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Financial Stylized Facts and the Taylor-Effect in Stochastic Volatility Models

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Abstract

According to the Taylor-Effect the autocorrelations of absolute financial returns are larger than the ones of squared returns. In this work, we analyze in detail, for two different asymmetric stochastic volatility models, how the Taylor-Effect relates to the most important model characteristics: the asymmetry, the volatility persistence and the kurtosis. We also realize Monte Carlo experiments to infer about possible biases of the sample Taylor-Effect and we fit the models to the return series of the Dow Jones.

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1 Introduction

Taylor (1986), Granger et al. (1999) and Dacorogna et al. (2001) showed, among others, that the absolute autocorrelations of financial return series are usually larger than the ones of squared observations. This phenomena is known as *Taylor-Effect* and it was first defined by Granger and Ding (1995). Recently, Malmsten and Teräsvirta (2004) reported evidence that the GARCH(1,1) model rarely generates series that display the *Taylor-Effect* since it does not satisfy this property theoretically. However, the EGARCH(1,1) does not have difficulties in reproducing empirically this property.

The aim of this paper is threefold: First, we explain in detail how the asymmetry, the volatility persistence and the kurtosis affect the *Taylor-Effect* in the context of symmetric and asymmetric stochastic volatility models with short and long memory. We consider long memory stochastic volatility models due to the relevance of fractionally integrated volatility processes in fitting the slow decay of the autocorrelation functions of the absolute and squared financial returns (see, for example, Baillie et al., 1996).

Second, we perform Monte Carlo experiments in order to infer about the models' ability to generate this property and, finally, we fit the stochastic volatility models to the Dow Jones and we report which model reproduces better the *Taylor-Effect*. In this way, we are indirectly proposing extra diagnosis to the stochastic volatility models considered that may be important in the decision of which model to use for empirical purposes such as volatility forecasting or value at risk, among others.

The paper is organized as follows: In the next section, we present the models and their autocorrelation structures. We run Monte Carlo experiments in Section 3. In Section 4, we report the estimation results and in Section 5, we conclude.

2 Stochastic Volatility Models and the *Taylor-Effect*

In this section, we review the asymmetric extension of the LMSV specification of Breidt et al. (1998) proposed by Ruiz and Veiga (2008) and denoted asymmetric ARLMSV model, in which the volatility persistence is capture by a fractional integrate process. Formally, let

$$y_t = \varepsilon_t \sigma \exp\left(\frac{h_t}{2}\right) \tag{1}$$

$$(1-\phi L)(1-L)^d h_t = \eta_t, \tag{2}$$

where σ denotes a scale parameter, $\sigma_t^2 = \sigma^2 exp\,(h_t)$ is the variance of y_t and ε_t is $NID(0,1)$. Moreover, L stands for the lag operator, d is the parameter of fractional integration, h is an unobservable latent variable that is stationary for $|\phi| < 1$ and $d \in (0, 0.5)$, and η_t follows a $NID(0, \sigma_\eta^2)$. Ruiz and Veiga (2008) assumed additionally that $(\varepsilon_t, \eta_{t+1})'$ followed the bivariate normal distribution

$$\begin{pmatrix} \varepsilon_t \\ \eta_{t+1} \end{pmatrix} \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta\sigma_\eta \\ \delta\sigma_\eta & \sigma_\eta^2 \end{pmatrix} \right), \tag{3}$$

where δ, the correlation between ε_t and η_{t+1}, induces correlation between returns and changes in volatility, (see Taylor, 1994; Harvey and Shephard, 1996).

On the other hand, equation (1) and

$$(1 - \phi L)h_t = \eta_t, \tag{4}$$

equation (4) together with the hypothesis that $(\varepsilon_t, \eta_{t+1})'$ follows a bivariate normal distribution similar to (3) specifies the asymmetric ARSV model.

