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Abstract: Instead of modeling asset price and currency risks separately, this paper derives the international hedge portfolio, hedging asset price and currency risk simultaneously for estimating the dynamic international optimal hedge ratio. The model estimation is specified in a multivariate GARCH setting with vector error correction terms and estimated for the commodity and stock markets of the U.S., the U.K., and Japan.

Keywords: Optimal Hedge Ratio, International Hedging, Multivariate GARCH, Currency Risk, Hedge Effectiveness and Efficiency, Spot and Futures Markets

I. Introduction

Globalization has brought additional choices of investing instruments, but that additional choice can also bring confusion. For a U.S.-based company that has a spot-asset risk exposure, management may be interested in not only hedging dollar-denominated risk, but also foreign exchange risk. For example, a U.S. firm that has a copper asset may want to contract with a U.K.-based firm to sell the asset. This not only exposes the parties to the price risk of copper, but also currency risk. For the U.S. firm, a futures contract on copper can be sold in either the U.S. or U.K. This may hedge the price risk of copper, but there is still the outstanding issue of the foreign exchange risk. At any given time, these alternative hedging "bundles" (commodity and currency futures) should be priced according to the law of one price. Transaction costs can drive a wedge between the theoretical prices that should prevail. In this paper, we examine which hedging bundle is optimal. In a very real way, this is equivalent to the covered versus uncovered interest rate parity theory of exchange rates, but instead of interest rates, we use the returns on commodities.

Despite the increased importance of international investing, the optimal hedge ratio is rarely estimated between countries' markets. A number of studies estimate the domestic optimal hedge ratio in commodity markets, currency markets, and financial markets.[1] There are a few notable studies regarding the optimal hedge ratio estimation between international financial markets.[2] Financial market liberalization has stimulated demand for international hedging. In addition to the enormous growth in the hedge fund industry, importers, exporters and multinational corporations utilize financial derivative markets to hedge risks across

countries and/or between international markets. Growing demand for international hedging has led to assets being traded in different countries' markets with more than one currency.

Currency risk plays a significant role in international hedging. De Santis and Gérard (1998) find that currency risk is priced on several major developed stock markets. Antell and Vaihekoski (2007) argue that currency risk can play a very important role in many small developed and/or emerging markets. Similar results have been found for small developed markets (see, Vaihekoski, 2007) and for emerging markets (see, Jacobsen and Liu, 2008; Phylaktis and Ravazzolo, 2004). Consequently, the volatility of currency should be considered in international hedging.

Earlier studies, (e.g., Kerkvliet and Moffett (1991), among others), investigate the hedging portfolio where an agent hedges the currency risk of a foreign denominated asset in the currency derivative markets separate from the risk of the underlying asset. Gardner and Stone (1995) have pointed out a deficiency of conducting two hedging operations, namely that the optimal solutions can be highly unstable due to the poor forecasting power of currency derivatives, such that the international hedging portfolio might bear double risks in both foreign asset markets and currency derivative markets. On the one hand, the investor must determine two sets of hedge ratios: one set for the optimal weights of the original foreign assets and the other for the hedging of currency risk; on the other hand, the investor must pay two sets of transaction fees.

This study proposes an alternative approach to deriving the international portfolio hedge without involving an additional hedge operation in the currency derivative market. To

accomplish this, we simultaneously hedge the currency risk within the same transaction being used to hedge the spot market risk via the foreign futures contracts. In other words, both asset and currency risks are hedged in one operation.

This paper is structured as below. Section II reviews related literature. Section III develops the simultaneous hedge theory. Section IV specifies empirical econometric models. Section V describes data. Section VI reports empirical results. Section VII concludes this paper.

II. Related Literature

This paper extends the mean-variance expected utility framework to derive the international optimal hedge ratio.[3] There are, numerous methods for determining the optimal hedge ratio: minimum variance, maximum Sharpe ratio, and minimum generalized semi-variance.[4] Interestingly, Lee et al. (2006) and Kroner and Sultan (1993) document that under martingale and joint-normality assumptions, various optimal hedge ratios are identical to the minimum variance hedge ratio. However, within the context of simultaneous hedging, the IOHR formulation of this study is not equivalent to those approaches.

We use a multivariate Generalized Autoregressive Conditional Heteroscedasticity model with vector error correction terms (VEC-MGARCH) to estimate the simultaneous IOHR. Many previous studies capture the time-varying feature of the second moments of financial time–series using GARCH models.[5] These studies report that the optimal hedge ratio is time-varying if the true joint distribution of spot and futures returns is changing over time.

III. Hedge Theory

Let s_t and f_t^* be changes in the spot market prices denoted in domestic currency and in the futures prices denoted in foreign currency at time t, respectively. Also, let e_t be the exchange rate at time *t* in domestic currency per unit of foreign currency. Suppose an agent invests in an international hedge portfolio consisting of being long one unit of the spot asset and short *h* units of the corresponding foreign futures assets priced in the foreign currency. The random cash flow to this hedged portfolio, Π, is

$$\Pi_t = s_t - h\left(f_t^* e_t\right) - hc \tag{1}$$

where *c* is the fixed transaction cost per futures contract. Assume the agent has constant absolute risk aversion utility defined over cash flows of the hedged portfolio. The agent's problem is to choose an optimal *h* to maximize the expected utility. If the portfolio return is normally distributed conditional on currently available information, $\Omega_{t-1,}$ this problem is equivalent to maximizing the following mean-variance expected utility function:

$$\underset{h}{Max}\, E(\Pi_t|\Omega_{t-1}) - \frac{\lambda}{2} Var(\Pi_t|\Omega_{t-1}) \tag{2}$$

where, λ is the coefficient of absolute risk aversion, $E\left(\Pi_t|\Omega_{t-1}\right) = E(s_t|\Omega_{t-1}) - hE(f_t^* e_t|\Omega_{t-1}) - hc$, and $Var\left(\Pi_t|\Omega_{t-1}\right) = Var(s_t|\Omega_{t-1}) + h^2 Var(f_t^* e_t|\Omega_{t-1}) - 2hCov(s_t, f_t^* e_t|\Omega_{t-1})$.

Solving this optimization problem for equation (2), we obtain the international optimal hedge ratio as h^*. The notation of conditional information, Ω_{t-1} is suppressed in the rest of this paper for simplicity.

$$h^* = \frac{\lambda Cov\left(s_t, f_t^* e_t\right) - E\left(f_t^* e_t\right) - c}{\lambda Var\left(f_t^* e_t\right)} \tag{3}$$

The exact variance and covariance formulation of product of random variables in Equation (3) was originally developed in Bohenstedt and Goldberger (1969). Its application in a different setting can be found in Kerkvliet and Moffett (1991), who studied the hedging of uncertain currency cash flow.

Equation (3) can be compared to a purely domestic hedge:

$$h_D = \frac{\lambda Cov(s_t, f_t) - E(f_t) - c}{\lambda Var(f_t)} \qquad (4)$$

Equations (3) and (4) differ in terms of the futures contract: does any covariance between the exchange rate and the futures contract price help determine the optimal hedge ratio? This relation is empirically examined in the section V.

McCurdy and Morgan (1988) and Kroner and Sultan (1993) show the empirical evidence that the futures rate follows a martingale. The martingale property implies $E(f) = 0$, but we will not assume that property for purposes of empirical testing.

Higher transaction costs reduce the international optimal hedge ratio, as expected. However, Jickling (2006), in the report of Congressional Research Service for Congress, states a relatively small transaction cost for derivative markets, which is between 0.01% and 0.003% of the transaction value. Moon, et. al. (2009) reports an even lower transaction cost around 0.00072% in the KOSTAR futures market. To be conservative, we will assume a transaction cost of 0.01%.

This study makes an assumption of λ=4, based on the main findings in the literature, e.g. Grossman and Shiller (1981), Kroner and Sultan (1993), Chou (1988), and Poterba and Summers (1986).

If the futures asset price is uncorrelated with the exchange rate, or the exchange rate is fixed, the international optimal hedge ratio is identical to the traditional domestic optimal hedge ratio. In other words, if the law of one price applies continuously, such that $f_t = f_t^* e_t$, then the domestic and international hedge ratios would be identical. Furthermore, a negative correlation between the exchange rate and the foreign futures asset price leads to a larger hedge ratio than if there is a positive correlation, everything else being held constant. The correlation between the spot price and the exchange rate, nonetheless, has no direct effect on the hedge ratio.

