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Financial Imperfections, Inequality and Capital Accumulation

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Abstract

Aghion, P. and P. Bolton (1997, "A Theory of Trickle-Down Growth and Development," Review of Economic Studies, 59, 151-172) provide a model analyzing the effect of capital accumulation on income inequality. We integrate two additional features to a modified version of this model. The first one is a costly financial contract enforcement which represents the second type of credit market imperfection in addition to moral hazard. The second one is enabling wealthy agents to undertake larger investment projects relatively to other agents. I show that inequality increases in a first stage of development and, contrarily to Aghion and Bolton (1997), remains constant or increases in a second stage (depending on the deposit interest rate ceiling).

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1. Introduction

This paper investigates the relationship between inequality and capital accumulation in the presence of credit market imperfection which reflects among others the weakness of the judicial institutions. For the banking system, which still dominates the financial system of most developing countries, the legal framework is particularly important. Indeed, in case of borrower's default the bank often has the right to seize collateral. However, the implementation of this right in practice depends on the efficiency of the judicial system. If the judicial system is weak, banks are willing to finance only entrepreneurs providing sufficient collateral. Considering 56 countries over the period 2002-04, regressing the entrepreneurship density on judicial efficiency we found positive and highly significant coefficient (t-statistic, 5.18) and R^2 of 0.20 (figure 1).

An increase of the judicial efficiency of 1 is associated, on average, with a 18.83 per mil increase in the entrepreneurship density. This is a large quantitative effect which signifies that an economy may suffer from low entrepreneurship due to the weakness of its judicial system. A possible explanation of this positive relationship between the judicial efficiency and the entrepreneurship is credit rationing. Credit rationing may accentuate the income inequality in a given economy. In deed, as mentioned by Banerjee and Duflo (2005) "two firms facing the exact same technological options may end up choosing very different methods of production. In particular, one person may start a large or more technologically advanced firm because he has money and another may start a small and backward one because he does not". As a consequence, banks will compete to lend for the wealthy entrepreneurs which become highly leveraged and need more monitoring. As noted by Banerjee and Duflo (2005) this may leads to a high cost of monitoring, low interest rate for savers and higher returns for entrepreneurs which increases the income inequalities. In order to investigate this relationship we regressed the GINI index on the Judicial Efficiency (JE) over the period 1999-2001 for 42 countries. As shown in figure 2, an increase of the judicial efficiency reduces the inequality captured through the GINI index.

Many papers (e.g. Bertola (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996)) revealed that inequality is negatively associated with growth. Recently, Banerjee and Duflo (2005) presented empirical evidence that the growth is an inverted U– curve of inequality. However, the effect of credit market imperfection (here judicial inefficiency) on the relationship between inequality and capital accumulation was rarely analyzed in a theoretical model. Galor and Moav (2004) analyze the effect of income inequality on the development process and distinguish three stages. In the first stage, inequality enhances the process of development by channeling resources towards individuals endowed with higher marginal propensity to save. In the second stage of development, inequality reduces the investment in human capital and lowers economic growth, in the presence of credit constraints. Finally in a third stage, credit constraints become less binding and the aggregate effect of income distribution on the growth process becomes less significant. Aghion and Bolton (1997) is the departure model of our research. They developed a theoretical model analyzing the relation between inequality and development when banks face a moral hazard problem when financing entrepreneurs. They showed that the capital accumulation process begins by widening the inequalities but reduces them in later stages.

Departing from Aghion and Bolton (1997) I integrate two additional features that change their results about the relationship between capital accumulation and inequality.

The first feature is including a costly contract enforcement (judicial inefficiency) as a second type of credit market imperfection in addition to moral hazard. The second feature is enabling, contrarily to Aghion and Bolton (1997), wealthy agents to undertake larger projects which is coherent with the above cited intuition of Banerjee and Duflo (2005). I show that inequality increases in a first stage of development and, contrarily to Aghion and Bolton (1997), remains constant or increases in a second stage (depending on the deposit interest rate ceiling).

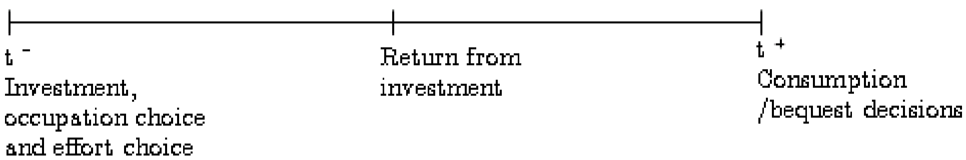
The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 and 4 analyze the optimal lending contract and the occupational choice respectively. I investigate the evolution of the wealth inequality in section 5. Finally, section 5 concludes.

2. Model

The economy is closed and contains a sequence of one-period-lived overlapping generations. An initial generation of old entrepreneurs coexists with young agents at date $t = 0$. Each generation is composed of a continuum of mass 1 agents indexed by i. Each agent has one offspring and works or invests. Agents are risk-neutral and their utility depends only on consumption and bequest. Hence, an agent divides the income he receives between consumption and bequest. The only source of heterogeneity among agents is their inherited wealth w_t^i. Each agent i is endowed with one unit of effort ($l^i = 1$). He may choose to undertake a project requiring a minimum fixed investment of $\widetilde{w} > 1$ that generate an uncertain revenue $\kappa^i(w^i)$ from an investment $w^i \geq \widetilde{w}$.

$$\kappa^i(w) = \begin{cases} aw^i & \text{with probability } p^i \\ 0 & \text{with probability } 1 - p^i \end{cases}$$

where $a \in \,]1, 2[$ and $p^i = l^i$ denotes the probability of success which is equal to 1 if the agent supplies his entire effort. We assume that there is an effort cost $C(l^i) = \frac{a\widetilde{w}}{2}(l^i)^2$. The chronology of an agent's decisions in his life is shown by the following graphic:



At the beginning of its life the agent decides the effort to supply and how to invest his inherited wealth w_t^i. At the end of its lifetime, the individual allocates his net final wealth between consumption and bequest. As in Aghion and Bolton (1997) agents are assumed to have Leontieff preferences over consumption and bequest. Therefore, the optimal bequest is a linear function of end of period wealth w_{t+}^i and is given by $b_{t+1}^i = w_{t+1}^i = (1 - \delta) w_{t+}^i$ where $1 - \delta$ is the saving propensity of individuals.

