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INFORMATION TRANSMISSION IN THE ABSENCE OF COMMITMENT *

Carlos Maravall Rodríguez [1]

Abstract

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Keywords: Candidate Entry; Candidate Commitment; Asymmetric Information.

JEL Classification: C72, D72, D82

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1 Introduction

The rationale behind asuming that candidates cannot commit is clear: if candidates have preferences for policy, why should they implement any but their preferred policy if elected? However, it is not clear what this assumption entails. This paper presents an implication of it. In particular, I study information transmission from candidates to voters when both have preferences for policies and candidate entry is open. I show that in a general case, where a policy is a vector, preferences are some normed distance in this space and the state translates ideal points, information is of no consequence for the election (i.e. even in a decision-theoretic sense) if candidates cannot commit.

A model where information is of no consequence can be seen as peculiar, its results a particular feature of the variable that defines the state of the world. This is not what is behind this result. The policy space is common to any cheap talk model, for example those of Crawford and Sobel (1982), Austen-Smith and Riker (1987), or Battaglini (2002). The absence of commitment is what drives the above. In contrast with the above result I show, in the one dimensional case, that when candidates can commit this is a standard signaling game (with multiple informative and uninformative equilibria).

The assumption of costless commitment was first criticised by Ferejohn (1986) and Alesina (1988). The citizen-candidate models of Osborne and Slivinski (1996) [OS] and Besley and Coate (1997) [BC] consider this critique and construct a model with agents who are voters, can decide to enter as candidates, but cannot commit to a policy. In both there is no private information (or state variable). OS is a model with "sincere" voters used to analyze the different implications of plurality rule compared to a runoff system. In BC the authors establish the existence of multiple equilibria in a model where candidates compete in a multidimensional space, are elected according to plurality rule, and voters are "strategic".

While OS and BC are not nested in a model to make a head-to-head comparison between the assumptions of commitment and no commitment, they suggest that the set of equilibria differs depending on whether candidates can commit or not (in the entry positions, number of candidates, etc). The interest of this paper is that it shows that these two assumptions have very different implications, in the presence of asymmetric information, beyond those predicted by OS and BC. With commitment, if candidates have private information, we face a standard signaling game with multiple equilibria (some informative -separating- and some uninformative -pooling). Instead, if candidates cannot commit, it is not only the case that this is not a signaling game anymore. If candidates cannot commit, their private information is of *no consequence* for the election (the set of equilibria is the same irrespective of whether voters observe the true state or not). This difference does not arise from picking a state variable that, by construction, satisfies our needs. As stated above, it is common with many other models with private information.

The next section presents a model general enough to allow all the different issues analyzed in this paper to be raised within it. Section 3 shows that if voters observed the true state (i.e. when the realization is common knowledge), all the realization does is translate entry positions without affecting voters' strategies. The following sections analyse the implications when only candidates observe the true state. Section 4 presents a general result when candidates cannot commit. Section 5 presents this result in a one dimensional policy space with sincere voters and compares it to the case when candidates can commit. Section 6 concludes.

2 The Model

An election takes place with a set of voters and possible candidates distributed on compact sets $\mathbf{I}, \mathbf{J} \subseteq \Re^n$ defined by their ideal points $\mathbf{z}^i$ and $\mathbf{z}^j$. Both candidates and voters face a realization of a state of nature that translates their ideal point: for state $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subseteq \Re^n$, their ideal points become $\mathbf{z}_i + \boldsymbol{\theta}$ and $\mathbf{z}_j + \boldsymbol{\theta}$. Preferences over the winning policy of the election, $\mathbf{x}$, are given by some normed distance in this vector space. That is, $u\left(\mathbf{x}, \mathbf{z}^i + \boldsymbol{\theta}\right) = -\left(\sum \left|x_k - \left(z_k^i + \theta_k\right)\right|^p\right)^{\frac{1}{p}}$ and $u\left(\mathbf{x}, \mathbf{z}^j + \boldsymbol{\theta}\right) = -\left(\sum \left|x_k - \left(z_k^j + \theta_k\right)\right|^p\right)^{\frac{1}{p}}$. Each voter's and possible candidate's preferences over lotteries are represented by their expected payoff. Preferences and ideal points irrespective of the state (i.e. $\mathbf{z}^i$, $\mathbf{z}^j$) are common knowledge among the players. Voters do not observe the realization of the state prior to the election but possible candidates do. While informed, possible candidates are infinitesimally small relative to voters, i.e. $\frac{|\mathbf{J}|}{|\mathbf{I}|} = 0$. [1]

The election is a three stage game. In the first stage possible candidates decide whether or not to enter at a position in $\Re^n$ and be considered a candidate for the election. A possible candidate who enters incurs a utility cost $c > 0$ and, if she wins, derives the benefit $b > 0$. In the second stage, after all possible candidates have simultaneously made their entry decisions, voters cast their votes. Whether voting is "strategic" or "sincere" is specified later. In the last stage, the candidate who wins decides the policy to implement. The set of positions which possible candidates can implement ex-post, $\mathbf{P}_j$, depends on their capacity to commit. This is specified in later sections. If no possible candidate enters the race all players obtain negative utility $-d << 0$. I only consider pure strategy equilibria of this game.

[1] The reason why we differ from BC and OS and create an exogenous distinction between voters and possible candidates, is that the latter are informed of the realization of the state.

3 If Voters Were Informed

Irrespective of candidates being committed or not, when both voters and possible candidates observe the true state it translates the set of equilibrium positions by its realization $\boldsymbol{\theta}$.

Proposition 1 *When voters observe the true state, (i) voter strategies do not depend on* $\boldsymbol{\theta}$ *and (ii) entry positions are translated by* $\boldsymbol{\theta}$*.*

Proof. The set of equilibria with no state is identical to there being a single realization normalized to $\boldsymbol{\theta} = \mathbf{0}$. It follows that all the state does is translate positions by $\boldsymbol{\theta}$. ■

4 A General Result when Candidates Cannot Commit

Assume that no candidate has a technology to commit to any policy but his or her ideal point: $\mathbf{P}_j = \{\mathbf{z}^j + \boldsymbol{\theta}\}$. That is, as candidates are unable to commit, once elected they will implement their preferred point given the realization of the state. With this knowledge, voters do not need to infer the realization of the state. They know that candidates will take into account the translation implied by the state when picking their policy. This is because the state translates candidates' and voters' ideal points identically, i.e. it is a common value or, alternatively, it implies a complete alignment between the interests of candidates and voters. For this reason, voters only take into consideration the ideal point of candidates irrespective of the state. That is, they will vote for an ideal point without conditioning on the realization of the state.

Proposition 2 *When voters do not observe the true state and candidates cannot commit (i) voter strategies do not depend on* $\boldsymbol{\theta}$ *and (ii) entry positions are translated by* $\boldsymbol{\theta}$*.*

Proof. *Policy Choice*: When candidates cannot commit to any point, they will implement their ideal point $\mathbf{z}^j + \boldsymbol{\theta}$ if they enter and are elected.