Although the series of returns is a martingale difference and, consequently, an uncorrelated sequence, it is not independent. Next, we provide the expressions of the first order autocorrelations of the absolute and squared returns for the asymmetric ARLMSV model derived in Ruiz and Veiga (2008) and Pérez et al. (2009). We simplify the analysis by considering first order autocorrelations throughout. Therefore, let the first order autocorrelations of squared $\left(corr\left(y_t^2, y_{t+1}^2\right)\right)$ and absolute $\left(corr\left(|y_t|, |y_{t+1}|\right)\right)$ observations be

$$\rho_2(1) = \frac{\exp\left(\sigma_h^2 \rho_h(1)\right)\left[1 + \delta^2 \sigma_\eta^2\right] - 1}{3\exp\left(\sigma_h^2\right) - 1} \tag{5}$$

and

$$\rho_1(1) = \frac{\exp\left(\frac{\sigma_h^2 \rho_h(1)}{4}\right)\left[\exp\left(-\frac{\sigma_\eta^2 \delta^2}{8}\right) + \frac{\sqrt{\pi}}{2\sqrt{2}}\sigma_\eta \delta\left[2\Phi\left(\frac{\sigma_\eta \delta}{2}\right) - 1\right]\right] - 1}{\frac{\pi}{2}\exp\left(\frac{\sigma_h^2}{4}\right) - 1}, \tag{6}$$

respectively, when y_t follows an ARLMSV model. Remember that σ_h^2 is the variance of the volatility factor h_t that is given by $\sigma_h^2 = \sigma_\eta^2 \frac{\Gamma(1-2d)}{[\Gamma(1-d)]^2} \cdot \frac{F(1,1+d;1-d;\,\phi)}{(1+\phi)}$ ($F(.,.;.;.)$ denotes the hypergeometric function) and $\rho_h(1)$ is the autocorrelation of order 1 of h_t that it is equal to $\frac{d}{1-d} \cdot \frac{F(1,d+1;1-d+1;\phi) + F(1,d-1;1-d-1;\phi) - 1}{(1-\phi)F(1,1+d;1-d;\phi)}$. The analogous values for the asymmetric ARSV model can be obtained by making $\sigma_h^2 = \frac{\sigma_\eta^2}{1-\phi^2}$ and $\rho_h(1) = \phi$.

Finally, the kurtosis of y_t exist in both specifications if $\phi < 1$ and $d \in (0, 0.5)$ (ARLMSV model) and $\phi < 1$ (ARSV model). It is given by $K_y = 3\exp(\sigma_h^2)$, where σ_h^2 is replaced by the respective expressions presented above. Furthermore, the autocorrelation of order one of squares, for the ARSV model, can be expressed as a function of the kurtosis as follows:

$$\rho_2(1) = \frac{\left(\frac{K_y}{3}\right)^{\phi}\left(1 + \delta^2 ln\left(\frac{K_y}{3}\right)^{(1-\phi^2)}\right) - 1}{K_y - 1}.$$

In fact, an increase of kurtosis for high values of it leads to a decrease of the autocorrelation of order 1 of squares keeping ϕ constant. This means that a low first order autocorrelation of squares and high persistence can coexist in these models if kurtosis is high. Similar results could be obtained for the ARLMSV model.

The autocorrelation of order one of absolutes can also be expressed as a function of the kurtosis as follows:

$$\rho_1(1) = \frac{\left(\frac{K_y}{3}\right)^{\phi/4}\left[\exp\left(-\frac{a\delta^2}{8}\right) + \frac{\sqrt{\pi}}{2\sqrt{2}}a^{0.5}\delta\left[2\Phi\left(\frac{a^{0.5}\delta}{2}\right) - 1\right]\right] - 1}{\frac{\pi}{2}\left(\frac{K_y}{3}\right)^{1/4} - 1},$$

where $a = ln\left(\frac{K_y}{3}\right)^{1-\phi^2}$. The first order autocorrelation of the absolutes decreases less than the analogous of the squares for an increase of kurtosis when it is very high. This implies that an increase of kurtosis leads to an increase of the *Taylor-Effect*, which is measured by $\rho_1(1) - \rho_2(1)$ with $\rho_1(1)$ and $\rho_2(1)$ given by equations 6 and 5, respectively.

