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Optimise initial spare parts inventories: an analysis and improvement of an electronic decision tool

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Abstract

Control of spare parts is very difficult as demands can be very low (once in a few years is no exception), while the consequences of a stockout can be severe. While in the past many companies choose to have very large spares inventories, one now observe trends in areas with good transportation connections to keep spare parts at the suppliers. Hence it is very important to make a good selection of which spare parts to stock at the start-up of new plants. To this end Shell Global Solutions has developed an electronic decision tool, called E-SPIR. In this report we analyse the decision rules used in it. We consider stockout penalties and advise to use criticality classifications instead. Furthermore, we investigate minimum stock levels, demand distributions and order quantities.

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Summary

Spare part inventories can be very high when poor inventory systems are used. On the other hand the costs associated to not having a spare part when needed can be significant. A good balance between holding costs associated to having parts and penalty costs associated to stockouts is therefore needed. This balance is especially important for slow moving items. Inventory systems often contain methodologies to achieve the optimum inventory cost based on historical consumption information. For projects aiming at the construction of new plants this information is not available. A software tool called E-SPIR has been developed by Shell Global Solutions to collect the spare parts information in a standard format and facilitate the review and ordering of the spare parts. E-SPIR stands for Electronic – Spare Parts and Interchangeability Record, which is the electronic version of the SPIR. In this report the spare parts selection procedure present in the E-SPIR is analysed and improved. This is achieved by addressing the following four main questions.

1. To stock or not to stock an item?

In general, items should only be stocked if the benefits of direct availability outweigh the costs of holding the items in stock. The decision to stock is especially important for slow moving parts as wrong decisions have very long-lasting effects. A comparison of costs associated with stocking one spare part and those associated to not stocking at all will give an answer on to stock or not. An easy to follow rule was developed by Olthof and Dekker (1994). This decision rule is based on four variables: consumption rate, price, leadtime and penalty costs. The first three variables should be given by the supplier of the item, the last by the user/company. The first parameter is difficult to estimate by either supplier or user/Company, yet an estimate can be given. The fourth variable, penalty costs, is also difficult to estimate. In the old version of the E-SPIR the user/company has to give an explicit value for these costs. In practice this appeared to be too difficult as the engineers involved have little insight into cost consequences. In the new version of the E-SPIR an item belongs to one of three criticality classifications: Vital (High), Essential (Medium) or Auxiliary (Low). To each of these classes a range of penalty costs is associated. Now the user/company only has to choose a range for each criticality classification once,

at the start of a new plant, and subsequently has to give each item a criticality classification. In the current situation the penalty costs for all the criticality classifications are values per day. In practice it is found that for the classification Auxiliary (Low) a one-time penalty cost will be sufficient. An extra possibility is given to take o f a period before which costs are incurred (so-called zero-cost days) for the classifications Vital (High) and Essential (Medium). This is done on project level and thus needs to be given by the user/company once, at the start of a project.

2. How many to order at once?

When the decision has been made to stock an item, the next question to answer is how many to order at once. At the start of a new plant, this is the initial purchase order. To determine an optimal order quantity, a well-known result in the inventory control area is used, viz. the classical economic order quantity (EOQ) formula. The EOQ is essentially an accounting formula that determines the inventory level at which the combination of order costs and holding costs are the least. The input variables needed are the annual consumption, the order costs and the holding costs. Not all assumptions of the EOQ hold in the case of slow moving items. Because an integer number is needed, the EOQ is rounded off. When an item needs to be stocked, at least one item is ordered. Further it is shown that the normal rounding rules can be applied.

3. When to release a new order?

The moment to release a new order is usually named the re-order point. The minimum stock is the level of inventory on stock before a replenishment order is placed (this is the re-order point plus one). Additional costs associated with a wrong decision on the minimum stock can be high. Having too many items on stock can result in high holding costs. On the other hand, having too few items on stock can result in high penalty costs. The minimum stock will be determined considering the consumption during the lead time. This consumption gives the estimated number of items needed during the lead time of the ordered items. The consumption during the lead time depends on the distribution of this consumption. Three different methods are investigated. First the what we call Factor Variance Method. In this method the consumption during the leadtime is considered to be normally distributed. Here the minimum stock consists of the mean demand during the leadtime plus a safety factor times the standard deviation of the demand during the lead time. For different levels of minimum stock the total costs (holding costs + penalty costs) are considered and the level with the lowest costs is chosen. For slow moving items the normal distribution is not very suitable, because of a significant probability of negative demand (which is impossible in reality, of course). The second method is based on the assumption that the time between two successive demands follows an Erlang -k distribution. The larger the shape parameter k, the less variance such a distribution will have. Given the value for k, the yearly consumption and the lead time, the probability of a stock out can be computed for different levels of minimum stock. Again the level with the lowest total costs is chosen. The third method is a very simple estimation of the number of needed items during a period. This method is not a scientific method but it is often used as a "common sense" idea. The idea of the method is to increase the lead time with a safety factor to reach a higher level of minimum stock. A main disadvantage of this approach is that it does not take holding costs and penalty costs into account.

The conclusion is that the best method for slow moving spare parts is the Erlang - k method. Although a choice for the appropriate k is difficult, we advise to use k=4 in case parts are installed in one or two pieces of equipment only and the plant can be re -supplied in a quick way. In all other cases we advise to be on the safe side and use a value of k=1.

4. How to cope with penalty values?

The minimum stock discussed above is based on costs. Penalty costs express the importance of having an item on stock. Another method to express this importance is the use of service levels. In the service sector service levels are often used. It is a relatively easy way to express the risk one is willing to take. In a call centre for example one can require that 99 of 100 people will not get a busy line. We will discuss the pros and cons of working with service levels and with stockout costs. When these costs are known, they can be used to balance the expected total costs. For the criticality classification Vital (High) and Essential (Medium) these costs are expressed in days. For the third criticality classification Auxiliary (Low) these are one time penalty costs. On a project level, each of these classifications needs to be given a pre -defined range of costs.

Besides these four main questions the input variables needed for the inventory system are discussed in detail. It is very important to know exactly what each variable means and how to determine its value. For the holding costs, a holding cost rate is used. This rate is multiplied by the purchase costs to compute the holding costs. Because this rate is difficult to determine, a separate chapter is devoted to it. In a sensitivity analysis it is concluded that the economic order quantity and the holding cost rate are quite robust to this cost rate.

1. Motivation and background

1.1 Introduction

Developments of modern information technology open up new possibilities for a more efficient control of inventory systems. Most organizations can reduce their costs substantially with this knowledge. In this research the initial spare part inventory procedures of a group of companies in the oil and gas sector are analysed. In this chapter the motivation and background of this research is presented. In section 1.2 the purpose of this research is explained. Next some information is given about the E-SPIR program (section 1.3). In section 1.4 the problems addressed in this research are formulated. Subsequently the input variables needed in the program are summed up (section 1.5). In section 1.6 the requirements of the program are shortly stated. This chapter ends with a preview of the following chapters (section 1.7).

1.2 Purpose of this research

This research was motivated by Shell Global Solutions International to improve the inventory model of spare parts. For collecting the spare parts information in a standard format they use the E-SPIR program. One of the strengths of this program is the facility to advise the inventory level at the start of a plant. Slow moving items are the most crucial in this respect. These are items with a consumption of at most 2 items per year. When an error is made in the minimum stock, these items stay on stock for a long time. Costs of stocking items too long can be high (e.g. due to obsolescence). The difficulty faced however, at the moment of the start of plant is the lack of historical information, on which the decision can be based of how many to stock.

The reason for having initial spare part inventories is to immediately provide parts when needed due to failures. It is essential that the chance of a part being out of stock when required is kept low. However, because inventory is expensive and can become obsolete as equipment models change, companies do not want to hold excessive amounts of stock. Therefore, understanding the inventory versus costs due to failures and striking a reasonable balance are a firm's primary concern and the main motivation for this research. The purpose of this research is to make improvements to the present initial spares selection procedure.

1.3 E-SPIR program

E-SPIR stands for Electronic – Spare Parts and Interchangeability Record, which is the electronic version of the SPIR, see e.g. http://www.e-spir.com. The program shows the relation between Equipment and Spare parts and the interchangeability between Spare parts in equipment installed. This program was developed by Shell Global Solutions International to assist in reviewing and selecting spare parts for new installations. The program makes use of data submitted by suppliers, completed with available information from the project/company. In this research the spare selection procedure present in the E-SPIR will be analysed and improved. Additional information required from the user/company should be minimal to keep the program user-friendly.

1.4 Problem formulation

Four separate questions will be addressed in this research:

1. To stock or not to stock an item?
2. How many to order at once?
3. When to release a new order?
4. How to cope with penalty values?

In the E-SPIR program a stock recommendation gives project users advise whether to stock or not to stock the spare part. This rule was developed by Olthof and Dekker (1994). In this research some adjustments to this rule are studied. When the decision to stock an item has been made, the next question is how many and when to order. The number to order at a time is currently based on an estimated quantity to cover the delivery time of the relevant spare part. Costs were not included in the present estimate in the E-SPIR. In this report a scientific basis for an economic order quantity (EOQ) in the E-SPIR program is provided. To answer the question when to order, different demand distributions are discussed to determine the minimum stock level. The fourth question is of a different kind, but in this research at least as important. Penalty values are important throughout the report. It is important to know how to determine the penalty values and what this means for the stocking decision and for the minimum stock. Also the difference between using penalty values and service levels is discussed.

1.5 Input data E-SPIR

The input data of the E-SPIR program is split over three levels. These levels are project, equipment and parts. A project is for example the construction of a gas plant, a whole or part of a refinery or an oil platform. Each project involves a large amount of equipment and per equipment many parts are involved, usually more than 1000 different pieces. The three different levels are introduced because of the different information needed per level. Some information can be asked once and used throughout the entire project. For each level we give below a summary of the data needed to answer the main four inventory questions (see section 1.4). In chapter 2 these variables are explained in more detail.

Project/Company:

- A price surcharge (in percentages) for all parts (e.g. for handling and import duties)
- A lead time surcharge because of the ordering, transport and handling time of the plant (in weeks)
- Average costs to place one order
- Holding Cost Rate
- Minimum period to cover
- Maximum period to cover
- Penalty values and zero-cost-days for the different criticalities of equipment

Equipment

- ➢ Criticality classification

Parts:

- ➢ Price of the item
- ➢ Lead time (days) required to replenish inventory
- ➢ Estimated consumption rate (items a year)

1.6 Requirements of the program

Any changes to the existing program as well as the additional information needed from the user/ company should be easy to implement, in the sense that

a. The stock quantity decision process does not depend on difficult-to-estimate parameters.
b. The policies lead to closed-form expressions for stocking parameters that can be computed with minimal user input in a spreadsheet.
c. The program provides a minimum and maximum stock recommendation.

The challenge is to make appropriate use of available data with minimal manual interaction allowing stocking policies to be easily and effectively implemented for the spare parts selection and ordering process in projects.

1.7 Literature overview

A review of the scientific literature (see Kennedy et al. (2002) reveals that the E-SPIR program is unique in its kind. There are several programs (e.g. Sparecalc, a program from Shell Expro, Relex OpSim software optimising spare parts), that can calculate the right number of a certain kind of spare. However, they usually ask a lot of questions and take relatively much time to give an advise. They should primarily be used for the very expensive (price > 10,000 USD) spares. Accordingly, they can not be used to scan the multitude of spares which are advised by a supplier in a large project. There are yardsticks based on a VED

(Vital, Essential and Desirable) analysis (see e.g. Mukhopadhyay (2003)), but they are very superficial and the coding directly implies the stocking decision (see e.g. Botter and Fortuin (2000)). There is a Spares calculater (see Mickel and Heim (1990)), but that is a manual instrument only. Already in the research of Olthof and Dekker (1994) it became clear that easy data entry and handling, easy and quick parameter estimation, fast computation and finally a good coding of the parts, are the crucial aspects in assessing stocking decisions for many spare parts. The present E-SPIR program is especially developed for this purpose.

1.8 Overview of this report

This report is divided in nine chapters. After this introduction on the research the different input variables are discussed in chapter 2. Chapter 3 explains one of the variables in more detail, namely the holding cost rate. The first question, to stock or not to stock an item, is addressed in chapter 4. The old decision rule is discussed and changes are suggested. In chapter 5 the second question of how much to order is investigated. The Economic Order Quantity (EOQ) will be introduced. The third question of when to re-order is reviewed in chapter 6. Different demand probabilities are suggested to compute a minimal stock. The reorder point is the moment when the inventory level falls down this minimum stock. In chapter 7 the penalty values are discussed, also in respect with service levels. Because different methods are discussed and used, a sensitivity analysis is given in chapter 8. This analysis is done for the Economic Order Quantity (EOQ) and the Holding Cost Rate. Finally, in chapter 9 conclusions and recommendations will be given.

