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Yucel, Eray and Tokel, Emre
Central Bank of Turkey, Central Bank of Turkey

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ERAY YUCEL[†] AND O. EMRE TOKEL[‡]*

[†] *(Corresponding Author) Communications and Foreign Relations Department*
Central Bank of the Republic of Turkey
Istiklal Cad. 10
06100-Ulus Ankara, Turkey;
Phone: +90-532-543-5888; Email: eray.yucel@gmail.com

[‡] *Informatics Technology Department*
Central Bank of the Republic of Turkey
Istiklal Cad. 10
06100-Ulus Ankara, Turkey;
Phone: +90-532-674-7909; Email: etokel@gmail.com

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Abstract

All decisions are practically made within a chainwise social setup named a decision-making chain (DMC). This paper considers some cases of an idea (a project proposal) propagating through an organizational DMC. Survival of a proposal through successive links of the DMC depends on the relative power of those links, in addition to proposal's intrinsic value. Then it is not impossible to reject a good proposal or to fail to reject a bad proposal, either of which may generate undesired, though not detrimental, outcomes. We consider here a simple metric to assess quality of decision-making. The notion of quality here derives from "not declining (not accepting) a project that is of high (poor) intrinsic value". As Fibonacci series establish the mathematical basis of our proposed metric, metric is simply named a Fibonacci metric.

***JEL Classification**:* A1, C00 and Z1.

***Key Words**:* Decision making chains, Innovation, Fairness metric, Fibonacci series.

** YUCEL: (Corresponding Author) Communications and Foreign Relations Department, Central Bank of the Republic of Turkey, Istiklal Cad. 10, 06100-Ulus Ankara, Turkey; Phone: +90-532-543-5888; Email: eray.yucel@gmail.com, TOKEL: Informatics Technology Department, Central Bank of the Republic of Turkey, Istiklal Cad. 10, 06100-Ulus Ankara, Turkey; Phone: +90-532-674-7909; Email: etokel@gmail.com All the views expressed in this paper belong to the authors and do not represent those of the Central Bank of the Republic of Turkey, or its staff.

1.Introduction

There is no doubt that decision-making is one of the key problematic areas of social life. Regardless of its importance, impact on people's lives or even on countries' performance, there are practically infinite instances at the heart of which there is a decision-making problem. Indeed, society itself can be viewed as a trivial means of facilitating complex and interrelated decision making problems.

There is vast literature devoted to analyze decision-making processes. Individual preferences and aggregation of them lie at the heart of the social choice theory. The economic literature, hence, deals with how individual preferences are translated to social (or aggregate) preferences. This is mostly done with specific reference to structure of individual preferences, voting systems (putting formally, social choice rules) and their nexus, which is termed as outcomes. It is important to highlight that these problems entail a set of individual preferences that yield an aggregate decision at the end of a one-shot vote, without loss of generality.

On the other hand, most decision-making instances include a sequence of approvals or vetoes. In this scheme, regardless of the evaluation method(s), whether an initial idea (a project proposal, for instance) will be implemented eventually requires a full sequence of approvals. It is then intuitive that one veto is enough to decline the implementation of the initially proposed idea. There are usually multiple decision makers in any organization and they can be structured in different forms. Although flat (horizontal) structures can be observed especially in smaller organizations, in bigger ones like government, the decision makers are arranged within a given hierarchy, which is well addressed by the term bureaucracy. Leaving for a while the details, any bureaucratic system is meant to simplify complex decision making problems by casting it in terms of a decision making chain (DMC). A decision making chain may be visualized as a linear path in an organizational chart, which is drawn from bottom to top including the related nodes. It is again trivial that, relative impacts of successive links in a DMC are proportional to their designated (or a priori defined) powers.

This paper elaborates the DMCs with specific reference to above-mentioned impact on decision-making and proposes a fairness measure for numerical assessments of the decision-making quality. The notion of quality here derives from “not declining a proposal that is of high quality and/or value added” or “not accepting a proposal that is low quality and/or not value added”. For simplicity this notion will be referred to as the “quality criterion”. This naming implies that decline of a valuable project proposal or approval of an irrational project proposal shall ultimately yield a lower quality outcome, which is supposed not to increase overall welfare. In other words, accepting or declining is basically testing a hypothesis. In testing hypotheses, there can be statistical and systematic errors. Since systematic errors are caused by non-random fluctuations from an unknown source, the decision makers should concentrate on the statistical errors. The statistical errors are of two types: False rejection (Type I) and false acceptance (Type II). Although the Type I errors seem to be more significant, the opportunity cost of Type II errors must be taken into account as well. Hence false acceptance of a low quality proposal is as important as false rejection of a high quality one and statistical errors of both types should be minimized as possible. In that sense, for an organization of any scale, not only encouraging innovation and entrepreneurship/intrapreneurship, but also true allocation of the limited resources is vital.