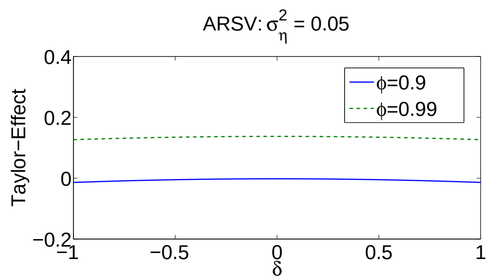

Figure 1: Relationship between the *Taylor-Effect* ($\rho_1(1) - \rho_2(1)$) and the parameter of asymmetry (δ).

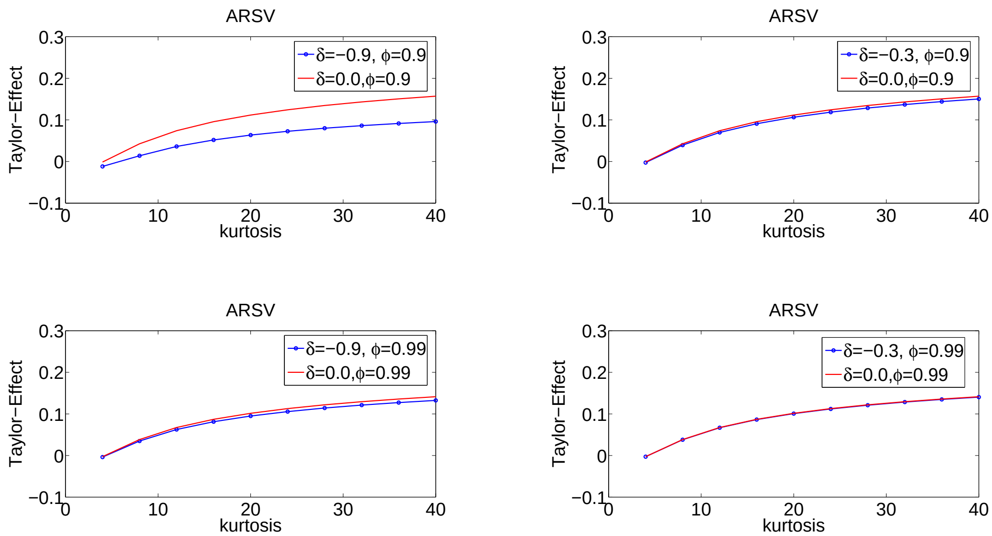

Figure 2: Relationship between the *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$ and the kurtosis.

Figure 1 and 2 exhibit the relationship among the *Taylor-Effect* and the parameter that captures the correlation between the volatility factor and the return process, δ, the volatility persistence and the kurtosis, in ARSV specifications. The values chosen for the parameters are the ones more frequently founded empirically. From Figure 1, we observe a positive relationship between the parameter that induces volatility persistence, ϕ, and the *Taylor-Effect*. In particular, the Figure shows that the curve shifts upward with the increase of ϕ. Furthermore, the highest *Taylor-Effect* is generated for values of δ closer to zero (independently of the value of ϕ).

The same relationship described above is found for the ARLMSV model, that is, the *Taylor-Effect* increases with the increase of the parameter of long memory, d (see Figure 3). However, we observe that a similar absolute variation of the parameter d (compared to the one of ϕ in the previous specification) has a larger impact in the value of the *Taylor-Effect*. In order to understand this event we plot the autocorrelation functions of absolute and squared observations for the symmetric ARSV and ARLMSV models (Figures 5 and 6). The same results would be obtained if we plot the ACF for the asymmetric versions of the models. We observe that an increase of the parameter d in the ARLMSV specifications has a larger impact in the ACF of the absolutes than in the ACF of the squares, which makes

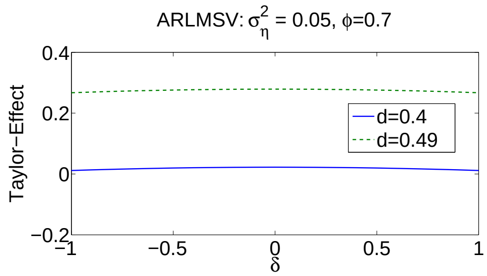

Figure 3: Relationship between the *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$ and the parameter of asymmetry (δ).