We assume that at date $t = 0$, a proportion π of the young agents has a low inherited wealth $\underline{w}_0 < \widetilde{w}$ (resulting from the initial old generation's bequests) and constitutes the class $i = l$. The remainder proportion $1 - \pi$ has a high inherited wealth $\widetilde{w} > \overline{w}_0$ and constitutes the class $i = h$. An agent of the class $i = l, h$ born at date $t \geq 0$ with an initial wealth w_t^i such that $w_t^i \geq \widetilde{w}$ could self-finance his project but may have an incentive to ask for a bank loan in order to enlarge it. The self-financing capital is considered as collateral for the bank and wealthy agents may be more likely to obtain a loan. Even if his project succeeds, an agent may have an incentive to default on the loan. In this case, the bank seizes a fraction $\lambda \in [\frac{1}{2}, 1]$ of the produced output. The unseized fraction $1-\lambda$ corresponds to an enforcing repayment cost which could be interpreted as the inefficiency's level of the judicial system. An agent with an inherited wealth $w_t^i \leq \widetilde{w}$ who is unable to obtain a loan has no choice but depositing his wealth in the bank in return of a certain (gross) return $1 \leq r_t^d \leq a$.

3. Optimal lending contract

An agent of class i can self-finance a project if his inherited wealth w_t^i is superior to the minimum fixed investment $\widetilde{w}$. He may also ask for a bank loan d_t^i for an additional investment. As in Aghion and Bolton (1997) since the incentive problem is a moral hazard problem with limited wealth constraints, an optimal investment contract between this agent and the bank specifies the repayment schedule R_t^i for every agent asking for an external financing

$$R_t^i = \begin{cases} r_t^i d_t^i & \text{with probability } p_t^i \\ 0 & \text{with probability } 1-p_t^i \end{cases}$$

where r_t^i is the unit repayment rate. In order to prevent the borrower's default, the repayment should be at most equal to the default's cost. The default cost is equal to the output the bank seizes in case of success of the project which is $\lambda a\left(w_t^i + d_t^i\right)$. Hence, we should impose $R_t^i \leq \lambda a\left(w_t^i + d_t^i\right)$ which gives $r_t^i d_t^i \leq \lambda a\left(w_t^i + d_t^i\right)$. Therefore, the maximum amount of loan the bank grants to the agent of class i is $\overline{d_t^i}$ defined by

$$\overline{d_t^i} = \frac{\lambda a w_t^i}{r_t^i - \lambda a} \tag{1}$$

Since the unit repayment rate r_t^i the borrower chooses the effort l^i to supply and the amount of loan d_t^i in order to maximize his expected revenue net of both repayment and effort cost

$$(\wp)\begin{cases} \underset{\substack{l_t^i > 0 \\ d_t^i \geq 0}}{Max} \; p_t^i\left[a(w_t^i + d_t^i) - r_t^i d_t^i\right] - C(l_t^i) \\ \text{subject to} \\ p^i = l_t^i \\ d_t^i \leq \overline{d_t^i} \\ w_t^i + d_t^i \geq \widetilde{w} \\ l_t^i \leq 1 \end{cases} \tag{2}$$

Given that the economy comprises at each date t a continuum of agents belonging to the two classes (the class of low inheriting wealth and the high inheriting one) and that the random returns on each risky project are independently and identically distributed the proportion of successful projects is p^i for the class i. Hence, the return of the bank could be interpreted as deterministic and is given by $p^i r_t^i$. Assuming a competitive banking system, this gross return rate is equal in equilibrium to the deposit return (gross) rate: $r_t^d = p^i r_t^i$.

Proposition 1

The probability of success of the project and the loan granted to an agent i are given by

$$p_t^i = \begin{cases} \frac{1}{2}[1+\sqrt{1-4\frac{\widetilde{w}-w_t^i}{a\widetilde{w}}r_t^d}] & \text{if} \quad w_{mt} \leq w_t^i < \frac{r_t^d}{a}\widetilde{w} \\ \frac{r_t^d\widetilde{w}}{2\lambda a}[1-\sqrt{1-\frac{4(1-\lambda)w_t^i}{r_t^d\widetilde{w}}\lambda a}] & \text{if} \quad \frac{r_t^d}{a}\widetilde{w} \leq w_t^i \leq w_{rt} \\ 1 & \text{if} \quad w_t^i \geq w_{rt} \end{cases} \tag{3}$$

and

$$d_t^i = \begin{cases} \widetilde{w} - w_t^i & \text{if} \quad w_{mt} \leq w_t^i < \frac{r_t^d}{a}\widetilde{w} \\ \overline{d_t^i} = \dfrac{\lambda a w_t^i}{r_t^d - \lambda a} & \text{if} \quad w_t^i \geq \frac{r_t^d}{a}\widetilde{w} \end{cases} \tag{4}$$

where $w_{mt} = \widetilde{w}\left(1 - \frac{\lambda(1-\lambda)a}{r_t^d}\right)$ and $w_{rt} = \frac{1}{4}\frac{r_t^d\widetilde{w}}{\lambda(1-\lambda)a}$.

