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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-75

INTERIM EFFICIENT MECHANISMS FOR PUBLIC DECISION MAKING IN A DISCRETE FRAMEWORK **

Mikel Pérez-Nievas *

Abstract

In this paper, I characterize the set of *Bayesian incentive compatible anonymous mechanisms* in a discrete public good problem when preferences are private information. With this result in hand, I characterize the set of *interim incentive efficient mechanisms* as voting schemes in which votes are weighted according to the tax paid by each agent.

Keywords: Public goods, voting mechanisms, interim efficiency.

* **Pérez-Nievas**, Departamento de Economía, Universidad Carlos III de Madrid and Universidade de Vigo. E-mail: mikel@eco.uc3m.es

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Glosary of notation

1. BIC : Bayesian Incentive Compatibility.
2. $EXABB$: Ex-ante budget balance.
3. $EXPBB$: Ex-post budget balance.
4. $\mathcal{D}$: Set of all possible distributions of N agents among the K valuations in the set $S = \{v_1, v_2, ..., v_k, ...v_K\}$.
5. $\mathcal{Q}$:. Set of all provision rules $q : \mathcal{D} \rightarrow [0, 1]$.
6. $\mathcal{Q}^R$: Set of all provision rules $q \in \mathcal{Q}$ such that the monotonicity condition $(T.1.1)$ in Theorem 1 is satisfied.
7. $\mathcal{T}$: Set of all tax functions $t : \mathcal{D} \rightarrow \Re^K$.
8. $\mathcal{M}$: Set of mechanisms $m \equiv (q, t) \in \mathcal{Q}^A \times \mathcal{T}$ satisfying BIC and $EXPBB$.

1. Introduction

This paper focuses on problems where a group of individuals, each of them having incomplete information about the others' characteristics, has to decide whether to undertake a public project or not and, if undertaken, how to distribute the costs of the project among the members of the group. Agents' valuations of the project are private information, and it is common knowledge that each agent's valuation is generated by a discrete probability distribution. The paper characterizes the set of *interim incentive efficient mechanisms* in this setting.

The notion of interim incentive efficiency (Holmstrom and Myerson, 1983) differs from the classical notion of efficiency in two important aspects. First of all, it focuses the *interim* stage (*i.e.*, the stage in the decision process at which the individuals know their private information but does not know the others' private information) as the relevant stage for welfare evaluation of mechanisms. Second of all, it does not refer to *feasible decisions* but to *incentive feasible decision mechanisms.* The notion of incentive feasibility takes into account both the technological (or classical) constraints arising because undertaking the public project is costly, and the incentive constraints arising due to the informational structure of the problem. That is, feasible mechanisms are *Bayesian incentive compatible* and *budget balanced* (or *budget balanced in expectations).* Thus, a Bayesian incentive compatible, budget balanced mechanism is interim incentive efficient (or simply, interim efficient) if there does not exist another Bayesian incentive compatible, budget balanced mechanism that, in the interim stage, makes some agents better off without making other agents worse off.

The early literature on decision mechanisms in environments with private information (see e.g., d'Aspremont and Gerard-Varet (1979)) identified Bayesian Incentive Compatible, budget balanced mechanisms that are efficient *ex-post.* However, the fact that a mechanism selects Pareto optimal decisions doest not mean that such mechanism is Pareto superior (in the interim stage) to any other any other mechanism. In fact, it can be shown that the class of $ex-post$ efficient mechanisms form a proper subset of the set of interim efficient mechanisms. The interest of characterizing the whole set of interim efficient mechanisms (and not just a particular class of mechanisms in this set) arises from both normative and positive considerations.

One the normative side, there might be situations in which the set of mechanisms available is restricted by voluntary participation constraints, imposing that a mechanism provides all agents with at least the same expected interim utility they obtain with a given decision. In many problems (see e.g., Mailath and Postlewaite (1990), or Dearden (1997)), *ex-post* efficient mechanisms fail to satisfy voluntary participation constraints. Therefore, exploring other mechanisms satisfying certain social objectives is interesting. Note that, if a group of individuals unanimously agree on altering a *statu quo* and use a mechanism instead, selecting an interim efficient mechanism exhausts the possibilities for further improvements.

On the positive side, one might be interested in characterizing the whole set of interim efficient mechanism to explore the properties of mechanisms that arise in practice. If all decisions (including whether to use a particular mechanism) are taken at the interim stage, then the set of interim efficient mechanisms consists on those incentive compatible mechanisms for which there is no other mechanism that generates unanimous improvement. Thus, we would expect that if an agreement on a particular mechanism is reached, then that mechanism should be interim efficient.

In general Bayesian Collective choice problems (see Myerson (1985)), interim efficient mechanisms can be characterized as the solutions to a welfare optimization problem. More precisely, a mechanism is interim incentive efficient if it maximizes a weighted average of the agents' interim utilities (subject to Bayesian incentive compatibility and budget balance constraints), where this average is evaluated with respect to a probability (or welfare) distribution defined on the set of types. Several authors (for example, Wilson (1985) and Myerson (1986), in the context of private goods; and Ledyard and Palfrey (1998, 1999) in the context of public goods) have explored the properties of the solutions to such welfare optimization problems in environments with continuous sets of types.

In discrete framework, the main contribution is that by Ledyard and Palfrey (1994), who have chacterized the class of interim efficient mechanisms in a public goods setting in which the individuals' valuations take only two possible values. An interesting finding of this paper is that interim efficiency can allways be accomplished by a majority voting scheme in which the individuals pay different taxes depending on the alternative they support. It is also shown that for an appropiate selection of the welfare distribution associated to an interim efficient mechanism, *interim* efficiency can be accomplished by a *referendum,* that is, a pure voting scheme that does not require transfers among the individuals. Furthermore, such *referendum* yields *ex-post* efficient decisions.

In the present paper, I extend Ledyard and Palfrey´s characterization to environments in which the set of types is represented by an arbitrary discrete probability distribution. I also show that interim efficiency can also be implemented by relatively simple mechanisms, namely, weighted voting shcemes in which votes are weighted according to the tax paid by the individuals. Furthermore, in some special cases with, interim efficiency can be implemented by a referendum that does not charge taxes on the participants.

To be more precise, the results obtained can be summarized as follows:

Bayesian incentive compatibility

The paper provides a characterization of the class of *Bayesian incentive compatible anonymous mechanisms.* According to this characterization, interim utilities obtained by the agents participating in a mechanism in this class is completely determined by (1) the *provision rule*[1] associated to the mechanism, (2) *a prespecified sharing rule* that specifies how the *surplus* that each agent obtains by reporting his true valuations is distributed among all participants, and (3) a lump sum term that determines the *ex.ante* expected total taxes associated to the mechanism. (Section 3)

Interim incentive efficiency

The results above are used to characterize the class of *interim efficient mechanisms.* The provision rule of every mechanism in this class is associated to a welfare distribution defined on the set of types. This result is used to characterize interim efficient mechanisms as *weighted voting schemes* to decide whether or not the public project should be undertaken. Althoug, in some special cases, interim efficiency can be accomplished by a *referendum,* voting schemes arising with more than two types are in general different from those arising in the two-type case. By paying different taxes, the participants decide not only the alternative they support but also the strength to which they support such alternative. (Section 4)

2. The problem

A group of individuals, denoted by $I = \{1, 2, ..., i, ...N\}$, has to decide whether to undertake a costless[2] public project or not. Denote by $d \in \{0, 1\}$ the decision taken ($d = 1$ means that the project is undertaken while $d = 0$ means that

[1]The provision rule of a mechanism, defined in section 1, gives the probability that the public project is undertaken as a function of agents' reported valuations.

[2]The model can be easily adapted to problems in which the project is costly.

the project is not undertaken). Since preferences on the public project differ among the individuals, a tax system that collects taxes from some individuals to subsidize others is also devised. An allocation is a pair $(d,t) \in \{0,1\} \times \Re^N$, that specifies the decision taken and the tax t_i paid by each individual. Each individual is assumed to have preferences representable by utility functions $u^i : \{0,1\} \times \Re^N \to \Re$ defined, for all $(d,t_i) \in \{0,1\} \times \Re$, by $u_i(d,t) = v^i d - t_i$, where v^i is a real number that represents individual i's *valuation* of the public project. The individuals' valuations are independent realizations of a discrete random variable $\mathcal{V}$ with values on an ordered set $\mathcal{S} \equiv \{v^1, v^2, ..., v_k, ...v_K\}$, where $v_k < v_{k+1}$ for all $k = 1, 2, ..., k, ...K$, and $v_1 < 0 < v_K$.

Thus, the set of possible types of each agent is $\{1, 2, ..., k, ...K\}$. For each $k \in \{1, 2, ..., k, ...K\}$, let π_k and F_k be defined, respectively, by

$$\pi_k = \Pr[\mathcal{V} = v_k]$$

and

$$F_k = \sum_{j=1}^{k} \pi_j.$$

Individuals' valuations are assumed to be private information, and a mechanism to elicit the individuals' preferences is devised. As in Ledyard and Palfrey (1990), attention is restricted to anonymous mechanisms.[3] Let $\mathfrak{n} : \mathcal{S}^N \to (\aleph \cup \{0\})^K$ be a function specifying, for each $v \in \mathcal{S}^N$ and each $k \in \{1, 2, ..., K\}$, the number $\mathfrak{n}^k(v)$ of agents whose valuation equals v. That is,

$$\mathfrak{n}(v) = (\mathfrak{n}^1(v), \mathfrak{n}^2(v), ..., \mathfrak{n}^K(v)),$$

where $\mathfrak{n}^k : \mathcal{S}^N \to \aleph \cup \{0\}$ is defined, for all $v \in \mathcal{S}^N$ and each $k \in \{1, 2, ...K\}$, by

$$\mathfrak{n}^k(v) = \# \{ i \in I : v^i = v_k \}.$$

In other words, $\mathfrak{n}$ determines, for each profile of valuations, a distribution of the N individuals among the different valuations in $\mathcal{S}$. Let $\mathcal{D}$ denote the range of $\mathfrak{n}$, that is,

$$\mathcal{D} \equiv \left\{ n = (n_1, n_2, ...n_k, ...n_K) \in \aleph^K : \sum_{k=1}^{K} n_k = N \right\}.$$

[3]Restriction to revelation mechanisms can be justified by the revelation principle (See e.g., Myerson, 1981). A justification for the use of anonymous mechanism can be found in Mailath and Postlewaite (1990, p.361) or Ledyard and Palfrey (1992, p.349).

