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A NON-WELFARIST SOLUTION FOR TWO-PERSON BARGAINING SITUATIONS *

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Abstract

In this paper we present a non-welfarist solution which is applicable to a broad spectrum of two-agent bargaining problems, such as exchange economies, location problems and division problems. In contrast to welfarist bargaining solutions, it depends only on the agents` preferences, not on their specific utility representation, and takes explicitly into account the underlying space of alternatives. We offer a simple sequential move mechanism, without chance moves, that implements our solution in subgame perfect equilibrium. Moreover, an axiomatic characterization of the solution is provided. It is shown that the solution coincides with the Kalai-Rosenthal bargaining solution after choosing a suitable utility representation of the preferences. When applied to exchange economies with equal initial endowments for both agents, the solution generates envy-free, Pareto efficient egalitarian equivalent allocations.

Keywords: Bargaining, Nash program, welfarism, non-welfarism, exchange economies, location problems, implementation.

Journal of Economic Literature: C78, D74

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1. Introduction

This paper deals with two-party disputes, in which both parties hold preferences over a set of alternatives and attempt to reach an agreement on one of them. If the agents fail in reaching an agreement, they fall back to some fixed *status quo point.* In the remainder, we refer to such situations as bargaining problems. In the literature, there exist two significantly different approaches to this class of problems: a *welfarist* and a *non-welfarist approach.* In the former, agents' preferences are represented by utility functions and solutions depend exclusively on the set of feasible utility pairs and the status quo (or disagreement) utilities. An advantage of this approach lies in its flexibility, since such solutions can be applied to a broad spectrum of choice problems. Shapley (1969), however, has shown that the welfarist approach does not allow for two-agent bargaining solutions which are both strictly individually rational and ordinal (i.e. invariant with respect to monotone transformations of the utility functions).

In the non-welfarist approach, the solution works directly on the agents' preferences. One advantage is that such solutions are, by definition, ordinal. However, existing non-welfarist solutions are defined for a specific context only, making them considerably less general than bargaining solutions of the welfarist type. In this paper, we attempt to reconcile the flexibility of the welfarist approach with ordinality. To this purpose, we present a solution, depending solely on the agents' preferences - not on their specific utility representation - which is general enough to be applied to a broad class of two-person bargaining problems, such as exchange economies, location problems and division problems.

In contrast to the usual assumption in bargaining, the agents' preferences are assumed to be *ordinal.* This means that preferences are defined on the set of alternatives only, without making any assumption regarding the agents' evaluations of lotteries on this set. This implies that the set of alternatives contains *all* objects which may be relevant in the bargaining problem. In particular, if agents' evaluations of lotteries are important for the outcome of the bargaining process, then these lotteries should already be incorporated into the set of alternatives. In this case, our model allows agents to have preferences over lotteries different from the von-Neumann-Morgenstern type.

The solution we present is best illustrated by means of an example. Consider an exchange economy with two agents and two perfectly divisible goods. Suppose that the agents have quasi-concave preferences, which are strictly monotonic in both goods. Let e be the initial endowment. For a given r between zero and one, consider the reduced Edgeworth box $A(r)$, in which only a fraction r of the initial endowment can be traded. The solution proposed in this paper selects the unique efficient allocation a^* in the original exchange economy, for which we can find a reduced economy $A(r)$ making each agent indifferent between a^* and his best allocation in $A(r)$. (See the figure below.)

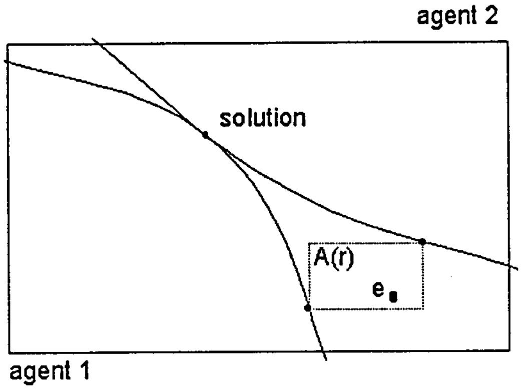

Figure 1.1

In our general setup, the set of alternatives is an abstract, compact and convex set A. We consider a family of reduced subsets of alternatives $A(r)$, which can be viewed as an appropriate generalization of the reduced Edgeworth boxes defined above. More precisely, for every r between zero and one, $A(r)$ is a contraction of A around the status quo point, such that $A(0)$ contains only the status quo point, $A(1)$ is the entire set A and $A(r)$ is monotonically increasing in r. The solution is then defined in the same way as above: it selects those efficient alternatives a^* for which some r can be found, such that both agents are indifferent between a^* and their best choice in $A(r)$.