Voting: For voter i, if a generic candidate j defined by her ideal point $\mathbf{z}^j$ is elected, his utility is $u\left(\mathbf{z}^j + \boldsymbol{\theta}, \mathbf{z}^i + \boldsymbol{\theta}\right) = -\left(\sum \left|\left(z_k^j + \theta_k\right) - \left(z_k^i + \theta_k\right)\right|^p\right)^{\frac{1}{p}} = -\left(\sum \left|z_k^j - z_k^i\right|^p\right)^{\frac{1}{p}} = u\left(\mathbf{z}^j, \mathbf{z}^i\right)$, a function that does not depend on the realization of the state, $\boldsymbol{\theta}$. This is true for any j and i. Thus, their decision to vote does not depend on the realization $\boldsymbol{\theta}$.

Entry: A given ideal point, $\mathbf{z}^j$, is part of an equilibrium configuration unconditioned on the realization of the state, as the chance of it winning or changing the outcome of the election is not affected by the realization.

Thus, the set of ideal points that belong to a given equilibrium configuration is determined irrespective of the state, so the state solely translates entry positions by its realization, $\boldsymbol{\theta}$, as in Proposition 1. ■

Proposition 2 shows that the set of *ideal points* that belong to an equilibrium configuration is unaltered by the state variable, as voters do not condition their vote for a given ideal point on the realization of the state. However, it makes no reference to the *candidates* who implement them. Recall that voters do not value candidates *per se* but their ideal points. Thus, whilst the set of ideal points that belong to equilibria remains unaltered by the state, the set of candidates can be conditioned on the state. This is the only sense in which the state variable can affect the set of equilibria, altering who is the *particular* candidate entering without altering the *particular* entry point or voters' strategies as, among those candidates who have the same ideal point, the entry of a particular one can be conditioned on the state. Information is of no consequence because, even if the realization of the state can condition who enters, it has no impact on voters.

Note that no assumption is made on whether voters are strategic or sincere, or on the set of possible realizations, $\mathbf{\Theta}$.

5 A Comparison

Given Proposition 2, we are interested in finding if it is a result of the assumption of no commitment, or other features of the model. In particular, our working strategy is to find out what are the features of the game *with* commitment. However, *with commitment*, as is well known, it is difficult to have equilibria if candidates' policies are defined in a many dimensional space (see Austen-Smith and Banks, 1999). For this reason, I restrict the model to a one-dimensional policy space (i.e. $n = 1$) and, for simplicity, assume sincere voters.

With candidates who cannot commit, and a realization observed by all, this is the setup of OS with a state variable that translates ideal points. For this reason I remind the reader how the equilibria of OS look like given Proposition 1. Given Proposition 2, this is exactly how the set of equilibria look like if voters do not observe the true state. I then consider the equilibria of the same game when candidates can commit. First when voters observe the true state. Finally I analyze what happens in this latter case when voters do not observe the true state. This is a standard signaling game and I give examples of the types of equilibria that can arise, but I do not refine the set of Perfect Bayesian equilibria. The reason is that I want to emphasize the contrast between the cases of commitment and no commitment, rather than the predictions given a particular equilibrium refinement. The paper is long enough as it is.

5.1 A Particular Case of the General Model

The set of policies where preferences are defined is the interval $[0, r]$ on the real line $\Re$, where both voters and possible candidates are equally distributed. Call F that common distribution and assume it has a unique median m. For simplicity, I assume that there is just a two-state of the world, where both states are equally likely. The state translates ideal points w to the left or to the right, i.e. $\theta \in \{-w, w\}$. Thus, if the realization is "w" the set of ideal points becomes the interval $[w, w+r]$, and if it is "$-w$" it is the interval $[-w, -w+r]$. The intervals overlap if $-w + r > w$. We will say w is large or small depending on whether $2w - r > 0$, or $2w - r < 0$. Preferences over the winning policy of the election, x, by a voter with ideal point z_i are represented by the function $-|x - (z_i + \theta)|$. Voters are "sincere": conditional on their belief of the true state, voters vote for the candidate whose position x_j is closest to their ideal point, $z_i + \theta$; and two candidates at a given position share its support equally.

Possible candidates have preferences centered at $z_j + \theta$. Without commitment, candidates cannot commit to any policy ex-ante and will implement their preferred point $z_j + \theta$ if elected. With commitment, the set of positions which possible candidates can implement, P_j, is $[0, r] \pm w$, i.e. $[-w, w+r]$, the smallest convex interval that includes all ideal points given the two states.

The timing of the game is the same. First nature picks the state θ and possible candidates observe the realization. Second, possible candidates simultaneously decide whether or not to enter the race and at which position, i.e. their action is represented by $E_j = \{\emptyset, P_j\}$, with $E = E_1 \times ... \times E_J$. After seeing all the possible candidates' action choices (but not the true state of nature), each voter i simultaneously chooses an action $v_i \in V_i = [0,1]^{|\{j|e_j \neq \emptyset\}|}$. I consider profiles of strategies $(e_j(s), v_i(e))_{\forall i \in I, j \in J}$ and a common belief function for voters

$\mu(s|e)$, that are perfect Bayesian equilibria in this game with sincere voters.

5.2 Results with Commitment

Given what we have assumed above, sincere voters and a one dimensional policy space, if candidates cannot commit and there is no state of the world, the model is that of Osborne and Slivinski (1996). This is also the case if there is a state and its realization is $\theta = 0$.

5.2.1 Equilibria if the state was observed

However, if there is a state with realization $\theta \neq 0$ and it is observed by all agents then the set of equilibria only differ in that entry positions are translated by θ (Proposition 1 above).

Equilibria with one or two candidates

Proposition 3 *Equilibria where candidates take a single position exist if and only if* $b \leqslant 2c$ *and* $b - c > -d$. *If* $b \in [c, 2c]$, *then the candidate's position is* $m + \theta$. *If* $b < c$, *then the entry position can be any within the distance* $\frac{c-b}{2}$ *of* $m + \theta$.

Proof. Proposition 1 above and Proposition 1 in OS. ■

Proposition 4 *(i) Two-candidate equilibria exist if and only if* $b \geqslant 2(c - e_p(F))$. *(ii) In any two-candidate equilibrium the candidates' ideal positions are* $(m + \theta) - \epsilon$ *and* $(m + \theta) + \epsilon$ *for some* $\epsilon \in (0, e_p(F)]$, *(iii) An equilibrium in which the candidates' positions are* $(m + \theta) - \epsilon$ *and* $(m + \theta) + \epsilon$ *exists if and only if* $\epsilon > 0$, $\epsilon \geqslant c - \frac{b}{2}$, $c \geqslant |(m + \theta) - s(\epsilon, F)|$, *and either* $\epsilon < e_p(F)$ *or* $\epsilon = e_p(F) \leqslant 3c - b$.

Proof. Proposition 1 above and Proposition 2 in OS, where $e_p(F)$ and $s(\epsilon, F)$ are defined in OS. ■

5.2.2 Equilibria when the state is not observed

Proposition 2 makes this section redundant: the set of entry positions is the same as in Propositions 3 and 4.

5.3 Results with No Commitment

5.3.1 Equilibria if the state was observed

However, the case when candidates can commit has not been analyzed. [2] For this reason we analyze it first in this subsection. In particular, we look at the set of equilibria where candidates only enter at one or two positions. We assume that the state is observed and normalize its value to zero (i.e. as if there was no state).

