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# THE RATE OF PROFIT IN A SPECIAL CASE OF JOINT PRODUCTION

Estratto da

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1. In the sixties consensus has emerged, that capital cannot be treated as a single aggregate which plays the dual role of a financial and physical entity at the same time. That is the reason why many capital theorists have proceeded to examine models that deal with multisector and multicommodity worlds. In the beginning of the seventies these worlds were - following the preparatory work of Torrens, Ricardo, Marx, Von Böhm-Bawerk, Von Neumann and Sraffa - formalized in the form of (linear) reproduction models. In this respect one should mention Hicks,[1] Morishima,[2] Van Schaik,[3] Pasinetti,[4] Abraham-Frois and Berrebi,[5] Lowe,[6] Steedman[7] and Wolfstetter.[8]

2. The formal framework of the reproduction model can be used for various purposes. We mention the reconstruction of verbal reasonings in order to clarify their contents and to test their consistency and the generalizing of issues raised during the so-called reswitching debate of the sixties.

[1] J. Hicks [3].
[2] M. Morishima [5].
[3] A.B.T.M. van Schaik [10].
[4] L. Pasinetti [6].
[5] G. Abraham-Frois – E. Berrebi [1].
[6] A. Lowe [4].
[7] I. Steedman [8].
[8] E. Wolfstetter [11].

However, it is a pity that - despite its mathematical beauty - the operational content of modern capital theory is rather meagre. It is not enough to repeat continuously, by means of the neoclassical theory of the choice of techniques, that capital aggregation is theoretically unsound. Neither is it enough to repeat endlessly that both Ricardo and Marx are to be praised for their concern with the distribution of income between social classes. We think that in modern capitalist economics problems of long-term disequilibrium announce themselves. In providing the building-stones of general multisector dynamic models for studying and solving *these* problems, capital theory has an important task. That is the reason why we shall not discuss sterile issues such as the transformation problem. What we intend to do is to apply reproduction theory to concrete reality.

3. A simple example of such an application is the clay-clay vintage model for the Netherlands, as it has been developed by the Central Planning Bureau in The Hague.[9] With this model it can be shown that the slow down of employment growth in the Netherlands can be related to the level and development of real wages. After 1964 the labour share in the value added by enterprises has risen considerably. In other words, real wages have grown faster than (average) labour productivity. In the context of a clay-clay vintage model this development finds concrete shape in a continuous shortening of the (economic) life span of oldest equipment installed. As a result growth of (potential) employment tended to slow down.

The stagnation of employment can be offset by the preservation of old jobs and/or by the creation of new jobs. The first alternative would demand a noticeably moderation of the increase in real wages. The second alternative would require appreciable amount of (new) investment. In the light of this second alternative it is important to describe - given the conditions of a capitalist society - the size and the development of the rate of profit. And nowadays we have the reproduction model to describe such a development in a more or less satisfactory way.

4. In *Reproduction and Fixed Capital*[10] we have shown that the traditional (clay-clay) vintage model can be regarded as a special case of the general reproduction model with fixed capital of unequal efficiency. The traditional vintage model is characterized by the assumptions that the capital output ratio is constant over time and uniform for all vintages and that there is only

[9] Cfr. H. Den Hartog – H.S. Tjan [2].
[10] A.B.T.M. Van Schaik [10].

labour saving technical progress embodied in new machines. In other words, labour requirements per unit of equipment decrease with a constant relative rate the younger a vintage of equipment is; labour requirements, however, become fixed at the moment of installation of new equipment. Usually, the economic life span or the year of installation of the oldest vintage kept in operation is found with the aid of the rule of thumb that equipment wilt be scrapped if its revenue is lower than its wage sum. Formally speaking this means that the cost equation of the oldest vintage is characterized by the equality of labour costs and revenues. Moreover it is assumed that both real wage rate and net rate of profit are uniform over all vintages.

5. Following the working hypothesis of the joint production method as laid down in *Reproduction and Fixed Capital*[11] the cost equations of the traditional vintage model assume the form:

*depreciations + net profit = capital returns*.

The number of cost equations, i.e. the economic life time (*m*), is - given the real wage rate - a known variable. As explained in *Appendix A* of this paper such a price system then consists of *m* -1 equations with which m -1 unknowns can be solved, that is to say

the net rate of profit

and

*m* - 2 relative prices of old machines.

6. A well-known theorem from the theory of reproduction is that the distribution of income cannot be determined by pure economic factors. But given for instance real wages all relative prices and the rate of profit can be found. Under certain conditions - as explained in *Reproduction and Fixed Capital*[12] - it can then further be seen that both relative prices and the rate of profit only depend on technical data. It is shown in *appendix A* of this paper that the same is true for the special case of the traditional clay-clay vintage model.