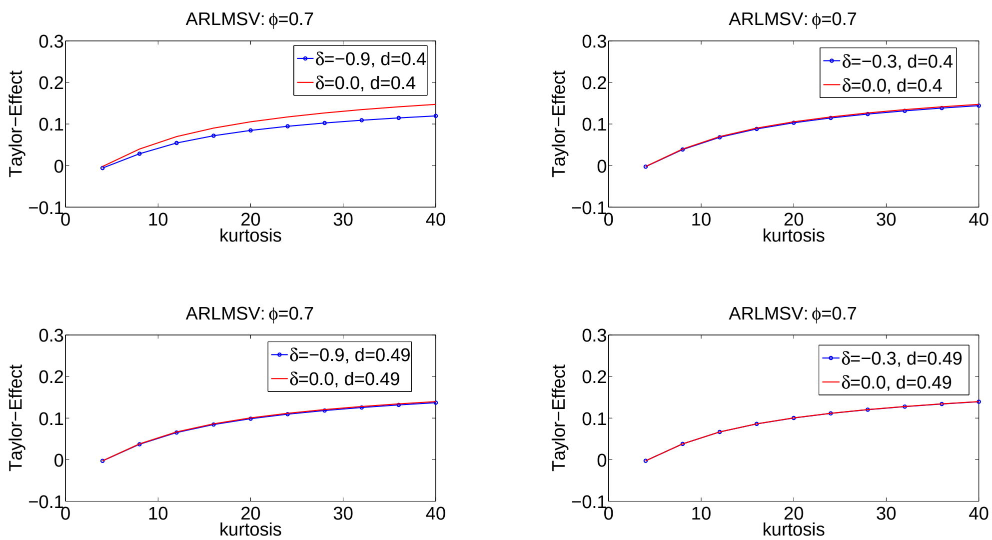

Figure 4: Relationship between the *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$ and the kurtosis.

this later model to generate a quite positive *Taylor-Effect* for high values of the long-memory parameter.

Moreover, we observe from Figure 3 and 7 that there is no significant increase in the *Taylor-Effect* when we increase the parameter ϕ for high values of the parameter of long memory, d. This is due to the fact that the autocorrelation function of the absolute observations shifts upward infinitesimally and the autocorrelation function of the squared returns decays much faster towards zero and reports a first order autocorrelation slightly smaller

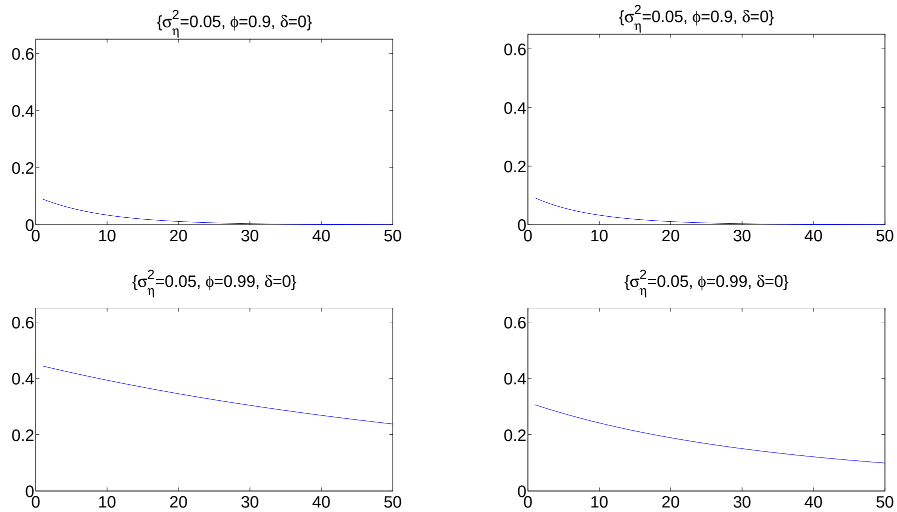

Figure 5: ACF of the absolute observations (left column) and ACF of the squared observations (right column) for two symmetric ARSV specifications.

