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DURABLE CAPITAL AND ECONOMIC GROWTH

BY

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1. INTRODUCTION

Capital and Time is the third book on the theory of capital, which Hicks has written in course of time.[1] The topic is difficult as well as controversial so that one needs a clear framework to build upon. Hicks has found such a framework in his reliance on the so-called Neo-Austrian method. The basic idea underlying this method is, that in certain periods primary factors of production are used as inputs, whereas outputs of final goods appear in principle later on, The difference with traditional Austrian theory is, that outputs are not solely realized in one period. In the Neo-Austrian approach the flow of outputs covers several periods, their number depending on the lifetime of the project. In other words, the traditional theory of Austrian origin is based on circulating capital; on the other hand Hicks studies the characteristics of fixed or durable capital.

In connection with this study by Hicks it seems appropriate to refer to the well-known distinction between pure and applied theory. Both aspects are covered in the book. In this review article we shall concentrate on the contributions to pure theory made by Hicks. With regard to the application of capital theory we restrict ourselves to a summing-up of a few topics: (1) measurement of the stock of capital; (2) problems of aggregation and time with regard to the concept of a macro-economic production function; (3) clarification of insights about the distribution of income. Hicks pays attention to all this in order to weaken the controversial character of the theory of capital. In addition, the author pays attention to the interpretation of the classics, especially to Ricardo and Mill.

In our opinion the effort of Hicks to clarify the theory of capital is successful. However, as a consequence of this, capital theory emerges more clearly as an incomplete theory in the sense that

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its possibilities to explain the facts of life are very limited indeed. That Hicks is aware of this aspect appears from several passages in the book and perhaps most clearly from the very last sentence of the main text: ‘A reminder that the Distribution of Income is not, in the short-run, a well-founded economic concept is perhaps not the least important point which has emerged from our inquiry.’ (page 184). In the opinion of Hicks the distribution is determined, in the long-run, by the well-known Cambridge savings function, which wilt be discussed in section 2 of this article. However, the long-run is conceived as a path of balanced growth (steady state), which serves at best as a reference path for more realistic developments. Precisely for this purpose the concept of a steady state is used by Hicks in his analysis of the Traverse, the movement of the system from an old technique towards a superior one. In order to explain the distribution of income in this case, the author has to rely on the assumption of full employment, which limits the scope of the analysis.

As said above, we shall pay attention to the theoretical side of Hicks’ work. In section 2 we shall discuss the method of analysis and the model of a steady state economy. Section 3 is devoted to the problems of the traverse from one steady state position to another caused by a technological change. So in this case we have to do with paths of non-balanced growth. It should be kept in mind that we do not give a complete review of the book. In particular we shall not go into the many generalisations and elaborations which are treated by the author. In addition, we want to emphasize that where our approach appears to be critical that detracts nothing from our appreciation. *Capital and Time* is in many respects a stimulating and seminal book.

2. BALANCED GROWTH

Hicks distinguishes three methods with regard to the way fixed capital is treated in economic literature. In the case of the so-called Method of Sectoral Disintegration there are two sectors, namely a consumption-goods industry and a sector for the production of capital-goods. Characteristic for this method is the simplistic manner, in which depreciation is introduced by means of a fixed percentage of the existing stocks. In contrast the *time aspect* of fixed capital is adequately taken into account in the method of Von Neumann.[2] In Von Neumann’s way of

[1] John Hicks, *Capital and Time*, A Neo-Austrian Theory, Oxford, 1973.
[2] For a more complete treatment along these lines see A. van Schaik, *Reproduktie en vast kapitaal* (doct. diss.), Tilburg, 1973. The present article draws heavily on the results of this study with regard to the analysis of linear models with joint production and fixed capital.

presentation machines appear at the end of each production period as joint products together with the original output. This implies that there are just as many types of machines of a certain kind (i.e. new, one-year old, two-years old, etc.) as there are years for the machine to live. As a result the time that elapses between the production of a new machine and its final elimination from production can be described with the aid of a chain of technical activities.

Hicks' objection against this manner of analysis is the great detail, which makes the economic interpretation more difficult. For this reason the author prefers a method in which the vertical integration of the production process stands central. In the case of a complete vertical integration there appear only inputs of primary factors (for instance labour) and outputs of final goods for consumption. Inputs and outputs are separated from each other with respect to time in such a way that as a general rule inputs are preceding outputs. So we have come to the Neo-Austrian model which is - as said before - favoured by Hicks.

It should be noted, that the distinction made between the three methods is not entirely functional for the following reasons. The distinction between sectors of production as such is not the same as the introduction of the phenomenon of fixed capital. What is more important, however, is that the method of the fixed depreciation rate and the Neo-Austrian method are particular cases of the joint-production method introduced by Von Neumann.[3] Besides these, there are of course more simple cases conceivable of the Von Neumann model. For this reason an appropriate choice is only possible after a thorough investigation of the characteristics of the model. We proceed as follows.