In accordance with the Nash program, we provide an axiomatic characterization of the solution and a mechanism which implements it. The mechanism proposed is a simple offer-and-counter-offer procedure, containing two rounds only. Being a sequential move mechanisms with perfect information, its outcomes can be predicted naturally by backward induction. Moreover, the mechanism does not contain chance moves. This should be so, since we have no information about agents' preferences over lotteries on A. In contrast to similar mechanisms proposed by Moulin (1984) and Crawford (1979), the role of first mover is given exogenously to one of the players, in this case player 1. Despite this fact, the outcome of the mechanism is anonymous, since the same outcome would be obtained if the role of first mover would be given to player 2.

The two key axioms in the characterization of the solution are *blow-up monotonicity* and *weak Maskin monotonicity.* The former states that by contracting the set of alternatives around the status quo point no agent should be better off. Or, stated equivalently, blowing up the set of alternatives should be beneficial for both agents. This axiom can be interpreted as an equity principle: blowing up the set of alternatives, in many examples, increases the opportunities for both agents symmetrically, and therefore both agents should benefit from it. In an exchange economy, for instance, blowing up the Edgeworth box means allowing the agents to exchange a bigger proportion of the initial endowment. The opportunities for both agents are therefore increased symmetrically.

Weak Maskin monotonicity states that an alternative, selected by the solution, should remain a solution outcome if the agents revise their preferences without changing the ranking of this particular alternative. It is a weaker version of Maskin monotonicity (1977), as the latter takes into account all preference transformations that do not decrease the ranking of the solution outcome. Together with efficiency and a uniqueness property, the above axioms characterize the

We define a particular solution ψ in the following way. For every $r \in [0,1]$, let

$$A(r) = \{(1-r)e + ra|\ a \in A\}$$

be a reduced set of alternatives. Geometrically, $A(r)$ can be seen as a contraction of A around the status quo point e. Since A is convex we have that $A(r) \subset A$. Obviously, $A(r) \subset A(r')$ if $r \leq r'$, $A(0) = \{e\}$ and $A(1) = A$. Note that moving from the set $A(r)$ to the set $A(r')$ where $r' > r$, can be interpreted as "blowing up" the set of alternatives, while leaving the shape unchanged. For every $r \in [0,1]$, let $b_i(r)$ be a maximal element for agent i in the set $A(r)$.[1]

Definition. *The solution ψ is the correspondence assigning to every bargaining problem $\mathcal{B} = (A, e, \succeq_1, \succeq_2)$ the set*

$$\psi(\mathcal{B}) = \{a \in A|\ a \text{ efficient and } \exists r \text{ such that } a \sim_i b_i(r) \text{ for both } i\}.$$

Hence, in the solution ψ, both agents are indifferent between the solution outcome and their best alternative in some reduced set $A(r)$, where $A(r)$ is the same for both agents.

2.2. Properties

First of all, we show that $\psi(\mathcal{B})$ is always nonempty. Consider, to this purpose, an arbitrary utility representation $u = (u_1, u_2)$ of the preferences and the corresponding set $u(A)$ of feasible utility pairs.

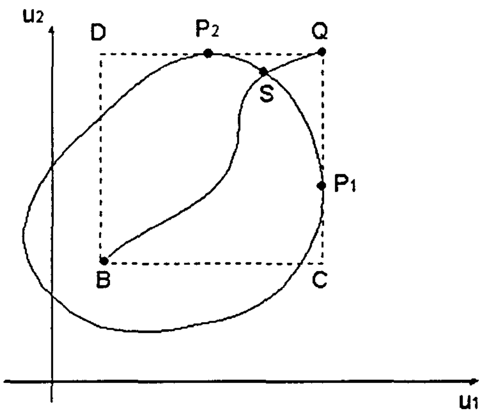

Figure 2.1

Let U_1, U_2 be the agents' maximal feasible utilities. Let $B = (u_1(e), u_2(e))$ be the status quo utilities. Let $P_1 = (U_1, u_2) \in EFF(u(A))$ and $P_2 = (u_1, U_2) \in EFF(u(A))$, where EFF denotes the set of efficient alternatives. Since the efficient frontier in $u(A)$ is, by assumption, a connected set, it connects P_2 with P_1 through a continuous, strictly decreasing curve. (See Figure 2.1).

[1]Such a maximal element exists since $A(r)$ is compact, and agent i's preferences can be represented by a continuous utility function.

for all agent 1 strategies s_1' and

$$a(s) \succeq_2 a(s_1, s_2')$$

for all agent 2 strategies s_2'. The strategy profile s is a subgame perfect equilibrium in Γ if it constitutes a Nash equilibrium in every subgame of Γ. An alternative $a \in A$ is said to be a subgame perfect equilibrium outcome in Γ if there is a subgame perfect equilibrium s with $a = a(s)$.

