profits. In our model, the competitive advantage period granted to the pioneer by the disclosure lag can be sufficient for a FMA.[24] Moreover, our simulation shows that the area in which the intermediate equilibrium grants a FMA increases with ρ. While faster technical progress reduces the adoption cost sunk by both firms, higher values of ρ favor the first mover (who bears the entire cost of R&D) more than the follower. For example, when $\Delta = 2.5$ and $\rho = 0.07$, we have a FMA intermediate equilibrium for $\theta \in (0.082, 0.578]$; when $\rho = 0.04$, we have a FMA equilibrium for $\theta \in (0.069, 0.368]$, and for $\rho = 0.01$ we have a FMA equilibrium only for $\theta \in (0.058, 0.192]$. Thus, when the technical progress parameter is low, we may expect a SMA equilibrium for realistic values of the disclosure lag.

The case of a major innovation ($x = 0.50A = 0.50$) is portrayed in Figure

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[24]Note that even when Δ shrinks to zero, there is still a difference between our framework and Hoppe's. We obtain earlier investment dates because the leader's payoff is not harmed by the level of spillover.

7. Here, the late equilibrium never prevails. A high value of x greatly favors the early equilibrium, as mentioned in Fudenberg and Tirole (1985). However, in our framework, an early equilibrium arises only for moderate values of the spillover parameter. When an intermediate equilibrium exists, it tends to prevail over the early equilibrium for two reasons. First, a high value of θ negatively influences the first mover's payoff in the early interval because it anticipates the follower's investment date (see Equation (5)). Second, as the spillover increases in the intermediate interval, the second mover's payoff gets larger. This effect further reduces the leader's incentive to invest. Thus, milder competition implies higher payoffs for both firms, inducing their selection of the intermediate equilibrium. (Refer, for example to Figure 3, Panel (B_1))

[Figure 7 about here]

The thresholds for θ yielding a subgame perfect intermediate equilibrium are slightly higher for a major than for a minor innovation, due to the strong incentive to be a pioneer provided by a major innovation. Note that, with a major innovation, the areas in which the intermediate equilibrium is of the FMA type are still relevant. Again, the area where a FMA type prevails is larger, the faster is the technical progress.

Our calculation show that – for low spillover levels – the leader's payoff tends to be higher in the intermediate equilibrium, than in an early one with no spillover. In fact the leader pays lower R&D costs, and benefits from the reduction in the follower preemption incentive. Accordingly, leaders may be induced to create mechanisms for spillover where these are lacking. For example, the firm may choose a geographical location close to their competitors or adopt a policy allowing its researchers to take part in scientific workshops and conferences. This point is similar to the one raised by Pacheco-de-Almeyda and Zemsky (2008), who – in a model of time consuming technology development built in the spirit of Ruir-Aliseda and Zemsky (2008) – challenge the view that inter-firm spillover are detrimental to the leader.[25]

In sum, our analysis of the equilibrium selection process suggests that the

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[25] In their model, the leader provides some (low) level of spillover to induce the follower to shift from a simultaneous to an imitative development strategy. Such a strategy requires that the follower starts developing his technology only once he has observed the outcome of the leader's effort. Accordingly, the leader benefits from some reduction in R&D costs, due to the slowing down of her development activity, and from the increase in her cost advantage period. In our model, instead, the lower preemption incentive is crucial in delivering the result.

intermediate equilibrium is subgame perfect in large portions of the parameter space. Notice, moreover, that for both types of innovations, the portion of the parameter space in which the intermediate equilibrium is of the FMA type increases with ρ.

6   Welfare analysis

In order to assess the welfare properties of our equilibria, we design and solve the benevolent planner problem, under the constraint that it can only manage a proportional subsidy on investment, financed by lump-sum taxes. In particular, we determine the parameter configurations in which the social welfare is positively affected by the investment subsidy. There, the market equilibrium implies under-investment.[26]

In our application, the planner's decisions are based on the instantaneous welfare levels attained by the Cournot decentralized solution, that are computed in Appendix B. Because the market game often does not have a closed form solution, to appreciate the welfare effect of a marginal subsidy, we need to rely on numerical simulations, which are based on the same parameterization we used in the previous Section. Our computations allow to obtain the following results:

i) Whenever the early equilibrium is subgame perfect, the market solution calls for a tax on investment, and hence implies overinvestment.

ii) Symmetrically, when the late equilibrium prevails, the decentralized solution involves a too low level of investment, requiring a subsidy.

iii) When the intermediate equilibrium dominates, it implies underinvestment, but for a small parameters sub-set, the size of which is increasing in the speed of the exogenous technical progress, and decreasing in the size of the innovation.

While the first two results are intuitive, the third deserves more attention. To understand why an overinvesting intermediate equilibrium is less likely the larger is the innovation size, consider that both the instantaneous social

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[26] Notice that in our second best perspective neither the number of firms acting in the market nor the way they compete in the second stage quantity game lies within the regulatory power of the benevolent planner. This is standard in the literature: see e.g. Stenbacka and Tombak (1994), Hoppe (2000), and Weeds (2002). In our framework, the first best equilibrium for an omnipotent planner would imply the presence of only one firm: whenever there are non-decreasing returns in the innovation size or probability, it is optimal to have only one firm to innovate and cover the entire market at the marginal (post-innovation) cost.

welfare and the firms profits increase more than proportionally with the size of the innovation (refer to Appendix B). Because the social welfare is larger than the pioneer's profit, also the wedge between the social and the private incentives to innovate increases with x, which acts against the possibility of overinvestment with a large innovation.

In the market game, a steeper cost reduction profile has strong effects on the innovation dates. In fact, an increase in ρ benefits the leader more than the follower, who bears only a share $1 - \theta$ of the cost. This provides an incentive to second comer's preemptive behaviors, which may lead to overnvestment.

Although the portion of the parameter space with overinvestment is the wider, the larger is ρ (and the lower is x), even in the case $\rho = 0.07$, the overinvestment area is small. When $x = 0.05$, and $\Delta = \{2, 4\}$ the intermediate equilibrium implies overivestment for $\theta \in [0.073, 0.104]$, and for $\theta \in [0.098, 0.209]$, respectively. When $x = 0.50$ and $\Delta = 2$ the intermediate equilibrium never implies overinvestment, while when $\Delta = 4$ it involves an excessive investment for $\theta \in [0.170, 0.199]$.[27]

Hence, not only the intermediate equilibrium prevails for most of the parameter configurations (as shown in Sub-section 5.1), but it also implies underinvestment in R&D, with the exception of small parameters areas. In particular, this result applies when the innovation size is large. Therefore, with major innovations, the duopolistic market equilibrium calls for public policies aimed at increasing the research activity, unless the inter-firm spillover is low so that the incentives to hasten innovation are high enough that the early equilibrium prevails. Notice that the natural indicators of a highly competitive environment, namely a diffusion equilibrium and rent equalization, do not necessarily imply that the R&D investment is excessive from the social planner's perspective.

When we focus on minor innovations – the case in which the market equilibrium underinvests, according to the earlier literature – our results imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, because intermediate equilibrium, even when it underinvests, is closer to the social optimum than the late equilibrium.

