

E C O N O M I C S   B U L L E T I N

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## Testing for asymmetry in Okun's law: A cross-country comparison

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### *Abstract*

Most specifications of Okun's law assume a symmetric relationship between changes in unemployment and real output. We test this assumption for seven OECD countries (Australia, Canada, Germany, Japan, New Zealand, the United Kingdom and the United States). We find that failure to take account of asymmetries would see a rejection of the hypothesis that there exists a long-run relationship between unemployment and output in countries such as the United States and New Zealand. We also find that short-run output and unemployment adjustments to disequilibrium usually differ according to whether up-turns or down-turns in the business cycle are considered. These results could not have been obtained using standard estimates of Okun's law based on a symmetric approach.

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## 1. Introduction

Okun's law - the relationship between changes in unemployment and output - is an important concept in macroeconomics both theoretically and empirically. Theoretically, Okun's law is the link between the aggregate supply curve and the Phillips curve. Empirically, Okun's coefficient is a useful "rule of thumb" in forecasting and policy-making. Indeed, Blinder (1997, p.241) has remarked that Okun's law "closes the loop between [US] real output growth and changes in unemployment with stunning reliability". Despite the theoretical and empirical usefulness of Okun's law, most specifications assume a symmetric relationship. This assumption, which implies that expansions and contractions in output have the same absolute effect on unemployment, may not always be appropriate. Given the link between Okun's law and the Phillips curve, it is not surprising that the renewed interest in modelling and testing for Phillips curve asymmetry is being matched by a similar interest in Okun's law. (See, for example, Debelle and Laxton 1997, Laxton *et al.* 1999, Lee 2000 and Virén 2001).

Testing for asymmetry in the output-unemployment relationship is important for at least four reasons. First, it could assist in discriminating among alternative theories of joint labour and goods market behaviour. Secondly, it would probably strengthen the case for an asymmetrical Phillips curve if a country's Okun relationship is also asymmetrical, and conversely. Thirdly, knowledge about the extent of asymmetry in the output-unemployment relationship could be useful for both structural policies (for example, labour market reforms) and stabilisation policies (for example, appropriate monetary policy responses). Fourthly, ignoring asymmetry in Okun's law, when it is present, could lead to forecasting errors.

Courtney (1991) and Palley (1993) are among the initial contributors to the idea that Okun's coefficient may be different in expansions and contractions. Taking an aggregate production function approach, Courtney's explanations for asymmetry include factor substitution during cycles (involving non-constant relationships among hours, labour force participation and capital), fluctuations in multi-factor productivity and changes in the distribution of sectoral growth rates. He concludes from his empirical work with United States data that "imposing symmetry in the Okun's law regression leads to serious underestimates of unemployment rate increases in contractions and overestimates of decreases in the unemployment rate during expansions" (Courtney 1991, p.285). Palley's explanations for asymmetry in Okun's law also include changes in sectoral growth rates and labour force participation rates.

Lee (2000), Mayes and Virén (2000) and Virén (2001) use contemporary econometric techniques to consider asymmetry in Okun's law. The papers involving Virén estimate a model whereby changes in unemployment ( $\Delta u$ ) are determined by changes in output ( $\Delta y$ ), with the latter split into positive and negative values. The long-run part of their model is a relationship between unemployment, employment, population and a time trend. Their approach is different to ours since we follow Attfield and Silverstone (1996, 1998) and model both the long and short-run Okun relationship. Lee is similar to Virén but omits the error-correction component. In particular, he regresses  $\Delta y$  on  $\Delta u$  with the latter split on the basis of whether  $\Delta u$  is positive or negative.

Our approach is more general than both Virén and Lee because we base asymmetries on the error correction component (that is, the component that says the Okun relationship is

above or below long-run equilibrium) rather than just positive or negative changes in either  $\Delta y$  or  $\Delta u$ . In short, we use a methodology that has a worked-out theoretical specification underlying it, namely, an error-correction model that captures long and short-run relationships between  $u$  and  $y$ . We believe our results are more robust in the sense that they are based on a tighter specification about positive and negative values of  $\Delta y$  and  $\Delta u$ . Despite these different specifications, however, they all point to a non-linear, asymmetric relationship between (changes in) output and (changes in) unemployment.

Section 2 of our paper briefly outlines Okun's law and our approach to estimating the relationship. Section 3 tests for cointegration between unemployment and output while Section 4 provides estimates of an asymmetric error-correction model. Section 5 has some thoughts about asymmetry and Section 6 concludes the paper.

