The Economic Effects of Restrictions on Government Budget Deficits: Imperfect Private Credit Markets

Christian Ghiglino
Department of Economics
Queen Mary, University of London
Mile End Road, London E1 4NS UK
c.ghiglino@qmul.ac.uk

Karl Shell
Department of Economics
402 Uris Hall
Cornell University
Ithaca, NY 14853-7601 USA
ks22@cornell.edu

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Abstract and Headnote

We consider a pure-exchange overlapping-generations model with many consumers per generation and many goods per period. As in Ghiglino and Shell (2000), there is a government that collects taxes, distributes transfers and faces budget deficit restrictions. We introduce, for realism and symmetry with the government, imperfection in the private credit markets. We find that with constraints on individual credit and anonymous (i.e., non-personalized) lump-sum taxes, strong (or "global") irrelevance of the government budget deficit is not possible, and weak irrelevance can hold only in very special situations. With credit constraints and anonymous consumption taxes, weak irrelevance holds provided the number of tax instruments is sufficiently large and at least one consumer's credit constraint is not binding. *Journal of Economic Literature* Classification Numbers: D51, E62, E52, H62, H63, O23.

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Correspondent: Christian Ghiglino, Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS UK. E-mail: c.ghiglino@qmul.ac.uk; telephone: +44-20-78827809; fax: +44-20-89833580.

Running Head: Credit Rationing and Deficit Irrelevance

1. Introduction and summary

In Ghiglino and Shell (2000), we analyzed the economic effects of constitutional or other restrictions on the government budget deficit. We assumed that private agents have access to perfect markets for borrowing and lending. This is a non-trivial assumption. If government borrowing is restricted but private borrowing is unconstrained, then the government can ease the effects of its own borrowing restrictions by in effect "borrowing off the books" by increasing the early-life taxes on some individuals while at least partially offsetting this by increasing late-life subsidies to the same individuals. In the real world, some consumers do face binding credit constraints or other imperfections in the borrowing market. It is natural to ask how these affect the government's ability to avoid restrictions on its deficit.

In the present paper, we assume that the government and individuals face credit restrictions. The restrictions on the government are from the constitution or other law, or from international borrowing agreements. The reasons for private credit constraints include imperfect collateral and other "moral hazards". The sources of private credit rationing will not be analyzed here. We simply assume that there are exogenously given private credit constraints which possibly differ across individuals.

Following Ghiglino and Shell (2000), we adopt a pure-exchange overlapping-generations model with several consumers per generation and several commodities per period. We allow for non-distorting lump-sum taxes and distorting consumption taxes. We also allow for the fact that tax schedules cannot be made perfectly individual-specific. In this paper, we focus for simplicity on the perfectly anonymous case in which each consumer from the same generation faces the same tax situation.

We use – with apology – the classic economic definitions [1] of *relevance* and *irrelevance* applied in this case to government-budget-deficit restrictions. The government-budget-deficit restriction is said to be *irrelevant* if the set of achievable allocations is unaffected by the restriction. Otherwise, the restriction is said to be *relevant*. Of course, saying that the restriction is irrelevant is not saying that the restriction does not matter. If the restriction either directly or indirectly affects expectations in such a manner that it affects the selection of the equilibrium, then the restriction does matter.

If private credit is unconstrained, and there are lump-sum taxes, then budget deficit restrictions are (globally) irrelevant [2]. If private credit is constrained but these constraints are not binding on any individual and the only tax instruments are anonymous lump-sum taxes, then we have weak (local) irrelevance of the government budget restrictions. If some credit constraints are binding, then with only anonymous lump-sum taxes even local irrelevance is unlikely or impossible.

If there are private credit constraints, the case with only consumption taxes is more interesting. Surprisingly, consumption taxes, although distorting, are more likely to pro-

[1] See Barro (1974). See also Ghiglino and Shell (2000) and the references therein.

[2] See Ghiglino and Shell (2000), Proposition 5.

vide (at least some form of) irrelevance than lump-sum taxes. With credit constraints and anonymous consumption taxes, there is weak (local) irrelevance if the number of tax instruments is sufficiently large and at least one consumer's credit constraint is not binding. This generalizes the result for the case with no private credit constraints [3]. In particular, we show that if the number of commodities is no less than the sum of the number of individuals *plus* the number of individuals for whom credit rationing is binding, then the deficit restriction is weakly (locally) irrelevant [4].

Why is weak (i.e. local) budget-deficit irrelevance more likely with anonymous consumption taxes than with anonymous lump-sum taxes? Their advantage to the government is that they can be used to transfer income from one consumer to another in the same generation, even when tax rates are anonymous. Such transfers are impossible with only anonymous lump-sum taxation.

2. The model

We employ a pure-exchange overlapping-generations model in which there are n different consumers per generation and ℓ perishable commodities per period. We suppose without loss of generality that consumers live for two periods. The government collects taxes, distributes transfers (negative taxes), and finances government consumption. We focus on two types of government instruments: (non-distorting) lump-sum taxes and (distorting) consumption taxes. We assume that lump-sum taxes and consumption tax rates must be the same for every member of a given generation, but that consumption taxes can vary freely over the l commodities. For the general case, see Ghiglino and Shell (2000), which allows for more general consumer tax classes and more general commodity tax classes. We assume that government consumption of commodities is exogenously determined. It is denoted by the sequence $g = (g^1, ..., g^t, ...)$ with $g^t \in \mathbb{R}^l_+$ for $t = 1, 2, ...$ It is assumed that use of capital markets is constrained, viz. each individual faces exogenously given constraints on his borrowing.

Our set-up is based on the Samuelson (1958) overlapping-generations model presented in Balasko and Shell (1980, 1981, 1986), but new tax instruments and the individual credit constraints must be defined. As in Balasko and Shell (1981), let $m^s_{th} \in \mathbb{R}$ be the lump-sum money transfer to consumer h of generation t in period s; if m^s_{th} is negative, then the consumer is paying a lump-sum tax. Following Ghiglino and Shell (2000), we let $\tau^{si}_{th} \in \mathbb{R}$ be the present (or nominal) tax rate levied on consumer h of generation t on his consumption of commodity i in period s.

Let $x^s_{th} = (x^{s1}_{th}, ..., x^{si}_{th}, ..., x^{sl}_{th}) \in \mathbb{R}^\ell_{++}$ be the vector of consumption in period s by indi-

[3] See Ghiglino and Shell (2000), Proposition 12.