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[11] *Ibid.*
[12] *Ibid.*

7. It is not an easy task to analyse the relation between the net rate of profit on the one hand and the economic life time and the technical data on the other hand by means of usual methods. That is the reason why we resorted to numerical experiments by computer. *Appendix B* of this paper contains some of the results. The results seem very plausible. We mention:

1. Given the technical data, *i.e.* the capital coefficient and the rate of technical progress, the relation between real wage and rate of profit is negative.

2. Given the rate of technical progress and the economic life span the relation between the capital coefficient and the rate of profit is negative.

3. Given the capital coefficient and the economic life time the relation between the rate of technical progress and the rate of profit is positive.

From this specification it follows that the net rate of profit is a measure for the 'productivity' which is inherent to a given technology. Another important result, known already from Sraffa[13] is, that a falling rate of profit is always associated with falling relative prices of old machines!

8. With the aid of the clay-clay vintage model for the Netherlands[14] it can be seen that the economic life time of capital goods in the sixties and early seventies has decreased from something like 30 years to *circa* 20 years. Given a capital coefficient of 2 and a rate of technical progress of 5 % this comes to a fall in the net rate of profit from 35 % to 25 %. Nevertheless this figure of 25 % is much higher than the rate of profit one obtains by dividing the (net) profit share in income by the capital coefficient, as it is usually done.

To give an example.[15] In a steadily growing economy, characterized by a (constant) economic life span of 20 years, a capital coefficient of 2, a rate of technical progress of 5 %, a growth rate of investment of 5 % and full utilization of capital, the wage share is *circa* 60 %. Dividing the profit share by the capital coefficient this results in a gross rate of profit (i.e. the sum of net rate of profit and rate of depreciation) of 20 %. The gross rate of profit calculated with the aid of the reproduction model comes to 35 %. (*Appendix C* of this paper contains some numerical

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13 P. Sraffa [7].
14 H. Den Hartog – H.S. Tjan [2].
15 Numerical examples of vintage – economies are also found in Th. Van de Klundert – A. Van Schaik [9].

examples to illustrate this and the problem of measuring profits as a surplus). The difference can be traced back to the different ways in measuring the value of capital.

We might say that the fall of the rate of profit in the Netherlands is considerable. On the other hand it has to be remarked that the size of the rate of profit - calculated with the aid of a reproduction model - remains substantial. It therefore remains to be seen whether the present-day stagnation in the creation of new jobs for the most part can be traced back to a rate of profit which has reached a critical low level.

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# *Appendix A*

## The clay-clay vintage model

The reproduction model as described in van Schaik[16] is:

quantity system: $Xy = (1 + g)Ay + c$

$a^* = ay$

$y^* = ey$

price system: $pX = (1 + r)pA + wa$

The symbols have the following meaning:

| | |
|---|---|
| $a$ | row vector of direct labour input coefficients |
| $A$ | (flow-) input matrix |
| $a^*$ | total labour requirements (potential employment) |
| $c$ | column vector of net surpluses e row unit vector |
| $e$ | row unit vector |
| $g$ | net rate of growth (of investment) |
| $p$ | row vector of prices |
| $r$ | net rate of profit |
| $w$ | nominal wage rate |
| $X$ | (flow-) output matrix |
| $y$ | column vector of process levels |
| $y^*$ | total production capacity |

In case of a clay-clay vintage model the matrix A becomes a square diagonal matrix of capital coefficients *b*. The square matrix *X* consists of a top-row unit vector and a left-hand sub-diagonal of capital coefficients. The other elements of the matrices *A* and *X* are zero. The vectors *p* and *y* are semipositive. The vector *c* consists of zero elements except the first one which is positive (a positive output of consumer goods).

[16] A.B.T.M. van Schaik [10].

In case of an economic life time of 4 years the catalogue of activities of a clay-clay vintage model can be described by

| | *vintage* | *input of* |
|---|---|---|
| | 0 1 2 3 | |
| $a =$ | $[\ a^0\ a^1\ a^2\ a^3\ ]$ | labour |

$$A = \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \begin{matrix} \text{new machines} \\ \text{one-year-old machines} \\ \text{two-year-old machines} \\ \text{three-year-old machines} \end{matrix}$$

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \end{bmatrix} \begin{matrix} \text{output of new goods} \\ \text{one-year-old goods} \\ \text{two-year-old goods} \\ \text{three-year-old goods} \end{matrix}$$

