Neubauer, Silke

Working Paper

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Discussion papers // WZB, Wissenschaftszentrum Berlin für Sozialforschung, Forschungsschwerpunkt Marktprozeß und Unternehmensentwicklung, No. FS IV 97-34

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discussion papers

FS IV 97 - 34

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Silke Neubauer

December 1997

ISSN Nr. 0722 - 6748

**Forschungsschwerpunkt
Marktprozeß und Unternehmensentwicklung**

**Research Area
Market Processes and
Corporate Development**

Zitierweise/Citation:

Silke Neubauer, **The Consequences of Endogenous Timing for Diversification Strategies of Multimarket Firms,** Discussion Paper FS IV 97 - 34, Wissenschaftszentrum Berlin, 1997.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,
Reichpietschufer 50, 10785 Berlin, Tel. (030) 2 54 91 - 0

ABSTRACT

The Consequences of Endogenous Timing for Diversification Strategies of Multimarket Firms

by Silke Neubauer

When firms diversify into new markets in spite of the existence of diseconomies of scope, not only firms' profits are affected, but also potential welfare is reduced. Nevertheless, multimarket competition is the outcome of a game when players move simultaneously. A Cournot model is developped where players can choose the timing of their action before deciding over quantities. This helps firms to avoid the inefficiencies that ocur with multimarket competition. Whenever the timing game has an impact on the outcome of the basic game, the consequences for welfare are positive.

ZUSAMMENFASSUNG

Die Bedeutung sequentieller Angebotsentscheidungen für diversifizierte Unternehmen

Die Diversifikation von Unternehmen in neue Märkte führt zu Effizienzverlusten und Gewinneinbußen, wenn dadurch Spezialisierungsvorteile nicht mehr genutzt werden können. Dennoch ist Diversifikation das Ergebnis eines simultanen Cournotspiels, in welchem sich zwei Unternehmen auf zwei Märkten als potentielle Wettbewerber gegenüberstehen. Die Einführung einer Vorstufe zum Cournotspiel, in welcher Unternehmen den Zeitpunkt ihrer Angebotsentscheidung in beiden Märkten wählen können, kann einen Teil dieser Ineffizienzen beseitigen. Unternehmen konzentrieren sich auf je einen Markt. Wegen der Existenz potentieller Konkurrenz sind die Wohlfahrtswirkungen trotz der resultierenden monopolistischen Marktstruktur im Vergleich zum Ausgangsspiel positiv.

1. Introduction

The reason for conglomerate firms to be active in several markets can often be found in the existence of economies of scope. Especially when firms diversify from a particular technology base to produce a wider range of outputs, they might be able to take advantage of cost complementaries and thus increase their cost-efficiency. But firms enter new markets not only when there are synergies on the cost- or demand side. In fact, it may be rational for firms to compete in several markets even when there are diseconomies of scope. In oligopolistic markets, marginal gains arising from entering a second market may outweigh the marginal negative impact on production costs caused by diseconomies of scope such that the result is multimarket competition.[1] The total effect on profits might well be negative in comparison to the situation, where each firm specializes in one market. Taking the latter situation (specialization) as a benchmark, the effect on welfare is twofold: there is a competition effect of diversification that might lead to higher total output in each of the markets, but there is also a negative efficiency effect reducing potential welfare.

A crucial reason for multimarket competition in the presence of diseconomies of scope is the lack of commitment power when firms choose their strategies simultaneously. If they were able to commit to specialize in one market, each firm would be able to increase its profits.

One way of modeling commitment power is through introducing sequential play. In Stackelberg- or Price-Leadership games, a leading firm has the opportunity to commit to an action before the other firms move. Usually, one of the roles - the leader- or the follower-role - is preferred to the other. Whenever there are no institutionalized reasons for one firm being the leader or the follower, this raises the question, who determines the order of moves.

The theory on endogenous timing tries to solve this question by considering Stackelberg- or Price-Leadership equilibria as the outcomes of an extended game, where firms first decide over the timing of their action and then play the basic game (Cournot- or Bertrand) according to the chosen timing.[2] It turns out that the conditions for sequential play being the outcome of an endogenous timing game are very restrictive. In standard Cournot models, simultaneous play remains the result of the extended game.

Empirical evidence predicts, that multipoint competition may help firms to assume market leadership in different products catered to different markets. For example, Proctor & Gamble and Kimberley Clark divided up leadership in the markets for disposable diapers and feminine napkins.[3] Brandenburger / Nalebuff (1996) describes the case of two railroads competing to service public utilities that use the laying of tracks as a first stage move in dividing markets.

