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Baseline Rationing*

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Abstract

The standard problem of adjudicating conflicting claims describes a situation in which a given amount of a divisible good has to be allocated among agents who hold claims against it exceeding the available amount. This paper considers more general rationing problems in which, in addition to claims, there exist baselines (to be interpreted as objective entitlements, ideal targets, or past consumption) that might play an important role in the allocation process. The model we present is able to accommodate real-life rationing situations, ranging from resource allocation in the public health care sector to international protocols for the reduction of greenhouse emissions, or water distribution in drought periods. We define a family of allocation methods for such general rationing problems - called baseline rationing rules - and provide an axiomatic characterization for it. Any baseline rationing rule within the family is associated with a standard rule and we show that if the latter obeys some properties reflecting principles of impartiality, priority and solidarity, the former obeys them too.

JEL numbers: *D63.*

Keywords: *rationing, baselines, claims, priority, solidarity.*

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1 Introduction

On February 27, 2009, California Governor Arnold Schwarzenegger proclaimed a state of emergency and ordered all government agencies to implement the state's emergency plan and provide help for people, communities and businesses impacted by the third consecutive year of drought. The governor called for a statewide water conservation campaign and asked all urban water users to immediately reduce their individual water use. Prompted by this, and only a month after Governor Schwarzenegger's message, customers of the Contra Costa Water District faced a plan to implement mandatory rationing and hence reduce water consumption. The plan amounted to send a letter to each district customer telling them how much water they used on average in 2005, 2006 and 2007 for each billing period and asking them to cut that usage by 15 percent. In this paper, we present (and study) a formal model of what we call *baseline rationing* inspired by the previous story.

The simplest (and seminal) model in Economics to analyze rationing problems is due to O'Neill (1982) who provided an ideal framework to discuss fairness (as well as strategic) issues when a group of individuals have conflicting claims.[1] His model describes a situation in which a given amount of a divisible good has to be allocated among agents who hold claims against it whose sum exceeds the amount available. This framework is able to accommodate many real-life situations, such as the division of an estate that is insufficient to cover all the debts incurred by the deceased, the collection of a given tax from taxpayers, the allocation of equities in privatized firms, the distribution of commodities in a fixed-price setting, or sharing the cost of a public facility.

The standard rationing model, however, fails to accommodate more general rationing situations in which not only claims, but also individual *baselines* (reflecting objective entitlements, ideal targets, or past consumption) might play an important role. The water rationing case described above is a good case in point. Indeed, customers were asked to reduce their water usage according to past consumption (i.e., their baseline) and not their current desired consumption (i.e., their claim) which will typically be higher. A similar rationale was behind the greenhouse gas emissions targets set by the Kyoto Protocol and the more recent Copenhagen Accord. Another instance is the case of resource allocation in the public health care sector (see e.g., Chalkley and Malcomson (2000) for a general discussion of health care budgeting

[1] Another important early contribution dealing with this same model is Aumann and Maschler (1985). The reader is referred to Moulin (2002) or Thomson (2003, 2006) for recent surveys of the sizable related literature.

procedures). Hospital departments budgets are typically determined according to a production target (baseline activity). By the end of each year the actual number of services delivered is recorded (the claim). On the basis of these measures, and the overall health care budget (which typically will not cover the full claim but often exceed the budgeted activities) the final department funding is settled by allocating any residual claim, i.e., actual expenses minus baseline budget. Similar procedures also exist at universities and related institutions in which objective entitlements of agents as well as actual claims play a crucial role in the allocation process (e.g., Pulido et al., 2002).

The formal (extended) model of rationing we analyze here enriches the standard model described above upon assuming the existence of a baselines profile, aimed to complement the claims profile of a rationing problem. We take first a direct approach to analyze this new model.[2] That is, we single out a natural class of baseline rationing rules which aims to encompass the real-life rationing situations mentioned above. In short, rules within this class tentatively allocate each agent with their baselines and then adjust this tentative allocation upon using a standard rationing rule to distribute the remaining surplus, or deficit, relative to the initially available amount. We then take an axiomatic approach and study the implications of axioms reflecting ethical or operational principles in this context. More precisely, we provide an axiomatic characterization for the class of baseline rationing rules just described. We also study the robustness of the class, by showing that the rules within the class inherit some important basic properties from the associated standard rationing rules reflecting principles of impartiality, priority and solidarity.

The rest of the paper is organized as follows. In Section 2, we describe the basic framework of the standard rationing model, as well as the new one to address baseline rationing problems. Section 3 is devoted to the main results of this paper. We conclude in Section 4 with some further insights. For a smooth passage, we defer some proofs and provide them in the appendix.

2 Model and basic concepts

2.1 The benchmark framework

We study rationing problems in a variable population model. The set of potential claimants, or *agents*, is identified with the set of natural numbers $\mathbb{N}$. Let $\mathcal{N}$ be the set of finite subsets of

[2]The terminology is borrowed from Thomson (2006)

$\mathbb{N}$, with generic element N. Let n denote the cardinality of N. For each $i \in N$, let $c_i \in \mathbb{R}_+$ be i's *claim* and $c \equiv (c_i)_{i\in N}$ the claims profile.[3] A *standard rationing problem* is a triple consisting of a population $N \in \mathcal{N}$, a claims profile $c \in \mathbb{R}^n_+$, and an amount to be divided $E \in \mathbb{R}_+$ such that $\sum_{i\in N} c_i \geq E$. Let $C \equiv \sum_{i\in N} c_i$. To avoid unnecessary complication, we assume $C > 0$. Let $\mathcal{D}^N$ be the set of rationing problems with population N and $\mathcal{D} \equiv \bigcup_{N\in\mathcal{N}} \mathcal{D}^N$.

Given a problem $(N, c, E) \in \mathcal{D}^N$, an *allocation* is a vector $x \in \mathbb{R}^n$ satisfying the following two conditions: (i) for each $i \in N$, $0 \leq x_i \leq c_i$ and (ii) $\sum_{i\in N} x_i = E$. We refer to (i) as *boundedness* and (ii) as *balance*. A *standard rationing rule* on $\mathcal{D}$, $R\colon \mathcal{D} \to \bigcup_{N\in\mathcal{N}} \mathbb{R}^n$, associates with each problem $(N, c, E) \in \mathcal{D}$ an allocation $R(N, c, E)$ for the problem. Each rule R has a dual rule R^* defined as $R^*(N, c, E) = c - R(N, c, C - E)$, for all $(N, c, E) \in \mathcal{D}$.