Behavior of Candidates in Equilibrium To characterize equilibria, it is important to understand what kind of behavior candidates can have in them. Candidate behavior is identified by how many of them can sit at a particular position and their performance in the election: potentially they can lose, win, or tie. Obviously, in a given equilibrium we cannot observe candidates who are tying and others who win, but the particular structure assumed in this model ("sincere" voting and a one dimensional policy space) restricts the behavior of candidates in equilibrium.

A first important simplification implied by our assumption that two or more candidates at a given position share voters' support equally, is that all of them must have the same behavior: losing, winning, or tying. With this in mind, we first analyze the behavior of sure winners.

[2] Feddersen, Sened, and Wright (1990) analyze a one-dimensional model with commitment but with strategic voters and candidates who only care for winning.

Winners Recall the above: candidates at the same position must have the same behavior. It follows that a winner can only sit on her own, as if some other "winner" was sitting at the same position, they would both be tying.

Lemma 5.1 *Winners can only sit at a position on their own.*

Things become more interesting with tying candidates.

Tying candidates There can always be a candidate who is tying and sitting at some position by herself. However, if voters are sincere, then there cannot be more than two tying candidates sitting at a given position. The reason is that if there were three or more, a new possible candidate could enter on some side of that position and win the election. Moreover, even with only two tying candidates sitting at a position, new entry is possible as long as the candidates do not sit at the median of their voter support. That is, they cannot get support of more than one half on one side and less than a half on the other.

Lemma 5.2 *Candidates that are tying (i) sit at a position on their own, or share it with another tying candidate, and (ii) if two tying candidates share a given position, both sit at the median of their voter support. Moreover, (iii) tying candidates who sit on their own cannot sit at any of the extremes of a candidate configuration, and (iv) the set of winners cannot consist of only two tying candidates at one position.*

Proof. (i) If three or more candidates tie at a given position, then at least half of that support must be on one of the two sides. It follows that if a possible candidate were to enter very close to them on that side, she would win for sure.

(ii) The same logic as for (i) applies. With half the share of votes two tying candidates get, it is enough to tie. However, if they are not sitting at the median of their support, then there would be more than half support on one side and a possible candidate could enter on that side close to them and win.

(iii) If a single tier was sitting by herself at the extreme of a candidate configuration, she could move an arbitrarily small amount towards her single neighboring candidate and win the election. The gain from winning compared to tying is a discrete jump in utility, this is to be compared to the marginal disutility (if there is!) that might result by moving in the direction of her neighbor.

(iv) Even if they sit at the median of their voter support, if the set of winners consisted of only two tying candidates, then a candidate could sit arbitrarily close to them and win. However, if the set of winners consists of more than these two, the new candidate faces the prospect of a worse lottery: the two tiers and her would lose, but there might still be other candidates belonging to the set of winners whose vote share is unaffected. ■

Finally, I consider the behavior of sure losers.

Sure losers Sure losers can come up to three at a time: sitting on their own, sharing the position with another sure loser, or sharing with two other sure losers.

Lemma 5.3 *There are at most three sure losers sitting at a given position.*

Proof. The only reason why two or more candidates who are losing for sure might want to share a position is because if one of them jumped, the remaining ones would belong to the set of winning candidates. However, we know that there cannot be more than two candidates sitting at a given position belonging to the set of winning candidates. The voter support needed to support more would ensure another candidate would enter very close to them and win. ■

Note that the behavior ruled out in Lemma 5.1 and Lemma 5.2 (i), (ii), and (iv) does not rely on the assumption that candidates can commit to any point. It is driven by the assumptions that voters are sincere and that candidates at a given position share their vote equally. However, Lemma 5.2 (iii) does rely on some ability of candidates to commit beyond their ideal point, and Lemma 5.3 does rely on full commitment.

Equilibria with candidates at one or two positions

All candidates at one position The next proposition states that the single equilibrium where all candidates take a single position is one with a sure winner. In fact, it shares the exact same equilibrium position with a single candidate as the single candidate equilibrium in Osborne and Slivinski (1996).

Proposition 5 *Equilibria where candidates take a single position exist if and only if* $b \leqslant 2c$ *and* $b - c > -d$*. If* $b \in [c, 2c]$*, then the candidate's position is* m*. If* $b < c$*, then the entry position can be any within the distance* $\frac{c-b}{2}$ *of* m*.*

Proof. If $b \geqslant 2c$ then another candidate would be willing to enter the race and either win (if the initial candidate is not entering at m) or share the victory. If $2c > b \geqslant c$, then another candidate is willing to enter if she wins the race, but would not be willing to enter to tie. The former is avoided if the initial candidate enters at m. Finally, if $b - c > -d$, then a candidate prefers to enter rather than see no candidate entering but, between entering implementing her preferred policy and somebody else entering and implementing her preferred policy, candidates prefer the latter, as long as it is not too far from their ideal point. Thus, the only question is whether the policy being implemented is too far away from one's ideal position that it merits entry (if there is the possibility of winning). If the distance from the median is within $\frac{c-b}{2}$, then this is not the case. Not only the median, but any candidate closer to the median than the entering candidate would not want to enter under these conditions. ■

All candidates at two positions It is well known that, when candidates only care for winning, there is no equilibrium where only two candidates enter the race. The result below shows that this also applies to the case where candidates have preferences for policies and is even extended beyond that, as there are no equilibria where candidates only enter at two positions, at least for the most common voting distributions.

Proposition 6 *There are no equilibria where candidates enter at two positions if the voting distribution is uniform or single-peaked.*

Proof. Armed with Lemma 5.1 and the assumption that there are only two positions at which candidates can sit at, then either all of the candidates at one of the positions are winning and the other losing, or both are tying. We first show that no equilibria of the latter type can arise. (i) Candidates at both positions are tying. Both positions must consist of tying candidates. However, by Lemma 5.2 (iii), no single tying candidates can sit at the extremes. Hence, both positions must consist of two candidates on top of each other who are tying. If the voting distribution has zero or one peaks (i.e. the uniform or all other distributions used in practice), this is impossible.

i.i. Uniform. Divide the interval of voter support in 4. With two couples of tiers at the extremes of the configuration, then one couple must sit at the lower quartile and the other at the upper quartile. However, a candidate who enters at the median would win the election.

i.ii. Single Peaked. First note that, by Lemma 5.2 (ii) twin tiers must sit at the median of their voter support.

Second, couples must enter on each side from the peak of the distribution. If not, the couple closest to the peak would win. For consider the interval in between both couples; because of sincere voting, in this interval the subinterval of support of each couple is of equal length. But both couples must sit at the median of their support, so they must get at least equal voter support in these subintervals. This is not possible if the peak is not between both couples.

Finally, even if the peak is in between both couples, there is one candidate that can enter between them and win for sure. Consider again the interval of support between both couples and note that there is enough vote share for two candidates to tie. Moreover, from sincere voting, if a new candidate entered somewhere in that interval, her interval of support would be a half of this interval. If this candidate decides to sit at the peak of the distribution, then she is getting more than enough to win. For she gets half of the interval where there is enough votes for two to tie, and she is sitting at the point where there is most support and from where support decreases monotonically towards both couples.