Suppose, a certain commodity can be produced with the aid of labour and machines. Machines have a (technical) life time of two production periods, so that one has to distinguish between new and old machines. As a consequence of this our commodity can be produced by new and old machines. There exist in this case two activities - or processes - but also two produced goods, viz. final products and old machines. The final products are *multiple purpose goods*, i.e. they are applicable as new machines and also as consumption goods. The 'catalogue of activities' can now be described with the aid of the vector of labour input coefficients (**a**), the matrix of input coefficients (**A**) and the matrix of output coefficients (**X**):

[3] See A. van Schaik, *op. cit.*, chapter III.

proces 1 proces 2

$\mathbf{a} = \begin{bmatrix} a & a \end{bmatrix}$ input of labour

$\mathbf{A} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$ input of new machines / input of old machines

$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ b & 0 \end{bmatrix}$ output of new machines and consumption goods / output of old machines

As indicated above, each column vector of the 'catalogue of activities' represents a process. One unit of the final product has been chosen as a norm of both vectors. Further it has been assumed that old and new machines have equal efficiency with regard to the production of final products. The consequences of this assumption are twofold. In both processes the final product can be produced with the aid of an equal amount of labour. Moreover old and new machines have the same (capital) productivity. So there is no need to make a distinction between the input coefficients of both types of machines. The possibility of a variable or economic life span is excluded by the assumption of equal efficiency. This will be explained later on. Moreover, old machines are mortal ('sudden-death' assumption). Consequently, old machines do not appear as an output in process 2.

In case of balanced growth (steady state) the *quantity system* of the model can now be formulated as follows:

$$\mathbf{Xy} = (1 + \mathrm{g})\mathbf{Ay} + \mathbf{c} \qquad (1)$$

In this equation **y** symbolizes the column vector of process levels (scale variables), g the net rate of growth and **c** the column vector of consumption goods.[4] If old machines are not consumed, the second element of the vector **c** equals zero. By writing out the equations of the quantity system according to the 'catalogue of activities' given above we get

$$y_1 + y_2 = (1 + g)by_1 + c_1 \qquad (2)$$

$$by_1 = (1 + g)by_2 \qquad (3)$$

[4] The difference between the gross and the net rate of growth will be explained later on.

The left-hand side of equation (2) symbolizes the production of our final product, whereas the right-hand side shows the demand for gross investment in new machines and consumption goods. Equation (3) relates the supply of old machines (LHS) to the demand for gross investment in old machines (RHS). Equations (2) and (3) can be solved for the net rate of growth g and the corresponding *ratio* of process levels if c_1 is known. If required, the absolute level of the scale variables can be determined by means of a given quantity of labour $(\bar{l})$:

$$ay_1 + ay_2 = l = \bar{l} \qquad (4)$$

It is also conceivable that g is fixed, for example in accordance with an exogenously determined rate of growth of the labour supply. In that case the process levels and the level of consumption can be calculated from the equations (2) through (4).

The dual or *price system* can be formalized in an analogous way as follows:

$$\mathbf{pX} = (1 + r)\mathbf{pA} + w\mathbf{a} \qquad (5)$$

The symbol **p** indicates the row vector of prices, whereas r stands for the net rate of profit and w for the (nominal) wage rate. According to the 'catalogue of activities' given above we get by matrix multiplication from formula (5):

$$\begin{aligned} p_1 + p_2 b &= (1 + r)p_1 b + wa \\ p_1 &= (1 + r)p_2 b + wa \end{aligned} \qquad (6)$$

Both equations can better be understood by re-writing them in the following way:

$$(p_1 - p_2)b + rp_1 b + wa = p_1 \qquad (6)'$$

$$(p_2 - 0)b + rp_2 b + wa = p_1 \qquad (7)'$$

The three terms on the left hand side of both equations symbolize consecutively depreciation, net profit and (direct) labour cost (all measured per unit of final product). Relative prices can be found from (6) and (7) by fixing the net rate of profit r. In addition to this absolute prices can be determined by specifying a *numéraire*. Another possibility for solving (6) and (7) is the fixation of a real wage rate, which leads to certain results for r and the price ratio. The price of old machines in terms of final products can be calculated as $(1 + r)/(2 + r)$. From this result we learn that the relative price cannot become negative for non-negative values of the net rate of profit r. That is the reason why an economic life span of one year is not possible in this case. Old machines will always be used, because they can yield the same net rate of profit as new machines.[5]

By eliminating the term p_2b the price system can be reduced to the form:

$$\frac{(1+r)^2}{2+r} p_1 b + wa = p_1 \tag{8}$$

The expression in r is an annuity, which can be interpreted as the gross rate of profit, viz. the sum of the net rate of profit r and the rate of depreciation $\delta = 1/(2 + r)$. Equation (8) can also be looked upon as the formula for the wage-profit relationship of the technique under discussion.[6]

A similar reduction of the quantity system is possible also, but appears to be more complicated. Final products are produced in both processes, so their total volume equals:

$$x_1 = y_1 + y_2 \tag{9}$$

Substitution of (3) into (9) results in:[7]

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[5] The life time of equipment may depend on economic factors, provided there is unequal efficiency which takes the form of higher operating costs of old machinery. See Th. van de Klundert and A. van Schaik, 'On Shift and Share of Durable Capital,' *Research memorandum* 46, Tilburg Institute of Economics, January 1974.