Some classical rules are the *constrained equal awards* rule, which distributes the amount equally among all agents, subject to no agent receiving more than she claims; the *constrained equal losses* rule, which imposes that losses are as equal as possible, subject to no one receiving a negative amount; and the *proportional* rule, which yields awards proportionally to claims.[4]

Rules are typically evaluated in terms of the properties they satisfy. The literature has provided a wide variety of axioms for rules reflecting ethical or operational principles. Here we shall concentrate on those formalizing the principles of impartiality, priority, and solidarity, which have a long tradition in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006).

Impartiality refers to the fact that ethically irrelevant information is excluded from the allocation process. In this context, it is modeled by the axiom of *Equal Treatment of Equals*, which requires allotting equal amounts to those agents with equal claims. Formally, a rule R satisfies equal treatment of equals if, for all $(N, c, E) \in \mathcal{D}$, and all $i, j \in N$, we have $R_i(N, c, E) = R_j(N, c, E)$, whenever $c_i = c_j$.

The principle of *priority* requires imposing a positive discrimination (albeit only to a certain extent) towards individuals with higher needs. In this context, needs are reflected by claims and as such, priority is modeled by the axiom of *Order Preservation*, which says that agents with larger claims receive larger awards but face larger losses too. That is, $c_i \geq c_j$ implies that $R_i(N, c, E) \geq R_j(N, c, E)$ and $c_i - R_i(N, c, E) \geq c_j - R_j(N, c, E)$, for all $(N, c, E) \in \mathcal{D}$, all $i, j \in N$. If only the first condition is satisfied then the axiom is referred to as *Order Preservation in Gains*. If, on the other hand, only the second condition is satisfied then the

[3]For each $N \in \mathcal{N}$, each $M \subseteq N$, and each $z \in \mathbb{R}^n$, let $z_M \equiv (z_i)_{i\in M}$.

[4]The reader is referred to Moulin (2002) or Thomson (2003, 2006) for their formal definitions as well as further details about these rules.

axiom is referred to as *Order Preservation in Losses.*

The principle of *solidarity*, with a long tradition in the axiomatic literature, can be modeled in various related ways. *Resource Monotonicity* says that when there is more to be divided, other things being equal, nobody should lose. Formally, a rule R is resource monotonic if, for each $(N, c, E) \in \mathcal{D}$ and $(N, c, E') \in \mathcal{D}$ such that $E \leq E'$, then $R(N, c, E) \leq R(N, c, E')$. *Claims Monotonicity* says that if an agent's claim increases, ceteris paribus, she should receive at least as much as she did initially. Formally, a rule R is claims monotonic if, for all $(N, c, E) \in \mathcal{D}$ and all $i \in N$, $c_i \leq c_i'$ implies $R_i(N, (c_i, c_{N\setminus\{i\}}), E) \leq R_i(N, (c_i', c_{N\setminus\{i\}}), E)$. The dual property says that if an agent's claim and the amount to divide increase by the same amount, the agent's award should increase by at most that amount. Formally, a rule satisfies *Linked Monotonicity* if, for all $(N, c, E) \in \mathcal{D}$ and $i \in N$, $R_i(N, (c_i + \varepsilon, c_{N\setminus\{i\}}), E + \varepsilon) \leq R_i(N, c, E) + \varepsilon$.[5] The next axiom amounts to simultaneous changes in the amount available and the population. It says that the arrival of new agents should affect all the incumbent agents in the same direction. In other words, agents cannot benefit from a change (either in the available wealth or in the number of agents) if someone else suffers from it. Formally, a rule R satisfies *Resource-and-Population Uniformity* if for all $(N, c, E) \in \mathcal{D}$ and $(N', c', E') \in \mathcal{D}$ such that $N \subseteq N'$ and $c'_N = c$, then, either $R_i(N', c', E') \leq R_i(N, c, E)$, for all $i \in N$, or $R_i(N', c', E') \geq R_i(N, c, E)$, for all $i \in N$. It is straightforward to show that this axiom implies resource monotonicity. As a matter of fact, it also satisfies the following axiom that relates the solution of a given problem to the solutions of the subproblems that appear when we consider a subgroup of agents as a new population and the amounts gathered in the original problem as the available amount to be distributed. *Consistency* requires that the application of the rule to each subproblem produces precisely the allocation that the subgroup obtained in the original problem.[6] More formally: A rule R is consistent if, for all $(N, c, E) \in \mathcal{D}$, all $M \subset N$, and all $i \in M$, we have $R_i(N, c, E) = R_i(M, c_M, E_M)$, where $E_M = \sum_{i \in M} R_i(N, c, E)$. It turns out that consistency and resource monotonicity together imply resource-and-population uniformity.

[5] For any given property α, α^* is the *dual property of* α if for each rule R, R satisfies α if and only if its dual rule R^* satisfies α^*. A property is said to be self-dual if it coincides with its dual. Equal treatment of equals, order preservation and resource monotonicity are instances of self-dual properties.

[6] See Thomson (1996) for an excellent survey of the many applications that have been made on the idea of consistency.

2.2 The extended framework

We now enrich the model to account for individual baselines that will be part of the rationing process. An *extended rationing problem* (or problem with baselines) will be a tuple consisting of a population $N \in \mathcal{N}$, a *baselines* profile $b \in \mathbb{R}^n_+$, a claims profile $c \in \mathbb{R}^n_+$, and an amount to be divided $E \in \mathbb{R}_+$ such that $\sum_{i \in N} c_i \geq E$. We denote by $\mathcal{E}^N$ the set of extended problems with population N and $\mathcal{E} \equiv \bigcup_{N \in \mathcal{N}} \mathcal{E}^N$. For each extended problem $(N, b, c, E) \in \mathcal{E}$, let $\widetilde{b}$ denote the corresponding truncated baseline vector, i.e., $\widetilde{b} = \{\widetilde{b}_i\}_{i \in N}$, where $\widetilde{b}_i = \min\{b_i, c_i\}$, for all $i \in N$. For ease of notation, let $\widetilde{B} = \sum_{i \in N} \widetilde{b}_i$.

Given an extended problem $(N, b, c, E) \in \mathcal{E}^N$, an (extended) *allocation* is a vector $x \in \mathbb{R}^n$ satisfying the following two conditions: (i) for each $i \in N$, $0 \leq x_i \leq c_i$ and (ii) $\sum_{i \in N} x_i = E$. An *extended rationing rule* on $\mathcal{E}$, $\widetilde{R} \colon \mathcal{E} \to \bigcup_{N \in \mathcal{N}} \mathbb{R}^n$, associates with each extended problem $(N, b, c, E) \in \mathcal{E}$ an (extended) allocation $x = \widetilde{R}(N, b, c, E)$ for the problem.