In this paper the possibility to endogenously time Cournot strategies is introduced in a

[1] See Dixon (1992), but also Bulow/ Geneakoplos/ Klemperer (1985) who implicitly adress this topic.
[2] Another way to decide about the order of moves is by applying an indirect evolutionary approach. See Güth (1997) for an evolutionary analisis of bargaining rules.
[3] See Hughes / Kao (1997) for this example.

context, where two two-product firms facing negative cost-linkages across markets compete against each other in each market. It thus combines two lines of research: The theory of multiproduct firms and multimarket contact and the theory of the endogenous determination of simultaneous or sequential play in a Cournot game.

The former line of research has concentrated so far on two aspects:

One is the potential interrelation of markets through cost-or demand-linkages. The concept applied on the cost-side is that of (dis-)economies of scope.[4] It describes the situation, where the scope of the firm determines the costs incurred in each product-line. On the demand-side, there may be intermarket-linkages caused by products being complements or substitutes, reputation- and bandwagon-effects etc. Because of intermarket-relations, firms cannot decide independently in each market, but have to take into account the potential effects in the related market(s).[5]

The second aspect dealt with in the theory of multiproduct firms is the effect of multimarket contact on collusion. When the same firms meet in several markets, they are able to retaliate against aggressive strategies of the other firm in any of the markets, where they compete. This might foster tacit collusion whenever there is ”slack enforcement power” which can be transferred from one market to another.[6]

The theory of endogenous timing evolved only recently. Hamilton /Slutzky (1990) were among the first who formally developed a duopoly-game in which players can choose to move ”early” or ”late” in a first stage and decide over their quantity after having observed the other player’s timing in a second stage. Robson (1990) considered endogenous timing in a price-setting game when there are time-dependent costs of precommitment in a second stage. Other authors took up the idea of endogenous timing and introduced imperfect information about costs (Albaeck (1990)) or demand (Mailath (1993)) or applied it to the field of Political Economy (Leininger (1993)). Nevertheless, the implications of endogenous timing in a two-market context have been neglected so far.

Building on these two research lines, this paper addresses the following questions:

* Does the possibility to endogenously time Cournot strategies affect the incentives to diversify in other markets if diseconomies of scope are present?

* Does the multimarket context affect firms’ incentives to move sequentially instead of simultaneously?

* What are the welfare effects of conglomerate diversification in this extended game with respect to a) conglomerate diversification without timing possibility, and b) two separated monopolies in each market?

The main results and the organization of this paper are as follows: In chapter 2 the original game of Hamilton/ Slutzky and its results are presented. It turns out that the conditions for sequential play being the outcome of an endogenous timing game are very restrictive. In

[4] See Baumol/Panzar/ Willig (1982) for a detailed overview.
[5] See Bulow/ Geneakoplos / Klemperer (1985).
[6] See Bernheim/ Whinston (1990) for a formal and detailed discussion of this topic.

standard Cournot models, simultaneous play remains the result of the extended game.

The implications of the timing-game in a two-market-context with diseconomies of scope are considered in chapter 3. After having introduced the basic model, it is shown, that, depending on importance of the diseconomies, there are different equilibria of the timing game: as long as the negative cost effect is small, firms still choose to move simultaneously as in the one-stage game. In a middle range, there are three potential equilibria of the timing game: one with firms playing simultaneously at the early timing, and two symmetric sequential play equilibria with each firm choosing "early" in one (distinct) market and late in the other. Nevertheless, by allowing pre-play communication, the simultaneous play equilibrium can be ruled out: both firms prefer the sequential play equilibria. When the parameter expressing the level of diseconomies of scope reaches a critical value, simultaneous play ceases to be an equilibrium. But in addition to the sequential play equilibria with each firm leading in one market, there are two other (symmetric) equilibria, where one firm leads in both markets. In this range, firms cease to compete in both markets: there is a mutual entry threat, but both firms actually concentrate each on one market. Finally, when the diseconomies of scope are very high, the timing game ceases to have an impact on the outcome of the basic game. Each firm acts as a monopolist in one of the markets regardless of the timing strategies of the first stage.

In chapter 4, the effects of the extended game on welfare are analyzed. It can be shown that whenever the timing-opportunity leads to outcomes different from a one-stage game without timing possibility, firms' profits as well as consumer surplus is increased. Social surplus is also bigger than or equal to surplus generated by an institutionalized two-market-monopoly with two firms. Consequently, the timing-game helps approaching the first-best-solution whenever there are diseconomies of scope.