Assuming that the rate of labour saving technical progress equals the net rate of growth the vector of labour coefficients can be specified as follows:

$$a = [a^0, (1 + g)a^0, (1 + g)^2a^0, (1 + g)^3a^0].$$

The price system then becomes:

| *vintage* | $pX$ | | $(1 + r)pA$ | | $wa$ |
|---|---|---|---|---|---|
| 0: | $p^0 + p^1b$ | = | $(1 + r)p^0b$ | + | $wa^0$ |
| 1: | $p^0 + p^2b$ | = | $(1 + r)p^1b$ | + | $wa^0(1 + g)$ |
| 2: | $p^0 + p^3b$ | = | $(1 + r)p^2b$ | + | $wa^0(1 + g)^2$ |
| 3: | $p^0$ | = | $(1 + r)p^3b$ | + | $wa^0(1 + g)^3$. |

Rearranging terms results in:

| *vintage* | *depreciations* | | *net profit* | | *capital returns* |
|---|---|---|---|---|---|
| 0: | $(p^0 - p^1)b$ | + | $rp^0b$ | = | $p^0 - wa^0$ |
| 1: | $(p^1 - p^2)b$ | + | $rp^1b$ | = | $p^0 - \mathrm{wa}^0(1 + g)$ |
| 2: | $(p^2 - p^3)b$ | + | $rp^2b$ | = | $p^0 - wa^0(1 + g)^2$ |
| 3: | $(p^3 - 0)b$ | + | $rp^3b$ | = | $p^0 - wa^0(1 + g)^3$. |

The scrappage condition

$$w/p^0 = 1/a^3 = 1/(1 + g)^3a^0$$

implies $p^3 = 0$.

Using this condition the price system comes to the following three equations:

| *vintage* | *depreciations* | | *net profit* | | *capital returns* |
|---|---|---|---|---|---|
| 0: | $\left(1-\frac{p^1}{p^0}\right)b$ | + | $r\ \ \ b$ | = | $1 - (1 + g)^{-3}$ |
| 1: | $\left(\frac{p^1}{p^0}-\frac{p^2}{p^0}\right)b$ | + | $r\frac{p^1}{p^0}b$ | = | $1 - (1 + \mathrm{g})^{-2}$ |
| 2: | $\left(\frac{p^2}{p^0}-0\right)b$ | + | $r\frac{p^2}{p^0}b$ | = | $1 - (1 + \mathrm{g})^{-1}$. |

These equations can be solved for

$r$, $p^1/p^0$ and $p^2/p^0$.

From the structure of this price system it can immediately be seen that the payback period increases if:

a) the economic life time decreases (given *b* and *g*);

b) the rate of technical progress decreases (given *b* and *m*);

c) the capital coefficient increases (given *g* and *m*).

It should be noticed that in empirical investigations the numerical value of the capital coefficient is also dependent on the choice of the base year of constant prices. Choice of another base year implies changing the capital coefficient. To avoid the consequences of this for the results of the price system the capital returns should be adapted accordingly. In other words, given a technology ($a$, $A$, $X$), the direction of the process vectors (activities) should remain unchanged.

## *Appendix B*

*Numerical examples concerning the relation between net rate of profit on the one hand and economic life span, capital output ratio and rate of technical progress on the other hand*

I) *Variable economic life time* ($b$ = 2 and $g$ = 0,05)

| $m$ | $r$ (x 100) | $p^1/p^0$ (x 10) | $p^9/p^0$ (x 10) |
|---|---|---|---|
| 11 | 3,27 | 8,40 | 0,23 |
| 12 | 7,26 | 8,65 | 0,64 |
| 13 | 10,66 | 8,85 | 1,17 |
| 14 | 13,62 | 9,01 | 1,77 |
| 15 | 16,20 | 9,15 | 2,39 |
| 16 | 18,49 | 9,25 | 3,01 |
| 17 | 20,53 | 9,34 | 3,60 |
| 18 | 22,37 | 9,42 | 4,15 |
| 19 | 24,04 | 9,48 | 4,66 |
| 20 | 25,56 | 9,53 | 5,12 |
| 21 | 26,96 | 9,58 | 5,54 |
| 22 | 28,25 | 9,62 | 5,92 |
| 23 | 29,45 | 9,65 | 6,25 |
| 24 | 30,56 | 9,68 | 6,56 |
| 25 | 31,59 | 9,71 | 6,83 |
| 26 | 32,56 | 9,73 | 7,07 |
| 27 | 33,47 | 9,75 | 7,30 |
| 28 | 34,33 | 9,77 | 7,49 |
| 29 | 35,15 | 9,79 | 7,67 |
| 30 | 35,89 | 9,80 | 7,84 |