This paper will focus on a certain class of extended rationing rules, *induced* by standard rationing rules, that will be called *baseline rationing rules*. More precisely, these rules are constructed such that agents are first allocated their truncated baselines, and then the resulting deficit or surplus is further allocated using a standard rationing rule to the resulting standard problem after embedding baselines into claims. More specifically, a potential deficit is allocated according to the amounts already received by the agents while a potential surplus is allocated according to the gap between their claims and what has already been allocated to them. Formally,

$$R^b(N, b, c, E) = \begin{cases} \widetilde{b} - R(N, \widetilde{b}, \widetilde{B} - E) & \text{if } E \leq \widetilde{B} \\ \widetilde{b} + R(N, c - \widetilde{b}, E - \widetilde{B}) & \text{if } E \geq \widetilde{B} \end{cases} \tag{1}$$

We shall refer to R^b as the baseline rationing rule induced by (standard rationing rule) R.

Note that, if $b = 0$, then $R^b \equiv R$. More interestingly, note that, for any standard rationing rule R, and any extended problem (N, b, c, E), the induced baseline rationing rule results in an allocation x satisfying

$$x_i \leq \widetilde{b}_i \text{ for all } i \in N \text{ if and only if } E \leq \widetilde{B},$$

$$x_i \geq \widetilde{b}_i \text{ for all } i \in N \text{ if and only if } E \geq \widetilde{B}.$$

In other words, baseline rationing rules impose a rationing of the same sort for each individual and the whole society according to the profile of baselines.

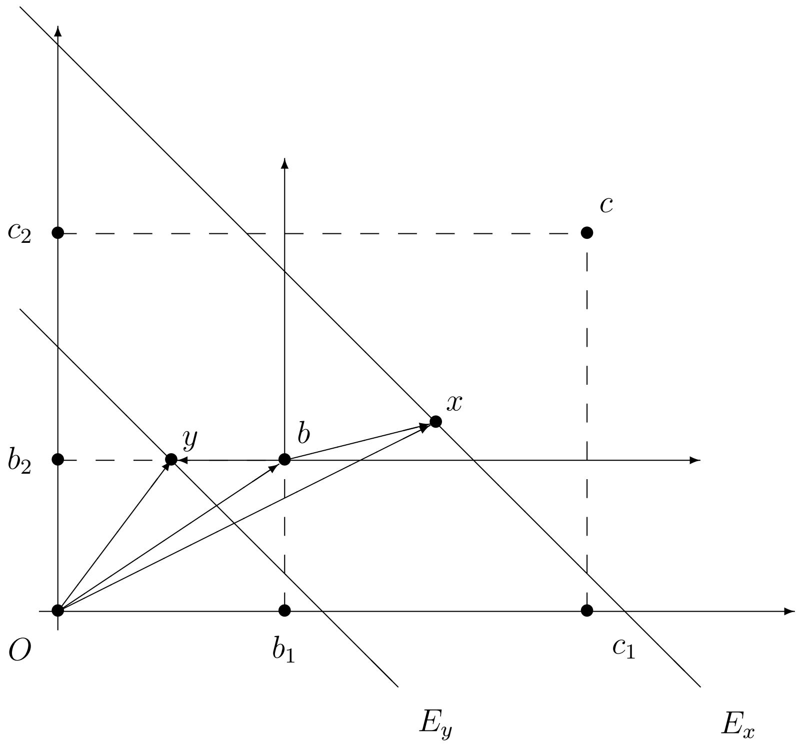

Figure 1: Baseline rationing rules in the two-claimant case. This figure illustrates how baseline rationing rules behave for $N = \{1, 2\}$, and $b, c \in \mathbb{R}^N_+$, with $c_i > b_i$, for $i = 1, 2$. If the amount to divide is $E_x > b_1 + b_2$, then the proposed solution x can be decomposed as $b + (x - b)$ where $x - b$ is to be interpreted as the solution for the standard rationing problem arising after adjusting claims (and available amount) down by the baselines, which implies that b is the new origin, i.e., $x = b + R(N, c - b, E_x - b_1 - b_2)$. If, however, $E_y < b_1 + b_2$ then the proposed solution y can be decomposed as $b - (b - y)$ where $b - y$ is to be interpreted as the solution for the standard rationing problem arising after replacing claims by baselines and the available amount by the difference between the aggregate baseline and the original available amount, i.e., $b - y = R(N, b, b_1 + b_2 - E_y)$.

It is not difficult to show that the following expression is equivalent to (1).

$$R^b (N, b, c, E) = \begin{cases} R^*(N, \widetilde{b}, E) & \text{if } E \leq \widetilde{B} \\ \widetilde{b} + R(N, c - \widetilde{b}, E - \widetilde{B}) & \text{if } E \geq \widetilde{B} \end{cases} \tag{2}$$

By (2), it is straightforward to show that if each individual baseline is exactly one half of each individual claim, then the induced baseline rule by the constrained equal losses rule described above is precisely the so-called *Talmud* rule (e.g., Aumann and Maschler, 1985). Similarly, the induced baseline rule by the constrained equal awards rule is precisely the so-called *Reverse Talmud* rule (e.g., Chun et al., 2001). If instead of one half, baselines are any other fixed proportion of claims, then the induced baseline rule by the constrained equal losses rule is a member of the so-called TAL-family of rules (e.g., Moreno-Ternero and Villar, 2006), whereas the induced baseline rule by the constrained equal awards rule is a member of the so-called Reverse TAL-family (e.g., van den Brink et al., 2008). Thus, a wide variety of existing standard rationing rules can actually be seen as members of the family of baseline rationing rules presented here.

3 Results

3.1 A characterization result

We mentioned above some usual axioms for rules in the standard rationing model. Obviously, none of them referred to baselines. We now provide a list of new axioms conveying natural ways of taking baselines into account while designing the rationing scheme.

Baseline Truncation is a property that requires baselines to be disregarded to the extent that they are above claims.[7] Formally, an extended rule $\widetilde{R}$ satisfies baseline truncation if, for each $(N, b, c, E) \in \mathcal{E}$, $\widetilde{R}(N, b, c, E) = \widetilde{R}(N, \widetilde{b}, c, E)$.

Truncation of Excessive Claims is a somewhat dual property that requires to disregard the amount of a claim exceeding its corresponding baseline, whenever all (truncated) baselines cannot be covered. Formally, an extended rule $\widetilde{R}$ satisfies truncation of excessive claims if, for each $(N, b, c, E) \in \mathcal{E}$ such that $E \leq \widetilde{B}$, $\widetilde{R}(N, b, c, E) = \widetilde{R}(N, b, \widetilde{c}, E)$, where $\widetilde{c}_j = \min\{c_j, b_j\}$, for all $j \in N$.