Proof. See the appendix. ∎

It is easy to see from (3) that the lower the entrepreneur's self-financing w_t^i the less effort he devotes to increase the probability of success of his project when his wealth is inferior to the threshold w_{rt}. Particularly, the mass π of agents having low inherited wealth w_t^l will have less incentive to supply effort compared to the class of mass $1-\pi$ having high inherited wealth w_t^h. Consequently, (as shown in the equation (19) of the Annex), the unit repayment is higher for the low wealthy class of agents $r_t^l > r_t^h$ when their initial endowment is inferior to a determined threshold w_{rt}. This is also due to the high amount of loan they need in order to undertake the project $\widetilde{w} - w_t^l > \widetilde{w} - w_t^h$. When their wealth is sufficiently high (superior to w_{rt}) they support the same unit repayment cost. However, the maximum amount of loan they obtain is always inferior to that of the high wealthy class $\overline{d_t^h} > \overline{d_t^l}$.

4. Occupational choice

Let $\widehat{w}_t$ denotes the initial wealth endowment of an agent of generation t who is indifferent between undertaking a project and depositing his wealth in a bank. Hence, only the agents with an initial endowment $w_t < \widehat{w}_t$ prefer strictly becoming depositors. Those with $w_t > \widehat{w}_t$ prefer becoming entrepreneurs. The threshold $\widehat{w}_t$ is determined by the condition $p_t[a(\widehat{w}_t + d_t) - r_t d_t] - C(l_t) = r_t^d \widehat{w}_t$ which could be written using $p_t r_t = r_t^d$ as following

$$p_t a(\widehat{w}_t + d_t) = r_t^d(\widehat{w}_t + d_t) + \frac{a\widetilde{w}}{2}(p_t)^2 \tag{5}$$

Lemma 1

i) The threshold $\widehat{w}_t$ exists for $r_t^d \in]r_\lambda, a[$ and is given by $\widehat{w}_t = \frac{a\left(r_t^d - \lambda a\right)\widetilde{w}}{2\left(a - r_t^d\right)r_t^d}$ where $r_\lambda \in]\max(1, \lambda a), a[$ verifies $\frac{\partial r_\lambda}{\partial \lambda} > 0$ and is solution of the equation:

$$2\lambda(1-\lambda)[(\tfrac{r_\lambda}{a}) - \lambda]/[1 - (\tfrac{r_\lambda}{a})] = (\tfrac{r_\lambda}{a})^2$$

ii) The threshold $\widehat{w}_t$ exceeds $\widetilde{w}$ if and only if $r_t^d \geq r'_\lambda$ where $r'_\lambda = \frac{a}{4}\left(1 + \sqrt{1 + 8\lambda}\right) < a$

Proof. See appendix.

5. Wealth dynamic and inequality

Proposition 2

The wealth inequality widens between the two classes of agents in a first stage of development. Outside, it remains constant or widens depending on the deposit interest rate ceiling.

Proof. The dynamic of wealth accumulation is given by

$$w_{t+1}^i = (1-\delta) w_{t+}^i$$

where $\delta \in]0, 1[$ is the consumption fraction and w_{t+}^i the wealth of an agent i at the end of his life. When the agent is a depositor we have $w_{t+}^i = r_t^d w_t^i$. If he is an entrepreneur w_{t+}^i is given by

$$w_{t+}^i = \begin{cases} a(w_t^i + d_t^i) - r_t^i d_t^i & \text{with probability} \quad p_t^i \\ 0 & \text{with probability} \quad 1 - p_t^i \end{cases}$$

From lemma 1 it is clear that $\partial\widehat{w}_t / \partial r_t^d > 0$ and the limit of $\widehat{w}_t$ is $+\infty$ when r_t^d tends to a. Hence, when the initial wealth of the low-inheriting agent w_t^l is strictly inferior to that of the high-inheriting agent w_t^h there exists always a value of r_t^d such that $w_t^l \leq \widehat{w}_t < w_t^h$ making the low inheriting agents always preferring depositing and the high inheriting agent preferring undertaking a project. Hence, even if the low inheriting agents could self-finance their project ($w_t^l > \widetilde{w}$) there exists a deposit interest rate such that $w_t^l < \widehat{w}_t$ making them preferring depositing their wealth in a bank. Besides, note that a high inheriting agent who couldn't self-finance a project ($w_t^h < \widetilde{w}$) could be rationed. This occurs if the total saving collected from the low-inheriting agents is not sufficient to satisfy the total demand of loan by the wealthy class of agents:

$$\pi w_t^l < (1-\pi)\left(\widetilde{w} - w_t^h\right) \tag{6}$$

In this case, the probability ϕ of a wealthy agent to be credit rationed. is determined by:

$$\pi w_t^l + \phi(1-\pi)w_t^h = (1-\phi)(1-\pi)\left(\widetilde{w} - w_t^h\right)$$

