Comparison of several demand systems

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Abstract:

Using Monte-Carlo simulation, we compare the most popular demand systems including the LES, AIDS, BTL, QES, QUAIDS and AIDADS, and find that different models actually have different advantages in estimating different elasticities. Specifically, QES, AIDS and AIDADS models are the best in income, own-price and cross-price elasticities, respectively. Overall, AIDADS model has the best performance. The results indicate that the rank three models are not necessary always better than the rank two models.

Key Words: Comparison, Demand Systems, Monte Carlo, AIDS

1. Introduction

Since the invention of the linear expenditure system (LES) by Stone in 1954, a number of new demand systems have been developed and applied in empirical analysis, and the price and income elasticities obtained from the empirical studies have been widely used for projection and policy making. Hence after, the development of the demand systems include more flexible functional forms, such as the Translog family and the AIDS (Almost Ideal Demand System), and more Engel-flexibility, such as the QES (Quadratic Expenditure System), the QUAIDS (Quadratic Almost Ideal Demand System) and the AIDADS (An Implicitly, Directly Additive Demand System).

Different models may result in different estimations. For instance, Flood et al. (1984) find substantial differences in estimated elasticities between the AIDS and the Indirect Translog model. Moreover, Fisher et al. (2001) find that the relation between commodities could switch from substitutes to complements in the eight different demand systems using the US consumption data. In a summary of the current literature in food demand for developing countries, Abler (2010) finally finds that the empirical results are quite inconsistent and the inconsistency in part is explained by model differences. For instance, the AIDS model is likely to perform poorly in price and income elasticities as income changes, and particularly the income elasticity tends to be smaller as income increases.

Model comparison is very important for identifying the advantages and disadvantages for each model, and is helpful for improving the reliability of studies. Quite a few empirical studies have employed different models

2

and also shed light on model comparisons, such as Parks (1969), Deaton (1974), Pollak and Wales (1978, 1980), Klevmarken (1979), Guilkey et al. (1983), Lewbel (1989), Hansen and Sienknecht (1989), Chambers (1990), Alston and Chalfant (1993), Lee et al. (1994), Katchova and Chern (2003), Cranfield et al. (2003), and Fousekis and Revell (2003). These studies for model comparisons only show which model fits the data better via R-squared, information criteria, nesting or non-nesting tests, and so far the results are not consistent. For instance, Cranfield et al. (2003) compared the project ability between three rank three demand systems (QES, QUAIDS and AIDADS) with two rank two demand systems (LES and AIDS), and find that the rank three models are in general better than the rank two models, while the difference between the rank three models are not significant; while Kotchova and Chern (2003) find that the AIDS model is more suitable than QES in studying Chinese food demand.

As we know, the estimated elasticity parameters are of particular importance for projection, so that identifying the bias for each model and discovering which model's estimators are much closer to the true ones are much more attractive for researchers than simple model tests. As the true parameters are unknown, the Monte Carlo Simulation becomes an important tool for carrying out such a task.

Recently Barnett and Seck (2008) compared different types of elasticities with a Monte Carlo Simulation between the AIDS model and the Rotterdam model, and find that both the Rotterdam and AIDS models perform well when the substitution between goods are low, and only the AIDS model performs well when the substitution is high.

However, price and income elasticities are of different importance in projection or policy-making for different purposes. For instance, most agricultural trade models are more interested in income elasticities than price elasticities in a long run as they presume income will play more important roles, while others are more interested in price elasticities than income elasticities in a short run, as they presume that price effects such as tariff changes are more important. For instance, Abler (2010) suggests the AIDS model performs well in a short run as income is assumed to be constant, while Yu et al. (2003) propose that the AIDADS demand system outperforms several other models in projecting long-run world food demand. It is plausible that some models perform well in price elasticities while some others are in income elasticities. In light of this, the intrinsic biases for each model should be analyzed separately for price and income elasticities, which however is neglected in the current literature.

3

The main objective for this study is to identify the intrinsic biases in price and income elasticities separately for different demand systems by using Monte Carlo simulations. The models used in the comparison are the prevalent demand systems of the past 50 years, including the LES model, the Basic Translog (BTL), the AIDS model, the QES model, the QUAIDS model and the AIDADS model. In addition, the number of observations, and the number of commodities are also influential on estimations, and these factors are also taken into consideration in the simulation.

The paper starts with a brief review of the different demand systems then introduces the methods and data generation process, and followed by the simulation results, and finally presents discussion and draws the conclusion.