Normal workings of any DMC, if designed for human beings' well being, are expected to yield high-quality outcomes. In specific, innovative ideas are supposed to be accepted and there is a good probability that these ideas shall include Pareto improvements. But deviations do occur; i.e. culture, institutional inertia, etc. may well induce these deviations. Indeed, deviations themselves -as they are referred to in this study- are mainly deviations from an ideal scheme. Nevertheless, these deviations are at the same time establishing the norm in many cases. Namely, resistance to new ideas and strict obedience to status quo may already be the norms of a society.

Some examples (or hypothetical cases) may be useful for concretizing what this paper is actually about. Decision-making at the country level can be a good starting point. Suppose that a certain segment of the society demands a certain large-scale project (P) to be undertaken. Under normal circumstances, the political party representing the views of this social segment is expected to make a formal proposition in attempt to legislate P. Then, the proposition P follows an array of expert committees, commissions and chambers of the parliament before it is presented to the prime minister or the president for final approval. Moreover, the Supreme Court, for instance, or other higher courts may be further required to approve the legislated project. This can be accepted as a well-known DMC. Unless the country at hand is not democratic, such a chain can be trivially formulated. As a matter of fact, one should not undermine the fact that the proposition of the project P by a political party also requires a sequence of approvals. This time, the DMC is located in an intra-party manner. In other words, separate DMCs can form a DMC-at-large, which is by definition a DMC.

Design projects can establish another good example. Most design projects are initiated by clients' demand. Following the needs analysis, a conceptual design framework or alternative frameworks are presented to superintendents for initial approval. Assuming that the initial proposal is approved, it should be approved again and again until the project is authorized. In such a case, the client has the strongest jurisdiction to finally approve or decline the proposal. Some other design proposals originate from incubators, as in the case of many R&D companies. However, the origin of the initial idea is not supposed to change the nature of the DMC. The requirement that "a full sequence of approvals is needed" remains intact.

This paper aims to reach a measure of the extent to which a DMC might generate low-quality outcomes, i.e. it declines a favorable proposal or accepts an unfavorable one. By this angle, the study aims to propose a metric in order to evaluate the past decision making inefficiencies as well as latent inefficiencies inherent to decision-making chains. Our approach resorts to extensive use of a very well known mathematical construct, the Fibonacci series, while defining our measure. As the current approach does not entail an optimizing aspect, paper seems fairly eclectic.

In the next section, we introduce the proposed metric. Section 3 elaborates possible applications of this metric and some further research directions.[1]

2.Developing the Model

2.1.The Decision Making Mechanism

Suppose that there are n links in a DMC, each of which is marked with L_j where $j = 1, 2, \ldots, n$. L_j's are located in the DMC in an increasing order of their importance (i.e. impact) in decision making.

A decision instance starts with initiation of a new idea (equivalently a proposal) at a certain link of the DMC. This can arbitrarily be selected as L_1, namely the starting link of the DMC. So the starting link can be thought of as the person (or a unit within an organization) who has initiated the idea.

We assume the following rule, for simplicity: For a proposal to propagate across the DMC, the initiator has to persuade the next link of the DMC, probably the nearest workmate or the nearest superior. If the idea is rejected by the second link then it is dismissed from the DMC. However, in the case that the second link gives credit to the initial idea, the second link now has the responsibility to defend it in later stages. Hence, the remaining steps are trivially replications of the first step of persuasion. That is, the second link tries to persuade the third, the third tries to persuade the fourth and so on. If the idea reaches L_n and gets accepted there, then it is approved and the project is launched.

[1] A brief overview of Fibonacci series is provided along with its applications in natural sciences and art in Appendix A.

2.2.The Assignment of Powers

Upon the decision-making mechanism introduced in the previous subsection, now we are ready to add "power" to picture. This is where we resort to elements of a Fibonacci series. We take the recurrence relation defined in Appendix A, starting from the two non-zero elements which are both 1. The values of the first n nonzero members of sequence are assigned to successive links of the DMC, which is of length n. We may denote these individual powers as:

For n=1, $$E(n)=1 \quad (1)$$

For n=2, $$E(n)=1 \quad (2)$$

Then for n>2, $$E(n)=E(n-1)+E(n-2) \quad (3)$$

These values indicate the persuasion power (or potential impact on the final decision) of each link of the DMC. These assigned powers can be interpreted as the importance of each link within the DMC and this picture borrows largely from our everyday bureaucratic experiences: If an initial idea does not get a full array of approvals, it does not eventually get implemented.