Denoting W_t^i the aggregate wealth of the class i we have

$$\begin{aligned} W_t^l &= \pi w_t^l \\ W_t^h &= (1-\pi)w_t^h \end{aligned}$$

and its dynamic is given by

$$\begin{aligned} W_{t+1}^l &= r_t^d \pi w_t^l = r_t^d W_t^l \\ W_{t+1}^h &= (1-\phi)(1-\pi)p_t^h\left[a(w_t^h + d_t^h) - r_t^h d_t^h\right] + \phi(1-\pi)r_t^d w_t^h \end{aligned}$$

where $(1-\phi)(1-\pi)p_t^h$ represents the proportion of entrepreneurs with successful projects. Therefore, the per capita wealth dynamic is given by

$$\begin{aligned} w_{t+1}^l &= r_t^d w_t^l \\ w_{t+1}^h &= (1-\phi)p_t^h\left[a(w_t^h + d_t^h) - r_t^h d_t^h\right] + \phi r_t^d w_t^h \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{w_{t+1}^h}{w_{t+1}^l} &= \frac{(1-\phi)p_t^h\left[a(w_t^h + d_t^h) - r_t^h d_t^h\right] + \phi r_t^d w_t^h}{r_t^d w_t^l} \\ &= \frac{w_t^h}{w_t^l}\frac{(1-\phi)p_t^h\left[a + (a - r_t^h)\frac{d_t^h}{w_t^h}\right] + \phi r_t^d}{r_t^d} \\ &> \frac{w_t^h}{w_t^l} \end{aligned} \tag{7}$$

Hence, the wealth inequality between the two types of agents increases from period t to period $t+1$. The last inequality results from the fact that for agents h undertaking a project is preferred then the bank deposit $p_t^h[a(w_t^h + d_t^h) - r_t^h d_t^h] > r_t^d w_t^h$. Note that if the credit rationing disappears (ie (6) is no more verified) we have to replace ϕ by zero in the above dynamic.

Let's now analyse the case where the deposit interest rate ceiling is strictly inferior to a and is given by $\overline{r} \in \left]\lambda a, r'_\lambda\right[$ where r'_λ is defined in lemma 1 . In this case, the threshold $\widehat{w}_t$ defined in lemma 1 couldn't exceed $\widehat{\widetilde{w}} = a\left(\overline{r} - \lambda a\right)\widetilde{w}/(2\left(a - \overline{r}\right)\overline{r}) < \widetilde{w}$. Therefore, when the wealth of the two types of agents exceeds the threshold $\widehat{\widetilde{w}}$ they will prefer strictly becoming entrepreneurs. In this case, the credit rationing is equally faced by the two categories of agents if their initial endowment w_t^i is strictly inferior to $\widetilde{w}$ and we obtain

$$\begin{aligned} w_{t+1}^l &= (1-\phi')p_t^l\left[a(w_t^l + d_t^l) - r_t^l d_t^l\right] + \phi' r_t^d w_t^l \\ w_{t+1}^h &= (1-\phi')p_t^h\left[a(w_t^h + d_t^h) - r_t^h d_t^h\right] + \phi' r_t^d w_t^h \end{aligned}$$

where ϕ' the probability of an agent to be credit rationed is defined by the equality between the total amount of deposit and the total amount of granted loans:

$$\phi'\left[\pi w_t^l + (1-\pi)w_t^h\right] = (1-\phi')\left[\pi d_t^l + (1-\pi)d_t^h\right]$$

Therefore, we obtain

$$\begin{aligned}
\frac{w_{t+1}^h}{w_{t+1}^l} &= \frac{(1-\phi')p_t^h\left[a(w_t^h+d_t^h)-r_t^h d_t^h\right]+\phi' r_t^d w_t^h}{(1-\phi')p_t^l\left[a(w_t^l+d_t^l)-r_t^l d_t^l\right]+\phi' r_t^d w_t^l} \\
&= \frac{w_t^h}{w_t^l}\frac{(1-\phi')p_t^h\left[a+(a-r_t^h)\frac{d_t^h}{w_t^h}\right]+\phi' r_t^d}{(1-\phi')p_t^l\left[a+(a-r_t^l)\frac{d_t^l}{w_t^l}\right]+\phi' r_t^d} > \frac{w_t^h}{w_t^l} && (8) \\
&< \frac{w_t^h}{w_t^l}\frac{(1-\phi')p_t^h\left[a+(a-r_t^h)\frac{d_t^h}{w_t^h}\right]+\phi' r_t^d}{r_t^d} && (9)
\end{aligned}$$

Comparing (7) and (9) it is clear that the dynamic of wealth inequality is slower in the case where the deposit interest ceiling is $\overline{r}$ rather then a. Moreover, from proposition 1 it follows that $p_t^i = 1$ and $\frac{d_t^h}{w_t^h} = \frac{d_t^l}{w_t^l}$ when $w_t^i \geq w_{\overline{r}} = \frac{1}{4}\frac{\overline{r}\widetilde{w}}{\lambda(1-\lambda)a}$. In this region, we conclude from (8) that the wealth inequality remains constant. This result remains when $w_t^i > \widetilde{w}$ since we could straightforwardly show that $p_t^i = 1$ and $w_{t+1}^i = a(1-\delta)w_t^i$.

6. Conclusion

Departing from Aghion and Bolton (1997) I integrate new features that modify their results about the relationship between capital accumulation and inequality. The first feature is including a costly contract enforcement as a second type of credit market imperfection in addition to the moral hazard problem between banks and borrowers. The second feature is enabling, contrarily to Aghion and Bolton (1997), wealthy agents to undertake larger investment projects. Our results show that inequality increases in a first stage of development and, contrarily to Aghion and Bolton (1997), remains constant or widens in a second stage.