2. Summary of the prevalent models

We selected six demand systems for comparison in our study: LES, BTL, AIDS, QES, QUAIDS, and AIDADS. Following the definition of Lewbel (1991), the demand systems are ordered by their ranks: the first three are the rank two demand systems and the last three are the rank three models.

2.1 The linear expenditure System (LES)

It is the only linear demand system in expenditure relative to price, which fulfills the regularity conditions of demand theory. The linearity and the little number of independent parameters ($2k - 1$, where k is the number of commodities) makes its application easy but also imposes some limiting constraints. For instance, all goods are Hicksian substitutes, and cross-price derivatives are proportional to expenditure derivatives, and expenditure elasticities are always positive (no inferior goods). In addition, the Engel-flexibility is limited because of constant marginal budget shares. The LES can be estimated with the formula:

$$p_i q_i = p_i a_i + b_i (m - \sum_{j=1}^{n} p_j a_j)$$

where p_i, q_i and m are, price, quantity and expenditure, respectively. The underlying utility function makes the following assumption necessary:

$$q_i > a_i$$

The model satisfies homogeneity and symmetry automatically. For adding-up it is necessary to implement $\sum_{i=1}^{n} b_i = 1$ b_i and > 0.

4

2.2 Almost Ideal Demand System (AIDS)

The AIDS model, introduced by Deaton and Muehllbaur (1980a) can be derived from a second order approximation of any cost function, implying that it has a flexible functional form. It has enough independent parameters ($\frac{1}{2}k(k+3) - 2$) that all the elasticities can be identified. The Engel-flexibility is limited to linearity in logarithms. The model in budget share form is given by:

$$w_i = a_i + \sum_j c_{ij} \log p_j + b_i \log\left(\frac{m}{P}\right)$$

$$\log P = a_0 + \sum_i a_i \log p_i + \frac{1}{2}\sum_i \sum_j c_{ij} \log p_i \log p_j$$

For adding-up, homogeneity and symmetry, the following restrictions must be satisfied:

$$\sum_{i=1}^n a_i = 1$$

$$\sum_{i=1}^n b_i = 0$$

$$\sum_{i=1}^n c_{ij} = \sum_{j=1}^n c_{ij} = 0$$

$$c_{ij} = c_{ji}$$

2.3 Basic Translog (BTL)

Because of its importance in the past, another model with locally flexible functional form is included. The BTL is part of the Translog family. Its Engel-flexibility has the same limitation as the AIDS. The expenditure share function is:

$$w_i = \frac{a_i + \sum_j b_{ij} \log\left(\frac{p_j}{m}\right)}{1 + \sum_k \sum_j b_{kj} \log\left(\frac{p_j}{m}\right)}$$

For Homogeneity, Symmetry and Adding-up it is necessary to include the following restrictions in the estimation:

$$\sum_i a_i = 1$$

$$\sum_i \sum_j b_{ij} = 0$$

$$b_{ij} = b_{ji}$$

$$i, j, k = 1, \dots, n$$

2.4 Quadratic Expenditure System （QES）

The next three models are the generalizations of the LES and the AIDS to more Engel-flexibility.

The QES was first introduced by Howe, Pollak and Wales (1979). The additional squared expenditure terms upgrade the basic LES to a rank three demand system. However the Engel-flexibility is still limited because of the linearity of marginal expenditure. For the simulation we use the Σ-QES from Pollak and Wales (1992) in budget share form:

$$w_i = \frac{p_i b_i}{m} + a_i \left(1 - \sum_{j=1}^n \frac{p_j b_j}{m} \right) + \frac{\left(p_i c_i - a_i \sum_{j=1}^n p_j c_j \right)}{m} \prod_{j=1}^n p_j^{-2a_j} \left(m - \sum_{j=1}^n p_j b_j \right)^2$$

Again homogeneity and symmetry are included because of the composition. $\sum_{i=1}^n a_i = 1$ is a necessary restriction for Adding-up.