(ii) Candidates at one position win and at the other lose.

ii.i. If candidates at the winning position were two candidates tying. This possibility is ruled out by Lemma 5.2 (iv).

ii.ii. If one of the candidates is a sure winner and the rest are sure losers. If the set of sure losers was a single candidate, she could drop out of the race, save the cost of entering, and the outcome would not change. If the set of sure losers is composed of two or three candidates then they must prefer the winning position rather than the one they are sitting at to divert votes. Moreover, the distance between both positions must be less than the cost of entering the race, for them to find it worth it to enter. However, if such a benefit exists for a candidate (from the set of sure losers) to enter and lose, then that same benefit exists for another candidate who is not entering and whose preferred policy is the one at which the sure losers are entering at. All this candidate has to do is enter at the position where the sure winner is claimed to sit, to make her preferred position win. ■

5.3.2 Equilibria when the state is not observed

When the state is observed by candidates but not by voters, this becomes a signaling model and it is intuitive to expect that we will have several types of equilibria depending on how much information is revealed. Analyzing equilibria along these classes is a staple of all signaling models, but in this environment we are interested in analyzing if the presence of imperfect information can determine that candidates position themselves in ways that would never arise if there was none or a single state. Thus, whilst we restrict ourselves to two or less locations, we are interested in whether these differ spatially by more than the translation implied by the state, whether the number of candidates entering at them is different, and whether the equilibrium behavior of candidates that enter at them is different.

We divide the following analysis in three categories. There are equilibria that rely on w being large, where 'large' refers to whether the two intervals of ideal points under each state overlap. Then we analyze how these equilibria survive when w becomes small. Finally, we give an example of an equilibrium that does not rely on the value of w. We do not give conditions that characterize the whole set of equilibria, but instead focus on examples that are interesting because they could never arise if the state was observed by all.

Large w $(2w - r > 0)$

When no information is revealed (posterior is the prior) We start by showing that, when $r < 2w$, for there to be equilibria where the state is not revealed, the beliefs cannot be the prior in and out of equilibrium.

Proposition 7 *If* $2w - r > 0$*, there are no equilibria where the belief is the prior in and out of equilibrium.*

Proof. Our proof relies on the following contradiction: if voters held a belief equal to the prior no matter what, then all candidates would like to sit at a particular point conditioned on the realization of the state. However, if all candidates sit at a point conditioned on the realization of the state, then the belief cannot be the prior.

Lemma: If $2w - r > 0$ and voters' belief is the prior, then all points in the interval $[-w + r, w]$, and only these, belong to the set of most preferred points for all voters.

Proof of Lemma:

The set of preferred policies for a generic voter i, if his belief is the prior, is $[z_i - w, z_i + w]$, as

$$\{x \in [z_i - w, z_i + w]\} = \arg\max - \left[\frac{1}{2}|x - (z_i - w)| + \frac{1}{2}|x - (z_i + w)|\right].$$

We then need to prove that $[-w + r, w] \subseteq [z_i - w, z_i + w]\ \forall i$. Alternatively, that

$$z_i - w \leqslant -w + r < w \leqslant z_i + w, \forall i.$$

Now, if $2w - r > 0$ the above is satisfied for $z_i = 0$, as $-w \leqslant -w + r < w \leqslant w$, and for $z_i = r$, as $r - w \leqslant -w + r < w \leqslant r + w$. Moreover, one of each of the two weak inequalities is binding for these two ideal points, so it follows that no point outside $[-w + r, w]$ is the most preferred point for all voters if their belief is the prior.□

Thus, if voters' belief is the prior, and a candidate sits at a point in $[-w + r, w]$, she gets a larger share of votes than any candidate not sitting in $[-w + r, w]$. Moreover, at all points in $[-w + r, w]$ she gets the same share of votes. It follows that, if voters' belief is the prior in and out of equilibrium, a candidate would like to sit at her preferred point in $[-w + r, w]$. Given that $2w - r > 0$, this is w for all candidates, if the state is w, and $-w + r$, if the state is $-w$. However, sitting at such points, depending on the realization of the state, is in contradiction with voters' belief being the prior. ■

It follows that when $2w - r > 0$, if there are any pooling equilibria, there must exist a discontinuity in the set of beliefs on the equilibrium path relative to the out-of-equilibrium path. With this in mind, we construct two examples where candidates enter to form configurations that would never arise if the state was observed: more than one candidate entering at a single location, and candidates entering at two locations with all of them tying. In both equilibria candidates sit at one of the extremes of the interval of ideal points (for some state).

Example 1 *An equilibrium where many candidates enter at a single location (see figure 1).*

$$Beliefs : \mu\left(s = w/e\right) = \begin{cases} \frac{1}{2} & \begin{array}{r} \textit{if } e_j = -w + r,\ \forall j \\ \textit{or } \exists j, k \textit{ s.t. } e_j > -w + r \\ e_k < -w + r \end{array} \\ \\ 0 & \begin{array}{r} \textit{if } \exists j \textit{ s.t. } e_j > -w + r \\ \textit{but } \nexists j \textit{ s.t. } e_j < -w + r \end{array} \\ \\ 1 & \begin{array}{r} \textit{if } \exists j \textit{ s.t. } e_j < -w + r \\ \textit{but } \nexists j \textit{ s.t. } e_j > -w + r \end{array} \end{cases}$$

$$Strategies \ : \ e_j = -w + r,\ \forall j = 1, ..., n$$

$$Parameters \ : \ \begin{cases} n \geqslant 2 \textit{ s.t. } \frac{b}{n} > c > \frac{b}{n+1} \\ \\ 2w - r > 0. \end{cases}$$

In English, the beliefs are "the prior, if all candidates sit at $-w + r$ or candidates enter on both sides from $-w + r$; w if some candidate enters to the left of $-w + r$ and none to the right, $-w$ if some candidate enters to the right of $-w + r$ and none to the left". The actions of candidates are "all candidates enter at $-w + r$, and the number of candidates who enter is determined by the benefit of winning relative to the cost of entry".

Example 2 *An equilibrium where candidates enter at two locations and all candidates tie (see* *figure 2).*

$$Beliefs \quad : \quad \mu\left(s=w/e\right)=\begin{cases} \frac{1}{2} & \textit{if } \left|\left\{j:e_j\in\left[-w,-w+r\right]\right\}\right|=\left|\left\{j:e_j\in\left[w,w+r\right]\right\}\right| \\ 0 & \textit{if } \left|\left\{j:e_j\in\left[-w,-w+r\right]\right\}\right|>\left|\left\{j:e_j\in\left[w,w+r\right]\right\}\right| \\ 1 & \textit{if } \left|\left\{j:e_j\in\left[-w,-w+r\right]\right\}\right|<\left|\left\{j:e_j\in\left[w,w+r\right]\right\}\right| \end{cases}$$

$$Strategies \quad : \quad e_j=\begin{cases} -w+r & j=1,...,n/2 \\ w & j=n/2,...,n \end{cases}$$

$$Parameters \quad : \quad \begin{cases} n\geqslant 2 \textit{ and even, and} \\ \textit{(i) } \frac{b}{n}-c>\frac{1}{2}\left(2w-r\right)>-\left(\frac{b}{n}-c\right), \\ \textit{(ii) } -\frac{1}{2(n+1)}\left(2w-r\right)>\frac{b}{n+1}-c, \\ \textit{(iii) } c>\frac{1}{2}\left(2w-r\right), \\ 2w-r>0. \end{cases}$$

In English, the beliefs are "the prior, if an equal number of candidates enter on $[-w,-w+r]$ and $[w,w+r]$; otherwise the true state is the opposite side of where a majority enters". The actions of candidates are "n candidates enter, where n is even, $\frac{n}{2}$ entering at $-w+r$, and $\frac{n}{2}$ at w". The parameters of the example ensure that (i) no candidate who is already entering wants to deviate, and no candidate who is not entering wants to enter and (ii) tie or (iii) lose but change the outcome of the election. It follows that $\frac{b}{n}-c>0$ but $\frac{b}{n+1}-c<0$.