[6] From the wage-profit relationship

$$\frac{w}{p_1} = \frac{1 - \frac{(1+r)^2}{2+r} b}{a}$$

we can see that the real wage is positive if the net rate of profit is fixed in such a manner that

$$\frac{2+r}{(1+r)^2} > b.$$

Moreover a positive r is possible if $b < 2$. This condition is generalised in A. van Schaik, *op.cit.*, chapter III.

[7] The share of process 1 (i.e. the contribution of new machines) in the total production of final products is larger than ½ if $g > 0$ and, as g increases, approaches the number 1 asymptotically.

$$x_1 = \frac{2+g}{1+g} y_1 \tag{10}$$

Taking into account (9) and (10) equation (2) changes into:

$$\frac{(1+g)^2}{2+g} bx_1 + c_1 = x_1 \tag{11}$$

Furthermore from (4), (9) and (11) can be derived:

$$\frac{(1+g)^2}{2+g} abx_1 + ac_1 = l \tag{12}$$

The term in g is an annuity, which can be interpreted in this case as a gross rate of growth, i.e. the combined effect of a net rate of growth g and a rate of replacement $\overline{\delta} = 1/(2 + g)$. Formula (12) can be seen as an expression of the consumption-growth relation of the Von Neumann model as specified by us.

It appears that the solutions of the Neo-Austrian model are similar to the reduced form equations of the Von Neumann model. The only difference relates to the assumption made by Hicks, in accordance with the Austrian tradition, that capital goods are produced by labour only. For this reason capital goods are called goods of higher order in Austrian Theory. If the production of capital goods is different from that of consumption goods the ‘catalogue of activities’ must be expanded. In addition a distinction must be made between the output of new machines and the output of consumer goods. If one assumes that new machines are produced solely by labour the ‘catalogue of activities’ can be written as:

	process 1	process 2	process 3	
a = [	a'	a	a]	input of labour
	0	0	0	input of consumption goods
A = [	0	b	0]	input of new machines
	0	0	b	input of old machines
	0	1	1	output of consumption goods
X = [	b	0	0]	output of new machines
	0	b	0	output of old machines

Again, each column vector of the 'catalogue of activities' represents a process. Process 1 describes the possibility to produce new machines. It can be called the capital-goods industry. The processes 2 and 3 produce the same consumption good. It should be noted, that an industry or sector can be defined here as the set of those processes producing the same final product. So the set of processes 2 and 3 can be called the consumption-goods sector. In this sector the final product (consumption goods) can be produced with the aid of labour and machines. Machines have a (technical) life span of two production periods. Further it has been assumed that old and new machines have equal efficiency with regard to the production of consumption goods. The possibility of a variable or economic life span is excluded by this assumption. Moreover we used the 'sudden-death' assumption again. In the capital-goods industry the labour coefficient is different from that in the consumption-goods sector ($a' \neq a$).

Applying the equations (1) and (5) to the technology formulated above we get the following model.

quantity system

$$y_2 + y_3 = c_1 \tag{13}$$

$$by_1 = (1 + g)by_2 \tag{14}$$

$$by_2 = (1 + g)by_3 \tag{15}$$

price system

$$p_2b = wa' \tag{16}$$

$$p_1 + p_3b = wa + (1 + r)p_2b \tag{17}$$

$$p_1 \quad = wa + (1 + r)p_3 b \tag{18}$$

According to the Austrian way of analysis it has to be assumed that new and old machines are not consumed ($c_2 = c_3 = 0$). Further, the quantity system has to be completed with the relation that specifies the demand for labour (l). This equation can now be written as

$$a'y_1 + ay_2 + ay_3 = l$$

The price system of the Neo-Austrian model can easily be reduced to:

$$\frac{(1+r)^2}{2+r} wa' + wa = p_1 \tag{19}$$

The reduction of the quantity system is in this case only possible by substituting the solutions for the process levels in terms of c_1 into the demand equation of labour. Consumption goods and machines are now different commodities, but for the production of both commodities the homogeneous factor labour is needed. Accordingly, for the reduced form of the quantity system the following expression can be found:

$$\frac{(1+g)^2}{2+g} a' c_1 + ac_1 = l$$

The relations (19) and (20) show up in the mathematical appendix of Hicks' book, *viz.* with the numbers (6.1) and (6.2). It concerns here the so-called simple profile, preferred by Hicks when problems of a somewhat complicated nature have to be analysed. For the sake of convenience we have assumed in contrast to Hicks that the (technical) life time (n) of machines is two years. For $n > 2$ the relations (19) and (20) remain essentially unchanged.[8] Further it appears from a comparison

[8] The general notation of the annuities in the relations (19) and (20) is:

$$(\delta + r) = \frac{r}{(1+r)^n - 1} + r = \frac{r(1+r)^n}{(1+r)^n - 1} \text{ (=gross rate of profit)}$$

of the relations (19) with (8) and of (20) with (12), that the introduction of a separate process for the production of machines brings about only a few changes. The coefficient *b* disappears from the reduced form equations, because machines have their own unit of measurement. The reduced price equation (8) contains the term p_1b, which symbolizes the value of the capital stock in the production of x_1, over which depreciations and profits are to be calculated. In equation (19) this term has been replaced by *wa*', being the cost price of one machine needed for the production of one unit of c_1. In this case too depreciations and profits are calculated over the value of capital. The reduced forms of the quantity systems can be compared in an analogous way. The term abx_1 in (12) reflects the volume of labour, required for the production of the physical capital stock bx_1.[9] Multiplication of this term with the gross rate of growth gives the amount of labour, which is necessary for the production of capital goods. Adding the amount of labour used in the production of consumption goods, results in the total demand for labour, as described by means of formula (12). Formula (20) can be explained in the same way, on the understanding that $a'c_1$ units of labour are used for the production of the physical capital stock of the corresponding case.[10]