What we introduce further is the cumulative power of each link in the DMC. The cumulative power of the first link equals her individual power, by definition. The second link, once she approves the initial idea of the first link, gains the power assigned to the first link. Similarly, the third gains the power of the second, so and so forth, as long as the latter approves the proposal of the former. Indeed, the cumulative power series is nothing but another Fibonacci series:

For n=1, $$E^c(n)=E(n) \quad (4)$$

Then for n$\geq$2, $$E^c(n)=E(n)+E(n-1) \quad (5)$$

It is clear that individual power sequence of {1, 1, 2, 3, 5, 8, …} generates the cumulative power sequence of {1, 2, 3, 5, 8, 13, …}

When L_1 has a project proposal, following the DMC defined earlier, she consults L_2. At this point, $E^c(1)$ is compared to $E(2)$. When L_2 accepts the proposal she consults L_3. At this point, $E^c(2)$ is compared to $E(3)$. General step is such that, L_j, having accepted the proposal, consults L_{j+1}, where $E^c(j)$ is compared to $E(j+1)$.

2.3.What Happens in Each Turn?

Every time a lower rank link consults a higher rank link, an isolated decision instance is experienced. As to the structure and rules of this isolated instance, we are not prescriptive. The decision can be made against any technically plausible background. For instance, if the proposal involves a measure of return, the upper rank link may compare the projected return of project to appropriate benchmarks and may evaluate the associated risks to reach a conclusion. What she is assumed not to do is to arbitrarily reject or accept the proposal. In other words, the proposal will not be subject to the full discretion of the evaluating body. This is equivalent to saying that a project's chance to being accepted (or rejected) is not altered by the relative powers of successive links.

When we use the power assignments of the previous subsection as well as the resulting cumulative powers, immunity of the probability of a project being accepted (or rejected) is provided by the equality of $E^c(j)$ and $E(j+1)$. Satisfaction of $E^c(j)=E(j+1)$ does not necessarily mean that every proposal will be accepted (or rejected). Only thing that we know is nothing but the chance of a proposal (based on its intrinsic value) shall remain intact throughout the evaluation process that involves a sequence of comparisons.

The case that $E(j+1)>E^c(j)$ represents discretion of the evaluating body, i.e. she might arbitrarily decline a proposal without any solid reference to its intrinsic value. The opposite extreme involves

$E^c(j) > E(j+1)$ and it corresponds to the case of a supervisor with practically no power. The probability of a proposal to be accepted is altered downwards (upwards) in the former (latter) case. As noted earlier in the introduction, each of these cases has its own associated costs.

Note that existence of these cases does not necessarily mean that every single proposal will be arbitrarily accepted (or rejected). Rather, their chance of survival will be changed compared to the case of $E^c(j) = E(j+1)$. The extents to which chances are altered are not explicitly formulated.

Based on our treatment up to this point, the notion of fairness for a DMC is defined. Fairness is defined as the endurance of a proposal's acceptance probability to evaluating links' power assignments. Then, a DMC is the "least unfair of fair schemes" or "fairest of unfair schemes" if it has power assignments that derive from the Fibonacci series, introduced in subsection 2.2. We name a DMC with this power scheme as a Fibonacci DMC (FDMC). The deviations from FDMC and implications of these deviations are discussed in the next subsection.

2.4.Examples

As mentioned earlier, we formulate a decision making mechanism in fairly informal terms. In order to enhance our arguments, so, some examples might prove to be useful. These examples are presented in Table 1 and except the first one; they are established pedagogically without any specific references.

In Table 1, case I represents the FDMC. Case II is intended to depict a highly progressive assignment of powers. In case III, we try to assess what happens in a DMC involving 'equals'. In case IV, we introduce a break at a single link and show how an FDMC deviates from the ideal case once the break is there. Finally, case V depicts the limiting case of FDMC.