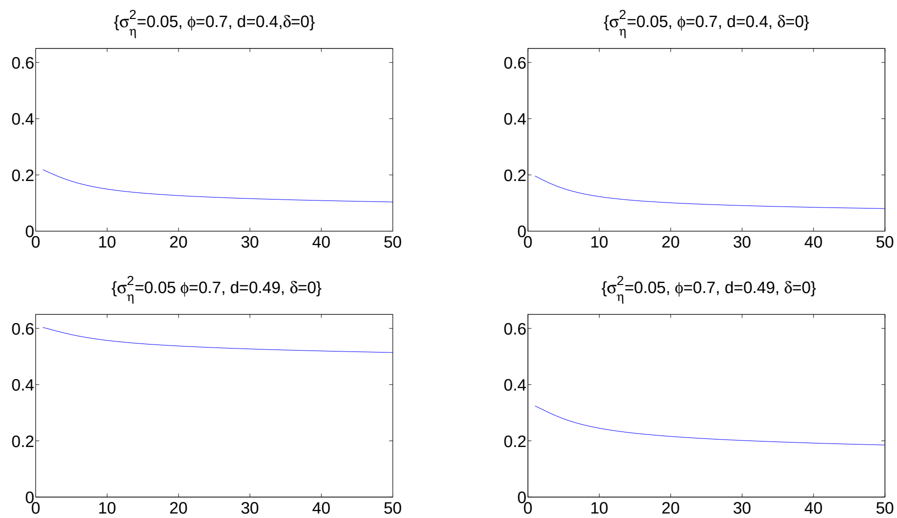

Figure 6: ACF of the absolute observations (left column) and ACF of the squared observations (right column) for two symmetric ARLMSV specifications.

than the one of the specification $\{\sigma_\eta^2 = 0.05, \phi = 0.7, d = 0.49\}$, (0.3229 versus 0.3244).

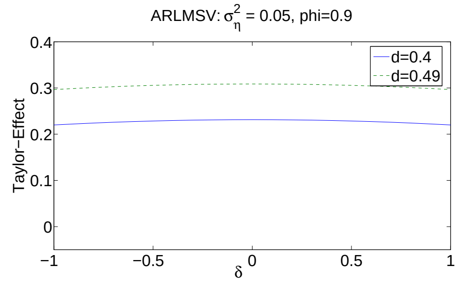

Figure 7: Relationship between the *Taylor-Effect* ($\rho_1(1) - \rho_2(1)$) and the parameter of asymmetry (δ).

Therefore, there is evidence, at least for the ACF of the squared returns, that an increase of the persistence parameter, ϕ, for high values of the long memory parameter, seems to cancel out the persistence of the squared observations. On the other hand, for the model to generate high kurtosis, ϕ should be relatively large (for a constant σ_η^2). This evidence seems to confirm the results by Chernov et al. (2003). In their work, they reported evidence that stochastic volatility models with one factor of volatility are not able to fit simultaneously the fat tails of the return distribution and the volatility persistence.

Finally, Malmsten and Teräsvirta (2004) reported evidence that for low values of the kurtosis the EGARCH is not able to reproduce the *Taylor-Effect*. Although, the symmetric ARSV is able to generate it for small values of kurtosis. Figures 2 and 4 show that there is a positive relationship between kurtosis and the *Taylor-Effect* in asymmetric stochastic volatility models with short and long memory. We also observe that a more negative correlation between ε_t and η_{t+1} leads to a smaller *Taylor-Effect*, keeping constant the other parameters.

3 Finite Sample Properties

So far we have seen that the two stochastic volatility models do not always generate the theoretical *Taylor-Effect*, in particular, for low values of persistence and long memory. However, we know nothing about the performance of these asymmetric models in capturing the sample *Taylor-Effect*.