2. Information needed

2.1 Introduction

In this chapter all the inputs are discussed, which are needed for giving a stocking advise for a large capital project. First the parameters for the whole project are explained in section 2.2, thereafter (section 2.3) the parameters for the equipment are discussed. In section 2.4 the parameters for the individual parts are explained. The penalty costs need to be discussed in more detail, and this is done in section 2.5. The results of this chapter are summarized in section 2.6.

2.2 Parameters for the whole project

2.2.1 Introduction

As described in section 1.5, the different parameters for the whole project are: a percentage to adjust prices (2.2.2), values to adjust lead times (2.2.3), order cost (2.2.4), holding cost rate (2.2.5), a minimum period to cover (2.2.6) and a maximum period to cover (2.2.7). The other parameter penalty values are discussed in more detail in section 2.5.

2.2.2 Percentage to increase prices

For every project the user/company can give a certain surcharge (in percentages) to increase the prices for all the parts. This is because for some projects there may be handling and import duties, or other reasons why the price (as given by the supplier) is not equal to the purchase costs of the part. For example: a given percentage of 25% will change the purchase costs of a certain item with a price of 100 dollar to 125 dollar.

2.2.3 Values to adjust (increase) lead times

The lead time as given by the supplier may be not correct for a certain project, due to ordering, transport or (custom) handling time. This is why the user/company can give a certain value to adjust this lead time. The given value (in weeks) will be used to increase the lead times of all the parts. For example: for a certain item the supplier gives a lead time of 8 weeks. The user/company estimates the total extra time of his plant at 2 weeks. This value of two weeks will be added to the lead time of all spare parts, the lead time of the item becomes 10 weeks.

2.2.4 Order cost

This is the sum of the fixed costs that are incurred each time a number of spare parts is ordered. These costs are not associated with the quantity ordered but primarily with the physical activities required to process the order. These activities are: specifying the order, selecting a supplier, issuing the order to the supplier, receiving the ordered goods, handling, checking, storing and payment. Determining these total fixed costs precisely by evaluating all the activities involved is a difficult and very time consuming method and the results of this type of measuring are rarely even close to accurate. In this study a more effective method is proposed. First determine the percentage of time within the plant consumed performing the specific activities and multiply this by the total labor costs for a certain time period (usually a month) and then dividing by the number of orders placed during that same period.

If, for example, during a month 150 orders are placed and the estimated total time needed to process these orders is 300 hours, while the labor costs per hour is $80, the estimated order costs for this project is $160 per order.

In case of e -commerce/EDI ordering the costs of placing an order can be a lot lower, for example a few dollars per order. However, the costs of handling and receiving the goods are still unchanged.

2.2.5 Holding cost rate

Holding costs express the costs (direct or indirect) to keeping parts on stock in a warehouse. To compute them, a holding cost rate is used. The yearly holding cost of an item is obtained by multiplying this rate with the purchase cost. It is not easy to define this rate and because of this, chapter 3 is dedicated to the holding cost rate. The user/company can give a rate for the whole project. The same rate is used for all the items.

2.2.6 Minimum Period to cover

To have enough inventories on hand, it is common business practice to keep the amount that will be consumed during a certain period as a base for the minimum stock. For this period the user/company often chooses a factor by which the lead time is multiplied to get the minimum period to cover. The consumption during this certain period can be used as the minimum stock level. For the different criticality classifications, different factors can be given. For example for an item of very low importance and a lead time of 10 weeks a factor of ½ can be chosen. This means that for this item the minimum period to cover becomes 5 weeks, and the estimated consumption of 5 weeks will be kept on stock. A more sophisticated way to determine the minimum stock is discussed in chapter 6.

2.2.7 Maximum Period to cover

The inventory of a certain part should not be too large. It can be, for example, that the equipment or plant is only in use for a few years. It may also be that there are inconsistencies or errors in the input provided by the user/company. Therefore the user/company can set a certain time period as the maximum period to cover. This period is used as an upper bound for the inventory of the parts and can be set, for example, at 2 years. The maximum period to cover is used to determine the maximum number of a spare part to stock. This means the decision to stock is already made. If the decision is made to stock an item, at least one item will be stocked. The maximum period to cover cannot undo this. Of course this maximum period can not be lower than the minimum period to cover (section 2.2.5).

2.3 Parameters for the equipment

The existing rule gave the possibility to fill in the penalty costs for every part. These penalty costs were the costs for every day that this part was not available when needed. In practice this is very difficult to estimate. It is better to use a few criticality classes each with a separate penalty value. The equipment should be classified in one of these classes. At this moment there is already a classification code available per equipment: Vital (high), Essential (Medium) and Auxiliary (Low) (for definitions see section 2.5). For every part one assesses in which equipment it is installed. The part gets the criticality of the highest criticality level of the equipment in which this part is installed.

2.4 Parameters for the parts

2.4.1 Introduction

In this section the different parameters for the parts (as introduced in section 1.5) are discussed. These parameters are: the price (2.4.2), the lead time (2.4.3) and the consumption rate (2.4.4).

2.4.2 Price

The price of the spare part is the price as set by the supplier. The purchase cost of an item is the price adjusted with the percentage as explained in paragraph 2.2.2. In this report the US Dollar is chosen as the currency for calculations.

2.4.3 Lead time

The lead time is the time needed to replace the part as indicated by the supplier, adjusted with the extra time as explained in section 2.2.3. It starts from the moment the supplier is informed until he delivers the part on site. For some projects the lead time is adjusted with the percentage as described in section 2.2.3.

2.4.4 Consumption rate

This is the consumption rate in units per year of the part evaluated. Two approaches can be used to specify this rate:

1. Per equipment
2. For the total quantity installed

In the first case an estimate is made per equipment (for example a pump) for the number of parts needed in a year (for example 1). For a new plant this is a very rough estimate because of the lack of historical data. To determine the total number of parts needed that year the outcome is multiplied with the total number of pumps in the project (for example 20). The outcome of the example is: Consumption rate for the pumps = 1 x 20 = 20 per year. In the second case the estimate is made for the total number of parts installed (in the example: 20 pumps). The estimate is based on this quantity and is likely to be lower then the estimate in the first case (for example: estimated needed number of parts in a year: 10 – 15 for the total of 20 pumps). This is because the estimation per equipment is more difficult to make and people then tend to estimate a higher value. A better overview exists when all equipment is considered.

The difference between the two approaches can be large. This is especially true for parts for which a large number is installed. The second method is recommended because the first will too easily lead to an overestimation of the consumption rate.

Another remark should be made concerning the difference between planned maintenance and unplanned maintenance. If all the pumps installed each year need replacement parts at a planned moment it will not be necessary to keep these all in stock because it is better to order them a lead time before the pumps will be overhauled. In this study only demand for parts originating from unplanned maintenance is considered.

2.5 Penalty Values

Three criticality levels are distinguished: Vital (High), Essential (Medium) and Auxiliary (Low). In the Manual “Guidelines for spare parts” (September 2002) the following description is used:

Vital (High) Equipment
Breakdown of such equipment would cause an immediate and unacceptable penalty, or create an immediate danger to health, safety, environment or reputation; such equipment is required to safeguard the technical integrity of the facility

Essential (Medium) Equipment
Equipment, operated in systems/subsystems, the unavailability of which would induce a significant production deferment or significant financial penalty. The purchase of spare parts for essential equipment may be justified by calculated penalty costs

Auxiliary (Low) Equipment
Equipment which, in view of its function, can be allowed to remain temporarily out of operation without having a serious effect on operations and without reducing safety below an acceptable level.

The penalty costs per class should be filled in per project. For the first two classifications, Vital (High) and Essential (Medium), the penalty costs are per day. The user/company can also enter a number of days for which there are no penalty costs. There is a possibility that costs only occur after some days, for example due to the time required before an item is delivered either from stock or a local supplier.
For the last classification, Auxiliary (Low), one can fill in the one-time costs incurred. This is because not having a part classified as auxiliary is inconvenient but not really damageable to the plant. So the one-time costs are for inconvenience of an item not being stocked.

An estimate for calculation purposes of the penalty costs for the different classes is:

Vital – $ 24,000 per item short per day

Essential – $ 4,800 per item short per day

Auxiliary – $ 50 per item short

All three estimates can be adjusted in the program.

2.6 Conclusion

In this chapter the different parameters were discussed in more detail. The parameters on the project level need to be entered once by the user/company at the start of a new project. Default values are set to help the user/company but it is important to know what these values mean and to change them when needed. Penalty values only need to be chosen for three criticality classifications. To make it easier to comprehend ranges can be chosen for these costs. On the equipment level, equipment needs to be given a criticality classification. On the part level, the price and lead time is given by the supplier. The user/company needs to make an estimation of the yearly consumption. Because the holding cost rate needs more research, the next chapter is about the determination of the holding cost rate.

3. Holding Cost Rate

3.1 Introduction

The holding cost rate is the percentage of the purchase costs of a part that is used for the inventory costs. Inventory costs are also called holding costs, costs associated with having inventory on hand. In this chapter a more detailed investigation is done on this holding cost rate. In section 3.2 some theory about the holding costs is discussed. In section 3.3 the situation at an oil company is analysed. New items on stock are considered separately (section 3.4). Different classifications and the problem of volume are briefly discussed in section 3.5. The chapter is finished with some conclusions (section 3.6).

3.2 Theory

In the formula of the basic Economic Order Quantity (EOQ) one of the parameters is the annual holding cost per unit. In practice a percentage of the purchase costs is used as an estimation of the holding costs (Nahmias, 2001). To define the holding costs this percentage, the holding cost rate, needs to be computed. It is important to define this parameter because of the influence it has on the order quantity. In this respect the assumption is made that there is enough capacity to store the needed inventory. Of course in reality this is not the case, but in this study it is assumed that all storage capacity will never be totally used and to know the optimal significance of the EOQ, the maximal capacity of the plant is left out. Holding costs are primarily made up of the costs associated with the inventory investment and storage costs. These costs also include the costs of obsolescence.

The holding cost rate is computed by dividing the annual cost of holding goods in stock by the average inventory investment. These holding costs typically range from 20% to 30% of average inventory investment. The reason for calculating holding costs for the EOQ formula is to include the extra cost, in terms of either time of money, for holding goods in stock.

Cost items that belongs to the holding costs:

1. Interest
2. Obsolescence
3. Storage space
4. Insurance on stock
5. Deterioration
6. Salary costs
7. System costs
8. Maintenance (protection) costs

Typically, the holding cost rate remains fairly constant unless there is a change in the character of the stockroom or warehouse facility, or if the interest rates change significantly, or if obsolescence risk increases substantially. However, the holding cost rate should be recalculated periodically – perhaps yearly. What happens if the real holding cost rate differs from the estimated holding cost rate? The consequence of underestimating the holding cost rate is a somewhat larger stock investment above appropriate level, because it increases the EOQ. For a detailed sensitivity analysis see Chapter 7.

3.3 Stocking costs in an Oil Company

What is the holding cost rate in an oil company and what are the values of the different components? In the guide to procurement and logistics management (Shell, 1995) there is a quite detailed explanation of the holding cost rate. It is said to be the cost of carrying one unit of currency's worth of inventory for one year. It includes

- fixed-cost elements that generally do not change if one more item is taken into stock, such as
  - cost of space claimed by the warehouses
  - cost of running a store (rent, heating, lighting, maintenance)
  - insurance cost of the stock and warehouses
- variable cost elements that do change if one more item is taken into stock, such as
  - capital tied up in inventories

- handling of stock
- damage
- deterioration
- obsolescence

As a guide to determining these costs the following averages can be used (in each case, the figures represent a percentage of an item's value per year):

- cost of capital (either the local money-market interest rate or the company's return on investment target): say 10% - 18%
- cost of storage: 1% - 3%
- loss from damage or deterioration: 1% - 2%
- obsolescence: 3% - 5%
- handling: 2% - 3%
- insurance: 0.5% - 1.5%

To get a holding cost rate, the averages of the above percentages are summed up. This sum is 25, which means the holding cost rate is set to be 25%. This estimation is seen as a good business practice, but the percentage may vary from company to company and from location to location.

3.4 New items on stock

It is important to make a distinction between stocking the first item and stocking an extra piece of an item you already had in stock. Why is this? If an item is stocked for the first time, there are more administration costs because of introducing a number, a storage place etcetera. The storage place itself also costs money. This means that the holding cost rate is more than 25% for the first time stocking. What does this mean for the EOQ? In chapter 7 (Sensitivity analysis) the sensitivity of the estimated holding cost rate is considered in detail. As it will be seen the EOQ is not that sensitive to changes in its input parameters. Besides this, the model will become too complex when a different value for the first item is taken into account. It only counts for the first time ordering and we incur this higher EOQ for the simplicity of the model.

3.5 Classifications and volume

Two other issues need to be discussed briefly. First, it needs to be considered if different classifications of products need different holding cost rates. Based on research at logistics companies connected to the IMCC (Inventory Management Competence Centre) in The Netherlands several possibilities were proposed to make such a classification. Some of these are:

- Turnover rate
- Cost price
- Volume

For simplicity, in this research no different classifications are used. Further research needs to be done to investigate if such classifications are efficient.

