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On the Dynamics of Competing Energy Sources

by

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On the dynamics of competing energy sources

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Abstract

We characterize the dynamics of energy markets in which energy is derived from polluting (fossil) and clean (solar) resources. The analysis is based on geometric optimal control considerations. An important feature of solar energy technologies is that their cost of supply is predominantly due to upfront investment in capital infrastructure (rather than to actual supply rate) and this feature has important implications for the market allocation outcome. In particular, it gives rise to a threshold behavior in that solar energy is adopted only when the price of fossil energy exceeds a certain threshold. Under this condition solar technologies will (eventually) dominate energy supply by driving fossil energy altogether out of the energy sector. A tax on fossil energy can have a substantial impact since it changes the threshold price. A quantity restriction (e.g., a cap on fossil energy) allows for the coexistence of clean and polluting energy technologies also in the long run, and its effect on the use of fossil energy is more moderate.

Keywords: fossil and solar energy, optimal processes, characteristic curves, price thresholds, environmental regulation.

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1 Introduction

Fossil fuels are often mentioned as the main culprit for an impressive list of undesirable consequences, including acid rain, smog, increased atmospheric concentration of greenhouse gases and financing of failed states and terror organizations. Yet, they continue to be the primary source of energy generation worldwide, fueling over 80% of total energy production and this share is not expected to decline anytime soon (International Energy Agency 2008). The obvious reason is that the market price of fossil energy is in most places cheaper than any of the alternative energy sources available. Market prices, however, ignore externalities and the adverse consequences listed above are all external effects par excellence. Regulating these external effects requires understanding the underlying market forces that determine the allocation of energy generation between fossil and alternative sources. We characterize the dynamics of the market allocation processes and use this framework to study market-based regulation in the form of taxes or production caps on fossil energy.

The economic processes underlying the energy sector are of interest because of the social relevance associated with environmental consequences and because their study reveals a rich plethora of dynamic behavior, calling for novel techniques of characterization and interpretation (Haurie 2005, Bencheckroun et al. 2005, Haurie and Moresino 2006). The recent Special Issue of Automatica (Haurie and Malhamé 2008) attests to the increasing attention they receive from the Dynamic Optimization community (see, in particular Bahn et al. 2008, Leizarowitz 2008). The present work contributes to this line of research by presenting a complete analytic characterization of the multi-dimensional energy allocation processes via geometric optimal control considerations.

We study an economy in which energy is a primary input of production along with the traditional labor and capital inputs. Energy can be derived from fossil fuels or from alternative sources, e.g. solar, wind or hydro, referred to generically as solar energy. Solar energy entails none of the external effects listed above and also differs in another important respect: while fossil energy generation depends on supply of fuels that give rise to a substantial variable cost component, solar energy generation is based on capital designated especially for that purpose. Once the solar infrastructure (wind turbines, solar thermal collectors, photovoltaic panels) has been installed, the generation of solar energy entails very little additional cost. This distinguishing feature is important for understanding the market forces underlying the energy sector and the ensuing market allocation of fossil and solar energy. In particular it gives rise to a threshold fossil energy price below which solar energy will never be used. In contrast, if the price of fossil energy exceeds this threshold, investment in solar energy capital will begin at some finite time and gradually increase until eventually driving fossil energy out of the energy sector altogether. The threshold effect, in turn, renders the market allocation sensitive to the details of the regulation policy designed to restrict the use of fossil energy.

The economy, described in Section 2, consists of a final good sector, an energy sector, and households owning labor and capital. The energy sector consists of fossil energy firms and solar energy firms. This structure extends that of Tsur and Zemel (2008) by treating solar energy as an endogenous sector of the economy.[1] In Section 3 the dynamic market allocation processes are

[1]In Tsur and Zemel (2008) solar energy is purchased at a given (exogenous) price.

characterized (the long-run equilibrium as well as the transition path leading towards it). This task is carried out by analyzing the geometric relations between the characteristic curves (Tsur and Zemel 2005, 2007) that give rise to the turnpike of this problem. Here, however, an essential input (energy) consists of a combination of state and control variables, hence the usual characteristic curves (the singular arc and the locus of feasible equilibria) must be complemented by a third curve, measuring the demand for energy when the latter is supplied at the fossil price. The intersection of the curves determines the above-mentioned threshold fossil price which is shown to provide a necessary and sufficient condition for the adoption of solar technologies. Economies that satisfy this condition, referred to as solar-based economies, begin to invest in solar capital at some finite time, gradually increasing the share of solar technologies in total energy generation until eventually driving fossil energy out of the energy sector altogether. In economies that fail to satisfy this condition (referred to as fossil-based economies) investment in solar capital never takes place unless induced by some form of regulation. In order to focus attention on the difference in cost structure between the two energy sources and the associated threshold, the analysis in this section abstracts from other important features of the energy market such as the technological constraints associated with solar energy as well as trends and fluctuations in the price of fossil energy and regulation measures to address the externalities entailed by its use.

Two common forms of energy regulation, namely fossil emission taxes and production caps, are studied in Section 4. As expected, both policies reduce the use of fossil energy in the long run. However, the threshold feature of the market allocation outcome implies that an emission tax can have a drastic effect if set at a rate that gives rise to an effective fossil energy price above the threshold energy price, in which case the economy's type switches from fossil-based to solar-based. Under the fossil energy cap policy, the use of fossil energy will not diminish below the imposed cap, hence the effect of this regulatory tool is more moderate than that of the emission tax.

The literature on energy economics and the competition among various technologies is vast and no attempt is made to review it here. Early concerns revolved around scarcity of fossil resources and the limit it imposes on economic growth (Barnett and Morse 1963). Technological progress and discoveries of new coal, oil and gas reserves on the one hand, together with rapidly deteriorating environmental quality on the other, have swung the pendulum towards environmental concerns. R&D efforts to develop a backstop substitute for fossil fuels have been suggested as an answer to both the scarcity and environmental concerns (Nordhaus 1973, Dasgupta and Heal 1974, 1979, Dasgupta and Stiglitz 1981, Tsur and Zemel 2003, and references they cite). The recent Stern (2007) and IPCC4 (2007) reports added urgency to the environmental concerns and renewed interest in threats associated with advancing occurrence of catastrophes of global scale (Clarke and Reed 1994, Tsur and Zemel 1996, Alley et al. 2003, Nævdal 2006, Roe and Baker 2007, Weitzman 2009). The regulation literature deals primarily with tradeoffs between prices (carbon tax) and quantity (cap-and-trade) measures (see Stern 2007, Bushnell et al. 2008, Dietz and Maddison 2009, and reference cited there). The present effort studies the market forces underlying the penetration of solar energy technologies and provides the analytic framework to compare these regulation measures within a dynamic context.

2 The economy

The economy consists of a final good sector, an energy sector and households. We discuss each in turn.

2.1 Final good

Firm i uses capital K_i, energy $X_i = X_i^f + X_i^a$ and labor L_i to produce output Y_i according to the constant-returns-to-scale technology $Y(K_i, X_i, L_i)$, where X_i^f is fossil energy and X_i^a is energy derived from alternative sources, such as solar and wind, which serve as perfect substitutes. We refer to these alternative sources generically as 'solar energy'. Thus,

$$Y_i = L_i y(k, x) \tag{2.1}$$

where $y(k, x) \equiv Y(k, x, 1)$, $k \equiv K_i/L_i$ and $x \equiv X_i/L_i$ are the same across firms that use the same technology, hence the firm subscript i can be dropped. The production function $y(\cdot, \cdot)$ satisfies the standard properties

$$\begin{gathered} y(0,x) = 0;\ y(k,0) = 0;\ y_k(k,x) > 0;\ y_x(k,x) > 0;\ y_k(0,x) = \infty; \\ y_{kk}(k,x) < 0;\ y_{kx}(k,x) > 0;\ y_{xx}(k,x) < 0; \\ y_{kk}(k,x) y_{xx}(k,x) - y_{kx}^2(k,x) > 0, \end{gathered} \tag{2.2}$$

where k and x subscripts signify partial derivatives with respect to k and x.

Firms take as given the capital rental rate r, the prices of fossil and solar energy, p^f and p^a, and the wage rate w and plan production in order to maximize instantaneous profit

$$L_i[y(k, x) - (r + \delta)k - p^f x^f - p^a x^a - w] \tag{2.3}$$

where x^f and x^a are, respectively, the per worker fossil and solar energy inputs and δ is the capital depreciation rate. Necessary conditions for profit maximization include

$$y_k(k, x) = r + \delta \tag{2.4}$$

and

$$y_x(k, x) = p \equiv \min(p^f, p^a). \tag{2.5}$$

As fossil and solar energy are perfect substitutes, firms will use only the cheaper source if $p^f \neq p^a$ and will be indifferent between the two sources if $p^f = p^a$. Thus, the per capita energy cost can be expressed as

$$p^f x^f + p^a x^a = px. \tag{2.6}$$

2.2 Energy

Aside from the external effects listed in the introduction (and addressed in Section 4), fossil and solar energy differ in one main respect: while fossil energy depends on the supply of fuels that give rise to a substantial variable cost component, solar energy supply is based on capital designated especially for that purpose (wind turbines, solar thermal collectors, photovoltaic panels). Once the solar infrastructure has been installed, the generation of solar energy entails hardly any further cost. When solar capital is irreversible (cannot be rented in and out), this feature implies that the decisions of managers of solar energy firms are of an intertemporal investment type. These considerations are explicitly addressed below.

2.2.1 Fossil energy

Let ζ represent the unit cost of fossil energy, assumed constant. The supply curve of fossil energy is therefore horizontal and the competitive price of fossil energy is

$$p^f = \zeta. \tag{2.7}$$

When the price of energy is ζ, final-good firms will demand the energy input x such that (cf. (2.5))

$$y_x(k, x) = \zeta. \tag{2.8}$$

For any capital stock k, we denote by $x^\zeta(k)$ the energy input x that satisfies (2.8).