The mechanism M is said to *implement* a solution φ in *subgame perfect equilibrium* if for every pair $(\succeq_1, \succeq_2)$ of preference relations, $\varphi(A, e, \succeq_1, \succeq_2)$ coincides with the set of subgame perfect equilibrium outcomes in $(M, \succeq_1, \succeq_2)$.

Let the pair (A, e), consisting of the set of alternatives and the status quo point, be given. Consider the following mechanism, which we call mechanism M^*.

Round 1. Agent 1 chooses a number $r \in [0,1]$ and proposes an alternative a^1 in the reduced set $A(r)$. Agent 2 can accept or reject the proposal.

Round 2. If agent 2 has rejected the proposal, then agent 2 makes a new proposal $a^2 \in A(r)$. The final outcome is a^2. STOP

If agent 2 has accepted the proposal, then agent 2 makes a new proposal a^2 which can be any alternative in A. Agent 1 can accept or reject agent 2's proposal a^2. If he accepts, the final outcome is a^2. STOP

If he rejects, the final outcome is a^1. STOP

This mechanism can be seen as some simple version of *step-by-step negotiation* as proposed by Kalai (1977). If agents do not reach an agreement at the second round, they fall back to the outcome reached at the first round. Note that the bargaining process in each round is very asymmetric: one agent makes a proposal and the other agent may simply accept or reject. However, the overall mechanism turns out to be anonymous.

Theorem 3.1. *Let (A, e) be given and let M^* be the mechanism described above. Then, M^* implements the solution ψ in subgame perfect equilibrium.*

A formal proof of this theorem is given in the Appendix. In order to give the reader an intuition why the mechanism leads to the solution ψ, we provide a graphical argument here. Let (u_1, u_2) be an arbitrary utility representation of the preferences and let $u(A)$ be the set of feasible utility pairs. Let $d = (u_1(e), u_2(e))$ be the utilities of the status quo point. Then, $(u(A), d)$ represents a "welfarist" bargaining problem, consisting of a set of feasible utilities and a disagreement point in this set.

Let $u_1(r) = u_1(b_1(r))$ and $u_2(r) = u_2(b_2(r))$ for every $r \in [0,1]$. Consider Figure 3.1, which represents the feasible set of utilities and the curve $r \longmapsto (u_1(r), u_2(r))$.

this proposal and choose a final outcome yielding utilities $(u_1(r^*), u_2(r^*))$. This outcome will be accepted by agent 1. Hence, the outcome of the mechanism coincides with the solution ψ.

4. Axiomatic Characterization

In this section, we show that the solution ψ is characterized by four axioms: efficiency, uniqueness, blow-up monotonicity and weak Maskin monotonicity.

Axiom 1. *Efficiency (EFF). A solution φ is called efficient if for every bargaining problem $\mathcal{B}$, every $a \in \varphi(\mathcal{B})$ is efficient.*

Axiom 2. *Uniqueness (UN). A solution φ is called unique if for every bargaining problem $\mathcal{B}$, every $a \in \varphi(\mathcal{B})$ and every $b \in A$ it holds: $b \in \varphi(\mathcal{B})$ if and only if $b \sim_i a$ for both i.*

Uniqueness can thus be decomposed into two parts. First, it states that the solution always provides a unique outcome in terms of utilities. Moreover, every alternative which is equivalent, for both agents, to a solution outcome, should be a solution outcome itself. There should be no distinction, therefore, among alternatives yielding the same utility pairs.

The following axiom states that blowing up the set of alternatives, without changing the shape of it, should be beneficial for both agents.

Axiom 3. *Blow-up monotonicity (BMON). A solution φ is called blow-up monotonic if for every bargaining problem $\mathcal{B}$, every $r \in [0,1]$, each $a \in \varphi(\mathcal{B})$ and each $b \in \varphi(\mathcal{B}(r))$ we have $a \succeq_i b$ for both i.*