Another issue is the possibility of great volume. As can be seen in section 3.4 the cost of storage is estimated at approximately 1% - 3% of an item's value per year. When an item has a big volume compared to its price, the cost of storage can be much higher. This is especially important for low cost items. Because the estimated holding costs are quite low, the EOQ can be quite high. In this case, it may be better to use cost of storage per square meter. In this research this option is not considered. These kinds of considerations need to be addressed manually, although the maximum period to cover may act as a check in case the EOQ formula advises too much inventory.

3.6 Conclusions

To compute the holding costs, in practice the holding cost rate is used. This rate is multiplied by the purchase costs of an item to have the holding costs of an item per year. Several cost items are taken into account and summed up, the holding cost rate of 25% seems the most appropriate value. This percentage may be evaluated regularly, say, yearly. Yet, small changes of 2% will hardly have an effect. Other considerations can be to use a higher rate for new items, use different rates for different product classifications and to use a higher rate for items with high volume. In this research one rate is used and the above considerations need to be handled manually.

4. To stock or not to stock

4.1 Introduction

In general, items should only be stocked if the benefits of direct availability outweigh the costs of holding the items in stock. The decision to stock is especially important for slow moving parts. As the estimated consumption for these parts is low, it is unlikely that more than one part of each has to be stocked. Having surplus of slow moving stock represents waste, both in terms of capital employed and resources devoted to its acquisition and care. The E-SPIR program computes if an item should be stocked or not according to the decision rule developed by Olthof and Dekker (1994). This stocking decision will still be used, but with some adjustments. In section 4.2 the variables needed for the decision rule are given. In section 4.3 the old decision rule is briefly explained. In section 4.4 changes are made to the rule with respect to the classification Auxiliary (low). Finally, the conclusion is given in section 4.5.

4.2 Variables

In this section the input variables of the decision rule are explained. The changes of these variables in relation to the current decision rule are also discussed. In section 4.3 the rule itself is explained.

The decision rule is based on four variables:

- Consumption rate
- Purchase costs (price plus surcharge)
- Lead time (supplier lead time plus surcharge)
- Penalty costs per day

These variables have been explained earlier in section 2.3.

The lead time here is used to compute the total penalty costs for the case there is a stock out. It is used to determine the number of days that penalty costs are incurred. Because of the new possibility by the user/company to give a number of zero -cost days, these days are subtracted from the lead time. What is left is the number of days until the new item arrives. In these days penalty costs are incurred. The variables lead time and purchase costs are only changed in definition because of the possibility to give a surcharge for the whole project. Instead of giving the penalty costs explicitly, one of three classifications can be chosen now. Per project the penalty costs for each classification must be given. All of these changes have an influence on the input variables of the decision rule.

4.3 Stocking decision rule

The decision rule is based on a comparison of costs associated with stocking one spare part and those associated not to stock at all. The yearly holding costs of stocking one part are given by the holding cost rate times the purchase price:

$$0.25 * \text{Purchase Costs}$$

Note that in section 2.5 the percentage of 25% is explained.
In case the part is not stocked
The expected costs per year in case the part is not stocked are given by:

$$\text{Yearly Consumption} * \text{Penalty Costs}$$

The total penalty costs are the penalty costs per day times the stock out period (in days). The stock out period equals the lead time minus the number of days for which no costs are incurred. Every time an item is needed, these penalty costs occur. The yearly consumption tells us how many times these penalty costs per item are incurred on a yearly basis. The product of these two factors results in the expected costs per year when the part is not stocked.

Comparing the yearly holding costs with the expected costs in case the part is not stocked yields a simple decision rule for stocking an item.

In formula the decision rule reads: "Stock at least one item if":

$$0.25 * P < C * PEN * (L - D)$$

Where

P = Purchase costs of an item in dollars

C = consumption rate per year

PEN = (estimated) penalty costs in dollars per day that equipment is not able to function, due to unavailability of the part when needed.

L = (estimated) time (in days) required to obtain the spare part, if it is needed, here called the lead time.

D = Zero-cost-days, that is the number of days before penalty costs occur.

When computer software is used to make the stocking decision, the formula above can be used. When it is important for the user/company to know how to make the decision, a more simple computation is desired.

By taking logarithms, one can reduce the multiplication to a summation. A final simplification is obtained by presenting the user/company with pre-specified ranges from which he can select. To each range an index (natural number $\in \mathbb{N}$) is associated. These indices are:

- CsI (for Consumption-Index),
- PrI (for Price-Index),
- PenI (for Penalty Costs-Index),
- LtI (for Lead time-Index)

The variables L and D are combined into one index, the lead-time demand index (LtI). The formulas and the tables for the indices are given in Appendix 1 and 2A, respectively. A brief note needs to be given about the index tables. The tables stated in the appendix have other ranges than the tables created by Olthof and Dekker (1994), but the same formulas are used to create the tables. Other ranges are used because in practice these ranges seem more appropriate. In the Guide Lines for Spare Parts of Shell (2002) the index table for the lead time is slightly changed. This is the responsibility of each company itself. When adjustments seem necessary not just the tables must be changed, but research about the underlying formulas for the indexes need to be done.

The decision rule now reads: "Stock the part if":

$$CsI + PrI + PenI + LtI > 0$$

In Table 4.1 some examples of this rule are given.

Item	**Consum-ption (p. year)**	**Lead time (years)**	**Price ($)**	**Penalty costs ($ per day)**	**Sum Index**	**Stock Yes/No**	**Item nr. (see Olthof)**
A	1	2/3	21120	10240	9	Yes	7
B	1/15	1/26	2640	160	-2	No	4
C	1	1/6	330	40960	15	Yes	8

Table 4.1. Examples of the decision rule by Olthof.

Throughout the report of Olthof and Dekker (1994) a few examples are used. In the last column above the numbers are given which refer to those examples.
In this case, item A and C are advised to be stocked.

4.4 Auxiliary (Low) one-time costs

In the current decision rule all penalty costs incurred are per day. For the classification Auxiliary (Low) there needs to be an adjustment, because the costs that occur here are quite low and we would like to incur one-time costs instead of costs per day. The index of the penalty costs is per day so a change needs to be made here.
The tables of the Penalty Costs Index and the lead time index will be combined to one new table. In the basic decision rule, the lead time is used to determine the total of penalty costs. This means the penalty costs per day times the lead time is equal to the total penalty costs. In the case of the Auxiliary (Low) specified items, the lead time doesn't have influence. If the lead time is set to one day, the total penalty cost is equal to the penalty costs per day times one and this is equal to one-time penalty costs. This means the lead time index for one day is added to the index of the penalty costs per day and a new table is

created which is called the One -time Penalty Costs Index. For this index table other ranges are chosen than for the old index table. These new ranges work better for the classification Auxiliary (Low).

Lead time Index for one day: $LtI = -2$

New table (Table 4.1):

One-time Penalty Cost ($)	ArI
0 – 400	-5
400 - 1500	-2

Table 4.2. Index one-time penalty cost

This new table will replace the index tables of PenI and LtI of Appendix 2.A. For a good overview the tables needed for the classification auxiliary (low) can be found in Appendix 2.B.

Example

An arbitrary item is chosen with a price of $ 375, a consumption rate of 1 per 2 years and one -time penalty costs of $ 200. To give a comparison the old decision rule is also used to compute the index. In this case we take the lowest value of the penalty costs, being $ 200 per day. Because penalty costs of $ 200 per day fall in the lowest class, this same amount is used and a lead time of 7 days is used.

New decision rule

		PrI	CsI	ArI
Price	$ 375	5		
Consumption Rate	1 per 2 years		0	
Penalty Cost	one-time $ 200			-5

Applying the decision rule: (PrI + CsI + ArI) = (5 + 0 - 5) = 0

Recommendation: *Reconsider*

Old decision rule

		PrI	CsI	PenI	LtI
Price	$ 375	5			
Consumption Rate	1 per 2 years		0		
Penalty Cost	$ 200 per day			-2	
Lead time	7 days				1

Applying the decision rule: (PrI + CsI + PenI + LtI) = (5 + 0 - 2 + 1) = 4

Recommendation: *Stocking*

Notice that with the adjusted (new) decision rule the sum of the indices is lower. It follows that fewer items of the auxiliary (low) classification, will be stocked.

How does the rule work?

When the holding costs per year of holding one item on stock are computed, it will be as follows:

$$\text{Holding costs} = 0.25 * 375 = \$\, 94 \,/\, \text{year}$$

In the first case the penalty cost of not having an item is $ 200. This penalty will occur approximately once every two years, because of the given consumption rate. So the average penalty costs will be $ 100 per year.
The holding cost of stocking one item is about the same as the penalty paid when not having an item. The advise is therefore to reconsider stocking.
In the second case (of the old decision rule) the total penalty cost of not having an item is:

$$\text{Penalty cost} = 7 days * \$200 / day = \$1400$$

This again will occur approximately every two years. In this case the estimated penalty costs per year ($ 700 per year) are far greater than the holding costs per year ($ 94 / year). Evidently the advise is to stock the item.

In the decision rules indices are used to make the computations easier. The use of these indices creates a small error, which is acceptable because of the uncertainty about the real values. Notice how important it is to know how the penalty costs are made up in practice.

4.5 Conclusion

For the decision to stock or not to stock the decision rule of Olthof and Dekker (1994) should be used with a few changes. First of all some definitions of the input variables are changed, but this has no effect to the rule itself. The classification Vital (high) and Essential (medium) still use the same rule and index tables. In practice it is preferred to use one-time penalty costs for the Auxiliary (low) classification. A new table is created for this last classification Auxiliary, which replaces two tables in the old rule. The rules are still easy to implement.

5. Economic Order Quantity

5.1 Introduction

After the question to stock or not to stock, another question comes forward: how many items need to be ordered at once. The issue at stake here is the initial purchase at the start of a new plant. In this chapter first some general theory about inventory models is given (section 5.2). In section 5.3 some detailed theory about the EOQ is stated. Next something is said about the different variables needed to compute the EOQ (section 5.4). This is followed by the rounding of the EOQ (section 5.5), some examples (section 5.6), the maximum period to cover (section 5.7) and finished with a short conclusion (section 5.8).

5.2 Theory

The amount of items to order at once is called the batch or order quantity Q. An important assumption made here, is that the future demand is deterministic and given. Deterministic demand (causally determined and not subject to random chance) may seem to be a very unrealistic assumption, because of the stochastic (random) variations in demand. Also in case of stochastic demand it is often feasible to use deterministic lot sizing. The determination of Q should, also in a stochastic case, essentially mean that the ordering and holding costs are balanced. A standard procedure is to first replace the stochastic demand by its mean and use a deterministic model to determine Q. Given Q a stochastic model is then, in a second step, used to determine the reorder point R (see chapter 6).

Why is it that, in these days of advanced information technology, many companies are still not taking advantage of these fundamental inventory models? Part of the answer lies in poor results received due to inaccurate data inputs. Accurate product costs, activity costs, forecasts, history, and lead times are crucial in making inventory models work. Software adva ncements may also in part to blame. Many ERP packages come with built in calculations for EOQ which (that) calculate automatically. Often the users do not understand how it is calculated and therefore do not understand the data inputs and system set-up that controls the output. Because of this, it is simply ignored.

5.3 EOQ: theory and formula

The most well known result in the inventory control area may be the classical Economic Order Quantity (EOQ) formula. This simple rule has had and still has an enormous number of practical applications. The EOQ is essentially an accounting formula that determines the point at which the combination of order costs and holding costs are the least. The result is the most cost-effective quantity to order. Assumptions underlying the EOQ formula are :

- Demand is constant and continuous in time .
- Ordering and holding costs are constant over time.
- The order quantity does not need to be an integer.
- The whole order quantity is delivered at the same time.
- No shortages are allowed.

These assumptions do not hold all in the case of slow moving items. In this case an integer number of items is needed so rounding is needed (see section 5.5). Shortages are allowed, so this assumption is released. There is also a possibility of including q uantity discounts, but this is left out of this research.

The basic Economic Order Quantity (EOQ) formula is as follows:

$$\text{EOQ} = \sqrt{\frac{2 * (\text{Annual usage in units})(\text{Order costs})}{(\text{Annual holding cost per unit})}}$$

From hereon we will use symbols for this formula:

$$\text{EOQ} = \sqrt{\frac{2CA}{H}}$$

where

C = Annual consumption rate in units

A = Order costs

H = Annual holding cost per unit

The annual holding cost per unit H will be defined as

$$H = i * P$$

where

i = Holding cost rate

P = Purchase cost

Note that it may be possible that the EOQ formula gives a value of more than 2 while on the other hand the stocking decision is negative. This will be the case if the order costs are high, but the criticality of the items is low. Consider for example an auxiliary item with astockout penalty of $ 200, which costs $ 3000 and has a consumption of 2 per year. The new decision rule gives -5 + 2 + 2 = -1 and advises therefore not to stock. If the order costs are $ 500 per occasion, the EOQ, however, equals v2.6 = 1.65, which will be rounded up to 2. Notice that by ordering 2 items after a demand instead of one, we save once a replenishment order of $ 500, which is more than the holding cost of the second item for half a year (0.25 x 0.5 x $ 3000 = $375). This can be considered equivalent to a one-time penalty of $ 500. The latter corresponds to an index of -2, in which case the rule advises to stock.