Table 1. Examples

i →	1	2	3	4	5	6	7	8
	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8
Case	**Assigned Powers**[(a)]							
I	1	1	2	3	5	8	13	21
	1	2	3	5	8	13	21	34
II	1	2	4	8	16	32	64	128
	1	3	6	12	24	48	96	192
III	1	1	1	1	1	1	1	1
	1	2	2	2	2	2	2	2
IV	1	1	2	3	6[(b)]	9	15	24
	1	2	3	5	9	15	24	39
V	1	φ =1.618	φ^2 =2.618	φ^3 =4.236	φ^4 =6.854	φ^5 =11.090	φ^6 =17.944	φ^7 =29.034
	φ [(c)] 1.618[(c)]	$1+\varphi$ =2.618	$\varphi+\varphi^2$ =4.236	$\varphi^2+\varphi^3$ =6.854	$\varphi^3+\varphi^4$ =11.090	$\varphi^4+\varphi^5$ =17.944	$\varphi^5+\varphi^6$ =29.034	$\varphi^6+\varphi^7$ =46.979

Notes: (a) For each case, the upper and lower rows display the individual and cumulative power assignments, respectively. (b) Indicates the break in power sequence. (c) Indicates the exceptional treatment of the first link.

Case I: First case is the aforementioned case of Fibonacci DMC. It displays the fairness property of subsection 2.3. Initiating a proposal at link 1, this scheme ensures that original probability of acceptance of this proposal not to be altered throughout the DMC.

Case II: In the second case the power assignments increase progressively throughout the DMC. This case might correspond to an organization in which a high degree of strict authority is exercised. In this case, practically every proposal is destined to arbitrary assessments and downward alteration of acceptance probabilities. However, the workings of the DMC in reverse direction have no complication. Real life case of a military command chain can be a trivial example.

Case III: The third case is a degenerate one as every single link in the DMC has the exact same power. To the best of our imagination, it is hard to find a real life and large-scale organizational counterpart for this scheme. A power scheme involving complete equality can, at best, be valid for small offices or small task forces. On informal grounds, rumor networks of daily life can also be a workable example. In case III, acceptance probabilities are likely to be altered upward.

Case IV: The power scheme of case IV is intended to see what happens when we allow for a single break in an FDMC. As seen in Table 1, L_5 has a power $E^*(5)=6$ instead of $E(5)=5$. Apart from this small irregularity, the power assignments are done in line with equations (1)-(3). This case is worth to underline: Any proposal passing through L_2 to L_4 might get stuck at L_5 as $E(5)>E^c(4)$. A real life example can be constructed by imagining a mid-level manager who has a power assignment disproportionate with her predecessors in the DMC.

Cases II through IV exemplify the outcomes of the deviations from FDMC. While the likelihood of acceptance of a proposal is altered downward in cases II and IV, it is altered upward in case III. By construction, only case I yields fair outcomes, in the sense that a proposal will not be subject to discretion of any assessing unit. As the FDMC is established to represent the ideal case, its limiting case might provide a simpler presentation of power assignments. Case V serves this purpose.

Case V: Up to this point, we have assumed that new proposals are initiated at L_1. Indeed, this simplifies the pedagogy a lot. However, it is possible for a new proposal to be initiated at any link L_j where $j>1$. Then the relevant power assignments start no more from L_1. Owing to the construction of the FDMC, the decision making mechanism do not malfunction.

If we start from a late segment of the FDMC, suppose L_{10} (not shown in Table 1), the ratio $E(j+1)/E(j)$ converges to a certain value as j tends to infinity. We are familiar with this ratio (see Appendix A) as it is the well-known golden ratio (φ), which reads out 1.6180339887…

Based on the observation on the golden ratio, it is possible to reformulate the power assignments starting from L_1: Suppose we set $E(1)=1$, $E(2)=\varphi$ and obtain the sequence of powers by means of equations (1)-(3). The resulting power scheme is demonstrated in Table 1 as case V. Case V seems to be a valid limiting representation of case I, provided that we exceptionally set $E^c(1)$ to φ. It is obvious that the DMC gets stuck at L_2 once the exception is not made.

Case V, at the end, represents a simpler FDMC which possesses the fairness property of the FDMC of case I, by definition. More importantly, it has a simpler verbal expression: To obtain an FDMC, assign a power of unity to the first link, then assign the golden ratio to the second, square of the golden ratio to the third, and so on. In other words, the power assignment of your nearest superior should be golden ratio times your power assignment, no more or no less.

In the remaining sections, the term FDMC will be used as equivalent to the DMC of case V.