Here, $\mathcal{B}(r)$ denotes the bargaining problem induced by the reduced set of alternatives $A(r)$. The axiom, together with uniqueness, is equivalent to saying that the utility curve $r \longmapsto u_i(\varphi(\mathcal{B}(r)))$ should be increasing in r for both agents i, where $u_i(\varphi(\mathcal{B}(r)))$ should be read as the unique utility for agent i induced by the alternatives in $\varphi(\mathcal{B}(r))$. The intuition behind the axiom is that, in most examples, blowing up the set of alternatives (i.e. changing from $A(r)$ to $A(r')$ where $r' \geq r$) equally increases the opportunities for both agents. No agent should therefore be hurt by such a "fair" transformation. This idea of blowing up the domain is also present in the bargaining literature. Kalai (1977), for instance, considers blow-ups of the set of feasible utilities, leaving its shape invariant, and calls a bargaining solution homogenous if it is covariant with respect to these blow-ups. Also Wiener and Winter (1999) consider monotonically increasing sets of feasible utilities. Their motivation is that agents usually follow a gradual process in order to reach an agreement. They propose a solution, called the gradual solution, which assigns an outcome not only to the big pie, but to any nested pie belonging to some fixed sequence of pies, approximating the big one. Our approach is different from Kalai's and Wiener and Winter's, since we consider explicitly the underlying set of alternatives. It is important to note that the shape of the set of feasible utilities may change when blowing up the set of alternatives.

The last axiom states that an alternative pertaining to the solution should remain a solution outcome if agents revise their preferences without changing the indifference set, upper contour set and lower contour set with respect to this alternative.

Now, suppose that φ is a solution satisfying EFF, UN, BMON and WMMON. We show that $\varphi = \psi$. Let $\mathcal{B}$ be a bargaining problem and $a^* \in \varphi(\mathcal{B})$. By UN of φ and ψ, it suffices to show that $a^* \in \psi(\mathcal{B})$. Let $R_1 = \{r \in [0,1] |\ a^* \sim_1 b_1(r)\}$. By BMON and the fact that $A(0) = \{e\}$, we have that $a^* \succeq_1 b_1(0) = e$. Then, by using a simple continuity argument, it is easily seen that R_1 is a non-empty, closed interval. By assumption, a^* is efficient. Hence, in order to prove that $a^* \in \psi(\mathcal{B})$, it remains to show that $a^* \sim_2 b_2(r)$ for some $r \in R_1$. Suppose not. Then, by a continuity argument, either $a^* \succ_2 b_2(r)$ for all $r \in R_1$ or $a^* \prec_2 b_2(r)$ for all $r \in R_1$.

Case 1. Let $a^* \succ_2 b_2(r)$ for all $r \in R_1$. Let r^* be the maximal element in R_1. Suppose that $r^* = 1$. Since, by assumption, $a^* \succ_2 b_2(r^*)$, it would follow that $a^* \succ_2 b_2(1)$. However, this is not possible since $b_2(1)$ is agent 2's best alternative in A. So, we must have $r^* < 1$. By construction, $a^* \prec_1 b_1(r)$ for all $r > r^*$. Since $a^* \succ_2 b_2(r^*)$, we can find some $\tilde{r} > r^*$ with $a^* \prec_1 b_1(\tilde{r})$ and $a^* \succ_2 b_2(\tilde{r})$. Now, choose preferences $\succeq_2'$having a continuous utility representation, such that for all $b \in A$

$$[a^* \succeq_2 b \Leftrightarrow a^* \succeq_2' b] \text{ and } [a^* \preceq_2 b \Leftrightarrow a^* \preceq_2' b]$$

and moreover

$$a \sim_2' b \text{ for all } a, b \in A(\tilde{r}).$$

Such preferences can be found since $a^* \succ_2 b$ for all $b \in A(\tilde{r})$. Let $\mathcal{B}' = (A, e, \succeq_1, \succeq_2')$ be the bargaining problem where agent 2's preferences are substituted by the ones above. Since in $\mathcal{B}'$, agent 2 is indifferent between all alternatives in $A(\tilde{r})$, it is clear that every efficient point in $A(\tilde{r})$, with respect to $\succeq_1, \succeq_2'$, must be equivalent, for agent 1, to $b_1(\tilde{r})$. By EFF, we know that $\varphi(\mathcal{B}'(\tilde{r}))$ chooses only efficient alternatives in $A(\tilde{r})$, so for every $\tilde{a} \in \varphi(\mathcal{B}'(\tilde{r}))$, we have that $\tilde{a} \sim_1 b_1(\tilde{r})$. By BMON of φ, it follows that, for every $a \in \varphi(\mathcal{B}')$, $a \succeq_1 b_1(\tilde{r})$. Since $\mathcal{B}$ and $\mathcal{B}'$ satisfy the conditions stated in the definition of WMMON, we know, by WMMON of φ, that $a^* \in \varphi(\mathcal{B}')$. However, this implies that $a^* \succeq_1 b_1(\tilde{r})$, which is a contradiction to the assumption above, that $a^* \prec_1 b_1(\tilde{r})$. Hence, Case 1 is not possible.