[4] If none of the private credit constraints are binding, this inequality reduces (as it should) to the one in Proposition 12 of Ghiglino and Shell (2000)

vidual h of generation t and $\omega_{th}^{s} = (\omega_{th}^{s1}, ..., \omega_{th}^{si}, ..., w_{th}^{s\ell}) \in \mathbb{R}_{++}^{\ell}$ be the vector of commodity endowments in period s of individual h from generation t for $t = 0, 1, \ldots, s = 1, 2, \ldots$, and $h = 1, ..., n$. Let $m_t^s \in \mathbb{R}$ be the money transfer in period s to each consumer from generation t, and $\tau_t^s = (\tau_t^{s1}, ..., \tau_t^{si}, ..., \tau_t^{sl}) \in \mathbb{R}^{\ell}$ be the vector of anonymous consumption tax rates in period s for consumers from generation t. Consumers from generation 0 are alive in period 1, while consumers from generation t ($t = 1, 2, \ldots,$) are alive in periods t and $t+1$. Hence it is convenient to define the following vectors:

$$x_{0h} = x_{0h}^{1} \in \mathbb{R}_{++}^{\ell}, \quad x_{th} = (x_{th}^{t}, x_{th}^{t+1}) \in \mathbb{R}_{++}^{2\ell},$$

$$\omega_{0h} = \omega_{0h}^{1} \in \mathbb{R}_{++}^{\ell}, \quad \omega_{th} = (\omega_{th}^{t}, \omega_{th}^{t+1}) \in \mathbb{R}_{++}^{2\ell},$$

$$m_0 = m_0^1 \in \mathbb{R}, \qquad m_t = (m_t^t, m_t^{t+1}) \in \mathbb{R}^2,$$

and

$$\tau_0 = \tau_0^1 \in \mathbb{R}^{\ell}, \qquad \tau_t = (\tau_t^t, \tau_t^{t+1}) \in \mathbb{R}^{2\ell}.$$

Let $p^s = (p^{s1}, ..., p^{si}, ..., p^{s\ell}) \in \mathbb{R}_{++}^{\ell}$ be the vector of present (before-tax) prices for commodities available in period s and let

$$q_t^s = (q_t^{s1}, ..., q_t^{si}, ..., q_t^{s\ell}) \in \mathbb{R}_{++}^{\ell}$$

be the present after-tax vector of commodity prices facing consumers of generation t in period s. Define the after-tax present price vectors facing consumers of generation $t = 0, 1, 2, \cdots$ by

$$\begin{aligned} q_0 \quad &= \quad q_0^1 = p^1 + \tau_0 \in R_{++}^{\ell} \text{ for } t = 0 \\ &\quad \text{and} \\ q_t \quad &= \quad (q_t^t, q_t^{t+h}) = (p^t, p^{t+1}) + (\tau_t^t, \tau_t^{t+1}) \in R_{++}^{2\ell} \text{ for } t = 1, 2, ... \end{aligned} \tag{2.1}$$

Then define the following quantity and price sequences: $x = ((x_{0h})_{h=1}^{h=n}, ..., (x_{th})_{h=1}^{h=n}, ...)$, $\omega = ((\omega_{0h})_{h=1}^{h=n}, ..., (\omega_{th})_{h=1}^{h=n}, ...)$, $p = (p^1, ..., p^t, ...)$, $m = (m_0, ..., m_t, ...)$, $\tau = (\tau_0, ..., \tau_t, ...)$ and $q = (q_0, ..., q_t, ...)$.

We assume that the preferences of consumer h from generation t can be described by the utility function u_{th} defined over the consumption set of all strictly positive x_t's (i.e. $\mathbb{R}_{++}^{\ell}$ or $\mathbb{R}_{++}^{2\ell}$) with the properties:

(i) u_{th} is twice differentiable with strictly positive first-order derivatives and with corresponding negative definite Hessian

and

(ii) the closure of every indifference surface of u_{th} is in the consumption set (i.e. $\mathbb{R}^{\ell}_{++}$ or $R^{2\ell}_{++}$).

These rather standard assumptions simplify the comparative statics [5]. Note that we have also assumed that the endowment of the consumer lies in his consumption set, i.e. we have ω_{th} is in $\mathbb{R}^{\ell}_{++}$ or $\mathbb{R}^{2\ell}_{++}$.

Let $b^s_{th} \in \mathbb{R}_+$ be the maximum credit available in money units in period s to consumer h from generation t. The behavior of consumer h ($h = 1, 2, ..., n$) from generation t ($t = 1, 2, ...$) is then described by

$$\text{maximize } u_{th}(x^t_{th}, x^{t+1}_{th})$$

$$\text{subject to}$$

$$\begin{aligned} q^t_{th} \cdot x^t_{th} + x^{tm}_{th} &= p^t \cdot \omega^t_{th} + m^t_{th}, \\ q^{t+1}_{th} \cdot x^{t+1}_{th} + x^{t+1,m}_{th} &= p^{t+1} \cdot \omega^{t+1}_{th} + m^{t+1}_{th}, \\ x^{tm}_{th} &\geq -b^t_{th}, \\ &\text{and} \\ x^{tm}_{th} + x^{t+1,m}_{th} &= 0, \end{aligned} \tag{2.2}$$

where $x^{sm}_{th} \in \mathbb{R}$ is the gross money holding in period s by consumer h of generation t. The last equation in (2.2) is the requirement that the consumer's indebtedness be zero in his final period of life. The borrowing constraint is not binding on consumer h if in equilibrium $x^{tm}_{th} > -b^t_{th}$. The inequality in (2.2) is the credit constraint. We have implicitly assumed in writing (2.2) that the borrowing constraint of at least one consumer is not binding, so that we can use the usual no-arbitrage argument to establish that the present price of money is constant, i.e.,

$$p^{t,m} = p^{t+1,m} = p^m \in \mathbb{R}_+ \tag{2.3}$$

[5] See Balasko (1988). See Balasko and Shell (1980, 1981) for their application in overlapping-generations models.

where $p^{s,m} \in \mathbb{R}_+$ is the present price of money in period $s = 1, 2, \cdots$. Assuming that the economy is in proper monetary equilibrium, we can without loss of generality set $p^m = 1$.

The nominal (coupon) rate of interest on money is assumed without loss of generality to be zero. Hence the only return on holding money is the capital gain relative to commodities. Condition (2.3) is thus that money appreciate in value relative to any commodity at the commodity rate of interest. For consumers for which the credit restriction is not binding, Condition (2.3) allows us to rewrite (2.2) somewhat as Balasko and Shell (1981) to yield

$$\begin{aligned} &\text{maximize } u_{th}(x_{th}^t, x_{th}^{t+1}) \\ &\text{subject to} \\ &q_t^t \cdot x_{th}^t + q_t^{t+1} \cdot x_{th}^{t+1} \\ &= p^t \cdot \omega_{th}^t + p^{t+1} \cdot \omega_{th}^{t+1} + m_t^t + m_t^{t+1} \end{aligned} \tag{2.4}$$

for $h = 1, 2, ..., n$ and $t = 1, 2, ...,$ where by choice of numeraire we set $q_{01}^1 = 1$. The transfers $m_t = (m_t^t, m_t^{t+1}) \in \mathbb{R}^2$ affect the behavior of the consumer only through the lifetime transfer $\mu_t = m_t^t + m_t^{t+1} \in \mathbb{R}$.

It remains to describe the behavior of the older generation ($t = 0$) in period 1. Consumer $0h$ maximizes his utility subject to his one-period budget constraint:

$$\begin{aligned} &\text{maximize } u_{0h}(x_{0h}^1) \\ &\text{subject to} \\ &q_0^1 \cdot x_{0h}^1 + x_{0h}^{1m} = p^1 \cdot \omega_{0h}^1 + m_0^1 \\ &x_{0h}^{1m} = 0, \\ &\text{and} \\ &x_{0h}^1 \in \mathbb{R}_{++}^{\ell}. \end{aligned} \tag{2.5}$$

3. Fiscal policy

We assume in this paper that the government has at its disposal either anonymous lump-sum taxation *or* anonymous consumption taxation. Thus, the government's fiscal policy is either the sequence of anonymous lump-sum transfers m or the sequence of the consumption tax rates τ.

