E C O N O M I C S B U L L E T I N

Multilateral negotiations with private side−deals: a multiplicity example

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Abstract

We study a multilateral negotiation procedure that allows for "partial agreements" in which responders are told only their own shares. Applications of our model include negotiations under "joint and several liability." Unlike previous models of multilateral bargaining with exit, we find that there are multiple equilibrium outcomes.

We thank Jerry Green, Amy Salsbury, Kathy Spier, Mike Wheeler and Robert Wilson for their comments. Serrano also acknowledges an Alfred P. Sloan Foundation research fellowship.
Citation: Baliga, Sandeep and Roberto Serrano, (2001) "Multilateral negotiations with private side−deals: a multiplicity example." *Economics Bulletin,* Vol. 3, No. 1 pp. 1−7
Submitted: February 12, 2001. **Accepted:** February 19, 2001.
URL: http://www.economicsbulletin.com/2001/volume3/EB−01C70003A.pdf

1 Introduction

In this paper we study a multilateral negotiation procedure with two essential features: (1) the proposals are made "privately," that is, each responder knows only the share offered to him or her when it is his or her turn to respond, and (2) the possibility of reaching multilateral consensus through "partial agreements" among fewer parties at different points in time is introduced. These two features facilitate the interpretation of our multilateral procedure as one in which private "side-deals" can be worked out.

Although we could write down the standard model of bargaining over a surplus, we shall study the formally equivalent problem of negotiations in the presence of "joint and several liability."[1] Suppose that two or more players (the defendants) are trying to settle a dispute where they have to divide an amount of C dollars that they owe to a claimant. We assume that they face "joint and several liability": first, they are each liable for the whole amount in dispute; second, if some players manage to settle earlier, the others are left to divide up any part of C that remains to be paid. In this situation, proposals among the defendants could be made "privately," as a way to avoid further disputes. We assume each of the defendants has enough funds to settle individually but would, of course, prefer that one of the others paid up. In addition, each defendant prefers to settle sooner rather than later in order to avoid interest charges: the claimant is owed C dollars to be paid immediately and thus has the right to collect interest. We have in mind cases in which a firm is trying to collect damages from a group of other firms, or a landlord who, at the end of the leasing period, may impose a penalty on the tenants that were occupying the apartment. We shall not model the role of the claimant explicitly: it will be limited to collect the amount C, whenever agreed.

A description of our procedure follows. One defendant offers a division of the cost C to the others. Each of the other defendants is told only the amount he or she pays. Therefore, the negotiations are conducted "piecemeal," in the sense that the proposer tries to convince each responder that his or her offered share is reasonable (regardless of the shares offered to others). Next, the responders accept or reject the offer. If any defendant agrees to pay the amount he is offered, he tells the others how much he is paying. If there remain defendants who have rejected the first proposer's offer, one of them, determined by some protocol, proposes a division of the

[1] In this context, feature (2) above is based on section 4(a) of the Uniform Contribution among Tortfeasors Act.

remaining cost to the first proposer and the other defendants who rejected the initial offer. The game continues in this fashion potentially ad infinitum till everyone has agreed to a division of C. An acceptor pays the amount agreed once final agreement has been reached on how to split the total cost. That is, all payments to the claimant must be made in the same period and therefore even the earlier acceptors are subject to interest charges.

For the case of two players, our extensive form reduces to Rubinstein's (1982) game of alternating offers. A question that arises in the multilateral setting is the equilibrium concept to employ. A player may receive an out-of-equilibrium offer and not know the offers to the other players. The equilibrium concept we work with is perfect Bayesian equilibrium (PBE). Unlike the case of perfect information where an offer is made publicly to all responders, our main result is that there are multiple equilibrium outcomes (Example 1). However, there is a unique PBE outcome supported by "independent" beliefs, which coincides with the unique equilibrium outcome of the game with public offers.