When information is revealed (posterior is the true state) We have shown that candidates can enter at one position in many numbers, and two positions that tie with each other, in equilibria where voters do not update their prior belief. Here we show another situation that cannot arise when there is perfect information: equilibria where candidates enter at two positions with candidates at one of them losing for sure. The reason why they remain in the race is to signal the true state.

Example 3 *An equilibrium where candidates enter at two locations, one of them to lose for sure (see figure 3).*

$$Beliefs : \mu\left(s=w/e\right)=\begin{cases} 1 & \begin{array}{l} \text{if } |j:e_j=-w|\geqslant 2 \text{ and } |\{j:e_j=w+r\}|<2, \text{ or} \\ \text{if } |j:e_j=w+r|=1 \text{ and } |\{j:e_j=-w\}|=0, \end{array} \\ \\ 0 & \begin{array}{l} \text{if } |j:e_j=w+r|\geqslant 2 \text{ and } |\{j:e_j=-w\}|<2, \text{ or} \\ \text{if } |j:e_j=-w|=1 \text{ and } |\{j:e_j=w+r\}|=0, \end{array} \\ \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

$$Strategies \quad : \quad e=\begin{cases} \begin{array}{l} e_j=-w \\ e_k=w+m \end{array} & if\ \theta=w \\ \\ \begin{array}{l} e_j=w+r \\ e_k=-w+m \end{array} & if\ \theta=-w \end{cases} \qquad for\ j=1,2,k=1.$$

$$Parameters \quad : \quad \begin{cases} \text{(i) } 2w+\min\{m,r-m\}>c, \\ \\ \text{(ii) } 2w-\max\{m,r-m\}>b, \\ \\ \text{(iii) } 2c\geqslant b, \\ \\ 2w-r>0. \end{cases}$$

In English, the beliefs are "if two or more candidates enter at $-w$ $(w+r)$ and less than two enter at $w+r$ $(-w)$, then the true state is w $(-w)$; if one candidate enters at $w+r$ $(-w)$ and none at $-w$ $(w+r)$, then w $(-w)$ is the true state; otherwise the belief is the prior". All candidates that enter have a preferred ideal point $w+m$ if the state is w, and $-w+m$ if the state is $-w$. Their actions are "if the state is w, two candidates enter at $-w$ to signal that w is the true state and a third candidate enters at $w+m$; whilst if the state is $-w$, two candidates enter at $w + r$ to signal the true state is $-w$ and a third candidate enters at $-w + m$". The parameters of the example ensure that under each possible realization, none of the candidates signaling the true state wants to deviate and (i) not enter, or (ii) enter at a different position where he would win for sure, and (iii) no candidate wants to enter, even if he would not alter the belief of voters, but just to tie. All other incentives to deviate are ruled out as either implied by these two conditions, or automatically satisfied.

Small w $(2w - r < 0)$ In this subsection we consider examples when the two intervals of voter ideal points under each state overlap, i.e. when $r > 2w$. We show that as w becomes smaller relative to r and the two sets overlap more, the set of equilibria that can arise that differ from when $w = 0$ (and there is none or a single state) becomes ever smaller. However, the value of w small enough such that a particular type of equilibrium does not exist any more is different for different types. Whilst not proved here, as we do not characterize the complete set of equilibria, it seems that eventually, as w becomes close to 0, the only equilibria that can arise are analogous to those when the state is observed.

For the purposes of tractability, whilst the above intuition is good beyond any particular voter distribution, it is difficult to characterize the bounds of equilibria for *any* voter distribution. For this reason, in this section we assume that F is uniform.

When no information is revealed (posterior is the prior) With the general principle above in mind, our first result shows up to which point it is possible to find the equilibrium configuration shown in Example 1 when the two intervals overlap. Note that what makes all candidates sitting at a single position possible in Example 1 is the belief that, if one candidate enters to one side of them, the opposite side is the true state. However, when the intervals overlap, even if the belief is the opposite side of where a candidate enters, there is still some support she will gather, as some voters will prefer her even under the opposite state. Thus, the number of candidates that can be supported at a single position decreases as w becomes smaller relative to r. The lower bound, i.e. when only two candidates can be supported and none could if w was smaller, is when $r = 6w$. Thus, if $r > 6w$ the configuration of Example 1 where candidates enter at a single location is not an equilibrium.

Example 4 *Assume F is uniform. The following is an equilibrium where the two intervals of voter support overlap, yet two candidates enter at a single location (see figure 4).*

$$Beliefs \;:\; \mu\left(s = w/e\right) = \begin{cases} \frac{1}{2} & \begin{array}{l} \textit{if } e_j = -w + r,\ \forall j \\ \textit{or } \exists j, k \textit{ s.t. } e_j > -w + r \\ \qquad\qquad\quad e_k < -w + r \end{array} \\ \\ 0 & \begin{array}{l} \textit{if } \exists j \textit{ s.t. } e_j > -w + r \\ \textit{but } \nexists j \textit{ s.t. } e_j < -w + r \end{array} \\ \\ 1 & \begin{array}{l} \textit{if } \exists j \textit{ s.t. } e_j < -w + r \\ \textit{but } \nexists j \textit{ s.t. } e_j > -w + r \end{array} \end{cases}$$

$$Strategies \;:\; e_j = -w + r,\ j = 1, 2$$

$$Parameters \;:\; \begin{cases} \frac{b}{2} > c > \frac{b}{3}, \\ \\ 2w < r < 6w. \end{cases}$$

In English, the beliefs are "the prior, if all candidates sit at $-w + r$ or candidates enter on both sides from $-w + r$; w if some candidate enters to the left of $-w + r$ and none to the right, $-w$ if some candidate enters to the right of $-w+r$ and none to the left". The actions of candidates are "two candidates enter at $-w+r$, and the benefit of winning relative to the cost of entry is sufficient for them to remain".

Next, with Example 2, in analogy with Example 1, there are configurations that can survive if $r-2w>0$, but not if w is much smaller. As before, the smaller is w, the lesser the number of candidates that can enter under this configuration. In fact, $r=\frac{30}{7}w$ is the smallest w can be relative to r for this to be an equilibrium. Two candidates enter at w and two other at $-w+r$.