It may be concluded that the Neo-Austrian model -- as represented by Hicks - does *not* differ in essence from the Von Neumann model. The distinction in the production-technical sense between a consumption-goods sector on the one side and a capital-goods sector on the other hand con easily be built into the Von Neumann model, but why should this be done? ‘This admits just two kinds of “firms”: those which make capital goods, now identified as “machines”, and those which use them. The accounting distinction between Consumption and Investment is converted into an industrial division. But the accounting division is not an industrial division,’ as Hicks observes, discussing the above mentioned Method of Sectoral Disintegration (page 5). This statement seems also applicable to the Neo-Austrian model. However, it should be noted, that the distinction

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and

$$\left(\bar{\delta}+g\right)=\frac{g}{(1+g)^n-1}+g=\frac{g(1+g)^n}{(1+g)^n-1} \text{ (= gross rate of growth)}$$

In these expressions the symbols δ and $\bar{\delta}$ should be considered respectively as the rates of depreciation and replacement. For a derivation of these formulas from linear models with fixed capital, see A. van Schaik, *op.cit*., chapter III.

[9] According to (9) we have: $bx_1 = by_1 + by_2$. It appears from matrix **A** of the Von Neumann model that by_1 is equal to the stock of new machines, whereas by_2 represents the stock of old machines in the production process.

[10] The psychical capital stock is equal to: $by_2 + by_3 = bc_1$. In order to produce *b* units of new machines a quantity a' of labour is required. Consequently the total stock can be produced with $(a'/b)bc_1 = a'c_1$ units of labour.

between both sectors renders the author good services in his analysis of non-balanced growth paths, as wilt be made clear in section 3.

As a result of the above comparison of models the question arises whether the outlined reduction procedure is always possible. The answer to this question is straightforward. If the complete model can be written in the form of equalities, the reduction can be carried through without problem. If we are forced on the other hand to introduce inequalities into the quantity system as well as into the price system, reduction becomes problematic. In principle inequalities should be introduced in two cases. A first possibility is that old machines are consumed as well. To give an example of this case we can use the Von Neumann model and put $c_2 > 0$. The quantity system can be written as follows:

$$(1 + g)by_1 + c_1 \leq y_1 + y_2 \qquad (2)'$$

$$(1 + g)by_2 + c_2 \leq by_1 \qquad (3)'$$

Given the net rate of growth and making use of (4) to determine the level of the scale variables, this quantity system is not solvable before we know the ratio between c_1 and c_2. For instance, if this ratio is equal to

$$\frac{c_2}{c_1} = \frac{b}{1-(1+g)b} > 0$$

the solution for y_1 is positive whereas y_2 equals zero.[11] This result can be explained by the fact that old machines are used as consumption goods in such an amount that there are no old machines left as capital goods. In this case process 2 has to be eliminated from the production system. Once this has been done the model can be written again in the form of equalities.[12]

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[11] This can be seen by calculating y_1 and y_2 from the quantity system

$$\mathbf{y} = [\mathbf{X} = (1 + g)\mathbf{A}]^{-1}\mathbf{c}$$

as

$$y_1 = \frac{\frac{1+g}{2+g}c_1 + \frac{1}{(2+g)b}c_2}{1-\frac{(1+g)^2}{2+g}b} \quad \text{and} \quad y_2 = \frac{\frac{1}{2+g}c_1 + \frac{1-(1+g)b}{-(2+g)b}c_2}{1-\frac{(1+g)^2}{2+g}}$$

and substituting the ratio between c_2 and c_1 as proposed above into these results.

[12] This example can be used to illustrate the restricted validity of the labour theory of value and the non-substitution theorem in case of the presence of joint production possibilities.

A second possibility is that old machines are less efficient than new ones in the production of final products (the input coefficients of both processes in the Von Neumann model are not equal in this case). In the latter case it is possible that old machines shift from one sector to the other for economic reasons. Hicks is aware of such complications, as the following quotations from his first chapter demonstrate: ‘There are in fact two ways in which an actual economy can react to technical change. It may on the one hand, transfer appliances, which were acquired in the past to take part in production on one technique ... applying them, as best it can, to serve as instruments for a purpose for which they were net designed. There is no doubt that this happens; and quite often the transfer is fairly easy ... Alternatively, the funds which would have been used for replacement of capital goods of the old sort, or for investment in such capital goods, may be transferred to finance the production of capital goods of the new kind. There is again no question that this happens.’ (page 11). Which kind of adjustment is the more important should be decided by means of an appeal to empirical facts. Hence, Hicks concludes his first chapter with an apology: ‘So it is unwise to commit ourselves, finally, to the one route or the other. I may well be felt to have committed myself, in Part II, too firmly to the latter route.’ (pages 11/12).