2.2.2 Solar energy

Production of solar energy uses capital designated solely for that purpose such that the energy output of solar energy firm j is

$$X_j^a = bA_j, \tag{2.9}$$

where A_j is the firm's stock of solar capital and b is a technological parameter indicating the rate of energy output per unit of capital. Solar capital depreciates at the same rate δ as capital k, but is irreversible in that it cannot be rented in and out. Thus, the firm is locked with its existing capital and will supply the solar energy it produces at the going market price $p(t)$, obtaining the revenue flow $p(t)bA_j(t)$.

Based on the market prices $p(t)$ (of energy) and $r(t)$ (of capital) the investment rate of the solar firms, $I_j(t)$, is determined according to:

$$\max_{\{0 \le I_j(t) \le \bar{I}_j\}} \int_0^\infty [p(t)bA_j(t) - I_j(t)] e^{-\int_0^t r(\tau)d\tau} dt \tag{2.10}$$

subject to

$$\dot{A}_j(t) = I_j(t) - \delta A_j(t) \tag{2.11}$$

and $A_j(0) = 0$. The upper bound $\bar{I}_j$ on the investment rate is due to physical and financial constraints.[2]

We let $I(t) = \sum_j I_j(t)$ represent aggregate investment in solar energy capital, $A(t) = \sum_j A_j(t)$ denotes the aggregate stock of solar capital, $X^a(t) = bA(t)$ is the aggregate solar energy supply rate and $\iota(t)$, $a(t)$ and $x^a(t) = ba(t)$ denote their per-capita counterparts. The per-capita solar capital evolves in time according to

$$\dot{a}(t) = \iota(t) - \delta a(t) \tag{2.12}$$

where $\iota(t)$ is constrained by the upper bound $\bar{\iota}$, and the per-capita total energy supply rate is

$$x(t) = x^f(t) + x^a(t) = x^f(t) + ba(t). \tag{2.13}$$

2.3 Households

The household income at time t consists of wage income $w(t)$ plus interest on savings $r(t)k(t)$ plus revenues from solar energy firms (owned by the households) $p^a(t)x^a(t)$ minus the investment costs of solar firms $\iota(t)$. In equilibrium, the wage rate that clears the labor market gives vanishing profits to the final good producers, implying, noting (2.3) and (2.7),

$$w(t) = y(k(t), x(t)) - \zeta x^f(t) - p^a(t)x^a(t) - [r(t) + \delta]k(t). \tag{2.14}$$

The household income, thus, equals $y(k(t), x(t)) - \zeta x^f(t) - \iota(t) - \delta k(t)$, which the household allocates between consumption $c(t)$ and saving, giving rise to the intertemporal budget constraint

$$\dot{k}(t) = y(k(t), x^f(t) + ba(t)) - \zeta x^f(t) - \iota(t) - \delta k(t) - c(t). \tag{2.15}$$

[2]The exact value of the upper bound is insignificant so long as it is large enough to avoid feasibility restrictions on the optimal processes.

The utility from consuming at the rate c is $u(c)$, assumed increasing, strictly concave and satisfying $u(0) = -\infty$, so that some positive consumption is essential. A consumption stream $c(t),\ t \geq 0$, generates the payoff

$$\int_0^\infty u(c(t))e^{-\rho t}dt, \tag{2.16}$$

where ρ is the pure (utility) rate of discount. The household seeks the consumption stream $c(t)$ that maximizes (2.16) subject to (2.15), given $k(0) = k_0$. In doing so households take firms (energy and final good) decisions exogenously.

2.4 Equilibrium

The economy is in equilibrium when all actors (households, managers of final good firms, managers of fossil energy firms and managers of solar energy firms) act rationally and none has an incentive to modify decisions. In equilibrium, the energy and capital price processes, $\{p(t), r(t),\ t \geq 0\}$, clear the energy and capital markets, i.e., at each point of time energy demand by the final good firms just equals the energy supply of fossil and solar firms and households savings just equal the capital demand of the final good firms.

Absent market failures, the competitive equilibrium processes are socially optimal in that they maximizes (2.16) among all feasible processes. We use this property to characterize the market allocation processes in the next section. In Section 4 we study market-based environmental regulation.

3 Market allocation

Without external effects and other sources of market failure, the market mechanism will give rise to an optimal allocation. Thus, the market allocation under these conditions can be characterized by solving

$$\max_{\{c(t),x^f(t),\iota(t)\}} \int_0^\infty u(c(t))e^{-\rho t}dt \tag{3.1}$$

subject to (2.12) and (2.15), $c(t) \geq 0$, $x^f(t) \geq 0$, $\iota(t) \in [0, \bar{\iota}]$, $k(0) = k_0 > 0$ and $a(0) = 0$. This intertemporal optimization problem has two states (the ordinary and solar capital stocks k and a) and three controls (consumption c, investment in solar capital ι, and fossil energy input x^f). We refer to the solution of (3.1) as the market allocation processes. We characterize these processes by means of three characteristic curves, defined in the capital-energy (k, x) plane.

3.1 Characteristic curves

We specify three curves that divide the (k, x) plane into distinct regions, in each of which the market allocation is restricted to a particular behavior. The first curve corresponds to the "singular" policy under which

$$y_k(k(t), x(t)) = by_x(k(t), x(t)), \tag{3.2}$$

i.e., the marginal products of k and of a are kept equal during a non-vanishing time interval. Condition (3.2), called the singular condition, defines a curve in the (k, x) plane, denoted $x^s(k)$ and referred to as the singular curve. The term "singular" comes from the property that problem (3.1) is linear in the solar investment rate $\iota(t)$. This implies that the optimal $\iota(t)$ process can either assume the corner values $\iota = \bar{\iota}$ or $\iota = 0$ or a singular, intermediate value (see Appendix B). The latter policy $\iota = \iota^s$ is optimal when the (k, x) process proceeds along the singular curve with $x^f = 0$. Invoking (2.12)-(2.15) this requires

$$\iota^s = \frac{x^{s\prime}(k)[y(k, x^s(k)) - c - \delta k] + \delta x^s(k)}{b + x^{s\prime}(k)}, \tag{3.3}$$

where $x^{s\prime}(k) \equiv dx^s/dk$.[3]

A second curve in the (k, x) plane, denoted $x^e(k)$, is defined by the condition

$$y_k(k, x) = \rho + \delta. \tag{3.4}$$

Points along this curve satisfy the Ramsey condition (Ramsey 1928) for a steady state, hence we refer to it as the steady state curve.

The third curve, denoted $x^\zeta(k)$, corresponds to energy demand when the unit price of energy is ζ. It is defined by condition (2.8), which relates the marginal product of energy to the unit cost of fossil fuel. This curve depicts the demand for energy as a function of capital k when some fossil energy is used.

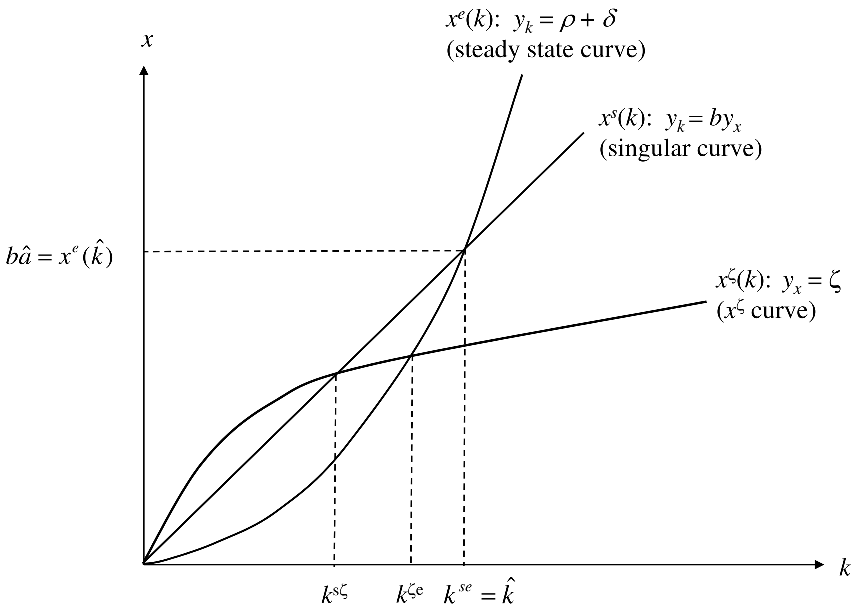

Figure 1: Characteristic curves for solar-based economies

Figures 1 and 2 display the three curves for a Cobb-Douglas technology

[3]The singular policy ι^s is feasible when the optimal consumption rate c yields $\iota^s \in [0, \bar{\iota}]$.