IV. Model Specification

In this study, a multivariate GARCH model with vector error correction terms (VEC-MGARCH) is specified to estimate the optimal hedge ratio over three hedging horizons. The optimal hedge ratio is adjusted continuously based on conditional information and thus calculated from conditional variances and covariances (see Lee et al. (2008)). In addition, we also follow earlier studies, using the asset returns to estimate conditional variances and covariances. Empirical studies have documented a cointegrating relationship (i.e., long memory relationship) between spot and futures markets. These previous studies specify error correction models to account for the cointegrating relationships.[6]

The mean equation of the VEC-MGARCH model is as follows

$$\mathbf{r_t} = \mathbf{u} + \varphi \mathbf{z_t} + \boldsymbol{\varepsilon_t} \qquad (5)$$

where, $\mathbf{r_t} = \begin{pmatrix} r_{s_t} \\ r_{f_t^*} \\ r_{e_t} \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} u_{s_t} \\ u_{f_t^*} \\ u_{e_t} \end{pmatrix}, \varphi = \begin{pmatrix} \varphi_s \\ \varphi_{f^*} \\ \varphi_e \end{pmatrix}$, $\boldsymbol{\varepsilon_t} = \begin{pmatrix} \varepsilon_{s_t} \\ \varepsilon_{f_t^*} \\ \varepsilon_{e_t} \end{pmatrix} \Big| \Omega_{t-1} \sim N(0, \mathbf{H}_t)$, $\mathbf{H}_t$ is the conditional variance-covariance matrix. r_t is the asset returns at time t. $\mathbf{z}_t$ is the vector of the error correction terms.

The VEC-MGARCH model in this paper is estimated by the techniques of Diagonal BEKK, Quasi-Maximum Likelihood (QML), and the Marquardt algorithm. The BEKK GARCH parameterization, proposed by Engle and Kroner (1995), circumvents restrictions in Bollerslev et al. (1988), such as the large number of parameters to be estimated, difficulties in obtaining a stationary covariance process, and the problems in achieving a positive-definite (co)variance matrix. In order to simplify the estimation process for the variance equations, we adopt the covariance stationary specification of Ding and Engle (2001), which was also applied in e.g., De Santis and Gérard (1998).

In order to evaluate the performance of the different hedged portfolios, we construct the hedged portfolios based on the optimal hedge ratios for the domestic and international strategies. The IOHR hedge portfolio, estimated by the proposed approach in this study, simultaneously hedges both asset and currency risks within one operation and one set of estimated optimal hedge ratios. The IOHR hedge portfolio is compared to the traditional domestic hedge strategy, which separately hedges the asset price risk and the foreign exchange risk. For comparison purposes, we denote the domestic (or conventional) portfolio as COHR.

V. Data

We examined the following asset markets: 1) commodity markets: copper spot and futures markets of London Metal Exchange (LME) and New York Mercantile Exchange-Commodity Exchange (NYMEX-COMEX); 2) stock index markets: spot index and associated index futures markets of S&P 500 in the Chicago Mercantile Exchange (CME); FTSE100 in the London Stock Exchange (LSE) and London International Financial Futures Exchange (LIFFE); and the TOPIX in Tokyo Stock Exchange (TSE). These commodities and stock indices are traded in major financial markets and well studied.

For the sake of simplicity, the naming conventions of "spot-futures" markets are used to express the hedged portfolio structure with more than one currency. For example, COMEX-LME represents that the hedged portfolio consists of the copper spot asset traded in COMEX (U.S. dollar denominated) and the copper futures contract traded in LME (British pound denominated). FTSE100-TOPIX forms a hedged portfolio of the FTSE100 stock index traded in LSE as a spot asset denominated in British pounds and the TOPIX index futures contract traded in TSE as a futures asset in Japanese Yen.

We used daily, weekly and monthly data from January 1998 to June 2008 taken from Datastream, including exchange rates. USDGBP, USDJPY and GBPJPY are the exchange rates between the US dollar and the British pound, the US dollar and Japanese Yen, the British pound and Japanese Yen, respectively.

Table I summarizes the statistical characteristics for these assets and currencies. Augmented Dickey Fuller (ADF) tests show that asset and currency returns are stationary.

The p-values of Jarque-Bera tests reject the null hypothesis of normal distributions for asset log levels, asset return series (except for the S&P 500 and TOPIX), and the currency variables (except USDGBP). The statistical results of the Ljung-Box test and the ARCH-LM test provide the evidence of the persistent autocorrelation patterns.

Panel C in Table I shows significant correlations between currencies and assets. The correlations between currencies and futures commodity contracts are much stronger than those between currencies and stocks. This phenomenon implies that covariance between the exchange rate and the futures contract price help determine the optimal hedge ratio in Equation (4). We only report the correlations for daily data since weekly and monthly correlations are similar to daily correlation between assets and currencies.

[Table I about here]

VI. Empirical Results

Model Estimation Results

Table II exhibits the results of estimated parameters from the VEC-MGARCH model. The term, $\boldsymbol{\mu}$, is statistically significant in all mean equations and horizons. The coefficients of vector error correction terms are statistically significant for both spot and futures markets, which shows the cointegration relations between international markets. The highly significant parameters (a_i and b_i) in the variance equations for the GARCH process indicate the significance of the second moment and make the conditional variance process

clearly time-varying. Moreover, the estimates of the b_i coefficients (which link second moments to their lagged value) are considerably larger than the corresponding estimates of the a_i which link second moments to their past innovations (shocks). Additionally, based on the methods in Harris et al. (2007) and Andersen and Bollerslev (1998), this paper executes the robust test of conditional unbiasedness (not reported, upon request) which shows the overall satisfactory in most of the estimations.

[Table II about here]

Hedge Performance

Table III reports the results of hedge effectiveness measured by the variance reduction and the expected utility. The results show that all of the hedge portfolios achieve substantial variance reduction relative to the unhedged portfolios, in line with prior studies in earlier sections. Moreover, the IOHR hedge portfolios obtain a slightly larger variance reduction than the COHR hedge portfolios in most cases with a few exceptions. This empirical result supports that the simplified and simultaneous approach in this study, which hedges both asset and currency risks in one operation and with one set of optimal hedge ratios, attains at least as much effectiveness as the COHR hedge portfolios in which the currency risk is completely and separately hedged in the currency derivative markets. In the cases of the few exceptions, the IOHR hedge portfolios realize larger expected utilities than the COHR hedge portfolios. In this regard, the IOHR hedge portfolios still retain a hedge effectiveness slightly superior to the COHR hedge portfolios. On the other hand, Table III shows that the hedge effectiveness

of the realistic situation of the POHR hedge portfolios is consistently inferior to the IOHR hedge portfolios by the comparison of both the variance reduction and the expected utility.[7]

[Table III about here]

However, as observed in Table III, the variance differences between the IOHR and COHR hedge portfolios are fairly minor. Thus, whether the variance differences are significant for the superiority of the IOHR hedge portfolios should be questioned. Table IV reports the significant tests of the variance difference of each hedge portfolio relative to the IOHR hedge portfolio. The test results demonstrate that the variance differences are insignificant in most cases between the IOHR and COHR hedge portfolios, whereas significant between the IOHR and POHR hedge portfolios. This evidence challenges the validity of the superiority of the IOHR hedge portfolios to the COHR hedge portfolios, even though the IOHR hedge portfolios perform persistently better than the POHR hedge portfolios.

[Table IV about here]

The insignificant variance difference requests further examination of the hedge performance between these hedging strategies. Table V displays alternative measures of the hedge performance that reveal both the efficiency and effectiveness of a hedge portfolio. The results in Table V show that the IOHR hedge portfolios gain the higher hedge efficiency due

to the smaller EOHR value.[8] The better hedge efficiency implies the practical usefulness of the stable optimal hedge ratios, the lower cost, less funding, etc., for an international hedging. For instance, the smaller volatility of the estimated IOHR suggests that the IOHR hedged portfolio is capable of lowering transaction and rebalancing costs, especially compared to the traditional separate hedge strategy involving additional costs in the currency derivative markets. Likewise, in Table V, the IOHR hedge portfolios with the smaller EER[9] ratios exhibit better comprehensive hedge performance than the COHR hedge portfolios, despite the insignificant variance difference between the IOHR and COHR hedge portfolios. Indeed, the alternative measures of the EOHR and EER provides useful insight for the hedge performance evaluation and supplements the conventional methods.