Example 5 *Assume F is uniform. The following is an equilibrium where candidates enter at two locations and all candidates tie (see figure 5).*

$$Beliefs \ : \ \mu(s=w/e)=\begin{cases} \frac{1}{2} & \textit{if } |\{j:e_j\in[-w,w]\}|=|\{j:e_j\in[-w+r,w+r]\}| \\ 0 & \textit{if } |\{j:e_j\in[-w,w]\}|>|\{j:e_j\in[-w+r,w+r]\}| \\ 1 & \textit{if } |\{j:e_j\in[-w,w]\}|<|\{j:e_j\in[-w+r,w+r]\}| \end{cases}$$

$$Strategies \ : \ e_j=\begin{cases} -w+r & j=1,2 \\ w & j=3,4 \end{cases}$$

$$Parameters \ : \ \begin{cases} 2w<r<\frac{30}{7}w, \\ \textit{(i) } \frac{b}{4}-c>\frac{1}{2}(r-2w), \\ \textit{(ii) } c>\frac{1}{2}(r-2w). \end{cases}$$

In English, the beliefs are "the prior, if an equal number of candidates enter on $[-w,w]$ and $[-w+r,w+r]$; otherwise the true state is the opposite side of where a majority enters". The actions of candidates are "4 candidates enter, 2 entering at w, and 2 at $-w+r$". The parameters of the example ensure that (i) no candidate wants to exit, which implies that no candidate wants to enter at a different location, and (ii) no candidate wants to enter to lose but change the outcome of the election. Compared to Example 2, note that now no candidate can enter in between the rest of the candidates and tie (or win) unless $r = \frac{30}{7}w$ ($r > \frac{30}{7}w$).

We conclude the analysis where no information is revealed with a result similar to Proposition 5. As in Proposition 5, it is good for any distribution beyond the uniform.

Proposition 8 *If $2w - r < 0$, there are no one or two candidate equilibria where the belief is the prior in and out of equilibrium.*

Proof. (one candidate) If a single candidate entered she would condition her position on the true state, as no other candidate is entering and it would strictly increase her utility. However, to condition her position on the true state is in contradiction with a belief equal to the prior.

(two candidates) No single tying candidate would sit at the position where she is, as she could win by getting closer to the other tying candidate. No candidate who is losing for sure when there is only another candidate entering and winning for sure would remain in the race, as exiting would not change the outcome of the election. Finally, if there are only two candidates entering who are tying sitting at a single position, then another candidate will enter and win the election. ■

When information is revealed (posterior is the true state) Whilst one could entertain the possibility that only equilibria where the belief is the prior are sensitive to the value of w relative to r, this is not the case. In particular, here I present the parameter values such that the equilibrium of example 3 collapses as w becomes small relative to r.

Example 6 *Assume F is uniform. The following is an equilibrium where candidates enter at two locations, one of them to lose for sure (see figure 6).*

$$Beliefs : \mu\left(s = w/e\right) = \begin{cases} 1 & \begin{array}{l} if\ |j : e_j = -w| \geqslant 2\ and\ |\{j : e_j = w + r\}| < 2,\ or \\ if\ |j : e_j = w + r| = 1\ and\ |\{j : e_j = -w\}| = 0, \end{array} \\ & \\ 0 & \begin{array}{l} if\ |j : e_j = w + r| \geqslant 2\ and\ |\{j : e_j = -w\}| < 2,\ or \\ if\ |j : e_j = -w| = 1\ and\ |\{j : e_j = w + r\}| = 0, \end{array} \\ & \\ \frac{1}{2} & otherwise. \end{cases}$$

$$Strategies \quad : \quad e = \begin{cases} \begin{array}{l} e_j = -w \\ e_k = w + m \end{array} & if\ \theta = w \\ & \\ \begin{array}{l} e_j = w + r \\ e_k = -w + m \end{array} & if\ \theta = -w \end{cases} \qquad for\ j = 1, 2,\ k = 1.$$

$$Parameters \quad : \quad \begin{cases} 4w > r > 2w \\ \\ (i)\ 2w + \min\{m, r - m\} > c \\ \\ (ii)\ 2w - \max\{m, r - m\} > b \\ \\ (iii)\ 2c \geqslant b. \end{cases}$$

In English, the beliefs are "if two or more candidates enter at $-w$ $(w + r)$ and less than two enter at $w + r$ $(-w)$, then the true state is w $(-w)$; if one candidate enters at $w + r$ $(-w)$ and none at $-w$ $(w + r)$, then w $(-w)$ is the true state; otherwise the belief is the prior". All candidates that enter have a preferred ideal point irrespective of the state equal to m (i.e. $w + m$ if the state is w, and $-w + m$ if the state is $-w$). Their actions are "if the state is w, two candidates enter at $-w$ to signal that w is the true state and a third candidate enters at $w + m$;

whilst if the state is $-w$, two candidates enter at $w+r$ to signal the true state is $-w$ and a third candidate enters at $-w+m$". The parameters of the example ensure that under each possible realization, none of the candidates signaling the true state wants to deviate and (i) not enter, or (ii) enter at a different position where he would win for sure, and (iii) no candidate wants to enter, even if he would not alter the belief of voters, but just to tie. All other incentives to deviate are ruled out as either implied by these two conditions, or automatically satisfied. When $r \geqslant 4w$, then a new candidate can enter and win the election implementing her preferred policy, even if it is close but different from the median, as when $r \geqslant 4w$ the two candidates who enter to signal the true state start to take away votes from the candidate at the median.

An equilibrium for any w and F The example above has shown that some equilibria where the true state is revealed are sensitive to the value of w relative to r. However, this is not true for all such equilibria. In particular, our last example is an equilibrium that does not depend on the value of w relative to r and is good for any distribution F. Part of the reason for its robustness is that the parameter restrictions imposed are the same as those imposed when w equalled zero and there was none or a single state. That is, I conclude by showing that the parameter values needed to ensure that an equilibrium exists with one or two candidates when $w = 0$ and there is none or a single state, are sufficient to ensure that an equilibrium exists when $w > 0$ and there are two states of the world. This is a fully revealing equilibrium with a candidate entering at the median consistent with the true state.

Example 7 *The following is a fully revealing equilibrium with a single candidate entering at* *the median consistent with the true state.*

$$Beliefs \ : \ \mu\left(s=w/e\right)=\begin{cases} 1 & \textit{if } |j:e_j=w+m|>|\{j:e_j=-w+m\}|, \\ 0 & \textit{if } |j:e_j=w+m|<|\{j:e_j=-w+m\}|, \\ \frac{1}{2} & \textit{otherwise.} \end{cases}$$

$$Strategies \ : \ e_j=\begin{cases} w+m & if\ \theta=w \\ -w+m & if\ \theta=-w \end{cases} \quad for\ j=1.$$

$$Parameters \ : \ \begin{cases} b-c>-d \\ 2c>b \end{cases}$$

Note that the parameter restrictions are the same as those in Proposition 3.

6 Conclusion

In this paper we have shown that when candidates and voters face an election with a state of the world that translates their ideal points, the strategies of voters are affected by the ability of candidates to commit or not to a policy ex-ante, if candidates are informed of the true state. With no commitment the strategies of voters are not affected by the information candidates have whatever is the realization of the state. Instead, when candidates can commit, we have shown that equilibria where candidates enter at one or two positions differ depending on whether there is a state of the world or not. This is a standard result of any signalling model.