5.4 Input variables

As can be seen above four parameters are needed to compute the EOQ. These variables are already explained in section 2.2 and 2.3. The calculations are fairly simple, but the task of determining the correct data inputs to accurately represent the inventory and operation is more difficult. Exaggerated order costs and carrying costs are common mistakes made in EOQ calculations. Accuracy here is crucial but small variances in the data inputs generally have very little effect on the outputs. See Chapter 7 for a sensitivity analysis.

5.5 Rounding

The EOQ will almost never give an integer answer of how many items to stock. Because of this a rule of rounding is needed. For every number between zero and one the batch quantity is rounded to one. For the other numbers the following rule is used. Assume that the EOQ lies between the numbers n and n+1, that is, $n < EOQ^* < n+1$, where n is an integer. Since the cost function:

$$F = \frac{EOQ^*}{2}H + \frac{C}{EOQ^*}O$$

is convex the best choice for EOQ is either n or n +1. The right choice should be made as follows. Choose EOQ = n if $EOQ^* / n \leq (n+1) / EOQ^*$. Otherwise, choose EOQ = n + 1 (Axsäter, 2000).

In reality some suppliers will only offer batch quantities as a multiple of some number. For instance, it is possible that one can order only multiples of ten items. This means another kind of rounding is necessary. This has to be kept in mind, but it is left out in this research.

5.6 Examples

In tables 5.1 and 5.2 one can see what this formula does.

Ordercost	**Purch / item**	**Consum / year**	**EOQ**	**Order quantity**
36	1	4	33.94	34
36	6	4	13.86	14
36	100	0.5	1.2	1
36	100	4	3.39	3
36	1000	0.5	0.38	1
36	1000	4	1.07	1
36	2500	0.5	0.24	1
36	2500	4	0.68	1

Table 5.1. Costs and choice of rounding; Order cost is $ 36

The higher the purchase costs per item are, the lower the EOQ. This is because the higher the holding costs are. And the higher the consumption rate is, the higher the EOQ. Notice that an EOQ less than one is always rounded up to one.

Ordercost	Purch / item	Consum / year	EOQ	Order quantity
200	1	4	80	80
200	6	4	32.66	33
200	100	0.5	2.83	3
200	100	4	8	8
200	1000	0.5	0.89	1
200	1000	4	2.53	3
200	2500	0.5	0.57	1
200	2500	4	1.6	2

Table 5.2. Costs and choice of rounding; Order cost is $ 200

From the two tables above it can be seen that the economic order quantity will go up when the order costs are higher. This can also be seen from the formula itself. Another point is that when the normal rounding rules were applied, the same answers would be gained. This means that the interval (0.0, 0.5) will be rounded down and the interval [0.5, 1.0) will be rounded up. Notice again that quantities lower than one are always rounded up to one.

5.7 Dealing with maximum period to cover

Another variable has an influence on the initial purchase order. The user/company can set a maximum period to cover and with this variable a maximum stock can be computed (see chapter 6). This results in an upper bound for the purchase order. It is important to keep in mind that the EOQ is related to minimum costs and the maximum period to cover is a manual input variable. A maximum period to cover is a good variable but its influence needs to be understood.
See for an example and further explanation section 6.

5.8 Conclusion

To create an ordering policy it is important to find an efficient order quantity. The formula used to compute this quantity needs to be easy to use and implement. In general it is known that many logistics companies use a simple formula known as the Economic Order Quantity. It is easy to use in practical applications and easy to implement in current systems. The advise here is to use this formula to balance the ordering and holding costs.

6. Minimum Stock

6.1 Introduction

An important feature in inventory control is the determination of the minimum number of items that should be on stock. In this report the following definition of minimum stock is used: the minimum level of inventory on stock which can be tolerated (that is, before a replenishment order is placed). An order is placed when the inventory level drops below this level. The level of stock at which an order is placed is called 're -order point' and is therefore the minimum stock minus 1. Using Q for the number of items ordered at once and S for the level of minimum stock, this strategy is also called (S-1,S+Q-1)-strategy.

In this chapter it will be explained how to determine this minimum stock. In section 6.2 first will be explained why it is important to determine this number and what the difficulties are. Section 6.3 will discuss three possible methods to compute the minimum stock. The total costs of these methods will be computed in section 6.4. Section 6.5 gives a suggestion on how to decide which method to choose and section 6.6 will be about dealing with the maximum stock. Finally a conclusion in section 6.7 will summarize the results of this chapter.

6.2 Determining the Minimum Stock

A company needs to decide how many items at least it wants to keep on stock. This is an important decision because it can cost a large amount of money when the wrong decision is made. This decision can be wrong in two ways: too much or too little. In the first case the company holds too many items on stock and this will result in high holding costs. In the other case there is a large probability of needing an item when it is not on stock, which results in (high) penalty costs. Therefore it is necessary that a company makes the best possible decision. In this chapter methods are described to support this decision-making process. Because there are many other terms and methods to define the number of items on stock it is important to get clear what this chapter is dealing with.

The objective is to compute the minimum level of inventory for which it is necessary to place a new order when the stock drops below this level. The minimum stock will be determined considering the consumption during the lead time. This consumption gives the estimated number of items needed during the lead time of the ordered items. When a new order is released (for example for a number equal to the EOQ) at the moment that the inventory level reaches this number of items it means that we would like to reach an inventory of 0 exactly at the moment the new order arrives. This is of course the optimal strategy. But this simple strategy is impossible to implement because the consumption is almost never exactly known. An example will make this clear (note that the parameters for this example are chosen for their simplicity, not because of practical interest):

Suppose you have an item X with an estimated yearly consumption of 12 items. The lead time of item X is 1 month. It seems to be optimal to order a new amount of items at the moment that the inventory level drops from 2 to 1 item (say: the minimum stock is 2 and reorder when the inventory drops below this level). During the month the new items have not arrived there will be an estimated consumption of 1 item (12 per year / 12 months = 1 per month) and therefore the new items arrive exactly at the moment that there are no items X on stock.
But what happens if during this month not 1 but 2 or 3 or more items are consumed? This will result in a stock out with penalty costs because there are not enough items on stock. Maybe it is less expensive to keep an extra stock of 1 or 2 items than to risk the probability of getting out of stock. So the minimum stock of item X not only depends on the estimated consumption during the lead time, but also on the probability of a higher (or lower) consumption and also on the penalty costs and the holding costs of item X. Knowing this probability, an estimation of the penalty and the holding costs can be made for different minimum stock levels.

This example shows that the minimum stock depends on several factors:

1. the lead time of the part
2. the costs of keeping an item on stock

3. the importance of having the part (the criticality of the equipment) expressed (in this section) in penalty costs
4. the consumption rate of the part and the distribution of this rate

For factors 1 and 2 the values are taken as the user enters them. For factor 3 there are two methods, which will be explained in this chapter. In the next chapter a more detailed investigation will be ma de and a recommendation will be given. Factor 4 is the most complex factor in the determination of the minimum stock. Therefore the next section will give an explanation of the difficulty of this factor and a description of the methods to solve this.

6.3 Distribution of the consumption

6.3.1 Introduction

Before computing the minimum stock, a short overview of the importance of knowing the distribution of the consumption rate will be given in this section. The probability of needing x items has to be determined. For example, what is the probability that the consumption of an item during the lead time will be 5, 6 or 7 items instead of an expected amount of 4? This can be different for every company and real data is needed to compute this probability. With these probabilities, a distribution for the consumption can be estimated. This chapter will give a theoretical description of the minimum stock method. The minimum stock will be computed with three different distributions to show the effect of different distributions on the minimum stock. It is difficult to make a practical investigation for one specific company, based on real data, because there are no real data available, but this chapter will give a recommendation of which method to use in which situatio n.

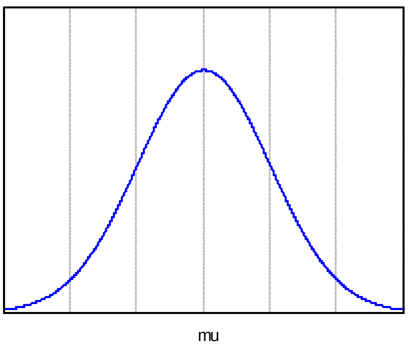
The company can choose the specific distribution that is appropriate for its situation. In the sections 6.3.3, 6.3.4 and 6.3.5 these three methods will be explained. But first something will be stated about the number of items replaced at once (section 6.3.2).

6.3.2 Number of items replaced at once

Only estimating the expected number of items that will be needed during a certain time period is not enough. There is a big difference between the scenario of needing 4 items all at one time (e.g. when a pump has four gaskets, which are all replaced when the pump is overhauled) or needing 4 times 1 item equally divided over a period. In the first case a stock of 2 or 3 will not be enough while in the second case, depending on the lead time, a stock of 1 might be enough. A simple rule will be sufficient to deal with this 'problem'. When the user fills in the estimated consumption rate, he also has to fill in the expected number of items needed at one time. From there on this number of items will be consideredas one unit for the consumption rate. After computing the minimum stock this number of units on stock will be multiplied by the number of items replaced in one time. This approach is only possible when the user knows this number.
If this is not known on forehand, this approach will not work. From now on the assumption is made that one item at a time is requested.

6.3.3 Factor Variance Method

The first method assumes that the consumption of items during the leadtime is normally distributed for which a mean and variance is known. It is described in Silver and Peterson (1998, p. 269) and we will refer to it as the Factor Variance method. A picture of the Normal Distribution is shown in Figure 6.1 to show the properties of the Normal Distribution.



Information

This picture shows the probability density function (pdf) of the Normal Distribution.

Properties of the Normal Distribution:

- Bell-shaped
- Mean mu
- Standard deviation sigma

x-axis: x
y-axis: pdf, f(x)

Figure 6.1 The Normal Distribution, with mean mu and variance = 1

This distribution can be suitable to model the consumption because this is a well-known and widely used probability distribution function. Especially for consumable goods, items with a coefficient of variation (standard deviation divided by the mean) lower than 0.5 a reasonable goodness of fit is reported in a number of applications (Minner, 2000). For slow moving items the distribution is less suitable, because there will be a probability of negative demand. This can simply be seen in Figure 6.1: imagine an average consumption (mu) of 1 item and the bell-shape will be around 1 and the left part of the shape will be on the negative side, which corresponds to a negative demand.

The formula to compute the minimum stock is:
Minimum stock = Mean consumption + (Factor * Standard deviation)

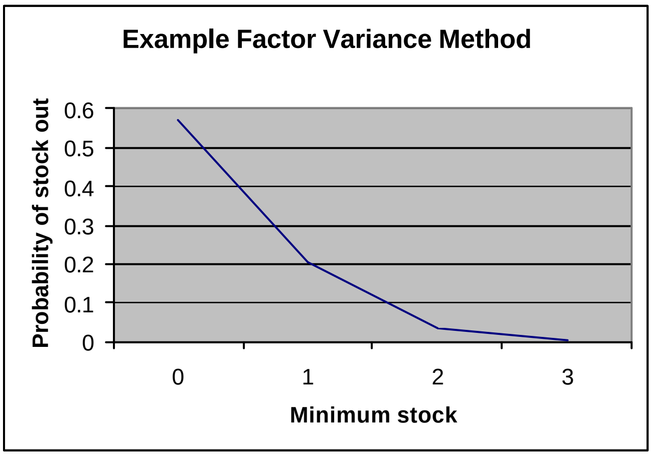
The standard deviation is the square root of the variance. In case of an initial stock estimate no data is available. Therefore the variance (and the standard deviation) is set at 1. The formula therefore becomes:

Minimum stock = Mean consumption + Factor

The Factor is a safety factor, when this factor is 0, for example, the minimum stock equals the estimated mean consumption. There is always a risk of consuming more then the estimated consumption, so the larger the safety factor, the larger the minimum stock and the smaller the probability of a stock out. The probability of a stock out is the probability of a higher consumption than the minimum stock and can be computed with the help of the Normal Distribution. In Appendix 3A is explained how this probability is computed. An example will make this more clear. Consider an item with a consumption rate of 1 per year and a lead time of 2 months, so the mean consumption during the lead time is 1/6 item. For different levels of minimum stock the corresponding factor and the corresponding probability of a stock out are computed and placed in Table 6.1. A picture of the table is shown in Figure 6.2.