In the above-mentioned analysis the quantity system has been treated apart from the price system. Both systems are connected through the equilibrium condition, implying that savings and investments are equal. The investments result from the quantity system, whereas savings also depend on the price system. In his discussion of the steady state Hicks introduces the well-known savings function, whereby savings are a fixed proportion of capital income. The author writes the equilibrium condition in that case as $r = g/s$, whereby s symbolizes the savings-ratio of net capital income or profit. However, this condition is not quite correct if durable capital is taken into account. In this case the proper net capital income per unit is not equal to r, because the depreciation rate $\delta = 1/(2 + r)$ (assuming $n = 2$) diverges from the replacement rate $\bar{\delta} = 1/(2 + g)$. These rates are only equal in case $r = g$. In the more general case the equilibrium condition can be written as:

$$s_b(\delta + r) = (\bar{\delta} + g).$$

It is thus assumed, that a constant proportion (s_b) of gross capital income is saved.[13] Gross savings are indiscriminately used to finance both net investment and replacement investment. Herewith the model is closed.

With regard to the solution of the model Hicks makes a distinction between the Fixwage assumption and the Full Employment assumption. In the first case the real wage is given (elastic supply of labour), so that the net rate of profit r can immediately be determined on the basis of the relevant technique. The net rate of growth then results from the equilibrium relation. In the other case the net rate of growth is given. The net rate of profit is then determined by means of the equilibrium condition, after which the real wage can be found on the basis of the given technique.

It is of course possible that more techniques (i.e. more processes than goods) are available to produce final commodities. In such a case the choice of technique depends on the net rate of profit c.q. on the real wage in the steady state. Discussing the technological possibilities Hicks also pays attention to the well known problem of the *reswitching* of techniques. Reswitching is excluded if the time-profile, i.e. the number of processes of the capital-goods sector and the number of processes forming the consumption-goods industry, of each technique is the same. This result holds for both models treated in this article. In case of a uniform life time of capital goods the gross rate of profit, as presented in equations (8) and (19), can be used as a measure for capital cost in comparing different techniques. The relation between the real wage and the gross rate of profit which can be constructed on basis of formula (8) is linear. Reswitching is therefore impossible. The corresponding relationship on the basis of formula (19) is hyperbolical. However, it is easily shown that also in this case there exists only one switching point.[14]

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[13] It should be noted, that Hicks' condition $r = g/s$ *implies* a certain gross savings-ratio in the equilibrium situation. This gross savings-ratio can be found by substituting the above condition into the equilibrium relation as formulated by us:

$$s_b = \frac{\dfrac{(1+g)^2}{2+g}}{\dfrac{(1+g/s)^2}{2+g/s}}.$$

Given. the net rate of growth g and the ratio s the gross savings ratio s_b can be determined with the aid of this formula. It is evident that the reverse way can also be taken, *viz.* by fixing s_b to deduce the corresponding s. However, this does not alter the fact that the condition as formulated by Hicks is subject to criticism.

[14] From (19) it can be seen that in a switching point the following equality must hold:

$$\frac{1}{a + a'(\delta + r)} = \frac{1}{\bar{a} + \bar{a}(\delta + r)} \therefore \delta + r = \frac{a - \bar{a}}{\bar{a} - a'}$$

The coefficients a and a' belong to the same technique. The same is true for the coefficients $\bar{a}$ and $\bar{a}'$, which characterize the other technique. The same result is obtained with the Method of Sectoral Disintegration if it is assumed that no capital is used for the production of capital goods. The net rate of profit in a switching point then follows from:

As observed by Hicks this result can be formulated in another way. If a technique is determined by two parameters reswitching cannot occur. If on the other hand more than two parameters are used to characterize a technique reswitching of techniques is possible. The realisation of this possibility depends on the nature of the technological differences. Hicks illustrates this conclusion with the aid of his Neo-Austrian model, whereby he assumes that machines have an infinite life time. Techniques are different with regard to the gestation period of capital goods, the labour required to produce these goods and the labour required to produce consumption goods. In this case reswitching can only occur if the technique with the longest production period of capital goods is using more labour in the production of consumption goods (and of course less labour in the production of capital goods). Hicks comments upon this result as follows: 'This is no more than an illustration; but it is highly suggestive. Reswitching, in itself, is no more than a curiosum.' (page 44). This does not take away the fact that in economic literature the possibility of reswitching of techniques has been demonstrated for a great number of models. However, there is no need to pay so much attention to this phenomenon, if problems of technical change are put in the forefront, as is done by Hicks. The possibility of reswitching belongs to the problems of substitution in case of a given technology.