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__Example 1__ (examples with 2w-r > 0)

candidates @ 1 location

(true state is not revealed)

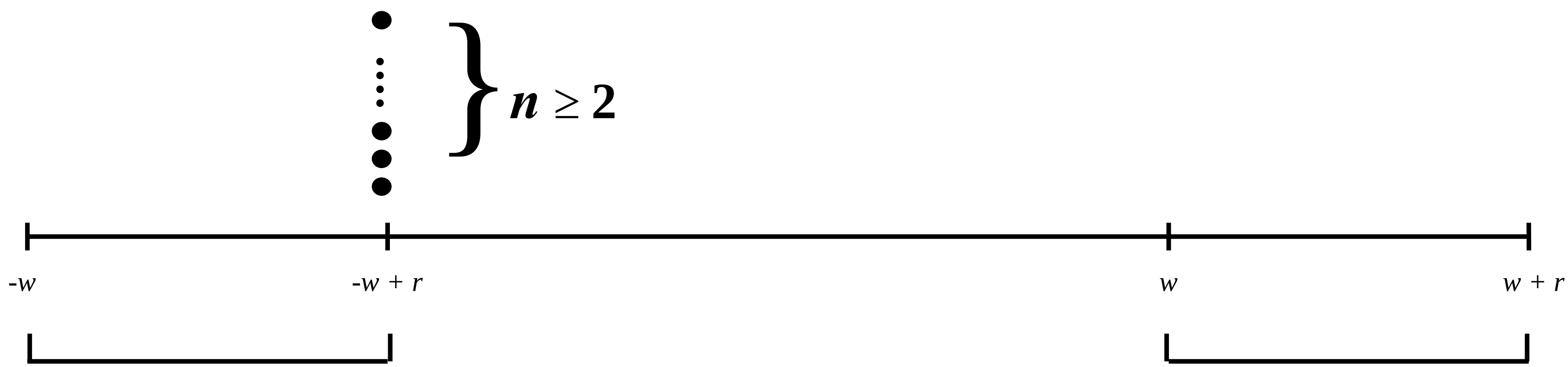

Figure 1

Example 2 (examples with $2w\text{-}r > 0$)

candidates @ 2 locations, all tying

(true state is not revealed)

$\boldsymbol{n \geq 4}$

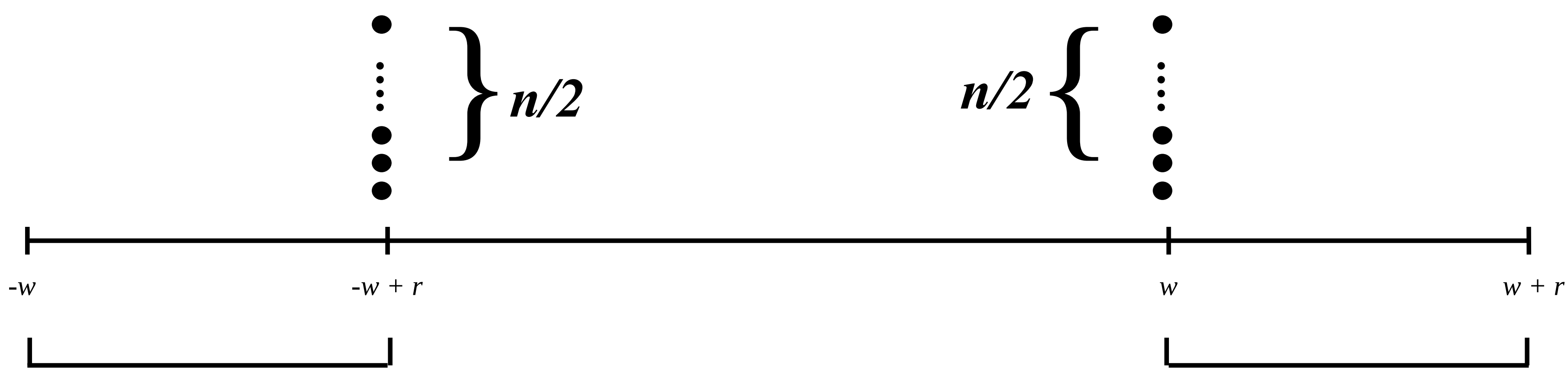

Figure 2

<u>Example 3</u> (examples with 2w-r > 0)

candidates @ 2 locations, 1 winning

(true state is revealed)

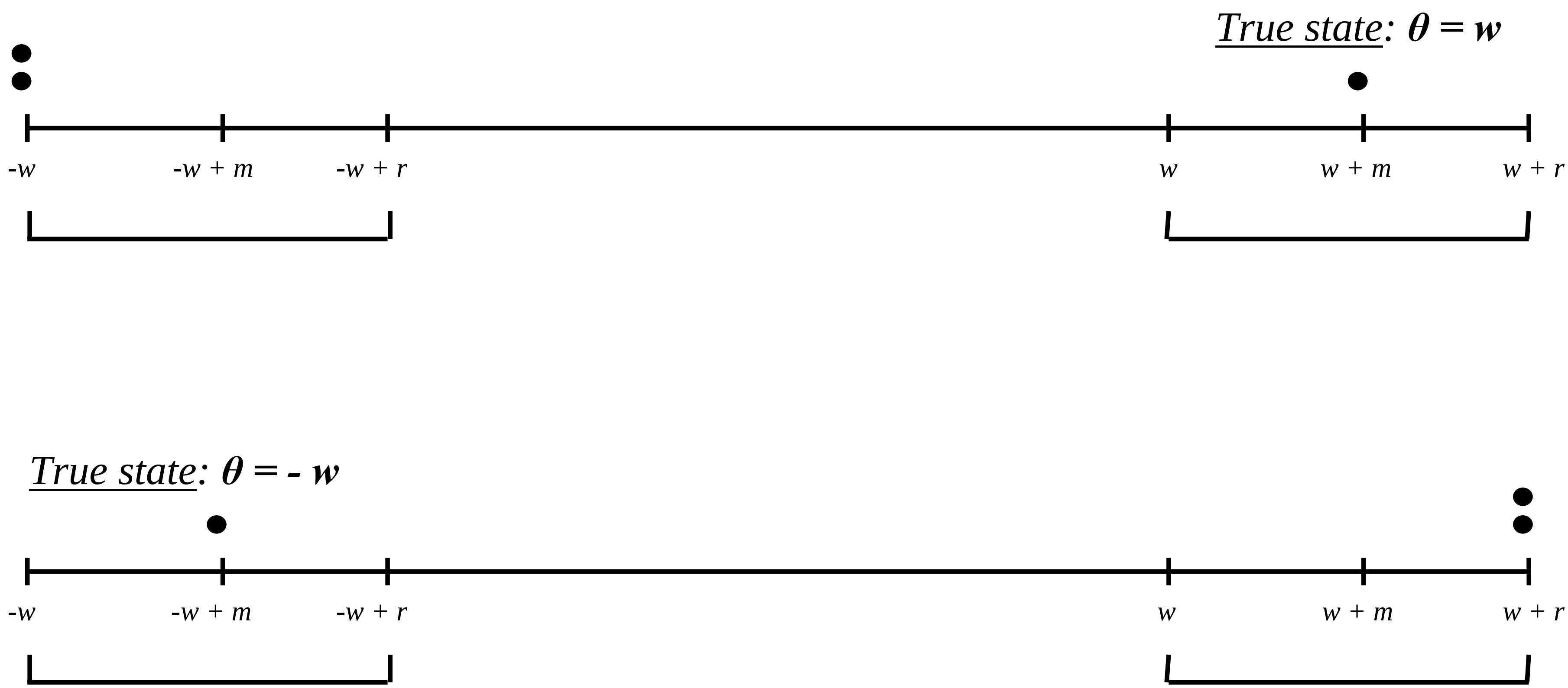

Figure 3

Example 4 *(examples with 2w-r < 0)*

2 candidates @ 1 location

(true state is not revealed)