An anonymous mechanism (or, simply, a mechanism) is a pair of functions

$$m \equiv (q,t) : \mathcal{D} \longrightarrow [0,1] \times \Re^K$$

specifying, for each $n \in \mathcal{D}$ and $k = 1, 2, ..., K$,

- the probability $q(n)$ that the public project is undertaken,
- the tax $t_k(n)$ paid by an agent reporting a valuation v_k.

Given a mechanism, the function $q : \mathcal{D} \to [0,1]$ will be referred to as the provision rule of the mechanism, and the function $t : \mathcal{D} \longrightarrow \Re^K$ will be referred to as the tax function of the mechanism.

Since mechanisms depend only on how agents are distributed among the different valuations, the outcome selected by a mechanism will depend only on the realization of the discrete random vector $\mathfrak{n}(V)$. Its probability function, denoted by $P{:}\mathcal{D} \to [0,1]$, is given by

$$P(n) = \Pr\left[\mathfrak{n}(V) = n\right] = \binom{N}{n_1, n_2, ..., n_K} \left(\prod_{k=1}^{K} \pi_k^{n_k}\right).$$

The conditional probability that a particular n occurs will be different, however, for all agents participating in the mechanism, since these agents are aware of their own valuations. More precisely, the probability that n occurs for an agent with valuation v_k is given by

$$P_k(n) = \begin{cases} 0, & \text{if } n^k = 0, \\ \binom{N-1}{n_1, n_2, \ldots n_k - 1, \ldots, n_K} \pi_k^{n_k - 1} \left(\prod_{j=1, j\neq k}^{K} \pi_j^{n_j}\right) & \text{otherwise.} \end{cases}$$

Equivalently,

$$P_k(n) = \left(\frac{n_k}{N\pi_k}\right) P(n).$$

Let $m \equiv (q,t)$ be a mechanism. For each type $k \in \{1, 2, ..., K\}$, let $\overline{q}_k(q)$ and $\overline{t}_k(t)$ denote, respectively, the expected probability of provision and the expected tax paid for an agent reporting v_k, when all other agents report their true valuations, that is,

$$\overline{q}_k(q) = \sum_{n \in \mathcal{D}} q(n) P_k(n) = \sum_{n \in \mathcal{D}} \left(\frac{n_k}{N\pi_k}\right) q(n) P(n),$$

and

$$\bar{t}_k(t) = \sum_{n\in\mathcal{D}} t_k(n)P_k(n) = \sum_{n\in\mathcal{D}} \left(\frac{n_k}{N\pi_k}\right) t_k(n)P(n).$$

Also, let $\overline{T}(t)$ denote the expected per capita tax of a mechanism $m \equiv (q,t)$, under the assumption that all agents report their true valuations. That is,

$$\overline{T}(t) = \frac{1}{N}\left[\sum_{n\in\mathcal{D}}\left(\sum_{k=1}^{K} t_k(n)\right)P(n)\right].$$

Agents valuations are private information, and mechanisms must induce agents to report their true valuations. A mechanism $m = (q,t)$ satisfies *Bayesian incentive compatibility (BIC)* if for any two $k, \widehat{k} \in \{1,2,...,K\}$ one has

$$v_k\bar{q}_k(q) - \bar{t}_k(t) > v_k\bar{q}_{\widehat{k}}(q) - \bar{t}_{\widehat{k}}(t);$$

that is, a mechanism m satisfies BIC if reporting the true valuation is a Bayesian equilibrium of the revelation game induced by the mechanism.

Note that nothing in the definition of a mechanism guarantees that the decision is *feasible*, in the sense that taxes paid by all agents are non-negative. A first condition that is discussed, called *ex-post feasibility,* is relevant if the group of agents involved in the collective decision does not have access to outside resources and therefore taxes paid by all agents must be non-negative in every "state of nature". More precisely, a mechanism $m \equiv (q,t)$ is feasible *ex-post* $(EXPF)$ if for each $n \in \mathcal{D}$ one has

$$\sum_{k=1}^{K} n_k t_k(n) \geq 0. \qquad (EXPF)$$

If agents have access to an outside insurance market, the relevant feasibility condition is *ex ante feasibility* $(EXAF)$, that requires that a mechanism (q,t) satisfies

$$\sum_{n\in\mathcal{D}}\left(\sum_{k=1}^{K} n_k t_k(n)\right)P(n) \geq 0. \qquad (EXAF)$$

It should be noticed that condition $EXAF$ above is meaningless when applied to a mechanism for which BIC is not satisfied since there is no reason to presume that the expression

$$\sum_{n\in\mathcal{D}}\left(\sum_{k=1}^{K} n_k t_k(n)\right)P(n) = 0.$$

corresponds to the actual expected taxes of a mechanism.

A stronger condition that is used in the literature is that of *budget balance*, that requires that the mechanism be feasible and *non-wasteful*, in the sense that taxes paid by some of the agents do not exceed the transfers received by others. A mechanism $m \equiv (q, t)$ satisfies *ex-post budget balance* if for each $n \in \mathcal{D}$ one has

$$\sum_{k=1}^{K} n_k t_k(n) = 0. \qquad (EXPBB)$$

If the individuals have acces to outside resources, the relevant budget balance condition is *ex-ante budget balance*, that requires that a mechanism (q, t) satisfies

$$\sum_{n \in \mathcal{D}} \left(\sum_{k=1}^{K} n_k t_k(n) \right) P(n) = 0 \qquad (EXABB)$$

Note that a risk neutral external agent who acts as a broker in a mechanism satisfying $EXABB$ and BIC, subsidizing agents when a deficit arises and collecting any surplus, breaks even in expectations.

3. Incentive compatible mechanisms

This section characterizes the set of Bayesian incentive compatible mechanisms. Denote by $\mathcal{Q}$ and $\mathcal{T}$, respectively, the set of all provision rules $q : \mathcal{D} \rightarrow [0, 1]$ and tax functions $t : \mathcal{D} \rightarrow \Re^K$.

Theorem 1. *A mechanism $m \equiv (q, t) \in \mathcal{Q} \times \mathcal{T}$ satisfies BIC if and only if*

$(T.1.1)$ *for every $k \in \{1, 2, 3, ..., K-1\}$ one has*

$$\overline{q}_{k+1}(q) \geq \overline{q}_k(q),$$

and

$(T.1.2)$ *there exists a vector $\beta \equiv \{\beta_k\}_{k=1}^{K-1} \in [0, 1]^{K-1}$ such that for every $k \in \{1, 2, 3, ..., K-1\}$ one has*

$$\overline{t}_{k+1}(t) - \overline{t}_k(t) = [\beta_k v_{k+1} + (1 - \beta_k) v_k] \left(\overline{q}_{k+1}(q) - \overline{q}_k(q) \right).$$

3.1. Sharing rules

Theorem 1 provides two conditions characterizing the set of mechanisms satisfying BIC. The intuition behind these conditions is the following. Let $m \equiv (q, t)$ be an arbitrary mechanism. For any arbitrary type k, the term

$$v_{k+1}\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right]$$

is the *utility gain* or *surplus,* if no taxes were paid, obtained by an agent with valuation v_{k+1} when he reports his true valuation instead of reporting v_k. Analogously, the term

$$v_k\left[\overline{q}_k(q) - \overline{q}_{k+1}(q)\right] = -v_k\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right]$$

is the utility obtained by an agent with valuation v_k when he reports his true valuation instead of reporting v_{k+1} (if no taxes were paid). Notice that

$$(v_{k+1} - v_k)\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right]$$

is the joint surplus obtained jointly by any two agents of types k and $k+1$ if they report their true valuations instead of reporting the other's valuation. Thus, condition $(T.1.1)$ ensures that, before any taxes are paid, agents of types k and $k+1$ obtain a non negative joint surplus by reporting their true valuations. In what follows, the set of provision rules $q \in \mathcal{Q}$ satisfying $(T.1.1)$ will be denoted by $\mathcal{Q}^R$.

Taking into account the taxes paid under the mechanism m, the actual surplus obtained by an agent with valuation v_{k+1} when he reports his true valuation instead of reporting v_k is given by,

$$v_{k+1}\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right] - \left(\overline{t}_{k+1}(t) - \overline{t}_k(t)\right).$$

Substituting $\left(\overline{t}_{k+1}(t) - \overline{t}_k(t)\right)$ by its expression in $(T.1.2)$ yields

$$v_{k+1}\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right] - \left(\overline{t}_{k+1}(t) - \overline{t}_k(t)\right) = (v_{k+1} - v_k)\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right](1 - \beta_k).$$

Analogously, the surplus obtained by an agent with valuation v_k when he reports his true valuation instead of reporting v_{k+1} is given by

$$v_k\left[\overline{q}_k(q) - \overline{q}_{k+2}(q)\right] - \left(\overline{t}_k(t) - \overline{t}_{k+1}(t)\right) = (v_{k+1} - v_k)\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right]\beta_k.$$

Call any vector $\beta \in [0,1]^{K-1}$ a *sharing rule.* Thus, condition $(T.1.2)$ states that any incentive compatible mechanism m defines implicitly at least one (and possibly many) sharing rule β that determines how any two types k and $k+1$ share the joint surplus obtained by reporting their true valuations instead of the valuation corresponding to the other type. Since condition $(T.1.1)$ guarantees this joint surplus is non-negative, both types of agents obtain non negative utility gains by reporting their true valuations instead of reporting the valuation corresponding to the other type.