Minimum stock	Safety Factor	Probability of stock out during lead time
0	-0.17	0.568
1	0.83	0.203
2	1.83	0.034
3	2.83	0.002

Table 6.1. Safety Factor and Probability of a Stock out during lead time for different levels of minimum stock



Information

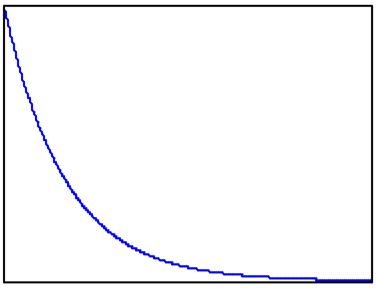
This picture shows for different levels of minimum stock the probability of a stock out during the lead time. Note that the larger the minimum stock, the smaller the probability of a stock out.

Figure 6.2 Example of the Factor Variance Method

For the different levels of minimum stock the according total costs can be computed. The minimum stock with the lowest total costs will be chosen. How these total costs can be computed will be discussed in the next chapter.

6.3.4 Erlang-k Method

The second method is based on the assumption that the time between two successive demands follows an Erlang-k distribution, which is the sum of k independent exponential distributions (so the Erlang-1 distribution is the same as an exponential distribution). Tijms (1994) claims that any probability distribution can be approximated by linear combinations of such distributions. This distribution is widely used in inventory models (Minner 2000). Figure 6.3 shows a picture of the Exponential distribution.



Information

This picture shows the probability density function (pdf) of the Exponential Distribution.
This distribution has only positive values, so the probability of a negative value is zero.

x-axis: x
y-axis: pdf, f(x)

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Figure 6.3 The Exponential Distribution

The k factor is a factor which indicates whether the distribution is memory less. If k=1 then an event is always equally likely to happen, even if there was no event for a long time. The higher the k factor the more peaked the distribution is, hence there is a low probability that two events occur quickly behind each other. In other words, for k > 1, the spread a round the mean value will be k times smaller than for k = 1. The larger the shape parameter k, the less variance the distribution will have. For k growing to infinity the distribution converges to a deterministic value. Given the value for k, the yearly consumption and the lead time, the probability of a stock out during the lead time can be computed for different levels of minimum stock. Details are given in Appendix 3.B. With this probability the stock with the lowest total costs can be calculated. An example is given to show the effect of different parameters k.

Table 6.2 gives the probabilities for an item X with a lead time of 2 months and an estimated yearly consumption rate of 1 item. These are the probabilities of a consumption of 0, 1, 2, 3 or 4 items during the lead time for different values of the shape parameter of the Erlang-k distribution (k = 1 to 10). From this table can be seen that for k = 7, 8, 9 and 10 the probabilities are almost the same. Therefore from now on only the values for k = 1 to 7 will be taken into account.

m	k=1	k=2	K=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
0	0.846	0.955	0.986	0.995	0.998	0.999	1.000	1.000	1.000	1.000
1	0.141	0.044	0.014	0.005	0.002	0.001	0.000	0.000	0.000	0.000
2	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 6.2 Probabilities of consumption of m items for different values of k

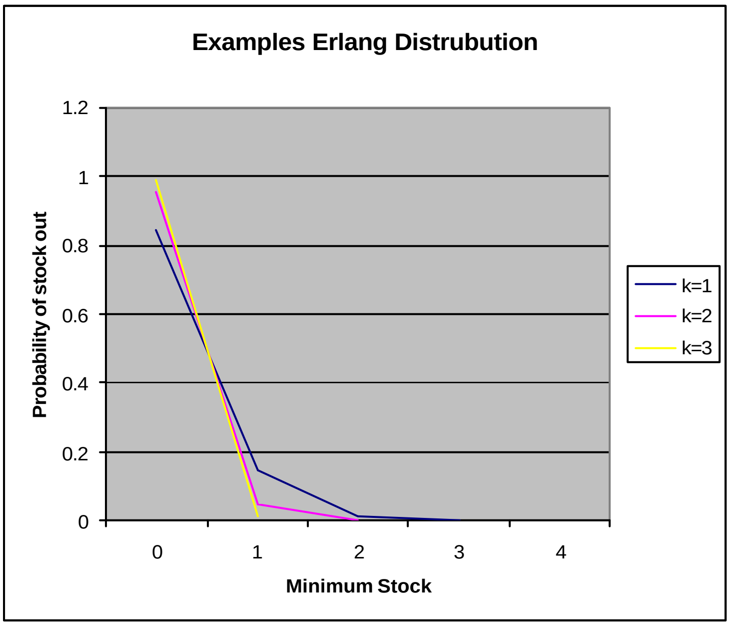
For example for k = 1 the probability of a consumption of 1 item during the lead time of 2 months is calculated as 0.141. For k = 4 this probability is reduced to 0.005 and for k = 10 this probability is 0.000. The Erlang-k distribution is suitable for the distribution of the consumption rate for many items, because it is not so heavy-tailed as most other distributions. By varying k, the shape can easily be controlled. It is more peaked for larger k's.

From Table 6.2, the probabilities of a stock out for different values of minimum stocks during the lead time can also be computed. For example, when the minimum stock is 0, the probability of a stock out during the lead time is the probability of a consumption of more than 0 items, so for k=1 this probability is 0.153. The probabilities of a stock out for the different levels of minimum stock are computed in Table 6.3.

m	k=1	k=2	K=3	k=4	k=5	k=6	k=7
0	0.154	0.045	0.014	0.005	0.002	0.001	0.000
1	0.012	0.000	0.000	0.000	0.000	0.000	0.000
2	0.001	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 6.3 Probabilities of a stock out during the lead time for different levels of minimum stock (m) and for different values of k

The results of Table 6.3 are also shown in a picture. To get a clear picture, only the results of k=1, k=2 and k=3 are shown.



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This figure shows the relationship between the level of minimum stock and the probability of a stock out during the lead time, for different k's of the Erlang-k distribution. Note again that the larger the minimum stock and the larger the k, the smaller the probability of a stock out.

Figure 6.4 Probabilities of a stock out for different levels of minimum stock

Considering Table 6.3 and Figure 6.4 a few remarks about the values k of the Erlang-k distribution should be made. The probabilities of a stock out for k = 5, 6 and 7 are very low, even for slow moving items. Because in practice this is expected not to be that low, we recommend choosing k not larger than 4. It is difficult to exactly explain the differences between these values, only the difference between k =1 and k > 1 is explicit. When the spare part is installed in a lot of machines (say more than 5), the probability of failure is independent of the time the parts are installed, because every moment there is a probability of a failure of an individual part. This means that a k of 1 should be chosen. For k > 1 the Erlang-k distribution becomes a sort of wear-out model. The probability of failure depends on the time a part is installed. The older the part, the higher the probability of failure. Therefore the probability of failure in a period after installing an item and during the lead time of the ordered part will be smaller. This results in a lower level of minimum stock. The larger the k, the smaller the probabilities on mulitple failures. But a k > 1 should only be chosen when the part is installed in one (or a few) machine(s).

To say it simply: take a k of 1 when the purpose is to avoid any risk, or in a situation of areas with highly uncertain logistics (for example Middle East), or when the item is installed in many machines which are maintained independently. Take a k of 4 in case of areas where the item is installed in a few machines only and when the spare parts logistics is trustworthy (for example in the Rotterdam or Houston areas).

6.3.5 Simple Minimum Stock Method

The third method is a very simple estimation of the number of needed items during a period. This method is not a scientific method but it is often used as a "common sense" idea. The idea of the method is to increase the lead time with a safety factor to reach a higher level of minimum stock.

The formula is:
Minimum Stock = C * L * factor.
With C = Consumption rate and L = lead time. The factor is the minimum period to cover (see also section 2.2.6) and can be different for the different classes of equipment. An example will show how this method works. An item X has the following properties: Lead time 2 months, a Consumption rate of 1 per year, Criticality is Vital (High) with a safety factor 3 and a Purchase costs of $1000. The minimum stock is the consumption during the lead time multiplied by the safety factor. This computation results in a minimum stock of 1 * 1/6 * 3 = ½ item. This will be rounded to 1. This example shows an important property of this method: the purchase costs of the item have no influence on the minimum stock. This is the disadvantage of this method; it doesn't take holding and penalty costs into account. Because this method is not based on a distribution of the consumption rate there are no calculations needed to compare the total costs for different minimum stock levels, as in the other two methods. Therefore this method will not return in the section about total costs computations.

6.3.6 Conclusion

In this section three different methods have been described to determine the minimum stock level. The first is the Factor Variance Method, based on the

Normal Distribution. This method is especially suited for parts with a relative high demand. The second method is the Erlang-k distribution, based on the assumption that the time between successive demands n is the sum of k exponential distributions. This method is suited for all kinds of consumption rates and has smaller probabilities for larger consumptions than the estimated consumptions. The third method is a simple rule to increase the period for which there should be enough stock with a certain factor, different for the criticalities.

A choice between the different methods is difficult to make as simpleness has to be compared with scientific rigourness. A scientifically justified choice can only be made if enough demand data is available, which unfortunately, is not the case in case of initial ordering of spare parts. To our opinion, however, the Erlang -k method is the best method to base the minimum stock for slow moving spare parts, as it is most founded in scientific theory. The choice for the appropriate k is important but difficult if there is not enough data available. The choice for $k = 1$ is appropriate for parts that are installed in several machines, say more than 5. Choose $k > 1$ when the failure of the part is more like wear-out, then the probability of failure depends on the time a part is installed. This is only possible when the part is installed in one (or a few) machine(s).

6.4 Total Costs Minimum Stock

6.4.1 Introduction

To make a choice between several minimum stock levels a computation has to be made for the total costs at different levels of stock. The total costs are holding costs and penalty costs. In this section the computations are made for holding costs (section 6.4.2) and penalty costs (section 6.4.3) for the Factor Variance Method and the Erlang-k Method.

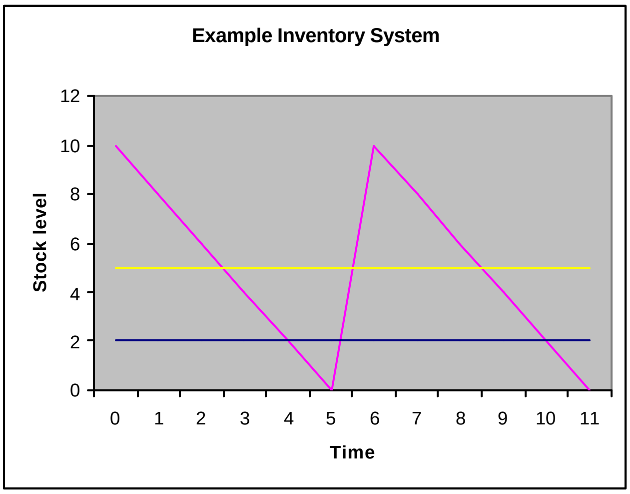
6.4.2 Holding costs

The total yearly holding costs are the average yearly inventory multiplied by the holding cost rate and the purchase costs of an item. The holding cost rate and the purchase costs are known. The average inventory depends on the minimum stock, the order quantity and the consumption during the lead time.

The average inventory can easily be computed by the following reasoning: The average stock is the minimum stock (M) added by the order quantity (Q) divided by a half minus the demand during the lead time (C*L). In a formula:

$$\text{Average stock level} = M + Q/2 - C * L$$

This can be shown in a picture, see Figure 6.4. This figure shows an inventory system of an item with a minimum stock of 2. When the stock level drops below this minimum stock, an order of 10 items is placed and received a lead time of 2 periods later. The consumption rate is 2 items per period, so at period 4 the first order is placed and the stock level reach the level of 10 at period 6. The average stock level is 5 (the yellow line in Figure 6.4). This is equal to 2 + 10/2 – 2.



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This picture shows an example of an inventory system. The initial inventory of 10 items is decreasing over the time and when this level drops below the minimum stock of 2 an order of 10 items is placed. After a lead time these items arrive and the stock level is again 10 items, and this cycle continues.

Figure 6.4 Example of an inventory system with Minstock is 2, Order Quantity is 10, Lead time is 2 periods and the consumption rate is 2 per period.

Now the holding costs of the item of the example of section 6.3 will be computed for the different methods.

Remember that this item has the following properties: Lead time 2 months, a Consumption rate of 1 per year and Purchase costs of $ 1,000. The penalty costs of this item with criticality Vital (High) are set at $ 30,000 per day.

The holding cost rate of 0.25/year and the purchase costs imply that the holding cost are: 0.25/year * $1,000 per item = $ 250 per item per year. For the order quantity the Economic Order Quantity as explained in Chapter 5 will be used. In this example the EOQ is 1. The consumption during the lead time is 1/6 item.

The average inventory is the same for both methods and differs only for the different levels of minimum stock. Table 6.4 shows the different average inventories and the according total yearly holding costs for different levels of minimum stock.