Case 2. Let $a^* \prec_2 b_2(r)$ for all $r \in R_1$. Let $R_2 = \{r \in [0,1] |\ a^* \sim_2 b_2(r))\}$. Then, by construction, $a^* \succ_1 b_1(r)$ for all $r \in R_2$. This case leads to a contradiction in the same manner as Case 1. One simply has to exchange the roles of agent 1 and 2.

Hence, we may conclude that there must be some $r \in R_1$ with $a^* \sim_2 b_2(r)$. Hence, $a^* \sim_i b_i(r)$ for both i and a^* is efficient. This means that $a^* \in \psi(\mathcal{B})$. By UN of φ and ψ, it follows that $\varphi(\mathcal{B}) = \psi(\mathcal{B})$.■

5. Examples

5.1. Pure exchange economy

Consider a pure exchange economy with two agents and two goods. Let $e_1 = (1,0)$ and $e_2 = (0,1)$ be the initial endowments of agents 1 and 2 respectively. Let $e = (e_1, e_2)$ be the status quo point. The space of alternatives is $A = \{((x_1,x_2),(y_1,y_2)) \in \mathbb{R}_+^{2\times 2} |\ x_1 + y_1 = x_2 + y_2 = 1\}$. For every $r \in [0,1]$, the reduced Edgeworth box $A(r)$ is equal to

$$\begin{aligned} A(r) &= \{(1-r)e + ra |\ a \in A\} \\ &= \{((x_1,x_2),(y_1,y_2)) \in A |\ x_1 \in [1-r,1], x_2 \in [0,r]\}. \end{aligned}$$

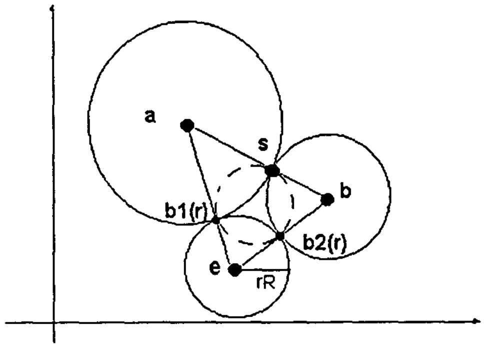

Figure 5.2

The indifference curve for agent i passing through $b_i(r)$ contains exactly the points that have the same distance with respect to the peak as $b_i(r)$. Since the solution is efficient, it selects the unique r for which the intersection point between these indifference curves hits the line connecting the two peaks. The agreed upon location is exactly this intersection point. Figure 5.2 illustrates the solution, denoted by the point s.

The solution for this location problem can be computed as follows. Let the solution be the location $s = \lambda a + (1 - \lambda)b$, where $\lambda \in [0, 1]$. It remains to compute λ. From the picture, it can be seen that

$$\begin{aligned} d(a, b_1(r)) &= d(a, s) = (1 - \lambda)d(a, b), \\ d(b, b_2(r)) &= d(b, s) = \lambda d(a, b) \text{ and} \\ d(e, b_1(r)) &= d(e, b_2(r)) = rR. \end{aligned}$$

Since

$$\begin{aligned} d(a, b_1(r)) + d(e, b_1(r)) &= d(a, e) \text{ and} \\ d(b, b_2(r)) + d(e, b_2(r)) &= d(b, e) \end{aligned}$$

we obtain λ by solving the system:

$$\begin{aligned} (1 - \lambda)d(a, b) + rR &= d(a, e) \\ \lambda d(a, b) + rR &= d(b, e). \end{aligned}$$

The unique solution of the system is equal to

$$r^* = \frac{d(a, e) + d(b, e) - d(a, b)}{2R}, \ \lambda^* = \frac{1}{2} + \frac{1}{2}\frac{d(b, e) - d(a, e)}{d(a, b)}$$

Note that the facility is located exactly in the middle between the two peaks if they are equally distant from the status quo point. If agent i's peak is closer to the status quo point than the other agent's peak, the facility will be located nearer to agent i's peak.

For every $r \leq 1$ we have that $h(r) = \left(\frac{1}{3}r, \frac{2}{3}r\right), w(r) = \left(\frac{3}{4}r, \frac{1}{4}r\right)$ and the corresponding utilities are given by $u_h(r) = \sqrt[3]{\frac{4}{27}}r$ and $u_w(r) = \sqrt[4]{\frac{27}{256}}r$. The solution, in terms of utilities, corresponds with the intersection point between the curve $r \mapsto (u_h(r), u_w(r))$ and the curve of utility pairs induced by $b + s = 1$. Therefore, the solution (b^*, s^*) and the selected r^* are obtained by solving the system

$$b^{\frac{1}{3}} s^{\frac{2}{3}} = \sqrt[3]{\frac{4}{27}}r, \; b^{\frac{3}{4}} s^{\frac{1}{4}} = \sqrt[4]{\frac{27}{256}}r \text{ and } b + s = 1.$$

The solution is $(b^*, s^*) = (0.544, 0.456)$ and $r^* = 0.914$.