Let d_t be the present (also the dollar) value of the government budget deficit in period t. Hence we have for the case of lump-sum taxation

$$d^t = p^t g^t + n\left(m_{t-1}^t + m_t^t\right)$$

for $t = 1, 2, ...$, where n is the number of consumers per generation. For the case of consumption taxes

$$d^t = p^t g^t - \sum_{h=1}^{n} \sum_{i=1}^{l} \left(\tau_{t-1}^{ti} x_{t-1,h}^{ti} + \tau_t^{ti} x_{th}^{ti}\right)$$

for $t = 1, 2, ...$ Let d denote the sequence $(d^1, ..., d^t, ...)$. Let δ^t be the present (and nominal) value of the constitutionally imposed deficit restriction (assumed for convenience in the form of an equality) in period t. Let δ denote the sequence $(\delta^1, ..., \delta^t, ...)$. The budget deficit restriction is then

$$d = \delta.$$

4. Equilibrium

We maintain throughout this paper some strong assumptions. We suppose perfect-foresight on the part of consumers and the government. We also suppose that the government is able to perfectly commit to its announced fiscal policy.

Next we define equilibrium in the economy with taxes.

Definition *Given the sequence of endowments ω, the feasible fiscal policy m or τ, the exogenous consumption g, the behavior of consumers described by the systems (2.2), (2.4) and (2.5), the numeraire choice yielding $p^{11}=1$, the (further) monetary normalization yielding $p^m = 1$ and the deficit-restriction sequence δ, a constitutional competitive equilibrium is defined by a positive price sequence p and the allocation sequence x such that markets clear, i.e. we have*

$$g^t + \sum_{h=1}^{h=n} (x_{t-1,h}^t + x_{t,h}^t) = \sum_{h=1}^{h=n} (\omega_{t-1,h}^t + \omega_{t,h}^t)$$

and the deficit restriction $d = \delta$ is satisfied.

From Balasko and Shell (1980), one might expect that the existence of competitive equilibrium to be guaranteed in "nice" overlapping-generation models, but this does not extend to our Definition. There are three reasons that competitive equilibrium as defined above could fail to exist. The first reason is because we are seeking a *proper* monetary equilibrium, one for which the price of money is strictly positive. For a proper monetary equilibrium to exist the fiscal policy must be bonafide[6]. The second reason applies only to commodity taxation. It might not be possible to equilibrate supply and demand while maintaining the positivity of the two price sequences p and q. The third reason is that equilibrium may fail to exist because of excessive government consumption.

When the model is stationary, i.e., preferences, endowments, and government consumption are constant across generations, one is tempted to focus on equilibria in which allocations are constant across periods. We provide separate definitions of the steady state for the two the tax regimes.

Definition L (Steady state with lump-sum taxes): *Let $p = (p^1, \cdots, p^t, \cdots) \in (\mathbb{R}^l_{++})^\infty$ be the equilibrium sequence of commodity prices when the fiscal policy is given by the sequence of lump-sum transfers $(m_0^1, m_1^1, m_1^2, \cdots, m_t^t, m_t^{t+1}, \cdots)$. These describe a steady-state equilibrium if there is a vector $\mathbf{p} \in \mathbb{R}^l_{++}$ and a scalar $\beta \in \mathbb{R}_{++}$ such that*

$$\begin{aligned} p^t &= \beta^{t-1}\mathbf{p} \\ m_t^t &= \beta^{t-1}\mathbf{m}^1 \end{aligned}$$

and

$$m_t^{t+1} = \beta^t \mathbf{m}^0$$

for $t = 1, 2, \cdots$, where $\mathbf{m}^1 = m_1^1$ and $\mathbf{m}^0 = m_0^1$.

Definition C (Steady state with consumption-taxes): *Let $p = (p^1, \cdots, p^t, \cdots) \in (\mathbb{R}^l_+)^\infty$ be an equilibrium vector of before-tax commodity prices when the fiscal policy is given by the sequence of consumption taxes $(\tau_0^1, \tau_1^1, \tau_1^2, \cdots, \tau_t^t, \tau_t^{t+1}, \cdots) \in (\mathbb{R}^l)^\infty$. These describe a steady-state equilibrium if there is a vector $\mathbf{p} \in \mathbb{R}^l_{++}$ and a scalar $\beta \in \mathbb{R}_{++}$ such that*

$$\begin{aligned} p^t &= \beta^{t-1}\mathbf{p} \\ \tau_t^t &= \beta^{t-1}\boldsymbol{\tau}^1 \end{aligned}$$

and

$$\tau_t^{t+1} = \beta^t \boldsymbol{\tau}^0$$

for $t = 1, 2, \cdots$, where $\boldsymbol{\tau}^1 = \tau_1^1$ and $\boldsymbol{\tau}^0 = \tau_0^1$.

When focusing on steady states it makes sense to focus on budget deficits $d = (d^1, \cdots, d^t, \cdots)$ that are constant in current terms, so that we have for a scalar $\mathbf{d} \in \mathbb{R}$

$$d^t = \beta^{t-1}\mathbf{d}$$

for $t = 1, 2, ...$ From Walras's law and market clearing, we have the two steady-state relations:

[6]See Balasko and Shell (1981, 1986, 1993) and Ghiglino and Shell (2000).

$$(\beta - 1)\left[\sum_{h=1}^{n}\left(\mathbf{p}\left(x_h^1 - \omega_h^1\right)\right) - n\mathbf{m}^1\right] + \mathbf{d} = 0 \tag{SS-L}$$

for lump-sum taxation, and

$$(\beta - 1)\sum_{h=1}^{n}\left[\mathbf{p}\left(x_h^1 - \omega_h^1\right) + \sum_{i=1}^{l}\boldsymbol{\tau}^{1i}x_h^{1i}\right] + \mathbf{d} = 0 \tag{SS-C}$$

for consumption taxation, where $\boldsymbol{\tau}^{1i} \in \mathbb{R}$ is the ith component of $\boldsymbol{\tau}^1$. The steady-state conditions (SS-L) and (SS-C) do not directly involve government consumption, but steady-state $\mathbf{g} = g^t$ for $t = 1, 2, \cdots$ is implied through the equilibrium allocations and prices. If $d = 0$, from (SS-L) and (SS-C), we have the familiar OG steady-state result that either the interest rate is zero ($\beta = 1$) or aggregate savings is zero. Our aim is to find conditions under which the government is able to "avoid" the restrictions on its deficit with changing neither its own consumption nor the consumption of any private consumer. When this is possible the deficit restriction is said to be *irrelevant*. We recall the formal definitions given in Ghiglino and Shell (2000).

Definition. Irrelevance of the deficit restriction. *Let g be government consumptions and let x be an allocation that can be implemented as a competitive equilibrium with some feasible fiscal policy m (resp. τ) and with the resulting deficits given by the sequence d. The deficit restriction $d = \delta$ is said to be irrelevant if for any other deficit restriction sequence δ' there exists a feasible fiscal policy m (resp. τ) that implements the allocation x as a competitive equilibrium and is compatible with g, but with the resulting deficit given by the sequence δ'.*

The above notion of irrelevance is very strong because it involves any possible deficit sequence other than the pre-reform, or baseline, deficit d. In many situations such an irrelevance fails to obtain.