7 Conclusions

We analyze a duopoly game of innovation characterized by exogenous technological progress. In this setting, firms take into account a technological

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[27]Notice that Assumption 7 applies only if $\theta > \theta'''(\Delta)$, i.e. when the intermediate equilibrium already implies underinvestment. Hence, it is not crucial for these results.

spillover process that lowers the second mover's innovation cost. The spillover only affects R&D costs after a certain number of years, a period called the disclosure lag. Both features affect the firms' incentive to pioneer the innovation (i.e., become the first mover) or imitate the new technology. We show that an equilibrium arises in which R&D investments are undertaken at dates *in between* the early-type and late-type equilibria already studied in the literature.

This intermediate equilibrium is subgame perfect for a wide range of parameters, and may give rise to either a first mover advantage (FMA) or a second mover advantage (SMA). While the incentive to be the pioneer is inversely related to the spillover level, it is also directly influenced by the speed of technical progress and the length of the disclosure lag. In particular, we have found that a FMA persists for realistic levels of information spillover and disclosure lags consistent with the competitive advantage periods observed in reality. Finally, the intermediate equilibrium is socially inefficient, implying a low level of investment in R&D.

These results were obtained in a framework that complements Hoppe's (2000) analysis. In her model, instantaneous informational spillovers produced a SMA over a large portion of the parameter space. We have shown that when a disclosure lag is added to the model, equilibrium results are less favorable to the second comer.

Our analysis raises some policy implications. First, R&D subsidies should be helpful for a wide range of market configurations. In our duopoly model, we rarely obtain over-investment. Under-investment happens even in the presence of major innovations, which imply a large incentive to invest in R&D. When the innovation size is small, the dominance of the intermediate equilibrium calls for R&D subsidies that are less intense than suggested in the earlier literature. Policies designed without taking into account the inter-firm spillover can be oversized even when the spillover is quantitatively modest. The intermediate equilibrium calls for moderate policies, which may prove easy to implement from a political perspective.

Second, research joint ventures (RJV) should be assessed in more favorable terms than those implied by the literature following d'Aspremont and Jacquemin (1988), and Kamien, Muller, and Zhang (1992). While a RJV – moderating the competitive pressure to invest in R&D – may induce a too sharp reduction in investment in comparison to the early equilibrium, it is likely to improve social welfare over the underinvesting intermediate equilib-

rium. Notice, moreover, that the joint R&D activity may grant information flows that are faster or larger than those characterizing the decentralized solution, which acts in favour of a welfare-improving RJV

Our setting can be extended in several directions.

The leader's payoff tends to be higher in the intermediate equilibrium with low spillover than in an early equilibrium with no spillover. In the former case, the leader pays lower R&D costs and benefits from the follower's reduced preemption incentive. Accordingly, as suggested by Pacheco-de-Almeyda and Zemsky (2008), the leader would like to create a spillover if none is present. However, even should a firm commit to knowledge dissemination, it would have to balance the positive effects of spillover against the negative effects of a shorter disclosure lag. An analysis of this trade-off is left for future research.

It would also be interesting to consider a stochastic inter-firm spillover, where the probability of information diffusion depends upon how much time has elapsed since the introduction of the innovation and on the follower's imitation effort. Both extensions would require heavy use of numerical techniques. Moreover, the analysis of Section 5 suggests that the effects of the disclosure lag on the dominance areas are weak. Hence, our main result should not be undermined by the adoption of a richer framework.

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A   Appendix A: Proofs

Proof of Proposition 1

As a preliminary, notice that Assumptions 4 and 5 guarantee that the interval $[0, T_F^* - \Delta]$ is non-empty for $\theta \in [0, \bar{\theta}], \Delta \in [0, \bar{\Delta}]$. Moreover, notice that $\bar{T} > T_F^* - \Delta$ for $\theta \geq 0$.

Proof of part (a): the payoff at time 0 for the second firm, when it invests at t_F, is given by (4).

Suppose that the leader has sunk the innovation cost at time $t_L \in [0, T_F^* - \Delta]$, and that the second mover decides to wait more than Δ years to grasp the inter-firm spillover. In this case, according to Eq. (2), the innovation cost is $C_F(t_F) = (1 - \theta)\gamma e^{-\rho t_F}$. A few straightforward calculations then show that T_F^*, as given by (5), maximizes $V_F(t_L, t_F)$.

Alternatively, the second comer could decide not to wait Δ periods. In this case he should invest at (7). This course of action implies that $T_F' \in [t_L, t_L + \Delta)$. If this restriction were not satisfied, the innovation follower would have chosen to benefit from the spillover. Since $T_F' > T_F^*$, whenever $t_L \in [0, T_F^* - \Delta]$ the innovation follower grasps the imitation benefits and invests at T_F^*.

Because of this condition, the follower's payoff can be written as

$$V_F(t_L, T_F^*) = \frac{\pi_0}{r} - \left(\frac{\pi_0 - \pi_1^F}{r}\right) e^{-rt_L} + \frac{\rho\left(\pi_2 - \pi_1^F\right)}{r(r + \rho)} \left[\frac{\pi_2 - \pi_1^F}{\gamma(r + \rho)(1 - \theta)}\right]^{\frac{r}{\rho}},$$

which implies $\frac{\partial V_F(t_L, T_F^*)}{\partial t_L} > 0$ and $\frac{\partial^2 V_F(t_L, T_F^*)}{(\partial t_L)^2} < 0$ over the whole interval $[0, T_F^* - \Delta]$. Also notice that $\frac{\partial V_F(t_L, T_F^*)}{\partial \theta} > 0$ for every $t_L \in [0, T_F^* - \Delta]$. (This explains the behavior of $V_F(t_L, T_F^*)$ for $t_L \in [0, T_F^* - \Delta]$ in Figures 2, 3, and 5)

Proof of part (b): when $t_L > T_F^* - \Delta$, the innovation follower will never wait more than Δ years, simply because $t_L > T_F^* - \Delta$. Hence, his available strategies are

(1) wait exactly Δ periods to grasp the benefit of the spillover,

(2) invest immediately after the innovation leader, and

(3) wait for a span shorter than Δ to exploit the cost benefits of exogenous technological progress, and then invest (renouncing the inter-firm spillover).

First we compare what the follower obtains by waiting Δ periods (strategy 1) with what he gains by investing immediately (strategy 2). That is, we determine

when $V_F(t_L, t_L + \Delta) \geq V_F(t_L, t_L)$. This inequality immediately boils down to

$$\frac{\pi_2 - \pi_1^F}{r} e^{-rt_L} - (1-\theta)\gamma e^{-(r+\rho)(t_L+\Delta)} \geq \frac{\pi_2 - \pi_1^F}{r} e^{-r(t_L+\Delta)} - \gamma e^{-(r+\rho)t_L},$$

which is satisfied when $t_L \leq \bar{T}$. Hence, the innovation follower never chooses strategy 2 for any $t_L \in (T_F^* - \Delta, \bar{T}]$.

Next we compare strategy 1 with strategy 3: first for $t_L \in [T_F^* - \Delta, T_F' - \Delta]$, then for $t_L \in (T_F' - \Delta, T_F']$, and finally for $t_L \in (T_F', \bar{T}]$.

As a preliminary, notice that the inequality $\bar{T} \geq T_F'$ is satisfied for $\theta \geq \theta'(\Delta)$.