Minimum Stock	Average Inventory	Yearly holding costs
m = 0	0.33	$83
m = 1	1.33	$333
m = 2	2.33	$583
m = 3	3.33	$833
m = 4	4.33	$1.083

Table 6.4 Average inventory and total yearly holding costs for different levels of minimum stock

For example: for a minimum stock of 2 the average stock becomes:

$$\text{Average stock level} = M + Q/2 - C * L = 2 + 1/2 - 1/6 = 2.33$$

The corresponding total yearly holding costs are 2.33 multiplied by 250 and this is $583 /year.

6.4.3 Penalty costs

The penalty costs are the costs of not having a part when needed and depends on the criticality + classification of the needed item and the number of days of not having the part. The exact formulas, to compute these penalty costs for the different methods, are shown in Appendix 3.C. In this section the penalty costs of the item of the example of section 6.3 will be computed for the different methods. First, the expected total number of yearly penalty days should be computed, depending on the probabilities of a stock out for the different levels of minimum stock. Table 6.5 shows this number of days for the different methods.

Minimum stock	Factor Variance	Erlang k=1	Erlang k=2	Erlang k=3	Erlang k=10
0	56.395	65.632	62.195	61.271	60.834
1	8.274	5.058	1.369	0.438	0.007
2	0.773	0.270	0.008	0.000	0.000
3	0.037	0.011	0.000	0.000	0.000
4	0.001	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000

Table 6.5 Total number of yearly penalty days for different methods and different levels of minimum stock

This total number of penalty days should be multiplied by the penalty values and this results in the total yearly penalty costs. For the different methods and levels of minimum stock the results are shown in Table 6.6.

Minimum stock	Factor Variance	Erlang k=1	Erlang k=2	Erlang k=3	Erlang k=10
0	1,691,850	1,968,975	1,865,841	1,838,133	1,825,009
1	248,231	151,754	41,081	13,142	9
2	23,204	8,098	241	9	0
3	1,097	329	1	0	0
4	22	11	0	0	0
5	0	0	0	0	0

Table 6.6 Total yearly penalty costs ($) for different methods and different levels of minimum stock

6.5 Advise

To determine the optimal minimum stock the example of the last sections will be examined here. The total costs consist of holding and penalty costs. Therefore Table 6.7 gives the total costs for the different levels of minimum stock for the different methods. For every method the optimal minimum stock (with according lowest total costs) is bold.

Minimum stock	Factor Variance	Erlang k=1	Erlang k=2	Erlang k=3	Erlang k=10
0	1,691,933	1,969,058	1,865,924	1,838,217	1,825,093
1	248,564	152,087	41,414	13,475	**343**
2	23,788	8,681	**824**	**592**	583
3	1,930	1,163	834	833	833
4	**1,106**	**1,094**	1,083	1,083	1,083
5	1,333	1,334	1,333	1,333	1,333

Table 6.7 Total costs ($) for different levels of minimum stock for the different methods

Compare these values for the minimum stock with the computed minimum stock of 1 item with the Simple Minimum Stock Method (as computed in section 6.3.5). The difference is due to the high value of the penalty costs, which are not present in the Simple Minimum Stock Method.

6.6 Maximum stock

When the maximum stock (the number of expected items needed during the maximum period to cover as described in section 2.2.7) is lower than the advised minimum stock, an inconsistency arises. In the example of section 6.5 it could be possible that the maximum period to cover was set at 2 years. In that case, the maximum stock would be 2 items. However using the Erlang-1 method, the recommended minimum stock is 4 items. What quantity should be chosen? Choosing the maximum stock of 2 items implies neglecting the results of the computation of the minimum stock based on total costs. On the other hand, choosing the recommended minimum stock of 4 items can result in a large inventory after 2 years with an associated risk of obsolescence. We recommend to advise the lowest value in this case while giving an indication in the program to the user that the minimum economic stock is higher. He/she can in that case always overrun the maximum period (as that is set for a group of items, not for a particular item). Note that we also recommend to round the maximum stock up to 1 if the expected demand over the maximum period to cover is less than one.

6.7 Conclusion

There are several ways of determining the minimum number of items on stock. In this chapter three are described. After evaluating these three methods one conclusion is that the Simple Minimum Stock Method is not a good solution, when compared with the other methods. The method does not take costs into account and therefore often stocks too many or too few items. It is therefore strongly suggested not to use this method in determining the minimum stock. The best method for the determination of the minimum stock for slow moving spare parts is the Erlang-k method. The choice for the according k is difficult to make, but with the guidelines provided in this chapter the choice is easier to make.

7. Penalty Values

7.1 Introduction

In chapter 6 the minimum stocks are determined based on the lowest total costs. The importance of having a part on stock is expressed in penalty costs. There is also another way to express this importance: with service levels. This chapter will investigate the difference between these two methods. In this report penalty costs have been chosen instead of service levels and this chapter will show why. Therefore first the definitions of penalty costs and service levels will be explained in section 7.2. To compare the minimum stocks based on penalty costs and based on service levels a few examples are computed in section 7.3. Section 7.4 will explain why the method based on penalty costs is advised. Finally section 7.5 will summarize the results of this chapter.

7.2 Penalty values and Service levels

Chapter 2 gives the definition of penalty costs and chapter 6 explains how to compute the total costs for different levels of minimum stock. The level of minimum stock with the lowest total costs is advised. Different criticality classifications result in different penalty values. The user/company has to give a range of penalty values to each criticality classification, on project level. To make it as easy as possible, pre-defined ranges are given, which the user/company can choose from. In Table 7.1 these ranges are given.

Classification	Range ($)			
Auxiliary	0	-	400	
	400	-	1,500	
Essential	0	-	1,000	per day
	1,000	-	7,500	per day
	7,500	-	30,000	per day
Vital	30,000	-	60,000	per day
		>	60,000	per day

Table 7.1 Pre-defined ranges for each classification

For service levels many definitions are available. In this report we define the service level as the probability that there is no stock out during the lead time.

This is also called the cycle service level, see e.g. Chopra and Meindl (2001). So the service level is the probability that the consumption during the lead time is less than the minimum stock. For different levels of minimum stock the corresponding service levels can be computed. The minimum stock is the first level of minimum stock for which the service level is greater than or equal to the required service level. The required service level can be different for the different criticalities. The following service levels might be considered:
Criticality Vital (High) – 99% or 98%
Criticality Essential (Medium) – 95%
Criticality Auxiliary (Low) – 90%
These service levels give the proportion of requests for stock items which are satisfied on demand. It provides an indication of how well the inventory is meeting user requirements.

7.3 Example

The question is whether the service levels as given in section 7.2 will result in the same levels of minimum stock as with the method using the penalty costs. In this section the minimum stock and the service level will be computed for different items.

Because this section has the purpose to show the difference between a computation based on penalty costs and based on service levels, there is no need to show differences between the computations based on the different distributions of the consumption rate. Therefore the computations in this section will be made with the Erlang-1 method (note that the results of the Erlang-1 method are almost the same as the Factor Variance method, as shown in Chapter 6).

For different items the minimum stock will be computed based on service levels and based on penalty values. Most of the properties of the items are the same: the consumption rate is 1 item per year, the lead time is 2 months, the criticality classification is Vital (High), the yearly holding costs are 25% of the purchase costs of the item, the order costs are set at $ 75 per order. The varying property is the price, which varies from $ 10 to $ 100,000.

The minimum stock levels for these items are shown in Table 7.2. The computations are made as described in Chapter 6 and Appendix 3. Remember that the criticality classification of the items is Vital (High), which means a daily penalty value of $ 30,000 or $ 100,000 and a service level of 98% or 99%.

	Penalty values ($ per day)		**Service levels**	
Price ($)	30,000	100,000	98%	99%
10	4	5	2	3
100	4	4	2	3
1,000	4	4	2	3
10,000	3	3	2	3
1,000,000	2	3	2	3

Table 7.2 Minimum stock for different prices

Table 7.2 shows that the minimum stock based on target service levels does not depend on the item price. A minimum stock based on penalty costs will be larger the lower the price (because the holding costs are lower for lower prices. From the examples can be concluded that there are large differences between basing the minimum stock on service levels and basing the minimum stock on penalty values.

7.4 Argumentation

In this section we will argue that it is better to base the decision how many items to stock on penalty values. The most important reason is the fact that the service level does not take costs into account. For the expensive and for the less expensive items the same decision will be made, only based on target service levels and the demand rates. Another problem with service levels concerns the period or the group of items for which the service level is computed. It is possible, theoretically, to reach for a few items a very high service level and for a few items a very low service level. On average this may result in the required service level. The main reason for a lot of companies to use service levels may be the ease of using and determining the service levels and the fact that you can easily check the target service level with the actually reached one, but the latter does not apply for slow moving spares. To our opinion there need to be a strong warning that these service levels may result in an inventory position that is far from optimal with respect to total costs. Of course the use of penalty values also has disadvantages: in general it is very difficult to estimate the real costs of not having a part when needed. To make it easier, it is convenient to choose penalty values for the different criticality classifications instead of for all the individual parts.

7.5 Conclusion

In this chapter it is recommended to use penalty values instead of service levels for determining the minimum stock, despite the difficulties in the estimation of the penalty values. This is because the method of penalty values takes costs into account and is therefore most suited for to the minimization of total costs.

8. Sensitivity analysis

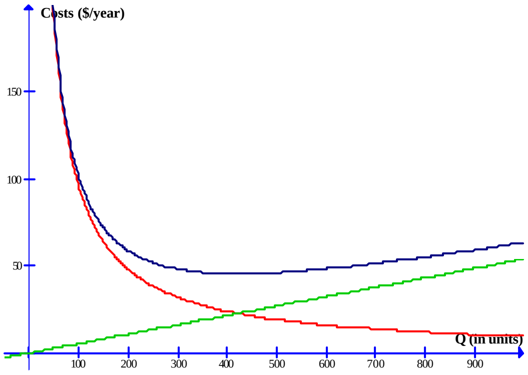
8.1 Introduction

In this chapter a sensitivity analysis is given for the Economic Order Quantity and for the Holding Cost Rate. The analysis starts with the EOQ (section 8.2). In that section first the sensitiveness and theory is discussed. There will be made a distinction between the sensitivity of the order quantity Q and the different parameters. In section 8.3 the analysis of the holding cost rate is discussed. Finally, some conclusions are given in section 8.4.

8.2 Sensitivity analysis EOQ

A well known and important feature of the basic EOQ model is that average inventory cost is relatively insensitive to changes in order quantity in the region of the optimum. In practice, actual order quantities may vary from the relevant model optimum for a number of reasons: for example, failure to make use of the appropriate inventory control model, preference for 'round number' order quantities, or error in estimating model parameters.

Silver (1998) gives a clearly and easy to follow sensitivity analysis about the EOQ. He has shown that the costs are insensitive to errors in selecting the exact size of a replenishment quantity. The total cost curve is quite shallow in the neighbourhood of the EOQ (see Figure 8.1).



Information

x-axis: Quantity (in units)

y-axis: Costs ($ per year)

Green line: Holding costs

Red line: Order costs

Blue line: Total costs

Figure 8.1. Cost curves

This indicates that reasonable-sized deviations from the EOQ will have little impact on the total relevant costs incurred.
It should be emphasized that the equality of the two cost components at the point where their sum is minimized is a very special property of the particular cost functions considered here; it certainly does not hold in general.

Mathematically, suppose a quantity Q' is used that deviates from the EOQ according to the following relation:

$$Q' = (1 + p)EOQ$$

that is, $100p$ is the percentage deviation of Q' from the EOQ. The percentage cost penalty (PCP) for using Q' instead of the EOQ is given by

$$PCP = \frac{TRC(Q') - TRC(EOQ)}{TRC(EOQ)} \times 100$$

where TRC is the total relevant costs per unit time, that is, the sum of those costs per unit time, which can be influenced by the given parameter. The dimensions are $/unit time.

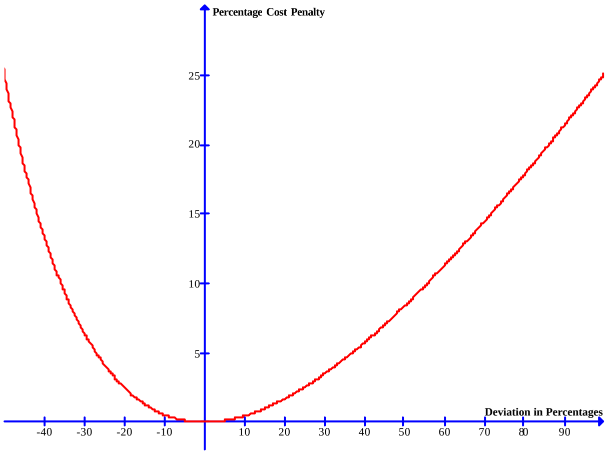
$$TRC(Q') = CO / Q' + Q'H / 2$$

$$TRC(EOQ) = \sqrt{2COH}$$

As shown in appendix 4,

$$PCP = 50\left(\frac{p^2}{1+p}\right)$$

This expression is plotted in Figure 8.2. It is clear that even for values of p significantly different from zero, the associated cost penalties are quite small.