We put forward a weaker notion of irrelevance. The first characteristic of the weaker deficit restriction is that it only applies to a finite number of periods. Define $\delta(T) = (\delta^1, \delta^2, \ldots, \delta^t, \ldots, \delta^T) \in \mathbb{R}^T$ as a deficit restriction of (finite) length T. For a competitive equilibrium to be weakly constitutionally feasible the deficit in period t, d^t, must be equal to δ^t if $t = 1, 2, \ldots, T$, but for $t > T$, the deficit is unrestricted. The second characteristic of the weaker deficit restriction is that only restrictions "near to" the base-line deficit are considered, i.e., only period-by-period deficits that are not too different from the baseline deficits are considered. In other words, only a neighborhood of the original sequence is considered (in any topology, since the sequence is finite). According to the weaker notion of irrelevance only restrictions of finite length $\delta(T)$ that belong to a first T-period neighborhood of the base deficit vector $d = (d^1, d^2, \ldots, d^t, \ldots, d^T, \ldots)$, denoted $\mathcal{D}^T(d)$, are considered.

Definition. Weak irrelevance of the deficit restriction *Let g be the government consumptions and let x be an allocation that can be implemented as a competitive* *equilibrium with some feasible fiscal policy m (resp. τ) and with the resulting deficits given by the sequence d. A deficit restriction is said to be weakly irrelevant if for any T there is a set $\mathcal{D}^T(d)$ such that for all $\delta \in \mathcal{D}^T(d)$ there is a fiscal policy m' (resp. τ') that implements the allocations x and g, but with the resulting deficit given by the sequence δ.*

Note that the time horizon of the deficit specification is arbitrary and may be any *finite* number T.

5. Relevance of government budget deficit restrictions with lump-sum taxes

The relevance of government deficit restrictions is first investigated in economies in which some consumers face credit constraints and only lump-sum taxation is available. It is shown that deficit restrictions are likely to be relevant unless the government can use non-anonymous taxes. In the leading example considered below, the government uses only anonymous lump-sum taxes and transfers and thus has no way to treat differently the consumers, so that the likely outcome is that some consumers are hurt or benefited by the fiscal scheme.

For general overlapping-generations economies with perfect borrowing markets and lump-sum taxes and transfers, restrictions on the government budget have no impact on the set of equilibrium allocations (see Ghiglino and Shell (2000), Proposition 5). The reason for this is that in these economies only the present value of taxes and transfers, not their timing, matters to consumers. In this case, the government can "borrow off the books" from taxpayers by adjusting the timing of individual taxes and transfers.

When credit restrictions are included, weak (or local) irrelevance of the budget deficit restriction obtains if non-anonymous taxes can be personalized to some consumer whose constraint is not binding. Being able to personalize taxes is not always possible. If this is not possible, then matters dramatically change. This is illustrated in the following example. For simplicity, a stationary equilibrium is considered. This amounts to ignoring the transition path. In other words, in this example we will assume that a suitable money transfer is made so that the economy "starts" at the steady state and only deficit specifications from $t = 2$ onward are considered[7].

Example (Relevance of government deficit restrictions when consumer credit is constrained): Let the economy be stationary with one commodity per period and two consumers. Perfectly anonymous lump-sum taxation is available. No other tax instruments are available. The two consumers, 1 and 2, have log-linear utility functions

[7]Another, equivalent, way to view steady states is to consider a model with no beginning as well as no end (see Ghiglino and Tvede (1995)).

$$u_{th} = 1/2 \log x_{th}^{t} + 1/2 \log x_{th}^{t+1}$$

for $h = 1$, 2 and $t = 1, 2, \ldots$. Endowments are given by

$$\omega_{t1} = (\omega_{t1}^{t}, \omega_{t1}^{t+1}) = (1, 20),$$

and

$$\omega_{t2} = (\omega_{t2}^{t}, \omega_{t2}^{t+1}) = (0.75, 1).$$

Consider generation t. Suppose that the borrowing of consumer 1 is unconstrained, $b_{t1}^{t} = \infty$, but that consumer 2 cannot borrow, $b_{t2}^{t} = 0$. At a steady state the individual demands of the consumers depend on the interest factor β. When the credit constraint is not binding, the demands must satisfy

$$x_{t,1}^{t} = \frac{1 + 20\beta}{2},$$

$$x_{t1}^{t+1} = \frac{1 + 20\beta}{2\beta},$$

$$x_{t,2}^{t} = \frac{0.75 + \beta}{2},$$

and

$$x_{t,2}^{t+1} = \frac{0.75 + \beta}{2\beta}.$$

For $\beta \geq 0.75$, we have

$$\left(x_{t,2}^{t}, x_{t,2}^{t+1}\right) = (0.75, 1)$$

because then the credit restriction is binding.

Suppose that $g^{t} = 1$ for $t = 1, 2, \ldots$ Without credit restrictions $\beta = 0.89498$ and $\beta = 0.093112$ solve the equilibrium equations, but $\beta = 0.89498$ is not an equilibrium interest factor because the borrowing constraint for consumer 2 is violated. The steady-state equilibrium interest factor is $\beta = 0.89408$ and the corresponding equilibrium allocations are

$$x_{t1} = (x_{t1}^{t}, x_{t1}^{t+1}) = (9.4408, 10.5592)$$

and

$$x_{t2} \;=\; (x_{t2}^{t}, x_{t2}^{t+1}) = (0.75, 1).$$

Since the government is not taxing any consumer, the associated deficit is $d^t = p^t g^t = p^t = (0.89408)^{t-1}$ in present or nominal units.

Suppose now that the government is required to balance its budget in every period, so that $d^t = \delta^t = 0$ for $t = 2, 3, \ldots$. We will show that the new restriction on the deficits leads to a modification of the existing allocation. First, note that in order to keep unchanged the consumption of consumer 1, β should be unchanged at $\beta = 0.89408$. Now, the government can either tax the young or tax the old. Suppose first that consumers are taxed in their youth and receive transfers in their old age. The procedure is similar to that used in the proof of Proposition 5 in Ghiglino and Shell (2000). Since the consumers of the same generation are perfectly anonymous for tax purposes, suppose that we tax each young equally with a lump-sum tax $-m_2^2 > 0$ and no tax on the consumers born in the first period, $m_1^2 = 0$. The government budget constraint is then $\beta^{t-1}g^t + 2m_2^2 = 0$ so that $m_{21}^2 = m_{22}^2 = -\beta/2$ (or 1/2 in current terms) in order that the deficit be zero, $d^2 = 0$. Note in the next period, these same consumers have to be compensated by a positive transfer of $\beta/2$ in present terms, or $1/(2\beta)$ in current terms. After the transfer, the endowments (in current terms) of consumer 2 are $(0.75 - 0.5, 1 + 0.5\beta)$. At $\beta = 0.89408$, consumer 2 would still like to borrow. However, due to the borrowing constraint his first period consumption is now 0.25. The equilibrium allocation has been affected by the fiscal policy. The other possibility is to subsidize the young and tax the old. A similar reasoning shows that also in this case the fiscal policy affects the equilibrium allocation. Therefore, the deficit sequence is relevant. □

The previous example shows that when some consumers are credit-constrained, anonymous lump-sum taxes are not powerful enough to achieve irrelevance of the government budget deficit. This is generalized in the following

Proposition (Relevance of the government deficit restrictions with credit constraints): Let the allocation x be implemented as a constitutional competitive equilibrium with a fiscal policy consisting only of lump-sum taxes and transfers compatible with the deficit restriction δ. If at least one consumer's credit constraint is binding then the deficit sequence δ is *weakly* (and strongly) *relevant.* Otherwise, it is *weakly irrelevant.*

Proof: If no consumer's credit constraint is binding, then Proposition 5 in Ghiglino and Shell (2000) applies. However, in general as the government only employs anonymous taxes any transfer changes his actual borrowings (or savings) and therefore affects his demand for commodities. Indeed, assume there are two consumers and that $h = 2$ is the consumer whose credit constraint is binding (if the credit constraint of consumer 1 is also binding then the deficit restriction is obviously relevant). Consider consumer 2 first.