Baseline Invariance is a property that requires the allocation to be invariant with respect to additive common shifts in baselines and claims, whenever all (truncated) baselines can be covered. Formally, an extended rule $\widetilde{R}$ satisfies baseline invariance if for each $(N, b, c, E) \in \mathcal{E}$ such that $E \geq \widetilde{B}$, and $k \in \mathbb{R}^n_+$ such that $k_j \leq \min\{c_j, b_j\}$, for all $j \in N$, then $\widetilde{R}(N, b, c, E) = k + \widetilde{R}(N, b - k, c - k, E - \sum_{i \in N} k_i)$.

Baseline Duality is a property inspired by the influential notion of duality in the standard model of rationing. It states that a rule should allocate awards for a problem with null baselines in the same way as it allocates losses for a problem in which baselines are equal to claims. Formally, an extended rule $\widetilde{R}$ satisfies baseline duality if, for each $(N, c, E) \in \mathcal{D}$, $\widetilde{R}(N, 0, c, E) = c - \widetilde{R}(N, c, c, C - E)$.

It turns out, as the next theorem shows, that these four axioms together characterize our family of baseline rationing rules introduced above.

Theorem 1 *An extended rationing rule satisfies Baseline Truncation, Truncation of Excessive Claims, Baseline Invariance and Baseline Duality if and only if it is a baseline rationing rule.*

Proof. It is straightforward to see that, for any standard rationing rule R, R^b satisfies the four axioms. Thus, we focus on the converse implication. Let $\widetilde{R}$ be an extended rule satisfying the four axioms from the list and let (N, b, c, E) be a given extended problem. We distinguish two cases.

Case 1. $E \geq \widetilde{B}$.

[7] This property is reminiscent to the so-called invariance under claims truncation axiom in the standard rationing model (e.g., Hokari and Thomson, 2003).

In this case, by baseline invariance, $\widetilde{R}(N,b,c,E)=\widetilde{b}+\widetilde{R}(N,b-\widetilde{b},c-\widetilde{b},E-\widetilde{B})$. It is straightforward to show that, if $b_i \leq c_i$ then $b_i-\widetilde{b}_i=0<c_i-\widetilde{b}_i=c_i-b_i$, whereas if $b_i \geq c_i$ then $b_i-\widetilde{b}_i=b_i-c_i>0=c_i-\widetilde{b}_i$. Thus, by baseline truncation, $\widetilde{R}(N,b-\widetilde{b},c-\widetilde{b},E-\widetilde{B})=\widetilde{R}(N,0,c-\widetilde{b},E-\widetilde{B})$.

Case 2. $E \leq \widetilde{B}$.

In this case, by baseline truncation, $\widetilde{R}(N,b,c,E)=\widetilde{R}(N,\widetilde{b},c,E)$. And, by truncation of excessive claims, $\widetilde{R}(N,\widetilde{b},c,E)=\widetilde{R}(N,\widetilde{b},\widetilde{b},E)$. Finally, by baseline duality, $\widetilde{R}(N,\widetilde{b},\widetilde{b},E)=\widetilde{b}-\widetilde{R}(N,0,\widetilde{b},\widetilde{B}-E)$.

To summarize,

$$\widetilde{R}\left(N,b,c,E\right)=\begin{cases}\widetilde{b}-\widetilde{R}(N,0,\widetilde{b},\widetilde{B}-E) & \text{if } E \leq \widetilde{B} \\ \widetilde{b}+\widetilde{R}(N,0,c-\widetilde{b},E-\widetilde{B}) & \text{if } E > \widetilde{B}\end{cases}. \tag{3}$$

Let $R:\mathcal{D}\rightarrow\bigcup_{N\in\mathcal{N}}\mathbb{R}^n$ be such that, for any $(N,c,E)\in\mathcal{D}$,

$$R\left(N,c,E\right)=\widetilde{R}(N,0,c,E).$$

In other words, R assigns to each standard rationing problem the solution that $\widetilde{R}$ yields for the corresponding extended problem in which baselines are null. It is straightforward to see that R is a well-defined (standard) rationing rule. Hence, (3) becomes

$$\widetilde{R}\left(N,b,c,E\right)=\begin{cases}\widetilde{b}-R(N,\widetilde{b},\widetilde{B}-E) & \text{if } E \leq \widetilde{B} \\ \widetilde{b}+R(N,c-\widetilde{b},E-\widetilde{B}) & \text{if } E > \widetilde{B}\end{cases},$$

which implies that $\widetilde{R}\equiv R^b$. ■

3.2 Robustness to baseline rationing

We now focus on the robustness to baseline rationing of the standard axioms described above. We say that a standard axiom is robust to baseline rationing if whenever a standard rule R satisfies it, then the induced baseline rule R^b satisfies the corresponding *extended* version of the axiom.[8] Note that, as mentioned above, if $b=0$, then $R^b\equiv R$. Thus, saying that "a property is robust to baseline rationing" is indeed equivalent to saying that "a standard rationing rule R satisfies a property if and only if R^b satisfies the extended version of such property".

We shall distinguish between a direct version of robustness and two indirect versions in which additional (but mild) properties have to be considered.

[8]For ease of exposition, we skip the straightforward definitions of the extended versions of each axiom introduced above. Just as an illustration, we say, for instance, that an extended rule $\widetilde{R}$ satisfies claims monotonicity if for each $(N,b,c,E),(N,b,c',E)\in\mathcal{E}$ such that $c_i\leq c'_i$ for some $i\in N$, and $c'_{N\setminus\{i\}}\equiv c_{N\setminus\{i\}}$, we have that $\widetilde{R}_i(N,b,c,E)\leq\widetilde{R}_i(N,b,c',E)$.

3.2.1 Direct robustness

Our first result says that many of the well-known axioms in the benchmark framework are not robust to baseline rationing.

Proposition 1 *If a property is not self-dual then it is not robust to baseline rationing.*

Proof. Let P be a property and P^* be its dual. Let R be a rule satisfying P, but not P^*. Then, R^*, the dual rule of R, satisfies P^* but not P.

If P is "punctual" then there exists a problem $(N, c, E) \in \mathcal{D}$ for which R^* violates P.[9] We then consider the corresponding extended problem $(N, b, c, E) \in \mathcal{E}$ in which $b = c$. It then follows that $R^b(N, b, c, E) = R^*(N, c, E)$ and hence we conclude that R^b violates P.