Uniform distr. ***r = 6w*** **(≥ 2w)**

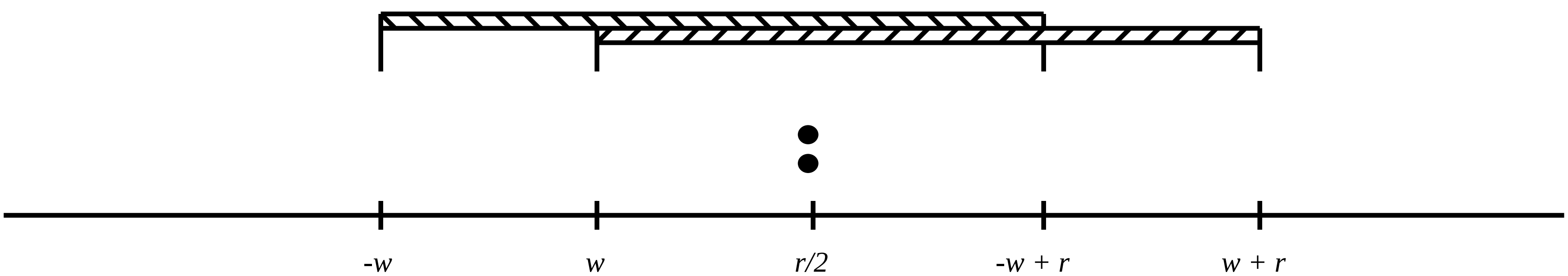

Figure 4

<u>Example 5</u> (examples with 2w-r < 0)

4 candidates @ 2 locations, all tying

(true state is not revealed)

Uniform distr. ***r = 30/7 w*** **(≥ 2w)**

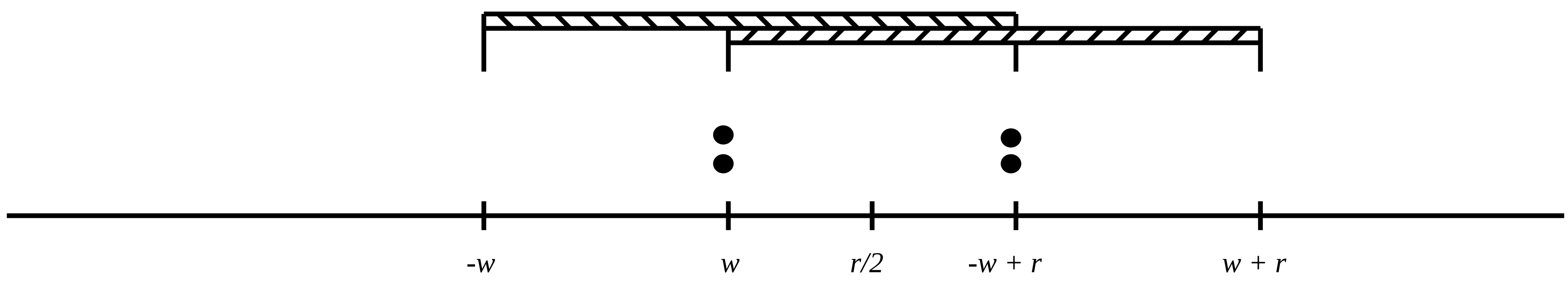

Figure 5

Example 6 (examples with $2w$-$r < 0$)

candidates @ 2 locations, 1 winning

(true state is revealed)

$\boxed{\textit{Uniform distr. } \boldsymbol{r \geq 4w} \ (\geq 2\mathbf{w})}$

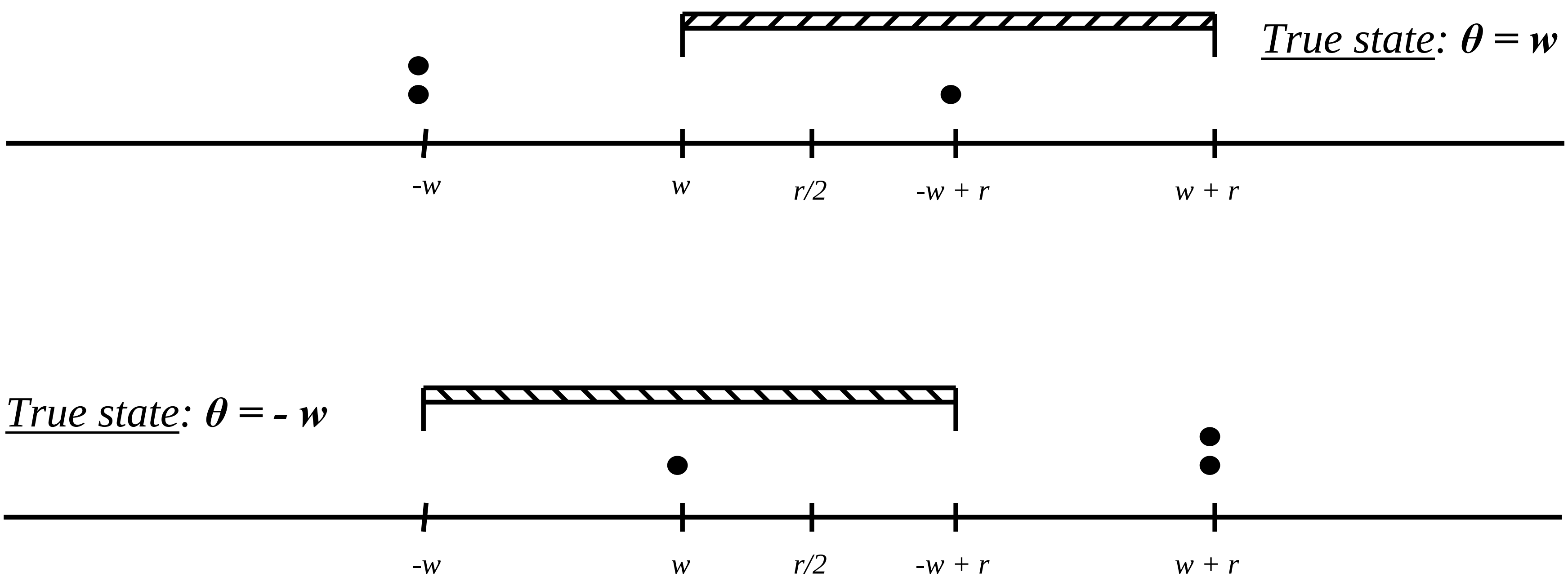

Figure 6