His demands are the solutions to the problem

$$\text{maximize } u_{2t}(x_{2h}^{t}, x_{2h}^{t+1})$$

$$\text{subject to}$$

$$\begin{aligned} p^{t} \cdot x_{2h}^{t} + x_{t2}^{tm} &= p^{t} \cdot \omega_{t2}^{t} + m_{t}^{t}, \\ p^{t+1} \cdot x_{t2}^{t+1} + x_{t2}^{t+1,m} &= p^{t+1} \cdot \omega_{t2}^{t+1} + m_{t}^{t+1}, \qquad (5.1) \\ x_{t2}^{tm} &= -b_{t2}^{t}, \end{aligned}$$

$$\text{and}$$

$$x_{t2}^{tm} + x_{t2}^{t+1,m} = 0.$$

Writing down the first-order conditions, it is clear that the allocation is unchanged provided the normalized prices and normalized first and second period incomes are unchanged by the new fiscal policy. This implies that both p^{t}/p^{t1} and $p^{t+1}/p^{t+1,1}$ are unaffected by the policy change. The similar condition for the incomes implies that

$$-b_{t2}^{m} + m_{t}^{t} = p^{t,1} W_{t} \quad \text{and} \quad -b_{t2}^{m} + m_{t}^{t+1} = p^{t,1} W'_{t+1}$$

where $W_{t}, W'_{t+1} \in R^{l}$ are unaffected by the policy change. On the other hand, the government budget deficit is

$$d_{t} = m_{t}^{t} + m_{t-1}^{t} + p^{t,1} g^{t}$$

Then, we obtain

$$\delta_{t} = d^{t} = p^{t,1} W_{t} + b_{2}^{t} + p^{t,1} W'_{t-1} - b_{2}^{t-1} + p^{t,1} g^{t}$$

which yields

$$\delta^{t} = p^{t,1}(W_{t} + W'_{t-1}) + b_{2}^{t} - b_{2}^{t-1} + p^{t,1} g^{t}$$

However, in order to keep consumer 1 unaffected by the fiscal policy, $p^{t+1,1}/p^{t,1}$ should also be kept constant and, as $p^{1,1} = 1$, the entire sequence of prices should remain unchanged. As a result, the deficit sequence δ is entirely predetermined. □

Remark: If the government were able to use personalized lump-sum taxes, then the deficit sequence δ would be *weakly irrelevant.* Indeed, renumber the consumers so that consumer *1* is the unconstrained consumer and reproduce the proof of Proposition 5 in Ghiglino and Shell (2000).

6. Restoring irrelevance with consumption taxes

In this section we assume that only anonymous taxes on consumption are available. The question is then whether the government is able to "avoid" the deficit restriction with these instruments even though some consumers are credit constrained. As in the case with unconstrained borrowing and lending, the answer depends on the number of tax instruments compared to the number of goals (consumers) and on the duration (in periods) of the restriction. We start our analysis by an example.

Example (Irrelevance of deficit restrictions in an economy with several tax instruments): Consider a stationary, overlapping-generations economy with four commodities per period ($\ell = 4$) and two consumers per generation ($n = 2$). Assume that the second consumer faces credit restrictions while the other has free access to the credit market. The government has a constant consumption in the first good only, $g^t = (g^{t1}, g^{t2}, g^{t3}, g^{t4}) = (3, 0, 0, 0)$. Preferences and endowments of consumer h are given by:

$$u_{th}(x_{th}^{t}, x_{th}^{t+1}) = \sum_{k=1}^{4} \alpha_{kh} \log x_{th}^{tk} + \sum_{k=1}^{4} \beta_{kh} \log x_{th}^{t+1k}$$

and

$$\omega_{th} = (\omega_h^{0k}, \omega_h^{1k})_{k=1}^{4}$$

with

$$\begin{aligned} \omega_1^{01} &= 300, \omega_1^{13} = \omega_1^{14} = \omega_2^{13} = 200, \omega_2^{14} = 230, \omega_2^{12} = 120, \omega_2^{01} = 250, \\ \omega_1^{12} &= 500,\ \omega_2^{02} = 1000,\ \text{and } \omega_i^{jk} = 100 \text{ for all other } h, j, k \end{aligned}$$

$$(\alpha_{kh})_{h=1,2,}^{k=1,\dots,4} = \begin{bmatrix} 1/8 & 5/8 & 1/8 & 1/8 \\ 1/8 & 4/7 & 1/7 & 9/56 \end{bmatrix},\ (\beta_{kh})_{h=1,2}^{k=1,\dots,4} = \begin{bmatrix} 1/4 & 1/4 & 1/5 & 6/20 \\ 1/5 & 1/4 & 1/4 & 6/20 \end{bmatrix}.$$

With anonymous consumption taxes we have

$$\tau_{t-1,1}^{tk} = \tau_{t-1,2}^{tk} = \tau_{t-1}^{tk} \text{ and } \tau_{t1}^{tk} = \tau_{t2}^{tk} = \tau_{t}^{tk} \text{ for } k = 1,..,4 \text{ and } t = 1, 2, ...$$

For convenience, we look at steady-state competitive equilibrium. As noted earlier, we will only consider the periods from $t = 2$ onward. First, we assume that the government finances its consumption by running a deficit, i.e. we look at a steady state with $\tau_t^{tk} = 0$ and $\tau_{t-1}^{tk} = 0$. By restricting our attention to prices of the form $p^{tk} = (\beta)^{t-1}\mathbf{p}^k$, $k = 1, ..., 4$, it can be shown that the following set of allocations and prices represents a steady state:

$$\mathbf{p}^2 = 1.06975, \mathbf{p}^3 = 1.30350, \mathbf{p}^4 = 1.51932, \beta = 1.03351$$

and

$$
\begin{aligned}
(x_1^{0k})_{k=1}^4 &= (120.5557, 563.4745, 92.4864, 79.3485)\\
(x_1^{1k})_{k=1}^4 &= (233.2935, 218.0816, 143.1801, 184.2615)\\
(x_2^{0k})_{k=1}^4 &= (154.2905, 659.3371, 135.2762, 130.5673)\\
(x_2^{1k})_{k=1}^4 &= (238.8603, 279.1068, 229.0573, 235.8228).
\end{aligned}
$$

The associated deficit is $p^{t1} = (\beta)^{t-1}\mathbf{p}^{1} = (\beta)^{t-1} = (1.03351)^{t-1}$ in present and nominal terms. In current units, the savings are -275.1889 for consumer 1, 367.7109 for consumer 2 producing an aggregate savings of 92.5221.