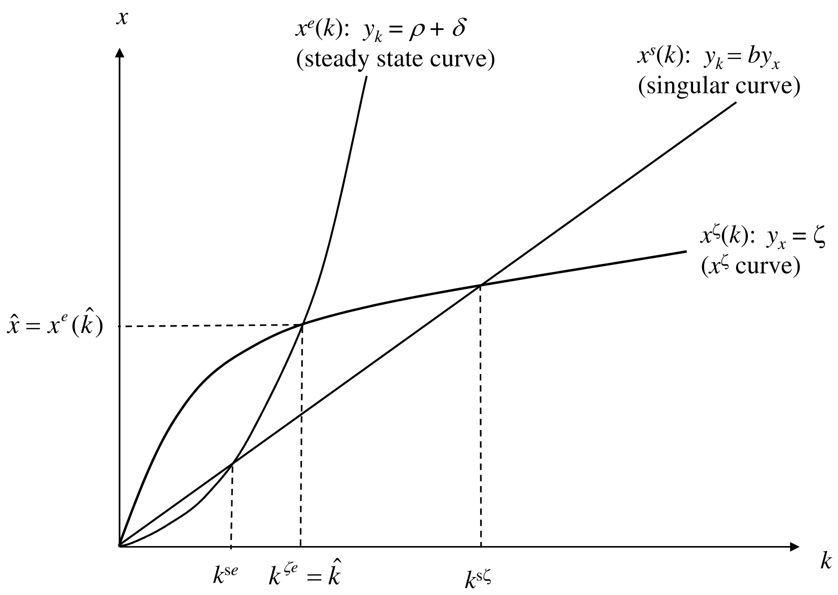

Figure 2: Characteristic curves for fossil-based economies

$y = y_0 k^\alpha x^\beta$ with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. As a matter of notation, we say that (k, x) is above or below $x^j(\cdot)$, $j = e, s, \zeta$, if $x > x^j(k)$ or $x < x^j(k)$, respectively. The geometrical relations among the three curves underlie the characterization of the market allocation processes. For example, a point above the singular curve represents a surplus of solar capital (relative to physical capital k) and implies the market outcome $\iota = 0$ (no solar investment). Using assumption (2.2), we verify in Appendix A the following properties:

Property 3.1. *The three characteristic curves* (i) *converge at the origin* $(0, 0)$, *and* (ii) *are increasing* $(i.e.\ dx^j(k)/dk > 0,\ j = e, s, \zeta)$.

Assuming that each pair of curves cross at least once away from the origin (i.e. with $k > 0$, $x > 0$), their relative geometry is completely determined:

Property 3.2. (i) $x^\zeta(\cdot)$ *crosses* $x^s(\cdot)$ *once from above.* (ii) $x^e(\cdot)$ *crosses* $x^s(\cdot)$ *once from below.* (iii) $x^\zeta(\cdot)$ *crosses* $x^e(\cdot)$ *once from above.*

Let k^{se} denote the k level at which $x^s(\cdot)$ and $x^e(\cdot)$ intersect, $k^{s\zeta}$ be the k level at which $x^s(\cdot)$ and $x^\zeta(\cdot)$ intersect, and $k^{\zeta e}$ be the k level at which $x^\zeta(\cdot)$ and $x^e(\cdot)$ intersect. Note that $k^{\zeta e}$ must always fall between the two other intersection points (Figures 1-2). In general, the three intersection points differ and the long term evolution of the economy depends on their relative positions. We investigate the long-run market allocation in the next subsection and study the transitional path in subsection 3.3.

3.2 Long-run market allocation

Define

$$\hat{k} = \max(k^{se}, k^{\zeta e}) \tag{3.5a}$$

and let

$$\hat{a} = \begin{cases} x^e(\hat{k})/b & \text{if } k^{se} > k^{\zeta e} \\ 0 & \text{otherwise} \end{cases}, \tag{3.5b}$$

$$\hat{x}^f = \begin{cases} 0 & \text{if } k^{se} > k^{\zeta e} \\ x^e(\hat{k}) & \text{otherwise} \end{cases}, \tag{3.5c}$$

$$\hat{y} = y(\hat{k}, \hat{x}^f + b\hat{a}), \tag{3.5d}$$

$$\hat{\imath} = \delta\hat{a} \tag{3.5e}$$

and

$$\hat{c} = \hat{y} - \zeta\hat{x}^f - \delta(\hat{k} + \hat{a}). \tag{3.5f}$$

Then (see proof in Appendix B):

Proposition 3.1. *The market allocation processes converge to the steady state specified by equations (3.5) from any capital endowment $k_0 > 0$ and $a_0 = 0$.*

From (3.5b)-(3.5c) we see that solar energy prevails in the long run if $k^{se} > k^{\zeta e}$. To see why, notice that $k^{se} > k^{\zeta e}$ implies $y_x(k^{se}, x^e(k^{se})) < \zeta$ (see Figure 1 and note that $y_x < \zeta$ above x^ζ). The singular and steady state curves intersect at $(k^{se}, x^e(k^{se}))$ where $y_k(k^{se}, x^e(k^{se})) = by_x(k^{se}, x^e(k^{se})) = \rho + \delta$. Thus, solar energy prevails in the long run (i.e., $k^{se} > k^{\zeta e}$) if and only if $\rho + \delta < b\zeta$ or

$$\frac{1}{b}(\rho + \delta) < \zeta. \tag{3.6}$$

The threshold energy price $(\rho+\delta)/b$ bears a simple economic interpretation. The solar capital stock $1/b$ generates a perpetual unit energy flow and inflicts the instantaneous cost of $1/b$ times the effective discount rate (the rate of interest plus the depreciation rate), which in the long run equals $\rho+\delta$.[4] Thus, $(\rho + \delta)/b$ is the long-run instantaneous cost of a perpetual flow of one unit of energy generated by solar technologies. The same unit energy flow can be derived from fossil sources at the instantaneous cost ζ. Thus, (3.6) is merely the condition under which solar energy is more cost effective (cheaper) in the long run, hence will (eventually) prevail. We summarize these considerations in:

Proposition 3.2. (i) *When the price of fossil energy ζ exceeds the threshold price $(\rho + \delta)/b$, the use of fossil energy gradually diminishes and long run production is based exclusively on solar energy.* (ii) *When ζ falls short of the threshold price $(\rho + \delta)/b$, in the long run energy is supplied exclusively from fossil sources.*

We refer to economies satisfying condition (3.6) as solar-based while economies for which the reverse condition holds are classified as fossil-based. Condition

[4]When $k^{se} > k^{\zeta e}$, Proposition 3.1 and equation (3.4) imply $y_k(\hat{k}, \hat{x}) = y_k(k^{se}, x^e(k^{se})) = \rho + \delta$.

(3.6) holds if and only if $k^{se} > k^{\zeta e}$, which in turn (noting Property 3.1(ii)) holds if and only if the singular curve crosses the steady state curve above the ζ-curve. Thus,

Remark 1. The economy is solar-based when the singular curve crosses the steady state curve above the ζ-curve (Figure 1) and is fossil-based when the singular curve crosses the steady state curve below the ζ-curve (Figure 2).

3.3 Transition path

Proposition 3.2 specifies the market allocation in the long run. Here we characterize the entire transitional path. Consider an economy with a capital endowment $k_0 < \min(k^{s\zeta}, k^{se})$ and a vanishing solar capital stock. Regardless of whether the economy is fossil-based or solar-based, initially the competitive market allocation entails no investment in solar capital ($\iota = 0$) and the economy grows along the x^{ζ} curve using fossil energy exclusively. For fossil-based economies (depicted in Figure 2), investment in solar capital never takes place (i.e., the $\iota = 0$ regime prevails indefinitely) and the economy approaches a steady state at the point $(k^{\zeta e}, x^{\zeta}(k^{\zeta e}))$, where the equilibrium and x^{ζ} curves intersect and conditions (3.4) and (2.8) are satisfied.

A solar-based economy (depicted in Figure 1) evolves along the x^{ζ} curve until it reaches $(k^{s\zeta}, x^{s}(k^{s\zeta}))$, where the x^{ζ} curve intersects the singular curve. Upon reaching this point, the solar investment policy switches to the singular regime, building up solar capital $a(t)$ at the singular rate

$$\iota^{s}(t) = y(k^{s\zeta}, x^{s}(k^{s\zeta})) - c(t) - \delta k^{s\zeta} - \zeta[x^{s}(k^{s\zeta}) - ba(t)], \qquad (3.7)$$

leaving k constant at $k^{s\zeta}$ and reducing the use of fossil energy such that the total energy use remains fixed at $x^{s}(k^{s\zeta})$. As soon as $a(t)$ is large enough to supply the entire energy demand, i.e., when $ba(t) = x^s(k^{s\zeta})$, both types of capital, $k(t)$ and $a(t)$, grow simultaneously along the singular curve (with solar investment given by (3.3)) towards the steady state $(k^{se}, x^s(k^{se}))$, where the singular curve intersects the steady state curve (see Figure 1) and conditions (3.4) and (3.2) are satisfied.

We summarize this behavior in:

Proposition 3.3. *The market allocation for an economy endowed with* $k(0) = k_0 < \min(k^{s\zeta}, k^{se})$ *and* $a(0) = 0$ *is characterized as follows:*

(*i*) *When* $(\rho + \delta)/b < \zeta$ *(solar based economies), the market processes evolve along the following three phases:* (*a*) *An initial fossil phase* (*with* $\iota(t) = a(t) = 0$)*, in which the economy grows along the* x^ζ *curve until it reaches the intersection point* $(k^{s\zeta}, x^s(k^{s\zeta})$ *with the singular curve* x^s*.* (*b*) *A coexistence phase, in which* $k(t)$ *and* $x(t)$ *are held fixed at* $k^{s\zeta}$ *and* $x^s(k^{s\zeta})$*, respectively, while fossil energy input* $x^f(t)$ *shrinks and solar energy input* $ba(t)$ *increases until the use of fossil energy vanishes.* (*c*) *A solar phase, in which* $x^f(t) = 0$ *and* $k(t)$ *and* $a(t)$ *grow together along the singular curve towards a steady state at the intersection point* $(k^{se}, x^s(k^{se})$ *of the singular and steady state curves.*

(*ii*) *When* $(\rho + \delta)/b \geq \zeta$ *(fossil based economies), no investment in solar energy ever takes place* ($\iota(t) = a(t) = 0$) *and the economy evolves along* x^ζ *towards a steady state at the intersection point* $(k^{\zeta e}, x^e(k^{\zeta e})$ *of* x^ζ *and the steady state curve.*

The proof is provided in Appendix B.

Remark 2. It follows from the explicit specification in Proposition 3.3 that the optimal policy is unique.