If P is "relational", a similar argument can be applied, although taking into account that the violation of P by R^* would involve several problems and thus the corresponding extended problems should be constructed so that R^b yields for them the allocations that R^* yields for the corresponding (standard) problems. ■

Contrary to what one might have guessed from the statement of Proposition 1, not all self-dual properties are robust to baseline rationing either. A straightforward counterexample is equal treatment of equals, which will not be satisfied by an induced baseline rationing rule if two agents with equal claims have different baselines. Nevertheless, it turns out that some other important self-dual properties are robust.

Proposition 2 *Resource monotonicity is robust to baseline rationing.*

Proof. Let R be a rule satisfying resource monotonicity. Let $(N, c, E), (N, c, E') \in \mathcal{D}$ be two problems such that $E < E'$. Let $b \in \mathbb{R}^n$ be a baseline profile and let $\widetilde{b}_j = \min\{b_j, c_j\}$ for all $j \in N$, and $\widetilde{B} = \sum_{j \in N} \min\{b_j, c_j\}$. Finally, let $i \in N$ be a given agent. The aim is to show that $R_i^b(N, b, c, E) \leq R_i^b(N, b, c, E')$. To do so, we distinguish three cases.

Case 1. $E < E' \leq \widetilde{B}$.

In this case, $R_i^b(N, b, c, E) = R_i^*(N, \widetilde{b}, E)$ and $R_i^b(N, b, c, E') = R_i^*(N, \widetilde{b}, E')$. Now, as resource monotonicity is a self-dual property, it follows that R^* satisfies resource monotonicity too and hence $R_i^*(N, \widetilde{b}, E) \leq R_i^*(N, \widetilde{b}, E')$, as desired.

Case 2. $\widetilde{B} \leq E < E'$.

[9] Following Thomson (2006), if a property applies to a rule for each problem separately, point by point, we refer to it as *punctual*. If, however, a property links the recommendations made by the rule for different problems that are related in a certain way, we call it a *relational* property. Instances of punctual properties are equal treatment of equals and order preservation. Instances of relational properties are the monotonicity axioms described above.

In this case,

$$R_i^b(N,b,c,E) = \widetilde{b}_i + R_i(N, c-\widetilde{b}, E-\widetilde{B}) \leq \widetilde{b}_i + R_i(N, c-\widetilde{b}, E'-\widetilde{B}) = R_i^b(N,b,c,E'),$$

where the inequality follows from the fact that R satisfies resource monotonicity.

Case 3. $E < \widetilde{B} < E'$.

In this case, the definition of baseline rationing guarantees that $R_i^b(N,b,c,E) \leq \widetilde{b}_i \leq R_i^b(N,b,c,E')$.

■

Proposition 3 *Consistency is robust to baseline rationing.*

Proof. Let R be a rule satisfying consistency. Let $(N,c,E) \in \mathcal{D}$ and $b \in \mathbb{R}_+^n$. Let $x = R^b(N,b,c,E)$. The aim is to show that, for any $M \subset N$,

$$R^b(M, b_M, c_M, \sum_{i\in M} x_i) = x_M.$$

Fix $M \subset N$ and let $E' = \sum_{i\in M} x_i$ and $\widetilde{B}' = \sum_{j\in M} \min\{b_j, c_j\}$. Then, it is straightforward to show that $E \leq \widetilde{B}$ if and only if $E' \leq \widetilde{B}'$. We then distinguish two cases.

Case 1. $E \leq \widetilde{B}$.

In this case, $x_i = \widetilde{b}_i - R_i(N, \widetilde{b}, \widetilde{B} - E)$ for all $i \in N$, and thus $\widetilde{B}' - E' = \sum_{i\in M} R_i(N, \widetilde{b}, \widetilde{B} - E)$. Therefore, $R_i^b(M, b_M, c_M, E') = \widetilde{b}_i - R_i(M, \widetilde{b}_M, \widetilde{B}' - E')$ for all $i \in M$. Now, as R is consistent, it follows that $R_i(N, \widetilde{b}, \widetilde{B} - E) = R_i(M, \widetilde{b}_M, \widetilde{B}' - E')$, for all $i \in M$, which concludes the proof of this case.

Case 2. $E \geq \widetilde{B}$.

In this case, $x_i = \widetilde{b}_i + R_i(N, c-\widetilde{b}, E-\widetilde{B})$ for all $i \in N$, and thus $\widetilde{B}' - E' = \sum_{i\in M} R_i(N, c-\widetilde{b}, E-\widetilde{B})$. Therefore, $R_i^b(M, b_M, c_M, E') = \widetilde{b}_i + R_i(M, c_M - \widetilde{b}_M, E' - \widetilde{B}')$ for all $i \in M$. Now, as R is consistent, it follows that $R_i(N, c-\widetilde{b}, E-\widetilde{B}) = R_i(M, c_M - \widetilde{b}_M, E' - \widetilde{B}')$, for all $i \in M$, which concludes the proof of this case. ■

The previous two propositions and the relationship among the axioms described in Section 2 guarantee the following result.

Corollary 1 *Resource-and-population uniformity is robust to baseline rationing.*

3.3 Indirect robustness

We have shown that, although some important properties are robust to baseline rationing, others are not. We now explore whether some of the latter are, at least, *indirectly robust*. We start with "robustness as a group". More precisely, we study whether when a standard rationing rule R satisfies a combination of some properties then the induced baseline rule R^b satisfies the corresponding combination of extended properties.

Proposition 4 *If a standard rationing rule R satisfies resource monotonicity, claims monotonicity, and linked monotonicity then the induced baseline rationing rule R^b satisfies the corresponding three extended properties.*

Proof. Let R be a rule satisfying resource monotonicity, claims monotonicity, and linked monotonicity. By Proposition 2, we only need to show that the induced baseline rationing rule R^b satisfies the extended properties of claims monotonicity and linked monotonicity.

Claims monotonicity. Let $(N,c,E),(N,c',E) \in \mathcal{D}$ be two problems such that, for some $i \in N$, $c_i \leq c'_i$, whereas $c_{N\setminus\{i\}} \equiv c'_{N\setminus\{i\}}$. Let $b \in \mathbb{R}^n_+$ be a baseline profile and let $\widetilde{b}_j = \min\{b_j, c_j\}$ for all $j \in N$, $\widetilde{b}'_j = \min\{b_j, c'_j\}$ for all $j \in N$, $\widetilde{B} = \sum_{j\in N} \widetilde{b}_j$, and $\widetilde{B}' = \sum_{j\in N} \widetilde{b}'_j$.[10] The aim is to show that

$$R^b_i(N,b,c,E) \leq R^b_i(N,b,c',E). \tag{4}$$

We distinguish several cases:

Case 1. $E \leq \widetilde{B}$.