2.5. Quadratic Almost Ideal Demand System (QUAIDS)

The QUAIDS is an extension of the AIDS and was first proposed by Banks, Blundell and Lewbel (1997). It is still consistent with consumer theory, but with adding a quadratic term to overcome the limitation of the flexibility in expenditure. The QUAIDS is specified as

$$w_i = a_i + \sum_{j=1}^n c_{ij} \log p_j + b_i \log \left[\frac{m}{\alpha(p)} \right] + \frac{d_i}{\beta(p)} \left\{ \log \left[\frac{m}{\alpha(p)} \right] \right\}^2$$

$$\log \alpha(p) = a_0 + \sum_{i=1}^{n} a_i \log(p_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \log p_i \log p_j$$

$$\beta(p) = \prod_{i=1}^{n} p_i^{b_i}$$

For the theoretical restrictions, like adding-up, homogeneity and symmetry, the following restrictions are necessary.

$$\sum_{i=1}^{n} a_i = 1$$

$$\sum_{i=1}^{n} b_i = 0$$

$$\sum_{i=1}^{n} c_{ij} = \sum_{j=1}^{n} c_{ij} = 0$$

$$c_{ij} = c_{ji}$$

$$\sum_{i=1}^{n} d_i = 0$$

$$i, j = 1, \dots, n$$

2.6 An implicitly, directly additive demand System (AIDADS)

The most recent model in the comparison is an additive-preference demand system, invented by Rimmer and Powell (1996) and simplified for empirical applications by Cranfield et al. (2000). The AIDADS nests the LES and overcomes the limited Engel-flexibility by imposing less restrictive marginal budget shares. Another advantage compared to the AIDS or BTL is its global regularity when at least the subsistence level is affordable by the consumers. The system has the following form:

$$q_i = c_i + \frac{a_i + b_i \exp(u)}{1 + \exp(u)} \left(m - \sum_{j=1}^{n} p_j c_j \right)$$

where u is the utility level. Additionally the constraints to fulfill adding-up are:

$$\sum_{i=1}^{n} a_i = 1$$

$$\sum_{i=1}^{n} b_i = 1$$

3. Data

The data requirements for a comparison of the intrinsic biases of the demand systems reduce to the global knowledge of the true elasticities. Additional requirements for consumption data like the regularity conditions can be implemented, but the little number of unknowns makes a choice necessary. There is a broad discussion about aggregated datasets and the validity of the regularity conditions. For instance the studies by Deaton and Muehllbaur (1980b) and Christiansen, Jorgenson and Lau (1975) rejected symmetry and homogeneity in aggregated datasets. Hence we restrict the regularity conditions implemented in the generated dataset to the relationship between prices, quantities and expenditure, namely the adding-up condition, and the Euler Equation.

For comparing the influence of numbers of commodities or observations on the demand systems, we use six scenarios. Hence we generate six datasets with respectively four, five or six commodities and 500 or 1000 observations. We explain the procedure for each case as follows.

3.1 The simulation Strategy

The simulation strategy of prices, quantities, expenditure and income can be summarized in three steps:

- **Defining the true elasticities.**

For the scenarios with the same number of commodities we replicate the structure of the estimated consumption behaviors of existing studies and implement them in our generated dataset as the regularity conditions are presumed to be satisfied. In the case of four commodities the price and income elasticities (Table 1) are taken from the paper of Moschini and Meilke (1989). Then, Banks, Blundell and Lewbel (1997) is the

8

source for the uncompensated elasticities (Table 2) of the five commodities scenarios. In the last scenario the price and income elasticities (Table 3) are taken from the paper of Rimmer and Powell (1989).

- **Quantities**

The simulated data must contain the relationship between the prices, quantities and income. In order to generate the quantities, we randomly generate the prices and incomes, both of which are required to be positive, so that we generate prices by a uniformly distribution between one and four and generate incomes by a log normal distribution with a mean of 5 and a variance of 0.3. Given the true elasticities, the demand for the goods is calculated by the following formula,

$$\log q_i = \sum_k \varepsilon_{ik} \log p_k + \eta_i \log I + u$$

where u is an added error term with normal distribution N(0,0.01).

- **Adding-up and expenditure.**

In this step Adding-up is implemented in the data by summing up the product of quantities and prices of all goods to the total expenditures.

3.2. Descriptive Statistics

The distribution of income and prices are set in the generation of the dataset. During the procedure we generate the associated demand and expenditure series. Their averages are reported in Table 4.

3.3. Comparison structure

In our simulation, we suppose the true elasticities are known, which allows us to compare the estimated elasticities with the true ones, and to calculate the deviations of the estimated elasticities from the true values.