Suppose now that the leader invests at $t_L \in [T_F^* - \Delta, T_F' - \Delta]$. During this interval, the payoff function for a follower who does not exploit the inter-firm spillover is always increasing. In fact, this function is concave with a global maximum at $t_F = T_F' \forall t_L$. Hence, it is optimal for the follower to invest with a delay no less than Δ, which implies that the spillover is actually exploited.

When $t_L \in (T_F' - \Delta, T_F']$, the optimal strategy for the innovation follower must be determined by comparing the outcomes of delaying for Δ periods and investing at time T_F'. Hence, we need to determine when $V_F(t_L, t_L + \Delta) - V_F(t_L, T_F') \geq 0$. This inequality immediately boils down to

$$\frac{\pi_2 - \pi_1^F}{r} \left[e^{-r(t_L+\Delta)} - e^{-rT_F'} \right] - \gamma \left[(1-\theta)e^{-(r+\rho)(t_L+\Delta)} - e^{-(r+\rho)T_F'} \right] \geq 0. \tag{A.1}$$

It is easy to show that the left-hand side of (A.1) is non-increasing in t_L over the whole interval $(T_F' - \Delta, T_F']$. We then evaluate equation (A.1) at $t_L = T_F'$, and exploit equation (7) to substitute out T_F' when convenient. The resulting inequality is

$$e^{-rT_F'} \frac{\pi_2 - \pi_1^F}{r} \left[e^{-r\Delta} - 1 - r(1-\theta)\frac{e^{-(r+\rho)\Delta}}{r+\rho} + \frac{r}{r+\rho} \right] \geq 0,$$

which is fulfilled when $\theta \geq \theta'(\Delta)$. Hence, under this restriction, the strategy of waiting Δ periods is chosen for any $t_L \in (T_F' - \Delta, T_F']$.

Finally, strategy 3 can never be optimal for $t_L \in (T_F', \bar{T}]$ simply because the payoff function for a follower who does not exploit the spillover is decreasing in $t_F \in (t_L, \bar{T}]$. Thus, there is no point in waiting once the leader has already invested; moreover, recall that the immediate investment strategy has already been proven to be dominated by the delay Δ.

Proof of part (c).

The proof of part (b) implies that the innovation follower will never wait Δ periods for any $t_L \geq \bar{T}$. Hence, for $t_L \in (\bar{T}, \infty)$, his available strategies are

(1) invest immediately after the innovation leader, and

(2) wait for a span shorter than Δ (to exploit exogenous technological progress) and then invest, renouncing the inter-firm spillover.

The Proof of part (b) implies that when the innovation follower decides to wait for a span shorter than Δ, he invests at time T_F' for any $t_L \in (T_F' - \Delta, T_F']$. In fact, the payoff function for the follower, $V_F(t_L, t_F)$, has a maximum at

T_F'. We have already noted that for $\theta > \theta'(\Delta)$, $\bar{T} \geq T_F'$. Hence, under this parameter restriction, the second innovator invests immediately after the innovation leader. It is never in the follower's interest to wait Δ periods, because $t_L > \bar{T}$ and $V_F(t_L, t_F)$ is decreasing in t_F over the whole interval $t_L \in [\bar{T}, \infty)$.

This completes the Proof. ■

Proof of Proposition 2

The proof of part (a) is similar to the proof of Proposition 1(a).

Proof for parts (b) and (c). From the proof for Proposition 1, we already know that when $\theta \in [0, \theta'(\Delta))$, $\bar{T} < T_F'$. Also, note that it is possible to prove that $T_F' - \Delta < \bar{T}$.

If the leader's entry occurs at time $t_L \in [T_F^* - \Delta, T_F' - \Delta]$, the follower's optimal strategy is again to wait Δ years then exploit the inter-firm spillover. The follower's payoff function $V_F(t_L, t_F)$ is increasing in $t_F \in [t_L, T_F' - \Delta]$.

When $t_L \in (T_F' - \Delta, \bar{T}]$, the follower's optimal strategy must be determined by comparing the outcomes of delaying for Δ periods or investing at T_F'. Unfortunately, it is not possible to analytically characterize the sub-intervals in which the two alternative strategies prevail. Let us denote by $\breve{T}_L$ the instant when $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$. $\breve{T}_L \in (T_F' - \Delta, \bar{T}]$ because $V_F(t_L, t_L + \Delta) - V_F(t_L, T_F')$ is non-increasing in t_L. $\lim_{\epsilon \to 0} [V_F(T_F' - \Delta + \epsilon, T_F' + \epsilon) - V_F(T_F' - \Delta + \epsilon, T_F')] > 0$ and $V_F(\bar{T}, \bar{T} + \Delta) - V_F(\bar{T}, T_F') < 0$, in fact, by definition, $V_F(\bar{T}, \bar{T} + \Delta) = V_F(\bar{T}, \bar{T})$, and $V_F(\bar{T}, \bar{T}) < V_F(\bar{T}, T_F')$. Hence, for $t_L \in (T_F' - \Delta, \breve{T}_L]$ strategy (1) is optimal, while for $t_L \in (\breve{T}_L, T_F']$ the innovation follower decides to innovate at T_F' (strategy 3).

Assumption 1 guarantees that $V_F(t_L, t_L + \Delta)$ has a maximum in $(T_F^* - \Delta, T_F']$.

Proof for Part (d). Because $\bar{T} < T_F'$, when $t_L \in (T_F', \infty)$ the follower invests immediately after the innovation leader because his payoff function is decreasing in t_F. ■

Proof of Corollary 3

$\breve{T}_L$ has been defined as the time instant such that $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$. It is immediate to verify that this equality, when $\theta = 0$, is satisfied for $\breve{T}_L + \Delta = T_F'$. Because when $\theta = 0$, then $T_F^* = T_F'$, the Proof is completed. ■

Proof of Proposition 4

Proof for Part (a). For $t_L \in [0, T_F^* - \Delta]$ the innovation leader's payoff is given by (3), where the innovation costs are provided by (1) and $t_F = T_F^*$.

Exploiting equation (5), we obtain

$$V_L(t_L, T_F^*) = \frac{\pi_0}{r} + \left[\frac{\pi_1^L - \pi_0}{r} - \gamma e^{-\rho t_L} \right] e^{-r t_L} - \frac{\pi_1^L - \pi_2}{r} \left(\frac{\pi_2 - \pi_1^F}{\gamma(r + \rho)(1 - \theta)} \right)^{\frac{r}{\rho}}.$$

A few calculations show that the restriction $\theta \leq \theta''(\Delta)$ implies $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$.

Notice also that $\frac{\partial V_L(t_L, T_F^*)}{\partial t_L} \gtrless 0$ when $t_L \lessgtr T_L^*$ (with T_L^* given by (8)). T_L^* need not be smaller than $T_F^* - \Delta$.

Proof for Part (b). When $\theta \in [\theta'(\Delta),\ \theta''(\Delta))$, the first mover is aware that if she invests later than $T_F^* - \Delta$, her competitor will invest with a delay of Δ periods (Proposition 1, part (b)). In this case one can show that the unique solution for the equation $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$, $T_L^{ip} = -\frac{1}{\rho} \ln \left(\frac{(\pi_1^L - \pi_1^F)(1 - e^{-r\Delta})}{r\gamma[1 - (1-\theta)e^{-(r+\rho)\Delta}]} \right)$, lies outside the interval $[T_F^* - \Delta, \bar{T}]$ (that is, $T_L^{ip} < T_F^* - \Delta$). Hence, the follower's payoff is lower than the leader's for $t_L \in (T_F^* - \Delta, \bar{T}]$.