Each of the phases described in Proposition 3.3 can be recast as a standard dynamic optimization problem with a single state variable. Solving for the optimal processes during each phase, and determining the durations of the phases by the transversality conditions associated with the transition from one phase to the other, the complete time dependence of the socially optimal processes is derived. We denote the market state processes by $k^*(t)$ and $a^*(t)$, and the associated consumption, investment and fossil energy processes by $c^*(t)$, $\iota^*(t)$ and $x^{f*}(t)$, respectively. The corresponding capital and energy prices, defined by (2.4) and (2.5), are denoted $r^*(t)$ and $p^*(t)$.

3.4 Competitive equilibrium

The allocation processes characterized above constitute a competitive equilibrium for the economy described in Section 2. This means that: (i) households anticipating the processes $\iota^*(t)$, $x^{f*}(t)$ and $a^*(t)$ will choose to consume $c^*(t)$ and save $k^*(t)$, (ii) final good firms facing the energy price $p^*(t)$ and the capital rental rate $r^*(t)$ will demand the inputs $k^*(t)$ and $x^*(t)$ to produce $y(k^*(t), x^*(t))$, (iii) fossil energy firms facing the energy price $p^*(t)$ and solar capital $a^*(t)$ will supply $x^{f*}(t)$, and (iv) managers of solar firms, anticipating the energy and capital price processes $p^*(t)$ and $r^*(t)$, will adopt the investment policy $\iota^*(t)$ which gives rise to the solar capital process $a^*(t)$.

Notice that households are not the only forward looking agents in the economy, since managers of solar firms (unlike managers of final good and fossil energy firms) make intertemporal investment decisions. The interaction between the various actors in the economy is thus more involved than the standard situation in which all firms maximize instantaneous profits. Nonetheless, the property that absent market failures, a competitive equilibrium is optimal is retained (see Tsur and Zemel 2009b, Section 4 for a verification).

4 Environmental regulation

The market allocation ignores external effects and is therefore suboptimal. If the economy is solar-based (Figure 1), the market failure will diminish over time as the economy builds up solar capital and gradually drives fossil energy out of production (cf. Proposition 3.3(i)). In fossil-based economies (Figure 2) the market failure persists and a correction requires regulatory measures. We focus on fossil-based economies and study two regulation policies: emission taxes (price regulation) and a cap on fossil energy use (quantity regulation). We find significant differences in the responses corresponding to each policy and relate these differences to how the policies affect the threshold condition (3.6).

We focus on external effects that are global in nature (e.g., emission of greenhouse gases) hence require international coordination and enforcement, and assume that the regulatory measure is exogenously imposed on the economy (e.g., by an international treaty). Even when the regulation is locally determined, its goal is set according to some global criteria and can be taken as exogenous to the economy under consideration.[5] We thus assume that a tax or a cap policy is exogenously imposed and study their effect on the competitive allocation.

[5] A case in point are the various policies aimed at reducing greenhouse gas emission that are implemented locally (see Bushnell et al. 2008) while international attempts to reach a global (post Kyoto) agreement drag on for reasons well understood (Barrett 2003).

4.1 A fossil energy tax

Consider a regulatory measure in the form of a tax β imposed on x^f (or, equivalently, on emission of greenhouse gases). The effective price paid by the final good firms for fossil energy becomes $\zeta + \beta$.[6] Thus, the fossil tax has the same effect as an increase in the cost of fossil fuel. The situation is depicted in Figure 3. Increasing the price of fossil energy tilts the x^ζ curve downward. This is so because the diminishing marginal productivity of energy requires reducing the energy input, for each capital input, when the price of energy is increased. The upper x^ζ-curve corresponds to the original, tax free situation. The middle and lower x^ζ-curves are the results of taxing fossil energy at the rates β_1 and β_2, respectively, with $\beta_1 < \beta_2$. Point A in figure 3 is the original, tax-free long-run equilibrium. Upon levying the fossil tax β_1, the x^ζ curve tilts downward and becomes the curve labeled $x^{\zeta+\beta_1}$ in Figure 3. The new ζ-curve intersects the steady state curve x^e at point B, which lies above point C where the singular and steady state curves intersect. Therefore, the economy maintains it fossil-based type (cf. Remark 1) and no incentive to invest in solar energy is created in response to increasing the price of fossil energy from ζ to $\zeta + \beta_1$. However, the shift to the new equilibrium (From point A to point B in Figure 3) entails a reduction in the use of fossil energy due to its higher price.

Imposing the higher tax rate β_2 tilts the ζ-curve further downwards to the curve $x^{\zeta+\beta_2}$ which crosses the steady state curve x^e at point D – below the intersection C of the singular and steady state curves. Recalling Remark 1, we

[6]We assume that the tax proceeds are not redistributed back in the economy but paid to an external fund (e.g., to buy emission permits from foreign countries or to finance an environmental super fund that handles the damage).

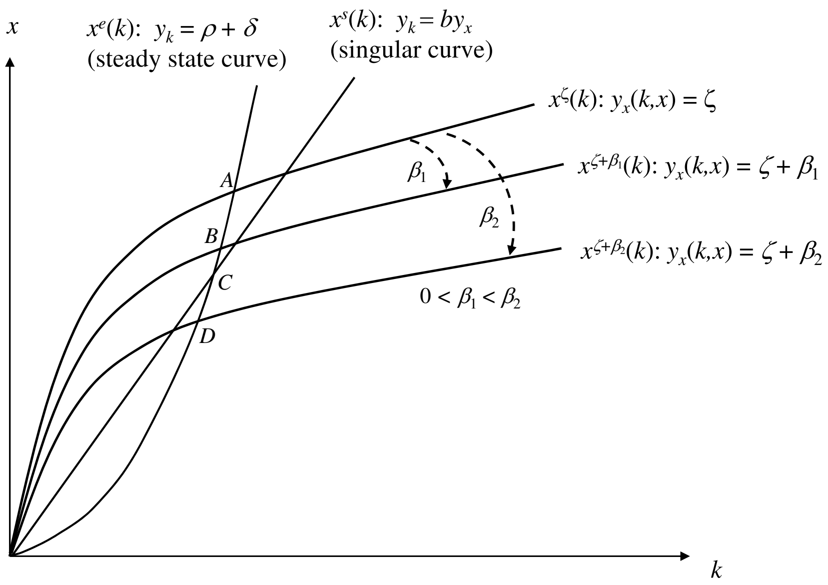

Figure 3: Consequences of taxing fossil energy.

find that the economy changes its type and becomes solar-based. As a result (Proposition 3.3(i)) it builds up solar capital gradually and eventually drives fossil energy out of the energy sector as it moves toward the new equilibrium point C. The tax reduces the profitability of fossil energy, the use of which will therefore diminish. We note that this regulation tool can take advantage of the high sensitivity of the market allocation to the fossil price when the latter is close to its threshold value. When this is the case, a relatively low tax rate can change the characterization of the economy and eliminate emissions altogether.

The dynamic characterization (Proposition 3.3) provides further insights on the implications of this policy. First, the lowering of the x^ζ curve implies a corresponding decrease in the steady state capital stock $\hat{k}$ and consumption rate $\hat{c}$. Thus, the benefits of the reduced emission can be weighted against the smaller objective for every value of β, helping to determine the optimal tax rate for the economy (when the regulator is free to do so). Second, the lowering of the x^ζ curve entails immediate reduction in emissions, since $x^\zeta(k)$ is lower at any k level along the curve and not only at the eventual steady state. This raises interesting possibilities regarding the time profile of the tax rate β. Early on, when $x^\zeta(k)$ falls short of the exogenous bound, a lower tax rate may be imposed, to be gradually increased as the economy expands and the demand for energy increases. A similar increase in time of the emission tax has been derived in Tsur and Zemel (2009a) as a tool to eliminate the hazard of catastrophic environmental events. We leave a more detailed study of these questions to future work.

4.2 A fossil energy cap

Suppose that a cap $\bar{x}_1^f$ on the per capita flow of fossil energy is imposed on a fossil-based economy. For the cap to be effective, it must be smaller than the equilibrium fossil energy flow under the cap-free economy, i.e., $\bar{x}_1^f < \hat{x}^f$ where $\hat{x}^f = x^\zeta(k^{\zeta e})$. The policy is enacted at $t = 0$, when the capital and energy inputs are k_0 and $x^\zeta(k_0)$, respectively (point A in Figure 4). If $x^\zeta(k_0) > \bar{x}_1^f$, the policy requires an immediate reduction of energy input to the allowed rate $\bar{x}_1^f$ (point B in Figure 4). Otherwise, the economy (and use of fossil energy) grow until the cap is reached. Upon reaching the allowed fossil energy cap, the economy evolves along the $x = \bar{x}_1^f$ line towards a steady state at point C, where the horizontal line intersects the steady state curve. The economy remains fossil-based, deriving energy solely from fossil sources, but the rate of energy use is reduced in compliance with the cap restriction.