3. NON-BALANCED GROWTH

Leaving the steady state the variables of the model must be given time indices. The quantity system can then be written in genera) as

$$\mathbf{A}\mathbf{y}(t + 1) = \mathbf{X}\mathbf{y}(t) - \mathbf{c}(t) \qquad (1a)$$

This equation expresses the fact that the capital stock in period (t + 1) equals gross investment in new and old machines in period (t). Gross investment is obtained by subtracting consumption from

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$$\frac{1}{\alpha_x + \alpha_k r} = \frac{1}{\overline{\alpha}_x + \overline{\alpha}_k r} \therefore r = \frac{\alpha_x - \overline{\alpha}}{\overline{\alpha}_k - \alpha_k}$$

The symbols α_x and α_k stand for respectively labour input in the consumption-goods sector and labour input in the capital-goods sector. It is assumed that capital goods are of infinite durability. *Cf.* Th. van de Klundert, 'Produktie, kapitaal en interest', *De Economist*, CXVIII (1970), pp. 563-588.

output. Of course, it should be remembered that the output component contains machines, which have become one year older in the meantime.

Applying formula (la) to the specification of the Neo-Austrian model, the following equations are obtained:

$$c_1(t) = y_2(t) + y_3(t) \quad (13a)$$

$$y_2(t + 1) = y_1(t) \quad (14a)$$

$$y_3(t + 1) = y_2(t) \quad (15a)$$

The dynamic system formed by the equations (13a) through (15a) describes the paths of consumption and old capital goods when $y_1(t)$ is known. Thee scale variable y_1, indicating the number of new machines, can be determined in various ways.

In studying the time path of the variable y_1 Hicks maintains the distinction between the Fixwage assumption and the Full Employment assumption. In the first case, as we have already seen, growth depends on savings out of capital income. It is assumed that wage-earners consume their whole income. Hicks compares the path of non-balanced growth with the steady state. The latter will be followed if no disturbance appears. Next the author assumes that consumption out of capital income is the same on both paths. It follows that the evolution of $y_1(t)$ is now determined, as wilt be demonstrated below.

With the real wage given, the flow of returns for capital-owners in terms of consumption goods is:

$q'y_1(t) = (- wa')y_1(t)$ (process 1)

$qy_2(t) = (1 - wa)y_2(t)$ (process 2)

$qy_3(t) = (1 - wa)y_3(t)$ (process 3)

If it is assumed now, that capital-owners do neither consume at all in the steady state nor in the situation of non-balanced growth and that savings equal investment, the following equation is valid:

$$q'y_1(t) + qy_2(t) + qy_3(t) = 0 \quad (21)$$

The first term of (21) represents the consumption of wage-earners in the capital-goods sector, which is produced by the set of processes forming the consumption-goods industry.[15] Substitution of (14a) and (15a) into (21) gives:

$$q'y_1(t) + qy_1(t - 1) + qy_1(t - 2) = 0 \qquad (22)$$

The characteristic roots of this difference equation are equal to:

$$\lambda_1 = \frac{-q - \sqrt{q^2 - 4qq'}}{2q'} \text{ and } \lambda_2 = \frac{-q + \sqrt{q^2 - 4qq'}}{2q'}$$

Since q' is negative and q is positive, we can conclude $\lambda_1 > 0$ and $\lambda_2 < 0$. At the same time λ_1 proves to be dominant. Consequently the scale variable y_1 converges in the long run to the net rate of growth $\lambda_1 - 1$.

Given the assumptions that capital-owners do not consume and that workers do not save the net rate of growth equals r, i.e. the net rate of profit in the steady state. Therefore we can write $\lambda_1 = 1 + r$. This equality can be shown in a formal way as follows. From (16) through (18) and $p_1 = 1$ it can be deduced:

$$q' + \frac{q}{1+r} + \frac{q}{(1+r)^2} = 0 \qquad (19a)$$

Multiplying all terms by $(1 + r)^2$ this yields a polynomial of degree two in $(1 + r)$, which is exactly similar to the characteristic equation of (22). With this the statement $\lambda_1 = 1 + r$ is proved!

So far the nature of the disturbance of the steady state was not under discussion. In his book Hicks analyses disturbances caused by the introduction of a new and better technique. When a superior technique with lower input coefficients becomes available, profit maximizing entrepreneurs will exclusively buy new machines, which embody the new technique. This implies that the path of balanced growth is no longer maintained. What is happening in the long run is described by equation (22) on the understanding that q' and q relate to the *new* technique.

[15] As said above, it is assumed that wage-earners do not save.

In the above analysis it has been assumed, that capital-owners do not consume. In contrast with this Hicks discusses the situation, in which the (positive) consumption of capital-owners is equal to their absolute level of consumption on the reference path. From a mathematical point of view this case is more general, but in an economic sense it is more particular. As a result of this assumption the growth rate of y_1 converges to the net rate of profit (r) of the new technique. However, the new rate of equilibrium growth is now approached *asymptotically*.[16] The consumption of capital-owners is positive, but is becoming less important as time expires. In fact, the *additional* capital income in comparison with the reference path is entirely invested. in each period.

It should be noted furthermore that equation (22) describes the evolution of y_1 after old machines have disappeared from the stage. Hicks calls this the Late Phase to distinguish it from the events in the Preparatory Phase and the Early Phase. In the Preparatory Phase new machines are constructed according to the latest technique, but not yet used. On the other hand machines embodying the newest technique are both constructed and used in the Early Phase; but there still are old machines in use too. At the moment that all old machines have disappeared the Late Phase sets in.[17] The complete development must therefore be studied on the basis of a three-stage dynamic construction. This will be illustrated in more detail in the course of our exposition.