6. Relation with the Kalai-Rosenthal Solution

In this section, we show that, for "regular" bargaining problems, the solution ψ has a close connection to the Kalai-Rosenthal solution (Kalai and Rosenthal, 1978). A bargaining problem $\mathcal{B} = (A, e, \succeq_1, \succeq_2)$ is called *regular* if for every $0 \leq r < r' \leq 1$ we have that $b_i(r) \prec_i b_i(r')$ for both i. Hence, blowing up the set of alternatives strictly increases the utility of the best alternative for both agents. For instance, a pure exchange economy with strictly monotonic preferences and interior initial endowment is always regular.

Consider a bargaining problem $\mathcal{B} = (A, e, \succeq_1, \succeq_2)$ and a utility representation $u = (u_1, u_2)$ of the preferences. Let $S = \{(u_1(a), u_2(a)) | \; a \in A\}$ be the set of feasible utilities and $d = (u_1(e), u_2(e))$ the utility pair corresponding to the status quo point. Then, (S, d) represents a "welfarist" bargaining problem.

For every r, let $\mathcal{B}(r)$ be the restriction of the bargaining problem $\mathcal{B}$ to $A(r)$ and $(S(r), d)$ the induced welfarist bargaining problem. Here, we use the same utility representation u, irrespective of r. Note that $e \in A(r)$ and therefore $d \in S(r)$ for all r.

We will show that for any regular bargaining problem $\mathcal{B}$, there is a utility representation u of the preferences such that for every r, the solution ψ applied to $\mathcal{B}(r)$ coincides with the Kalai-Rosenthal solution of the induced welfarist bargaining problem $(S(r), d)$. Hence, in particular, ψ applied to the 'large' problem $\mathcal{B}$ corresponds with the Kalai-Rosenthal solution of the induced welfarist problem (S, d). The solution ψ can therefore be viewed as an ordinal extension of the Kalai-Rosenthal solution.

The Kalai-Rosenthal solution is defined as follows. Let (S, d) be a welfarist bargaining problem with $d = (d_1, d_2)$. Let S be such that the efficient frontier in S is a strictly decreasing curve, connecting agent 2's best point with agent 1's best point. Let U_1 and U_2 be the maximal utilities for both agents in S. The Kalai-Rosenthal solution (KR) of (S, d) is the unique efficient point $(u_1, u_2) \in S$ for which

$$\frac{u_1 - d_1}{u_2 - d_2} = \frac{U_1 - d_1}{U_2 - d_2}.$$

Hence, the Kalai-Rosenthal solution is the intersection point between the line connecting d with the utopia point (U_1, U_2), and the efficient frontier of S.

Consider a pure exchange economy with two agents and m perfectly divisible goods. Assume that the aggregate endowment of each good is normalized to 1 and that the initial endowment (e_1, e_2) divides the total endowment equally among the agents; i.e. $e_1 = e_2 = \frac{1}{2}\mathbf{1}$, where $\mathbf{1}$ denotes the individual bundle containing one unit of each good. Suppose that both agents have continuous preferences which are strictly monotonic in each good.

The solution ψ selects an efficient allocation (a_1, a_2) and (implicitly) an $r^* \in [0,1]$ such that each agent i is indifferent between a_i and his best allocation in the reduced economy $A(r^*)$. Now, agent i's best choice from $A(r^*)$ is the bundle $e_i + r^*(\mathbf{1}-e_i) = (r^*+1)\frac{1}{2}\mathbf{1}$. Hence, both agents are indifferent between the proposed allocation and the egalitarian reference bundle $(1+r^*)\frac{1}{2}\mathbf{1}$. Since the solution ψ is efficient, the selected allocation is a Pareto efficient egalitarian equivalent allocation (PEEEA). The reference bundle, moreover, is a multiple of the equal division bundle $\frac{1}{2}\mathbf{1}$, which ensures that the allocation is envy-free (see Pazner and Schmeidler, 1978).[4] The following lemma is obtained.