Note that for any mechanism m and any two $\beta, \beta' \in [0,1]^{K-1}$ satisfying $(T.1.2)$ one has $\beta_k \neq \beta'_k$ only if $\overline{q}_{k+1}(q) = \overline{q}_k(q)$, where q is the provision rule of the mechanism. Thus, β and β' are equal except for those coordinates $k \in \{1, 2, ..., K-1\}$ for which the surplus obtained jointly by agents k and $k+1$ is zero. Given a provision rule $q \in \mathcal{Q}^R$, any two sharing rules $\beta, \beta' \in [0,1]^{K-1}$such that $\beta_k = \beta'_k$ whenever $\overline{q}_{k+1}(q) \neq \overline{q}_k(q)$ will be said to be *equivalent* (with respect to q). Thus, for any mechanism m satisfying BIC, the set of all sharing rules $\beta \in [0,1]^{K-1}$ satisfying $(T.1.2)$ are equivalent with respect to the provision rule of the mechanism. Among these, let $\beta(m)$ be the sharing rule β satisfying $(T.1.2)$ such that $\beta_k = 0$ whenever $\overline{q}_{k+1}(q) = \overline{q}_k(q)$. Such sharing rule is uniquely defined, and it will be referred to as the *sharing rule associated to m*.

3.2. Reduced form representation of mechanisms

Given a mechanism $m \equiv (q, t)$, let $\overline{U}_k(m)$ be the *interim expected utility* obtained by an agent of type k participating in the mechanism m, that is,

$$\overline{U}_k(m) = v_k \overline{q}_k(q) - \overline{t}_k(t).$$

Thus, the interim expected utilities obtained by agents are completely determined by its provision rule and the vector $\{\overline{t}_k(t)\}_{k=1}^K$ the interim expected tax paid by each type of agent.

Consider now an arbitrary pair $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$. Note that one can allways obtain a mechanism $m \equiv (q, t)$ satisfying BIC and $EXABB$ such that β is equivalent to the sharing rule $\beta(m)$. A particular construction is as follows. Given $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$, let $\{c_k^*\}_{k=1}^K$ be the solution to the system of K equations

$$c_{k+1} - c_k = [\beta_k v_{k+1} + (1-\beta_k) v_k] \left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right); k = 1, 2, ...K-1; \quad (3.2.1)$$

$$\sum_{k=1}^K \pi_k c_k = 0. \quad (3.2.2)$$

It is straightforward to check that such vector $\{c_k^*\}_{k=1}^K$ exists an it is unique. Let $t^* : \mathcal{D} \rightarrow \Re^K$ be defined, for each $n \in \mathcal{D}$ and each $k = 1, 2, ..., K$, by $t_k^*(n) = c_k^*$. Clearly, the mechanism (q, t^*) satisfies $(T.2.2)$ and, hence, it satisfies BIC.

Also, it follows from Proposition that for any mechanism $m' = (q', t')$ satisfying BIC and $EXABB$ such that $q' = q$ and $\beta(m) = \beta$, the vector $\left\{\bar{t}_k(t')\right\}_{k=1}^K$ specifying the interim expected taxes paid by each type of agent must be a solution to (3.2.1) and (3.2.2). Therefore, for each $k = 1, 2, ..., K$, one has

$$\bar{t}_k(t^*) = \bar{t}_k(t') = c_k.$$

Select now $c = c(q, \beta)$ in such a way that

$$\sum_{k=1}^{K} \pi_k \bar{t}_k(t^c) = 0.$$

Note that such $c = c(q, \beta)$ exists and it is unique. It follows that for every mechanism $m^* = (q^*, t^*)$ satisfying BIC and $EXABB$ such that $q = q'$ and $\beta(m) = \beta$ one has

$$\bar{t}_k(t^c) = t_k(t^*).$$

Summarizing, it is allways possible to construct a particular mechanism m satisfying BIC and $EXABB$ from a provision rule $q \in \mathcal{Q}^R$ and a sharing rule $\beta \in [0,1]^{K-1}$. Also, although many other mechanisms that differ only on the tax functions may satisfy these requirements, it follows from Theorem 1 that all of them yield the same interim expected taxes (and, hence, the same interim expected utilities) to all agents. In what follows, any two mechanisms yielding the same interim utilities to all agents will be said to be *interim equivalent*. Thus, Theorem 1 establishes an interim equivalence between the set $\mathcal{Q}^R \times [0,1]^{K-1}$ and the set of all mechanisms $m \in \mathcal{Q}^R \times \mathcal{T}$ satisfying BIC and $EXABB$.

Proposition 1 below establishes that for any mechanism $m \in \mathcal{Q}^R \times \mathcal{T}$ satisfying BIC and $EXABB$, there exists an interim equivalent mechanism $m' \in \mathcal{Q}^R \times \mathcal{T}$ satisfying BIC and $EXPBB$. That is, welfare properties that can be achieved by mechanisms satisfying BIC and $EXABB$ can also be achieved by mechanisms satisfying BIC and $EXPBB$.

Proposition 1. *For any mechanism $m \in \mathcal{Q}^R \times \mathcal{T}$ satisfying BIC and $EXABB$, there exists an interim utility equivalent mechanism $m' \in \mathcal{Q}^R \times \mathcal{T}$ satisfying BIC and $EXPBB$.*

In what follows, the set of mechanisms satisfying BIC and $EXABB$ will be denoted by $\mathcal{M}$. In view of the interim equivalence established by Theorem 1, the vector

$$m^R \equiv (q, \beta(m)) \in \mathcal{Q}^R \times [0,1]^{K-1}$$

associated to any $m \in \mathcal{M}$ will be called the *reduced form*[4] representation of m. Note also that for any

$$m^R \equiv (q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$$

one can use a construction like the one used above to identify a *structural form* $m \equiv (q,t) \in \mathcal{Q}^R \times \mathcal{T}$, which is unique except for transformations that preserve interim utilities. entify a *structural form* $m \equiv (q,t) \in \mathcal{Q}^R \times \mathcal{T}$, which is unique except for transformations that preserve interim utilities. For any mechanism in reduced form $(q,\beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$ and each $k = 1,2,...,K-1$, let

$$\overline{U}_k^R(q,\beta) = \overline{U}_k(m),$$

where $m \in \mathcal{M}$ is any mechanism such that $\beta(m) = \beta$.

4. Interim efficient mechanisms

This section explores the efficiency properties of mechanisms satisfying BIC and $EXPBB$. When the agents unanimously agree to give up a statu quo decision and use a mechanism, it seems plausible that in considering alternative mechanisms agents will agree on a mechanism from which unanimous improvements in interim utilities are not possible. In situations in which agents evaluate the consequences of using different mechanisms once they are aware of their valuations but not of other agents' valuations, mechanisms having this property have been referred to as *interim efficient mechanisms*[5]

Formally, a mechanism $m \in \mathcal{M}$ is interim incentive efficient if there does not exist any other mechanism $m' \in \mathcal{M}$ such that, for all $k \in \{1,2,...,k,...K\}$ one has

$$\overline{U}_k(m') \geq \overline{U}_k(m), \qquad (E.1)$$

[4]This use of the term *reduced form* is different from Ledyard and Palfrey's (1992), who define the reduced form of a mechanism as a vector $(\overline{q}, \overline{t}) \in [0,1]^K \times \Re^K$ representing the vector of expected probabilities of provision and expected taxes associated to a mechanism.

[5]See Holmström and Myerson (1983).

and, for some $j \in \{1, 2, ..., k, ..., K\}$,

$$\overline{U}_k(m') > \overline{U}_k(m). \qquad (E.2)$$

Let $\triangle S$ denote the set of probability distributions with support on the set of valuations, that is,

$$\triangle S \equiv \left\{ g = (g_1, g_2, ..., g_K) : g_k \geq 0; \sum_{k=1}^{K} g_k = 1 \right\}.$$

Note that the vector $\pi = (\pi_1, \pi_2, ..., \pi_k, ... \pi_K)$ is a member of $\triangle S$. A distribution $g \in \triangle S$ will be regarded as a welfare distribution defined on the set of valuations S. For each $g \in \triangle S$, let $G = (G_1, G_2, ..., G_K)$ be the vector specifying the "cumulative welfare" associated to each type, that is, for each $k \in \{1, 2, ..., k, ..., K\}$,

$$G_k = \sum_{j=1}^{k} g_j.$$

Holmström and Myerson (1983) has shown that an interim efficient mechanism can be "rationalized" as the solution to a constrained welfare maximization problem. More precisely, an interim incentive efficient mechanism maximizes a weighted average of an agents' interim utility, where the welfare weight given to each type of agent is determined by an arbitrary welfare distribution g on the set of valuations. That is, a mechanism $m^* \equiv (q^*, t^*) \in \mathcal{M}$ is interim efficient if it solves

$$\max_{m \in \mathcal{M}} \left\{ \sum_{k=1}^{K} g_k \overline{U}_k(m) \right\} \qquad (P.1)$$

Taking into account the interim utility equivalence established by Theorem 1 and Proposition 1, one obtains

Proposition 2. *A mechanism $m^* \equiv (q^*, t^*) \in \mathcal{M}$ is interim efficient if and only if the exists $g \in \triangle S$ such that its reduced form $(q^*, \beta(m^*))$ solves*

$$\max_{\mathcal{Q}^R [0,1]^{K-1}} \left\{ \sum_{k=1}^{K} g_k \overline{U}_k^R(q, \beta) \right\}. \qquad (P.2)$$

Observe that, for every $g \in \triangle S$, the set $\mathcal{Q}^R \times [0,1]^{K-1}$ of mechanisms in reduced form is compact and the function function $\sum_{k=1}^{K} g_k \overline{U}_k^R(\cdot, \cdot)$ is continuous.