## 2. Okun's Law

A typical version of Okun's law is a gap equation of the type:

$$y_t - y_t^* = a(u_t - u_t^*) + e_t \quad \alpha < 0 \quad (1)$$

where  $y$  is the log of observed real output,  $u$  is the log of observed unemployment,  $y_t^*$  and  $u_t^*$  are the corresponding potential values and  $e_t$  is a random error term. Okun (1962) found a value of -3 for the coefficient on the unemployment gap. The reason a one percentage point change in the rate of unemployment leads to a more than proportionate change in output, argued Okun, is because changes in unemployment are also associated with induced changes in labour force participation, labour hours and capital utilisation. Prachowny (1993) showed, formally, that these changes can be derived from the following natural log production function:

$$\begin{aligned} y_t &= \alpha(k_t + c_t) + \beta(\gamma n_t + \delta h_t) + \tau_t + \varepsilon_t \\ &= \alpha(k_t + c_t) + \beta[\gamma(l_t - u_t) + \delta h_t] + \tau_t + \varepsilon_t \end{aligned} \quad (2)$$

where  $y$  is real output,  $k$  capital input,  $c$  capital utilisation,  $n$  number of workers (labour force less number unemployed),  $h$  average hours worked,  $l$  labour force,  $u$  unemployment rate ( $l-n$ ),  $\tau$  disembodied technological progress,  $\gamma$ ,  $\delta$ ,  $\alpha$ ,  $\beta$  are output elasticities and  $\varepsilon$  is an error term. Equation 2 shows that labour services can be separated into three components: the labour force ( $l_t$ ), the unemployment rate ( $u_t$ ) and hours worked ( $h_t$ ). The substance of Okun's law, as expressed in equation 1, is to say that co-movements in output ( $y_t$ ) and unemployment dominate any adjustment in capital and its utilisation ( $k_t + c_t$ ), the labour force ( $l_t$ ), hours worked ( $h_t$ ) and technological progress ( $\tau_t$ ).

Equation 2 depicts the long-run equilibrium relationship between  $y_t$  and  $u_t$  (excluding the other inducement terms) whereas equation 1 is essentially a dynamic short-run version of Okun's law. The econometric approach currently favoured is to incorporate both short and long-run information through an error-correction model.

Since  $\log u$  and  $\log y$  (hereafter denoted  $u$  and  $y$ ) are potentially non-stationary variables, the relationship between them has to be estimated using cointegration. (See

Attfield and Silverstone 1996, 1998). This approach presupposes there is, at most, a single long-run relationship between  $u$  and  $y$ , that is,

$$u_t = \beta_0 + \beta_1 y_t + \beta_2 t + \varepsilon_t \quad (3)$$

where the time trend ( $t$ ) is included to allow for long-run linear growth which the model cannot explain. One reason for now having  $u_t$  on the left hand side of equation 3 is that subsequent tests establish that  $y_t$  is weakly exogenous. Since, however, we allow *a priori* both  $y_t$  and  $u_t$  to be endogenous, the ordering adopted in equation 3 is largely a matter of taste. Assuming  $u_t$  and  $y_t$  are both  $I(1)$ , Engle and Granger (1987) show that cointegration exists if  $\varepsilon_t \sim I(0)$ .

The long-run model set out in equation 3 is associated with a short-run error-correction model (ECM) based on symmetric adjustment, with the second-step Engle-Granger test for cointegration based on the OLS estimate of  $\rho$  in equation 4:

$$\Delta \hat{\varepsilon}_t = \rho \hat{\varepsilon}_{t-1} + v_t \quad v_t \sim IID(0, \sigma^2) \quad (4)$$

If the null hypothesis of no cointegration,  $H_0: \rho = 0$ , can be rejected in favour of  $H_1: \rho < 0$ , then equations 3 and 4 jointly imply the following ECMs:

$$A(L)\Delta u_t = B(L)\Delta y_{t-1} - (1 - \alpha_1)ec_{t-1} + \omega_t \quad \omega_t \sim IID(0, \sigma^2) \quad (5a)$$

$$A(L)\Delta y_t = B(L)\Delta u_{t-1} - (1 - \alpha_2)ec_{t-1} + \omega_t^* \quad \omega_t^* \sim IID(0, \sigma^2) \quad (5b)$$

where

$$ec_{t-1} = \hat{\varepsilon}_{t-1} = u_{t-1} - \hat{\beta}_0 - \hat{\beta}_1 y_{t-1} - \hat{\beta}_2 t$$

and  $A(L)$  and  $B(L)$  are polynomial lag operators.

Equation 5 implies that any short-run changes in unemployment and output due to disequilibrium ( $1 - \alpha_i$ ) are strictly proportional to the absolute value of the error-correction term. If, however, adjustment to disequilibrium is asymmetric, then Enders and Granger (1998) and Enders and Siklos (2001) show that an alternative specification for equation 4 (called the threshold autoregressive model, TAR) can be written as:

$$\Delta \hat{\varepsilon}_t = I_t \rho_1 \hat{\varepsilon}_{t-1} + (1 - I_t) \rho_2 \hat{\varepsilon}_{t-1} + v_t^* \quad v_t^* \sim IID(0, \sigma^2) \quad (6)$$

where  $I_t$  is the Heaviside indicator function based on the threshold value  $\tau$ .

$$I_t = \begin{cases} 1 & \text{if } \hat{\varepsilon}_{t-1} \geq \tau \\ 0 & \text{if } \hat{\varepsilon}_{t-1} < \tau \end{cases} \quad (7)$$

The asymmetric version of the ECM, then, replaces the single error-correction term in equation 5 ( $ec_{t-1}$ ) with two error-correction terms multiplied by  $I_t$  and  $(1 - I_t)$ , respectively.

### 3. Testing for Cointegration between $u_t$ and $y_t$

Real GDP growth and unemployment rates between 1978 and 1999 varied considerably across the seven countries in this study.<sup>1</sup> Japan, for example, experienced above average growth and low unemployment, Australia had high unemployment and high growth, while Germany, following reunification, experienced a rapid increase in unemployment and a significant slow-down in growth.