Suppose now that the more stringent cap $\bar{x}_2^f$ is imposed instead. The econ-

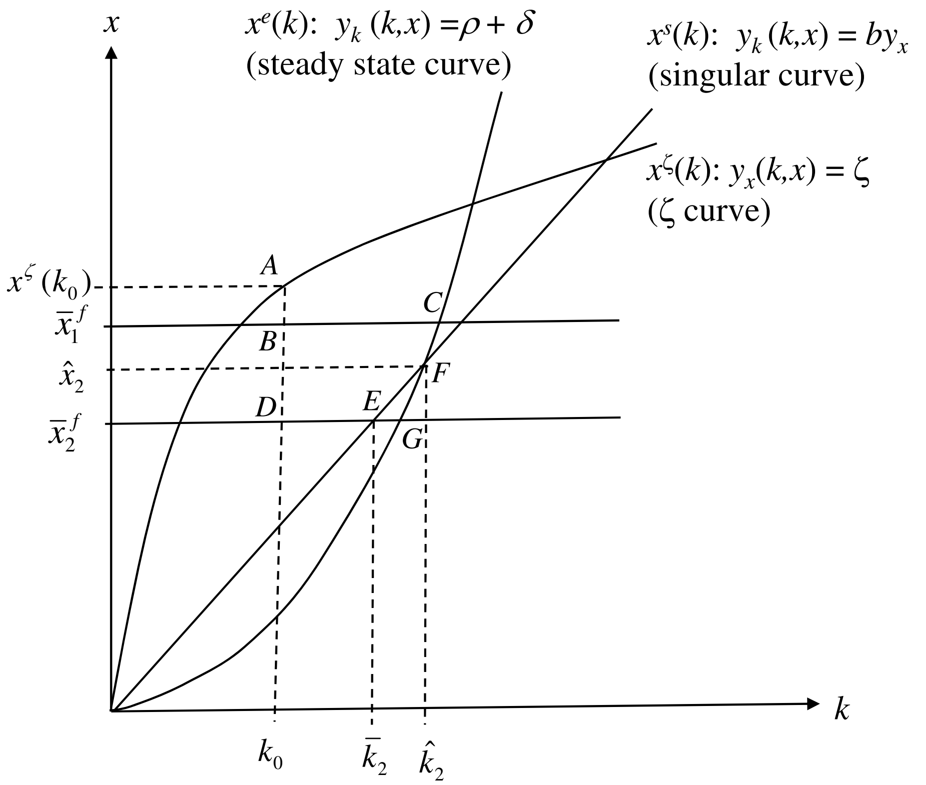

Figure 4: Consequences of imposing a cap on the use of fossil energy.

omy initially moves to point D by reducing the use of fossil energy to $\bar{x}_2^f$ and evolves along the $x = \bar{x}_2^f$ line until it reaches point E, where this line intersects the singular curve. At this point, with energy and capital inputs given by $\bar{x}_2^f$ and $\bar{k}_2$, respectively, yielding the capital rental rate $r = y_k(\bar{k}_2, \bar{x}_2^f) - \delta$ (cf. equation (2.4)), solar firms find it beneficial to start investing in solar capital. The economy then evolves along the singular curve towards a steady state at point F where the singular and steady state curves intersect. During this phase the economy continues to derive energy from fossil sources at the permitted rate $\bar{x}_2^f$ and augments it with solar power at the rate $x^s(k) - \bar{x}_2^f$.

We summarize these results in Proposition 4.1, making use of the following notation: (k^{se}, x^{se}) is the point of intersection between the singular and steady state curves, and k^{fe} is the capital stock at which the horizonal line $\bar{x}^f$ crosses the steady state curve $x^e(k)$.

Proposition 4.1. *suppose that a cap $\bar{x}^f$ is imposed on a fossil-based economy whose equilibrium energy input exceeds $\bar{x}^f$. (i) If $\bar{x}^f \geq x^{se}$ the economy approaches a steady state at $(k^{fe}, \bar{x}^f)$ and derives its energy input solely from fossil sources. (ii) If $\bar{x}^f < x^{se}$, the economy approaches a steady state at (k^{se}, x^{se}), where the energy input is divided between fossil sources (at the cap rate $\bar{x}^f$) and solar power (at the residual rate $x^{se} - \bar{x}^f$).*

The proof is outlined in Appendix C. In Figure 4, $\bar{x}_1^f > x^{se}$ corresponds to case (i) of the Proposition and the stringent cap policy $\bar{x}_2^f < x^{se}$ corresponds to case (ii). The horizontal segments BC (for $\bar{x}_1^f$) and DG (for $\bar{x}_2^f$) can be considered as the effective replacements of the ζ curve segment from A to $(k^{\zeta e}, x^{\zeta e})$ in forming the loci of the optimal trajectories under the imposed caps. Case (ii) offers the novel feature of simultaneous long term use of both energy sources. Without the cap, coexistence of the two energy types does not occur at all for fossil based economies, and can last only for a finite duration in solar based economies. The persistence of simultaneous use under the cap policy can be interpreted in terms of the threshold condition (3.6). Under this policy, the fossil price ζ is augmented by the shadow price associated with the constraint $\bar{x}^f - x^f \geq 0$. When the constraint is moderate, i.e. $\bar{x}^f > x^{se}$, even the augmented price is insufficient to reverse the inequality of the threshold condition and induce solar investments. The characterization of the economy as fossil based remains valid, but fossil energy use is fixed at the constrained rate. More stringent caps entail larger shadow prices, and the threshold condition (including the shadow price of the fossil cap constraint) implies a solar-based economy. However, the shadow price obtains a positive value only when fossil energy is used at the corner rate $x^f = \bar{x}^f$. The fossil rate x^f does not vanish in this case and the steady state must involve, therefore, the coexistence of both energy sources. Indeed, the cap restriction does not make fossil less profitable; it only restricts its quantity. Thus, if fossil energy is profitable without the restriction (which is the case for fossil-based economies), it will be used at the permitted rate also after the cap is imposed.

5 Conclusions

We study prospects for the penetration of solar energy technologies in a competitive economy, where energy (an essential factor of production) can be generated from polluting (fossil) or clean (solar) sources. We characterize the evolution of the market allocation processes and provide a necessary and sufficient condition for solar energy to prevail in the long run. This condition is specified in terms of the efficiency of solar energy generation (b), the price of fossil fuel (ζ) and the long term price of capital ($\rho+\delta$) and shows the effects of these parameters on the economic viability of solar energy in a competitive environment. The presence of the threshold condition implies large variations in the market response to price (tax) and quantity (cap) regulation policies that address the externalities associated with the use of fossil energy.

The analysis simplifies in a number of ways. First, no account is taken of fossil fuels scarcity. With scarcity included, the price of fossil energy will increase over time as the fossil reserves shrink, increasing the desirability of solar technologies. In this respect our analysis is somewhat overpessimistic about the prospects of solar energy. Second, the efficiency of solar energy generation can increase due e.g. to learning by doing or as a result of dedicated R&D efforts that consume resources (Dasgupta and Heal 1979, Chakravorty et al. 1997, Tsur and Zemel 2003, 2005, Gerlagh et al. 2009, and references they cite).

For example, learning by doing can be modeled by assuming that the solar efficiency parameter b is an (increasing) function of the aggregate solar capital A. Under the unregulated market allocation no investment in solar energy will take place and the fossil based economy will not realize the potential benefits of learning by doing. A possible remedy for this market failure comes in the form of a subsidy on investments in solar capital that will induce the solar firms to undertake such investments. The corresponding increase in $b(A)$ might suffice to meet the threshold condition, following which time solar investments will proceed even without the subsidy. A temporary subsidy (financed e.g., by the proceeds of an emission tax) can change the classification of the economy from fossil based to solar based and eliminate emissions in the long run.

Finally, effects of sustained economic growth can be considered (as in Tahvonen and Salo 2001, Smulders and de Nooij 2003, Tsur and Zemel 2009a). Any of these changes will constitute a valuable extension.

Acknowledgements

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Appendix

A Properties of the characteristic curves

Property 3.1

Proof. Suppose $x^e(0) = x_0 > 0$, then $y_k(0, x_0) = \rho + \delta$ violating (2.2) which states that $y_k(0, x) = \infty$ for all $x > 0$. Suppose that the $x^e(\cdot)$ curve crosses the k axis at some state $k_0 > 0$, then $y_k(k_0, 0) = \rho + \delta$ violating (2.2) which implies $y_k(k, 0) = 0$ for all $k > 0$. Therefore the $x^e(\cdot)$ curve must approach the origin. Similar considerations apply for the other two curves, establishing (i).

Taking the derivatives of (3.2), (3.4) and (2.8) we obtain

$$\frac{dx^s(k)}{dk} = \frac{by_{kx} - y_{kk}}{y_{kx} - by_{xx}} > 0, \tag{A.1}$$

$$\frac{dx^e(k)}{dk} = -\frac{y_{kk}}{y_{kx}} > 0 \tag{A.2}$$

and

$$\frac{dx^\zeta(k)}{dk} = -\frac{y_{kx}}{y_{xx}} > 0, \tag{A.3}$$

establishing (ii). □

Property 3.2:

Proof. Evaluated at a crossing point, (A.1) and (A.3) give

$$\frac{dx^s(k)}{dk} - \frac{dx^\zeta(k)}{dk} = \frac{y_{kk}y_{xx} - y_{kx}^2}{-y_{xx}(y_{kx} - by_{xx})} > 0.$$

Since multiple crossings imply alternating signs for the slope difference, this entails (i). A similar comparison of (A.1) and (A.2) at a crossing point gives

$$\frac{dx^s(k)}{dk} - \frac{dx^e(k)}{dk} = \frac{-b(y_{kk}y_{xx} - y_{kx}^2)}{y_{kx}(y_{kx} - by_{xx})} < 0,$$

establishing (ii). Finally

$$\frac{dx^{\zeta}(k)}{dk} - \frac{dx^{e}(k)}{dk} = \frac{y_{kk}y_{xx} - y_{kx}^2}{y_{kx}y_{xx}} < 0,$$

which verifies (iii). □

B Characterization of the market allocation

The market allocation processes are the outcome of

$$v(k_0) = \max_{\{c(t)\geq 0, x^f(t)\geq 0, \iota(t)\in[0,\bar{\iota}]\}} \int_0^{\infty} u(c(t))e^{-\rho t}dt \tag{B.1}$$

subject to (2.12)-(2.15), given $k(0) = k_0$ and $a(0) = 0$. The bound $\bar{\iota}$ is assumed to be large enough so that the singular policy is feasible, and the $k(\cdot)$ process decreases under the $\iota = \bar{\iota}$ regime for the relevant k domain.