Proof for Part (c). If $t_L \in (T_F^* - \Delta, \breve{T}_L]$, because $T_L^{ip} < T_F^* - \Delta$, it is obvious that $V_L(t_L, t_L + \Delta) > V_F(t_L, t_L + \Delta)$ for any $t_L \in [T_F^* - \Delta, \bar{T}]$. Hence, *a fortiori*, this condition is also true for any $t_L \in [T_F^* - \Delta, \breve{T}_L]$. When $t_L \in (\breve{T}_L, T_F']$, the follower innovates at time T_F' (Proposition 2, Part (c)).

In this case, it is convenient to define the function $Q(t_L, T_F') = V_L(t_L, T_F') - V_F(t_L, T_F')$, so that:

$$Q(t_L, T_F') = \frac{\pi_1^L - \pi_1^F}{r} \left(e^{-rt_L} - e^{-rT_F'} \right) - \gamma \left(e^{-(r+\rho)t_L} - e^{-(r+\rho)T_F'} \right).$$

Notice that $Q(t_L, T_F') \equiv V_L(t_L, T_F') - V_F(t_L, T_F')$ for $t_L \in (\breve{T}_L, T_F']$; notice also that $Q(T_F', T_F') = 0$. We have that $Q(T_F^* - \Delta, T_F') > 0$. In fact, from Part (a), $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$, which implies $V_L(T_F^* - \Delta, T_F') \geq V_F(T_F^* - \Delta, T_F')$, and therefore $Q(T_F^* - \Delta, T_F') > 0$. Because $Q(t_L, T_F')$ may have at most two zeroes (and $Q(T_F', T_F') = 0$), we have that $Q(t_L, T_F') \geq 0$, in the whole interval $[T_F^* - \Delta, T_F']$, and hence, *a fortiori*, in the interval $(\breve{T}_L, T_F']$. ∎

Proof of Proposition 5

As a preliminary, notice that $\theta'''(\Delta) \geq \theta''(\Delta)$ for $\Delta \in [0, \bar{\Delta}]$.

Proof for Part (a). Recall that for $t_L \in [0, T_F^* - \Delta]$, the innovation follower invests at T_F^* (Proposition 1), and that the leader is aware of this behavior. Notice that if $\theta \geq \theta^*(\Delta)$ we have $T_L^* \geq T_F^* - \Delta$, because the latter inequality requires $\pi_1^l - \pi_0 \leq \frac{\pi_2 - \pi_1^F}{1 - \theta} e^{\rho\Delta}$; hence, $\theta \geq 1 - \frac{\pi_2 - \pi_1^F}{\pi_1^L - \pi_0} e^{\rho\Delta}$. Because $\theta \geq \theta''(\Delta)$, we have $V_L(T_F^* - \Delta, T_F^*) \leq V_F(T_F^* - \Delta, T_F^*)$. Hence, $V_L(t_L, T_F^*)$ is increasing for $t_L \in [0, T_F^* - \Delta]$. Moreover, even when the leader's payoff is maximized (i.e., when $V_L(T_F^* - \Delta, T_F^*)$), the second mover's payoff is higher.

When $t_L \in (T_F^* - \Delta, \bar{T}]$, the follower innovates with a delay of Δ. Accordingly, the first innovator's payoff for $t_L \in [T_F^* - \Delta, \bar{T}]$ is

$$V_L(t_L, t_L + \Delta) = \frac{\pi_0}{r} + \left\{ \left[\frac{\pi_1^L - \pi_0}{r} - \gamma e^{-\rho t_L} \right] + \frac{\pi_2 - \pi_1^L}{r} e^{-r\Delta} \right\} e^{-rt_L}.$$

Notice that $V_L(t_L, t_L + \Delta)$ reaches its maximum at

$$t_L = \hat{T}_L = -\frac{1}{\rho} \ln \left[\frac{\left(\pi_1^L - \pi_0\right) - \left(\pi_1^L - \pi_2\right) e^{-r\Delta}}{\gamma(r + \rho)} \right];$$

a few calculations suffice to show that $\hat{T}_L > T_F^* - \Delta$. Hence, the maximum for the first mover's value function over the interval $[0, \bar{T}]$ lies in the sub-interval $(T_F^* - \Delta, \bar{T}]$.

Proofs for parts (b) and (c). For $\theta \geq \max\{\theta^*(\Delta), \theta''(\Delta)\}$, notice that T^{ip}, the unique solution for the equation $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$, belongs to the interval $[T_F^* - \Delta, \bar{T}]$. Then compute that $T^{ip} < \hat{T}_L$ when $\theta < \theta'''(\Delta)$. ∎

B Appendix B: A Cournot interpretation for payoffs and welfare levels

The two firms, i and j, composing the industry, in each (infinitesimally short) period, are involved in a two–stage interaction: first they decide whether to innovate or not, and then they compete *à la* Cournot. Market demand is linear and equal to: $P = a - Q$, where P is the market clearing price and $Q = q_i + q_j$ is the total quantity supplied. Each firm has a unit cost of production c.

Investment in R&D immediately yields a process innovation that shrinks the unit production cost by an amount x, with $x < c$. The post-innovation production cost of firm h is therefore $C(q_h) = (c - x)q_h$, $h = i, j$.

Each firm's payoff depends not only on its own adoption date, but also its rival's. If neither firm has invested prior to period t, their individual profits in the Cournot subgame at t are those of the pre-innovation stage, i.e.,

$$\pi_0 = \frac{A^2}{9}, \tag{A.2}$$

where $A = a - c$. The subscript indicates the number of firms which have already introduced the innovation. The instantaneous welfare (computed *à la Marshall*) is then equal to

$$W_0 = \frac{4A^2}{9}. \tag{A.3}$$

If only one firm (say, firm 1) invests in R&D at time t, it benefits from the efficiency advantage and obtains a higher market share. The market price at t decreases from its pre-innovation level, and the individual profits become

$$\pi_1^L = \frac{(A + 2x)^2}{9}; \pi_1^F = \frac{(A - x)^2}{9}, \tag{A.4}$$

Notice that $\pi_1^L > \pi_1^F$, $\pi_1^L > \pi_0$, and $\pi_1^F < \pi_0$. Because the quantity produced by the firm that has not innovated is $(A - x)/3$, to preserve the duopolistic structure of our market we need to assume that $A > x$. In a Cournot environment, this hypothesis implies that the cost-reducing innovation is non-drastic. For this asymmetric behavior at t, the welfare is:

$$W_1 = \frac{8A(A + x) + 11x^2}{18}, \tag{A.5}$$

with $W_1 > W_0$.