The issue is whether $(\tau_{t-1}^{t}, \tau_t^{t})_{k=1}^4$ can be used in order to meet the deficit requirement $\delta^t = 0$ in period t without disturbing the allocations given previously. Such a tax scheme must at least satisfy for each t ($t = 2, 3, \dots$) the following equations

$$
\begin{aligned}
x_{t-1,1}^{tk} &= (1-\alpha_1)\frac{\beta_{k1}W_{t-11}}{p^{tk} + \tau_{t-1}^{tk}} = x_1^{0k},\\
x_{t1}^{tk} &= \alpha_1\frac{\alpha_{k1}W_{t1}}{p^{tk} + \tau_t^{tk}} = x_1^{1k},\\
x_{t-1,2}^{tk} &= (1-\alpha_2)\frac{\beta_{k2}W_{t-12}}{p^{tk} + \tau_{t-1}^{tk}} = x_2^{0k},\\
&\text{and}\\
x_{t2}^{tk} &= \alpha_2\frac{\alpha_{k2}W_{t2}}{p^{tk} + \tau_t^{tk}} = x_2^{1k},
\end{aligned}
\tag{6.1}
$$

where

$$
W_{th} = \sum_{k=1}^{4} p^{tk}\omega_h^{0k} + \sum_{k=1}^{4} p^{t+1k}\omega_h^{1k}
$$

and

$$
\sum_{k=1}^{4}(x_1^{1k} + x_2^{1k})\tau_{t-1}^{tk} + \sum_{k=1}^{4}(x_1^{0k} + x_2^{0k})\tau_t^{tk} + 3p^{t1} = 0.
$$

A natural candidate for a solution to the first four equations of (6.1) is of the form $p^{tk} = (\beta)^{t-1}\mathbf{p}^k$, $\tau_t^{tk} = (\beta)^{t-1}\boldsymbol{\tau}^{0k}$, and $\tau_{t-1}^{tk} = (\beta)^{t-1}\boldsymbol{\tau}^{1k}$, where $\beta \in \mathbb{R}$ is the interest factor, $\boldsymbol{\tau}^{0k} \in \mathbb{R}$ is the present and nominal value of the tax rate on the young and $\boldsymbol{\tau}^{1k} \in \mathbb{R}$ is the present and nominal value of the tax rate on the old.

In this example, consumer 2 is assumed to face a binding credit constraint while the government cannot personalize his taxes. Then it is required that his saving or borrowing should remain exactly as they were in the untaxed situation. The budget equation for this consumer when young is

$$\sum_{k=1}^{4}(p_t^k + \tau_{t-1}^{tk})x_2^{0k} - x_{t2}^m = \sum_{k=1}^{4} p_t^k \omega_2^{0k}.$$

In the initial situation, with no taxes, this yields at the steady state

$$\sum_{k=1}^{4}(\beta^{t-1}\mathbf{p}^k x_2^{0k}) - x_{t2}^m = \sum_{k=1}^{4} \beta^{t-1}\mathbf{p}^k \omega_2^{0k}$$

which can be rewritten as

$$\beta^{t-1}(\sum_{k=1}^{4}\mathbf{p}^k(x_2^{0k} - \omega_2^{0k})) = x_{t2}^m$$

or as

$$x_{t2}^m = \beta^{t-1} x_2^m.$$

We should point out that in the absence of taxes, a strictly positive government consumption is financed through a permanent deficit (see Equation (SS-C)) implying that both aggregate and individual savings are non-zero and β is different from unity.

Since we assume that consumer's 2 savings are not affected by the fiscal policy, if $\widehat{\beta}$ is the value of the interest factor obtained with taxes, we should have

$$\sum_{k=1}^{4}(\widehat{\beta}^{t-1}p^k + \widehat{\beta}^{t-1}\tau^{0k})x_2^{0k} - \widehat{\beta}^{t-1}p^k\omega_2^{0k} = \beta^{t-1}x_2^m \text{ for any } t.$$

These equations imply that $\widehat{\beta}$ should remain unaffected by the introduction of taxes, $\widehat{\beta} = \beta$. By replacing the demand functions, savings are unaltered for consumer 2 if

$$\frac{1}{\beta^{t-1}}(\alpha_2 \sum_{k=1}^{4}\alpha_{k2}W_{t2} - \sum_{k=1}^{4}\beta^{t-1}\mathbf{p}^k\omega_2^{0k}) = \alpha_2\sum_{k=1}^{4}(\mathbf{p}^k\omega_2^{0k} + \beta\mathbf{p}^k\omega_2^{1k}) - \sum_{k=1}^{4}\mathbf{p}^k\omega_2^{0k} = x_2^m \quad \text{(CS)}$$

In this example the total system is composed of 11 equations; 7 normalized prices, 2 normalized incomes, the government budget deficit equation and Equation (CS) concerning the individual borrowing/savings constraint. On the other hand, there are 3 prices

(β is fixed) and 8 taxes. Hence a solution to the set of equations is likely to exist. The following set of values represents the steady-state equilibrium:

$$\mathbf{p}^2 = 1.2399, \mathbf{p}^3 = 1.24845, \mathbf{p}^4 = 2.64413$$

and

$$(t_1^{0k})_{k=1}^4 = (t_1^{1k})_{k=1}^4 = (0.22449, 0.07003, 0.3477, -0.7837).$$

Finally, the saving is now -367.7109 for consumer 1 and 367.7109 for consumer 2, so that aggregate savings are zero. This last result agrees with Equation (SS-C) as in the new situation a balanced budget ($\boldsymbol{\delta} = 0$) co-exists with β different from unity, implying that the aggregate savings are zero.□

The previous example illustrates the mechanism for irrelevance. Starting from a steady state with non-zero aggregate savings, there is a tax scheme which keeps the interest rate unchanged while achieving a zero government budget deficit. In the new situation, the government is taxing the consumers just enough to balance its budget. Moreover, this tax-transfer is such that aggregate, after tax, savings become zero. In this game, the change in the aggregate savings is completely done through the unrestricted consumer. A further important fact is that the taxes required to obtain irrelevance are age-anonymous in the sense that the tax to be applied on the young and on the old is the same.

The role played by the consumer whose credit constraint is not binding is crucial. Indeed, if all consumers would be kept at their initial levels of borrowing or saving, achieving irrelevance would be impossible. This is clearly seen by considering an economy consisting of only one consumer.

The budget constraint for generation $t-1$ consumer during his old age can be rearranged as

$$\tau_{t-1}^t x_{t-1}^t = -p_t x_{t-1}^t + p_t \omega_{t-1}^t - x_{t-1}^{tm},$$

while for the consumer in generation t, his first period constraint is

$$\tau_t^t x_t^t = -p_t x_t^t + p_t \omega_t^t - x_t^{tm}.$$

Adding the two, we obtain

$$\tau_t^t x_t^t + \tau_{t-1}^t x_{t-1}^t = p_t \left[\omega_t^t + \omega_{t-1}^t - x_t^t - x_{t-1}^t\right] - x_{t-1}^{tm} - x_t^{tm}.$$

Considering now the government budget deficit equation

$$\begin{aligned} d^t &= p^t g^t - \left[\tau_t^t x_t^t + \tau_{t-1}^t x_{t-1}^t\right] \\ &= p^t \left[g^t + x_t^t + x_{t-1}^t - \omega_t^t - \omega_{t-1}^t\right] + x_{t-1}^{tm} + x_t^{tm} \\ &= x_{t-1}^{tm} + x_t^{tm}, \end{aligned}$$

so that the deficit is completely determined by the aggregate borrowings and savings decisions of the consumers. If these are unaffected by the fiscal policy, the deficit will remain unchanged.