Prior testing - including tests for structural breaks - established that the data are basically non-stationary. (See Appendix Tables A1 and A2). Since  $u_t$  and  $y_t$  also follow asymmetric adjustment paths, equation 3 was estimated for each country and the residuals used to estimate equations 6 and 7. As the threshold value  $\tau$  in equation 7 is unknown (and there is no *a priori* reason to expect that it should be zero), the procedure suggested in Enders and Siklos (2001) was used to perform a grid-search. Specifically, the estimated residuals from equation 3 were sorted in ascending order and called  $\hat{\varepsilon}_1^\tau < \hat{\varepsilon}_2^\tau < \dots < \hat{\varepsilon}_T^\tau$  where  $T$  is the number of usable observations. The largest and smallest 15 percent of the  $\{\hat{\varepsilon}_i^\tau\}$  values were discarded and the remainder considered as possible thresholds. Equations 6 and 7 were then estimated for each possible threshold. The model with the lowest residual sum of squares was chosen in order to obtain the preferred value of  $\tau$ . Equation 6 was then used to test for cointegration using the  $t$ -Max and  $F$ -test proposed in Enders and Siklos. The results obtained from the estimation are presented in Table 1.

The long-run Okun coefficient for most of the countries lies between -0.39 and -0.5, with the UK and especially Japan as outliers.<sup>2</sup> For four countries, as shown in Table 1, it was necessary to include shift dummies (taking a value of 1) in the long-run model to account for regime shifts. Examples include the period between 1983:2 and 1986:1 in the UK, when hysteresis resulted in unemployment remaining very high despite a significant growth in output, and the post-unification period in Germany.<sup>3</sup> As to whether the estimates of equation 1 represent long-run stationary relationships, the  $t$ -Max and  $F$ -tests obtained from estimating asymmetric Dickey-Fuller equations both reject the null hypothesis of no cointegration at better than the five percent significance level in all countries (except Canada) using the critical values in Enders and Siklos.

Since the Enders and Siklos critical values are based on simulations with no trend in the long-run relationship (and no dummies in either the long-run model or the DF equation), Monte Carlo experiments were conducted. These experiments were based on the actual model structures used, including estimates of  $\tau$  and with  $u_t$  and  $y_t$  replaced by two

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<sup>1</sup> The data are consistent, quarterly, seasonally adjusted series (mostly) from the *OECD Quarterly National Accounts Database*. Data for West Germany after 1993 had to be linked to the series covering all Germany. Unemployment data prior to 1982 for the UK was obtained from *Economic Trends Annual Supplement* (published by HMSO). Statistics New Zealand and Chapple (1994) provided data on real GDP and unemployment for New Zealand. All data start from the first quarter of 1978 and end at 1998:3 for Australia, Japan, the UK and West Germany, 1998:4 for Canada and the USA and 1999:1 for New Zealand.

<sup>2</sup> Moosa (1997) provides similar estimates for several countries ranging from -0.49 and -0.46 for Canada and the USA to -0.10 for Japan. Most countries, however, had Okun coefficients between -0.38 and -0.49.

<sup>3</sup> Failure to include these shift-dummies results in non-rejection of the null hypothesis of no cointegration (in both the asymmetric and symmetric forms of the DF-test for cointegration).

variables constrained to equal random walks. The simulation was performed 10,000 times using  $N(0,1)$  serially uncorrelated pseudo-random numbers.<sup>4</sup> The significance levels for rejecting the null hypothesis for the  $t$ -Max and  $F$ -tests are reported alongside each test statistic in Table 1. The model structures used here (especially involving the time trend) do have an important effect on the size properties of the model. We are, however, still able to reject at around (or better than) the five percent significance level in all countries except Canada (using the non-normal  $F$ -test, which Enders and Siklos show has better power properties than the  $t$ -Max statistic). Lastly, having established that the  $\hat{\varepsilon}_{t-1}$  are stationary, it is possible to use the standard  $F$ -test to consider whether  $\rho_1 = \rho_2$ . This null is strongly rejected and asymmetry is again confirmed for each country except Canada.

In comparison, the symmetric Engle-Granger test, based on testing the residuals from equation 1 using equation 2, failed to reject the null of no cointegration for Canada, New Zealand and the USA (based on the critical values in MacKinnon 1991). The Johansen (1995) approach was also used (with a time trend constrained to enter the cointegration space) and the  $\lambda_{\max}$  and  $\lambda_{\text{trace}}$  tests (that the rank  $r = 0$ ) were unable to reject the null except in Japan, the UK and Germany.<sup>5</sup> These results show that when  $\rho_1$  and  $\rho_2$  have similar values, the symmetric Engle-Granger and Johansen models reject the null of no cointegration (when the asymmetric tests also reject). When asymmetry is important, these symmetric tests generally fail to detect the long-run Okun relationship.

#### 4. Asymmetric Error-Correction Model

Having established cointegration in the asymmetric model for all countries except Canada, it is possible to estimate an asymmetric version of equation 5. The results are shown in Tables 2 and 3. All the equations are well specified as shown by the various diagnostic tests including Chow tests for parameter stability. The  $t$ -statistics on the error-correction terms show that real GDP is weakly exogenous in the two smallest open-economy countries (Australia and New Zealand). The  $t$ -statistics on the  $\Delta u_{t-1}$  and  $\Delta y_{t-1}$  terms show that real GDP Granger-causes unemployment in New Zealand (but not the other way around), while unemployment Granger-causes real GDP in Australia, Japan and the USA.