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Appendix

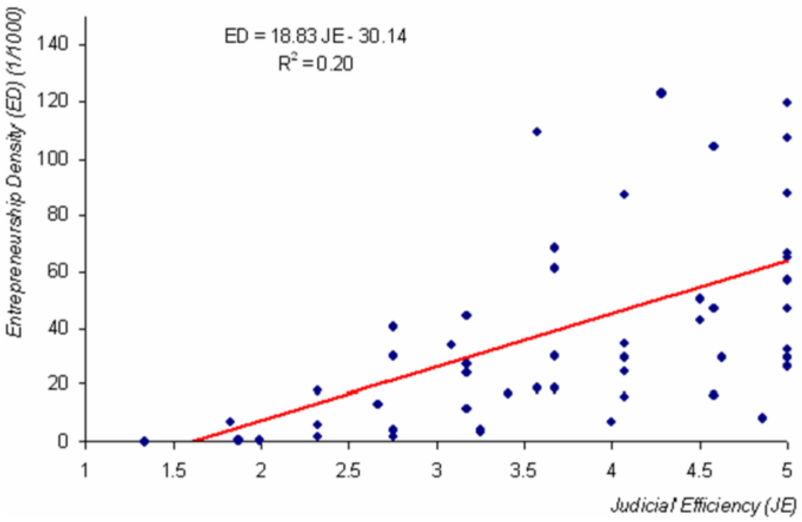

Figure 1: Judicial efficiency and entrepreneurship density
Source: *I used the entrepreneurship density from the World Bank Group Entrepreneurship Database* (2007) *and the judicial efficiency index of Laeven and Majnoni* (2005).

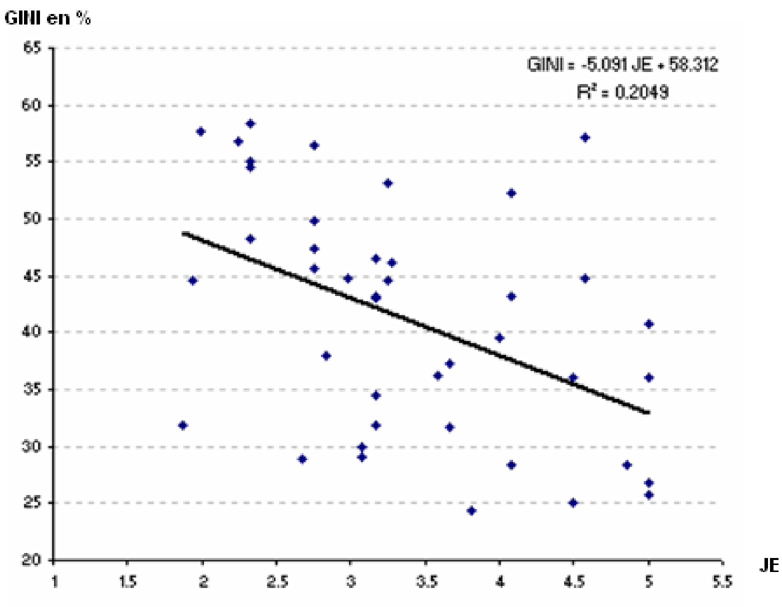

Figure 2: The judicial efficiency and the GINI index
Source: *I used the judicial efficiency index of Laeven and Majnoni* (2005) *and the GINI index from the WDI.*

Table 1. The Judicial efficiency and Entrepreneurship Density data.

Country	Judicial Efficiency	Entrepreneurship Density ‰
Albania	1.83	7.01
Argentina	4.08	15.67
Australia	5	66.35
Austria	5	29.57
Bangladesh	1.87	0.70
Belgium	4.58	47.00
Bolivia	2.75	4.41
Botswana	3.67	61.00
Canada	5	64.81
Chile	4.58	16.15
Colombia	1.99	0.67
Costa Rica	3.17	44.30
Croatia	3.08	34.06
Cyprus	4.08	244.51
Czech Republic	4.08	34.50
Denmark	5	56.80
El Salvador	3.25	3.48
Estonia	3.67	68.77
Finland	5	32.33
France	4.08	29.34
Germany	4.86	8.35
Greece	3.25	4.19
Haiti	1.33	0.08
Hong Kong	4.58	104.05
Iceland	5	107.51
Indonesia	2.33	1.68
Ireland	5	57.31
Israel	4.08	87.29
Italy	4.5	42.29
Japan	4.62	29.69
Jordan	3.67	30.13
Kenya	2.33	6.26
Latvia	3.58	109.91
Lebanon	3.17	24.24
Lithuania	3.17	27.16
Madagascar	2.75	1.88
Malta	4.29	122.62
Moldova	3.58	18.80
Morocco	4	7.13
Netherlands	5	87.86
New Zealand	5	119.41
Peru	2.75	30.14
Poland	3.67	18.39
Portugal	4.08	24.59
Russia	2.75	40.64
Slovak Republic	3.41	16.96
Slovenia	4.08	24.37
South Africa	2.33	18.07
Spain	3.67	68.42
Sri Lanka	2.75	3.82
Sweden	4.5	50.30
Switzerland	5	26.66
Ukraine	2.67	13.03
United Kingdom	5	46.81
United States	5	26.01
Zambia	3.17	11.55

Table 2. The Judicial efficiency and GINI index.