However, first we want to drop the Fixwage assumption. As a matter of fact we do not deny that this assumption has its merits if it concerns problems of less developed countries or the interpretation of Ricardo, as Hicks shows on page 49 of his study. But since we are more interested in the problems of growth in an economy confronted with a limited supply of labour, it seems desirable to reserve the remaining space for the analysis of non-balanced growth in case of labour scarcity.

Full employment - as assumed by Hicks - implies the following relationship

$$a'y_1(t) + ay_2(t) + ay_3(t) = \bar{l}(t) = \bar{l}(0)(1+g)^t \tag{23}$$

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[16] *Cf.* Hicks, *op.cit.*, pages 95 and 191. The solution can be written as:

$$y_1(t) = \bar{y}_1(1+g)^t + \eta_t(1+r)^t,$$

with $r > g$ and η_t converging to a constant.

[17] Hicks assumes the life time of machines to be equal to n, so that the Late Phase must be analysed with the help of a difference equation of order $n + 1$.

In this equation g represents the given rate of growth of the supply of labour. The amount of labour in period 0 can be put equal to one [$\bar{l}$ (0) = l]. Taking account of (14a) and (15a) equation (23) passes into:

$$a' y_1(t) + a y_1(t - 1) + a y_1(t - 2) = (1 + g)^t \qquad (24)$$

The characteristic equation of (24) has the solution:

$$\lambda_1 = \frac{-a + \sqrt{a^2 - 4aa'}}{2a'} \text{ and } \lambda_2 = \frac{-a - \sqrt{a^2 - 4aa'}}{2a'}$$

The roots are complex for $a < 4a'$. In this case the modulus equals $\sqrt{a/a'}$. Consequently the solution of (24) is stable for $a' > a$. The path to the new equilibrium situation with rate of growth g is then characterised in the Late Phase by an oscillating movement.[18] In the Preparatory Phase and the Early Phase (24) must be replaced respectively by

$$a' y_1(t) = \bar{a}' y_1(0)\,(1 + g)^t \qquad (25)$$

$$a' y_1(t) + a y_1(t-1) = \left(\bar{a}' + \frac{\bar{a}}{1+g}\right) y_1(0)(1+g)^t \qquad (26)$$

The barred coefficients refer to the old technique. In the Preparatory Phase the process levels $y_2(t)$ and $y_3(t)$ - appearing in equation (23) - are equal to their values in the steady state before the introduction of a new technique. The name holds for $y_3(t)$ in the Early Phase.

The adjustment process - which starts after a disturbance of the steady state by the introduction of a superior technique - can be analysed fully on the basis of the formulas (24) through (26). In his analysis Hicks makes a distinction between the various possibilities with regard to the bias of technical change. In the case of neutral technical progress both labour input

[18] According to his own statement Hicks is not in a position to describe in detail the path of adjustment in the Late Phase. The reason for this is that he discusses the more general case of a technical life time of machines of more than two years.

coefficients (a' and a) are proportionally lower in the new technique. As is easily imaginable, in this case the process of adjustment is already finished in the Early Phase. For, saving of labour in the Preparatory Phase leads to a larger production of new machines to such an extent that the decline in employment in the Early Phase is exactly compensated for. So, only two periods are required to complete the adjustment to a new steady state.

This is different in cases of non-neutral technical progress. Here a distinction can be made - following Hicks - between a backward bias and a forward bias. In the first case the relative decline of the labour input coefficient in the capital-goods sector is greater than that of the labour input coefficient in the consumption-goods industry, whereas the forward bias shows the reverse picture.[19] In both cases the adjustment process has not yet been completed after the Early Phase so that the dynamic characteristics of the Late Phase become effective.

That the adjustment process needs a longer time will be explained for the case of a forward bias. The additional machines from the Preparatory Phase are now insufficient in number to absorb in the Early Phase the amount of labour saved in the machine-using industry or consumption-goods sector. In Hicks' terminology there is not enough utilization employment. This means that labour must be re-allocated from the consumption-goods sector to the capital-goods sector (machine-constructing industry). In the next period employment in the machine-using industry is decreasing further, but there is now also a compensating effect in consequence of the re-allocation of labour in the preceding period. However, this compensation is insufficient, if $a' > a$.[20] In the meantime we have arrived at the beginning of the Late Phase, of which the characteristics have been investigated above. The starting position in the Late Phase shows a relatively too large amount of labour in the capital-goods sector. As already explained the adjustment process becomes cyclical from then on.

From the solution of $y_1(t)$ the time path of all other variables can of course easily be found. One of those variables is the consumption per head (c_1/l) which equals the real wage under the condition that wage-earners do not save and capital-owners do not consume. The development of c_1/l through time has been illustrated in Figure 1. The diagram relates to a numerical example with the following data:

[19] In addition, the author recognizes the possibility that one of the labour input coefficients increases while the other one decreases proportionally more. We will pay no attention to these cases of so-called strong biases. The same must be said with respect to changes, in which the technical life time of machines is at stake.
[20] For a proof of this proposition see Hicks, *op.cit.*, pages 104-105.