Table 2 shows that unemployment adjusts asymmetrically to disequilibrium in each country. Positive values of  $\hat{\varepsilon}_{t-1}$  are usually associated with short-run negative adjustments in the unemployment rate. This brings the long-run unemployment-output relationship back into equilibrium. Other things being equal, the speed of adjustment ( $1-\alpha_1$ ) indicates that adjustment is relatively fast in Japan, where some 35.7 percent of the disequilibrium is removed each quarter. It would thus take 8.5 months for the economy to return to its long-run trend in Japan but some 3.75 years for the UK to achieve equilibrium.

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<sup>4</sup> In common with this type of Monte Carlo experiment, we set the initial values of the two random walks at zero, and discarded the first 50 observations generated before computing  $t$ - and  $F$ -values.

<sup>5</sup> For each country, models were estimated where the residuals from each VECM pass the various diagnostic tests available in PcFiml (Version 9), such as no autocorrelation, no ARCH processes, normality, and homoskedasticity (including vector tests and tests for stability based on 1-step ahead residuals and Chow tests).

In contrast, (mostly) negative values of  $\hat{\varepsilon}_{t-1}$  only have a significant impact on short-run changes to unemployment in Australia, Japan and the UK, and in each case adjustment is in the ‘wrong’ direction. That is, when  $\hat{\varepsilon}_{t-1}$  is negative, and the economy is in the upturn of the business cycle, we would expect that since  $u_t < y_t$  in terms of long-run equilibrium,  $u_t$  should adjust upwards and  $y_t$  should fall. Since this does not happen, quantity adjustments in the labour market do not act to re-establish equilibrium. Presumably prices (which are not modelled as part of the Okun relationship) could compensate to bring about the long-run Okun relationship. With respect to adjustments in unemployment, the asymmetric model is clearly a superior specification except in Australia and Japan where the asymmetric disequilibrium terms are similar, and therefore not very different from the parameter estimate that is obtained from the symmetric ECM (Table 2, last row).

Turning to the results in Table 3, output apparently does not adjust to disequilibrium when unemployment (output) is above (below) its long-term trend relationship. The exception is the UK where, other things being equal, the speed of adjustment ( $1-\alpha_2$ ) indicates that some 25.6 percent of the disequilibrium is removed each quarter. It would therefore take about one year for the British economy to return to its long-run trend. In contrast, (mostly) negative values of  $\hat{\varepsilon}_{t-1}$  (indicating ‘boom’ conditions) have a significant impact on short-run changes to output in Japan, the UK, the USA and Germany. For Japan and the UK, adjustment is in the ‘wrong’ direction, so that quantity adjustments in output do not act to re-establish equilibrium. In the USA and Germany there are fairly strong output responses when the economy starts to ‘overheat’, with output adjusting downwards sufficiently to restore equilibrium within about four months in both countries. Lastly, with respect to adjustments in output, the asymmetric model is clearly a superior specification for those countries where changes in  $y_t$  resulting from disequilibrium are important.

Table 4 summarises the asymmetric response to disequilibrium in the unemployment-output relationship in terms of changes in unemployment and output. Clearly, there is substantial variation across the countries considered (with Canada lacking evidence of any long-run Okun relationship). In all countries, except Canada, unemployment adjusts in the expected manner during a downturn in the business cycle. The labour market continues to ‘tighten’ in upturns in those countries that experience a response to disequilibrium. Output responses to unemployment changes are uncommon during downturns, and often wrongly-signed in upturns. The USA is the most likely to experience changes in unemployment and output that restore long-run equilibrium, followed closely by Germany.

**Table 4. Adjustments to Disequilibrium**

| Country     | Upturn in Business Cycle |              | Downturn in Business Cycle |              |
|-------------|--------------------------|--------------|----------------------------|--------------|
|             | $\Delta u_t$             | $\Delta y_t$ | $\Delta u_t$               | $\Delta y_t$ |
| Australia   | ☒                        | –            | ☑                          | –            |
| Japan       | ☒                        | ☒            | ☑                          | –            |
| New Zealand | –                        | –            | ☑                          | –            |
| UK          | ☒                        | ☒            | ☑                          | ☑            |
| USA         | –                        | ☑            | ☑                          | ☑            |
| Germany     | –                        | ☑            | ☑                          | –            |

☑ correct response. ☒ incorrect response. – no significant response.

## 5. Asymmetry: Some Thoughts

What could explain asymmetry in the output-unemployment relationship? Courtney (1991), it will be recalled from the introduction, offered an explanation in terms of relationships within the aggregate production function. If we broaden Okun's law to include its link to the Phillips curve, then further explanations for asymmetry are available. Dupasquier and Ricketts (1998), for example, consider four models to explain Phillips curve asymmetry: capacity constraints, signal extraction, costly adjustment and downward nominal wage rigidity. Overall, they found it difficult to distinguish empirically among the possible models generating non-linearity in Canada's output-inflation relationship. It is likely, they say, that more than one model could be operating.