Country	JE	GINI index
Argentina	4.08	52.20
Bangladesh	1.87	31.79
Bolivia	4.58	44.70
Brazil	2.33	58.45
Bulgaria	3.17	31.91
Cameroon	1.94	44.57
Chile	4.58	57.10
China	2.98	44.73
Colombia	1.99	57.60
Costa Rica	3.17	46.50
Croatia	3.08	29.00
Egypt, Arab Rep,	3.17	34.41
El Salvador	3.25	53.20
Estonia	3.67	37.24
Ethiopia	3.08	29.97
Finland	5	26.88
Germany	4.86	28.31
Guatemala	2.33	48.32
Guyana	3.17	43.24
Honduras	2.33	55.00
Hungary	3.81	24.44
Italy	4.5	36.03
Jamaica	2.83	37.91
Lithuania	3.17	31.90
Madagascar	2.75	47.45
Mexico	2.33	54.60
Moldova	3.58	36.19
Morocco	4	39.50
Norway	5	25.79
Panama	2.75	56.40
Paraguay	2.25	56.80
Peru	2.75	49.80
Philippines	3.28	46.09
Poland	3.67	31.60
Russian Federation	2.75	45.62
Slovenia	4.08	28.41
Sweden	4.5	25.00
Thailand	4.08	43.15
Uganda	3.17	43.00
Ukraine	2.67	28.96
United Kingdom	5	35.97
Uruguay	3.25	44.60

Proof of proposition 1

The Lagrangian that corresponds to the problem $\wp$ defined by (2) is

$$L(l^i, d^i_t, \lambda^i, \mu^i) = l^i \left[a(w^i_t + d^i_t) - r^i_t d^i_t\right] - \frac{a\widetilde{w}}{2}\left(l^i\right)^2 + \beta^i\left(\overline{d^i_t} - d^i_t\right) + \mu^i\left(1 - l^i\right) + \gamma^i\left(w^i_t + d^i_t - \widetilde{w}\right)$$

Applying the Kuhn-Tucker Theorem the first-order conditions are

$$\begin{cases} \dfrac{\partial L}{\partial l^i} = 0 \\ \dfrac{\partial L}{\partial d^i} \leq 0, \quad d^{i*}_t \geq 0 \\ \dfrac{\partial L}{\partial \beta^i} \geq 0, \quad \beta^{i*} \geq 0 \\ \dfrac{\partial L}{\partial \mu^i} \geq 0, \quad \mu^{i*} \geq 0 \\ \dfrac{\partial L}{\partial \gamma^i} \geq 0, \quad \gamma^{i*} \geq 0 \end{cases}$$

with complementary slackness in each of the three last conditions.

$$\begin{cases} \left[a(w^i_t + d^{i*}_t) - r^i_t d^{i*}_t\right] - a\widetilde{w}l^{i*} - \mu^{i*} = 0 \\ l^{i*}\left[a - r^i_t\right] - \beta^{i*} + \gamma^{i*} \leq 0, \quad d^{i*}_t \geq 0 \\ \overline{d^i_t} - d^{i*}_t \geq 0, \quad \beta^{i*} \geq 0 \\ 1 - l^{i*} \geq 0, \quad \mu^{i*} \geq 0 \\ w^i_t + d^i_t \geq \widetilde{w}, \quad \gamma^{i*} \geq 0 \end{cases}$$

Case of the variable $d^{i*}_t > 0$

Then $l^{i*}\left[a - r^i_t\right] + \gamma^{i*} = \beta^{i*}$.

Subcase $0 < d^{i*}_t < \overline{d^i_t}$ Then $\beta^{i*} = 0$ and

$$l^{i*}\left[a - r^i_t\right] + \gamma^{i*} = 0 \tag{10}$$

- If $\gamma^{i*} = 0$ then $a = r^i_t$.
- If $\gamma^{i*} > 0$ then $w^i_t + d^{i*}_t = \widetilde{w}$. From (10) we need to have $r^i_t > a$ and we should verify that $d^{i*}_t = \widetilde{w} - w^i_t < \overline{d^i_t}$ which is the case if $w^i_t \geq w^i_{1t} = \left(1 - \dfrac{\lambda a}{r^i_t}\right)\widetilde{w}$.

Subcase $d^{i*}_t = \overline{d^i_t}$ Then $\beta^{i*} \geq 0$ and we should have

$$\left[a - r^i_t\right] + \gamma^{i*} \geq 0$$

or equivalently

$$r^i_t \leq \gamma^{i*} + a \tag{11}$$

- If $\gamma^{i*} = 0$ then the condition (11) becomes $r^i_t \leq a$ and we should verify that $w^i_t + \overline{d^i_t} \geq \widetilde{w}$ which is the case if $w^i_t \geq w^i_{1t}$.
- If $\gamma^{i*} > 0$ then $w^i_t + \overline{d^i_t} = \widetilde{w}$ which is the case if $w^i_t = w^i_{1t}$.

Case of the variable $\mu^{i*} > 0$

This signifies that $l^{i*} = 1$ which imposes that

$$\left[a(w_t^i + d_t^{i*}) - r_t^i d_t^{i*}\right] - a\widetilde{w} > 0 \tag{12}$$

Subcase $d_t^{i*} = \overline{d_t^i}$ Condition (12) requires

$$w_t^i > w_{2t}^i = \frac{r_t^i - \lambda a}{(1-\lambda) r_t^i}\widetilde{w}$$

Subcase $0 < d_t^{i*} < \overline{d_t^i}$ Then $\beta^{i*} = 0$ and

$$\left[a - r_t^i\right] + \gamma^{i*} = 0 \tag{13}$$

- If $\gamma^{i*} = 0$ Then $a = r_t^i$ and (12) becomes $w_t^i > \widetilde{w}$
- If $\gamma^{i*} > 0$ then $d_t^{i*} = \widetilde{w} - w_t^i$. The condition (12) becomes $w_t^i > \widetilde{w}$ which is impossible since it leads to $d_t^{i*} < 0$.

Subcase $d_t^{i*} = 0$ Condition (12) becomes $w_t^i > \widetilde{w}$.