The example above shows that when consumption tax instruments are sufficiently diversified, irrelevance of deficit restrictions may hold. However, in general as was the case with no credit restrictions (see Ghiglino and Shell (2000), Example 11), having several tax instruments is not sufficient for irrelevance, which also depends on the length and the magnitude of the deficit restriction. Indeed, even when there are enough instruments, it is not assured that $q_{th}^{sk} - \tau_{th}^{sk}$ is positive, i.e. we could have for some s ($s = 1, 2, \ldots$) and some k ($k = 1, \ldots, \ell$) that $\mathbf{p}^{sk} < 0$. This would be consistent with the formal model, but is, of course, inconsistent with free disposal of endowments. As a consequence, the next proposition which generalizes the former example, gives only a *necessary* condition for irrelevance and a *sufficient* condition for *weak irrelevance.*

Proposition (Relevance of deficit restrictions in economies with consumption taxes and consumer credit restrictions): Suppose that only anonymous consumption taxes are available and that the credit constraint of at least one consumer is not binding. Let x be an allocation that can be implemented as a constitutional equilibrium with a fiscal policy and deficit restriction δ and let $r, 0 \leq r < n$, be the number of consumers for which the credit constraint is binding. Then, if $n + r > \ell$ the deficit restriction is *weakly* (and strongly) *relevant*. On the other hand, for $n + r \leq \ell$ the deficit restriction is *weakly irrelevant.*

Proof: When consumers are potentially credit constrained, demand for commodities may depend on the individual borrowings or lendings, so that these must be kept constant when the policy changes. Formally, x_{th}^{tm}, with $x_{th}^{tm} = p^t \cdot \omega_{th}^t - q_t^t \cdot x_{th}^t$, is kept constant for constrained consumers. Denote this quantity by $\overline{b}_{th}^t$. Furthermore, since there is some consumer whose credit constraint is not binding, prices in successive periods are linked. Therefore, in period t the relevant system consists of $2\ell - 1$ conditions on prices and $r + n$ conditions on individual wealths. Let the consumers whose credit constraint is binding be denoted by $h = 1, \cdots, r$ while the remaining $h = r + 1, \cdots, n$ have non-binding credit restrictions. Taking into account the restriction on the deficit, the system of $2\ell + r + n$ equations can be written as

$$\widehat{p}^t + \widehat{\tau}^t_t = (p^{t1} + \tau^{t1}_t) R^t_t,$$

$$p^{t+1} + \tau^{t+1}_t = (p^{t1} + \tau^{t1}_t) R^{t+1}_t,$$

$$p^t \cdot \omega^t_{th} - \overline{b}^t_{th} = (p^{t1} + \tau^{t1}_t) W^t_{t,h}, \quad h = 1, \cdots, r$$

$$p^t \cdot \omega^t_{th} + p^{t+1} \cdot \omega^{t+1}_{th} = (p^{t1} + \tau^{t1}_t) W_{th}, \quad h = 1, \cdots, n$$

and

$$\sum_{h=1}^{h=n} \sum_{i=1}^{i=\ell} \tau^{ti}_t f^{ti}_{th}(R^t_t, R^{t+1}_t, W_{th}) + \tau^{t,i}_{t-1} f^{t,i}_{t-1,h}(R^{t-1}_{t-1}, R^t_{t-1}, W_{t-1,h}) = -\delta^t,$$

for $i = 1, \ldots, \ell$, where the R's, the W's, and the δ's are fixed. Suppose that $n \leq \ell$. In the Appendix, it is shown that it is useful to consider as "free" variables the last $\ell - n$ prices of period t: $p^{t,n+1}, \ldots, p^{t\ell}$ and the first n prices of period $t+1$: $p^{t+1,1}, \ldots, p^{t+1,n}$. This system, which is linear in 3ℓ unknowns, has a solution if and only if $n + r \leq \ell$. The usual sign restrictions on the p's apply so this condition is not sufficient for irrelevance. □

7. Conclusion

We have considered here a general OG exchange economy with anonymous taxes and transfers and constraints on individual borrowing. We ask whether or not the set of equilibrium allocations is affected by constitutional restrictions on the government's budget deficit.

The credit constraints are important. With credit constraints on individuals and only anonymous lump-sum taxes, strong (or global) irrelevance of deficit restrictions is impossible and weak (or local) irrelevance can obtain only in uninteresting circumstances. This strongly contrasts with the case without individual credit constraints, where deficit restrictions are globally (and weakly) irrelevant.

With credit constraints on individuals and only anonymous consumption taxes, global deficit irrelevance is impossible just as it is for the case without credit constraints. If there is a sufficient number of tax instruments and at least one consumer's credit constraint is not binding, then there is weak (or local) irrelevance of the deficit restriction. This generalizes a similar result for the model without consumer credit constraints.

Consumption taxes are better for avoiding deficit restrictions than are lump-sum taxes. Consumption taxes, even anonymous consumption taxes, provide a mean for "transferring income" from one individual to another in the same generation. With only anonymous lump-sum taxes, intra-generational transfers are not possible.

The present paper along with Ghiglino and Shell (2000) indicates that there can be limits on the government's ability to avoid the restrictions on its deficit. Of course, as

Kotlikoff (1993) and others have argued, there is still plenty scope for the government to evade (as opposed to "avoid") the deficit restrictions by altering in its books the timing of receipts and disbursements, by guaranteeing "off-the-books" private loans, and so forth.

We stress again that to say the deficit restriction is irrelevant is not to say that the deficit does not matter. It is likely to matter if individuals condition their (rational or non-rational) expectations on the deficit.

References

Balasko, Y., 1988, Foundations of the theory of general equilibrium, Academic Press, Boston, MA.

Balasko, Y., and K. Shell, 1980, The Overlapping-Generations Model, I: the case of pure exchange without money, Journal of Economic Theory, 23, 281-306.

Balasko, Y., and K. Shell, 1981, The Overlapping-Generations Model, II: the case of pure exchange with money, Journal of Economic Theory, 24, 112-142.

Balasko, Y., and K. Shell, 1986, Lump sum taxes and transfers: public debt in the overlapping generation model, in Equilibrium analysis: Essays in honor of Kenneth J. Arrow, Volume II,

Balasko, Y., and K. Shell, 1993, Lump sum taxation: the static economy, in General Equilibrium, Growth and Trade II, Essays in Honor of Lionel McKenzie, Academic Press.

Barro, R., Are government bonds net wealth?, 1974, Journal of Political Economy, 82, 1095-1118.

Ghiglino, C. and K. Shell, 2000, The Economic Effects of Restrictions on Government Budget Deficits, Journal of Economic Theory, 94 (1), 106-137.

Ghiglino, C., and M. Tvede, 1995, Endowments, stability and fluctuations in OG models, Journal of Economic Dynamics and Control, 19, 621-653.

Kotlikoff, L. J., 1993, From Deficit Delusion to the Fiscal Balance Rule, Journal of Economics, Suppl. 7, 17-41.

Samuelson, P. A., 1958, An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money, Journal of Political Economy, 66, 467-482.

Appendix: Rank computations

For notational convenience, we focus attention on the case $n \geq 2$. Let the consumers for whom their credit constraint is binding be $h = 1, ..., r$ while for the remaining, $h = r + 1, ...n$, their credit constraint is not binding. First, consider a consumer of generation 0. It is clear from Ghiglino and Shell (2000) that the set of free parameters left after imposing the condition of constant individual demands to these consumers is a set of dimension $l-n$. Let us then consider as free the last $l-n$ prices $p^{1,n+1},\ldots,p^{1,l}$.

Second, consider the consumers of generation t, ($t=1,2,\ldots$), with the constraint that the prices $p^{t1},\ldots,p^{tn}$ are already fixed (from previous-period conditions).