But given that we use the expenditure in the estimation and are interested in the effect of the income which is very important for long-run projection, we have to transfer the estimated expenditure elasticities into income elasticities:

$$\eta_i = \frac{\partial q_i}{\partial I}\frac{I}{q_i} = \frac{\partial q_i}{\partial m}\frac{m}{q_i}\frac{\partial m}{\partial I}\frac{I}{m} = \varepsilon_i * s$$

whereas η_i and ε_i are the income and expenditure elasticities, respectively and s can be estimated.

The estimation of the demand systems is repeated 1000 times for each scenario. To compare the resulting intrinsic errors of the models, we use the mean squared error (MSE) and mean absolute relative error (MRE):

$$MSE = \frac{1}{l}\sum_{i=1}^{l}(e - \hat{e})^2$$

$$MRE = \frac{1}{l}\sum_{i=1}^{l}\left|\frac{e - \hat{e}}{e}\right|$$

where l is the number of simulations, e and $\hat{e}$ are the true and estimated elasticities, respectively.

4. Empirical Results

The results of the estimation are presented in two parts. In the first part we aim to figure out which model has the best fit overall or in different elasticities. In the second part, we test the influence of the factors in a regression model. For each of the six models we estimate six simulation scenarios with different numbers of observations and commodities. Totally, we have 36 simulation scenarios.

4.1. Comparing the Results

As aforementioned the separated analysis of intrinsic errors in the income and price elasticities is the focus of the study. We will analyze them separately in this session.

- **Income elasticities**

Table 5 reports the mean of the MSE, MRE and the ranks for the simulation results of income elasticities. The rank three models are developed to increase the income flexibility of the demand systems, and theoretically they should perform better.

In the four commodities scenarios, the QES, which is a rank three demand system, has the smallest errors. However, in the five-commodity cases, the AIDADS performs best. In the model with six commodities the AIDS is the best and it however is a rank two demand system. The results show that no evidence supports that more Engel-flexibility is necessarily better. The AIDS model performs very well generally in different settings particularly when the number of commodities is large (k=6). Interestingly, when the number of commodities is small (k=4 and k=5), the AIDADS model performs very well, but it does not when the number of commodities is

large (k=6). It is worth to note that the other two rank two demand systems (BTL and LES) do not perform well in income elasticities.

- **Own-Price elasticities**

The intrinsic errors of the estimated own-price elasticities are reported in Table 6 and the results are also quite diversified. The general findings are (1) AIDS, BTL and QUAIDS are fairly good and robust overall cases, (2) AIDADS has consistently poor performances in all scenarios, (3) the results of LES and QES are changing with the number of commodities within different scenarios.

- **Cross-Price elasticities**

For the cross-price elasticities (Table 7) the results are diversified between MSE and MRE in the cases of five and six commodities. The diversification is caused by the different error structures for cross-price elasticities with small absolute values. It means that the measurement of the estimations could be significantly influenced by the scales of cross-price elasticities particularly when the absolute values of cross-price elasticities are very small, as a small estimator might lead to a huge MRE, which dominates the whole result. For instance, the estimated cross-price elasticity between good four and good one by the BTL is 0.052, whereas the true elasticity is 0.002, even though the estimated elasticity is still very inelastic. Calculating the MRE and MSE leads to values of 26 and 0.002, respectively. An absolute error of 0.05 does dominate the result in the first case extremely and does not in the second case. However, in terms of economic interpretations, we are more interested in MRE, as it standardizes the errors. In addition, we find that the results of MRE are more robust than those of MSE. Hence we restrict our interpretation to the MRE even though the results of the MSE are reported.

Based on the results of MRE, we find that (1) The number of commodities also influences the estimation significantly, particularly for the QES. When k= 4 , the QES is the best; and when k=6, the QES is also fairly well; but when k=5, the QES is the worst. (2) However, other models are consistent. The LES and AIDADS are good in estimation cross-price elasticities, and BTL, AIDS and QUAIDS are not so ideal.

- **Overall elasticities**

The results of overall performance for all elasticities are reported in Table 8 and it can help us justify the overall performance for each model. Similar to the results of the cross-price elasticities the results between MSE and MRE are quite diversified. However, the MRE standardizes the errors, so that the discussion in this session will be based on the results of the MRE. Similar to the findings in the cross-price elasticities, the general findings are

11

that (1) the LES model overall performs very well, and the AIDADS and AIDS perform fairly well; (2) However, QUAIDS and BTL are not good; (3) The results of QES are not robust and changes with the number of commodities.