Case of the variable $\mu^{i*} = 0$

Then

$$l^{i*} = \frac{a(w_t^i + d_t^{i*}) - r_t^i d_t^{i*}}{a\widetilde{w}} \leq 1 \tag{14}$$

Subcase $d_t^{i*} = \overline{d_t^i}$ Condition (14) becomes $w_t^i \leq w_{2t}^i$.

Subcase $0 < d_t^{i*} < \overline{d_t^i}$

- If $\gamma^{i*} = 0$ then $r_t^i = a$ and $w_t^i \leq \widetilde{w}$.
- If $\gamma^{i*} > 0$ then $d_t^{i*} = \widetilde{w} - w_t^i$. Condition (14) is verified when $w_t^i \leq \widetilde{w}$.

Subcase $d_t^{i*} = 0$ Condition (14) becomes $w_t^i \leq \widetilde{w}$ which is possible just when $w_t^i = \widetilde{w}$ the minimum required investment for undertaking the project.

Finally, combining the different cases the solution is characterized by the following

$$l_t^i = \begin{cases} \begin{cases} 1 - \dfrac{r_t^i(\widetilde{w} - w_t^i)}{a\widetilde{w}} & \text{if } w_{1t}^i \leq w_t^i \leq \widetilde{w} \\ 1 & \text{if } w_t^i \geq \widetilde{w} \end{cases} & \text{if } r_t^i > a \\ \begin{cases} \dfrac{r_t^i(1-\lambda)}{\widetilde{w}(r_t^i - \lambda a)} w_t^i & \text{if } w_{1t}^i \leq w_t^i \leq w_{2t}^i \\ 1 & \text{if } w_t^i \geq w_{2t}^i \end{cases} & \text{if } r_t^i \leq a \end{cases}$$

$$d_t^i = \begin{cases} \begin{cases} \widetilde{w} - w_t^i & \text{if } w_{1t}^i \leq w_t^i \leq \widetilde{w} \\ 0 & \text{if } w_t^i \geq \widetilde{w} \end{cases} & \text{if } r_t^i > a \\ \overline{d_t^i} & \text{if } w_t^i \geq w_{1t}^i \text{ and } r_t^i \leq a \end{cases}$$

where $\overline{d_t^i} = \frac{\lambda a w_t^i}{r_t^i - \lambda a}$, $w_{1t}^i = \frac{r_t^i - \lambda a}{r_t^i}\widetilde{w}$ and $w_{2t}^i = \frac{r_t^i - \lambda a}{(1-\lambda) r_t^i}\widetilde{w}$.

Since we have $r_t^d = p^i r_t^i$, then the required repayment rate r_t^i must satisfy

$$\begin{array}{ll} \left\{ r_t^i \left[a\widetilde{w} - r_t^i \left(\widetilde{w} - w_t^i \right) \right] = a\widetilde{w} r_t^d \right. & \text{if } w_{1t}^i \leq w_t^i \leq \widetilde{w} \text{ if } r_t^i > a \\ \begin{cases} \left(r_t^i\right)^2 \frac{(1-\lambda) w_t^i}{\widetilde{w}\left(r_t^i - \lambda a\right)} = r_t^d & \text{if } w_{1t}^i \leq w_t^i \leq w_{2t}^i \\ r_t^i = r_t^d & \text{if } w_t^i \geq w_{2t}^i \end{cases} & \text{if } r_t^i \leq a \end{array} \tag{15}$$

Let's consider the first equation of the system (15) $r_t^i \left[a\widetilde{w} - r_t^i \left(\widetilde{w} - w_t^i \right) \right] = a\widetilde{w} r_t^d$ if $w_{1t}^i \leq w_t^i \leq \widetilde{w}$ and $r_t^i > a$. It can be seen as a second degree equation in r_t^i with a solution

$$r_t^i = \left[1 - \sqrt{1 - 4\frac{\widetilde{w} - w_t^i}{a\widetilde{w}} r_t^d} \right] \frac{a\widetilde{w}}{2\left(\widetilde{w} - w_t^i\right)} \tag{16}$$

under the conditions

$$1 - 4\frac{\widetilde{w} - w_t^i}{a\widetilde{w}} r_t^d \geq 0$$

Which is equivalent to

$$w_t^i \geq \underline{w}_t = \widetilde{w}\left(1 - \frac{a}{4r_t^d}\right)$$

The second condition is

$$w_t^i \geq w_{1t}^i = \frac{r_t^i - \lambda a}{r_t^i}\widetilde{w}$$

which is equivalent to

$$\frac{r_t^i\left(\widetilde{w} - w_t^i\right)}{a\widetilde{w}} \leq \lambda$$

or equivalently $\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\frac{\widetilde{w} - w_t^i}{a\widetilde{w}} r_t^d} \leq \lambda$ which is verified only if

$$w_t^i \geq w_{mt} = \widetilde{w}\left(1 - \frac{\lambda(1-\lambda)a}{r_t^d}\right) \geq \underline{w}_t$$