[Table V about here]

Time-varying International Optimal Hedge Ratios

Table VI summarizes the statistical characteristics of the estimated time varying international optimal hedge ratio (TVIOHR). The estimated TVIOHR increases with the rise of the length of the hedge horizon, in line with the finding in Chen et al. (2004). For example, the international optimal hedge ratios of LME-COMEX hedged portfolio are 0.441, 0.779 and 0.868 for the daily, weekly and monthly horizons, respectively. The empirical results in Table VI also show that both the value and standard deviation of the TVIOHR are lower than the estimated optimal hedge ratio (OHR) of the conventional separate hedge strategy with a few exceptions in the monthly horizon. For instance, the IOHR value and standard deviation

of the weekly horizon of S&P500-FTSE100 are 0.5734 and 0.135, less than those of OHR, 0.6728 and 0.159, respectively. This feature is actually consistent with the finding in the hedge efficiency based on the EOHR measurement aforementioned.

[Table VI about here]

In fact, Figure 1 shows the different patterns of the dynamic optimal hedge ratios between horizons. For example, the dynamic optimal hedge ratios of the daily horizons apparently emerge the mean-reverting characteristics. The distinct movement tendency and fluctuation trend between portfolios and hedge horizons provide a hint of how investors can take advantage of the international optimal hedge ratios to construct hedge strategies. In this study, the revealed evidence supports that the results in Chen et al. (2004), which showed that both optimal hedge ratio and hedge effectiveness increase with the rise of the length of hedge horizons, are not only valid for domestic hedging, but also international hedging. Also, the insights are useful for decision-making of the hedging strategies.

[Figure I about here]

VII. Conclusions

Most prior studies have investigated hedging taking place within one domestic market. However, due to the rapid global expansion of the hedging industry and the growing demand for international hedging, more studies estimate dynamic hedge ratios in an international portfolio context. The previous researches, e.g., Bugar and Maure (2002),

Larsen and Resnick (2000), and Levy and Lim (1994), among others use separate transactions in currency derivative markets to hedge currency risk. This strategy has presented issues of instable optimal solutions, determination difficulty for two sets of hedge ratios, extra input and additional costs in currency derivative markets.

However, this paper diversifies the international hedge portfolio in a different context and proposes an alternative approach to estimate the international optimal hedge ratios. The proposed approach attempts to simultaneously hedge currency risk within the same transactions in which the spot asset risk priced in the domestic currency is hedged via the futures contracts priced in the foreign currency. In other words, both asset and currency risks are hedged in one operation. As an alternative approach, this method implies that international hedging can be accomplished with one set of hedge ratios, significantly reducing the determination difficulty of estimating two sets of hedge ratios. To some extent, this strategy makes both separate transactions and extra input in currency derivative markets unnecessary, and avoids the exposure to multiple markets' risks. Indeed, it presents a simplified approach for international hedging in practice.

The empirical result in this paper shows that the IOHR hedge portfolio attains a slightly larger variance reduction than the COHR portfolio, while its variance reduction is consistently greater than the POHR portfolio. However, the variance difference is insignificant between the IOHR and COHR hedge portfolios, whereas significant between the IOHR and POHR hedge portfolios. The insignificant variance difference generates the difficulty of comparing the superiority between IOHR and COHR hedge portfolios.

Therefore, this paper further examines their hedge performance by the efficiency and effectiveness measures. Unlike previous studies, this paper extends the hedge performance to incorporate both the hedge effectiveness and the hedge efficiency. The hedge efficiency measures the economic aspects of the hedge portfolio, such as the operation cost saving, the practicability, and the like, while the hedge effectiveness evaluates the validity of the risk-return adjustment of the hedge portfolio.

As a matter of fact, the constructed alternative measures show greater hedge efficiency and effectiveness for the IOHR hedge portfolios. The efficiency (EOHR) measurement implies that the IOHR hedge portfolios have better practicability, such as lowering transaction and rebalance costs, reducing the financing needs, etc. In addition, the efficiency-effectiveness (EER) measurement also shows better comprehensive hedge performance than the COHR hedge portfolios, despite the insignificant variance difference. Indeed, the alternative measures of the EOHR and EER provides useful insights for the hedge performance evaluation and serves as a supplement to the conventional methods. As a result, these empirical evidences support the proposed hedge approaches with certain advantages could be valid alternatives to the conventional hedge methods.

The results in this study should be very useful in practice for both companies and investors who are interested in international hedging. For instance, using the international hedge ratio, investors can simplify the international hedging process, reduce fund budgets, avoid extra capital input and separate operation in currency futures markets, as well as pursue better hedge performance. Additionally, this study shows insights for investors making

decisions regarding international hedge strategies. The evidence revealed in this study supports that the results in Chen et al. (2004), which demonstrated that both optimal hedge ratio and hedge effectiveness increase with the rise of the length of hedge horizons, are not only valid for domestic hedging, but also international hedging

Footnotes:

1. **Commodity futures markets**: Fernandez (2008), Power and Vedenov (2008), Hsu et al. (2008), Lien and Yang (2006a, 2008b), Lien (2007), Lee et al. (2008), Lee et. al.(2006), Bystrӧm (2003), Turvey and Nayak (2003), Lien and Tse (1998, 2000, 2001), Bera et al. (1997), De Jong et. al.(1997), Martinot et. al. (2000), etc.; **Foreign currency futures markets**: Henderson (2002), Hagelin and Pramborg (2004), Chan-Lau (2006), Lien and Yang (2006b), etc.; **Stock futures markets**: Park and Switzer (1995), Tong (1996), Gagnon et al. (1998), Yang (2001), Brooks et al. (2002), Alizadeh and Nomikos (2004), Harris et al. (2007), Dark (2007), Floros and Vougas (2007), Lee and Yoder (2007a, 2007b), etc.

2. See, Leuthold and Kim (2000), Booth et al. (1998), Maurer and Valiani (2004), and Kerkvliet and Moffett (1991), etc.

3. See Kroner and Sultan (1993), Chakraborty and Barkoulas (1999), and Gardner and Stone (1995), etc.

4. See Alizadeh and Nomikos (2004), Kooli et al. (2005), Turvey and Nayak, (2003), etc.

5. See Engle (1982), Bollerslev (1986, 1990), Park and Switzer (1995), Power and Vedenov (2008), Lee and Yoder (2007), Bystrӧm (2003), Hagelin and Pramborg (2004), and Chan-Lau (2006), etc.

6. See, Kroner and Sultan (1993), Lien and Yang (2008a), Watkins and McAleer (2004), Martinot et al. (2000), Fung et al. (2003), Floros and Vougas (2007), Yang (2001), Lien and Yang (2006b), Dark (2007), Ghosh (1993), Lien and Luo, 1993; Lien, 1996, etc.

7. COHR is an unrealistic case in which the currency risk is assumed completely hedged out. In contrast, POHR, a more realistic case, is assumed that the currency risk is partially hedged. In this paper, we assume POHR for currency risk by 90% hedge.

8. The efficiency of optimal hedge ratios (*EOHR*) is defined as

$$EOHR = E(h_t \mid \Omega_{t-1}) + \frac{\lambda}{2} Var(h_t \mid \Omega_{t-1})$$

9. The efficiency-effectiveness ratio (*EER*) is defined as

$$EER = \frac{EOHR}{VR}$$

where VR is the variance reduction of the hedge portfolio.

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Table I: Statistical Description

All data is taken from Datastream.USDGBP is the exchange rate between US dollar and UK pound. USDJPY is the exchange rate between US dollar and Japan Yen. GBPJPY is the exchange rate between UK pound and Japan Yen. Sample period is from January 1998 to June 2008, 2737 observations for daily horizon, 547 observations for weekly horizon and 126 observations for monthly horizon. p-value is reported for Q(12), Q^2(12), Q(26) and Q^2(26), normality test, unit root test and ARCH LM (4). Q( ) and Q^2( ) are the Ljung-Box (1978) statistics for the (squared) returns.