4.2. Controlling Influencing Factors

So far, we only reported the performance for each model with respect to income, own-price, cross-price, and overall elasticities. In this part, we test the impacts of different factors on the performance by regressions. In table 9, we report the results of the regression both for the MSE, and for the MRE on dummy variables including the type of model (AIDS, BTL, QES, QUAIDS, AIDADS), number of observations (n=1000), number of commodities (k=5, k=6) and negativity condition (neg). Additionally, the true elasticity is included in the regressions to account for the influence of the size on the error. An insignificant parameter would indicate that the result is independent of the scale of the elasticities. The models are tested against each other, to see if the results are significantly different.

We report the regressions both for MRE and MSE in Table 9. But the following discussion is based on the results of the regression for the MRE, as it standardizes the errors.

First, the results indicate that the AIDADS is the best in overall performance if other factors are controlled. However, QES, AIDS and AIDADS models are the best in income, own-price and cross-price elasticities respectively.

Second, the number of samples is important for the performance, and it can surprisingly increase the intrinsic errors particularly for income and own-price elasticities. It might be caused by the fact that most elasticities are calculated at the means of variables, so that an increase in the sample size will increase the disperse of variables, so to the errors.

Third, the size of the true elasticity has a significant influence on the overall MRE, and the sign depends on the type of elasticity. The impact on the income and cross-price elasticity is negative whereas the the impact on own-price elasticities is positive.

Fourth, the number of commodities also affects the accuracy of the estimated elasticities. Regarding the income and cross-price elasticities, the five-commodity model has the best performance, compared with cases of six-commodity and four-commodity. For the own-price elasticities, the results indicate that the performance becomes better as the number of commodities increases.

12

5. Conclusion

It is very important to identify the advantages and disadvantages for each demand model by model comparisons as there are many systems in the current literature. Without knowing the intrinsic errors in different models, the estimated elasticities in the current literature might bias the project of demand and mislead policy making. Using Monte-Carlo simulation, we compare the most popular demand systems including the LES, AIDS, BTL, QES, QUAIDS and AIDADS. The performances of the demand systems are compared with errors in estimated elasticities.

First, our simulations indicate that the number of commodities, the sample size and the real elasticities have significant impacts on the performances of different models for different elasticities, and the effects are quite diversified.

Second, after controlling sample size, scale of elasticities, and the number of commodities, we find that different models actually have different advantages in estimating different elasticities. Specifically, QES, AIDS and AIDADS models are the best in income, own-price and cross-price elasticities, respectively. Overall, AIDADS model has the best performance.

The results imply that the rank three models are not necessary always better than the rank two models. Therefore, it should be very careful to pick up different elasticities in the literature for projection with different purposes.

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Table 1: Price and income elasticities for four commodities.

E	1	2	3	4	Income
1	-0.983	-0.004	-0.124	-0.109	1.22
2	0.087	-1.015	-0.047	-0.066	1.041
3	-0.161	0.086	-0.09	-0.073	0.238
4	-0.182	-0.021	-0.092	-0.138	0.432

Table 2: Price and income elasticities for five commodities.

E	1	2	3	4	5	Income
1	-0.9593	0.0657	0.2474	0.2641	-0.186	0.568
2	0.313	-0.804	-0.4946	0.6087	-0.0988	0.4753
3	0.7129	-0.4546	-1.0535	-0.0075	-0.3358	1.1388
4	1.1543	0.672	-0.0441	-1.7212	-1.3397	1.2786
5	-0.3621	-0.0641	-0.062	-0.1491	-0.6831	1.2605

Table 3: Price and income elasticities for six commodities.

E	1	2	3	4	5	6	Income
1	-0.517	-0.011	-0.003	0.002	0.001	0.013	0.515
2	0.021	-0.642	0.021	0.042	0.085	0.257	0.215
3	0.017	0.02	-0.834	0.049	0.099	0.3	0.348
4	-0.083	-0.024	-0.006	-0.997	-0.003	0.017	1.096
5	-0.145	-0.053	-0.028	-0.032	-1.03	-0.195	1.483
6	-0.09	-0.027	-0.009	-0.001	-0.011	-1.006	1.144

Table 4: Mean of the quantity and income series with 1000 observations

	quantity_1	quantity_2	quantity_3	quantity_4	quantity_5	quantity_6	expenditure
k=4	4.57	2.14	0.07	0.21			15.21
k=5	0.52	0.32	7.93	22.27	5.57		88.72
k=6	0.22	0.06	0.11	2.7	14.18	3.31	44.46

