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A NOTE ON RAMSEY AND CORLETT-HAGUE RULES

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Abstract

Ramsey-type results dictate that an optimal pattern of taxes must tax more heavily those goods which have a more inelastic (compensated) demand. Corlett and Hague (1953) investigated the optimal revenue-neutral movements from an initial uniform tax. They obtained that the goods (relatively) more complementary to the untaxed good (leisure) should see their taxes increased—which in a revenue-neutral setting implies that the other goods see their taxes disminished. In a three-good economy (with only two goods being subject to taxation) the Ramsey-type rule and the Corlett-Hague result can be easily related.

Key Words:
Optimal taxation, Ramsey, Corlett-Hague, demand elasticity.

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A NOTE ON RAMSEY AND CORLETT-HAGUE RULES

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Ramsey-type results dictate that an optimal pattern of taxes must tax more heavily those goods which have a more inelastic (compensated) demand. Corlett and Hague (1953) investigated the optimal revenue-neutral movements from an initial uniform tax. They obtained that the goods (relatively) more complementary to the untaxed good (leisure) should see their taxes increased—which in a revenue-neutral setting implies that the other goods see their taxes disminished. In a three-good economy (with only two goods being subject to taxation) the Ramsey-type rule and the Corlett-Hague result can be easily related.

THE INVERSE ELASTICITY RULE, often referred to as the Ramsey rule, is one of the most well-known results in public finance. The most simple inverse elasticity rule is of the form $t_i/(p_i + t_i) = k/\epsilon_{ii}$, where t_i is the tax on good i, p_i is i's before-tax price, k is a constant, and ϵ_{ii} is i's own-price elasticity of demand. It ignores any cross effects between the taxed goods, and, if ϵ_{ii} is meant to be the marshallian demand price elasticity, any income effects. More complicated expressions are obtained when all the general equilibrium effects are properly taken into account. Corlett and Hague (1953) investigated the optimal revenue-neutral movements from an initial uniform tax. They obtained that the goods (relatively) more complementary to the untaxed good (leisure) should see their taxes increased—which in a revenue-neutral setting implies that the other goods see their taxes disminished.

Relating both themes, if the optimal pattern is a uniform *ad valorem* tax then the optimal Corlett-Hague movement would be no movement at all. Therefore, all goods must behave equally with respect to the untaxed good (and symmetrically between them). On the other hand, if the Corlett-Hague conceptual experiment implies that good j must see his tax burden increased and good k decreased, it has to be the case that the Ramsey-type rule suggests that j must be relatively more heavily taxed than k. An explicit relationship will be developed in this paper.

In the first section, I shall derive the measure of excess burden used in the paper. In section two I shall introduce Ramsey-type results, and in section three the Corlett-Hague model. Finally, in the last section I shall relate both results.

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1. MEASUREMENT OF LOSS FROM DISTORTING TAXES

I will introduce, first, the assumptions about the model used throughout the paper. These are:

- Three-good economy. Good 1, leisure, will be the numeraire and will be untaxed (otherwise proportional taxation would be optimal).
- Agents have no exogenous income (otherwise a lump-sum tax would be optimal) and have the same preferences.
- The technology is a linear one. Therefore, producer's prices, $\mathbf{p}$, are constant. Final consumer prices, $\mathbf{q}$, are the sum of the producer prices and the vector of taxes, $\mathbf{t}$. Thus, $\mathbf{q} = \mathbf{p} + \mathbf{t}$.

The loss for any given tax vector can be measured as the negative of the **compensating variation** after the revenue collected has been returned to the consumers. The compensating variation will be given by

$$CV = \mu(\mathbf{q}; \mathbf{q}, 0) - \mu(\mathbf{q}; \mathbf{p}, 0) = -\mu(\mathbf{q}; \mathbf{p}, 0);$$

where $\mu(\cdot)$ is the money-metric utility function.[1] Let $u^o \equiv v(\mathbf{p}, 0)$, then the burden $L(\mathbf{t})$ due to the tax vector $\mathbf{t}$ can be expressed as

$$\begin{aligned} L(\mathbf{t}) &= -\left(\sum_{i=2}^{3} t_i x_i(\mathbf{q}, e(\mathbf{q}, u^o)) - \mu(\mathbf{q}; \mathbf{p}, 0)\right) \\ &= e(\mathbf{q}, u^o) - \sum_{i=2}^{3} t_i h_i(\mathbf{q}, u^o) \\ &= \sum_{i=2}^{3} (q_i - t_i) h_i(\mathbf{q}, u^o) \\ &= \sum_{i=2}^{3} p_i h_i(\mathbf{q}, u^o); \end{aligned}$$

which is the value of the expenditure function at the after-tax price vector less the tax revenues collected[2] and (conceptually) returned lump sum. From the last expression, it can be seen that the loss equals the value, at pre-tax prices, of the cheapest post-tax bundle that yields the initial level of utility.