Finally, we need to compute the outcomes when both firms have innovated at t. In this case, being more efficient, they both produce more than they did in the *status quo*; the market price is therefore lower. Their individual profits at t are

$$\pi_2 = \frac{(A+x)^2}{9}. \tag{A.6}$$

Obviously, $\pi_1^L > \pi_2$. Notice that the difference between π_1^L and π_2 is increasing in x: when only one firm enjoys a cost advantage, she obtains a larger market share while benefiting from an higher price to cost margin. Moreover, notice that the instantaneous payoffs in (A.2), (A.4), and (A.6) fulfill *Assumption* 1. When both firms have innovated, the social welfare is

$$W_2 = \frac{4(A+x)^2}{9}, \tag{A.7}$$

with $W_2 > W_1$ since $A > x$.

When firms simultaneously invest in R&D, individual profits rise from (A.2) to (A.6) and welfare jumps from (A.3) to (A.7). Alternatively, firms may behave asymmetrically, so that one is an innovation leader and the other is a follower. Under these circumstances, individual profits first change from to (A.2) to (A.4) (and welfare from (A.3) to (A.5)). Then, when the follower invests in R&D, the profit changes from (A.4) to (A.6) (and welfare from (A.5) to (A.7)).

Follower's reply at:
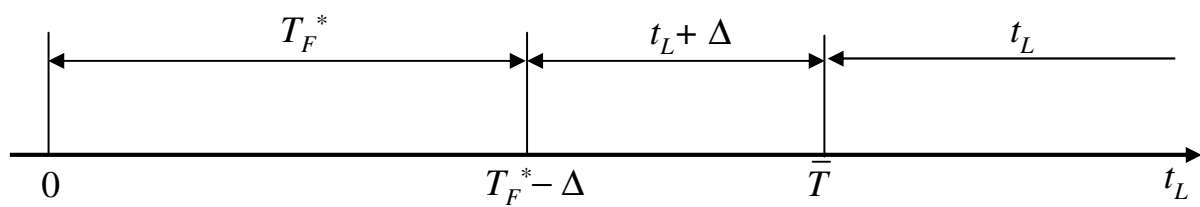
Figure 1: Follower's best reply as a function of t_L for $\theta \in [0, \theta''(\Delta)]$.
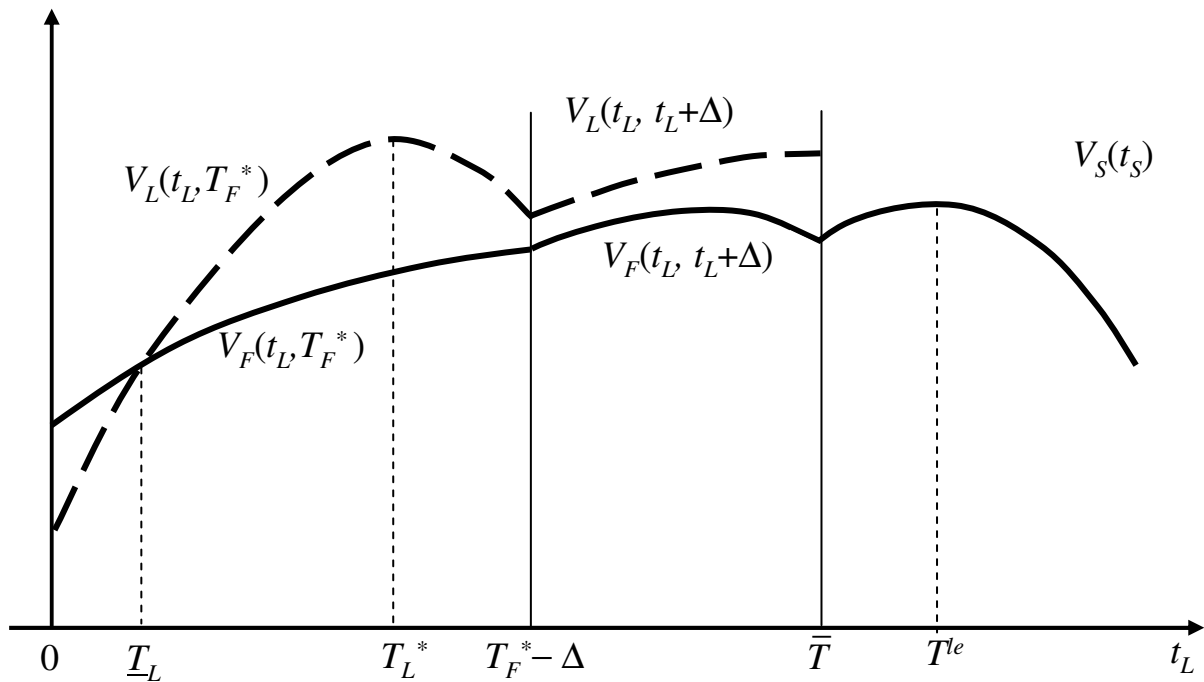
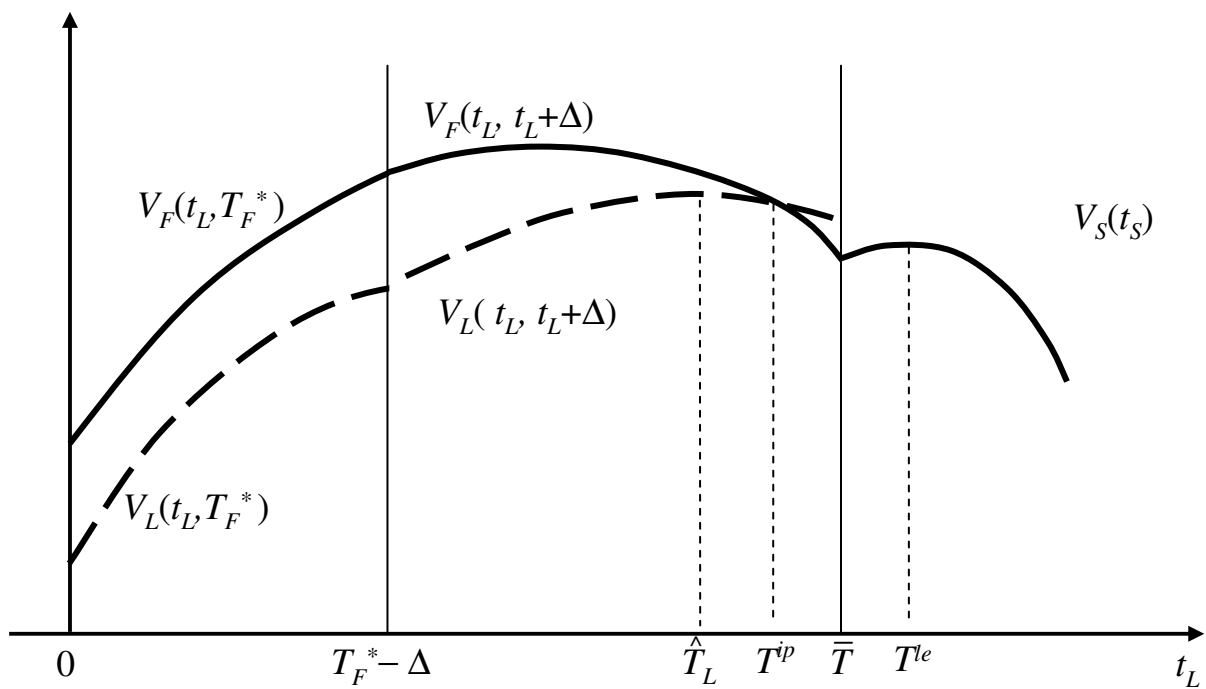Panel (A_1) : $\theta \in (\theta'(\Delta), \theta''(\Delta)]$
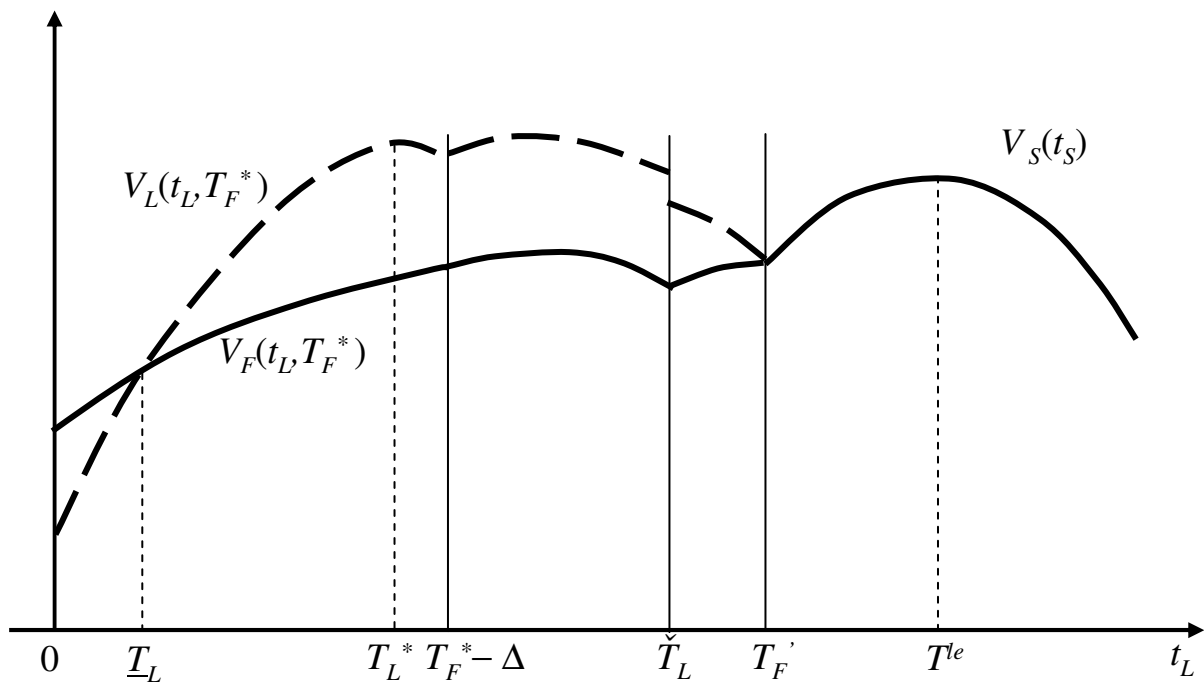
Panel (A_2) : $\theta \in (0, \theta'(\Delta)]$