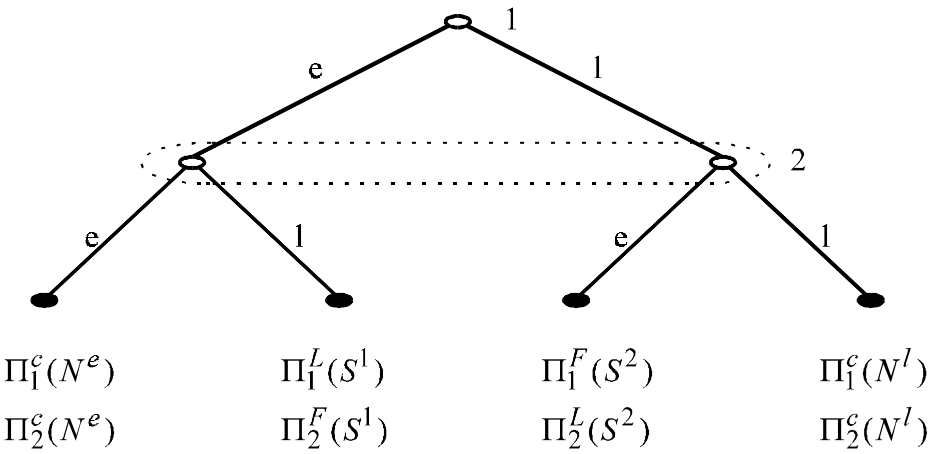
2. The Endogenous Timing Game of Hamilton/Slutzky

In the extended game with observable delay (Hamilton/ Slutzky (1990)) players choose the timing of their action in a stage prior to the "basic" quantity- setting game. Strategies at the first stage are either early (e) or late (l) move. This leads either to simultaneous play (N^e or N^l provided that both players chose either e, or l) or to sequential play (S^1 (S^2) provided that player 1 (2) played e whereas player 2 (1) other chose l). In accordance with the results of the timing game, players set their profit-maximizing quantities at the second stage. Considering a Cournot duopoly, this would lead to the known Cournot solution (with Π^C for both players) in the first case and the Stackelberg solution (with Π^L for the leader and Π^F for the follower) in the second (see fig. 2.1).

Hamilton/ Slutzky use the concept of subgame perfect equilibrium. They assume that the three equilibria of the basic game (N for simultaneous play, S^1 and S^2 for sequential play with player 1 or 2 being the leader) have a (unique)[7] equilibrium in pure strategies. When the leader-position is preferred to simultaneous play (which is always the case in a Cournot

[7] Amir (1996) shows, that the uniqueness of the equilibrium is not necessarily required for obtaining their results.

Figure 2.1: Reduced Extensive Form



game with monotone best responses)[8], the resulting equilibria of the extended game depend on the preferences regarding the follower position. The following conclusions are then drawn by Hamilton/ Slutzky in their theorems I-IV: [9]

(a)When both players prefer the simultaneous move outcome N to their payoff as a follower, the equilibrium of the game is N with both firms moving at the first opportunity.

(b) When both players prefer their follower profit to that of the simultaneous game, then the equilibria are S^1 or S^2.

(c) When there is one player preferring the follower position to simultaneous play while the other one prefers the leader profit to the Nash profit to the follower-profit, there is a unique equilibrium with the first player moving late, the other player moving early.

The conditions on cost- and demand functions that lead to the different outcomes in the Cournot case with homogeneous goods are formally set up by Amir (1996). They turn out to be rather restrictive, such that one would expect simultaneous play in most of the standard Cournot models.

If firms competed in two markets, there would be the possibility of a division of the leadership role, such that each would act like a leader in one market and the asymmetric results concerning leader- and-follower-profits of the one-market game were avoided. But with firms competing in several independent markets, the general results of Hamilton/ Slutzky do not change. The incentives to deviate from a timing strategy that induces sequential play stay the same irrespective of a second market. Thus, also the conditions on cost- and demand-functions stay the same for each market.

Nevertheless, if markets were interrelated through a common cost- or demand-function the conditions on cost- and demand-functions stated for one single market can not be applied any more. When there are diseconomies of scope, the timing game may help firms to commit to specialize and thus to avoid superfluous diversification costs.

[8] For a formal proof see Amir (1996), proof of Lemma 2.2.
[9] For proofs see Hamilton/ Slutzky (1990).