Panel A: Assets	Log Level							Return %								
	Mean	Std. Dev.	Skewness	Excess kurtosis	Normality Test	Unit root test I(0)	Unit root test I(1)	Mean	Std. Dev.	Skewness	Excess kurtosis	Normality Test	Q(26)	Q^2(26)	ARCH LM(4)	Unit root test I(0)
Daily Horizon																
LME																
S&Pot	7.358	0.527	0.799	2.096	<0.0001	0.984	<0.0001	0.053	1.509	0.029	4.240	<0.0001	0.011	<0.0001	<0.0001	0.0001
Futures	7.360	0.516	0.848	2.174	<0.0001	0.981	<0.0001	0.052	1.448	-0.042	4.475	<0.0001	0.122	<0.0001	0.032	0.0001
COMEX																
S&Pot	4.796	0.614	0.765	2.013	<0.0001	0.988	<0.0001	0.059	1.638	-0.200	7.285	<0.0001	0.005	<0.0001	<0.0001	0.0001
Futures	4.795	0.603	0.804	2.083	<0.0001	0.988	<0.0001	0.058	1.574	-0.153	7.234	<0.0001	0.028	<0.0001	0.002	0.0001
S&P500																
S&Pot	7.095	0.150	-0.502	2.655	<0.0001	0.202	<0.0001	0.0101	1.125	-0.029	5.713	<0.0001	0.022	<0.0001	0.097	0.0001
Futures	7.099	0.152	-0.493	2.638	<0.0001	0.203	<0.0001	0.0098	1.153	-0.048	6.228	<0.0001	0.016	<0.0001	0.035	0.0001
FTSE100																
S&Pot	8.595	0.162	-0.695	2.396	<0.0001	0.489	<0.0001	0.003	1.159	-0.176	5.656	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
Futures	8.603	0.167	-0.700	2.427	<0.0001	0.500	<0.0001	0.002	1.183	-0.128	5.577	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
TOPIX																
S&Pot	7.141	0.210	-0.180	2.151	<0.0001	0.492	<0.0001	0.0043	1.261	-0.226	4.869	<0.0001	0.037	<0.0001	0.075	0.0001
Futures	7.140	0.211	-0.177	2.151	<0.0001	0.410	<0.0001	0.0038	1.400	-0.342	7.030	<0.0001	0.065	<0.0001	0.048	0.0001
Weekly Horizon																
LME																
S&Pot	7.357	0.527	0.795	2.090	<0.0001	0.981	<0.0001	0.264	3.281	-0.197	3.698	0.001	0.368	<0.0001	0.601	0.0001
Futures	7.358	0.515	0.844	2.169	<0.0001	0.983	<0.0001	0.258	3.196	-0.231	3.714	<0.0001	0.325	<0.0001	0.539	0.0001
COMEX																
S&Pot	4.796	0.615	0.765	2.014	<0.0001	0.984	<0.0001	0.297	3.485	-0.222	3.658	0.001	0.072	<0.0001	0.550	0.0001
Futures	4.794	0.604	0.804	2.084	<0.0001	0.987	<0.0001	0.293	3.416	-0.227	3.681	<0.0001	0.238	<0.0001	0.527	0.0001
S&P500																
S&Pot	7.095	0.150	-0.508	2.676	<0.0001	0.269	<0.0001	0.055	2.479	-0.425	5.352	<0.0001	0.026	<0.0001	0.009	0.0001
Futures	7.099	0.153	-0.499	2.658	<0.0001	0.282	<0.0001	0.053	2.511	-0.491	5.618	<0.0001	0.014	<0.0001	0.007	0.0001
FTSE100																
S&Pot	8.595	0.163	-0.687	2.390	<0.0001	0.486	<0.0001	0.014	2.585	-0.564	5.819	<0.0001	0.138	<0.0001	0.002	<0.0001
Futures	8.603	0.167	-0.692	2.418	<0.0001	0.493	<0.0001	0.013	2.617	-0.515	5.938	<0.0001	0.194	<0.0001	0.006	<0.0001
TOPIX																
S&Pot	7.141	0.211	-0.173	2.151	<0.0001	0.521	<0.0001	0.027	3.100	-0.143	3.301	0.141	0.044	0.082	0.016	0.0001
Futures	7.139	0.211	-0.170	2.147	<0.0001	0.507	<0.0001	0.026	3.174	-0.144	3.366	0.085	0.042	0.054	0.011	0.0001
Monthly Horizon																
LME																
S&Pot	7.354	0.526	0.804	2.110	<0.001	0.970	<0.001	1.103	6.965	0.439	3.024	0.134	0.493	0.771	0.744	<0.0001
Futures	7.356	0.515	0.853	2.191	<0.001	0.973	<0.001	1.078	6.791	0.432	2.998	0.144	0.524	0.806	0.734	<0.0001
COMEX																
S&Pot	4.793	0.613	0.770	2.024	<0.001	0.979	<0.001	1.237	7.384	0.717	3.912	0.001	0.684	0.844	0.985	<0.0001
Futures	4.792	0.601	0.811	2.096	<0.001	0.983	<0.001	1.217	7.142	0.646	3.542	0.006	0.829	0.698	0.967	<0.0001
S&P500																
S&Pot	7.095	0.151	-0.466	2.525	0.056	0.411	<0.001	0.285	4.132	-0.275	3.280	0.370	0.946	0.001	0.738	<0.0001
Futures	7.100	0.154	-0.456	2.507	0.060	0.435	<0.001	0.279	4.096	-0.219	3.180	0.558	0.954	<0.001	0.680	<0.0001
FTSE100																
S&Pot	8.596	0.164	-0.690	2.357	0.002	0.602	<0.001	0.126	4.158	-0.742	4.260	0.000	0.605	0.057	0.995	<0.0001
Futures	8.604	0.169	-0.698	2.394	0.002	0.622	<0.001	0.120	4.198	-0.792	4.211	0.000	0.626	0.043	0.998	<0.0001
TOPIX																
S&Pot	7.145	0.213	-0.166	2.097	0.088	0.639	<0.001	0.154	5.018	0.017	3.082	0.980	0.672	0.867	0.326	<0.0001
Futures	7.143	0.214	-0.166	2.094	0.087	0.521	<0.001	0.144	4.973	-0.006	2.970	0.997	0.602	0.843	0.219	<0.0001

Panel B: Instruments	Return %								
	Mean	Std. Dev.	Skewness	Excess kurtosis	Normality Test	Q(26)	Q^2(26)	ARCH LM(4)	Unit root test I(0)
Daily Horizon									
USDGBP	0.0069	0.504	2.522	9.629	-0.040	3.693	<0.0001	0.673	<0.0001
USDJPY	0.0076	0.684	-2.763	13.058	-0.758	11.394	<0.0001	0.150	<0.0001
GBPJPY	0.0008	0.720	-0.306	13.746	-0.659	9.041	<0.0001	0.382	<0.0001
Weekly Horizon									
USDGBP	0.034	1.107	-1.778	7.981	0.096	3.225	0.368	0.166	0.479
USDJPY	0.039	1.620	-2.038	11.683	-1.103	11.468	0.000	0.378	<0.0001
GBPJPY	0.005	1.722	-0.247	12.420	-1.382	12.396	0.000	0.006	0.015
Monthly Horizon									
USDGBP	-0.140	2.125	-1.676	7.359	-0.175	2.920	0.715	0.008	0.010
USDJPY	-0.179	3.249	-2.144	11.255	-0.887	7.186	<0.0001	0.002	0.585
GBPJPY	-0.036	3.297	-0.427	11.420	-1.086	8.766	<0.0001	0.128	0.981