Figure 2: Alternative behaviors of the firms' discounted payoffs for $\theta \in [0, \theta''(\Delta)]$.
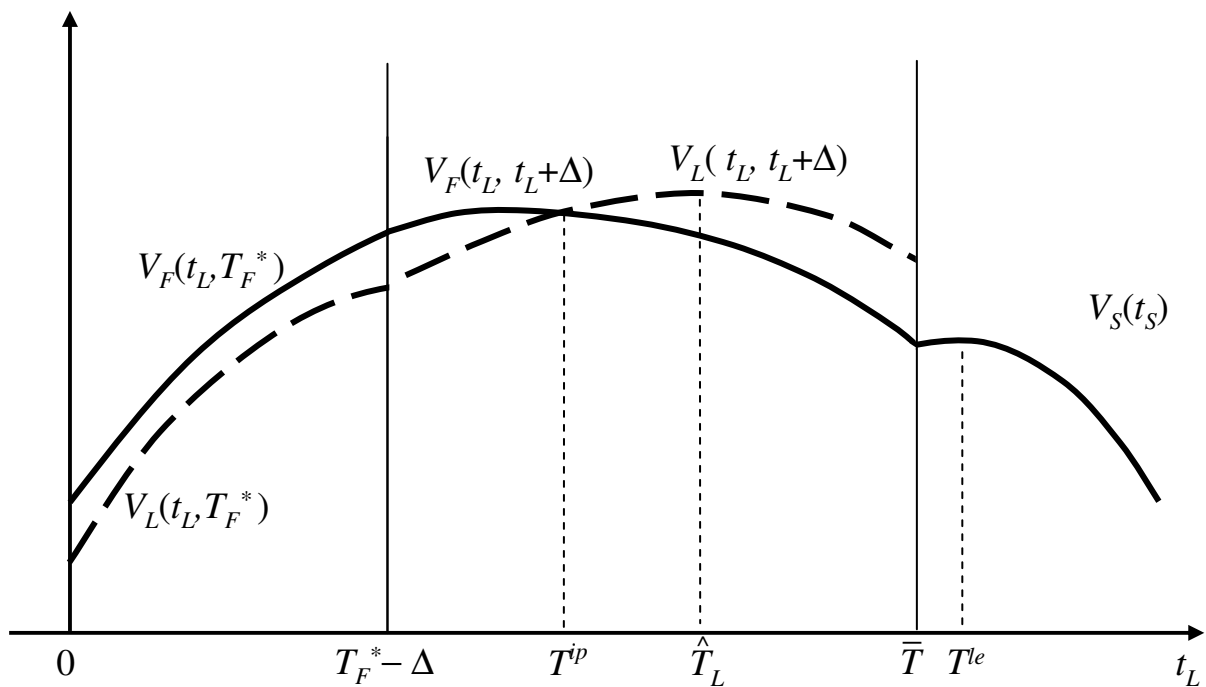
Panel (B_1) : $\theta \in$ ($\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$