$y_1(0) = 12.1$	$y_2(0) = 11.0$	$y_3(0) = 10.0$	
$\bar{l}\,(0) = 90.4$	$g = 0.1$		
$\bar{a}' = 4.0$	$\bar{a} = 2.0$	$a' = 3.8$	$a = 1.8$

It is assumed that the new technique is introduced in period 1.

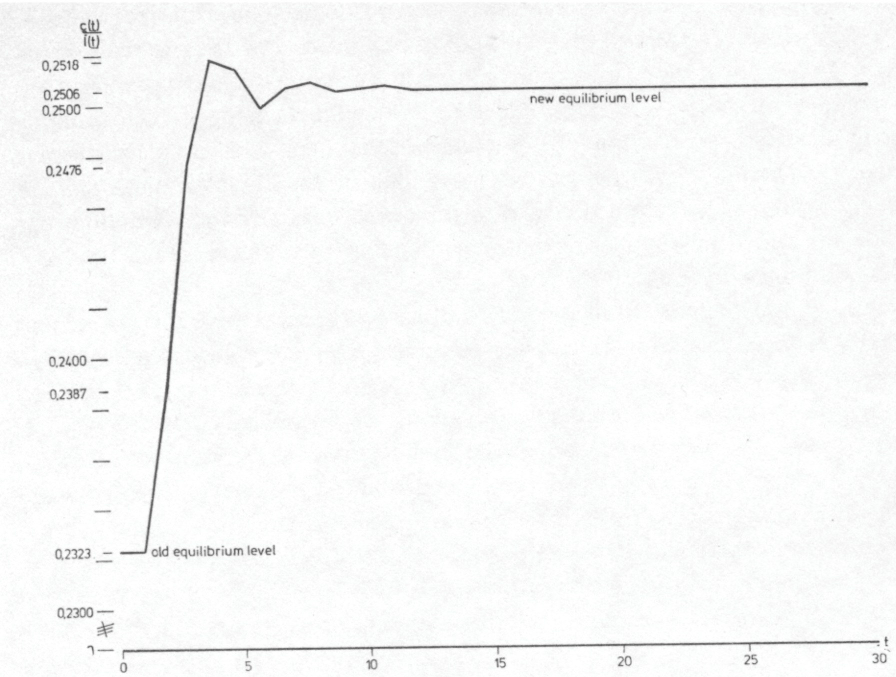

Figure 1

As can be seen from the figure, consumption per head increases considerably in the Preparatory Phase (period 1). This rise continues in the Early Phase (period 2). In the beginning of the Late Phase consumption per head lies under the new equilibrium level. This result is in accordance with the conclusion, that after the Early Phase there is relatively too much constructional employment and too less utilization employment. The diagram also illustrates the cyclical path of the adjustment process in the Late Phase, that comes to an end in period 11.[21]

Building upon the analysis of the Full Employment path with only one new and superior technique, Hicks discusses the possibility of substitution. This can only happen if a whole spectrum of new techniques becomes available. The corresponding wage-profit curves must intersect. When

[21] It should be noted, furthermore, that consumption per head temporarily surpasses the new equilibrium level. Hicks did not notice this aspect, because he was not in a position to analyse in detail the time path in the Late Phase (see his Fig. 12 on page 102).

in the course of the adjustment process the real wage rate then changes, it may be profitable to switch to another technique. However, there is a fundamental problem here, which is formulated by Hicks as follows: ‘Since each individual process extends over time, the choice should in general depend on expected wages as well as on current wages’ (page 110). Although aware of their unrealistic nature the author decides in favour of static expectations. This implies that entrepreneurs expect the ruling wage rate to remain unchanged. The resulting path is not an optimal one. ‘But in positive economics it has its place; there is no simple assumption which throws more light on the kinds of things that are likely to happen.’ (page 110). This is of course a legitimate assumption, hut also one which highlights the static character of the analysis along the Neo-Austrian Times.

Full employment and full utilization of capital are assumed, but we are not told how these states wilt be reached when they are initially disturbed. A positive theory of a dynamic economy must be based upon behavioural equations. The price to be paid for this is not only increasing complexity, but also greater freedom of choice with regard to the specification of the relevant relationships.[22] Consequently one has to resort to simulation and econometric analysis. The question then arises whether it makes still sense to deal with such specific problems as for instance the age distribution of the capital stock and its evolution in the course of time. In our opinion this is advisable only to a certain extent. In general equilibrium analysis disaggregration is inevitable. However, in analysing dynamic features it seems more appropriate to work with aggregaten. If possible, important results derived from general equilibrium models - for instance with regard to the durability of capital - should be incorporated in an adequate way. It remains to be seen where Neo-Austrian theory fits into this picture.

Summary

DURABLE CAPITAL AND ECONOMIC CROWTH

In *Capital and Time* Hicks explores Neo-Austrian capital theory. It is shown here that this approach is a special case of the more general method of treating fixed capital as introduced by Von Neumann. In the Neo-Austrian model the path of balanced growth is analysed with reduced form equations. The second part of this article discusses paths of non-balanced growth. If the steady state

[22] An example of such a specification in the framework of the Von Neumann model is given in A. van Schaik, *op.cit*., appendix IV-a.

is disturbed by the introduction of a superior technique Hicks needs a three-stage dynamic model for a full description of subsequent developments. Special attention is given to the Full Employment path.