II) *Variable capital output ratio*

$m$ = 20 and $g$ = 0,05

| $b$ | $r$ (x 100) | $p^1/p^0$ (x 10) | $p^9/p^0$ (x 10) |
|---|---|---|---|
| 0,5 | 117,33 | 9,65 | 6,11 |
| 1,0 | 56,59 | 9,62 | 5,79 |
| 1,5 | 36,07 | 9,58 | 5,45 |
| 2,0 | 25,56 | 9,53 | 5,12 |
| 2,5 | 19,06 | 9,49 | 4,83 |
| 3,0 | 14,56 | 9,44 | 4,56 |
| 3,5 | 11,22 | 9,40 | 4,33 |
| 4,0 | 8,62 | 9,35 | 4,13 |
| 4,5 | 6,51 | 9,31 | 3,95 |
| 5,0 | 4,75 | 9,27 | 3,78 |

III) *Variable rate of labour saving technical progress*

$b$ = 2 and $m$ = 20

| $g$ | $r$ (x 100) | $p1/p0$ (x 10) | $p9/p0$ (x 10) |
|---|---|---|---|
| 0,02 | 8,23 | 9,26 | 3,76 |
| 0,03 | 15,37 | 9,38 | 4,34 |
| 0,04 | 20,99 | 9,47 | 4,77 |
| 0,05 | 25,56 | 9,53 | 5,12 |
| 0,06 | 29,32 | 9,58 | 5,42 |
| 0,07 | 32,44 | 9,63 | 5,68 |
| 0,08 | 35,04 | 9,66 | 5,91 |
| 0,09 | 37,21 | 9,69 | 6,13 |
| 0,10 | 39,04 | 9,72 | 6,33 |

It can be proved that $r_{maximum} \rightarrow 1/b$ if $m \rightarrow \sim$

or $g \rightarrow \sim$

## *Appendix C*

*Depreciations and net profit*

In this appendix we show that in a growing economy a part of the net profit share should be devoted to finance the replacement of jobs.

Three examples wilt suffice. These examples have in common:

- an economic life time of 20 years;

- a capital coefficient of 2;
- a rate of technical progress of 0,05.

With the aid of the price system of the reproduction model it can be calculated then that the net rate of profit is 0,2556292.

For the quantity system it is assumed that the production capacity of the oldest vintage ($y^{19}$) = 100. We now distinguish:

Case I: net rate of growth is 0

Case II: net rate of growth is 0,05

Case III: net rate of growth is 0,10

Each table contains

column 1 = number of vintage (0, 1, 2, . . . , 18, 19)

column 2 = value of capital (*K*)

column 3 = depreciations (*D*)

column 4 = labour cost (*L*)

column 5 = net profit (*P*)

column 6 = capacity (*C*)

TABLE I

*Net rats of growth is* 0

| | K | D | L | P | C |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 200,00 | 9,30 | 39,57 | 51,13 | 100,00 |
| 1 | 190,70 | 9,70 | 41,55 | 48,75 | 100,00 |
| 2 | 181,00 | 10,10 | 43,63 | 46,27 | 100,00 |
| 3 | 170,90 | 10,50 | 45,81 | 43,69 | 100,00 |
| 4 | 160,40 | 10,90 | 48,10 | 41,00 | 100,00 |
| 5 | 149,50 | 11,28 | 50,50 | 38,22 | 100,00 |
| 6 | 138,22 | 11,64 | 53,03 | 35,33 | 100,00 |
| 7 | 126,59 | 11,96 | 55,68 | 32,36 | 100,00 |
| 8 | 114,63 | 12,23 | 58,47 | 29,30 | 100,00 |
| 9 | 102,40 | 12,43 | 61,39 | 26,18 | 100,00 |
| 10 | 89,97 | 12,54 | 64,46 | 23,00 | 100,00 |
| 11 | 77,43 | 12,52 | 67,69 | 19,79 | 100,00 |
| 12 | 64,91 | 12,34 | 71,07 | 16,59 | 100,00 |
| 13 | 52,57 | 11,94 | 74,62 | 13,44 | 100,00 |
| 14 | 40,63 | 11,26 | 78,35 | 10,39 | 100,00 |
| 15 | 29,37 | 10,22 | 82,27 | 7,51 | 100,00 |
| 16 | 19,15 | 8,72 | 86,38 | 4,90 | 100,00 |
| 17 | 10,42 | 6,63 | 90,70 | 2,67 | 100,00 |
| 18 | 3,79 | 3,79 | 95,24 | 0,97 | 100,00 |
| 19 | | | 100,00 | | 100,00 |
| | 1922,58 | 200,00 | 1308,51 | 491,49 | 2000,00 |