In the multilateral bargaining literature, ours is a model with exit and imperfect information. The first extension of Rubinstein's model to multilateral settings is due to Shaked, generalized by Herrero (1985).[2] These authors find that, although there is a unique stationary subgame perfect equilibrium (SPE), if players are sufficiently patient, every feasible outcome is supported by a SPE. Their game allows only unanimous agreements to be executed, and has perfect information (the proposal is publicly made and responses are sequential). Haller (1986) finds that this result is robust if responses are simultaneous. Jun (1987) and Chae and Yang (1994) study extensive forms with pairwise meetings among the agents, in which the possibility of exit or "partial agreements" is introduced. A unique SPE is found regardless of the discount factor. Krishna and Serrano (1996) present a model with exit where a proposal is made to all agents, and uniqueness also obtains. Two recent papers have established the robustness of this result based on exit. Vannetelbosch (1999) shows that uniqueness obtains even with a notion of rationalizability, weaker than SPE; and Huang (1999) establishes that uniqueness is still the result in a model that combines unanimity and exit, since offers can be made both conditional and unconditional to each responder. Finally, Baliga and Serrano (1995) introduce imperfect information in the unanimity game and multiplicity persists.

[2]For an account of Shaked's result, see Osborne and Rubinstein (1990).

2 Negotiating under "Joint and Several Liability"

We consider situations where a set of players $N = \{1, 2, \ldots, n\}$ negotiate over the way in which a cost of size C is divided among them. They face "joint and several liability," so that in principle each of them is liable for the whole amount. We shall assume that the claimant is owed C dollars to be paid immediately. If the defendants delay payments, the debt increases taking into account interest charges. Let $r > 0$ be the per period interest (this is a simplifying assumption that does not affect the results; we could have a "personalized" interest rate $r_i > 0$ for each defendant i).

We shall denote an *offer* to the set of players S by x_S. An offer to N will also be denoted simply by x: it is a vector $x = (x_1, x_2, \ldots, x_n)$ in which x_i is player i's share of the cost. The set of possible offers to N is

$$X = \{x \in \mathbb{R}^n | \text{for all } i, x_i \geq 0 \text{ and } \sum_{j \in N} x_j = C\}.$$

For some $S \subset N$, we write $\sum_{j \in S} x_j$ as $x(S)$; also $T \setminus S$ is the set of players who are members of T but are not members of S.

The extensive form of the bargaining procedure in which n defendants bargain over a cost of size C is denoted by $G_i(N; C)$ and is defined recursively. If $N = \{i\}$, $G_i(N; C)$ consists of an offer made (to himself) by player i, which is immediately accepted. For any $S \subseteq N$, $i \in S$ and $C' \leq C$, define $G_i(S; C')$ as follows. In period 0 player i makes an offer x_S. Each player j in $S \backslash \{i\}$ receives an envelope containing only his or her part of the cost x_j. All players $j \neq i$ respond simultaneously by accepting or rejecting x_j.[3] If player j accepts x_j, he or she pays $(1 + r)^t x_j$ after all the others agree to a division of C in period t.

Let $A \subseteq S \backslash \{i\}$ be the set of players who accept player i's offer in period 0. All the players who accept show their envelopes. If $A = S \setminus \{i\}$ then all players including i pay their shares immediately. If $\emptyset \neq A \subset S \setminus \{i\}$, then in period 1, $G_j(S \setminus A; C' - x_S(A))$ is played where j is the smallest index in $S \setminus A$ greater than i (modulo S). If $A = \emptyset$, then in period 1, $G_j(S; C')$ is played, where j is determined as above.

As for evaluating payoffs, player i's payoff from paying a share x_i of C in period t is $v_i((1 + r)^t x_i)$ where $v_i : \mathbb{R} \rightarrow \mathbb{R}$ is a strictly decreasing function, $v_i(\infty) = -\infty$. That is, while there are no personal discount rates, players prefer to settle earlier rather than later in order to avoid the interest

[3] This assumption is made to avoid problems of perfection at the response stage, as well as any further incidence of off-equilibrium path beliefs.

charges. The payoff to players from perpetual disagreement is assumed to be $-\infty$.

3 Equilibrium

With at least three defendants, our extensive form contains many continuation games that are not proper subgames: because the proposal is made "piecemeal" in personalized envelopes, each responder does not know the amounts offered to the others when it is his or her turn to respond. Thus, we have a game of imperfect information and the concept of SPE cannot help refine the set of Nash equilibria.

We adopt as our equilibrium notion that of perfect Bayesian equilibrium (PBE) that combines the following elements:

(I) Sequential rationality in the players' actions given their beliefs: at every information set and given the beliefs held at that point, every player uses a best response to the other players' strategies.