Panel (B_2) : $\theta \in (\theta'''(\Delta), \bar{\theta}]$

Figure 3: Alternative behaviors of the firms' discounted payoffs for $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$
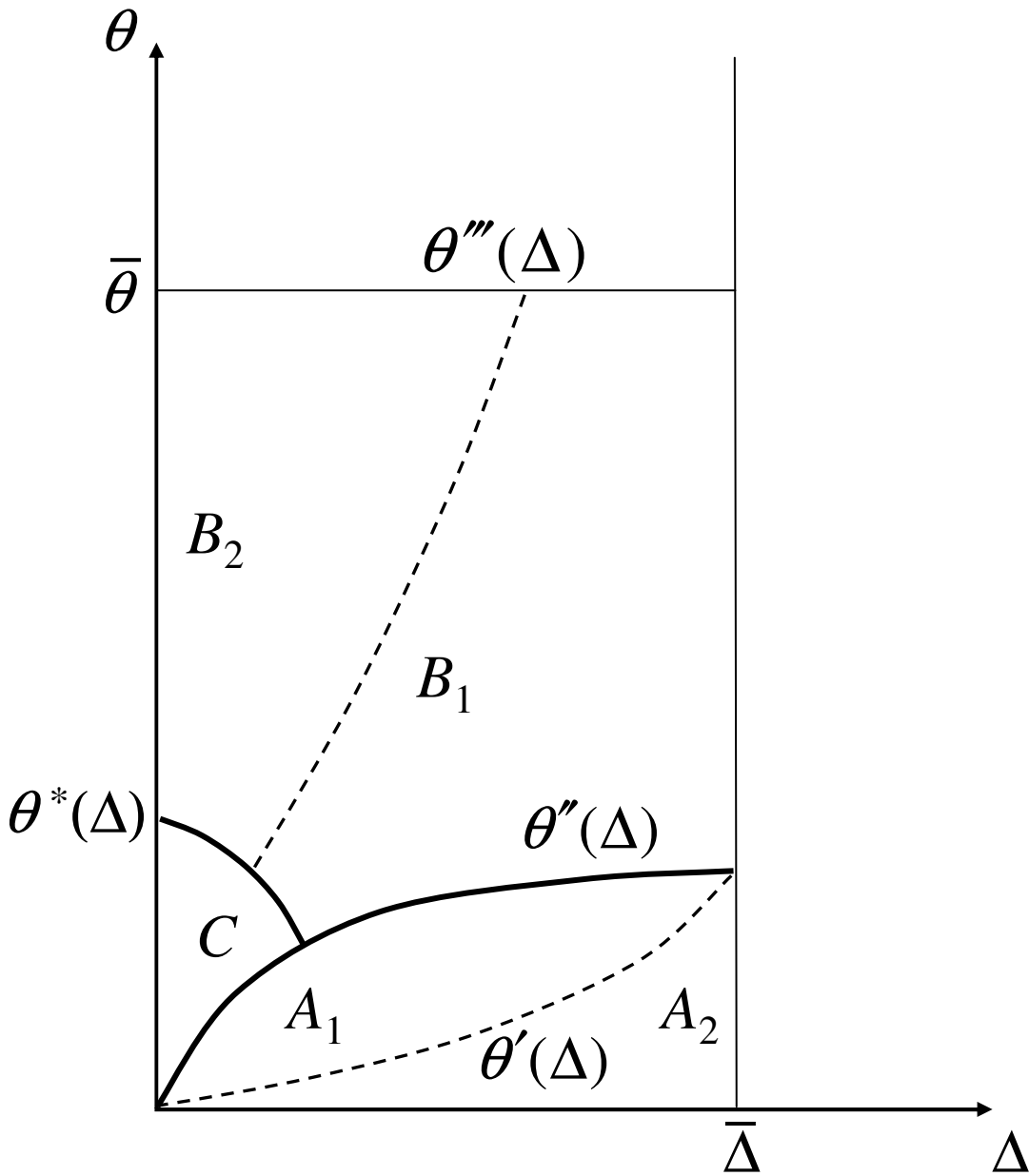
Figure 4. Parameter sets leading to alternative behaviors of the payoffs functions in the early and in the intermediate intervals.
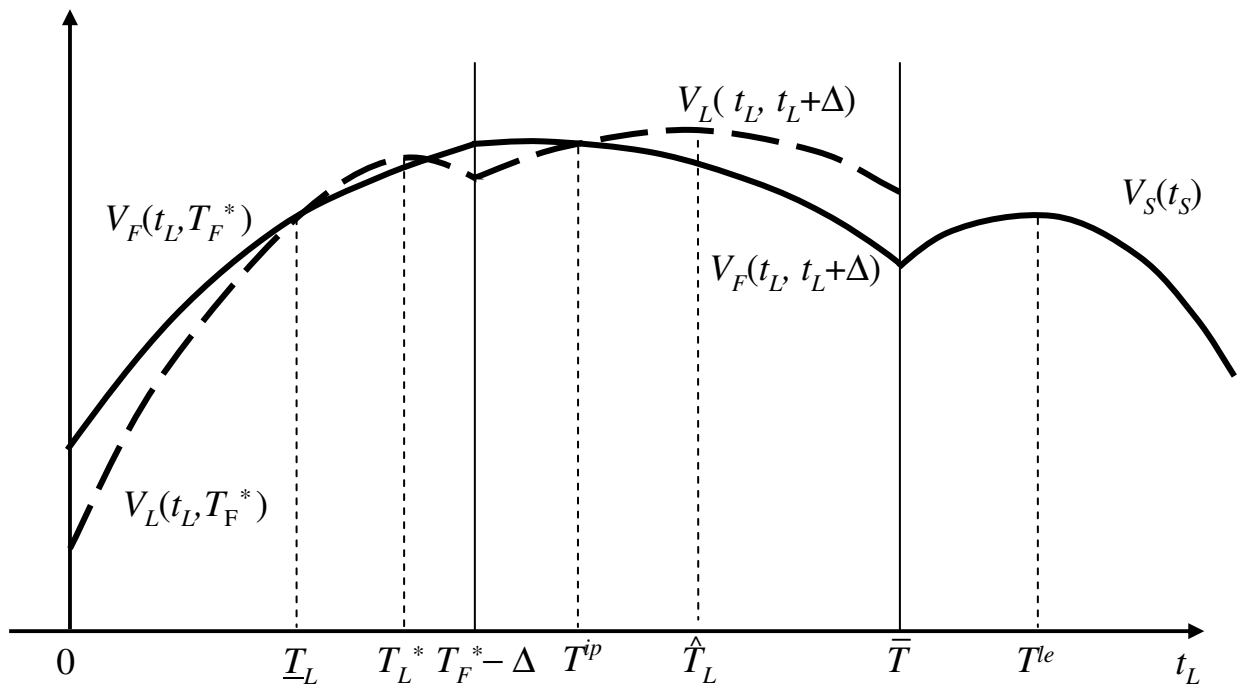
Figure 5: Behaviors of the firms' discounted payoffs for $\theta \in (\theta''(\Delta), \theta^*(\Delta)]$.

Panel (a): $\rho = 0.01$
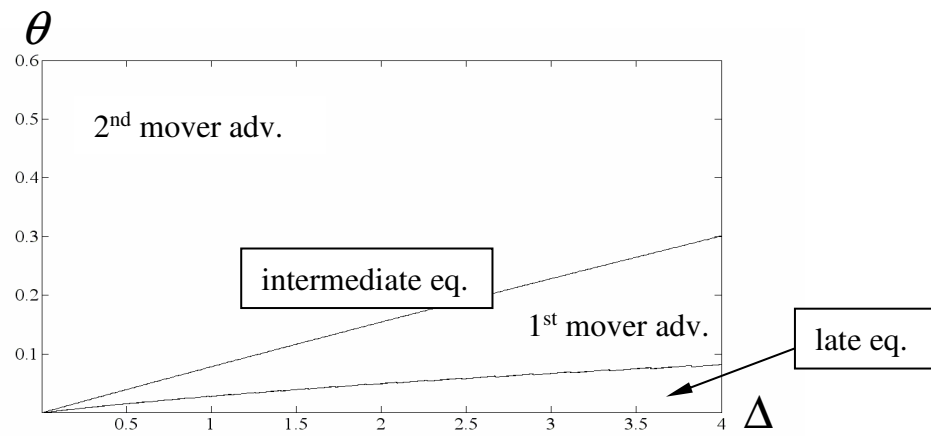
Panel (b): $\rho = 0.04$