For this purpose, we have run several Monte Carlo experiments. All results are based on 1000 replicates of the models. We have selected eight cases for each model and in all cases we have imposed a scale parameter, σ, of one. The results are presented in Tables 1-4. The first conclusion is that the biases exist and are larger for extreme values of the asymmetry ($\delta = -0.9$) and the first order autocorrelations of the squares, in the case of the ARSV

T=1000								
$\{\phi_1, \sigma_\eta^2, \delta\}$	TE	MC.TE	$\rho_1(1)$	$\rho_2(1)$	$MC_{\rho_1(1)}$	$MC_{\rho_2(1)}$	RB_1	RB_2
$\{0.9, 0.05, -0.9\}$	-0.0118	-0.0046	0.0979	0.1097	0.0916	0.0962	-0.0648	-0.1234
$\{0.9, 0.05, -0.5\}$	-0.0050	-0.0021	0.0925	0.0975	0.0879	0.0899	-0.0499	-0.0778
$\{0.9, 0.05, -0.3\}$	-0.0031	-0.0018	0.0909	0.0940	0.0864	0.0883	-0.0494	-0.0612
$\{0.9, 0.05, 0.0\}$	-0.0020	-0.00006	0.0900	0.0921	0.0859	0.0865	-0.0460	-0.0606
$\{0.99, 0.05, -0.9\}$	0.1286	0.1118	0.4485	0.3198	0.3609	0.2492	-0.1952	-0.2210
$\{0.99, 0.05, -0.5\}$	0.1346	0.1117	0.4451	0.3105	0.3621	0.2505	-0.1865	-0.1934
$\{0.99, 0.05, -0.3\}$	0.1364	0.1118	0.4442	0.3078	0.3637	0.2519	-0.1812	-0.1818
$\{0.99, 0.05, 0.0\}$	0.1373	0.1097	0.4436	0.3063	0.3692	0.2503	-0.1884	-0.1827

Table 1: TE denotes *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$, MC.TE denotes Monte Carlo finite sample *Taylor-Effect*, $\rho_1(1)$ and $\rho_2(1)$ denote first order autocorrelation of absolute and squared observations, respectively, $MC_{\rho_1(1)}$ and $MC_{\rho_2(1)}$ denote Monte Carlo finite sample first order autocorrelation of absolute and squared observations, respectively, RB_1 denotes relative bias respectively to the first order autocorrelation of absolute returns and RB_2 denotes relative bias respectively to the first order autocorrelation of squared returns, in ARSV models. T is the sample size.

T=5000								
$\{\phi_1, \sigma_\eta^2, \delta\}$	TE	MC.TE	$\rho_1(1)$	$\rho_2(1)$	$MC_{\rho_1(1)}$	$MC_{\rho_2(1)}$	RB_1	RB_2
$\{0.9, 0.05, -0.9\}$	-0.0118	-0.0061	0.0979	0.1097	0.0927	0.0988	-0.0534	-0.0999
$\{0.9, 0.05, -0.5\}$	-0.0050	-0.0035	0.0925	0.0975	0.0912	0.0947	-0.0138	-0.0292
$\{0.9, 0.05, -0.3\}$	-0.0031	-0.0030	0.0909	0.0940	0.0898	0.0928	-0.0119	-0.0129
$\{0.9, 0.05, 0.0\}$	-0.0020	-0.0016	0.0900	0.0921	0.0892	0.0907	-0.0096	-0.0142
$\{0.99, 0.05, -0.9\}$	0.1286	0.1314	0.4485	0.3198	0.4199	0.2885	-0.0637	-0.0979
$\{0.99, 0.05, -0.5\}$	0.1346	0.1341	0.4451	0.3105	0.4189	0.2848	-0.0589	-0.0826
$\{0.99, 0.05, -0.3\}$	0.1364	0.1358	0.4442	0.3078	0.4206	0.2847	-0.0531	-0.0750
$\{0.99, 0.05, 0.0\}$	0.1373	0.1326	0.4436	0.3063	0.4144	0.2818	-0.0659	-0.0800