$D/K = 0{,}10$ $L/C = 0{,}65$ $D/C = 0{,}10$

$P/K = 0{,}26$ $P/C = 0{,}25$ $\frac{D+P}{C \times b} = 0{,}17.$

TABLE II

*Net rate of growth is* 0,05

| | K | D | L | P | C |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 505,39 | 23,50 | 100,00 | 129,20 | 252,70 |
| 1 | 458,94 | 23,34 | 100,00 | 117,32 | 240,66 |
| 2 | 414,85 | 23,15 | 100,00 | 106,05 | 229,20 |
| 3 | 373,05 | 22,93 | 100,00 | 95,36 | 218,29 |
| 4 | 333,45 | 22,65 | 100,00 | 85,24 | 207,89 |
| 5 | 296,00 | 22,33 | 100,00 | 75,66 | 197,99 |
| 6 | 260,64 | 21,94 | 100,00 | 66,62 | 188,56 |
| 7 | 227,33 | 21,47 | 100,00 | 58,12 | 179,59 |
| 8 | 196,06 | 20,92 | 100,00 | 50,11 | 171,03 |
| 9 | 166,80 | 20,25 | 100,00 | 42,64 | 162,89 |
| 10 | 139,57 | 19,45 | 100,00 | 35,68 | 155,13 |
| 11 | 114,40 | 18,50 | 100,00 | 29,25 | 147,75 |
| 12 | 91,33 | 17,36 | 100,00 | 23,35 | 140,71 |
| 13 | 70,45 | 16,00 | 100,00 | 18,01 | 134,01 |
| 14 | 51,86 | 14,37 | 100,00 | 13,26 | 127,63 |
| 15 | 35,70 | 12,42 | 100,00 | 9,13 | 121,55 |
| 16 | 22,16 | 10,10 | 100,00 | 5,66 | 115,76 |
| 17 | 11,49 | 7,31 | 100,00 | 2,94 | 110,25 |
| 18 | 3,98 | 3,98 | 100,00 | 1,02 | 105,00 |
| 19 | | | 100,00 | | 100,00 |
| | 3773,45 | 341,97 | 2000,00 | 964,62 | 3306,59 |

$D/K = 0{,}09$ $L/C = 0{,}60$ $D/C = 0{,}10$

$P/K = 0{,}26$ $P/C = 0{,}29$ $\frac{D+P}{C \times b} = 0{,}20.$

TABLE III

*Net rate of growth is* 0,10

| | K | D | L | P | C |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1223,18 | 56,88 | 242,03 | 312,68 | 611,59 |
| 1 | 1060,27 | 53,93 | 231,02 | 271,04 | 555,99 |
| 2 | 914,86 | 51,06 | 220,53 | 233,86 | 505,45 |
| 3 | 785,27 | 48,26 | 210,50 | 200,74 | 459,50 |
| 4 | 670,01 | 45,52 | 200,93 | 171,27 | 417,72 |
| 5 | 567,72 | 42,82 | 191,80 | 145,13 | 379,75 |
| 6 | 477,18 | 40,16 | 183,09 | 121,98 | 345,23 |
| 7 | 397,29 | 37,52 | 174,76 | 101,56 | 313,84 |
| 8 | 327,06 | 34,89 | 166,81 | 83,61 | 285,31 |
| 9 | 265,61 | 32,24 | 159,23 | 67,90 | 259,37 |
| 10 | 212,15 | 29,57 | 151,99 | 54,23 | 235,79 |
| 11 | 165,98 | 26,84 | 145,09 | 42,43 | 214,36 |
| 12 | 126,49 | 24,05 | 138,20 | 32,33 | 194,87 |
| 13 | 93,13 | 21,15 | 132,20 | 23,81 | 177,16 |
| 14 | 65,43 | 18,14 | 126,18 | 16,73 | 161,05 |
| 15 | 42,99 | 14,97 | 120,45 | 10,99 | 146,41 |
| 16 | 25,48 | 11,61 | 114,98 | 6,51 | 133,10 |
| 17 | 12,61 | 8,03 | 109,75 | 3,22 | 121,00 |
| 18 | 4,17 | 4,17 | 104,76 | 1,07 | 110,00 |
| 19 | | | 100,00 | | 100,00 |
| | 7436,88 | 601,81 | 3224,59 | 1901,09 | 5727,49 |

$D/K = 0{,}08$ $L/C = 0{,}56$ $D/C = 0{,}11$

$P/K = 0{,}26$ $P/C = 0{,}33$ $\frac{D+P}{C \times b} = 0{,}22.$

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