Therefore a solution to $(P.2)$ exists. Given a distribution $g \in \Delta S$ and a mechanism in reduced form $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$, the number $\sum_{k=1}^{K} g_k \overline{U}_k^R(q, \beta)$ will be referred to as the mechanism's *average welfare* (with respect to the distribution g).

4.1. Interim incentive efficient mechanisms as voting schemes

Let $w \equiv (w_1, w_2, ..., w_k) \in \Re^K$ be a vector of weights such that

$$w_1 \leq w_2 \leq ... \leq w_k \leq w_{k+1} \leq w_K.$$

Also, let $q \in \mathcal{Q}^R$ be a provision rule satisfying, for each $n \in \mathcal{D}$,

$$q(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} w_k n_k < 0 : \\ 1, & \text{whenever } \sum_{k=1}^{K} w_k n_k > 0. \end{cases} \tag{4.1.1}$$

Consider now a mechanism $m \equiv (q, t) \in \mathcal{M}$ such that q satisfies (4.1.1) for some $w \in \Re^K$. Such mechanism can be seen as a *weighted voting scheme* (with weights $w \in \Re^K$) to decide whether or not the public project should be undertaken. When an individual reports his valuation, he is in fact choosing the strength to which he supports (or opposes if the corresponding coefficient is negative) the public project. Note that the tax paid by an agent depends upon the valuation he reports, and therefore an agent, when choosing the strength to which he supports or opposes the public project, must account for the cost (in taxes) of such a choice.

A particular family of provision rules in the class described above are *α-majority voting rules.* A provision rule q is an $\alpha-$ majority voting rule if there exists $\alpha \in [0,1]$ such that q satisfies, for all $n \in \mathcal{D}$,

$$q(n) = \begin{cases} 0, & \text{if } \sum_{k:v_k>0} n_k \leq \alpha(N - n_a), \\ 1 & \text{if } \sum_{k:v_k>0} n_k > \alpha(N - n_a); \end{cases}$$

where n_a denotes the number of individuals such that $v_a = 0$ (Thus, such number n_a will be equal to zero if $v_a \notin S$). Recall that for each $n \in \mathcal{D}$one has

$$\sum_{k:v_k>0} n_k + \sum_{k:v_k<0} n_k = N - n_a.$$

and therefore a majority voting rules satisfies

$$q(n) = \begin{cases} 0, & \text{if } (1-\alpha) \sum_{k:v_k>0} n_k - \alpha \sum_{k:v_k<0} n_k \leq 0, \\ 1 & \text{if } (1-\alpha) \sum_{k:v_k>0} n_k - \alpha \sum_{k:v_k<0} n_k > 0. \end{cases}$$

Thus, majority voting rules are indeed a particular class of weighted voting rules.

It is now shown that a mechanism (q, t) such that q is a $\alpha-$majority voting rule and t satisfies $t(n) = 0$ for each $n \in \mathcal{D}$, satisfies BIC. To see this, observe first that every $\alpha-$majority rule satisfies $(T.1.1)$. Thus, in order to show such a mechanism satisfies BIC, it only remains to be shown that there exists a sharing rule $\beta \in [0,1]^{K-1}$ such that, for each $k = 1, 2, .., K-1$,

$$\bar{t}_{k+1}(t) - \bar{t}_k(t) = [\beta_k v_{k+1} + (1-\beta_k) v_k] \left(\bar{q}_{k+1}(q) - \bar{q}_k(q)\right), \tag{4.1.2}$$

which, taking into account that $t \equiv 0$ is equivalent to

$$0 = [\beta_k v_{k+1} + (1-\beta_k) v_k] \left(\bar{q}_{k+1}(q) - \bar{q}_k(q)\right). \tag{4.1.3}$$

In order to show a sharing $\beta \in [0,1]^{K-1}$ exists, it is useful to distinguish between two possible cases. Suppose first there exists a type a such that $v_a = 0$. In this case, it is straightforward to check that a sharing rule $\beta \in [0,1]^{K-1}$ such that $\beta_{a-1} = 1$ and $\beta_a = 0$ satisfies (4.1.3). If there does not exist a such that $v_a = 0$, let p be the highest type such that $v_p < 0$ and let $\beta \in [0,1]^{K-1}$ be such that

$$\beta_p = \frac{-v_p}{v_{p+1} - v_p}.$$

Clearly, such a sharing rule satisfies (4.1.3). Thus, a majority voting scheme without taxes satisfies BIC and $EXPBB$. In what follows, such a mechanism will be referred to as a *referendum.*

Theorems 2 and 3 characterize the class of interim efficient mechanism as a particular class of weighted voting schemes. To obtain the weights associated to interim efficient voting schemes, let $w : \Delta\mathcal{S} \times [0,1]^{K-1} \to \Re^K$ be defined, for each $k \in 1, 2, ..., K-1$, by

$$w_k(g, \beta) = \begin{cases} v_1 + \frac{(F_1 - G_1)\beta_1^g (v_2 - v_1)}{\pi_1}, & \text{if } k = 1; \\ v_k + \frac{(F_{k-1} - G_{k-1})(1-\beta_{k-1}^g)(v_k - v_{k-1})}{\pi_k} + \frac{(F_k - G_k)\beta_k^g (v_{k+1} - v_k)}{\pi_k}, & \text{if } k = 2, ...K; \\ v_K + \frac{(F_{K-1} - G_{K-1})(1-\beta_{K-1}^g)(v_K - v_{K-1})}{\pi_K}, & \text{if } k = K. \end{cases}$$

Also, let $\beta^g \in [0,1]^{K-1}$ be a sharing rule satisfying, for each $k = 1, 2, ..., K-1$,

$$\beta_j^g = \begin{cases} 1, & \text{whenever } F_j - G_j < 0; \\ 0, & \text{whenever } F_j - G_j > 0. \end{cases} \tag{SR}$$

Finally, let $w^g = w(g, \beta^g)$. Note that w^g is well defined. For each $k = 1, 2, ..., K$, the number w_k^g will be referred to as type k's *virtual* weight.

Theorem 2. *Let $g \in \Delta S$ be such that $w_1^g \leq w_2^g \leq ... \leq w_k^g \leq w_{k+1}^g \leq \cdots \leq w_K^g$ and let (q^*, β^*) be such that the following two conditions are satisfied:*
(T.2.1) β^* is equivalent (with respect to q^*) to a sharing rule β^g satisfying (SR); *and*
(T.2.1) q^* satisfies, for each $n \in D$,

$$q^*(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} w_k^g n_k \leq 0, \\ 1, & \text{whenever } \sum_{k=1}^{K} w_k^g n_k > 0. \end{cases}$$

Then (q^*, β^*) solves (P.2).

Thus, if the welfare distribution g satisfies $w_1^g \leq w_2^g \leq ... \leq w_k^g \leq w_{k+1}^g \leq ... \leq w_K^g$, the provision rule of the inetrim efficient mechanism associated to g is a weighted voting scheme with weights given by w^g. To see how weights are determined, note first that each weight w_k^g can be written equivalently as

$$\begin{aligned} w_k^g &= v_k + \frac{(1 - G_{k-1}) - (1 - F_{k-1})}{\pi_k} (v_k - v_{k-1}) (1 - \beta_{k-1}^g) \\ &\quad + \frac{(1 - G_k) - (1 - F_k)}{\pi_k} (v_{k+1} - v_k) \beta_k^g. \end{aligned}$$

In order to understand the role of these weights, it might be useful to think of a social planner who has to decide the strength that each agent's vote should have in order to maximize the average interim utility (with respect to a distribution g) associated to a mechanism in reduced form. An intuitive way to proceed for this planner would be to increase the probability of provision whenever someone reports a positive valuation, and to decrease it when it is negative. However, increasing the probability of provision for an agent reporting a particular valuation v_k might have negative effects on expected taxes that can be collected from agents with higher valuations (if taxes paid by these agents were too high, they might have incentives to understate their valuations). Thus, the value that the planner assigns to valuation v_k needs to be modified. Notice that $(v_k - v_{k-1})(1 - \beta_{k-1})$ represents the effect (of increasing the probability of provision) on utility gains obtained by agents with valuation v_k if they report v_k instead of v_{k-1}, while $(v_{k+1} - v_k)\beta_k$ represents the effect on utility gains obtained by agents with valuation v_k if they report v_k instead of v_{k+1}. The corrections applied will be increasing in the expected number of agents with valuation higher than v_k (represented by $(1 - F_{k-1})$ and $(1 - F_k)$) and decreasing in cumulative welfare associated to these agents (represented by $(1 - G_{k-1})$ and $(1 - G_k)$). Also, the rule $(\beta_k^g, 1 - \beta_k^g)$ determining how any two agents of types k and $k+1$ share the joint surplus they obtain by reporting there true valuations depends on whether or not type k's cumulative welfare G_k is higher than its cumulative probability F_k, as condition (SR) states. Finally, corrections applied are proportional to $\frac{1}{N\pi_k}$, the inverse of the expected number of individuals of type k. It should be noticed that for every $g \in \triangle S$, the *ex-ante* expected value of agent´s virtual valuation equals

$$\sum_{k=1}^{K} \pi_k w_k^g = \sum_{k=1}^{K} g_k v_k.$$

4.2. Ex post efficiency

A commonly studied case is that for which $g \equiv \pi$. That is, the cumulative welfare corresponding to any valuation corresponds to its cumulative probability. Provision rules that maximize *average* welfare (with respect to the welfare distribution π) satisfy, for all $n \in \mathcal{D}$,

$$q(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} v_k n_k \leq 0, \\ 1, & \text{whenever } \sum_{k=1}^{K} v_k n_k > 0; \end{cases} \tag{4.1.4}$$

Therefore, the project is undertaken if the sum of the individuals' valuations is positive. This corresponds to the *ex-post efficiency criterium* and, therefore, *such provision rules select ex-post efficient decisions.* Furthermore, observe that since $F_k = G_k$ for each $k = 1, 2, ..., K-1$, any sharing rule $\beta \in [0,1]^{K-1}$ satisfies *condition* (SR) and, therefore, *any sharing rule maximizes average welfare* (with respect to $g = \pi$).