With respect to asymmetry in Okun's law, we expect that future research will need to consider an explanation centred on asymmetric responses in terms of job creation and job destruction when heterogeneous plants are faced with shocks. Campbell and Fisher (2000), for example, develop a model which shows how microeconomic asymmetries in adjustment costs can account for aggregate asymmetries in job creation and destruction with contracting plants responding more than expanding plants to positive external shocks.

## 6. Conclusion

Failure to take account of asymmetries would see a rejection of the hypothesis that there exists a long-run relationship between unemployment and real GDP in countries such as the United States and New Zealand. Using an asymmetric approach, it is possible to establish cointegration and to show that for all the countries studied (except Canada) short-run adjustment to disequilibrium differs according to whether up-turns or down-turns in the business cycle are considered. These results suggest that unemployment adjusts in the expected manner during a downturn in the business cycle (it falls), whereas in most countries the labour market continues to 'tighten' in upturns when there is disequilibrium between unemployment and output. Furthermore, output responses to disequilibrium are uncommon in downturns, and often take the economy further away from equilibrium in upturns.

Of the countries examined, the United States is the most likely to experience changes in unemployment and output that restore long-run equilibrium, while other countries appear to rely on short-run price-adjustments during upturns to restore market equilibria. These results could not have been obtained using standard estimates of Okun's law based on a symmetric approach. They suggest that using an asymmetric model should become the standard approach to estimating what is a well-established empirical link between the labour market and output in the macroeconomy.



**Table 1. Long-run Model and Tests for Cointegration**

| Variable  | Australia        | Canada          | Japan            | New Zealand     | UK               | USA             | Germany         |
|---|------------------|-----------------|------------------|-----------------|------------------|-----------------|-----------------|
| <i>Estimates of long-run model Equation 3<sup>a</sup></i> |                  |                 |                  |                 |                  |                 |                 |
| Constant  | 2.901 (17.49)    | 2.487 (11.33)   | 0.527 (13.89)    | 3.919 (13.38)   | 1.680 (5.22)     | 3.790 (18.82)   | 2.975 (18.12)   |
| $y_t$   | -0.501 (-17.10)  | -0.386 (-10.94) | -0.091 (-13.41)  | -0.406 (-13.31) | -0.263 (-5.06)   | -0.444 (-18.45) | -0.389 (-17.87) |
| $t$   | 0.004 (18.36)    | 0.002 (-11.17)  | 0.001 (19.53)    | 0.003 (18.61)   | 0.001 (4.37)     | 0.003 (17.26)   | 0.002 (15.30)   |
| Shift dummy date  | -                | -               | 87:3-98:3        | -               | 83:2-86:1        | 86:1-98:4       | 92:1-98:3       |
| <i>Asymmetric Dickey-Fuller Equation 4<sup>a</sup></i>    |                  |                 |                  |                 |                  |                 |                 |
| $I_t \varepsilon_{t-1}$                                   | -0.524 (-4.66)   | -0.075 (-1.068) | -0.377 (-3.17)   | -0.301 (-4.16)  | -0.095 (-3.40)   | -0.138 (-1.54)  | -0.241 (-2.92)  |
| $(1-I_t) \varepsilon_{t-1}$                               | -0.304 (-2.48)   | -0.049 (-0.97)  | -0.254 (-2.22)   | -0.054 (-0.73)  | -0.102 (-2.77)   | -0.324 (-4.10)  | -0.300 (-3.10)  |
| $\Delta \varepsilon_{t-1}$                                | -                | -               | -                | -               | 0.195 (4.70)     | -               | 0.136 (1.25)    |
| $\Delta \varepsilon_{t-2}$                                | -                | -               | -                | -               | -                | -               | 0.233 (2.08)    |
| $\Delta \varepsilon_{t-3}$                                | -                | -               | -                | -               | -                | -               | 0.275 (2.40)    |
| Dummies   | One              | None            | None             | One             | Eight            | None            | None            |
| $\tau$  | 0.004            | 0.008           | 0.001            | 0.006           | -0.003           | 0.003           | -0.003          |
| AR 1-5 $F(5, n-k-5)$                                      | 0.548            | 1.503           | 1.641            | 1.560           | 1.317            | 1.250           | 1.563           |
| ARCH $F(4, n-k-8)$  | 0.704            | 0.140           | 0.636            | 0.232           | 0.591            | 1.588           | 1.190           |
| Normality $\chi^2(2)$                                     | 0.095            | 3.140           | 0.360            | 0.116           | 0.651            | 0.167           | 0.074           |
| $X_1^2 F(2k, n-k-2k-1)$                                   | 1.655            | 0.710           | 0.395            | 0.826           | 0.826            | 1.997           | 0.937           |
| RESET $F(1, n-k-1)$                                       | 0.983            | 0.254           | 0.280            | 0.052           | 0.008            | 3.937           | 1.115           |
| $\rho_1 = \rho_2 F(1, n-k)$                               | 24.788**         | -               | 14.593**         | 11.760**        | 18.452**         | 14.963**        | 15.891**        |
| <i>Asymmetric Cointegration Tests<sup>b</sup></i>         |                  |                 |                  |                 |                  |                 |                 |
| CI $t$ -Max ( $p$ -value)                                 | -4.659 (0.000)   | -1.068 (0.925)  | -3.169 (0.035)   | -4.163 (0.000)  | -3.399 (0.091)   | -4.096 (0.002)  | -2.920 (0.086)  |
| CI $F$ -test ( $p$ -value)                                | 13.931 (0.000)   | 1.036 (0.827)   | 7.477 (0.035)    | 8.936 (0.005)   | 9.769 (0.058)    | 9.576 (0.009)   | 7.954 (0.057)   |
| <i>Symmetric Cointegration Tests</i>                      |                  |                 |                  |                 |                  |                 |                 |
| E-G $\varepsilon_{t-1}$ ( $t$ -value)                     | -0.423 (-0.08**) | -0.058 (-1.41)  | -0.375 (-5.68**) | -0.123 (-2.00)  | -0.149 (-6.57**) | -0.163 (-2.60)  | -0.265 (-3.98*) |
| Johansen $\lambda_{\max}$                                 | 13.57            | 11.64           | 23.39*           | 6.31            | 62.86**          | 15.70           | 24.46**         |
| Johansen $\lambda_{\text{trace}}$                         | 19.27            | 13.82           | 28.00*           | 9.64            | 76.14**          | 19.15           | 45.02**         |