When no risk of confusion arises, we suppress the time argument t. The current-value Hamiltonian corresponding to (B.1) is

$$\mathcal{H} = u(c) + \lambda[y(k, x^f + ba) - \zeta x^f - \iota - c - \delta k] + \gamma[i - \delta a] \tag{B.2}$$

where λ and γ are the current value costates of k and a, respectively. Defining

$$\phi \equiv \gamma - \lambda, \tag{B.3}$$

the necessary conditions for optimum include:

$$u'(c) = \lambda, \tag{B.4}$$

and

$$y_x(k, x) \leq \zeta, \text{ equality holding if } x^f > 0. \tag{B.5}$$

The necessary condition for ι gives

$$\iota = \begin{cases} \bar{\iota} & \text{if } \phi > 0 \\ 0 & \text{if } \phi < 0 \\ \iota^s & \text{if } \phi = 0 \end{cases} \tag{B.6}$$

where ι^s is the singular policy, defined by (3.3) or (3.7). The costate variables evolve according to

$$\dot{\lambda} = -\lambda[y_k(k, x) - (\rho + \delta)], \tag{B.7}$$

and

$$\dot{\gamma} = -\lambda b y_x(k, x) + \gamma(\rho + \delta), \tag{B.8}$$

The transversality condition is

$$\lim_{t \to \infty} \mathcal{H}(t)e^{-\rho t} = 0. \tag{B.9}$$

Define

$$\Lambda(k, x) \equiv y_k(k, x) - b y_x(k, x) \tag{B.10}$$

and combine (B.3), (B.7) and (B.8) to obtain

$$\dot{\phi} \equiv \dot{\gamma} - \dot{\lambda} = \Lambda\lambda + \phi(\rho + \delta), \tag{B.11}$$

which can be integrated to give (for any arbitrary time $t_0 \geq 0$)

$$\phi(t)e^{-(\rho+\delta)t} = \phi(t_0)e^{-(\rho+\delta)t_0} + \int_{t_0}^{t} \Lambda(k(\tau), x(\tau))\lambda(\tau)e^{-(\rho+\delta)\tau} d\tau. \tag{B.12}$$

The analysis is carried out in terms of the geometry of the market process vis-a-vis the three characteristic curves. Each of these curves divides the (k, x) plane to regions above and below it (i.e. with x exceeding or falling short of $x^j(k)$, $j = e, s, \zeta$, respectively). We say that the (k, x) process crosses a curve *from below* when it moves from the region below the curve to the region above it, even if the $x(\cdot)$ process decreases at the crossing time (in which case the crossing might be more appropriately described as *from the right*). We also say that some policy is maintained *indefinitely* if it is followed from some time onwards to $t \to \infty$. We refer to the solution of (B.1) interchangeably as "processes," "market processes" or "optimal processes".

Property B.1. *Under the optimal policy: (i)* $\dot{c} > 0$ *if* (k, x) *is above* $x^e(\cdot)$*; (ii)* $\dot{c} < 0$ *if* (k, x) *is below* $x^e(\cdot)$*; (iii) a steady state must reside on the* $x^e(\cdot)$ *curve.*

Proof. Taking the time derivative of (B.4) and using (B.7), we find

$$-u''(c)\dot{c}/u'(c) = y_k(k, x) - (\rho + \delta). \tag{B.13}$$

Condition (3.4) which defines the steady state curve and assumption (2.2) imply that $y_k(k, x) > \rho + \delta$ above $x^e(\cdot)$ and the reverse relation holds below $x^e(\cdot)$. Noting that $-u''/u' > 0$, we conclude that $\dot{c} > 0$ above $x^e(\cdot)$ and $\dot{c} < 0$ below $x^e(\cdot)$. A steady state entails $\dot{c} = 0$, hence it must reside on the steady state curve. □

Property B.2. *The optimal* (k, x) *process proceeds along or above* $x^\zeta(\cdot)$*. This is achieved by adjusting* x^f *such that*

$$x^f = \begin{cases} 0 & \text{if } \ x^\zeta(k) - ba \le 0 \\ x^\zeta(k) - ba & \text{if } \ x^\zeta(k) - ba > 0 \end{cases},$$

Proof. (*i*) According to (2.8) and (2.2), $y_x(k, ba) > \zeta$ when $x^\zeta(k) - ba > 0$. This situation violates (B.5) and $x^f = x^\zeta(k) - ba > 0$ must be invoked to augment ba and satisfy (B.5), shifting (k, x) to reside along the $x^\zeta(\cdot)$ curve. If $x^\zeta(k) - ba < 0$ then $y_x(k, ba) < \zeta$. However when $x^f > 0$, (2.2) implies $y_x(k, ba) > y_x(k, x^f + ba) = \zeta$, where the latter equality follows from (B.5). The contradiction implies that $x^f = 0$ holds above $x^\zeta(\cdot)$. □

The following corollary holds:

Property B.3. *Maintaining the* $\iota = \bar{\iota}$ *regime indefinitely cannot be optimal.*

Proof. According to Property B.2, the (k, x) process proceeds on or above $x^{\zeta}(\cdot)$. If the $\iota = \bar{\iota}$ regime is followed indefinitely, the decreasing k process will fall below $k^{\zeta e}$ at some finite time, following which $x^{\zeta}(k) > x^{e}(k)$ holds. Thus, $\dot{c} > 0$ (Property B.1) and, with $\iota = \bar{\iota}$, the capital stock k will be depleted in finite time, which (with $\eta > 1$) reduces utility to $-\infty$ and cannot be optimal. □

Property B.4. (i) *Under the singular regime the* (k, x) *process proceeds along the singular curve.* (ii) *If* $\iota = 0$ *at some time when* (k, x) *is below* $x^{s}(k)$ *then:* (a) *the process cannot switch to another* ι*-regime as long as* (k, x) *remains below* $x^{s}(k)$ *and* (b) *the* (k, x) *process must eventually cross* $x^{s}(k)$*.* (iii) *If* $\iota = \bar{\iota}$ *at some time when* (k, x) *lies above* $x^{s}(k)$ *then:* (a) *the process cannot switch to another* ι*-regime as long as* (k, x) *remains above* $x^{s}(k)$ *and* (b) *the* (k, x) *process must eventually cross* $x^{s}(k)$*.* (iv) *Except for the intersection point* $(k^{s\zeta}, x^{\zeta}(k^{s\zeta}))$*, a singular process must proceed with* $x^{f} = 0$*.*

Proof. (i) According to (B.6), the singular regime entails $\phi = \dot{\phi} = 0$, hence (B.11) implies $\Lambda(k, x) = 0$ which defines $x^{s}(\cdot)$.

(ii) The properties of $y(\cdot, \cdot)$ (see (2.2)) imply that $\Lambda(k, x)$ is negative or positive for (k, x) below or above $x^{s}(\cdot)$, respectively. Suppose that a (k, x) process is initiated at some time t_0 below $x^{s}(\cdot)$ with $\iota(t_0) = 0$, so that according to (B.6) $\phi(t_0) < 0$. With $\lambda > 0$ and $\Lambda(k, x) < 0$, (B.12) ensures that $\phi(t)e^{-(\delta+\rho)t}$ is bounded from above by the negative constant $\phi(t_0)e^{-(\delta+\rho)t_0}$ so long as the (k, x) process remains below the singular curve, establishing (a). To verify (b), notice that if (k, x) never crosses $x^{s}(\cdot)$, the policy $\iota = 0$ will be retained indefinitely (since ϕ remains negative), which cannot be optimal for the following reason. With $\gamma > 0$, we see that $\lambda(t)e^{-(\delta+\rho)t}$ is also bounded away from zero by a positive constant. Integrating (B.7) we find

$$\lambda(t)e^{-(\delta+\rho)t} = \lambda(t_0)e^{-(\delta+\rho)t_0} \exp\left[-\int_{t_0}^{t} y_k(k(\tau), x(\tau))d\tau\right],$$

which is bounded away from zero only if $k \to \infty$ at large t. Under the $\iota = 0$ regime, the state a decreases, hence eventually $x^{\zeta}(k) - ba > 0$ must hold and from that time on both k and x increase along $x^{\zeta}(\cdot)$ (Property B.2(i)). For large enough k, Property 3.2(*iii*) ensures that $x^{\zeta}(k) < x^{e}(k)$, where $\dot{c} < 0$ (Property B.1). However, the policy of keeping k and x constant, diverting the resources required to increase them to enhance consumption, is feasible and yields a higher value.

(*iii*) Suppose that a (k, x) process is initiated at some time t_0 above $x^s(\cdot)$ with $\iota(t_0) = \bar{\iota}$, so that $\phi(t_0) > 0$. Repeating the above argument, we show that $\phi(t)e^{-(\delta+\rho)t}$ is bounded away from zero by a positive constant as long as (k, x) is above $x^s(\cdot)$, hence the ι regime will be maintained. According to property B.3, this regime cannot hold indefinitely, hence the singular curve must be crossed.

(*iv*) The singular process proceeds along the singular curve $x^s(\cdot)$, which lies below or above $x^{\zeta}(\cdot)$ for $k < k^{s\zeta}$ or $k > k^{s\zeta}$, respectively. According to Property B.2, no process is optimal below $x^{\zeta}(\cdot)$, while $x^f = 0$ holds above $x^{\zeta}(\cdot)$. □

Property B.5. (*i*) *If a singular (k, x) process leaves the singular curve to the region above it, the corresponding ι regime changes from singular to $\iota = \bar{\iota}$ at the departure time.* (*ii*) *If a singular (k, x) process leaves the singular curve to the region below it, the corresponding ι regime changes from singular to $\iota = 0$ at the departure time.*

Proof. The singular (k, x) process proceeds with $\dot{\phi} = \phi = 0$. Suppose that by mistuning ι the (k, x) process is driven above the singular curve at some time t_0. With $\phi(t_0) = 0$ and $\Lambda(k(t), x(t)) > 0$, (B.12) implies that $\phi(t) > 0$ at t just after t_0, hence $\iota = \bar{\iota}$ is adopted above $x^s(\cdot)$. The same considerations show that leaving to the region below $x^s(\cdot)$ (where $\Lambda(\cdot, \cdot) < 0$) implies $\iota = 0$. □

Property B.6. *The optimal (k, x) process does not cross $x^s(\cdot)$ from above with $\iota = \bar{\iota}$.*

Proof. Under the $\iota = \bar{\iota}$ regime, $k(\cdot)$ decreases and $a(\cdot)$ increases. For the (k, x) process to cross $x^s(k)$ from above, its slope must exceed that of the singular curve, i.e. $\dot{x}/\dot{k} > x^{s\prime}(k) > 0$ must hold at the crossing time. Suppose $x^f = 0$ then $\dot{x}^f \geq 0$ hence $\dot{x} \geq b\dot{a} > 0$ while $\dot{k} < 0$, so crossing from above cannot occur.