As a particular case, suppose $K = 2$. Since any sharing rule associated to a referendum maximizes *average* welfare (with respect to the welfare distribution $g = \pi$), one might select a sharing rule $\beta_1 \in [0,1]$ such that

$$\beta_1^* = \frac{-v_1}{v_2 - v_1}.$$

Thus, a mechanism $m \equiv (q,t)$ such that

$$\bar{t}_2(t) - \bar{t}_1(t) = [\beta_1^* v_2 + (1-\beta_1^*)\, v_1]\,(\bar{q}_2(q) - \bar{q}_1(q)) = 0$$

is interim efficient. Thus, *interim efficiency (and ex-post efficiency), can be accomplished by a referendum*[6]. In fact, the welfare distribution $g = \pi$ is the only one for which a referendum maximizes *average* welfare in the two-type case. Note that for any other welfare function g one has $F_1 - G_1 = \pi_1 - g_1 \neq 0$ and therefore maximizing the *average* welfare requires a sharing rule $\beta_1^g \neq \beta_1^*$ that yields different taxes to agents voting in favor (or against) the public project.

Note that Theorem 2 does not apply to all welfare distributions $g \in \Delta S$ for which there exists k such that $w_k^g > w_{k+1}^g$, since in this case a provision rule with weights given by w^g might fail to satisfy the monotonicity condition $(T.1.1)$. Theorem 3 provides a procedure to compute interim efficient mechanisms associated to such distributions. Some additional notation is introduced first.

Let $\lambda = (\lambda_1, \lambda_2, ..., \lambda_{K-1}) \in \Re_+^{K-1}$ be arbitrary. Also, given $g \in \Delta S$, let $\times \Re_+^{K-1} \to \Re^K$ be defined, for each $(g,\lambda) \in \Re_+^{K-1}$ and each $k = 1,2,....,K$, by

$$\widetilde{w}_k^g(\lambda) = \begin{cases} w_1^g - \frac{\lambda_{k-1}}{\pi_1}, & \text{if } k = 1; \\ w_k^g + \frac{\lambda_k - \lambda_{k-1}}{\pi_k}, & \text{if } k = 2,...,K; \\ w_K^g + \frac{\lambda_K}{\pi_K}, & \text{if } k = K. \end{cases}$$

Theorem 3. *Let $g \in \Delta S$ be arbitrary. Then a mechanism in reduced form $(q^*, \beta^*) \in Q^R \times [0,1]^{K-1}$ solves (P.2) if and only if there exists $\lambda^* \in \Re_+^{K-1}$* such that

[6] Ex-post efficiency of referenda in the two-type case has been established by Ledyard and Palfrey (1994). It should be noticed, however, that they study a problem in which undertaking the project costs C units of a private good. Thus, when decisions are taken by using a referendum and the project is undertaken, it is necessary to charge positive taxes from the agents in order to cover costs. Taxes paid in this case, are lump sum.

$(T.3.1)$ *β^* is equivalent (with respect to q^*) to a sharing rule β^g* satisfying (SR);

$(T.3.2)$ *q^* satisfies, for each $n \in \mathcal{D}$,*

$$q^*(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^*) n_k < 0, \\ 1, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^*) n_k > 0; \end{cases}$$

$(T.3.3)$ *λ^* satisfies, for each $k = 1, 2, ..., K-1$,*

$$\lambda_k^* \left[\overline{q}_{k+1}(q^*) - \overline{q}_k(q^*) \right] = 0.$$

Notice that Theorem 2 follows straightforward as a corollary of Theorem 3. To see this, simply observe that if g satisfies $w_1^g \leq w_2^g \leq ... \leq w_K^g$, then a provision rule q^* satisfying $(T.2.1)$ yields, for each $k = 1, 2, ..., K-1$,

$$\overline{q}_{k+1}(q^*) - \overline{q}_k(q^*) \geq 0.$$

Also, the vector $\lambda^* \equiv 0$ yields, for each $k = 1, 2, ..., K-1$,

$$\widetilde{w}_k^g(0) = w_k^g.$$

Thus, the pair $(q^*, 0)$ satisfies $(T.3.3)$. Therefore the mechanism in reduced form (q^*, β^*) satisfying $(T.2.1)$ and $(T.2.2)$ maximizes *average* welfare (with respect to the welfare distribution g).

The intuition behind Theorem 3 is the following. Suppose a distribution g yields a system of weights w^g such that $w_k^g > w_{k+1}^g$ for some $k = 1, 2, ..., K-1$. Then a provision rule q^* satisfying $(T.2.2)$ might fail to satisfy the monotonicity constraints in condition $(T.1.1)$ and therefore $q^* \notin \mathcal{Q}^R$. Thus, in order to obtain the provision rule that maximizes welfare, each virtual weights w_k^g is modified whenever the monotonicity consntraint in $(T.1.1)$ is binding. Notice that for every $\lambda \in \Re_+^{K-1}$ one has

$$\sum_{k=1}^{K} \pi_k \widetilde{w}_k^g(\lambda) = \sum_{k=1}^{K} \pi_k w_k^g + \sum_{k=1}^{K-1} (\lambda_k - \lambda_{k+1}) + \lambda_K = \sum_{k=1}^{K} \pi_k w_k = \sum_{k=1}^{K} g_k w_k^g,$$

and therefore the vector $\widetilde{w}_k^g(\lambda^*)$ modifies w^g without altering the expected virtual weight of an individual. Thus, given an arbitrary g and a vector λ^* satisfying $(T.3.3)$, for each $k = 1, 2, ..., K$, the number $\widetilde{w}_k^g(\lambda^*)$ will be referred to as type k's *generalized virtual weight* (associated to g).

An important consequence of Theorem 3 is that *pooling of types* might appear in some interim incentive efficient mechanism. That is, there might exist an interim incentive efficient mechanism (q^*, t^*) and two types k, k' such that $\widetilde{w}_k^g(\lambda^*) = \widetilde{w}_k^g(\lambda^*)$. Note that such mechanisms one has $\overline{q}_k(q^*) = \overline{q}_{k'}(q^*)$ and, hence, $\overline{t}_k(t^*) = \overline{t}_k(t^*))$. Ledyard and Palfrey (1994) have shown that with two types of agents the only mechanisms for which pooling of types occur are either a constant mechanism for which the project is always undertaken, or a mechanism for which for all distributions of valuations the provision rule is a lottery between the two alternatives available. With more than two types, other interim incentive efficient mechanisms exhibiting pooling of types might exist.

4.3. Interim efficiency of Referenda

Recall that a voting rule without taxes is called a referendum. Also, recall that in problems with only two types, the only welfare distribution that can be maximized by a referendum is that distribution satisfying $F \equiv G$. Clearly, this result does not hold in problems with more than two types, since in this case the weighted voting rule that maximizes *average* welfare (with respect to $g = \pi$), yields different weights to any two agents with different types. Thus, a system of taxes is required to preserve incentive compatibility of such a mechanism.

Surpisingly, in problems with more than two types, there exists other welfare distributions that can be maximized by a referendum. For example, consider a problem described by

$$\mathcal{S} = \{-5, -3, 0, 3, \}$$

and

$$\pi_k = \frac{1}{4} \text{ for every } k = 1, ..., K.$$

Let $g = (0,\ 3/4, 0,\ 1/4)$. Note that such distribution yields

$$\begin{aligned} w_1^g &= -5; \\ w_2^g &= -3 + \frac{1/4}{1/4} - \frac{(1/4)3}{1/4} = -5; \\ w_3^g &= 0; \\ w_4^g &= 3 \end{aligned}$$

Thus, per capita welfare is maximized by a mechanism in reduce form (q^*, β^*) such that β^* is equivalent to a sharing rule $\beta^g = (0,1,0)$ and q^* satisfies, for each $n \in \mathcal{D}$,

$$q^*(n) = \begin{cases} 0, & \text{if } 3 \sum_{k:v_k>0} n_k - 5 \sum_{k:v_k<0} n_k \leq 0, \\ 1 & \text{if } 3 \sum_{k:v_k>0} n_k - 5 \sum_{k:v_k<0} n_k > 0; \end{cases}$$

or, equivalently,

$$q^*(n) = \begin{cases} 0, & \text{if } \frac{3}{8} \sum_{k:v_k>0} n_k - \frac{5}{8} \sum_{k:v_k<0} n_k \leq 0, \\ 1 & \text{if } \frac{3}{8} \sum_{k:v_k>0} n_k - \frac{5}{8} \sum_{k:v_k<0} n_k > 0. \end{cases}$$

Thus, the provision rule that maximizes per capita welfare is a $(5/8)-$majority voting rule. Also, since $\beta^g = (0,1,0)$, a mechanism (q,t) such that $q = q^*$ and $t \equiv 0$ is interim efficient.

However, under under weak assumptions on the distribution generating the agents' valuations, *referenda* are not interim efficient, as shown in Proposition 4 below.

Proposition 4. *Suppose $K \geq 4$, $v_2 < 0$ and $v_{K-1} \geq 0$. Then an interim efficient referendum mechanism does not exist.*

Thus, the possibility of achieving efficiency by a referendum is ruled out in problems in which the range of possible valuations for tyhe public project are sufficiently rich.