Table 5: The mean of the MSE and MRE of the income elasticities

	MSE						MRE					
	LES	AIDS	BTL	QES	QUAIDS	AIDADS	LES	AIDS	BTL	QES	QUAIDS	AIDADS
k=4												
n=500	1.951	1.892	4.975	0.018	1.895	0.32	1.682	1.415	4.639	0.215	1.425	0.693
	5	3	6	1	4	2	5	3	6	1	4	2
n=1000	1.949	1.887	4.955	0.017	1.89	0.419	1.684	1.402	4.633	0.21	1.412	0.716
	5	3	6	1	4	2	5	3	6	1	4	2
k=5												
n=500	0.4	0.105	0.108	0.227	0.11	0.045	0.487	0.22	0.249	0.387	0.252	0.139
	6	3	2	5	4	1	6	2	3	5	4	1
n=1000	0.442	0.104	0.099	0.239	0.11	0.031	0.524	0.211	0.241	0.4	0.252	0.137
	6	3	2	5	4	1	6	2	3	5	4	1
K=6												
n=500	0.217	0.001	1.41	0.075	0.062	0.488	1.076	0.048	1.274	0.596	0.582	0.742
	4	1	6	3	2	5	6	1	5	3	2	4
n=1000	0.217	0	1.462	0.077	0.063	1.239	1.075	0.036	1.294	0.601	0.586	0.617
	4	1	6	3	2	5	6	1	5	3	2	4

Note: All t-tests for the means were significant;

Table 6: The mean of the MSE and MRE of the own-price elasticity

	MSE						MRE					
	LES	AIDS	BTL	QES	QUAIDS	AIDADS	LES	AIDSS	BTL	QES	QUAIDS	AIDADS
k=4												
n=500	0.017 (5)	0.015 (2)	0.016 (4)	0.015 (1)	0.015 (3)	0.41 (6)	0.839 (5)	0.738 (3)	0.756 (4)	0.591 (1)	0.736 (2)	2.312 (6)
n=1000	0.017 (5)	0.015 (2)	0.015 (4)	0.014 (1)	0.015 (3)	0.455 (6)	0.841 (5)	0.735 (3)	0.756 (4)	0.593 (1)	0.732 (2)	2.395 (6)
k=5												
n=500	0.272 (5)	0.084 (1)	0.097 (2)	0.298 (6)	0.100 (3)	0.235 (4)	0.421 (5)	0.14 (1)	0.174 (2)	0.424 (6)	0.194 (3)	0.404 (4)
n=1000	0.271 (5)	0.082 (1)	0.096 (2)	0.324 (6)	0.098 (3)	0.221 (4)	0.422 (5)	0.134 (1)	0.171 (2)	0.442 (6)	0.193 (3)	0.386 (4)
K=6												
n=500	0.001 (1)	0.003 (4)	0.002 (3)	0.009 (5)	0.002 (2)	0.338 (6)	0.027 (1)	0.045 (4)	0.042 (2)	0.086 (5)	0.043 (3)	0.625 (6)
n=1000	0.001 (1)	0.003 (3)	0.003 (4)	0.038 (5)	0.002 (2)	0.339 (6)	0.025 (1)	0.043 (4)	0.042 (2)	0.109 (5)	0.042 (2)	0.617 (6)

Note: All t-tests for the means were significant;

Table 7: The mean of the MSE and MRE of the cross-price elasticity

	MSE						MRE					
	LES	AIDS	BTL	QES	QUAIDSS	AIDADSS	LES	AIDSS	BTL	QES	QUAIDSS	AIDADSS
k=4												
n=500	0.115	0.124	0.129	0.016	0.125	0.031	1.483	3.194	3.181	1.256	3.196	2.594
	3	4	6	1	5	2	2	5	4	1	6	3
n=1000	0.114	0.124	0.123	0.016	0.125	0.042	1.479	3.198	3.162	1.237	3.2	2.521
	3	4	6	1	5	2	2	5	4	1	6	3
k=5												
n=500	0.263	0.19	0.186	0.531	0.189	0.272	1.724	2.259	2.404	4.521	2.203	1.061
	4	3	1	6	2	5	2	4	5	6	3	1
n=1000	0.264	0.19	0.185	0.537	0.189	0.273	1.755	2.255	2.403	4.243	2.232	1.062
	4	3	1	6	2	5	2	4	5	6	3	1
K=6												
n=500	0.009	0.005	0.013	0.01	0.011	0.073	1.047	5.154	8.306	1.849	7.274	1.752
	2	1	5	3	4	6	1	4	6	3	5	2
n=1000	0.009	0.005	0.02	0.01	0.011	0.108	1.025	5.11	8.156	1.959	7.217	2.465
	2	1	5	3	4	6	1	4	6	2	5	3