In this case, $R^b_i(N,b,c,E) = R^*_i(N,\widetilde{b},E)$ and $R^b_i(N,b,c',E) = R^*_i(N,\widetilde{b}',E) = R^*_i(N,(\widetilde{b}_{N\setminus\{i\}},\widetilde{b}'_i),E)$. As R satisfies linked monotonicity, it follows that R^* satisfies claims monotonicity, from where we obtain (4).

Case 2. $E \geq \widetilde{B}'$.

In this case, $R^b_i(N,b,c,E) = \widetilde{b}_i + R_i(N,c-\widetilde{b},E-\widetilde{B})$, and

$$R^b_i(N,b,c',E) = \widetilde{b}'_i + R_i(N,c'-\widetilde{b}',E-\widetilde{B}') = \widetilde{b}'_i + R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},c'_i-\widetilde{b}'_i),E-\widetilde{B}-\widetilde{b}'_i+\widetilde{b}_i).$$

Let $\varepsilon = \widetilde{b}'_i - \widetilde{b}_i \geq 0$. Then, (4) is equivalent to

$$R_i(N,c-\widetilde{b},E-\widetilde{B}) \leq \varepsilon + R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},c'_i-\widetilde{b}'_i),E-\widetilde{B}-\varepsilon). \tag{5}$$

We then distinguish three subcases.

Case 2.1. $b_i > c'_i$.

Here, $\widetilde{b}_i = c_i < c'_i = \widetilde{b}'_i$ (and thus $\varepsilon = c'_i - c_i$). Then, (5) becomes

$$R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},0),E-\widetilde{B}) \leq \varepsilon + R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},0),E-\widetilde{B}-\varepsilon).$$

Now, by claims monotonicity (of R),

$$R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},0),E-\widetilde{B}) \leq R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},\varepsilon),E-\widetilde{B}).$$

And, by linked monotonicity (of R),

$$R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},\varepsilon),E-\widetilde{B}) \leq \varepsilon + R_i(N,((c-\widetilde{b})_{N\setminus\{i\}},0),E-\widetilde{B}-\varepsilon),$$

[10] Note that $\widetilde{b}'_{N\setminus\{i\}} \equiv \widetilde{b}_{N\setminus\{i\}}$, $\widetilde{b}'_i \geq \widetilde{b}_i$ and thus, $\widetilde{B}' \geq \widetilde{B}$.

which concludes the proof in this case.

Case 2.2. $c'_i \geq b_i \geq c_i$.

Here, $\widetilde{b}_i = c_i \leq b_i = \widetilde{b'}_i$ (and thus $\varepsilon = b_i - c_i$). Then, (5) becomes

$$R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, 0), E - \widetilde{B}) \leq b_i - c_i + R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, c'_i - b_i), E - \widetilde{B} - b_i + c_i).$$

Now, by claims monotonicity (of R),

$$R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, 0), E - \widetilde{B}) \leq R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, c'_i - c_i), E - \widetilde{B}).$$

And, by linked monotonicity (of R),

$$R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, c'_i - c_i), E - \widetilde{B}) \leq \varepsilon + R_i(N, ((c - \widetilde{b})_{N \setminus \{i\}}, c'_i - c_i - \varepsilon), E - \widetilde{B} - \varepsilon),$$

which concludes the proof in this case.

Case 2.3. $c_i > b_i$.

Here, $\widetilde{b}_i = b_i = \widetilde{b'}_i$ (and thus $\varepsilon = 0$), from where (5) trivially follows as a consequence of the fact that R satisfies claims monotonicity.

Case 3. $\widetilde{B} < E < \widetilde{B}'$. In this case, $R^b_i(N, b, c, E) = \widetilde{b}_i + R_i(N, c - \widetilde{b}, E - \widetilde{B})$, and $R^b_i(N, b, c', E) = R^*_i(N, \widetilde{b}', E) = \widetilde{b}'_i - R_i(N, (\widetilde{b}_{N \setminus \{i\}}, \widetilde{b}'_i), \widetilde{B}' - E)$. Thus, in order to prove (4), it suffices to show that

$$R_i(N, (\widetilde{b}_{N \setminus \{i\}}, \widetilde{b}'_i), \widetilde{B}' - E) + R_i(N, c - \widetilde{b}, E - \widetilde{B}) \leq \widetilde{b'}_i - \widetilde{b}_i \tag{6}$$

Note that Case 3 implies that $\widetilde{b}_i = c_i$ (otherwise, $\widetilde{b}_i = b_i$ and hence $\widetilde{B} = \widetilde{B}'$). Thus, by boundedness, $R_i(N, c - \widetilde{b}, E - \widetilde{B}) = 0$. Also, by balance and boundedness, $R_i(N, (\widetilde{b}_{N \setminus \{i\}}), \widetilde{b}'_i, \widetilde{B}' - E) \leq \widetilde{B}' - E = \widetilde{B} - E + \widetilde{b'}_i - \widetilde{b}_i \leq \widetilde{b'}_i - \widetilde{b}_i$, from where (6) follows.

Linked monotonicity. Let $(N, c, E) \in \mathcal{D}$ and $i \in N$. Let $b \in \mathbb{R}^n_+$ be a baseline profile and let $\widetilde{b}_j = \min\{b_j, c_j\}$ for all $j \in N$, and $\widetilde{B} = \sum_{j \in N} \min\{b_j, c_j\}$. Let $\varepsilon > 0$, $\widetilde{b}'_i = \min\{b_i, c_i + \varepsilon\}$ and $\widetilde{B}' = \widetilde{B} + \widetilde{b}'_i - \widetilde{b}_i$. Then, $\widetilde{b}'_i \leq \widetilde{b}_i + \varepsilon$ and $\widetilde{B} \leq \widetilde{B}' \leq \widetilde{B} + \varepsilon$. The aim is to show that

$$R^b_i(N, b, (c_i + \varepsilon, c_{N \setminus \{i\}}), E + \varepsilon) \leq R^b_i(N, b, c, E) + \varepsilon \tag{7}$$

We distinguish several cases:

Case 1. $E \leq \widetilde{B}' - \varepsilon$.