Panel C: Assets' Correlations

	SP500 Spot	SP500 Futures	FTSE Spot	FTSE Futures	TOPIX Spot	TOPIX Futures	USDGBP	USDJPY	GBPJPY
SP500 Spot	1	0. 9996	0. 894	0. 889	0. 872	0. 872	0. 355	0. 345	-0.059
SP500 Futures		1	0. 900	0. 895	0. 874	0. 874	0. 346	0. 340	-0.053
FTSE Spot			1	0. 999	0. 837	0. 836	0. 130	0. 044	-0.046
FTSE Futures				1	0. 831	0. 830	0. 116	0. 040	-0.035
TOPIX Spot					1	1. 000	0. 369	0. 196	-0.186
TOPIX Futures						1	0. 370	0. 196	-0.187
USDGBP							1	0. 357	-0.762
USDJPY								1	0.322
GBPJPY									1

	COMEX Spot	COMEX Futures	LME Spot	LME Futures	USDGBP
COMEX Spot	1	0.999	0.997	0.997	0.818
COMEX Futures		1	0.996	0.996	0.817
LME Spot			1	0.9995	0.785
LME Futures				1	0.779
USDGBP					1

Table II: Parameter Estimation Results

Spot Markets	Futures Markets	Hedged Portfolio	Mean Equations	Daily Horizon				Weekly Horizon				Monthly Horizon			
				μ	φ	a_i	b_i	μ	φ	a_i	b_i	μ	φ	a_i	b_i
COMEX	LME														
		OHR	r_{st}	-0.048**	-0.012**	0.021**	0.976**	0.0634*	0.0156	0.041**	0.954**	0.3219*	0.0786	0.0011*	1.014**
			r_{f*t}	0.0341**	0.0086**	0.024**	0.972**	0.2549**	0.0643**	0.041**	0.960**	0.8389**	0.2115**	0.0002*	1.012**
		IOHR	r_{st}	-0.046**	-0.011**	0.020**	0.978**	0.0889*	0.0222	0.032**	0.964**	0.4975*	0.1235	0.126**	0.879**
			r_{f*t}	0.0340**	0.0086**	0.022**	0.976**	0.2162**	0.0543**	0.032**	0.972**	0.6096**	0.1522**	0.081**	0.952**
			r_{et}	1.000111**		0.021**	0.948**	1.000158**		0.021**	0.966**	0.999656**		0.065**	0.908**
LME	COMEX														
		OHR	r_{st}	0.0278*	-0.0086*	0.023**	0.974**	0.2414**	-0.074**	0.020**	0.982**	0.7423**	-0.229**	0.0015*	1.014**
			r_{f*t}	-0.059**	0.0188**	0.021**	0.976**	0.0177*	-0.0045	0.026**	0.970**	0.0817**	-0.0213	0.0099*	1.004**
		IOHR	r_{st}	0.0279*	-0.0086*	0.021**	0.976**	0.2004**	-0.061**	0.022**	0.984**	0.4504**	-0.1376	0.076**	0.956**
			r_{f*t}	-0.058**	0.0182**	0.020**	0.978**	0.0290*	-0.0081	0.025**	0.972**	0.2414**	-0.0728	0.115**	0.889**
			r_{et}	0.999902**		0.019**	0.954**	0.999836**		0.028**	0.958**	1.000148**		0.064**	0.906**
S&P500	FTSE100														
		OHR	r_{st}	0.0039**	-0.014**	0.044**	0.950**	0.0093**	-0.033**	0.095**	0.913**	-0.000*	0.0117	0.077**	0.908**
			r_{f*t}	0.0001*	0.0006	0.075**	0.919**	0.0001*	0.0022	0.121**	0.887**	-0.030**	0.0999*	0.124**	0.863**
		IOHR	r_{st}	0.0038**	-0.013**	0.040**	0.954**	0.0098**	-0.035**	0.091**	0.917**	-0.014*	0.0476	0.0014*	0.994**
			r_{f*t}	0.0001*	0.0005	0.073**	0.921**	0.0008*	-0.0008	0.108**	0.900**	-0.026**	0.085*	0.01279	0.966**
			r_{et}	1.000111**		0.018**	0.960**	1.000105**		0.020**	0.952**	0.998557**		0.04074	0.921**
FTSE100	S&P500														
		OHR	r_{st}	0.0028*	-0.0015	0.073**	0.921**	0.0349*	-0.0198	0.103**	0.898**	0.2254**	-0.124**	0.126**	0.855**
			r_{f*t}	-0.018**	0.0105**	0.050**	0.944**	-0.018*	0.0108	0.084**	0.923**	0.0931*	-0.0503	0.069**	0.915**
		IOHR	r_{st}	0.0026*	-0.0013	0.071**	0.923**	0.0348*	-0.0195	0.096**	0.908**	0.2142*	-0.118**	0.01286	0.962**
			r_{f*t}	-0.018**	0.0104**	0.046**	0.948**	-0.013*	0.0082	0.079**	0.927**	0.1321*	-0.0727*	0.0014*	0.994**
			r_{et}	0.999891**		0.018**	0.960**	0.999764**		0.017**	0.944**	1.001443**		0.03797	0.921**
S&P500	TOPIX														
		OHR	r_{st}	0.0206**	-0.007**	0.046**	0.950**	0.0438*	-0.0162	0.078**	0.919**	0.1420**	-0.0526	0.09**	0.870**
			r_{f*t}	-0.014**	0.00584*	0.066**	0.931**	-0.123**	0.0470**	0.059**	0.929**	-0.275**	0.1051**	0.0047*	0.974**
		IOHR	r_{st}	0.0201**	-0.007**	0.046**	0.950**	0.0398*	-0.0153	0.074**	0.923**	0.2966**	-0.110**	0.219**	0.734**
			r_{f*t}	-0.012**	0.00493*	0.065**	0.931**	-0.132**	0.0505**	0.065**	0.919**	-0.199**	0.0781**	0.01115	1.042**
			r_{et}	0.999998**		0.013**	0.982**	1.000402**		0.060**	0.902**	0.9968**		0.033**	0.921**
TOPIX	S&P500														
		OHR	r_{st}	-0.007**	-0.004**	0.062**	0.927**	-0.067**	-0.043**	0.062**	0.925**	-0.142**	-0.093**	0.029**	0.952**
			r_{f*t}	0.0056**	0.0034**	0.053**	0.940**	0.0016*	0.0008	0.084**	0.917**	0.0436*	0.0257	0.086**	0.902**
		IOHR	r_{st}	-0.007**	-0.004**	0.060**	0.927**	-0.069**	-0.044**	0.066**	0.919**	-0.153**	-0.098**	0.025**	0.946**
			r_{f*t}	0.0055**	0.0033**	0.052**	0.942**	-0.000*	-0.00002	0.075**	0.925**	-0.002*	-0.0012	0.0025*	1.002**
			r_{et}	0.999978**		0.015**	0.980**	0.999557**		0.060**	0.900**	1.002674**		0.115**	0.898**
FTSE100	TOPIX														
		OHR	r_{st}	0.0302**	-0.007**	0.070**	0.925**	0.0712**	-0.0184*	0.121**	0.877**	0.3645**	-0.0933*	0.01511	0.978**
			r_{f*t}	-0.015*	0.0042	0.066**	0.921**	-0.119**	0.0304**	0.076**	0.910**	-0.169*	0.0442	0.0068*	1.022**
		IOHR	r_{st}	0.0279**	-0.007**	0.065**	0.929**	0.0707**	-0.0184*	0.116**	0.883**	0.3026**	-0.0776*	0.087**	0.904**
			r_{f*t}	-0.020**	0.0055**	0.054**	0.935**	-0.125**	0.0319**	0.080**	0.904**	-0.103*	0.0266	0.089**	0.846**
			r_{et}	0.999865**		0.021**	0.976**	0.999884**		0.042**	0.944**	0.998904**		0.01304	0.960**
TOPIX	FTSE100														
		OHR	r_{st}	-0.006**	-0.003**	0.057**	0.927**	-0.062**	-0.030**	0.080**	0.908**	-0.076*	-0.0370	0.0142*	0.998**
			r_{f*t}	0.0090**	0.0042**	0.066**	0.929**	0.0063*	0.0031	0.126**	0.874**	0.1050*	0.0504	0.0031*	0.994**
		IOHR	r_{st}	-0.008**	-0.004**	0.049**	0.938**	-0.064**	-0.031**	0.082**	0.904**	-0.077*	-0.0385	0.052**	0.966**
			r_{f*t}	0.0085**	0.0040**	0.061**	0.933**	0.0046*	0.0024	0.121**	0.879**	0.0781*	0.0367	0.0003*	1.002**
			r_{et}	1.000126**		0.021**	0.974**	1.000003**		0.042**	0.944**	1.000722**		0.0001*	0.942**

Notes: OHR: the portfolio, estimated from the conventional seperate hedge strategy with currency risk hedged in the currency derivative market. IOHR: the portfolio, estimated from the proposed approach of international optimal hedge ratio in this paper. ** and * represent the statistical significance at 5% and 10% levels, respectively. a_i and b_i are estimated coefficients from M-VEC-GARCH variance equations as in the equation (8). μ and φ are coefficients from VEC-MGARCH estimation. r represents the corresponding mean equation as in the equation (7). r_{et} specially is a gross return.