<sup>a</sup>  $t$ -values in parenthesis.

<sup>b</sup> based on Monte-Carlo simulations with structures set by equations 3 and 4. Enders and Siklos (1999) critical values at 5 percent level for the  $t$ -Max and  $F$ -test have a maximum value of -1.92 and 7.41, respectively (lower absolute values when equation 4 has less than 4 lagged terms).

\*\*(\*) rejects null at 1 (5) percent significance level.

**Table 2. Estimates of Asymmetric ECM for  $u_t$** 

| Variable   | Australia      | Japan          | New Zealand    | UK             | USA            | Germany        |
|--|----------------|----------------|----------------|----------------|----------------|----------------|
| <i>Estimates of Asymmetric Equation 5a (<math>\Delta u_t</math>)</i> |                |                |                |                |                |                |
| Constant   | -0.000 (-0.01) | 0.000 (0.49)   | 0.002 (3.14)   | 0.000 (0.63)   | 0.000 (0.90)   | 0.000 (1.65)   |
| $\Delta y_{t-1}$   | -0.020 (-0.55) | -0.003 (-0.28) | -0.103 (-2.74) | -0.034 (-1.22) | -0.013 (-0.30) | -0.035 (-1.56) |
| $\Delta u_{t-1}$   | 0.171 (1.83)   | -              | 0.248 (2.41)   | 0.337 (3.40)   | 0.576 (4.72)   | 0.599 (6.75)   |
| $\Delta u_{t-2}$   | 0.242 (2.70)   | -              | -              | 0.376 (4.24)   | -              | 0.144 (1.71)   |
| $I_t \varepsilon_{t-1}$  | -0.148 (-1.63) | -0.357 (-3.43) | -0.158 (-2.26) | -0.067 (-2.03) | -0.214 (-2.09) | -0.119 (-2.44) |
| $(1-I_t)\varepsilon_{t-1}$   | -0.186 (-2.02) | -0.394 (-3.74) | 0.036 (0.51)   | -0.104 (-3.49) | -0.022 (-0.23) | -0.006 (-0.10) |
| Dummies  | Two            | Two            | None           | One            | Three          | Three          |
| $R^2$  | 0.559          | 0.591          | 0.289          | 0.770          | 0.513          | 0.771          |
| AR 1-5 $F(5, n-k-5)$   | 1.013          | 2.088          | 2.026          | 0.309          | 1.210          | 2.248          |
| ARCH $F(4, n-k-8)$   | 0.530          | 0.342          | 0.286          | 1.359          | 1.598          | 0.227          |
| Normality $\chi^2(2)$  | 0.613          | 0.289          | 0.271          | 4.577          | 4.526          | 3.067          |
| $X_i^2 F(2k, n-k-2k-1)$  | 1.836          | 0.906          | 0.944          | 0.845          | 0.477          | 0.298          |
| RESET $F(1, n-k-1)$  | 0.003          | 0.062          | 0.729          | 0.005          | 0.008          | 0.301          |
| Chow $F(4, n-k-4)$   | 0.564          | 0.478          | 1.642          | 1.061          | 0.308          | 0.582          |
| Chow $F(14, n-k-14)$   | 0.509          | 0.736          | 0.975          | 1.067          | 0.407          | 0.625          |
| Chow $F(30, n-k-30)$   | 0.650          | 0.781          | 1.360          | 1.142          | 0.617          | 0.646          |
| $\alpha_2^1 = \alpha_2^2 F(1, n-k)$                                  | 7.333**        | 31.842**       | 2.030          | 17.047**       | 3.540          | 3.362          |
| <i>Estimates of Symmetric Equation 5a (<math>\Delta u_t</math>)</i>  |                |                |                |                |                |                |
| $\varepsilon_{t-1}$  | -0.167 (-2.72) | -0.375 (-5.66) | -0.080 (-1.90) | -0.087 (-4.24) | -0.116 (-1.85) | -0.086 (-2.40) |