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[1] I will be using Varian (1984) textbook's notation throughout the paper. Thus, the marshallian demands will be denoted by $x_i(\mathbf{p}, m)$; hicksian demands by $h_i(\mathbf{p}, u)$; expenditure function by $e(\mathbf{p}, u)$ and indirect utility function by $v(\mathbf{p}, m)$. Therefore, $\mu(\mathbf{q}; \mathbf{p}, m) \equiv e(\mathbf{q}, v(\mathbf{p}, m))$. Also, price vectors are row vectors while quantity vectors are column vectors.

[2] If the consumer were compensated so that she could stay at her initial utility level u^o, her demand would be $\mathbf{h}(\mathbf{q}, u^o)$ and, in turn, the taxes collected would be $\mathbf{th}(\mathbf{q}, u^o)$. Actual taxes collected, $\mathbf{tx}(\mathbf{q}, 0)$ are irrelevant here.

The marginal loss from a small change in one of the unit taxes is given by

$$\frac{\partial L(\mathbf{t})}{\partial t_k} = \frac{\partial e(\mathbf{q}, u^o)}{\partial t_k} - \frac{\partial \sum_{i=2}^{3} t_i h_i(\mathbf{q}, u^o)}{\partial t_k}$$

$$= -\sum_{i=2}^{3} t_i \frac{\partial h_i(\mathbf{q}, u^o)}{\partial t_k}. \tag{1}$$

The optimal pattern of taxes to raise T while minimizing the loss inflicted to the economy can be obtained by solving[3]

$$\min_{\mathbf{t}} \quad L(\mathbf{t})$$
$$\text{s.t.} \quad \sum_{i=2}^{3} t_i h_i(\mathbf{q}, u^o) = T.$$

Both demand and production are homogeneous of degree zero in $\mathbf{p}$ and $\mathbf{q}$ respectively. Therefore, two normalizations are allowed. It's useful to make $p_1 \equiv q_1 \equiv 1$, which implies $t_1 \equiv 0$.

The first order conditions give

$$\frac{\partial L}{\partial t_k} = -\sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_k} + \lambda\left(h_k + \sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_k}\right) = 0, \qquad k = 2, 3.$$

Rearranging, we get

$$\frac{\lambda}{1-\lambda} = \frac{\sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_k}}{h_k}$$

$$= \frac{\sum_{i=2}^{3} t_i \frac{\partial h_k}{\partial q_i}}{h_k} \tag{2}$$

$$\approx \frac{\sum_{i=2}^{3} t_i \frac{\Delta h_k}{\Delta t_i}}{h_k} \tag{3}$$

$$= \frac{\Delta h_k}{h_k}; \tag{4}$$

[3] If either the equivalent variation were used instead of the compensating one, or the indirect utility function were maximized subject to the revenue constraint, similar results would be obtained. However, $h(\cdot)$ would be evaluated at different points in the resulting formulae—i.e., at $h(\mathbf{q}, v(\mathbf{p}, 0))$ with the CV and at $x(\mathbf{q}, 0)$ with the EV. Thus, $\max_{\mathbf{t}} \quad v(\mathbf{q}, 0) + \lambda\left(\sum_i t_i x_i(\mathbf{q}, 0) - T\right)$ is equivalent to $\min_{\mathbf{t}} \quad L(\mathbf{t}) + \eta\left(\sum_i t_i h_i(\mathbf{q}, v(\mathbf{q}, 0)) - T\right)$; when $L(\mathbf{t})$ uses the equivalent variation measure. This holds because we have only one consumer (or any arbitrary number of them as long they all have the same preferences). In a many-person economy, the expenditure approach and the indirect utility function one do not give the same answers. Furthermore, in this context the concept of loss in not well defined.

for $k = 2, 3$. To exchange the approximately-equal sign for a plain-equal sign between equations (2) and (3), all we need is to have linear hicksian demands over the pertinent range.

Equation (2) makes evident that the distortionary effects of any tax pattern come through the substitution effects. Two alternative tax patterns that raise the same revenues must have the same income effects and, therefore, can only be compared on the grounds of the inefficiencies they arise by the reallocation of resources. Equation (4) suggests that the optimal tax structure involves an equal proportionate change in the hicksian demands for all goods.