In this case, $R^b_i(N, b, c, E) = R^*_i(N, \widetilde{b}, E)$ and $R^b_i(N, b, (c_i + \varepsilon, c_{N \setminus \{i\}}), E + \varepsilon) = R^*_i(N, \widetilde{b}', E + \varepsilon) = R^*_i(N, (\widetilde{b}'_i, \widetilde{b}_{-i}), E + \varepsilon)$. By claims monotonicity (of R^*), $R^*_i(N, \widetilde{b}', E + \varepsilon) \leq R^*_i(N, (\widetilde{b}_i + \varepsilon, \widetilde{b}_{-i}), E + \varepsilon)$. By linked monotonicity (of R^*), $R^*_i(N, (\widetilde{b}_i + \varepsilon, \widetilde{b}_{-i}), E + \varepsilon) \leq R^*_i(N, \widetilde{b}, E) + \varepsilon$, from where (7) follows.

Case 2. $E \leq \widetilde{B}$.

In this case, $R^b_i(N, b, c, E) = \widetilde{b}_i + R_i(N, c - \widetilde{b}, E - \widetilde{B})$ and $R^b_i(N, b, (c_i + \varepsilon, c_{N \setminus \{i\}}), E + \varepsilon) = \widetilde{b'}_i + R_i(N, (c_i + \varepsilon, c_{N \setminus \{i\}}) - (\widetilde{b}'_i, \widetilde{b}_{-i}), E + \varepsilon - \widetilde{B}')$. As $\widetilde{B}' = \widetilde{B} + \widetilde{b}'_i - \widetilde{b}_i$ and $c_i + \varepsilon - \widetilde{b}'_i = c_i - \widetilde{b}_i + \varepsilon - \widetilde{b}'_i + \widetilde{b}_i$, (7) follows from linked monotonicity (of R).

Case 3. $\widetilde{B}' - \varepsilon < E < \widetilde{B}$.

In this case, $R_i^b(N, b, (c_i + \varepsilon, c_{N\setminus\{i\}}), E + \varepsilon) = \widetilde{b}_i' + R_i(N, (c_i + \varepsilon - \widetilde{b}_i', (c - \widetilde{b})_{-i}), E + \varepsilon - \widetilde{B}')$, and $R_i^b(N, b, c, E) = \widetilde{b}_i' - R_i(N, \widetilde{b}, \widetilde{B} - E)$. Thus, (7) becomes

$$\varepsilon - \widetilde{b}_i' + \widetilde{b}_i \geq R_i(N, \widetilde{b}, \widetilde{B} - E) + R_i(N, (c_i + \varepsilon - \widetilde{b}_i', (c - \widetilde{b})_{-i}), E + \varepsilon - \widetilde{B}') \tag{8}$$

Now, by balance and boundedness, the right hand side of (8) is bounded above by $\widetilde{B} - E + E + \varepsilon - \widetilde{B}'$, which is precisely the left hand side of (8).

We conclude that R^b satisfies all the desired properties. ■

To conclude with this section, we take another indirect approach to robustness. More precisely, we now focus on properties that, while not directly robust to baseline rationing, are so with the help of additional (mild) conditions over baselines. We refer to that behavior as *assisted robustness*.

We start with equal treatment of equals. We say that baselines and claims are *uniformly impartial* if whenever $c_i = c_j$ then $b_i = b_j$. The following result, which straightforward proof we omit, is a first instance of assisted robustness.

Proposition 5 *If baselines and claims are uniformly impartial then equal treatment of equals is robust to baseline rationing.*

We now consider a slightly stronger condition. We say that baselines and claim-baseline differences are ordered like claims if whenever $c_i \leq c_j$ then $b_i \leq b_j$ and $c_i - b_i \leq c_j - b_j$. Then, we have the following:

Proposition 6 *If baselines and claim-baseline differences are ordered like claims then order preservation is robust to baseline rationing.*

Proof. Let R be a rule satisfying order preservation and let (N, b, c, E) be an extended problem for which baselines and claims are similarly ordered. Let $i, j \in N$ be such that $c_i \leq c_j$ (and hence $b_i \leq b_j$ and $c_i - b_i \leq c_j - b_j$). We have to show that

- $R_i^b(N, b, c, E) \leq R_j^b(N, b, c, E)$.
- $c_i - R_i^b(N, b, c, E) \leq c_j - R_j^b(N, b, c, E)$.

To do so, we distinguish two cases.

Case 1. $E \leq \widetilde{B}$.

In this case, $R_i^b(N, b, c, E) = R_i^*(N, \widetilde{b}, E)$ and $R_j^b(N, b, c, E) = R_j^*(N, \widetilde{b}, E)$. Now, as order preservation is a self-dual property, it follows that R^*, the dual rule of R, is order preserving too. As b and c are similarly ordered, it follows that $\widetilde{b}_i \leq \widetilde{b}_j$. Altogether, we have that $R_i^b(N, b, c, E) \leq R_j^b(N, b, c, E)$.

As for the second item, note that the fact that b and c are similarly ordered implies that $c_i - \widetilde{b}_i \leq c_j - \widetilde{b}_j$. Thus, as R^* is order preserving, it follows that

$$c_i - R_i^b(N, b, c, E) = c_i - R_i^*(N, \widetilde{b}, E) \leq c_j - R_j^*(N, \widetilde{b}, E) = c_j - R_j^b(N, b, c, E),$$

as desired.

Case 2. $E \geq \widetilde{B}$.

In this case, $R_i^b(N, b, c, E) = \widetilde{b}_i + R_i(N, c - \widetilde{b}, E - \widetilde{B})$ and $R_j^b(N, b, c, E) = \widetilde{b}_j + R_j(N, c - \widetilde{b}, E - \widetilde{B})$. Note that $\widetilde{b}_i \leq \widetilde{b}_j$, $c_i - \widetilde{b}_i \leq c_j - \widetilde{b}_j$, and R satisfies order preservation. Thus, both items above easily follow. ■

4 Final remarks

In this paper, we have explored an extended framework of the conventional rationing model including baselines profiles. An individual baseline can be interpreted as an objective entitlement (in budgeting situations), an ideal target (in greenhouse gas emissions), or past consumption (in water rationing problems). We have introduced a natural family of (extended) rationing rules and argued that this family encompasses a series of real-life rationing situations. It is somewhat surprising that with little or no structure on the arbitrarily chosen baseline profiles this family still proves relatively robust in preserving a series of well known and desirable properties from the standard rationing model.

As mentioned in the introduction we have followed a direct and an axiomatic approach to study rationing problems. There is yet a third approach, the so-called *game theoretic approach*, which consists in modeling rationing problems as a game and aims at identifying the likely outcome of such a game to the solution of the rationing problem. This is the approach taken, for instance, by Pulido et al., (2002, 2008) to analyze what they call *bankruptcy situations with references*, which are a specific case of the baseline rationing problems considered in this paper.