Table III: Hedge Effectiveness

			Daily Horizon				Weekly Horizon				Monthly Horizon			
Spot Markets	**Futures Markets**	**Portfolios**	Mean %	Variance %	Variance reduction %	Expected Utility	Mean %	Variance %	Variance reduction %	Expected Utility	Mean %	Variance %	Variance reduction %	Expected Utility
COMEX	LME													
		Unhedged	0.0661	0.0459		-0.117	0.3312	0.2108		-0.512	1.0972	0.9040		-2.519
		IOHR hedge	0.0278	0.0213	53.6	-0.057	0.0406	0.0386	81.7	-0.114	0.1971	0.0852	90.6	-0.144
		COHR hedge	0.0276	0.0214	53.3	-0.058	0.0654	0.0471	77.7	-0.123	0.1547	0.0979	89.2	-0.237
		POHR hedge	0.0286	0.0237	48.3	-0.066	0.0705	0.0545	74.1	-0.148	0.1337	0.1189	86.9	-0.342
LME	COMEX													
		Unhedged	0.0460	0.0405		-0.116	0.2296	0.1925		-0.540	1.2423	0.8261		-2.062
		IOHR hedge	0.0301	0.0173	57.4	-0.039	0.0373	0.0325	83.1	-0.093	0.1265	0.0579	93.0	-0.105
		COHR hedge	0.0271	0.0177	56.2	-0.044	0.0382	0.0405	79.0	-0.124	-0.0004	0.0883	89.3	-0.353
		POHR hedge	0.0261	0.0198	51.1	-0.053	0.0331	0.0474	75.4	-0.157	0.0206	0.1082	86.9	-0.412
SP500	FTSE100													
		Unhedged	0.0170	0.0265		-0.089	0.0890	0.1285		-0.425	0.1453	0.3915		-1.421
		IOHR hedge	0.0089	0.0118	55.6	-0.038	0.0385	0.0352	72.6	-0.102	0.1743	0.0594	84.82	-0.063
		COHR hedge	0.0071	0.0120	54.8	-0.041	0.0575	0.0311	75.8	-0.067	0.2137	0.0595	84.79	-0.024
		POHR hedge	0.0081	0.0137	48.4	-0.047	0.0626	0.0372	71.0	-0.086	0.1927	0.0761	80.6	-0.112
FTSE100	SP500													
		Unhedged	-0.0042	0.0276		-0.115	-0.0206	0.1363		-0.566	0.2652	0.3947		-1.314
		IOHR hedge	-0.0025	0.0122	55.7	-0.051	-0.0094	0.0360	73.6	-0.154	-0.0776	0.0568	85.6	-0.305
		COHR hedge	-0.0032	0.0126	54.3	-0.054	-0.0363	0.0345	74.7	-0.174	-0.2034	0.0596	84.9	-0.442
		POHR hedge	-0.0042	0.0144	47.9	-0.062	-0.0414	0.0409	70.0	-0.205	-0.1824	0.0761	80.7	-0.487
SP500	TOPIX													
		Unhedged	0.0177	0.0327		-0.113	0.0940	0.1680		-0.578	0.1063	0.5448		-2.073
		IOHR hedge	0.0118	0.0113	65.5	-0.033	0.0414	0.0533	68.3	-0.172	0.2381	0.1339	75.4	-0.297
		COHR hedge	0.0120	0.0115	65.0	-0.034	0.0251	0.0493	70.7	-0.172	0.2368	0.1302	76.1	-0.284
		POHR hedge	0.0131	0.0138	58.0	-0.042	0.0310	0.0607	63.9	-0.212	0.2100	0.1678	69.2	-0.461
TOPIX	SP500													
		Unhedged	-0.0033	0.0378		-0.155	-0.0127	0.2228		-0.904	0.3331	0.6834		-2.400
		IOHR hedge	0.0021	0.0143	62.1	-0.055	0.0130	0.0760	65.9	-0.291	0.1596	0.2102	69.2	-0.681
		COHR hedge	-0.0056	0.0169	55.4	-0.073	0.0004	0.0757	66.0	-0.302	0.0096	0.2079	69.6	-0.822
		POHR hedge	-0.0067	0.0196	48.1	-0.085	-0.0055	0.0897	59.7	-0.364	0.0364	0.2547	62.7	-0.982
FTSE100	TOPIX													
		Unhedged	0.0035	0.0353		-0.138	0.0184	0.1768		-0.689	0.0899	0.5557		-2.133
		IOHR hedge	0.0039	0.0127	64.1	-0.047	-0.0182	0.0542	69.3	-0.235	0.0385	0.1281	76.9	-0.474
		COHR hedge	0.0062	0.0132	62.6	-0.047	-0.0144	0.0525	70.3	-0.224	0.0862	0.1374	75.3	-0.463
		POHR hedge	0.0063	0.0158	55.3	-0.057	-0.0137	0.0642	63.7	-0.271	0.0809	0.1765	68.2	-0.625
TOPIX	FTSE100													
		Unhedged	0.0034	0.0392		-0.153	0.0218	0.2325		-0.908	0.1900	0.6913		-2.575
		IOHR hedge	0.0090	0.0145	63.0	-0.049	0.0007	0.0771	66.9	-0.308	0.1623	0.2004	71.02	-0.639
		COHR hedge	0.0100	0.0147	62.4	-0.049	-0.0140	0.0771	66.8	-0.322	0.0699	0.2001	71.05	-0.731
		POHR hedge	0.0099	0.0175	55.4	-0.060	-0.0147	0.0921	60.4	-0.383	0.0752	0.2468	64.3	-0.912

Mean and standard deviation value are in percentage. Unhedged is a portfolio without hedge. IOHR hedge is the hedged portfolio estimated by the proposed alternative approach in this paper. COHR hedge is the hedged portfolio in which the currency risk is completely and seperately hedged in currency future markets. POHR hedge is the hedged portfolio in which the currency risk is partially and seperately hedged in currency future markets. Variance reduction is in percentage form. The expected utility is computed by the equation (2) in the return form.

Table IV: the Significant Tests of the Vairance Difference

			Daily Horizon		Weekly Horizon		Monthly Horizon	
Spot Markets	**Futures Markets**	**Portfolios**	**Variance Difference**	**t-statistic**	**Variance Difference**	**t-statistic**	**Variance Difference**	**t-statistic**
COMEX	LME							
		COHR hedge	0.0002	0.17	0.0084**	7.34	0.0126**	3.88
		POHR hedge	0.0024**	2.64	0.0159**	13.84	0.0336**	10.29
LME	COMEX							
		COHR hedge	0.0005	0.78	0.0080**	8.38	0.0303**	18.63
		POHR hedge	0.0025**	4.21	0.0149**	15.65	0.0503**	30.88
SP500	FTSE100							
		COHR hedge	0.0002	0.40	-0.0041*	-1.86	0.0001	0.08
		POHR hedge	0.0019**	3.74	0.0020	0.93	0.0166**	13.07
FTSE100	SP500							
		COHR hedge	0.0004	0.56	-0.0016	-0.63	0.0027**	2.78
		POHR hedge	0.0021**	3.05	0.0048*	1.92	0.0193**	19.33
SP500	TOPIX							
		COHR hedge	0.0002	0.33	-0.0040	-1.41	-0.0036	-0.44
		POHR hedge	0.0024**	4.39	0.0073**	2.58	0.0339**	4.13
TOPIX	SP500							
		COHR hedge	0.0025**	5.05	-0.0003	-0.17	-0.0023	-0.51
		POHR hedge	0.0053**	10.55	0.0136**	7.48	0.0444**	9.78
FTSE100	TOPIX							
		COHR hedge	0.0005	0.68	-0.0017	-0.59	0.0092*	1.85
		POHR hedge	0.0030**	4.18	0.0100**	3.43	0.0483**	9.66
TOPIX	FTSE100							
		COHR hedge	0.0002	0.55	0.00003	0.01	-0.0002	-0.07
		POHR hedge	0.0029**	6.65	0.0150**	8.08	0.0464**	14.64