Equation (4) does not provide an explicit rule for the pattern of taxes but indicates what it must satisfy. The only way the first order conditions can be simultaneously satisfied is having goods (factors) whose compensated demands (supplies) are relatively inelastic subject to relatively higher taxes.[4]

2. RAMSEY-TYPE RESULTS

In our two-good economy, it's easy to manipulate (2) further to get

$$(5) \quad \begin{bmatrix} t_2 \\ t_3 \end{bmatrix} = \left(\frac{\lambda}{1-\lambda}\right) \left(\frac{\partial h_2}{\partial q_2}\frac{\partial h_3}{\partial q_3} - \left(\frac{\partial h_2}{\partial q_3}\right)^2\right)^{-1} \begin{bmatrix} \frac{\partial h_3}{\partial q_3} & -\frac{\partial h_2}{\partial q_3} \\ -\frac{\partial h_3}{\partial q_2} & \frac{\partial h_2}{\partial q_2} \end{bmatrix} \begin{bmatrix} h_2 \\ h_3 \end{bmatrix};$$

which implies that we are going to have $t_2/q_2 > t_3/q_3$ whenever

$$(6) \quad \frac{\partial h_2}{\partial q_2}\frac{q_2}{h_2} + \frac{\partial h_2}{\partial q_3}\frac{q_3}{h_2} > \frac{\partial h_3}{\partial q_3}\frac{q_3}{h_3} + \frac{\partial h_3}{\partial q_2}\frac{q_2}{h_3}.$$

Equation (6) confirms all the elasticity arguments discussed above and will be used to link up with the Corlett-Hague results. Let ϵ_{ij} denote the hicksian elasticity of good i with respect to j's price. Then, we can rewrite (6) as

$$\epsilon_{22} + \epsilon_{23} > \epsilon_{33} + \epsilon_{32};$$

in particular, if the cross-price elasticities were zero, we'd need

$$|\epsilon_{22}| < |\epsilon_{33}|.$$

Therefore, good 2 must be relatively more inelastic than good 3.

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4 Atkinson and Stiglitz (1972) investigated under which conditions equal (uniform) proportional taxation of all commodities (except labor, the numeraire) is optimal. They proved that uniform taxation is optimal if either (i) labor is in absolutely fixed supply—i.e., uniform taxation amounts to a lump-sum tax on labor rents as argued before—and (ii) preferences are homothetic. Also, if preferences have an additive representation then tax rates are inversely proportional to each commodity's income elasticity of demand. If preferences are additive in logarithms with equal coefficients, then all income elasticities equal one, and uniform taxation is optimal.

From equation (5) we can also get a related expression which captures the same ideas,

$$\frac{t_2}{q_2} = \frac{1}{\epsilon_{22}}\left(\frac{\lambda}{1-\lambda} - \frac{t_3}{q_3}\epsilon_{23}\right);$$

if cross-price effects are zero, this equation collapses to

$$\frac{t_k}{q_k} = \frac{\lambda}{1-\lambda}\frac{1}{\epsilon_{kk}};$$

which is the appealing inverse elasticity rule. Note that the relevant elasticity is the hicksian elasticity which won't necessarily agree with the marshallian elasticity unless income effects are zero. Also, the assumption that cross effects vanish is a rather heroic one.

We can also obtain expressions in terms of the (observable) marshallian demands. Using the identity $h_i(\mathbf{q}, u) \equiv x_i(\mathbf{q}, e(\mathbf{q}, u))$ we can derive Slutsky's equation,

$$\frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial q_i} = \frac{\partial h_k(\mathbf{q}, u^o)}{\partial q_i} - \frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial m} x_i(\mathbf{q}, e(\mathbf{q}, u^o));$$

which can be used to substitute $\partial h_k/\partial q_i$ in (2), we get

$$\frac{1}{x_k(\mathbf{q}, e(\mathbf{q}, u^o))}\sum_{i=2}^{3} t_i\left(\frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial q_i} + \frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial m} x_i(\mathbf{q}, e(\mathbf{q}, u^o))\right) = \frac{\lambda}{1-\lambda};$$

rearranging, we obtain

(7)

$$\frac{1}{x_k(\mathbf{q}, e(\mathbf{q}, u^o))}\sum_{i=2}^{3} t_i \frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial q_i} = \frac{\lambda}{1-\lambda} - \frac{\eta_k}{e(\mathbf{q}, u^o)}\sum_{i=2}^{3} t_i x_i(\mathbf{q}, e(\mathbf{q}, u^o));$$

or, dropping arguments to make it look more compact,

$$\frac{1}{x_k}\sum_{i=2}^{3} t_i \frac{\partial x_k}{\partial q_i} = \frac{\lambda}{1-\lambda} - \frac{\eta_k}{e(\mathbf{q}, u^o)}\sum_{i=2}^{3} t_i x_i;$$

where η_k is good's i income elasticty,

$$\eta_k = \frac{e(\mathbf{q}, u^o)}{x_k(\mathbf{q}, e(\mathbf{q}, u^o))}\frac{\partial x_k(\mathbf{q}, e(\mathbf{q}, u^o))}{\partial m}.$$