Note: All t-tests for the means were significant;

Table 8: The means of MSE and MRE of all the elasticities

	MSE						MRE					
	LES	AIDS	BTL	QES	QUAIDS	AIDADS	LES	AIDS	BTL	QES	QUAIDS	AIDADS
k=4												
n=500	0.463	0.456	1.075	0.016	0.457	0.165	1.394	2.347	2.988	0.915	2.350	2.158
	3	4	6	1	5	2	2	4	6	1	5	3
n=1000	0.461	0.455	1.068	0.016	0.456	0.200	1.392	2.346	2.975	0.903	2.349	2.135
	3	4	6	1	5	2	2	4	6	1	5	3
k=5												
n=500	0.287	0.158	0.158	0.441	0.161	0.228	1.301	1.566	1.673	3.149	1.543	0.798
	5	1	2	6	3	4	2	4	5	6	3	1
n=1000	0.295	0.157	0.156	0.452	0.161	0.224	1.328	1.561	1.67	2.969	1.562	0.795
	5	2	1	6	3	4	2	3	5	6	4	1
K=6												
n=500	0.037	0.004	0.211	0.019	0.017	0.170	0.905	3.694	6.121	1.451	5.285	1.447
	4	1	6	3	2	5	1	4	6	3	5	2
n=1000	0.037	0.004	0.224	0.024	0.017	0.303	0.889	3.661	6.017	1.501	5.245	1.976
	4	1	5	3	2	6	1	4	6	2	5	3

Note: All t-tests for the means were significant;

Table 9: Regression of MSE and MRE on model specification and other properties

	MSE				MRE			
	All	Income	own-price	cross-price	All	Income	own-price	cross-price
Constant	0.476	2.386	0.000	0.103	1.444	1.869	1.493	1.55
	0.000	0.000	0.995	0.000	0.000	0.000	0.000	0.000
AIDS	-0.101	-0.349	-0.097	-0.037	0.704	-0.48	-0.170	1.256
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BTL	0.142	1.085	-0.091	-0.034	1.772	0.95	-0.155	2.499
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
QES	-0.134	-0.925	-0.019	0.04	-0.263	-0.648	-0.126	-0.175
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.017
QUAIDS	-0.097	-0.345	-0.091	-0.035	1.418	-0.271	-0.15	2.258
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AIDADS	-0.046	-0.505	0.192	0.012	-0.398	-0.457	0.586	-0.586
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
n=1000	0.015	0.065	0.003	0.005	0.034	0.014	0.008	0.05
	0.001	0.009	0.05	0.000	0.165	0.021	0.015	0.168
k=5	-0.323	-2.186	-0.09	0.133	-0.916	-1.861	-0.519	-0.806
	0.001	0.000	0.002	0.000	0.097	0.000	0.000	0.326
k=6	-0.332	-1.465	-0.057	-0.066	1.551	-1.01	-0.571	2.305
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Elasticity	0.247	-0.474	-0.18	0.08	0.344	-0.028	0.901	-0.17
	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.002
Negativity	0.103	0.415	0.092	0.026	0.421	0.405	0.242	0.475
	0.299	0.463	0.001	0.375	0.445	0.003	0.001	0.563
AIDS=BTL	0.000	0.000	0.009	0.116	0.000	0.000	0.004	0.000
AIDS=QES	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AIDS=QUAIDS	0.584	0.903	0.006	0.192	0.000	0.000	0.000	0.000
AIDS=AIDADS	0.000	0.000	0.000	0.000	0.000	0.023	0.000	0.000
BTL=QES	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BTL=QUAIDS	0.000	0.000	0.988	0.734	0.000	0.000	0.367	0.000
BTL=AIDADS	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
QES=QUAIDS	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
QES=AIDADS	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
QUAIDS=AIDAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\tilde{R}2$	0.011	0.040	0.221	0.046	0.012	0.318	0.433	0.016
N	887127	148310	146906	591911	887127	148310	146906	591911