Using the relationship between the prices in periods t and $t+1$, the system of equations associated to a given demand can be written as

$$\begin{array}{rcll}
\hat{p}^t+\hat{\tau}_t^t &=& (p^{t1}+\tau_t^{t1})R_t^t & \\
p^{t+1}+\tau_t^{t+1} &=& (p^{t1}+\tau_t^{t1})R_t^{t+1} & \\
p^t\omega_{th}^t-\overline{b}_{th}^t &=& (p^{t1}+\tau_t^{t1})W_{th}^t & \text{for } h=1,\ldots,r\\
p^t\omega_{th}^t+p^{t+1}\omega_{th}^{t+1} &=& (p^{t1}+\tau_t^{t1})W_{th} & \text{for } h=1,\ldots,n\\
\sum_{h=1}^n\sum_{i=1}^l\tau_t^{ti}f_{th}^{ti}(R_t^t,R_t^{t+1},W_{th})+ && & \\
+\tau_{t-1}^{ti}f_{t-1,h}^{ti}(R_{t-1}^t,R_{t-1}^t,W_{t-1,h}) &=& \delta^t &
\end{array}$$

where the quantities $R_t^t\in\mathbb{R}^{l-1}$, $R_{t-1}^t\in\mathbb{R}^l$, W_{th}^t, W_{th} and δ^t are fixed. The system of $2l-1+n+r+1=2l+n+r$ equations becomes linear in $3l$ unknowns, $p^{t,n+1},\ldots,p^{t,l}$, $p^{t+1,1},\ldots,p^{t+1,n}$, τ_t^t and τ_t^{t+1}.

Introduce the vectors $P_0^t\in\mathbb{R}^{l-n}$ and $P_1^{t+1}\in\mathbb{R}^n$ defined by

$$P_0^t=\begin{bmatrix} p^{t,n+1}\\ p^{t,n+2}\\ \vdots\\ p^{t,l}\end{bmatrix} \text{ and } P_1^{t+1}=\begin{bmatrix} p^{t+1,1}\\ p^{t+1,2}\\ \vdots\\ p^{t+1,n}\end{bmatrix}.$$

In matrix form, the system can be written as $A_tz_t=b_t$ with

$$A_t=\begin{bmatrix}
0 & 0 & -\underline{R}_t^t & I_{n-1} & 0 & 0 & 0\\
I_{l-n} & 0 & -\overline{R}_t^t & 0 & I_{l-n} & 0 & 0\\
0 & I_n & -\underline{R}_t^{t+1} & 0 & 0 & I_n & 0\\
0 & 0 & -\overline{R}_t^{t+1} & 0 & 0 & 0 & I_{l-n}\\
\varpi_t^t & 0 & -W_t^t\cdot J_r & 0 & 0 & 0 & 0\\
\omega_t^t & \omega_t^{t+1} & -W_t\cdot J_n & 0 & 0 & 0 & 0\\
0 & 0 & \sum_{h=1}^n f_{th}^{t1} & \sum_{h=1}^n \hat{f}_{th}^t & & 0 & 0
\end{bmatrix}_{2l+n+r\times 3l},$$

$$\underline{R}_t^t=\begin{bmatrix} R_t^{t2}\\ R_t^{t3}\\ \vdots\\ R_t^{tn}\end{bmatrix}_{n-1\times 1},\ \overline{R}_t^t=\begin{bmatrix} R_t^{t,n+1}\\ R_t^{t,n+2}\\ \vdots\\ R_t^{tl}\end{bmatrix}_{l-n\times 1},\ \underline{R}_t^{t+1}=\begin{bmatrix} R_t^{t+1,1}\\ R_t^{t+1,2}\\ \vdots\\ R_t^{t+1,n}\end{bmatrix}_{n\times 1},$$

$$\overline{R}_t^{t+1}=\begin{bmatrix} R_t^{t+1,n+1}\\ R_t^{t+1,n+2}\\ \vdots\\ R_t^{t+1,l}\end{bmatrix}_{l+n\times 1},\ \varpi_t^t=\begin{bmatrix} \omega_{t1}^{t,n+1} & \omega_{t1}^{t,n+2} & \ldots & \omega_{t1}^{tl}\\ \omega_{t2}^{t,n+1} & \omega_{t2}^{t,n+2} & \ldots & \omega_{t2}^{tl}\\ \vdots & \vdots & \vdots & \vdots\\ \omega_{tr}^{t,n+1} & \omega_{tr}^{t,n+2} & \ldots & \omega_{tr}^{tl}\end{bmatrix}_{r\times l-n},$$

$$\omega_t^t = \begin{bmatrix} \omega_{t1}^{t,n+1} & \omega_{t1}^{t,n+2} & \dots & \omega_{t1}^{tl} \\ \omega_{t2}^{t,n+1} & \omega_{t2}^{t,n+2} & \dots & \omega_{t2}^{tl} \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{tn}^{t,n+1} & \omega_{tn}^{t,n+2} & \dots & \omega_{tn}^{tl} \end{bmatrix}_{n\times l-n}, \omega_t^{t+1} = \begin{bmatrix} \omega_{t1}^{t+1,1} & \omega_{t1}^{t+1,2} & \dots & \omega_{t1}^{t+1,n} \\ \omega_{t2}^{t+1,1} & \omega_{t2}^{t+1,2} & \dots & \omega_{t2}^{t+1,n} \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{tn}^{t+1,1} & \omega_{tn}^{t+1,2} & \dots & \omega_{tn}^{t+1,n} \end{bmatrix}_{n\times n}$$

and $z_t = \begin{bmatrix} P_t^t \\ P_t^{t+1} \\ \tau_t^{t1} \\ \tau_t^{t2} \\ \vdots \\ \tau_t^{t+1,l} \end{bmatrix}$.

Let also $W_t^t \in \mathbb{R}^n$ and $W_t \in \mathbb{R}^n$ be the vectors of individual wealths. The rank of the matrix A_t is equal to the rank of the matrix

$$\begin{bmatrix} 0 & 0 & -\underline{R}_t^t & I_{n-1} & 0 \\ I_{l-n} & 0 & -\overline{R}_t^t & 0 & I_{l-n} \\ \varpi_t^t & 0 & -W_t^t \cdot J_n & 0 & 0 \\ \omega_t^t & \omega_t^{t+1} & -W_t \cdot J_n & 0 & 0 \\ 0 & 0 & \sum_{h=1}^n f_{th}^{t1} & & \sum_{h=1}^n \hat{f_{th}^t} \end{bmatrix}_{l+n+r\times 2l}$$

plus l. Some tedious manipulations similar to those performed in Ghiglino and Shell (2000), show that generically the above matrix has maximal rank. Then, for $l = n + r$ the A_t matrix has full rank $3l$. In this case the system has always a solution. The same can be said for $n + r < l$.

Suppose now that $l + 1 = n + r$. Then the A_t matrix is a $3l + 1 \times 3l$ matrix which has generically maximal rank $3l$. Consider the square $3l + 1$ matrix associated to the augmented system, (A_t, b_t) and let us prove that Rank $(A_t, b_t) = 3l + 1$. Indeed, the last coordinate of b_t is a function of δ_t that can be written as

$$\delta^t - \sum_{h=1}^{n} \sum_{i=1}^{l} \tau_{t-1}^{ti} f_{it-1}^{ti}.$$

The determinant of (A_t, b_t) is a first degree polynomial expression in δ^t. Therefore, to prove that the relevant matrix has full rank for an open and dense set of values of δ^t it is enough that the coefficient of δ^t in the polynomial expression is nonzero, which can be seen to be generically true. Since $Rank(A) < Rank(A, b)$, the solution set is empty. This is the borderline case so the same result holds also whenever $n + r > l + 1$.