Information

x-axis: deviation from the EOQ (100p is the percentage deviation of Q' from the EOQ)

y-axis: Percentage Cost Penalty (PCP)

Figure 8.2. Percentage cost penalty (PCP)

The insensitivity of total costs to the exact value of Q used has two important implications. First, use of an incorrect value of Q can result from inaccurate estimates of one or more of the parameters, which are used to calculate the EOQ. Therefore, it is not worth making accurate estimates of these input parameters if considerable effort is involved; in most cases inexpensive, crude estimates should suffice. Second, certain order quantities may have additional appeal over the EOQ. The shallow nature of the total cost curve indicates that such values can be used provided that they are reasonably close to the EOQ.

8.3 Holding Cost Rate

What does it mean when the real value of *i* is different from the estimated value? First some mathematical computations about this are given.

Remember the formula for the EOQ:

$$EOQ = \sqrt{\frac{2CO}{H}} = \sqrt{\frac{2CO}{iP}}$$

Now let's take the real value of *i* as *x*, then

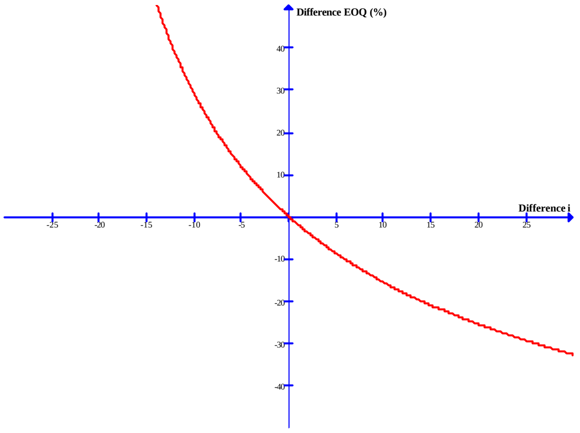
$$EOQ' = \sqrt{\frac{2CO}{\frac{x}{i} iP}} = \sqrt{\frac{i}{x}} * \sqrt{\frac{2CO}{iP}} = \sqrt{\frac{i}{x}} * EOQ$$

where *EOQ'* is the real value of the order quantity and EOQ is the order quantity with the estimated *i*.

Important to know is the difference in percentages. The real EOQ differs from the estimated one by:

$$100\sqrt{\frac{i}{x}} - 100 = \text{difference in \%}$$

If it is taken into account that i is 25%, a graph for the differences in the economic order quantity can be drawn. In Figure 8.3 the difference in i (in percentage points) against the difference in the order quantity (in percentages) is presented. This means that 1 in the x-axes stands for 26% of i instead of the estimated 25%.



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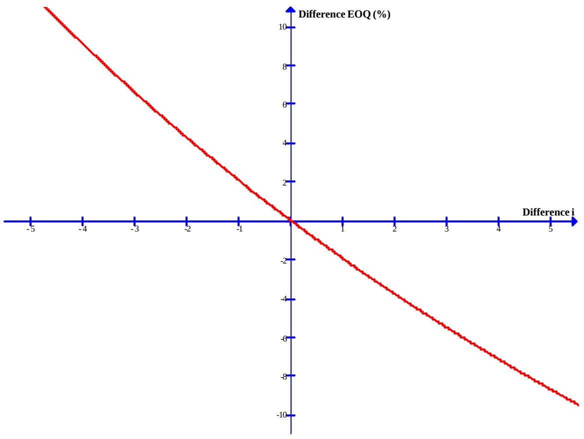
Difference of the holding cost rate (in percentage points) against the difference in the economic order quantity (in percentages).

x-axis: difference in i

y-axis: difference in EOQ

Figure 8.3. Difference of i (in perc points) against difference in order quantity (in perc)

In Figure 8.4 the figure above is zoomed in because the estimated i will not differ more than probably 5 percentage points above or below.



Information

Difference of the holding cost rate (in percentage points) against the difference in the economic order quantity (in percentages).

x-axis: difference in i

y-axis: difference in EOQ

Same as figure 8.3, but zoomed in.

Figure 8.4. Difference of i (in perc points) against difference in order quantity (in perc)

In Table 8.1, the exact numbers are given for the difference of i in a range of 10 percentage points. As can be seen the EOQ differs at most about 10% of the estimated EOQ and this is reasonable enough.

Difference i (absolute)	Difference EOQ (%) (relative)
-5	11,80
-4	9,11
-3	6,60
-2	4,26
-1	2,06
1	-1,94
2	-3,77
3	-5,51
4	-7,15
5	-8,71

Table 8.1. Difference of i against the difference in EOQ

8.4 Conclusion

It can be seen from sections 8.2 and 8.3 that the Economic Order Quantity and the Holding Cost Rate are quite robust. It is not worth making accurate estimates of the input parameters if considerable effort is involved; in most cases inexpensive, rough estimates should suffice.

9. Conclusions and recommendations

As stated in chapter 1 the purpose of this research is to make improvements to the present initial spares selection procedure. For collecting information, the standard format of the E-SPIR program is used. One of the strengths of the E-SPIR program is the facility to advise the inventory level at the start of a plant. Because of the lack of historical information at that moment, robust rules which do not need that data need to be applied. Slow moving items are the most important category of items in this respect. To improve the initial spare parts selection procedure four main questions are answered. We shall briefly answer each question and give recommendations when needed.

1. To stock or not to stock an item?

With some adjustments the current decision rule will still be sufficient. The cost rule itself (instead of the index tables) can also be applied if computations are done in the software package. The simple rule is mainly useful when a user needs to make the computations himself. The main adjustments made are the change in penalty values. In the new situation an item belongs to one of three criticality classifications: Vital (High), Essential (Medium) or Auxiliary (Low). Next each of these classes belongs to a range of penalty costs. The user/company only has to choose a range for each criticality classification once, at the start of a new plant, and subsequently has to give each item a criticality classification. In practice it is found that for the classification Auxiliary (Low) a one-time penalty cost will be more sufficient than costs per day. An extra possibility is given to give zero-cost days for the classifications Vital (High) and Essential (Medium). This is done on project level and thus needs to be given by the user/company once, at the start of a project.

2. How many to order at once?

An item is only ordered when the first question is answered with stocking. The economic order quantity (EOQ) is a simple formula and is one of the most well known results in the inventory control area. It is easy to compute, needs only a few relative easy parameters and is quite robust. These are the main reasons that this formula is chosen for computing the quantity to order at once. This is the initial order at the start of a plant. This quantity is somewhat overshadowed by the variable maximum period to cover. The company can enter this variable in the E-SPIR program. As can be seen at the third question, this variable results in a maximum stock. This means an upper bound of how many items to order. It is important to understand the effects of this maximum period to cover.

3. When to release a new order?

A minimum stock (or reorder point) needs to be computed to know when to release a new order. An important factor is the consumption during the lead time. From the probability distribution of this consumption, expected penalty costs and holding costs can be calculated. The stock with the lowest total costs needs to be chosen. We recommend using the Erlang-k method. Which k to use needs to be chosen by the company and this needs to be done on project level. To make this choice as easy as possible each k to choose from is explained in terms of expected consumption. Another decision variable also has an influence on the minimum stock. The company can give a maximum period to cover in the E-SPIR program. Using the given yearly consumption, this maximum period to cover results in a maximum stock. When the consumption is low and the daily penalty costs are high this maximum stock can be lower than the minimum stock following from the economic optimisation. That is why the maximum stock should always be rounded up if it is below one. Companies attach great value to this maximum stock. We advise also to show also the minimum stock in the E-SPIR program, so as to give the user most information.

4. How to cope with penalty values?

The minimum stock discussed above is determined based on costs. Penalty costs express the importance of having an item on stock. It is advised to use this method instead of fixing service levels. Total costs are ultimately more important than just considering the expected rate of availability of an item. For the criticality classification Vital (High) and Essential (Medium) these penalty costs are expressed in days. For the third criticality classification Auxiliary (Low) these

are one time penalty costs. On project level, each of these classifications needs to be given a pre-defined range of costs.
Besides these four main questions it is important that all the input variables are clear and are interpreted in the same way throughout the company. Additional research is done on the holding cost rate. This rate is not easy to determine. A percentage of 25% seems to be a good business practice. It is important to re-evaluate this rate regularly, say once a few years.

Further research is possible on certain issues mentioned in this report. More empirical research can be done on the Erlang-k, especially on which k to use in what case. It is important to realize that this distribution can be different for each company and for each item. Diversification is therefore possible for some items, but this makes the inventory control system more complex. More research can also be done on the holding cost rate and the penalty costs. One important question is whether the same holding cost rate applies to all different items. For the penalty costs it is important to know the costs of a failure when no spare part is available. Are the three criticality classifications enough to include all the items or need there to be more categories? These are some questions which can be investigated. All this additional information will make the inventory system more complex and should therefore only be used if considerable gains are expected.

We feel that this report establishes a good base for a better use of the inventory control system in the E-SPIR program. It is important that the company using this program understands the inputs and the outcome of the program to reach the optimal inventory level. Instead of making manual decisions with respect to inventory control, this program can be used to improve the inventory control at a company. Expert judgment is still very important to check whether the general rules of the program are appropriate for the individual spare parts.

References

Axsäter, S. (2000). *"Inventory control"*: Kluwer Academic Publishers, Boston.

Botter, R. & L. Fortuin (2000), "Stocking strategy for service parts – a case stuy", Int. J. Oper. & Prod. Mgmt 20(6), p. 656-674.

Chopra, S., & P. Meindl (2001). *"Supply chain management strategy, planning and operations"*: Prentice Hall, Upper Saddle River, NJ.

Evans, M., N. Hastings & B. Peacock (1993). *"Statistical distributions"*: John Wiley & Sons, New York.

Kennedy, W.J., J. Wayne Patterson & L.D. Fredenhall (2002), "An overview of recent literature on spare parts inventories", Int. J. Prod. Econ. 76, p. 201-215.

Mickel, L.S. & R.L. Heim (1990), "The spares calculator: a visual aid to provisioning", Proceedings Annual Reliability and Maintainability Symposium, p. 410-414, IEEE.

Minner, S. (2000). *"Strategic Safety Stock in Supply Chains"*: Spinger-Verlag, Berlin, New York.

Mukhopadhyay, S.K., K. Pathak, & K. Guddu (2003), "Development of Decision Support System for Stock Control at Area Level in Mines", IE(J) Journal-MN, 84, p. 11-16.

Nahmias, S. (2001). *"Production and operations analysis"* Boston: McGraw-Hill/Irwin.

Olthof, J.R.G.A., R. Dekker (1994). *Spare parts selection procedure.* Report Econometric Institute, Erasmus Universiteit Rotterdam.

Press, W.H., S.A. Teukolsky, W.T. Vetterling and B.P. Flannery (2004). "*Numerical Recipes in C++: The Art of Scientific Computing*", Cambridge University Press, Cambridge, UK.

Relex OpSim, see http://www.relexsoftware.com/products/opsim.asp

Shell (1995). *Guide to procurement and logistics management. Strategic issues.* Den Haag: Shell.

Silver, E.A., D.F. Pyke & R. Peterson (1998). *"Inventory management and production planning and scheduling"* New York: Wiley.

Tijm, H.C. (1994). *"Stochastic models: an algorithmic approach* " Chichester: Wiley.

Wackerly, D.D., W. Mendenhall & R.L. Schaeffer (1996). *"Mathematical statistics with applications"* Belmont: Duxbury Press

Appendix 1. Formulas for the indices of the stocking decision rule

Here the formulas for the indices are given. Each index consists of a logarithm of base two of a standardised value. The choice for the standard values, which thus have index 1, has been made as to facilitate the estimation. Moreover, the standards have been chosen in such a way that an index of 0 corresponds to a indifference (and hence cost equality) between stocking and nonstocking. More information can be found in Olthof and Dekker (1994).

Purchase Costs Index

$$\mathrm{PrI} = -{}^{2}\log\left(\frac{\mathrm{PurchaseCosts}}{10560\ \mathrm{dollars}}\right)$$

Lead time Index

$$\mathrm{LtI} = -{}^{2}\log\left(\frac{\mathrm{Lead\ time}}{4\ \mathrm{days}}\right)$$

Consumption Rate Index

$$\mathrm{CsI} = -{}^{2}\log\left(\frac{\mathrm{Consumption\ Rate}}{1\ \mathrm{per}\ 2\ \mathrm{years}}\right)$$

Penalty Cost Index

$$\mathrm{PenI} = -{}^{2}\log\left(\frac{\mathrm{Penalty\ Costs}}{1280\ \mathrm{dollars}}\right)$$

Appendix 2.A. Index values for the decision rule (Vital and Essential)

The intervals have been made with a range of a factor two. The indices are based on the midpoints of the intervals, except for the extreme values, where the intervals are unlimited.