(I.a) Proposers' information sets are always singletons: a proposer chooses the offer to be a best response to the other players' strategies, taking into account the continuation equilibrium.

(I.b) If $|N| = 2$, responders' information sets are always singletons and if $|N| > 2$, respondents' information sets are never singletons. Given their beliefs following any offer, they choose their response rules as a best reply to the other players' strategies, taking into account the continuation equilibrium.

(II) Updating of beliefs using Bayes' rule whenever possible: if $|N| > 2$ and a responder is offered his or her equilibrium share, he or she believes that the equilibrium proposal has been made.

(III) Arbitrary beliefs after off-equilibrium actions.

4 Result

Our main result is a counterexample to uniqueness. Indeed, there are multiple PBE outcomes in the game $G_i(N;C)$ when $|N| \geq 3$. This is the content of Example 1.

Example 1: let $N = \{1,2,3\}$ and consider the game $G_1(N;C)$. Let $0 \leq \epsilon \leq \frac{rC}{3(3+2r)}$ and $r^2 < 3$. The following splits can arise in a PBE of this game:

$$(\frac{C}{3+2r} + 2\epsilon, \frac{(1+r)C}{3+2r} - \epsilon, \frac{(1+r)C}{3+2r} - \epsilon).$$

The strategies and beliefs that support it are as follows:

- (i) In period 0, player 1 proposes the given split.
- (ii) In period 0, player $j = 2, 3$ accepts an offer x if and only if $x_j \leq \frac{(1+r)C}{3+2r} - \epsilon$.
- (iii) In period 0, if player $j = 2, 3$ is offered a share lower than the equilibrium one, he or she believes that the other responder is offered the equilibrium share; if he or she is proposed a higher share, he or she believes (if feasible) that so is the other responder.
- (iv) Following a rejection of only one responder in period 0, the unique SPE is played in the two-player continuation.
- (v) Following a unanimous rejection in period t, the same equilibrium is played in period $t+1$ with the natural permutation of roles (e.g., player 2 makes the new proposal in period 1).

To check that this is a PBE, it suffices to account for all one-time deviations. Consider period 0 (the exercise for any other period is identical). Player 1 compares his or her share to the well defined share (possibly increased by interest charges) that he or she gets from any deviation. Players 2 and 3, on the other hand, act optimally given their beliefs no matter what offer they receive. To check all this, the reader will find the following inequalities useful, together with the restriction on ϵ:

$$\frac{r}{3(3+2r)} < \frac{r(2+r)}{(3+r)(3+2r)} < \frac{r(2+r)^2}{(3-r^2)(3+2r)}.$$

Remark: Note that the usual multiplicity example, as constructed by Shaked, would not work here. That is, the extreme points of the payoff space cannot be supported by (or even approximated by) PBEs. For example, the split $(0, 0, C)$ can never be a PBE outcome of $G_1(N; C)$ because player 3 would have an incentive to deviate by rejecting player 1's proposal.

In Example 1, beliefs are correlated when responders are asked to pay more than the equilibrium amount, that is, they believe that the proposer deviated from the equilibrium in the two envelopes he or she wrote. This creates a "boot-strap" self-confirming theory, in which responder j rejects the offer because he or she believes that responder k will also reject it.

This poses the question of whether a refinement of PBE that rules out this correlation of off-equilibrium path beliefs would be effective. For example, consider the following requirement on off-equilibrium beliefs:

(III') Independent beliefs after off-equilibrium actions: Consider the game $G_i(N;C)$ with $|N| > 2$ and let x be the proposal made in equilibrium by player i; denote by y an off-equilibrium proposal. Following an off-equilibrium offered share $y_j \neq x_j$, if $y_j + x(N \setminus \{i,j\}) \leq C$, responder j believes that all other responders have been offered their components of x. If $y_j + x(N \setminus \{i,j\}) > C$, beliefs are unrestricted.

As shown in Baliga and Serrano (1998), there exists a unique PBE outcome of $G_i(N;C)$ compatible with independent beliefs. This outcome would also be the unique SPE outcome of the game with public offers (Krishna and Serrano (1996)). In this PBE, player i pays a share $y_i^* - \beta^* = \frac{C}{|N|(1+r)-r}$ and every player $j \neq i$ pays $y_j^* = \frac{(1+r)C}{|N|(1+r)-r}$.

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