Lemma 7.1. *Let $\mathcal{E}$ be a two-agent exchange economy with aggregate endowment $\mathbf{1}$, equal initial endowments for both agents and continuous, strictly monotonic preferences. Let a be the allocation selected by the solution ψ, and let r^* be such that both agents are indifferent between a and their best choice in $A(r^*)$. Then, a is an envy-free PEEEA allocation with egalitarian reference bundle $(r^*+1)\frac{1}{2}\mathbf{1}$.*

Crawford (1979) proposed a divide-and-choose mechanism which generates PEEEA in pure exchange economies. In view of the lemma above, the mechanism presented in this paper provides an alternative procedure to generate PEEEA. Applied to the special environment of an exchange economy with equal initial endowments, our mechanism works as follows. In the first round, agent 1 chooses an $r \in [0,1]$ and selects an allocation in the reduced economy $A(r)$, where only a fraction r of the initial endowment can be traded. On the equilibrium path, agent 1 will choose his best bundle in $A(r)$, which is $e_1 + r(\mathbf{1}-e_1) = (1+r)\frac{1}{2}\mathbf{1}$, leaving the bundle $(1-r)\frac{1}{2}\mathbf{1}$ to agent 2. If agent 2 rejects this proposal, he can take his best bundle in $A(r)$, which is $(1+r)\frac{1}{2}\mathbf{1}$. If he accepts, agent 2 will choose his optimal allocation in the whole economy A, subject to the restriction that agent 1 should not be worse off than in $(1+r)\frac{1}{2}\mathbf{1}$, since otherwise, he will reject. Hence, once r is chosen by agent 1, agent 2 has the choice between the reference bundle $(1+r)\frac{1}{2}\mathbf{1}$ and his best bundle in A, given that agent 1 should not be worse off than with $(1+r)\frac{1}{2}\mathbf{1}$.

In order to compare our mechanism with the Crawford procedure, we briefly describe the latter for the case of two agents. Crawford's divide-and-choose mechanism starts with an auction, in which both agents simultaneously bid some scalar multiple λ of the egalitarian bundle $\frac{1}{2}\mathbf{1}$.[5] The agent with the highest bid, say λ^*, has to privilege to be divider; the other player is assigned chooser. The divider proposes some allocation a and the chooser can choose between a and the

[4] In fact, Pazner and Schmeidler show that choosing the reference bundle equal to a multiple of the equal division bundle is the *only* way to generate envy-free PEEEA in all two-person economies with convex preferences. Pazner and Schmeidler use the term *fair* allocations instead of envy-free allocations.

[5] In the general version of the divide-and-choose mechanism, agents bid a multiple of some numeraire bundle x. We choose $x = \frac{1}{2}\mathbf{1}$, since this corresponds to the mechanism that generates envy-free PEEEA.

In this paper we argue, however, that there is a meaningful way to extend the notion of egalitarian equivalence to a more general class of bargaining situations. Suppose that the agents have agreed on an alternative $a \in A$. One can not directly adopt the egalitarian equivalence principle for exchange economies, and say that a should be equivalent, for both agents, to some "fair" alternative b, which may or may not be feasible in A. The notion of a "fair" alternative is meaningless in this general context.

A possible solution to this problem is by saying that agents are not indifferent between a and some other (hypothetical) outcome, but are indifferent between a and some "fair" *subset* of alternatives F. By this, we mean that each agent should be indifferent between the solution outcome and his best alternative in the subset F. Important is that every agent can choose from the same, "fair" subset F.

Defining "fair" subsets of alternatives in an abstract context is a problematic matter. Rather than exploring all possibilities, we have proposed a special family of subsets $A(r)$, namely those that are obtained by contracting the set A around the status quo point. In many examples, these reduced sets prove to give equal opportunities to both agents and can therefore be regarded as fair.

2. Extensions of the solution.

The properties of the solution, as well as its implementation by the mechanism and the axiomatic characterization, do not really depend on the specific choice of the reduced sets of alternatives $A(r)$. One could define a similar solution by choosing some other family of reduced sets $A(r)$, with the property that $A(r)$ is continuously, monotonically increasing with respect to r, the set $A(0)$ is equal to $\{e\}$ and $A(1)$ is equal to A. In this way, we actually obtain a class of bargaining solutions (one for each family of reduced sets) having the same properties as the solution presented in this paper. The same mechanism (up to the definition of $A(r)$) can be used to implement the solution, and the axiomatic characterization would be identical for all of them, after adapting the blow up monotonicity axiom to the new notion of $A(r)$. In many economic examples, however, the particular choice of the reduced sets used for the solution ψ seems a natural one, as the reduced problems turn out to have the same characteristics as the original problem. This does by no means exclude the possibility that in some bargaining environments, another choice of reduced sets of alternatives may be plausible.

A natural question is how the solution could be extended to the case of more than two agents. The answer to this question seems far from trivial, and is one of the objects of our present research.