Note: the variance difference is the difference between IOHR hedged and each of other strategies. * is statistically significant at 10% level. ** is statistically significant at 5% level

Table V: Alternative Measures of Hedge Efficiency and Effectiveness

	COMEX-LME		LME-COMEX		S&P500-FTSE100		FTSE100-S&P500		S&P500-TOPIX		TOPIX-S&P500		FTSE100-TOPIX		TOPIX-FTSE100	
	COHR	IOHR	COHR	IOHR	COHR	IOHR	COHR	IOHR	COHR	IOHR	COHR	IOHR	COHR	IOHR	COHR	IOHR
Daily Horizon																
EOHR	0.939	0.855	0.947	0.886	0.824	0.725	0.879	0.826	0.808	0.745	1.242	1.043	0.805	0.690	1.087	0.912
EER	1.763	1.595	1.684	1.544	1.503	1.304	1.618	1.482	1.243	1.136	2.243	1.679	1.285	1.077	1.743	1.448
Weekly Horizon																
EOHR	1.456	1.307	1.252	1.262	1.307	1.112	1.597	1.418	0.804	0.721	1.765	1.262	1.052	0.945	1.776	1.281
EER	1.875	1.600	1.585	1.518	1.725	1.531	2.137	1.927	1.138	1.056	2.674	1.916	1.497	1.363	2.657	1.917
Monthly Horizon																
EOHR	1.310	1.415	1.216	1.423	1.364	0.991	1.669	1.475	0.568	0.630	0.978	0.886	0.978	0.840	1.339	0.700
EER	1.469	1.562	1.361	1.530	1.609	1.169	1.959	1.723	0.747	0.836	1.406	1.280	1.300	1.091	1.885	0.985

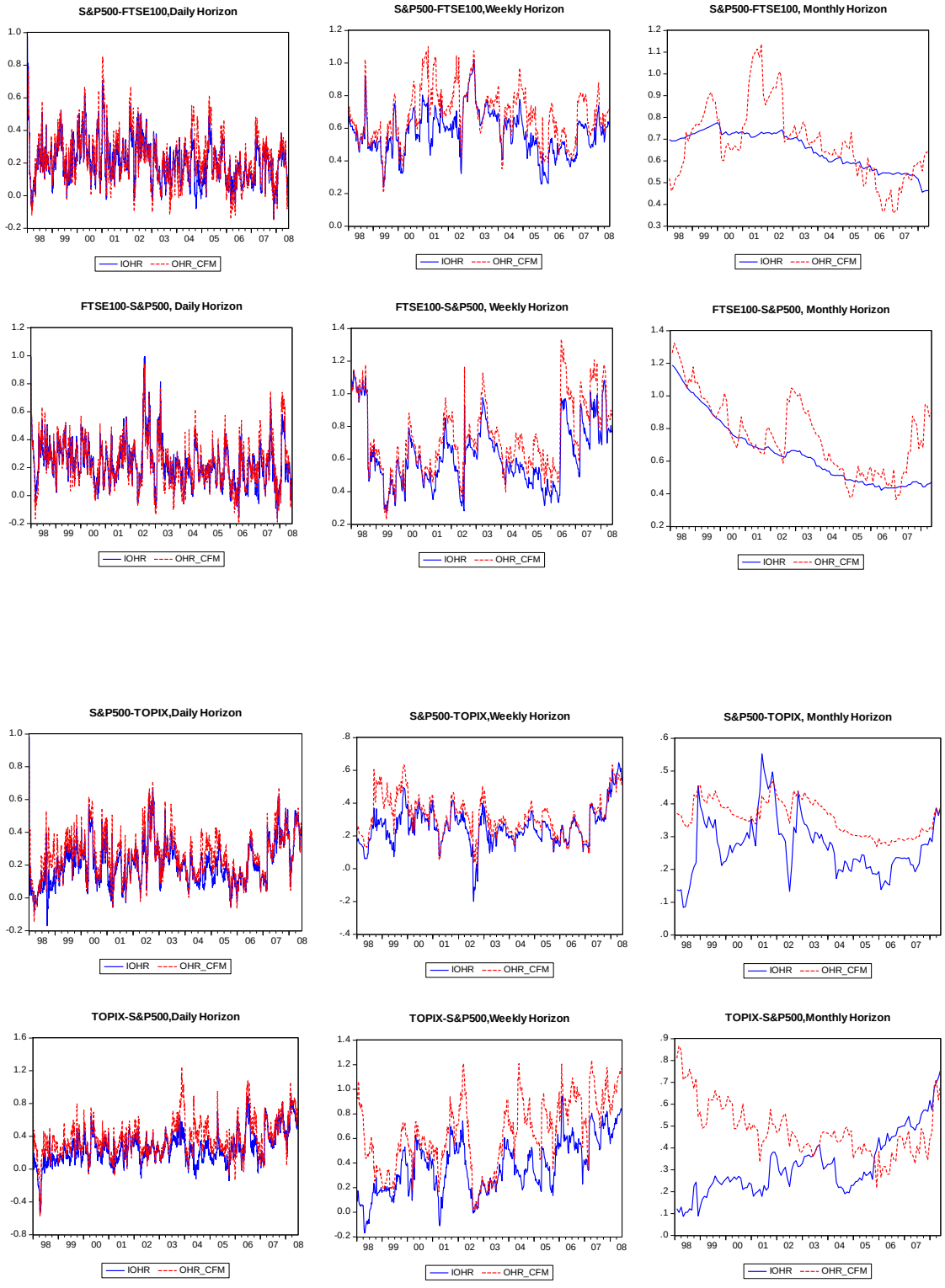
Notes: EOHR is the efficiency of the estimated optimal hedge ratio. EER is the efficiency-effectiveness ratio of the hedged portfolio.