Table 2: TE denotes *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$, MC.TE denotes Monte Carlo finite sample *Taylor-Effect*, $\rho_1(1)$ and $\rho_2(1)$ denote first order autocorrelation of absolute and squared observations, respectively, $MC_{\rho_1(1)}$ and $MC_{\rho_2(1)}$ denote Monte Carlo finite sample first order autocorrelation of absolute and squared observations, respectively, RB_1 denotes relative bias respectively to the first order autocorrelation of absolute returns and RB_2 denotes relative bias respectively to the first order autocorrelation of squared returns, in ARSV models. T is the sample size.

model. For this model and for the parameters chosen, we do not observe cases where the *Taylor-Effect* is only observed empirically and not in the population or viceversa. However, we do observe a case where the empirical *Taylor-Effect* estimated is stronger than the one in the population (see Table 2, specification {0.99,0.05,-0.9}).

In order to simulate the ARLMSV model, we use its infinite moving average representation and we truncated it at 1000.[1] According to Bollerslev and Mikkelsen (1996), this truncation procedure is able to generate high volatility persistence.

Once more, we observe that the specifications are able to reproduce empirically the *Taylor-Effect* although the biases are high. For this model, their origin is mainly the autocorrelations of absolutes.

[1]The infinite moving average is $h_t = \sum_{i=0}^{\infty} \psi_i \eta_{t-i}$, where ψ_i is a function of gamma functions.

T=1000								
$\{\phi_1, d, \sigma^2_\eta, \delta\}$	TE	MC.TE	$\rho_1(1)$	$\rho_2(1)$	$MC_{\rho_1(1)}$	$MC_{\rho_2(1)}$	RB_1	RB_2
$\{0.7, 0.4, 0.05, -0.9\}$	0.0135	0.0065	0.2255	0.2120	0.1535	0.1470	-0.3191	-0.3068
$\{0.7, 0.4, 0.05, -0.5\}$	0.0196	0.0102	0.2208	0.2012	0.1509	0.1407	-0.3165	-0.3007
$\{0.7, 0.4, 0.05, -0.3\}$	0.0213	0.0087	0.2194	0.1981	0.1496	0.1409	-0.3182	-0.2888
$\{0.7, 0.4, 0.05, 0.0\}$	0.0223	0.0106	0.2187	0.1964	0.1498	0.1392	-0.3148	-0.2910
$\{0.7, 0.49, 0.05, -0.9\}$	0.2693	0.0363	0.6069	0.3376	0.2330	0.1967	-0.6161	-0.4173
$\{0.7, 0.49, 0.05, -0.5\}$	0.2760	0.0386	0.6045	0.3285	0.2328	0.1942	-0.6149	-0.4088
$\{0.7, 0.49, 0.05, -0.3\}$	0.2779	0.0383	0.6038	0.3259	0.2316	0.1933	-0.6164	-0.4067
$\{0.7, 0.49, 0.05, 0.0\}$	0.2790	0.0362	0.6034	0.3244	0.2304	0.1942	-0.6182	-0.4015

Table 3: TE denotes *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$, MC.TE denotes Monte Carlo finite sample *Taylor-Effect*, $\rho_1(1)$ and $\rho_2(1)$ denote first order autocorrelation of absolute and squared observations, respectively, $MC_{\rho_1(1)}$ and $MC_{\rho_2(1)}$ denote Monte Carlo finite sample first order autocorrelation of absolute and squared observations, respectively, RB_1 denotes relative bias respectively to the first order autocorrelation of absolute returns and RB_2 denotes relative bias respectively to the first order autocorrelation of squared returns, in ARLMSV models. T is the sample size.