2.5.Quest for a Metric to Assess Deviations from FDMC

Despite we have avoided a full-fledged mathematical treatment in this paper, the background purpose of this study is nothing but to obtain a good numerical measure of how good or bad a given DMC is with respect to the fairness criterion of subsection 2.3. To this end, suppose that we are given an arbitrary DMC with $E=[E(1),E(2),...,E(n)]$. We compute first, for $j=1,2,...,n-1$:

$$R(j+1)=E(j+1)/E(j) \quad (6)$$

$$\Delta(j+1)=R(j+1)-\varphi \quad (7)$$

$$S(j+1) = Sgn(\Delta(j+1)) \quad (8)$$

Intuition behind these is straightforward: Equation (6) computes the ratios of successors' powers to predecessors' powers. Equation (7) is to compute the deviations of these ratios from the golden ratio. Finally, equation (8) is to obtain the signs of differences from equation (7). This paper advances no further in mathematical terms, yet we set the following principles for assessing a DMC's power assignments:

- If $S(j+1)$ is zero for all j then the DMC in question is an FDMC.
- If there are nonzero values of $S(j+1)$ then the DMC in question is not an FDMC and it might generate undesired outcomes in terms of our notion of fairness.
    - The fair portion of the DMC is only as long as the index j of the first nonzero $S(j+1)$ value.
        - If the first nonzero $S(j+1)$ value is +1, acceptance probability of a proposal at $j+1$ is altered downward; hence from this link onwards, fairness criterion is seriously violated.
        - If the first nonzero $S(j+1)$ value is -1, acceptance probability of a proposal at $j+1$ is altered upward; hence the proposal does not necessarily get stuck, despite the loss of fairness.

One may develop a sophisticated metric based on Euclidean distance and obtain a good summary measure workable for any given DMC.

2.6.Some Criticisms

There may be several criticisms of the model developed in the previous subsections. The below apology is intended to address some of them:

- The decision making mechanism is too simple or oversimplified.
- The notion of power is quite unclear.
- Voting or similar choice mechanics are omitted by the mechanism.
- Mathematical level is low, except the charm of the Fibonacci numbers and golden ratio.
- Despite there are several mentions of probability of acceptance (rejection) and the like, there are no formal probabilistic assessment of what is actually going on in the model.
- The model may be useful to understand some real life cases. It is not clear, yet, how to accomplish this, from where to find data, and so on.
- The link between the presented model and the earlier positional power literature is vague.
- The problem is interesting yet the current treatment is totally fruitless.

All these rightful criticisms sheds light on our further research agenda as presented in the last section of the paper.

3.Further Research: Proposed Metric at Work

3.1.Theoretical Directions

In order to obtain a concrete model with solid theoretical infrastructure, one can introduce first a formal statistical treatment of what is happening at each link. To this end, the intrinsic probability of acceptance (or rejection) can be defined as a prior probability and the posterior might be defined as a probability conditional upon $E(j+1)/E(j)$. The extent to which the posterior differs from the prior can be a good measure of aforementioned “alteration” of chance of acceptance (or rejection). Obviously, there will be a long array of assumptions on the shape of probability functions.

In order to further concretize the model, a concept of cost must be introduced to the model. Indeed, if there are no associated costs, the problem itself is degenerate. A minimization argument then is indispensable; i.e. a total cost function including costs of rejecting a good proposal or accepting a bad proposal can be minimized with respect to the sequence of ratios $E(j+1)/E(j)$. Whether the minimization confirms a Fibonacci-like assignment of powers is not that obvious.

Then more complex forms of the decision making rule along the DMC can be taken into consideration.

Endogenous prior probabilities can provide a more interesting direction of research. If a link (an individual or a unit) realizes the distortions on the chance that its proposal is accepted, in a repeated game fashion, it might consider tailoring the proposal so as to maximize the chance to survive through the DMC. Clearly such a treatment requires an explicit correspondence between the properties of a project and its chances of acceptance.

If one thinks of a DMC where each link is a voting body, a nexus between the current model and social choice rules can be established.

Leaving the further theoretical research directions aside, quest for better structured interpretations of the concepts of power, project evaluation and fairness within the current context establishes a good starting point for further research. In this regard, the power assignments of this paper should not necessarily be read as arbitrary attributions of ‘potential to impact final outcomes’. One is allowed to treat a power assignment as a perceived (or implied) potential to impact organizational decisions. In an imaginary organization, it is quite possible to have a *de jure* ideal distribution of power. Whereas, owing to cultural, technical and sometimes personal reasons, *de facto* distribution of power among links (of DMC) may be quite different from the *de jure* setup. In such a case, what really matters is the power of other links perceived by a certain link. Perceived power can also be named implied power.