5. Concluding Remarks

One of the main results in the paper characterizes the set of Bayesian incentive compatible mechanisms in discrete public good problems in which the set of valuations is also discrete. This characterization allows one to obtain the set of interim efficient mechanisms in this setting. Provision rules corresponding to interim incentive efficient mechanisms can be described as weighted voting schemes, and therefore interim efficiency can be achieved in a relatively simple way. Furthermore, in some cases, a simple referenda that does not impose any taxes on the agents might accomplish interim efficiency.

Several extensions that are worth exploring. One of them is to explore the assymptotic properties of interim efficient mechanisms in the presence of a large number of individuals, as studied by Ledyard and Palfrey (1998) in problems with a continuum of types.

In this paper, Ledyard and Palfrey have shown that, in problems with continuous sets of types, any interim efficient mechanism can be approximated by a referendum. The intuition behind this result is the following: in the presence of a large number of individuals, the probability that a particular agent is pivotal vanishes as the number of agents increases, and thus incentive compatibility constraints imply that the expected tax paid by each agent converges to zero. Hence, interim utilities obtained by agents participating in a mechanism depends only of the limit probability that the project is undertaken and, therefore, on the limit distribution of the agents' average virtual valuation. Therefore, a referendum in which votes in favor and against the project are weighted in such a way that the expected weight coincide with the expected virtual valuation associated to an interim efficient mechanism, will yield the same limit probability of provision.

Thus, the results obtained here suggests that the result obtained by Ledyard and Palfrey apply also to discrete frameworks. It should be noticed, however, that many other weighted voting schemes might also approximate interim efficient mechanisms in the presence of a large number of individuals. In addition, some of the problems associated to other weighted voting schemes in small economies (such as the necessity of designing a balanced tax function to ensure manipulation of votes) might become less important in the presence of a large number of individuals (for example, by designing the tax system in such a way that taxes are paid only when agents are pivotal). Therefore using other weighted voting schemes to approximate interim efficiency might be interesting.

Other possible extension would be to explore whether the class of mechanisms studied here might be applied also to environments with continuous sets of types. In addition, some of the conceptual framework proposed could be relevant in exploring the set of interim efficient mechanism in other problems, such as provision of excludable public goods or trading private goods.

Finally, another extension would be introducing voluntary participation constraints in the optimization problem associated to interim incentive efficiency, where these voluntary participation constraints refer to other (not necessarily constant) mechanism. This might be interesting for practical aspects of mechanism design, since it allows one to study whether or not all agents involved in the problem might agree to alter a statu quo when this statu quo is a well established mechanism (e.g., a referendum). In addition, voluntary participation constraints might restrict the set of interim incentive efficient mechanisms that agents might agree upon.

Appendix

Theorem 1. *A mechanism $m \equiv (q,t) \in \mathcal{Q} \times \mathcal{T}$ satisfies BIC if and only if*

T.1.1 *for every $k \in \{1,2,3,...,K-1\}$ one has*

$$\overline{q}_{k+1}(q) \geq \overline{q}_k(q),$$

T.1.2 *there exists a vector $\beta \equiv \{\beta_k\}_{k=1}^{K-1} \in [0,1]^{K-1}$ such that for every $k \in \{1,2,3,...,K-1\}$ one has*

$$\overline{t}_{k+1}(t) - \overline{t}_k(t) = [\beta_k v_{k+1} + (1-\beta_k) v_k] \left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right).$$

Proof: Necessity is proved first. Let $m \equiv (q,t) \in \mathcal{Q} \times \mathcal{T}$ be a mechanism satisfying BIC. Note that m satisfies, for any $k = 1,2,...,K-1$,

$$v_{k+1}\overline{q}_{k+1}(q) - \overline{t}_{k+1}(t) \geq v_{k+1}\overline{q}_k(q) - \overline{t}_k(t) \tag{A.1}$$

and

$$v_k\overline{q}_k(q) - \overline{t}_k(t) \geq v_k\overline{q}_{k+1}(q) - \overline{t}_{k+1}(t). \tag{A.2}$$

Adding up terms in each side of the inequalities one has

$$(v_{k+1} - v_k)\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right] \geq 0. \tag{A.3}$$

Hence, as $v_{k+1} > v_k$, $(A.3)$ yields

$$\overline{q}_{k+1}(q) \geq \overline{q}_k(q),$$

and therefore $(T.1.1)$ is satisfied. To show $(T.1.2)$ is satisfied, use again $(A.1)$ and $(A.2)$ to obtain

$$\overline{t}_k(t) + v_k\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right] \leq \overline{t}_{k+1}(t) \leq \overline{t}_k(t) + v_{k+1}\left[\overline{q}_{k+1}(q) - \overline{q}_k(q)\right]. \quad ((A.4))$$

Note that if $\overline{q}_{k+1}(q) = \overline{q}_k(q)$, then $\overline{t}_{k+1}(t) = \overline{t}_k(t)$. Let now $\beta \in [0,1]^{K-1}$ be defined, for all $k \in \{1,...,K-1\}$, by

$$\beta_k = \begin{cases} \dfrac{\overline{t}_{k+1}(t) - \overline{t}_k(t) - v_k\left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right)}{(v_{k+1} - v_k)\left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right)}, & \text{if } \overline{q}_{k+1}(q) > \overline{q}_k(q), \\ 0 & \text{if } \overline{q}_{k+1}(q) = \overline{q}_k(q). \end{cases} \quad ((A.5))$$

Clearly, such β satisfies $(T.1.2)$. Note also from $(A.4)$ that each $\beta_k \in [0,1]$.

To prove sufficiency, let m be a mechanism satisfying $(T.1.1)$ and $(T.1.2)$. Consider two arbitrary types $k, k' \in \{1, 2, ...K\}$ and assume, without loss of generality that $k > k'$. Then,

$$\begin{aligned}
\bar{t}_k(t) - \bar{t}_{k'}(t) &= \sum_{j=k'}^{k-1} \left(\bar{t}_{j+1}(t) - \bar{t}_j(t)\right) \\
&= \sum_{j=k'}^{k-1} \left[\beta_j v^{j+1} + \left(1 - \beta_j\right) v^j\right] \left[\bar{q}_{j+1}(q) - \bar{q}_j(q)\right].
\end{aligned}$$

Recall that $v_{j+1} > v_j$ for any $j : k' \le j < k$. Taking into account also that $\beta_j \in [0,1]$ and $\left[\bar{q}_{j+1}(q) - \bar{q}_j(q)\right] \ge 0$ one has

$$\begin{aligned}
\bar{t}_k(t) - \bar{t}_{k'}(t) &\le \sum_{j=k'}^{k-1} v^{j+1} \left[\bar{q}_{j+1}(q) - \bar{q}_j(q)\right] \\
&\le v_k \sum_{j=k'}^{k-1} \left[\bar{q}_{j+1}(q) - \bar{q}_j(q)\right] = v_k \left[\bar{q}_k(q) - \bar{q}_{k'}(q)\right].
\end{aligned}$$

Therefore,

$$v_k \bar{q}_k(q) - \bar{t}_k(t) \ge v_k \bar{q}_{k'}(q) - \bar{t}_{k'}(t),$$

therefore establishing that (q,t) satisfies BIC. This completes the proof of Theorem 1.||

the

Proposition 2. *For any* $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$, *there exists a tax function* $t \in T$ *such that*

$(P.2.1)$ *the pair* $m \equiv (q,t) \in \mathcal{Q}^R \times T$ *satisfies* BIC and $EXPBB$;

$(P.2.2)$ *its associated sharing rule* $\beta(m)$ *satisfies, for all* $k = 1, 2, ..., K-1$,

$$\beta_k(m) = \beta \text{ whenever } \bar{q}_{k+1}(q) \ne \bar{q}_k(q);$$

Proof. Let $(q,\beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$ be arbitrary and let $t^* \in \mathcal{T}$ be a tax function such that the mechanism (q,t) satisfies $EXABB$ and, for each $k = 1,2,...,K-1$,

$$\beta_k(m) = \beta_k.$$

Also, let $t : \mathcal{D} \to \Re^K$ be defined, for each $n \in D$ and each $k = 1,2,....,K$, by

$$t_k(n) = \frac{1}{(N-1)}\left(\sum_{j\neq k, j=1}^{K} n_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right),$$

Let $m \equiv (q,t)$. Notice that for $k \in \{1,2,...,K\}$ one has

$$\bar{t}_k(t) = \sum_{n\in\mathcal{D}} t_k(n)P_k(n).$$

Thus,

$$\bar{t}_k(t) = \frac{1}{(N-1)}\sum_{n\in\mathcal{D}}\left(\sum_{j\neq k, j=1}^{K} n_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right)P_k(n).$$

Note from the definitions that, for any $n \in \mathcal{D}$ such that $n_k > 0$, $P_k(n)$ equals the probability that the distribution $n' \equiv (n_1, n_2, ..., n_k - 1, ...n_K)$ occurs in a decision problem with $N-1$ agents. That is, let $\mathcal{D}'$ denote the set of all possible distributions of $N-1$ agents among the set of valuations S, and let $P' : \mathcal{D} \to [0,1]$ be its probability function. Then for all $n \in \mathcal{D}$ such that $n_k > 0$ one has

$$P_k(n) = P'(n_1, n_2, ..., n_k - 1, ...n_K).$$

Therefore,

$$\sum_{n\in\mathcal{D}}\left(\sum_{j\neq k, j=1}^{K} n_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right)P_k(n) = \sum_{n\in\mathcal{D}'}\left(\sum_{j\neq k, j=1}^{K} n_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right)P(n'),$$