9. Appendix

Proof of Theorem 3.1. Let (A, e) be given, let $(\succeq_1, \succeq_2)$ be a pair of preference relations on A and let $u = (u_1, u_2)$ be an arbitrary utility representation of $(\succeq_1, \succeq_2)$. We consider the bargaining problem $\mathcal{B} = (A, e, \succeq_1, \succeq_2)$. In order to reduce notation, we work in the feasible utility space induced by u. We adopt the following conventions:

By $S = \{(u_1(a), u_2(a)) | \ a \in A\}$ we denote the set of feasible utilities. A pair of utilities is typically denoted by (x, y), where x is agent 1's utility and y is agent 2's utility. If we write, *Case 2.* If $x^1 \geq X(Y)$. If agent 2 accepts, we know from round 2 that the final outcome will be $(x^1, Y(x^1))$. If agent 2 rejects, the final outcome is $(x, Y(r)) \in S(r)$.

Case 2.1. If $Y(x^1) > Y(r)$. Then, agent 2 will accept x^1 and the final outcome is $(x^1, Y(x^1))$.

Case 2.2. If $Y(x^1) < Y(r)$. Then, agent 2 will reject x^1 and the final outcome is some $(x, Y(r)) \in S(r)$.

Claim 2. Let $r \in [0,1]$ be such that $Y(X(r)) > Y(r)$. Then, agent 1 can guarantee $X(r)$ in a subgame perfect equilibrium.

Proof of claim 2. Let r be such that $Y(X(r)) > Y(r)$. Let agent 1 choose r and $(x^1, y^1) = (X(r), y) \in S(r)$ at round 1.

Case 1. If $X(r) < X(Y)$. Then, we know from case 1 at round 1 that the final outcome will be $(X(Y), Y)$. Since $X(Y) > X(r)$, agent 1 gets more than $X(r)$.

Case 2. If $X(r) \geq X(Y)$. Since $Y(X(r)) > Y(r)$, it follows from case 2.1 of round 1 that agent 2 will accept the proposal and the final outcome will be $(X(r), Y(X(r)))$.

So, in both cases agent 1 gets at least $X(r)$, which completes the proof of claim 1.

Claim 3. Agent 1 can never get more than $X(r^*)$ in any subgame perfect equilibrium.

Proof of claim 3. Consider any subgame perfect equilibrium in which agent 1 chooses r and $(x^1, y^1) \in S(r)$ at round 1.

Case 1. If $x^1 < X(Y)$. Then, the final outcome will be $(X(Y), Y)$. Since $(X(r^*), Y(r^*))$ is efficient, we have that $X(r^*) \geq X(Y)$. So, agent 1 gets $X(Y) \leq X(r^*)$.

Case 2. If $x^1 \geq X(Y)$.

Case 2.1. Agent 2 accepts (x^1, y^1). Then, the final outcome is $(x^1, Y(x^1))$. Suppose that $x^1 > X(r^*)$. By assumption on the preferences, the efficient frontier in S connects agent 2's best point with agent 1's best point through a strictly decreasing curve. Or, equivalently, the function $Y(x)$ is strictly decreasing for $x \geq X(Y)$. Since $x^1 > X(r^*)$, we have that $Y(x^1) < Y(X(r^*)) = Y(r^*)$. Since agent 2 can always guarantee $Y(r)$ by rejecting agent 1's proposal, it must hold that $Y(x^1) \geq Y(r)$. So, we have that $Y(r) \leq Y(x^1) < Y(r^*)$, implying that $r < r^*$. Since $(x^1, y^1) \in S(r)$, it holds that $x^1 \leq X(r)$. Using the fact that $r < r^*$, we have $x^1 \leq X(r) \leq X(r^*)$, which is a contradiction to the assumption that $x^1 > X(r^*)$. Hence, we may conclude that $x^1 \leq X(r^*)$.

Case 2.2. Agent 2 rejects (x^1, y^1). Then, agent 2 chooses $(x, Y(r)) \in S(r)$ and this is the final outcome.

Case 2.2.1. If $r \leq r^*$. Then, $x \leq X(r) \leq X(r^*)$.

Case 2.2.2. If $r > r^*$. Then, $Y(r) \geq Y(r^*)$ and $x \leq X(Y(r)) \leq X(Y(r^*)) = X(r^*)$.

So, in all the cases above, agent 1 gets less than $X(r^*)$, which completes the proof of our claim.

Since by Claim 1, $r^* = \max\{r \in [0,1] | \; Y(X(r)) \geq Y(r)\}$, claims 2 and 3 imply that in any subgame perfect equilibrium, agent 1 should obtain exactly $X(r^*)$.

Claim 4. In any subgame perfect equilibrium, agent 2 should get exactly $Y(r^*)$.

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