Panel (c): $\rho = 0.07$
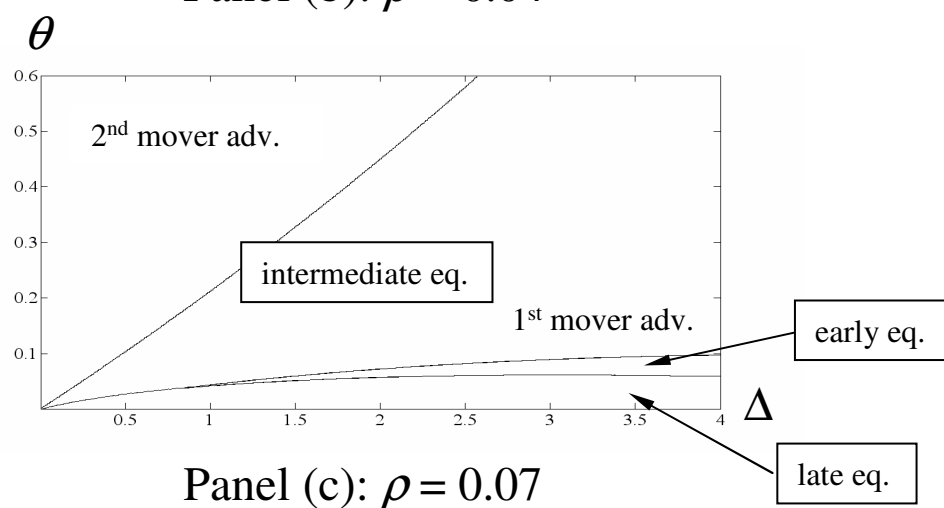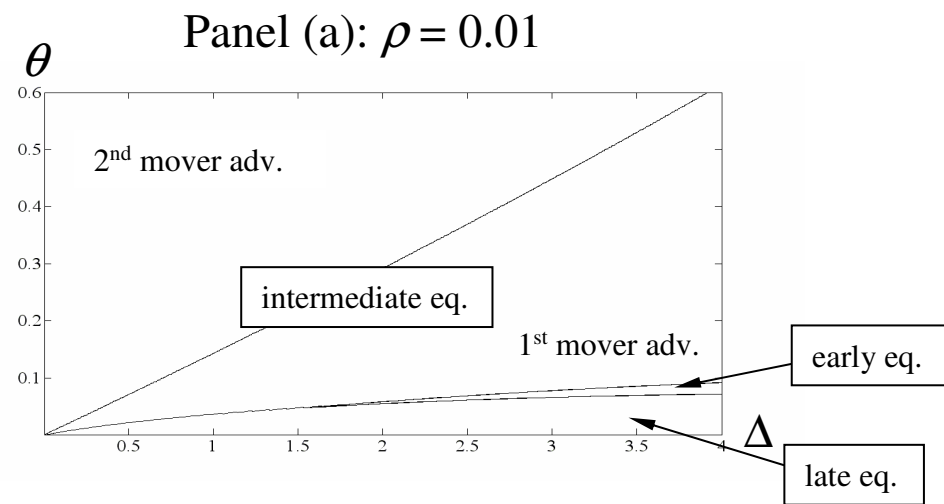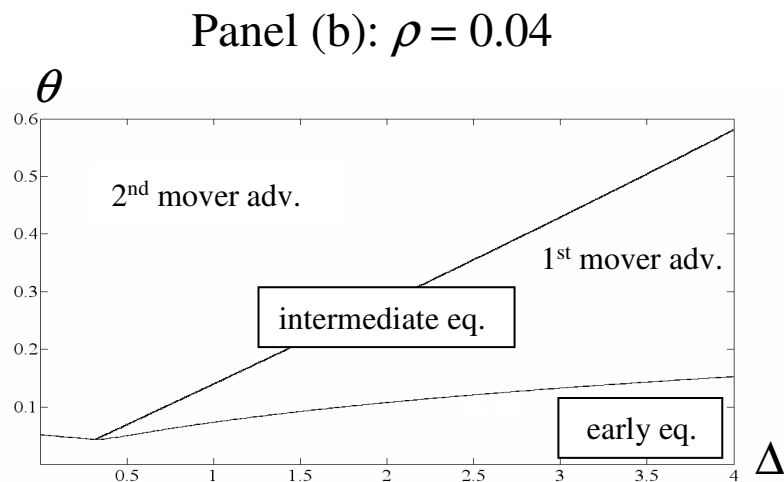
Figure 6 Equilibrium selection $-$ minor innovation ($x = 0.05$)

Panel (a): $\rho = 0.01$

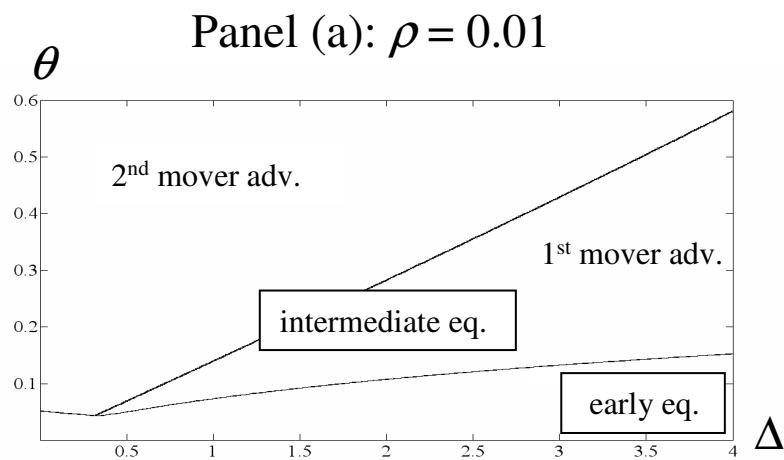
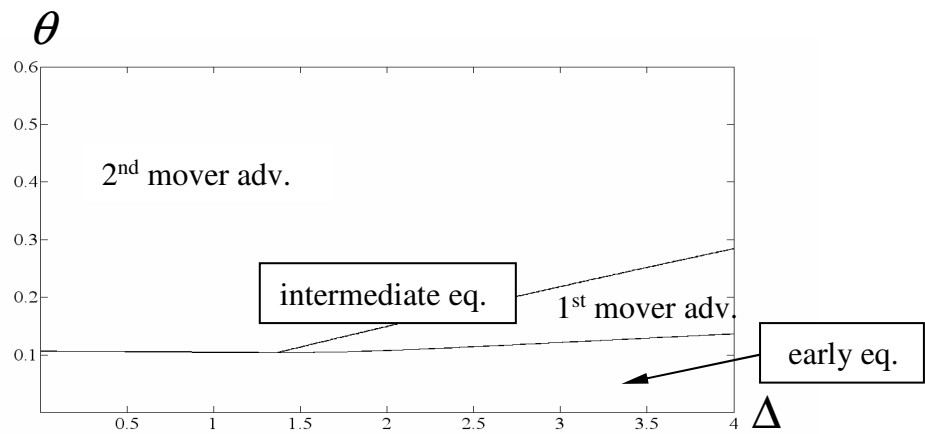
Panel (b): $\rho = 0.04$

Panel (c): $\rho = 0.07$

Figure 7 Equilibrium selection − major innovation ($x = 0.50$)