Also, from the properties of multinomial distributions yields

$$\begin{aligned}\sum_{n\in\mathcal{D}'}\left(\sum_{j\neq k, j=1}^{K} n_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right)P(n') &= (N-1)\left(\sum_{j\neq k, j=1}^{K} \pi_j\left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right]\right)\\ &= (N-1)\left[\bar{t}_k(t^*) - \sum_{j=1}^{K}\pi_j\bar{t}_j(t^*)\right].\end{aligned}$$

which taking into account that t^* satisfies $EXABB$ yields

$$\bar{t}_k(t) = \bar{t}_k(t^*).$$

Hence, (q,t) satisfies BIC and $\beta(m) = \beta$. To complete the proof, it only remains to be shown that m satisfies $EXPBB$. Total taxes equal, for all $n \in \mathcal{D}$,

$$\begin{aligned}\sum_{k=1}^{K} n_k t_k(n) &= \sum_{n\in D} \frac{1}{(N-1)} \left(\sum_{j\neq k, j=1}^{K} n_j \left[\bar{t}_k(t^*) - \bar{t}_j(t^*)\right] \right) \\ &= \sum_{k=1}^{K} n_k \bar{t}_k(t^*) - \frac{N-1}{N-1} \sum_{k=1}^{K} n_k \bar{t}_k(t^*) \\ &= 0.\end{aligned}$$

Thus, m satisfies $EXPBB$. This completes the proof of Proposition 2.||

Theorem 2. *Let $g \in \Delta S$ be such that $w_1^g \leq w_2^g \leq ... \leq w_k^g \leq w_{k+1}^g \leq ... \leq w_K^g$ and let $(q^*, \beta^*) \in \mathcal{M}^R$ be a mechanism in reduced form such that the following two conditions are satisfied:*
$(T.2.1)$ β^* is equivalent (with respect to q^*) to the sharing rule β^g;
and
$(T.2.1)$ q^* satisfies, for each $n \in D$,

$$q^*(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} w_k^g n_k \leq 0, \\ 1, & \text{whenever } \sum_{k=1}^{K} w_k^g n_k > 0. \end{cases}$$

Then (q^*, β^*) solves $(P.2)$.

Proof. Given $g \in \Delta S$ be such that $w_1^g \leq w_2^g \leq ... \leq w_k^g \leq w_{k+1}^g \leq ... \leq w_K^g$. Observe that these inequalities imply that $q^* \in \mathcal{Q}^R$ and therefore a pair (q^*, β^*) satisfying $(T.2.1)$ and $(T.2.2)$ is a mechanism in reduced form. It is now shown that any such pair solves $(P.2)$.

(1) First, it is shown first that for every $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$ one hasFirst, it is shown first that for every $(q, \beta) \in \mathcal{Q}^A \times [0,1]^{K-1}$ one has

$$\sum_{k=1}^{K} g_k U_K^R(q,\beta) = \frac{1}{N} \sum_{n\in D} \left(\sum_{k=1}^{K} w_k(g,\beta) n_k \right) q(n) P(n). \tag{A.6}$$

To prove that $(A.6)$ is satisfied, let $m \equiv (q,t) \in \mathcal{M}$ be a mechanism in structural form satisfying, for each $k = 1, 2, ..., K$,

$$\overline{U}_k(m) = \overline{U}_k^R(q,\beta)$$

From the definitions one has

$$\overline{U}_{k+1}^R(q,\beta) - \overline{U}_k^R(q,\beta) = v_{k+1}\overline{q}_{k+1}(q) - v_k\overline{q}_k(q) - \left[\overline{t}_{k+1}(t) - \overline{t}_k(t)\right]$$

and, since m satisfies $(T.1.2)$ and $\beta(m) = \beta$,

$$\begin{aligned}\overline{U}_{k+1}^R(q,\beta) - \overline{U}_k^R(q,\beta) &= v_{k+1}\overline{q}_{k+1}(q) - v_k\overline{q}_k(q) \\ &\quad - \left[\beta_k v_{k+1} + (1-\beta_k)\, v_k\right]\left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right).\end{aligned}$$

Therefore

$$\overline{U}_{k+1}^R(q,\beta) - \overline{U}_k^R(q,\beta) = \left[\overline{q}_{k+1}(q)\,(1-\beta_k) + \beta_k\overline{q}_k(q)\right](v_{k+1} - v_k). \qquad ((A.7))$$

Observe now that for for every distribution $g \in \Delta S$ one has

$$\begin{aligned}\sum_{k=1}^{K} g_k\overline{U}_k^R(q,\beta) &= \overline{U}_K^R(q,\beta)\left(\sum_{k=1}^{K} g_k\right) + \sum_{k=1}^{K} g_k\left(\overline{U}_k^R(q,\beta) - \overline{U}_K^R(q,\beta)\right) \\ &= \overline{U}_K^R(q,\beta) - \sum_{k=1}^{K} g_k \sum_{j=k}^{K-1}\left(\overline{U}_{j+1}^R(q,\beta) - \overline{U}_j^R(q,\beta)\right)\end{aligned}$$

Also,

$$\sum_{k=1}^{K} g_k\overline{U}_k^R(q,\beta) = \sum_{k=1}^{K}\pi_k\overline{U}_k^R(q,\beta) - \sum_{k=1}^{K}(g_k - \pi_k)\left(\sum_{j=k}^{K-1}\left(\overline{U}_{j+1}^R(q,\beta) - \overline{U}_j^R(q,\beta)\right)\right).$$

Substituting each term $\left[\overline{U}_{j+1}(m) - \overline{U}_j(m)\right]$ by its expression in $(A.7)$ yields

$$\sum_{k=1}^{K} g_k\overline{U}_k^R(q,\beta) = \sum_{k=1}^{K}\pi_k\overline{U}_k^R(q,\beta) + \sum_{k=1k}^{K}(F_k - G_k)\left[\overline{q}_{k+1}(q)\,(1-\beta_k) + \beta_k\overline{q}_k(q)\right](v_{k+1} - v_k).$$

Finally, taking into account that the structural form m satisfies $EXABB$ one has

$$\sum_{k=1}^{K}\pi_k\overline{U}_k^R(q,\beta) = \sum_{k=1}^{K}\pi_k\overline{U}_k(m) = \sum_{k=1}^{K} v_k\pi_k\overline{q}_k(q).$$

Therefore

$$\sum_{k=1}^{K} g_k \overline{U}_k^R(q,\beta) = \sum_{k=1}^{K} v_k \pi_k \overline{q}_k(q) + \sum_{k=1k}^{K} (F_k - G_k) \left[\overline{q}_{k+1}(q)(1-\beta_k) + \beta_k \overline{q}_k(q)\right](v_{k+1} - v_k), \tag{A.8}$$

Finally,recall from the definitions that for any $q \in \mathcal{Q}$ nd all $k = 1, 2, ..., K$ one has

$$\overline{q}_k(q) = \sum_{n \in \mathcal{D}} \frac{n_k}{N \pi_k} q(n). \tag{A.9}$$

Substituting each $\overline{q}_k(q)$ in $(A.8)$ by its expression in $(A.9)$ yields

$$\sum_{k=1}^{K} g_k U_k^R(q,\beta) = \frac{1}{N} \sum_{n \in \mathcal{D}} \left[\sum_{k=1}^{K} n_k w_k(g,\beta)\right] q(n) P(n),$$

which establishes $(A.6)$.

(2) It is now shown that for every mechanism in reduced form $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$ one has

$$\sum_{k=1}^{K} g_k U_k^R(q,\beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q,\beta). \tag{A.10}$$

To prove this statement, notice from $(A.9)$ that for each $q \in \mathcal{Q}^R$, the function $\sum_{k=1}^{K} g_k U_k^R(q,\cdot)$ is differentiable on with respect to β and it satisfies, for each $k = 1, 2, ..., K-1$,

$$\frac{\partial \left(\sum_{k=1}^{K} g_k U_k^R(q,\beta)\right)}{\partial \beta_k} = (F_k - G_k) \left[\overline{q}_k(q) - \overline{q}_{k+1}(q)\right] \left[v^{j+1} - v^j\right]. \tag{A.11}$$

Since for each $k = 1, 2, ..., K-1$ one has $\left[\overline{q}_k(q) - \overline{q}_{k+1}(q)\right] \leq 0$, $(A.11)$ implies that $\sum_{k=1}^{K} g_k U_k^R(q,\cdot)$ is non-increasing in β_k if $F_k - G_k > 0$, and it is non-decreasing in β_k if $F_k - G_k < 0$, which establishes $(A.10)$.

(3) To complete the proof, observe that since q^* satisfies $(T.2.2)$ one has, for every $q \in \mathcal{Q}^R$

$$\sum_{k=1}^{K} g_k U_k^R(q^*,\beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q,\beta^g), \tag{A.12}$$

which yields, for every β^* satisfying $(T.2.1)$ and every mechanism in reduced form $(q, \beta) \in \mathcal{Q}^R \times [0,1]^{K-1}$,

$$\sum_{k=1}^{K} g_k U_k^R(q^*, \beta^*) = \sum_{k=1}^{K} g_k U_k^R(q^*, \beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q, \beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q, \beta).$$

Therefore (q^*, β^*) maximizes $\sum_{k=1}^{K} g_k U_K^R(\cdot, \cdot)$, which establishes Theorem 2. ||

Theorem 3. *Let $g \in \Delta S$ be arbitrary. Then a mechanism in reduced form $(q^*, \beta^*) \in \mathcal{M}^R$ solves $(P.2)$ if and only if there exists $\lambda^* \in \Re_+^{K-1}$* such that

$(T.3.1)$ *β^* is equivalent (with respect to q^*) to a sharing rule β^g* satisfying (SR);

$(T.3.2)$ *q^* satisfies, for each $n \in \mathcal{D}$,*

$$q^*(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^g) n_k < 0 : \\ 1, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^g) n_k > 0, \end{cases}$$

$(T.3.2)$ *$\lambda^* \in \Re_+^K$ satisfies, for each $k = 2, 3, ..., K$,*

$$\lambda_k^* \left[\overline{q}_{k+1}(q^*) - \overline{q}_k(q^*) \right] = 0.$$

Proof.