Our model is also related to another extension of the standard rationing model provided to account for *multi-issue rationing problems*, i.e., rationing problems in which claims refer to different issues (e.g., Kaminski, 2006; Ju et al., 2007; Moreno-Ternero, 2009). In actual bankruptcy laws, the creditors' legal characteristics are typically divided among categories (e.g., secured claims, unsecured claims, taxes, and trustee expenses) which calls for a more complex model in which, rather than single claims, vectors of claims, each indicating the claim for a given characteristic, are considered (e.g., Kaminski, 2006). In the canonical case with only two issues, the underlying mathematical model is equivalent to our model, in which two profiles of individual characteristics (baselines and claims in our case) are included in each rationing problem. Our main departure from that case is to provide an asymmetric role to both profiles, considering baselines as tentative allocations, and claims as means to correct them.

Finally, it is worth mentioning that an alternative approach to baseline rationing, in which baselines are assumed to be an endogenous component of a standard rationing problem, can also be considered. In some rationing problems, it makes sense to postulate that the part of a claim that is above a certain threshold (e.g., the amount to divide) should be ignored. Somewhat polar to the previous idea is the principle that a minimal amount (a lower bound) should be ensured for each agent (e.g., Moreno-Ternero and Villar, 2004; Dominguez and Thomson, 2006). One could then define, for each standard rationing problem, a baseline profile, to be interpreted as a rule violating the requirement of balance described in the definition of the standard model. Formally, a baseline b would be a function $b\colon \mathcal{D} \to \bigcup_{N\in\mathcal{N}} \mathbb{R}^n$, associating with each problem $(N,c,E) \in \mathcal{D}$ a profile $b\,(N,c,E)$ satisfying $0 \leq b_i\,(N,c,E) \leq c_i$ for each $i \in N$. If $\sum_{i\in N} b_i\,(N,c,E) < E$, then $b\,(N,c,E)$ can be interpreted as a profile of lower bounds for that problem. If, on the other hand, $\sum_{i\in N} b_i\,(N,c,E) > E$, then $b\,(N,c,E)$ can be interpreted as a new claims profile for that problem. The analysis of such endogenous baselines, and their connections to the study of operators for the space of (standard) rationing rules (e.g., Thomson and Yeh, 2008), is undertaken in a companion paper (Hougaard et al., 2010).

5 Appendix

To save space, we have included in this appendix, which is not for publication, the proofs of some statements made in the text.

5.1 On the tightness of the characterization result

We now provide examples of extended rules showing the independence of the axioms in Theorem 1. In what follows, let A denote the (standard) constrained equal awards rule and P denote the (standard) proportional rule.[11]

- Let $\widetilde{R}$ be defined by

$$\widetilde{R}(N,c,b,E) = \begin{cases} P(N,\widetilde{b},E), & E \leq \widetilde{B} \\ \widetilde{b} + P(N,c-\widetilde{b},E-\widetilde{B}) & \text{if } E \geq \widetilde{B} \quad \text{and } c_i \geq b_i \text{ for all } i \in N \\ \widetilde{b} + A(N,c-\widetilde{b},E-\widetilde{B}) & E \geq \widetilde{B} \quad \text{and } c_i < b_i \text{ for some } i \in N, \end{cases}$$

  It is straightforward to show that $\widetilde{R}$ is a well-defined rule that satisfies Truncation of Excessive Claims, Baseline Invariance and Baseline Duality but not Baseline Truncation.

[11]Formally, for all $(N,c,E) \in \mathcal{D}$, $A\,(N,c,E) = (\min\{c_i,\lambda\})_{i\in N}$ where $\lambda > 0$ is chosen so that $\sum_{i\in N} \min\{c_i,\lambda\} = E$, whereas $P\,(N,c,E) = \frac{E}{C} \cdot c$.

- Let $\widetilde{R}$ be defined by

$$\widetilde{R}\,(N,b,c,E)=\begin{cases} P(N,c,E) & \text{if } \; E\leq \widetilde{B} \\ \widetilde{b}+P(N,c-\widetilde{b},E-\widetilde{B}) & \text{if } \; E\geq \widetilde{B} \end{cases}$$

  It is straightforward to show that $\widetilde{R}$ is a well-defined rule that satisfies Baseline Truncation, Baseline Invariance and Baseline Duality but not Truncation of Excessive Claims.

- Let $\widetilde{R}$ be defined by

$$\widetilde{R}\,(N,b,c,E)=\begin{cases} \widetilde{b}-P(N,\widetilde{b},\widetilde{B}-E) & \text{if } \; E\leq \widetilde{B} \\ P(N,c,E) & \text{if } \; E\geq \widetilde{B} \end{cases}$$

  It is straightforward to show that $\widetilde{R}$ is a well-defined rule that satisfies Baseline Truncation, Truncation of Excessive Claims and Baseline Duality but not Baseline Invariance.

- Let $\widetilde{R}$ be defined by

$$\widetilde{R}\,(N,b,c,E)=\begin{cases} \widetilde{b}-A(N,\widetilde{b},\widetilde{B}-E) & \text{if } \; E\leq \widetilde{B} \\ \widetilde{b}+P(N,c-\widetilde{b},E-\widetilde{B}) & \text{if } \; E\geq \widetilde{B} \end{cases}$$

  It is straightforward to show that $\widetilde{R}$ is a well-defined rule that satisfies Baseline Truncation, Truncation of Excessive Claims and Baseline Invariance but not Baseline Duality.

5.2 On the proof of Proposition 1

Illustration of the argument when the property is punctual. Let R be a rule satisfying order preservation in gains but not order preservation in losses. Then, its dual rule, R^*, violates order preservation in gains. More precisely, there exists a problem $(N,c,E)\in\mathcal{D}$ and $i,j\in N$ such that $c_i\leq c_j$, and for which it holds that $R_i^*(N,c,E)>R_j^*(N,c,E)$. Then, $R^b(N,c,c,E)=R^*(N,c,E)$, from where it follows that R^b violates the extended property of order preservation in gains.

Illustration of the argument when the property is relational. Let R be a rule satisfying claims monotonicity but not linked monotonicity. Then, its dual rule, R^*, violates claims monotonicity. More precisely, there exist two problems $(N,c,E),(N,c',E)\in\mathcal{D}$ such that $c_i\leq c_i'$ for some $i\in N$, and $c_{-i}'\equiv c_{-i}$, and for which it holds that $R_i^*(N,(c_i',c_{-i}),E)<R_i^*(N,(c_i,c_i),E)$. Then, $R^b(N,c,c,E)=R^*(N,c,E)$ and $R^b(N,c,c',E)=R^*(N,c',E)$. It then follows from here that R^b violates the extended version of claims monotonicity.

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