Crossing with $x^f > 0$ can take place place only along the x^ζ curve. The latter crosses the singular curve at $k^{s\zeta}$ from above, hence the crossing requires that $k(\cdot)$ increases, which cannot occur under this ι regime. □

Property B.7. *Above $x^s(\cdot)$, the optimal policy is to set $\iota = 0$.*

Proof. Suppose that $\iota = \bar{\iota}$ when (k, x) is above $x^s(\cdot)$. According to Property B.4, the (k, x) process must cross $x^s(\cdot)$ from above before changing the ι regime, violating property B.6. This rules out this regime. The singular regime can only be applied along $x^s(\cdot)$, so the only possibility left above the singular curve is $\iota = 0$. □

We now show that maximal solar investment can be optimal only at the initial phase:

Property B.8. *A switch to $\iota = \bar{\iota}$ from any of the other two ι regimes cannot be optimal.*

Proof. Above the singular curve, the $\iota = 0$ regime is optimal (Property B.7) hence a switch to $\iota = \bar{\iota}$ will not take place in this region. Proceeding along the singular curve cannot change the sign of $\phi(\cdot)$ (see (B.12)) while leaving it (with $\phi = 0$) to the region below implies $\iota = 0$ (property B.5). Below the singular curve, the singular regime never holds and the $\iota = 0$ regime cannot be switched (Property B.4). □

Property B.9. *If $k < k^{s\zeta}$ then the optimal policy is to set $\iota = 0$.*

Proof. When $k < k^{s\zeta}$ the $x^{\zeta}(\cdot)$ curve lies above the singular curve, hence the (k, x) process (which must proceed on or above x^{ζ} – see Property B.2) evolves above the singular curve. The optimal policy, then, is to set $\iota = 0$ (Property B.7). □

An immediate corollary of properties B.8 and B.9 is

Property B.10. *A small economy, with $k_0 < k^{s\zeta}$, will never adopt the $\iota = \bar{\iota}$ regime.*

Property B.11. *If $k < \min(k^{s\zeta}, k^{\zeta e})$ then the optimal $k(\cdot)$ process increases.*

Proof. When $k < \min(k^{s\zeta}, k^{\zeta e})$ the $x^{\zeta}(\cdot)$ curve lies above the other two curves, hence the (k, x) process (which must proceed on or above x^{ζ} – see Property B.2) evolves above the other two curves. This implies an increasing $c(\cdot)$ process (Property B.1) and $\iota = 0$ as the optimal choice (property B.9). Under this ι-regime, $a(\cdot)$ does not increase. We show that $k(\cdot)$ increases. Consider the function

$$D(t) \equiv y(k(t), x(t)) - c(t) - \zeta x^f(t) - \delta k(t) \tag{B.14}$$

With $\iota = 0$, $\dot{k} = D$. Taking the time derivative, we find

$$\ddot{k} = \dot{D} = [y_k - \delta]\dot{k} + [y_x - \zeta]\dot{x}^f + y_x b\dot{a} - \dot{c} \tag{B.15}$$

Now, the second term of (B.15) vanishes because $x^f = \dot{x}^f = 0$ above $x^\zeta(\cdot)$ and $y_x - \zeta = 0$ on $x^\zeta(\cdot)$. If k decreases, the first term is negative above $x^e(\cdot)$ because $y_k > \rho + \delta > \delta$. The third term is not positive when $\iota = 0$ while $\dot{c} > 0$ above the steady state curve. Thus, both $\dot{k}$ and $\ddot{k}$ are negative, implying that if k decreases it must vanish at a finite time, which cannot be optimal. □

Property B.12. *The optimal state trajectory does not cross the steady state curve $x^e(\cdot)$ from below with $\iota = 0$ or $\iota = \iota^s$.*

Proof. Crossing $x^e(\cdot)$ from below must occur at $k \geq k^{\zeta e}$, i.e. above or along the $x^\zeta(\cdot)$ curve. In the former case, $x^f = 0$ and $\iota = 0$ implies that a decreases, hence k must also decrease. (Otherwise, the (k, x) process moves away from $x^e(\cdot)$.) It follows that all the terms of (B.15) are negative or vanishing at and after the crossing time, hence $\ddot{k} < 0$. Thus, k will continue to decrease at an increasing rate and will inevitably fall below $\min(k^{s\zeta}, k^{\zeta e})$, violating property B.11. Crossing $x^e(\cdot)$ from below along $x^\zeta(\cdot)$ at $k^{\zeta e}$ also involves a decreasing k process, hence is ruled out using the same argument, which can also be used to rule out the crossing under the singular regime. □

B.1 Solar based economies

The economy is solar based when $k^{se} > k^{\zeta e}$ or equivalently, when $\rho + \delta < b\zeta$ (see the derivation of 3.6). In this case $\hat{k} = k^{se}$ and $\hat{a} = x^e(\hat{k})/b$, as depicted in Figure 1.

Property B.13. *The optimal state trajectory does not cross the singular curve $x^s(\cdot)$ at $k < \hat{k}$ from below with $\iota = 0$.*

Proof. Crossing the singular curve from below must occur at $k \geq k^{s\zeta}$, i.e. above or along the $x^{\zeta}(\cdot)$ curve. A crossing with $\iota = 0$ implies for both cases that both k and a do not increase. For $k < \hat{k}$, the crossing occurs above the steady state curve. It follows that all the terms of (B.15) are negative or vanishing at and after the crossing time, hence $\ddot{k} < 0$. Thus, k decreases at an increasing rate and will inevitably fall below $k^{s\zeta}$, violating property B.11. □

Property B.14. *When (k, x) is below $x^s(\cdot)$ and $k < \hat{k}$, then the optimal policy is to set $\iota = \bar{\iota}$.*

Proof. If $\iota = 0$ the (k, x) process must cross $x^s(\cdot)$ from below before the ι regime is switched (Property B.4). Increasing k only moves the (k, x) process further away (below) from $x^s(\cdot)$, hence the crossing must take place with $k < \hat{k}$, violating property B.13 and ruling out the $\iota = 0$ policy. The singular policy is also ruled out away from the singular curve, and the only possibility left is $\iota = \bar{\iota}$. □

B.1.1 More on singular processes

Property B.15. *A singular process cannot leave the singular curve while $k < \hat{k}$.*

Proof. In view of property B.5, driving a singular (k, x) process above the singular curve entails $\iota = \bar{\iota}$ above this curve, violating Property B.7. Driving a singular (k, x) process below $x^s(\cdot)$ entails $\iota = 0$ at $k < \hat{k}$, violating Property B.14. □

Property B.16. *A singular process with $k^{s\zeta} < k < \hat{k}$ must increase (i.e. both k and a increase).*

Proof. Above the ζ curve, $x^f = 0$ hence $x = ba$ and a and k vary in the same direction along the increasing singular curve. With $k < \hat{k}$, the singular process proceeds above the steady state curve, where $c(\cdot)$ increases. Thus, the process cannot settle at a steady state in this region. Suppose that it decreases, then it cannot reverse its direction, nor can it leave the singular curve. The decreasing process, then, must proceed towards $k^{s\zeta}$ where it is forced to leave the singular curve, violating property B.15. This leaves an increasing process as the optimal option. □

The crossing point $(k^{s\zeta}, x^{\zeta}(k^{s\zeta}))$ marks an exception to this rule because a time period during which both $k(t) = k^{s\zeta}$ and $x(t) = x^f(t) + ba(t) = x^{\zeta}(k^{s\zeta})$ remain fixed while the solar-fossil mix varies cannot be ruled out. Indeed, the solar investment rate (3.7) is adopted during this period. Once x^f vanishes and the solar component takes over, however, the process must leave the crossing point and increase along the singular curve with ι^s given by (3.3) in accordance with property B.16.

Property B.17. *A singular process with $k^{s\zeta} < k < \hat{k}$ must approach the intersection point $(\hat{k}, x^s(\hat{k}))$.*

Proof. While $k < \hat{k}$ the singular process cannot leave x^s (property B.15) or settle at a steady state hence it must increase (property B.16) towards $(\hat{k}, x^s(\hat{k}))$. □

Property B.18. *A singular process with $k > \hat{k}$ must decrease.*

Proof. Consider a singular process proceeding along the singular curve segment with $k > \hat{k}$, i.e. below the steady state curve. According to property B.1, $c(\cdot)$ must decrease along this process. Since the steady state curve will never be crossed (property B.8) this decrease in consumption will never reverse. If $k(\cdot)$ increases, $a(\cdot)$ must increase as well along the increasing singular curve. This behavior, however, is inconsistent with decreasing consumption, since the alternative policy of maintaining both capital stocks fixed, diverting the resources required to increase them to enhanced consumption is also feasible and yields a higher utility. A steady state cannot be optimal away from the steady state curve, hence the process must decrease. □

B.1.2 Convergence

The $\iota = \bar{\iota}$ regime can hold only during the initial phase of the optimal policy, and only for a final duration (properties B.3 and B.8). Thus there exists some finite time t_0 following which only the other two ι regimes can be optimal. (For small economies, $t_0 = 0$, see property B.10.) To study long term behavior, we restrict attention to $t > t_0$ hence consider only these other two regimes.