Only if. Let $g \in \Delta S$ be arbitrary and let $(q^*, \beta^*) \in \mathcal{M}^R$ be a solution to $(P.2)$. It is shown that there exists $\lambda^* \in \Re_+^{K-1}$. such that $(T.3.1)$ to $(T.3.3)$ are satisfied.

First, it is shown that (q^*, β^*) satisfies $(T.3.1)$. To prove this statement, suppose not. Then $(A.11)$ yields

$$\sum_{k=1}^{K} g_k U_k^R(q^*, \beta^g) > \sum_{k=1}^{K} g_k U_k^R(q^*, \beta^*),$$

a contradiction. Therefore (q^*, β^*) satisfies $(T.3.2)$.

Since β^* and β^g are equivalent, it follows that q^* solves the constrained optimization problem

$$\max_{q \in \mathcal{Q}} \sum_{k=1}^{K} g_k U_k^R(q, \beta^g)$$
$$s.t \; : \; \overline{q}_{k+1}(q) - \overline{q}_k(q) \geq 0; \; k = 2, 3, ..., K-1. \tag{P.3}$$

Let $\mathcal{L}_g : \mathcal{Q} \times \Re^{K-1} \to \Re$, be the Lagrangian function associated to $(P.3)$. The Lagrangian is defined, for all $(q, \lambda) \in \mathcal{Q} \times \Re^{K-1}$, by

$$\mathcal{L}_g(q, \lambda) = \sum_{k=1}^{K} g_k U_k^R(q, \beta^g) + \sum_{k=1}^{K-1} \lambda_k \left(\overline{q}_{k+1}(q) - \overline{q}_k(q)\right),$$

or, equivalently, by

$$\begin{aligned}
\mathcal{L}_g(q, \lambda) &= \frac{1}{N} \sum_{n \in \mathcal{D}} \left(\sum_{k=1}^{K} w_k^g n_k \right) q(n) P(n) + \frac{1}{N} \sum_{n \in \mathcal{D}} \sum_{k=2}^{K} \left(\frac{\lambda_{k-1} - \lambda_k}{\pi_k} \right) q(n) P(n) \\
&= \frac{1}{N} \sum_{n \in \mathcal{D}} \left(\sum_{k=1}^{K} \widetilde{w}_k^g(\lambda) n_k \right) q(n) P(n).
\end{aligned}$$

By the Sadle Point Theorem in Linear Programing, there exists λ^* such that, for every $(q, \lambda) \in \mathcal{Q} \times \Re_+^{K-1}$ one has

$$\mathcal{L}_g(q^*, \lambda) \geq \mathcal{L}_g(q^*, \lambda^*) \geq \mathcal{L}_g(q, \lambda^*). \tag{A.13}$$

Observe that since q^* maximizes $\mathcal{L}_g(\cdot, \lambda^*)$ one has, for each $n \in D$,

$$q^*(n) = \begin{cases} 0, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^*) n_k < 0, \\ 1, & \text{whenever } \sum_{k=1}^{K} \widetilde{w}_k^g(\lambda^*) n_k > 0. \end{cases}$$

Thus, the pair (q^*, λ^g) satisfies $(T.3.2)$. Finally, $(T.3.3)$ follows from the fact that λ^* minimizes the Lagrangian function on $\Re_+^{K-1}$. Thus, for every solution (q^*, β^*) to $(P.2)$, there exists $\lambda^* \in \Re_+^{K-1}$ such that $(T.3.1)$ to $(T.3.3)$ are satisfied, which establishes the *Only if* statement in Theorem 1.

If.

To prove sufficiency, suppose there exists a mechanism in reduced form $(q^*, \beta^*) \in Q^A \times [0,1]^{K-1}$ and a vector $\lambda^* \in \Re_+^{K-1}$ such that $(T.3.1)$ to $(T.3.3)$ are satisfied. To show (q^*, β^*) solve $(P.2)$, observe that the pair (q^*, λ^*) is a sadle point of the Lagrangian function associated to $(P.3)$ and therefore q^* solves $(P.3)$. Since β^* is equivalent to a vector (β^g) satisfying (SR) one has, for every mechanism $(q, \beta) \in Q^A \times [0,1]^{K-1}$,

$$\sum_{k=1}^{K} g_k U_k^R(q^*, \beta^*) = \sum_{k=1}^{K} g_k U_k^R(q^*, \beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q, \beta^g) \geq \sum_{k=1}^{K} g_k U_k^R(q, \beta).$$

Therefore (q^*, β^*) maximizes $\sum_{k=1}^{K} g_k U_K^R(\cdot,\cdot)$, which establishes that (q^*, β^*) solves $(P.2)$ and therefore completes the Proof of Theorem 3.||

Proposition 3. *Suppose $K \geq 4$, $v_2 < 0$ and $v_{K-1} \geq 0$. Then an interim incentive efficient referendum mechanism does not exist.*

Proof. By Theorem 3, it is sufficient to show there do not exist $\alpha \in [0,1]$ and $g \in \triangle S$ satisfying, for all $k \in \{1,2,...,K\}$,

$$w_k^g(\lambda^*) = -\alpha \text{ whenever } v_k < 0,$$

and

$$w_k^g(\lambda^*) = 1 - \alpha \text{ whenever } v_k > 0;$$

where $w^g(\lambda^*)$ is a vector of weights associated to a solution (q^*, β^*) to $(P.3)$.

To prove this statement, recall that for each $k \in \{1,2,...,K\}$,

$$\begin{aligned} w_k^g(\lambda^*) &= v_k + \left(\frac{F_{k-1} - G_{k-1}}{\pi_k}\right)(1 - \beta_{k-1})(v_k - v_{k-1}) \\ &\quad + \left(\frac{F_k - G_k}{\pi_k}\right)\beta_k(v_{k+1} - v_k) + \frac{\lambda_{k-1}^* - \lambda_k^*}{\pi_k}. \end{aligned}$$

Since $K \geq 4$, $v_2 < 0$ and $v_{K-1} > 0$, it follows that there exists $p \in \{2, ..., K-2\}$ such that $w_k^g = -\alpha$ for each $k \leq p$ and $w_k^g = 1 - \alpha$ for each $k > p$. Since λ_p^* satisfies

$$\lambda_k^* \left[\overline{q}_{k+1}(q^*) - \overline{q}_k(q^*)\right] = 0$$

one has $\lambda_p^* = 0$. Note also that $w_p^g(\lambda^*) - w_1^g(\lambda^*) = 0$, which by definition implies

$$\begin{aligned} 0 &= v_p - v_1 + \left(\frac{F_{p-1} - G_{p-1}}{\pi_p}\right)(1-\beta_p)(v_p - v_{p-1}) \\ &+ \left(\frac{F_p - G_p}{\pi_p}\right)\beta_p (v_{p+1} - v_p) + \frac{\lambda_{p-1}^*}{\pi_p} \\ &- \left(\frac{F_1 - G_1}{\pi_1}\right)\beta_1 (v^2 - v^1) + \frac{\lambda_1^*}{\pi_1}. \end{aligned} \tag{A.14}$$

Taking into account that, for each $k \in \{1, 2, ..., K\}$ one has $\beta_k = 0$ whenever $F_k - G_k > 0$ and $\beta_k = 1$ whenever $F_k - g_k < 0$ one obtains, for each $k = 1, 2, ..., K$,

$$\left(\frac{F_k - G_k}{\pi_k}\right)\beta_k (v_{k+1} - v_k) \leq 0$$

and

$$\left(\frac{F_k - G_k}{\pi_k}\right)(1 - \beta_k)(v_{k+1} - v_k) \geq 0.$$

Hence, $(A.16)$ is satisfied only if $F_p - G_p < 0$. It follows that $\beta_p = 1$ and therefore,

$$\begin{aligned} w_K^g - w_g^{p+1} &= v_K - v_p + \left(\frac{F_{K-1} - G_{K-1}}{\pi_K}\right)(1 - \beta_{K-1})(v_k - v_{K-1}) \\ &+ \frac{\lambda_{k-1}}{\pi_p} + \frac{\lambda_{p-1}}{\pi_p}. \end{aligned}$$

On the other hand one has

$$\left(\frac{F_{K-1} - G_{K-1}}{\pi_K}\right)(1 - \beta_{K-1})(v_K - v_{K-1}) \geq 0$$

hence $w_K^g > w_{p+1}^g$, a contradiction. This completes the proof of Proposition 3.||

References

[1] Dearden, J.(1997) "Efficiency and exclusion in Collective action allocations".*Economic Theory* ***12:*** 391.

[2] Green, J. and Laffont, J.-J.(1979) "Incentives in Public Decision Making". North Holland, Amsterdam/New York.

[3] Holmström B., and Myerson R.(1983), "Efficient and durable decision rules with incomplete information", *Econometrica* **51**: 1799-1819.

[4] Ledyard J., and Palfrey T.R (1994): "Voting and Lottery drafts as efficient Public Good Mechanisms", *Review of Economic Studies* ***61***: 327-355.

[5] Ledyard J., and Palfrey T.R (1996): "interim incentive efficiency in a Public Goods Problem", mimeo, California Institute of Technology.

[6] Mailath G., and Postlewaite A. (1990): "Asymmetric information bargaining problems with many agents" *Review of Economic Studies* **57**: 351-367.

[7] Myerson, R. (1984): "Two person bargaining problems with incomplete information", *Econometrica* **52**: 461-87.

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