T=5000								
$\{\phi_1, d, \sigma^2_\eta, \delta\}$	TE	MC.TE	$\rho_1(1)$	$\rho_2(1)$	$MC_{\rho_1(1)}$	$MC_{\rho_2(1)}$	RB_1	RB_2
$\{0.7, 0.4, 0.05, -0.9\}$	0.0135	0.0064	0.2252	0.2120	0.1810	0.1746	-0.1970	-0.1764
$\{0.7, 0.4, 0.05, -0.5\}$	0.0196	0.0114	0.2208	0.2012	0.1775	0.1661	-0.1961	-0.1746
$\{0.7, 0.4, 0.05, -0.3\}$	0.0213	0.0136	0.2194	0.1981	0.1749	0.1613	-0.2928	-0.1858
$\{0.7, 0.4, 0.05, 0.0\}$	0.0223	0.0140	0.2187	0.1964	0.1741	0.1601	-0.2036	-0.1845
$\{0.7, 0.49, 0.05, -0.9\}$	0.2692	0.0527	0.6069	0.3376	0.2940	0.2413	-0.5155	-0.2850
$\{0.7, 0.49, 0.05, -0.5\}$	0.2760	0.0571	0.6045	0.3285	0.2873	0.2302	-0.5247	-0.2993
$\{0.7, 0.49, 0.05, -0.3\}$	0.2779	0.0565	0.6038	0.3259	0.2885	0.2320	-0.5222	-0.2880
$\{0.7, 0.49, 0.05, 0.0\}$	0.2790	0.0582	0.6034	0.3244	0.2853	0.2271	-0.5271	-0.3000

Table 4: TE denotes *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$, MC.TE denotes Monte Carlo finite sample *Taylor-Effect*, $\rho_1(1)$ and $\rho_2(1)$ denote first order autocorrelation of absolute and squared observations, respectively, $MC_{\rho_1(1)}$ and $MC_{\rho_2(1)}$ denote Monte Carlo finite sample first order autocorrelation of absolute and squared observations, respectively, RB_1 denotes relative bias respectively to the first order autocorrelation of absolute returns and RB_2 denotes relative bias respectively to the first order autocorrelation of squared returns, in ARLMSV models. T is the sample size.

4 An Empirical Example

In this section, we take real data from the Dow Jones Industrial Index in order to determine whether the models are able to reproduce the empirical properties. The daily returns of the Dow Jones span the period 3/01/90 to 11/01/07 including a total of 4293 observations. The kurtosis of this series is 7.71 and the first order autocorrelations of the absolute and squared observations are 0.15968 and 0.15965, respectively, that implies a very small *Taylor-Effect* of 0.00003.

We estimated the models with the Efficient Method of Moments (EMM) by Gallant and Tauchen (1996). The estimated parameters together with the implied *Taylor-Effects* are presented in Table 5. The results show that the ARLMSV is able to generate an estimated *Taylor-Effect* closer to the empirical one (less biased) and that the ARSV model overestimates its magnitude.[2]

[2]We estimated two long memory models. The first was the ARLMSV(1,d,0) and the second the

	ϕ	d	σ_η	δ	σ	Estimated T.E.
ARSV	0.988		0.121	-0.831	1.027	0.011
	(0.001)		(0.006)	(0.021)	(0.044)	
ARLMSV		0.153	0.161	-0.802	0.937	-0.005
		(0.002)	(0.011)	(0.028)	(0.029)	

Table 5: EMM estimates of the parameters and in parenthesis numerical Wald standard errors. T.E. denotes *Taylor-Effect* $(\rho_1(1) - \rho_2(1))$. All parameters are statistical significant.

5 Conclusion

We show that not only the sign of asymmetry affects the *Taylor-Effect* but also the volatility persistence and kurtosis. In particular, a higher persistence and kurtosis lead to a more positive *Taylor-Effect*. These results are consistent with the ones found in the literature for the GARCH and EGARCH models.

We have also observed a case where the empirical *Taylor-Effect* estimated was larger than the one in the population which may us think that in other circumstances (with other parameter values, for instance) the *Taylor-Effect* may be a sampling result due to the biases in the sample autocorrelations and *viceversa*.

Finally, the empirical results report evidence, for the Dow Jones Industrial Index, that the ARLMSV is able to generate a slight accurate estimate of the sample *Taylor-Effect*.

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