Assuming the $\partial x_k/\partial q_i$'s are constant over the relevant range, the left-hand side of (7) gives the percentage change in the marshallian demand (supply) of the k^{th} good (factor). Goods with high income elasticities should change by greater amount. (Notice that this argument would suggest light taxation on necessities against the conventional wisdom based on the inverse elasticity rule that suggests heavy taxation on necessities.)

3. CORLETT-HAGUE RESULTS

In this section, I will consider revenue-neutral substitution among taxes to derive the Corlett-Hague results. Total revenue is given by

$$T = \sum_{i=2}^{3} t_i h_i(\mathbf{q}, u^o),$$

differentiating T with respect to $\mathbf{t}$, we get

$$\begin{aligned} dT &= \sum_{i=2}^{3} h_i dt_i + \sum_{i=2}^{3} t_i \sum_{k=2}^{3} \frac{\partial h_i}{\partial q_k} dt_k \\ &= \sum_{k=2}^{3} h_k dt_k + \sum_{k=2}^{3} \sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_k} dt_k \\ &= \sum_{k=2}^{3} \left(h_k + \sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_k} \right) dt_k . \end{aligned}$$

In particular,

$$\frac{\partial T}{\partial t_j} = h_j + \sum_{i=2}^{3} t_i \frac{\partial h_i}{\partial q_j}. \tag{8}$$

Setting $T = \bar{R}$ and $dT = 0$ determines all possible tax substitutions that keep revenue unchanged. Since we only have two taxes here, we must have

$$dT = \frac{\partial T}{\partial t_2} dt_2 + \frac{\partial T}{\partial t_3} dt_3 = 0,$$

or

$$\left. \frac{dt_2}{dt_3} \right|_{T=\bar{R}} = - \frac{\frac{\partial T}{\partial t_3}}{\frac{\partial T}{\partial t_2}}. \tag{9}$$

The marginal loss due to this change is given by

$$dL = - \sum_{i=2}^{3} t_i \left(\frac{\partial h_i}{\partial q_2} dt_2 + \frac{\partial h_i}{\partial q_3} dt_3 \right).$$

Consider (marginally) substituting a tax on good 3 by a tax on good 2, the proposed change does produce a movement towards efficiency if

$$\frac{dL}{dt_3} = \frac{\partial L}{\partial t_2} \left. \frac{dt_2}{dt_3} \right|_{T=\bar{R}} + \frac{\partial L}{\partial t_3} > 0.$$

Corlett and Hague (1953) examined the efficiency gains obtained by moving away from equal proportional taxes on two goods in this three-good economy. Assume initially $t_2 = \tau p_2$ and $t_3 = \tau p_3$, so $q_i = (1+\tau)p_i$. From the expression above,

$$\begin{aligned}\frac{dL}{dt_3} &= \frac{\partial L}{\partial t_2}\frac{dt_2}{dt_3}\bigg|_{T=\bar{R}} + \frac{\partial L}{\partial t_3} \\ &= -\left(t_2\frac{\partial h_2}{\partial q_2} + t_3\frac{\partial h_3}{\partial q_2}\right)\frac{dt_2}{dt_3}\bigg|_{T=\bar{R}} - t_2\frac{\partial h_2}{\partial q_3} - t_3\frac{\partial h_3}{\partial q_3}.\end{aligned}$$

The homogeneity conditions imply

$$\begin{aligned}\frac{\partial h_1}{\partial q_3} &= -q_2\frac{\partial h_2}{\partial q_3} - q_3\frac{\partial h_3}{\partial q_3} \\ &= -(1+\tau)\left(p_2\frac{\partial h_2}{\partial q_3} + p_3\frac{\partial h_3}{\partial q_3}\right),\end{aligned} \tag{10}$$

also

$$t_2\frac{\partial h_2}{\partial q_3} + t_3\frac{\partial h_3}{\partial q_3} = \tau\left(p_2\frac{\partial h_2}{\partial q_3} + p_3\frac{\partial h_3}{\partial q_3}\right);$$

so

$$\frac{dL}{dt_3} = \frac{\tau}{1+\tau}\left(\frac{\partial h_1}{\partial q_2}\frac{dt_2}{dt_3}\bigg|_{T=\bar{R}} + \frac{\partial h_1}{\partial q_3}\right).$$