Property B.19. *An optimal process converges to* $(\hat{k}, b\hat{a}) = (k^{se}, x^e(k^{se}))$.

Proof. Proceeding below the steady state curve, the optimal process can never cross to the region above it (property B.12), hence the consumption process must decrease indefinitely. To avoid vanishing consumption at a finite time, the rate of decrease must approach zero, hence the process must approach the steady state curve (property B.1). The point of approach cannot have $k < \hat{k}$, because this region implies the excluded regime $\iota = \hat{\iota}$ (property B.14). At $k > \hat{k}$, the $\iota = 0$ regime holds and with $x^f = 0$, $x(\cdot) = ba(\cdot)$ decreases exponentially, hence the $(k(\cdot), x(\cdot))$ process, restricted to the vicinity of the steady state curve, must converge to the intersection point with the singular curve, where the singular $\iota = \iota^s$ policy allows to maintain $a(\cdot)$ fixed at its steady state value.

Suppose that the $(k(\cdot), x(\cdot))$ process proceeds above the steady state curve. If it crosses this curve, it can never return back to the region above it, hence it will converge to the steady state as shown above. We can, therefore consider processes restricted to the region above the steady state curve indefinitely. In this region, the process must also proceed on or above the singular curve (outside the interval $[k^{s\zeta}, \hat{k}]$ the singular curve lies below one of the other curves so the process must proceed above it, while within this interval the region below the singular curve implies the excluded $\iota = \bar{\iota}$ policy, see property B.14). The process, then can proceed either above all three curves, or along the singular curve with $k(\cdot) \in [k^{s\zeta}, \hat{k}]$ or along x^ζ with $k(\cdot) < k^{s\zeta}$. In the former case, $x^f = 0$ and the $\iota = 0$ regime implies that $x(\cdot) = ba(\cdot)$ decreases exponentially, and since $k(\cdot)$ is bounded from below by $\min(k_0, k^{s\zeta})$ (see property B.11) the process must reach the singular or the ζ curve (whichever lies higher at the point of arrival) in finite time. Neither curve can be crossed, nor can the process return to the region above them. It can, however, either increase along x^ζ with $\iota = 0$, using fossil energy at the required rate to make up for the shrinking solar capital, or switch to the singular regime and increase along x^s towards the intersection point with the steady state curve (property B.16). If the ride along x^ζ takes place first, it must end up at $(k^{s\zeta}, x^s(k^{s\zeta}))$, where the singular regime takes over, eventually bringing the process along x^s to the steady state. □

Property B.20. *Characterization of the optimal process for a small solar-based economy.*

Consider a small economy endowed with $k_0 < k^{s\zeta}$ and $a_0 = 0$. The initial policy for this economy must proceed with $\iota = 0$ (property B.9) hence the $\iota = \bar{\iota}$ policy will never be adopted (property B.10). With $a(\cdot) \equiv 0$ the optimal process must increase (property B.11) along the ζ curve because the region above this curve requires that $x^f = 0$ and $ba(\cdot) = x(\cdot) > x^{\zeta}(k(\cdot))$ (property B.2). This ride along x^{ζ} must proceed until the singular curve is reached at $k^{s\zeta}$. The latter curve cannot be crossed, because the region below it implies the excluded $\iota = \bar{\iota}$ policy (property B.14). Neither can the process leave the ζ curve with $a = 0$ as stated above. Moreover, the process cannot stay at the crossing point under the $\iota = 0$ regime because a steady state is not allowed away from x^e. Thus a switch to the singular regime must occur upon reaching $k^{s\zeta}$. The process, however cannot leave the crossing point and increase along the singular curve (i.e. above x^{ζ}) so long as the solar stock $a(\cdot)$ falls short of $x^{\zeta}(k^{s\zeta})$, because otherwise a positive rate of fossil energy would be required above x^{ζ}, violating property B.2. Thus, a quasi-stationary and singular coexistence phase must take place to allow solar capital to build up, leaving $k(\cdot)$ and $x(\cdot)$ fixed but shrinking $x^f(\cdot)$ gradually to make room for the increasing use of solar energy. Once the use of fossil energy ceases, staying at $(k^{s\zeta}, x^s(k^{s\zeta}))$ is no longer possible, because this point does not qualify as a steady state. Leaving the singular curve is not possible below $\hat{k}$ (property B.15) hence the singular process must increase along this curve (property B.16) reaching towards the intersection point $(k^{se}, x^s(k^{se}))$ with the steady state curve. A further increase along the singular curve implies crossing the steady state and an indefinitely decreasing consumption process hence the process must settle at a steady state in the intersection point.

B.2 Fossil based economies

We consider now the case $\rho + \delta > b\zeta$ which implies $k^{se} < k^{\zeta e}$, so that $\hat{k} = k^{\zeta e}$, $\hat{x} = x^e(\hat{k})$ and $\hat{a} = 0$, as depicted in Figure 2.

Observe first that the crossing point $(k^{se}, x^e(k^{se}))$ of the steady state and singular curves that served as a steady state in the previous subsection lies here below the ζ-curve hence cannot belong to an optimal process (Property B.2). What other point might serve as a steady state? Obviously it must lie on the steady state curve, hence above the singular curve so the singular policy cannot be optimal in this state. The $\iota = \bar{\iota}$ regime cannot hold indefinitely, hence does not correspond to a steady state. With $\iota = 0$, a steady state implies also $a = 0$ so $x = x^f$. Thus, the state must lie on the ζ-curve, hence the only possibility is the crossing point $(k^{\zeta e}, x^e(k^{\zeta e}))$. Indeed, for this type of economies the following property holds

Property B.21. *If $k < k^{\zeta e}$ then $\iota = 0$ and $k(\cdot)$ increases.*

Proof. When $k < k^{\zeta e}$ the x^ζ curve lies above the other two curves, hence the (k, x) process (which must proceed on or above x^ζ, see Property B.2) evolves above the other two curves. The proof, then, follows that of properties B.9 and B.11. □

Next, we verify that the corresponding proofs are not affected by the change in geometry and all the properties established for the solar based economies (except those that deal with the interval $k^{s\zeta} < k < k^{se}$ which is not relevant for the present geometry) hold also in the the present case. Put together, they entail the following characterization

Property B.22. *Under the optimal policy, the* $(k(\cdot), ba(\cdot))$ *process converges to* $(\hat{k}, b\hat{a}) = (k^{\zeta e}, 0)$ *with* $\hat{x} = \hat{x}^f = x^e(\hat{k})$.

The proof follows closely the reasoning for the case of solar based economies, hence will not be reproduced here.

Property B.23. *Characterization of the optimal process for a small fossil-based economy.*

Consider a small economy endowed with $k_0 < k^{\zeta e}$ and $a_0 = 0$. The initial policy for this economy must increase with $\iota = 0$ (property B.21) along x^ζ (because $a(\cdot) = 0$ excludes the region above this curve). The process cannot increase beyond the intersection at $k^{\zeta e}$ because crossing the steady state line implies that the consumption process will decrease indefinitely, while settling at a steady state at the intersection point (diverting the resources required to increase $k(\cdot)$ and $x^f(\cdot)$ to enhanced consumption) is feasible and yields a higher value. A transition to the $\iota = \bar{\iota}$ regime is not allowed, nor can a switch to the singular regime take place away from x^s. The optimal process, then, is restricted to a fossil-based increase along x^ζ towards the steady state $(\hat{k}, b\hat{a}) = (k^{\zeta e}, 0)$ with $\hat{x} = \hat{x}^f = x^e(\hat{k})$, as asserted in property B.22.

Put together, properties B.19 and B.22 establish Proposition 3.1, and properties B.20 and B.23 establish Proposition 3.3.

C Cap regulation

Introducing the constraint $\bar{x}^f - x^f \geq 0$ to the optimization problem (B.1) adds the term $\nu(\bar{x}^f - x^f)$ to the Hamiltonian (B.2), where $\nu \geq 0$ is the shadow price associated with the new constraint. Maximization with respect to x^f

modifies (B.5) to

$$y_x(k,x) \begin{cases} > \zeta & \text{if } x^f = \bar{x}^f \\ = \zeta & \text{if } 0 < x^f < \bar{x}^f \\ < \zeta & \text{if } x^f = 0 \end{cases} \tag{C.1}$$

Property B.2, then must be extended to allow for the possibility that the optimal (k,x) process proceeds below $x^{\zeta}(\cdot)$ when $x^f = \bar{x}^f$. This is achieved by adjusting x^f such that

$$x^f = \begin{cases} 0 & \text{if } \; x^{\zeta}(k) - ba \le 0 \\ x^{\zeta}(k) - ba & \text{if } \; \bar{x}^f > x^{\zeta}(k) - ba > 0. \\ \bar{x}^f & \text{if } \; x^{\zeta}(k) - ba > \bar{x}^f \end{cases} \tag{C.2}$$

This change implies an extension of Property B.4(iv) so as to allow a singular process to proceed with $x^f = \bar{x}^f$. For a singular process, then

$$x^f = \begin{cases} 0 & \text{if } \; k > k^{s\zeta} \\ \bar{x}^f & \text{if } \; k < k^{s\zeta}, \end{cases} \tag{C.3}$$

with solar input $ba = x^s(k) - x^f$ supplying the residual energy required to keep the process on the singular curve.

The proof of Proposition 4.1, then, follows the arguments of Appendix B, when the intersection point of the $x = \bar{x}^f$ line and the singular curve replaces $(k^{s\zeta}, x^s(k^{s\zeta}))$ as the relevant point for policy change (from the fossil phase to the coexistence phase). The details are omitted to avoid duplication.

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