**Table 3. Estimates of Asymmetric ECM for  $y_t$** 

| Variable   | Australia      | Japan          | New Zealand    | UK             | USA            | Germany        |
|--|----------------|----------------|----------------|----------------|----------------|----------------|
| <i>Estimates of Asymmetric Equation 5b (<math>\Delta y_t</math>)</i> |                |                |                |                |                |                |
| Constant   | 0.008 (5.40)   | 0.009 (10.03)  | 0.005 (3.46)   | 0.005 (4.09)   | 0.006 (6.59)   | 0.002 (0.86)   |
| $\Delta u_{t-1}$   | -1.018 (-3.86) | -2.743 (-5.11) | -0.382 (-1.23) | -0.288 (-1.21) | -1.221 (-6.10) | 0.069 (0.15)   |
| $\Delta y_{t-1}$   | -              | -              | 0.217 (1.90)   | 0.124 (1.11)   | -              | 0.206 (1.66)   |
| $\Delta y_{t-2}$   | -              | -              | -              | -              | -              | 0.277 (2.45)   |
| $\Delta y_{t-3}$   | -              | -              | -              | -              | -              | 0.384 (3.42)   |
| $\Delta y_{t-4}$   | -              | -              | -              | -              | -              | -0.363 (-3.19) |
| $I_t \varepsilon_{t-1}$  | -0.400 (-1.35) | 0.364 (0.51)   | -0.190 (-0.89) | 0.256 (2.21)   | 0.330 (1.35)   | -0.052 (-0.22) |
| $(1-I_t)\varepsilon_{t-1}$   | -0.114 (-0.42) | 1.459 (2.09)   | 0.047 (0.22)   | 0.261 (2.44)   | -0.613 (-2.42) | -0.593 (-1.85) |
| Dummies  | Two            | Four           | One            | One            | Two            | None           |
| $R^2$  | 0.503          | 0.597          | 0.229          | 0.547          | 0.494          | 0.325          |
| AR 1-5 $F(5, n-k-5)$   | 0.375          | 0.164          | 1.656          | 1.131          | 1.955          | 0.586          |
| ARCH $F(4, n-k-8)$   | 1.669          | 0.900          | 0.651          | 0.251          | 1.112          | 0.792          |
| Normality $\chi^2(2)$  | 1.016          | 0.448          | 2.732          | 1.299          | 1.267          | 0.247          |
| $X_t^2 F(2k, n-k-2k-1)$  | 0.292          | 0.306          | 0.555          | 0.754          | 1.757          | 1.785          |
| RESET $F(1, n-k-1)$  | 0.322          | 0.057          | 1.264          | 0.018          | 0.451          | 0.656          |
| Chow $F(4, n-k-4)$   | 0.182          | 0.447          | 0.216          | 0.374          | 0.966          | 0.535          |
| Chow $F(14, n-k-14)$   | 0.524          | 0.742          | 0.507          | 0.513          | 0.410          | 0.719          |
| Chow $F(30, n-k-30)$   | 0.485          | 0.737          | 0.433          | 0.512          | 0.866          | 0.768          |
| $\alpha_2^1 = \alpha_2^2 F(1, n-k)$                                  | 1.141          | 4.781*         | 0.830          | 12.430**       | 0.847          | 3.460          |
| <i>Estimates of Symmetric Equation 5b (<math>\Delta y_t</math>)</i>  |                |                |                |                |                |                |
| $\varepsilon_{t-1}$  | -0.185 (-1.15) | 0.925 (2.22)   | -0.091 (-0.69) | 0.259 (3.56)   | 0.003 (0.02)   | -0.261 (-1.54) |

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## Appendix

**Table A1. Augmented Dickey-Fuller Tests for Unit Roots**  
Unemployment and real GDP, Selected OECD Economies, Seasonally Adjusted

| Variable                 | Test Statistic |          |            |         |
|--------------------------|----------------|----------|------------|---------|
|                          | Lag Length     | $\tau_r$ | $\tau_\mu$ | $\tau$  |
| <i>Australia</i>         |                |          |            |         |
| Unemployment rate, $u_t$ | 2              | -3.24    | -2.14      | -0.31   |
| Real GDP, $y_t$          | 3              | -2.88    | 0.28       | 3.33    |
| $\Delta u_t$             | 3              | -3.72*   | -3.70**    | -3.71** |
| $\Delta y_t$             | 3              | -4.69**  | -4.66**    | -2.44*  |
| <i>Canada</i>            |                |          |            |         |
| Unemployment rate, $u_t$ | 5              | -2.94    | -2.48      | -0.27   |
| Real GDP, $y_t$          | 1              | -2.39    | -0.25      | 2.77    |
| $\Delta u_t$             | 3              | -4.25**  | -4.08**    | -4.11** |
| $\Delta y_t$             | 1              | -4.54**  | -4.57**    | -3.34** |
| <i>Japan</i>             |                |          |            |         |
| Unemployment rate, $u_t$ | 3              | -0.16    | 0.67       | 1.60    |
| Real GDP, $y_t$          | 3              | -0.20    | -1.49      | 1.32    |
| $\Delta u_t$             | 1              | -4.98**  | -4.69**    | -4.33** |
| $\Delta y_t$             | 1              | -4.74**  | -4.28**    | -2.97** |
| <i>New Zealand</i>       |                |          |            |         |
| Unemployment rate, $u_t$ | 3              | -1.85    | -1.87      | -0.10   |
| Real GDP, $y_t$          | 3              | -1.92    | -0.22      | 2.45    |
| $\Delta u_t$             | 3              | -3.51*   | -3.40*     | -3.31** |
| $\Delta y_t$             | 2              | -4.82**  | -4.85**    | -3.71** |
| <i>United Kingdom</i>    |                |          |            |         |
| Unemployment rate, $u_t$ | 2              | -4.42**  | -3.85**    | -0.51   |
| Real GDP, $y_t$          | 3              | -3.13    | -0.38      | 1.92    |
| $\Delta u_t$             | 3              | -3.17    | -2.70*     | -2.71** |
| $\Delta y_t$             | 2              | -2.69    | -2.93*     | -1.99*  |
| <i>United States</i>     |                |          |            |         |
| Unemployment rate, $u_t$ | 2              | -3.21    | -1.82      | -0.66   |
| Real GDP, $y_t$          | 1              | -2.36    | 0.66       | 3.87    |
| $\Delta u_t$             | 3              | -3.96*   | -3.82**    | -3.83** |
| $\Delta y_t$             | 1              | -4.58**  | -4.49**    | -2.91** |
| <i>Germany</i>           |                |          |            |         |
| Unemployment rate, $u_t$ | 5              | -3.06    | -2.50      | 0.11    |
| Real GDP, $y_t$          | 4              | -2.40    | 0.32       | 2.75    |
| $\Delta u_t$             | 3              | -3.60*   | -3.57**    | -3.45** |
| $\Delta y_t$             | 3              | -4.00*   | -3.96**    | -2.73** |