Next, differentiating h_1 with respect to t_3 subject to the normalization and the revenue constraint,

$$\frac{dh_1}{dt_3} = \frac{\partial h_1}{\partial q_2}\frac{dt_2}{dt_3}\bigg|_{T=\bar{R}} + \frac{\partial h_1}{\partial q_3}.$$

Therefore, substituting this last expression we get

$$\frac{dL}{dt_3} = \frac{\tau}{1+\tau}\frac{dh_1}{dt_3}. \tag{11}$$

The sign of dh_1/dt_3 depends on the relative magnitude of the elasticities of the two goods with respect to leisure. Using equations (9) and (8), we have that

$$\begin{aligned}\frac{dh_1}{dt_3} &= -\frac{\partial h_1}{\partial q_2}\frac{h_3 + t_2\frac{\partial h_2}{\partial q_3} + t_3\frac{\partial h_3}{\partial q_3}}{h_2 + t_2\frac{\partial h_2}{\partial q_2} + t_3\frac{\partial h_3}{\partial q_2}} + \frac{\partial h_1}{\partial q_3} \\ &= -\frac{\partial h_1}{\partial q_2}\frac{h_3 - \frac{\tau}{1+\tau}\frac{\partial h_1}{\partial q_3}}{h_2 - \frac{\tau}{1+\tau}\frac{\partial h_1}{\partial q_2}} + \frac{\partial h_1}{\partial q_3} \\ &= \frac{-h_3\frac{\partial h_1}{\partial q_2} + h_2\frac{\partial h_1}{\partial q_3}}{h_2 - \frac{\tau}{1+\tau}\frac{\partial h_1}{\partial q_2}} \\ &= \frac{h_2 h_3}{h_2 - \frac{\tau}{1+\tau}\frac{\partial h_1}{\partial q_2}}(\epsilon_{31} - \epsilon_{21}).\end{aligned}$$

Thus,

$$\frac{dL}{dt_3} = \frac{\tau h_2 h_3}{(1+\tau)h_2 - \tau \frac{\partial h_1}{\partial q_2}}(\epsilon_{31} - \epsilon_{21}) \tag{12}$$

Assuming the denominator is positive,[5] whenever $\epsilon_{31} > \epsilon_{21}$ we'll have $dL/dt_3 > 0$ so the loss will decrease if t_3 decreases. If, on the other hand, $\epsilon_{31} < \epsilon_{21}$ then lowering the tax on good 2 will produce a movement towards efficiency.

Thus, the sign of dh_1/dt_3 depends on the substitution terms. In particular, if 2 and 1 are substitutes, 3 and 1 must be complements and $dh_1/dt_3 < 0$ so the loss decreases. Hence the good that is complementary to leisure should be taxed. If both were substitutes, the tax should be raised on the good relatively more complementary with leisure.

4. RELATING RAMSEY AND CORLETT-HAGUE RULES

Equation (6) stated the condition that would lead to an optimal tax pattern with $t_2/q_2 > t_3/q_3$, which was

$$\frac{\partial h_2}{\partial q_2}\frac{q_2}{h_2} + \frac{\partial h_2}{\partial q_3}\frac{q_3}{h_2} > \frac{\partial h_3}{\partial q_3}\frac{q_3}{h_3} + \frac{\partial h_3}{\partial q_2}\frac{q_2}{h_3};$$

using equation (10), it can be rewritten as

$$\frac{1}{h_2}\frac{\partial h_2}{\partial q_1} < \frac{1}{h_3}\frac{\partial h_3}{\partial q_1}; \tag{13}$$

or

$$\epsilon_{21} < \epsilon_{31}. \tag{14}$$

Thus, if both goods are leisure substitutes—in a hicksian sense—the above elasticities are positive and 2 would be more complementary—i.e., less substitute—than 3. The other way equation (13) might hold is with 2 being complementary to leisure and 3 being substitute, in which case the left hand side of the inequality is negative and the right hand side is positive. Finally, if the hicksian demand of good 2 (good 3) does not respond to changes in the price of leisure we need good 2 (good 3) to be substitute (complementary) to leisure.

In summary, equations (12) and (14) convey the same message that the good (relatively) more complementary to leisure should bear a (relatively) higher tax. Squash racquets, tennis shoes and good books are to be the innocent victims of Ramsey, Corlett and Hague.

[5] Equation (12) could give perverse results if goods 1 and 2 were hicksian-substitutes and it turned out that the denominator were negative.

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