3.2.Practical Directions

The current model can be a useful, yet *ad hoc*, template to empirically assess the real life organizations. One of the two trivial uses of the model is in comparing alternative DMCs. In that, based on the closeness of two DMCs to the FDMC, one can generate fairness scores. These scores, whenever computable, can be used to sort organizations.

Second, the model can be used to measure organizational change in terms of our notion of fairness. For a given organization and for its given decision making mechanism, a current score can be computed. Then changes in the organization’s structure and/or in its decision rules can be assessed against the gains (or losses) in fairness score.

With regard to the practical use of our model, there are some questions to study. The first is about the source of organizational data: Should/can one employ surveys to gather data (note that this is nothing but the ‘perceived power’ of the previous subsection) on power assignments of links? This seems to be a feasible alternative, yet survey outcomes can be quite biased by a number of social and/or psychological factors. If this turns out not to be a workable alternative, one may prefer revisiting the organizational records of project proposals in retrospect. That is, destinies of a long array of past proposals can be assessed in order to obtain the likelihood of acceptance (or rejection) at a certain link of DMC. This is equivalent to extracting a consolidated picture of odd ratios of acceptance based on historical data. By the very nature of many organizations, historical data on proposals might not exist, or might exist only for the accepted proposals.

The second question regards the intrinsic probability of acceptance of a proposal. This more complicated question requires some objective body to re-assess an organization's past data, which is quite an expensive process. However, such an assessment is indispensable for measuring the alterations in acceptance (rejection) probabilities.

Finally, a clear enough connection between the abstraction of the current model and actual organizational structure is necessary. Existence of feedback and/or internal audit mechanisms and the extent to which they are active may, indeed, alter the empirical conclusions.

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Appendix A. Fibonacci Numbers: A Short Background

Mathematics of Fibonacci Numbers

The Fibonacci numbers are defined as a sequence of numbers[2] in mathematics. The following simple recurrence relation defines the sequence:

For n=0, $F(n)=0$ (A1)

For n=1, $F(n)=1$ (A2)

Then for n>1, $F(n)=F(n-1)+F(n-2)$ (A3)

That is, after the two starting (or initial) values, each subsequent number is the sum of the two preceding ones. The first few members of the sequence are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377,...[3]

Like every sequence defined by linear recurrence, the Fibonacci numbers have a closed-form solution, which has become known as Binet's formula despite it was already known by Abraham de Moivre. The numbers in the sequence can be generated by:

$$F(n)=\varphi^{n}-(1-\varphi)^{n}\Big/\sqrt{5} \qquad \text{(A4)}$$

where φ is popularly known as the golden ratio.[4] Indeed, φ is the positive root of the equation:

$$x^{2}-x-1=0 \qquad \text{(A5)}$$

which is the generating polynomial (or characteristic equation) of the recursion.

The golden ratio, φ, is the limit[5] of the two consecutive Fibonacci numbers as the index of sequence tends to infinity. In formal terms:

$$\varphi=\lim_{n\to\infty} F(n+1)/F(n) \qquad \text{(A6)}$$

Fibonacci Numbers in Nature

There are several places and different contexts on earth where we observe that Fibonacci numbers are embedded in natural phenomena. Among these we can count branching in trees, fruitlets of a pine tree, breeding of rabbits, spirals of shells and curve of waves. We can expand two of these examples for better comprehension: In the case of branching in trees, the first (main) branch yields a second one in the next year, then each of the two yield one branch, and so on. Similarly in the case of breeding of rabbits, a rabbit has a baby rabbit in the first year, then each have a kid in the next year, and so on. Other examples are quite similar to these two.

In art, although the Fibonacci numbers themselves do not explicitly appear, the most refined product of Fibonacci numbers, namely the golden ratio appear in many places. A beautiful rectangle, for example, is defined as that having proportion of its sides closest to the golden ratio. In many Leonardo da Vinci paintings, the basic figure is enveloped within a golden rectangle in addition to the observation that several objects are fit into other golden rectangles, where, off course, the rectangles themselves are not visible on paintings.

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[2] Named after Leonardo of Pisa, known as Fibonacci, whose Liber Abaci published in 1202 introduced the sequence to Western European mathematics.
[3] Each third number in the sequence is an even number. Note that, the sequence begins with F(0)=0 by modern convention. The Liber Abaci, on the other hand, began the sequence with F(1) = 1 omitting the initial 0.
[4] Notice that 1- φ = -1/ φ .
[5] This convergence does not depend on the starting values chosen, except F(0)=F(1)=0.