It is simple to show that the condition $r_t^i > a$ is verified if $w_t^i < \frac{r_t^d}{a}\widetilde{w}$. Hence the solution (16) is accepted for $w_{mt} \leq w_t^i < \frac{r_t^d}{a}\widetilde{w}$. Let's consider the second equation of the system (15) $\left(r_t^i\right)^2 \frac{(1-\lambda) w_t^i}{\widetilde{w}\left(r_t^i - \lambda a\right)} = r_t^d$ if $w_{1t}^i \leq w_t^i \leq w_{2t}^i$ and $r_t^i \leq a$.It can also be seen as a second degree equation in r_t^i with a solution

$$r_t^i = \left[1 + \sqrt{1 - \frac{4(1-\lambda) w_t^i}{r_t^d \widetilde{w}} \lambda a} \right] \frac{r_t^d \widetilde{w}}{2(1-\lambda) w_t^i} \tag{17}$$

under the four conditions. The first one is

$$1 - \frac{4(1-\lambda) w_t^i}{r_t^d \widetilde{w}} \lambda a \geq 0$$

which is equivalent to

$$w_t^i \leq w_{rt} = \frac{1}{4}\frac{r_t^d \widetilde{w}}{\lambda(1-\lambda)a}$$

The second condition is

$$w_t^i \leq w_{2t}^i = \frac{r_t^i - \lambda a}{(1-\lambda)r_t^i}\widetilde{w}$$

which is equivalent to $\frac{(1-\lambda)r_t^i w_t^i}{\widetilde{w}\left(r_t^i - \lambda a\right)} \leq 1$ or $r_t^d \leq r_t^i$ which is verified. The third condition is

$$w_{1t}^i = \left(1 - \frac{\lambda a}{r_t^i}\right)\widetilde{w} \leq w_t^i \tag{18}$$

It is straightforward to show that this condition is verified for $\frac{r_t^d}{a}\widetilde{w} \leq w_t^i \leq w_{rt}$. Indeed, since we are in the case of $r_t^i \leq a$ then $1 - \dfrac{\lambda a}{r_t^i} < 1 - \lambda \leq \frac{1}{2}$. Besides, $r_t^d \geq 1 > \frac{a}{2}$ because $a \in \left]1, 2\right[$. Therefore, $w_{1t}^i < \frac{r_t^d}{a}\widetilde{w}$ and the condition (18) is verified. The final condition $r_t^i \leq a$ is verified for $w_t^i \geq \frac{r_t^d}{a}\widetilde{w}$. Hence the solution (17) is verfied for

$$\widetilde{w} \leq w_t^i \leq w_{rt}$$

Finally the third equation of the system (15) becomes $r_t^i = r_t^d$ and $w_t^i \geq w_{pt} = \frac{r_t^d - \lambda a}{(1-\lambda)r_t^d}\widetilde{w}$. Finally noting that $w_{pt} < w_{rt}$ and in order to limit the discontinuity of r_t^i the solution is given by

$$r_t^i = \begin{cases} \left[1 - \sqrt{1 - 4\frac{\widetilde{w} - w_t^i}{a\widetilde{w}} r_t^d}\right] \frac{a\widetilde{w}}{2\left(\widetilde{w} - w_t^i\right)} > a & \text{if} \quad w_{mt} \leq w_t^i < \frac{r_t^d}{a}\widetilde{w} \\ \left[1 + \sqrt{1 - \frac{4(1-\lambda)w_t^i}{r_t^d \widetilde{w}}\lambda a}\right] \frac{r_t^d \widetilde{w}}{2(1-\lambda)w_t^i} \leq a & \text{if} \quad \frac{r_t^d}{a}\widetilde{w} \leq w_t^i \leq w_{rt} \\ r_t^d & \text{if} \quad w_t^i \geq w_{rt} \end{cases} \tag{19}$$

where $w_{mt} = \widetilde{w}\left(1 - \frac{\lambda(1-\lambda)a}{r_t^d}\right)$ and $w_{rt} = \frac{1}{4}\frac{r_t^d \widetilde{w}}{\lambda(1-\lambda)a}$. Using the equality $r_t^d = p^i r_t^i$ the rest of the proof is straightforward.

Proof of lemma 1

From proposition 1 we can distinguish two cases

- If $\widehat{w}_t \leq w_{rt}$ then $d_t^i = \widetilde{w} - \widehat{w}_t$ and (5) becomes $p_t a\widetilde{w} = r_t^d \widetilde{w} + \dfrac{a\widetilde{w}}{2}(p_t)^2$ or equivalently $(1-p_t)^2 = 1 - 2\frac{r_t^d}{a}$ which is not possible since the second term of the equation is strictly negative ($2/a \in \left]1, 2\right[$ and since $r_t^d \geq 1$ then $2r_t^d/a > 1$).
- If $\widehat{w}_t > w_{rt}$ then $d_t^i = \overline{d_t^i} = \lambda a\widehat{w}_t/(r_t^d - \lambda a)$ and the solution to (5) is $\widehat{w}_t = a\left(r_t^d - \lambda a\right)\widetilde{w}/[2\left(a - r_t^d\right)r_t^d]$. Now, we should determine the condition on r_t^d in order to have $\widehat{w}_t > w_{rt} = w_{rt} = \frac{1}{4}r_t^d\widetilde{w}/[\lambda(1-\lambda)a]$. After some algebra we obtain the following condition $2\lambda(1-\lambda)[(\frac{r_t^d}{a}) - \lambda]/[1 - (\frac{r_t^d}{a})] > (\frac{r_t^d}{a})^2$. Therfore, we can show straightforwardly that $\widehat{w}_t > w_{rt}$ if and only if $\lambda \in \left]\frac{1}{2}, 1\right[$ and $r_t^d \in \left]r_\lambda, a\right[$ where $r_\lambda \in \left]1, a\right[$ verifies $\frac{\partial r_\lambda}{\partial \lambda} > 0$ and $2\lambda(1-\lambda)[(\frac{r_\lambda}{a}) - \lambda]/[1 - (\frac{r_\lambda}{a})] = (\frac{r_\lambda}{a})^2$.

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