Consumption Rate		CsI
12 or more	per year	5
6 to 11	per year	4
3 to 5	per year	3
1.5 to 3	per year	2
1 per	8-15 months	1
1 per	15-30 months	0
1 per	2.5-5 yrs	-1
1 per	5-10 yrs	-2
1 per	10-20 yrs	-3
less than 1 per	20 yrs	-4

Purchase Cost ($)	PrI
< 250	6
250 - 500	5
500 - 1,000	4
1,000 - 2,000	3
2,000 - 4,000	2
4,000 - 8,000	1
8,000 - 15,000	0
15,000 - 30,000	-1
30,000 - 65,000	-2
65,000 - 125,000	-3
> 125,000	-4

Penalty Cost ($ per day)	PenI
0 – 1,000	-2
1,000 – 7,500	1
7,500 – 30,000	4
30,000 – 60,000	5
> 60,000	6

Lead time			LtI
No lead time			-10
	<=1.5	day	-2
>1.5	<=3	days	-1
>3	<=6	days	0
>6	<=12	days	1
>12	<=21	days	2
>21	<=42	days	3
>1.5	<=3	months	4
>3	<=6	months	5
>6		months	6

Appendix 2.B. Index values for the decision rule (Auxiliary)

Consumption Rate		CsI
12 or more	per year	5
6 to 11	per year	4
3 to 5	per year	3
1.5 to 3	per year	2
1 per	8-15 months	1
1 per	15-30 months	0
1 per	2.5-5 yrs	-1
1 per	5-10 yrs	-2
1 per	10-20 yrs	-3
less than 1 per	20 yrs	-4

Purchase Cost ($)	PrI
< 250	6
250 - 500	5
500 - 1,000	4
1,000 - 2,000	3
2,000 - 4,000	2
4,000 - 8,000	1
8,000 - 15,000	0
15,000 - 30,000	-1
30,000 - 65,000	-2
65,000 - 125,000	-3
> 125,000	-4

One-time Penalty Cost ($)	ArI
0 – 400	-5
400 - 1500	-2

Appendix 3. Minimum Stock Computations

A Probability of Stock out for the Minimum Factor Method

In this Appendix an explanation of the derivation of the formula for the probability of a stock out in the Minimum Factor method is given. With this method the minimum stock is computed by estimating the average consumption during the lead time plus an extra minimum stock based on the average variance. This variance is based on the normal distribution, with mean µ and a variance of 1. The optimal minimum stock position, S, is equal to:

$$S = \mu + k * s$$

k = safety factor
µ = value of the consumption during the lead time
s = square root of the variance of the distribution of the consumption during the lead time

If we reformulate, we get a formula for k:

$$k = \frac{S - \mu}{s}$$

Define ? () as the cumulative distribution function of the standard Normal(0,1) distribution. This function is directly available in e.g. Excel™ as the function NORMDIST(.). Alternatively, good approximations are available in Press et al. (2004). Then, the probability of a stock out during the lead time is equal to:

$$P(\text{stockout}) = 1 - F\left(\frac{S - \mu}{s}\right)$$

If we fill in the safety factor k in this formula, we get the following equation for the probability of a stock out during the lead time:

$P(\text{stockout}) = 1 - F(k)$

For different values of S, we can compute the safety factor k and compute the probability of a stock out during the lead time.

It is also sometimes needed to compute the possibility of a demand of D items during the lead time instead of the probability of a stock out.

Probability of a demand D during the lead time = $P_{FV}^{d}(D) = F\left(\frac{D+1-\mu}{s}\right) - F\left(\frac{D-\mu}{s}\right)$

With this p robability, the probability of a stock out can also be represented as the sum of the probabilities of a consumption of S or more during the lead time.
Probability of a stock out during the lead time given a minimum stock S =

$$P_{FV}^{SO}(S) = \sum_{m=S}^{\infty} p_{FV}^{d}(m)$$

Note that writing out this formula results in 1-? (k), 1 minus the probability of a demand of more than S items during the lead time.

B Probability of Stock Out for the Erlang-k Method

In this Appendix a derivation for the probability of a stock out using the Erlang-k method is given. The Erlang-k distribution is the sum of k exponential distributions and has the following probability density function:

$$f_k(x) = ?^k * \frac{x^{k-1}}{(k - 1)!} * \exp(- ? * x), \qquad x > 0$$

Suppose we have a minimum stock of S items. We are interested in the probability that during the lead time more than S items are needed. For k = 1 this probability becomes:

$$\text{P(number of failures during the lead time = S)} = e^{-C*L} * \frac{(C * L)^S}{S!} = p_E^S$$

C = consumption rate (items per year)

L = lead time (years)

p_E^S = the probability of S failures during the lead time (for the Erlang method)

For k = 2 the distribution is no longer exponential so that computations become more complex. For every two exponential 'failures' only one is an 'item-failure'. So the new probability that the number of Erlang-failures is S is the sum of the Exponential probabilities of 2*S and (2*S + 1). The new formula becomes:

P(number of failures during the lead time = S) =

$$\exp(-2 * C * L) * \left[\frac{(2 * C * L)^{2*S}}{(2 * S)!} + \frac{(2 * C * L)^{2*S+1}}{(2 * S + 1)!}\right] = p_E^S$$

We can continue this for k = 3, 4 etc. In formulas:

P(number of failures during the lead time = S) =

$$p_E^S = \exp(-k * C * L) * \sum_{W=k*S}^{k*S+k-1} \frac{(k * C * L)^W}{(W)!}$$

With this probability, the probability of a stock out during the lead time given a minimum stock S can easy be computed by:

$$p_E^{SO}(S) = \sum_{m=S}^{\infty} p_E^m$$

C Penalty Costs for the two methods

In this appendix the penalty costs are computed for the two methods. The total penalty costs depend on the expected time of a stock out and the costs of a stock out. The first depends on the probability of a stock out and the second depends on the criticality of the item, with the according penalty costs. There is a *fixed penalty cost* for backordering an item with criticality classification Auxiliary (Low). There is a *daily penalty cost* for backordering an item with criticality classifications Vital (High) and Essential (Medium). There is also an extra possibility of having zero-cost days (days for which no penalty costs occur).
For both cases and methods, the computations are described below.

In the appendices A and B the probabilities of a stock out during the lead time and a certain demand during the lead time were developed for both methods. These formulas are shortly shown here again:

Factor Variance method:

Probability of a demand D during the lead time = $P_{FV}^{d}(D) = F\left(\frac{D+1-\mu}{s}\right) - F\left(\frac{D-\mu}{s}\right)$

Probability of a stock out during the lead time = $P_{FV}^{SO}(S) = \sum_{m=S}^{\infty} p_{FV}^{d}(m)$

In this formula, μ is the average consumption during the lead time and therefore can be computed by C multiplied by L. For s the value 1 is chosen.

Erlang-k method:

Probability of a demand D during the lead time = p_E^d

Probability of a stock out during the lead time = $p_E^{SO}(S) = \sum_{m=S}^{\infty} p_E^m$

From now on we will refer to the probability of a demand D during the lead time with p_{Method}^{d} and to the probability of a stock out during the lead time with p_{Method}^{SO} for an arbitrary method.

Waiting time

First two definitions are introduced:

inventory position = stock on hand + outstanding orders - backorders

inventory level = stock on hand – backorders

A demand at time T results in penalty costs when the current inventory level is zero (so there is a stock out and a demand at that moment). To compute the probability that this inventory level is zero, the time (T-L, T) has to be investigated. What was the inventory level at T-L and how many items are consumed during this time-interval? How long will the stock out endures? This depends on the minimum stock and the order quantity. In case of an (S-1,S)-strategy (order 1 item at the moment the inventory position drops below S) the computation is easy. Because in this report the (S-1, S+Q-1)-strategy is used (order an amount of Q items at the moment the inventory position drops below S), the computation becomes a lot more complicated. Therefore the (S-1, S) -strategy will be explained first and will be extended to the strategy as used in this report.

A replenishment order arrives a period of L after it is ordered. Assuming that the number of consumptions is uniformly divided over this time interval L (this is an assumption in case of the Factor Variance method and this can beproved in case of the Erlang-k method, as shown by Olthof and Dekker (1994, p.112)) gives an opportunity to compute the waiting time in case of a stock out.

Suppose a failure occurs at time T and there is no stock, so there is a stock out. Suppose that at this moment there already are B backorders. It is easy so see that a demand at time T is backordered if and only if the number of demands in time interval [T-L, T) is at least S, because the inventory position at any moment is S. This means that a total of S+B items are somewhere between the supplier and the plant. The question is how long it takes until the demand from time T will be satisfied (the number of penalty days). This is the number of days until the order arrives with item number B+1. Assuming that the number of consumptions is equally divided over the lead time L, this expected penalty time (in years) becomes $\frac{L(B+1)}{S+1+B}$.

In case of an order quantity > 1 the only difference is the total number of items that are ordered and that not have been arrived at time T. In case of the 'simple' strategy, the inventory position at any moment is S. In case of an order quantity > 1 the inventory position can be S, S+1, ... , S-1+Q. After the assumption that these positions are uniformly distributed (have equal probability) the same reasoning can be used as the 'simple' strategy, but the average has to be taken from the different inventory positions. The expected penalty time (in years) becomes: $\frac{1}{Q}\sum_{i=S}^{S+Q}\frac{L(b+1)}{i+b+1}$.

Finally, a formula for the total number of waiting time, given a certain S and Q can be given as follows. The expected penalty time needs to be multiplied by the probability that a number of S+B items is consumed during the lead time L. Summing over all possible values of B (from zero till infinity) give the total expected penalty time in years at a random moment of time. In formula:

$$\text{Penalty time in years} = T = \sum_{b=0}^{\infty}\left(\frac{1}{Q}\sum_{i=S}^{S+Q}\left(p_{Method}^{i+b} * \frac{L(b+1)}{i+b+1}\right)\right)$$

<u>Penalty costs</u>

After computing the penalty time, the formula for the total penalty costs can be derived for the different types of penalty values. First the case of fixed penalty costs is given.

Let F denote the fixed penalty cost. In this case the total time of a stock out has no influence on the total penalty costs, only the probability of a stock out and the number of stock outs. For this computation a part of the formula for the penalty time can be used, because that formula computes the total number of stock outs (and multiplies this with the expected time the stock out endures, but that part is not needed for the computation of the fixed penalty costs). This total number of stock outs multiplied with the consumption rate and with the fixed penalty costs, gives the total expected penalty costs per year. In formula the yearly penalty costs in case of fixed penalty costs P_f, given minimum stock S, becomes:

$$E_f = CP_f\sum_{b=0}^{\infty}\left(\frac{1}{Q}\sum_{i=S}^{S+Q}p_{Method}^{i+b}\right)$$

In case of penalty costs per year P_v, the total penalty time has to be multiplied by the yearly penalty costs and the (yearly) consumption rate. This is because the expected total waiting time in years at a random moment in time multiplied by the total number of consumptions in a year gives the expected total number of penalty years in a year. Multiplying this by the penalty costs per year gives the total yearly penalty costs.

Total yearly penalty costs = $E_v = CP_v T$

In case there are x days for which no penalty costs are computed, these x days need to be subtracted from the total waiting time. When these x days exceed the waiting time, the penalty costs are zero. Because the waiting time is in years, first the number of zero -cost-days need to be expressed in number of 'zero - costs-years'. So let x be the number of zero -cost-years. The formula for T now becomes:

Penalty time in years = $T = \sum_{b=0}^{\infty}\left(\frac{1}{Q}\sum_{i=S}^{S+Q}\left(p_{\text{Method}}^{i+b} * \max\left(\frac{L(b+1)}{i+b+1} - x, 0\right)\right)\right)$

Formula for the computation of the total yearly penalty costs stays the same:

Total yearly penalty costs = $E_v = CP_v T$

Appendix 4. Percentage Cost Penalty

The percentage cost penalty is

$$PCP = \frac{TRC(Q') - TRC(EOQ)}{TRC(EOQ)} \times 100$$

where

$$Q' = (1 + p)EOQ = (1 + p)\sqrt{\frac{2CO}{H}}$$

Substituting this *Q'* expression into the equation 5.3 representation of TRC, the following formula is obtained

$$TRC(Q') = (1 + p)\sqrt{\frac{COH}{2}} + \frac{1}{1+p}\sqrt{\frac{COH}{2}} = \sqrt{2COH}\frac{1}{2}\left(1 + p + \frac{1}{1+p}\right)$$

Also from equation #

$$TRC(EOQ) = \sqrt{2COH}$$

Substituting the results of the above two equations into the PCP equation gives

$$PCP = \left[\frac{1}{2}\left(1 + p + \frac{1}{1+p}\right) - 1\right] \times 100$$

or

$$PCP = 50\left(\frac{1}{1+p} - 1 + p\right) = 50\left(\frac{p^2}{1+p}\right)$$