Rejects the null hypothesis at \*\* 1 per cent and \* 5 percent levels, respectively.

**Table A2. Recursive, Rolling and Sequential Augmented Dickey-Fuller Tests for Unit Roots**  
Unemployment and Real GDP, Selected OECD Economies, Seasonally Adjusted

| Variable          | Recursive       | Rolling         | Mean-shift statistics |         | Trend-shift statistics |         |
|-------------------|-----------------|-----------------|-----------------------|---------|------------------------|---------|
|                   | min $\tau_\tau$ | min $\tau_\tau$ | Min $\tau_\tau$       | max $F$ | min $\tau_\tau$        | max $F$ |
| Australia         |                 |                 |                       |         |                        |         |
| $u_t$             | -3.27           | -3.30           | -2.20                 | 1.88    | -2.22                  | 1.05    |
| $y_t$             | -2.48           | -2.61           | -1.94                 | 2.57    | -1.74                  | 1.70    |
| Canada            |                 |                 |                       |         |                        |         |
| $u_t$             | -2.46           | -3.76           | -1.67                 | 2.13    | -1.64                  | 2.73    |
| $y_t$             | -2.28           | -3.66           | -2.01                 | 4.46    | -2.08                  | 4.05    |
| Japan             |                 |                 |                       |         |                        |         |
| $u_t$             | -1.76           | -2.17           | -0.95                 | 8.70    | -0.54                  | 3.41    |
| $y_t$             | -1.40           | -2.19           | -0.50                 | 3.05    | -0.77                  | 3.60    |
| New Zealand       |                 |                 |                       |         |                        |         |
| $u_t$             | -2.09           | -2.74           | -1.83                 | 4.48    | -2.11                  | 4.83    |
| $y_t$             | -1.87           | -3.15           | -1.52                 | 5.11    | -1.77                  | 4.59    |
| United Kingdom    |                 |                 |                       |         |                        |         |
| $u_t$             | -3.33           | -5.24*          | -2.40                 | 0.97    | -2.43                  | 2.11    |
| $y_t$             | -2.32           | -3.58           | -2.03                 | 5.42    | -1.99                  | 4.31    |
| USA               |                 |                 |                       |         |                        |         |
| $u_t$             | -2.84           | -2.81           | -1.95                 | 2.00    | -1.96                  | 1.11    |
| $y_t$             | -2.64           | -4.57           | -2.12                 | 4.11    | -2.01                  | 2.99    |
| Germany           |                 |                 |                       |         |                        |         |
| $u_t$             | -2.77           | -3.48           | -1.93                 | 2.56    | -1.99                  | 2.39    |
| $y_t$             | -1.85           | -3.19           | -1.68                 | 4.21    | -1.42                  | 3.39    |
| 5% critical value | -4.33           | -5.01           | -4.80                 | 18.62   | -4.48                  | 16.30   |

### Testing for Unit Roots in $u_t$ and $y_t$

Standard ADF-tests for unit roots are reported in Table A.1, based on the sequential testing procedure outlined in Perron (1988) which tests during the period under consideration is often mistaken by conventional ADF-tests as a persistent down from the drift plus trend model to the no drift, no trend model. The results indicate that both unemployment and output are non-stationary  $I(1)$  series.

Perron (1989) showed that a stationary series around a deterministic time trend that undergoes a permanent shift innovation to a stochastic trend. Thus the recursive, rolling and sequential approaches developed by Banerjee, Lumsdaine and Stock (1992) are used to test for unknown shifts in the trend and/or intercept in the ADF-test. The results are reported in Table A.2, and these show that even after allowing for structural breaks in the series,  $u_t$  and  $y_t$  are  $I(1)$ .

### Appendix References

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