Table VI: Time varying optimal hedge ratio statistical description

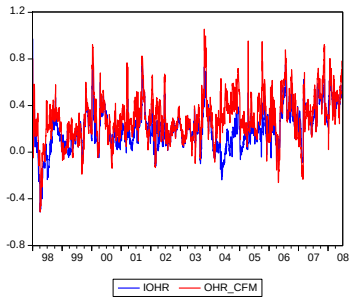
	COMEX-LME		LME-COMEX		S&P500-FTSE100		FTSE100-S&P500		S&P500-TOPIX		TOPIX-S&P500		FTSE100-TOPIX		TOPIX-FTSE100	
	OHR	IOHR	OHR	IOHR	OHR	IOHR	OHR	IOHR	OHR	IOHR	OHR	IOHR	OHR	IOHR	OHR	IOHR
Daily Horizon																
Mean	0.526	0.477	0.471	0.441	0.226	0.207	0.234	0.228	0.256	0.211	0.373	0.256	0.188	0.164	0.291	0.204
Median	0.526	0.475	0.473	0.444	0.214	0.202	0.221	0.216	0.249	0.195	0.339	0.226	0.169	0.141	0.275	0.195
Maximum	0.872	0.813	0.807	0.780	0.850	0.979	0.942	0.995	0.804	0.983	1.249	0.976	0.804	0.993	1.055	0.973
Minimum	0.188	0.172	0.180	0.136	-0.148	-0.141	-0.190	-0.163	-0.146	-0.170	-0.580	-0.527	-0.272	-0.189	-0.513	-0.513
Std. Dev.	0.103	0.094	0.119	0.111	0.150	0.129	0.161	0.149	0.138	0.133	0.217	0.197	0.154	0.132	0.199	0.177
Skewness	-0.035	0.139	0.020	-0.011	0.569	0.752	0.573	0.807	0.296	0.684	0.420	0.444	0.602	1.225	0.206	0.128
Kurtosis	2.784	3.110	2.593	2.932	3.835	4.661	3.824	4.881	2.990	3.532	4.479	4.240	3.798	5.783	4.112	3.978
Jarque-Bera	5.872	10.246	19.044	0.573	227.259	572.685	227.255	700.233	39.909	245.844	330.021	265.306	237.675	1567.05	160.446	116.529
Probability	0.053	0.006	0.000	0.751	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Weekly Horizon																
Mean	0.810	0.829	0.764	0.779	0.6728	0.5734	0.7293	0.6300	0.322	0.257	0.642	0.3753	0.339	0.296	0.5940	0.3739
Median	0.820	0.826	0.783	0.797	0.674	0.574	0.691	0.593	0.313	0.251	0.658	0.366	0.317	0.267	0.595	0.403
Maximum	1.184	1.109	0.975	1.007	1.101	1.024	1.335	1.143	0.636	0.648	1.236	0.945	1.023	0.950	1.563	1.031
Minimum	0.381	0.496	0.441	0.440	0.206	0.238	0.230	0.273	0.005	-0.200	0.024	-0.170	-0.039	0.005	-0.079	-0.276
Std. Dev.	0.162	0.119	0.122	0.121	0.159	0.135	0.217	0.197	0.121	0.116	0.281	0.222	0.178	0.162	0.295	0.227
Skewness	-0.316	-0.301	-0.891	-0.696	0.143	0.194	0.450	0.584	0.293	0.356	-0.100	0.065	1.108	1.180	0.133	-0.217
Kurtosis	3.326	3.324	3.360	3.083	2.864	3.418	2.571	2.604	2.822	5.002	2.208	2.372	4.969	4.670	2.641	2.661
Jarque-Bera	11.520	10.609	75.127	44.237	2.3	7.4	22.6	34.6	8.5	102.7	15.2	9.4	200.0	190.2	4.5	6.9
Probability	0.003	0.005	0.000	0.000	0.320	0.025	0.000	0.000	0.014	0.000	0.001	0.009	0.000	0.000	0.103	0.032
Monthly Horizon																
Mean	0.934	0.896	0.814	0.868	0.681	0.650	0.759	0.653	0.3582	0.2712	0.486	0.321	0.484	0.275	0.751	0.430
Median	0.947	0.904	0.843	0.897	0.662	0.692	0.731	0.628	0.362	0.272	0.469	0.280	0.411	0.2519	0.686	0.442
Maximum	1.064	1.208	0.947	1.062	1.137	0.776	1.324	1.189	0.468	0.552	0.867	0.758	0.785	0.560	1.166	0.557
Minimum	0.752	0.527	0.620	0.415	0.360	0.456	0.363	0.421	0.270	0.084	0.217	0.087	0.319	0.034	0.599	0.152
Std. Dev.	0.094	0.130	0.100	0.139	0.171	0.085	0.228	0.206	0.053	0.090	0.123	0.141	0.124	0.141	0.147	0.067
Skewness	-0.336	-0.427	-0.492	-1.263	0.521	-0.448	0.360	0.902	0.088	0.624	0.812	0.833	0.906	0.204	1.349	-2.231
Kurtosis	1.814	3.078	1.903	4.400	3.073	1.890	2.385	2.870	1.806	3.388	3.564	3.374	2.723	2.074	3.634	8.642
Jarque-Bera	9.7	3.8	11.3	43.4	5.7	10.6	4.7	17.0	7.6	8.9	15.4	15.2	17.5	5.330	40.0	269.5
Probability	0.008	0.148	0.004	0.000	0.059	0.005	0.096	0.000	0.022	0.012	0.000	0.001	0.000	0.1	0.000	0.000

Notes: IOHR is the international optimal hedge ratio estimated by the proposed alternative approach in this paper. OHR is the optimal hedge ratio estimated by the conventional separate hedge strategy by which the currency risk is seperately hedged in currency derivative markets.

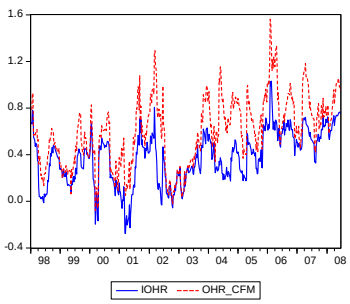
Figure 1: Time-varying international optimal hedge ratios. IOHR is the international optimal hedge ratio estimated by the proposed alternative approach in this paper. OHR-CFM is the optimal hedge ratio estimated by the conventional hedge strategy in which the currency risk is hedged in currency derivative markets.



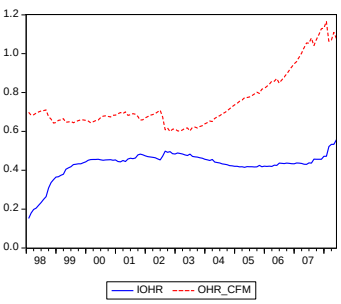
TOPIX-FTSE100, Daily Horizon



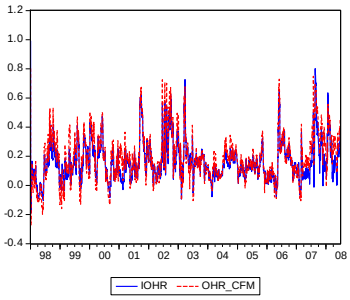
TOPIX-FTSE100,Weekly Horizon



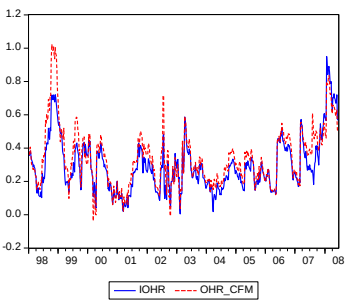
TOPIX-FTSE100,Monthly Horizon



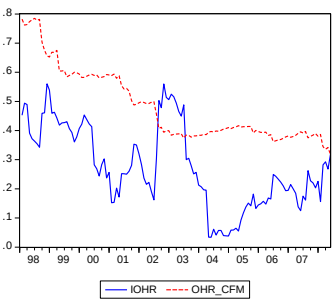
FTSE100-TOPIX,Daily Horizon



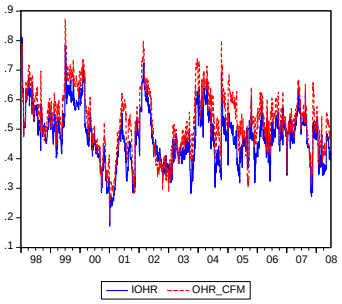
FTSE100-TOPIX,Weekly Horizon



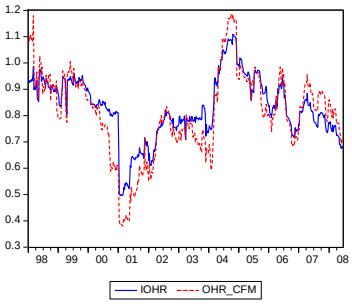
FTSE100-TOPIX,Monthly Horizon



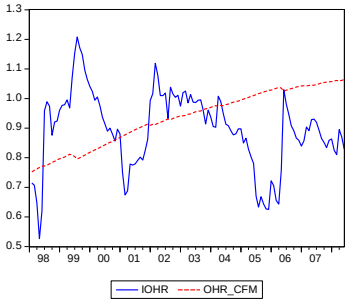
COMEX-LME,Daily Horizon



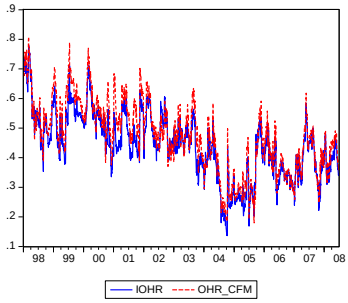
COMEX-LME,Weekly Horizon



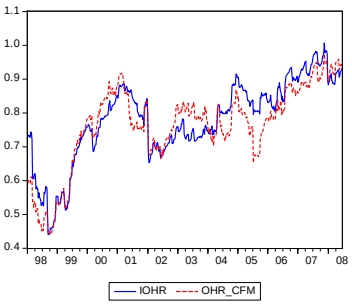
COMEX-LME,Monthly Horizon



LME-COMEX,Daily Horizon



LME-COMEX, Weekly Horizon



LME-COMEX,Monthly Horizon