3. Endogenous Timing and Diseconomies of Scope

I consider two firms ($i = 1, 2$) being active in two markets ($k = A, B$) . Whereas demand is independent, production costs are interrelated. The following cost-function is assumed:

$$C_i(x_i, y_i) = cx_i + gx_iy_i + cy_i \quad for \quad i = 1, 2.$$

where x is the quantity produced in market A, y the quantity produced in market B. I will normalize c to zero to simplify the analysis.[10] The parameter g is restricted to be positive and smaller than 2. In that case firms face diseconomies of scope by producing in two markets.[11] These can be due to switching costs when there are joint capacities or increased maintenance costs of flexible techniques. Other examples are increasing marginal opportunity cost of constrained capital or management skills or forgone learning-effects when producing smaller quantities of each product.

Demand is linear and can be expressed by the inverse demand-function:

$$\begin{aligned} p^A(x_1, x_2) &= a - x_1 - x_2 \qquad (market\ A) \\ p^B(y_1, y_2) &= a - y_1 - y_2 \qquad (market\ B) \end{aligned}$$

When diseconomies of scope are high ($1 \leq g < 2$), there are two potential equilibria of the one-stage Cournot game with simultaneous play. One is a boundary solution where firms choose to leave one market and concentrate each on one market. It implies firms choosing their optimal monopolistic output in one market and producing nothing in the other. Given this situation ($x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}$), it does not pay to invade the other market for neither firm: The marginal gain from entering the second market would be the monopoly price of $p^m = \frac{a}{2}$, whereas the marginal increase of production costs is $g\frac{a}{2}$, which equals or is bigger than p^m if $g \geq 1$.[12] Firms' profit in this boundary solution,

$$\Pi_i^b(N, N) = \frac{a^2}{4},$$

therefore equals the profit resulting from a monopoly in both markets with each firm being monopolist in one market.

But there is another potential equilibrium with both firms being active in both markets and choosing

$$x_i = y_i = \frac{a}{3 + g}, \quad i = 1, 2.$$

---

10 This does not alter the qualitative results obtained.

11 see Dixon (1992) for a similar approach. Bulow/Geanakoplos/Klemperer (1985) consider quadratic unit-costs of each single product.

12 This can also be seen by checking the derivatives of the profit function at $x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}$:
$\frac{\partial \Pi_i}{\partial x_i}(x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}) = \frac{\partial \Pi_j}{\partial y_j}(x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}) = 0$
$\frac{\partial \Pi_i}{\partial y_i}(x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}) = \frac{\partial \Pi_j}{\partial x_j}(x_i = \frac{a}{2},\ y_i = 0,\ x_j = 0,\ y_j = \frac{a}{2}) \leq 0,$
$i \neq j.$

It is the interior solution of the two-market game and yields

$$\Pi_i(N,N) = \frac{a^2(2+g)^2}{(3+g)^2}.$$

Nevertheless, from the players' viewpoint, the former (monopolistic) equilibrium dominates the equilibrium with two-market-competition. If players were able to coordinate on equilibria, they would unanimously choose to leave each one market. In a two-player-context, the ability to pre-communicate is a realistic assumption.[13] I will therefore assume players' ability to coordinate for the rest of the analysis.[14] Consequently,

$$\begin{aligned} \Pi_i^b(N,N) &= \Pi_i(M^1, M^2) = \frac{a^2}{4} \\ with \quad x_i &= y_j = \frac{a}{2} \quad and \\ x_j &= y_i = 0, \qquad i \neq j, \end{aligned}$$

is the relevant result of the one-stage Cournot game for $1 \leq g < 2$.

When firms move simultaneously and $0 < g \leq 1$, the boundary solution stated above is not an equilibrium. Firms have an incentive to produce in both markets in spite of the diseconomies of scope and the fact that they both would do better monopolizing one market each: departing from the situation where each firm acts like a monopolist in one market, the marginal gain from invading the other market ($= a/2$) offsets the marginal negative impact on the production costs ($= g\frac{a}{2}$). The equilibrium profit is thus $\Pi_i(N,N) = \frac{2a^2(1+g)^2}{(3+g)^2}$, which is smaller than $\Pi_i^b(N,N) = \frac{a^2}{4}$. Firms thus face a prisoner's dilemma when $0 < g \leq 1$.

Things might change if firms were given the possibility of commitment through deciding over the timing of their action: This possibility could help firms to produce asymmetrically in the two markets.

In accordance with the endogenous timing game of Hamilton/ Slutzky (1990) I introduce a stage prior to the quantity setting stage, in which firms can choose to set their quantity either early (e) or late (l) in each of the markets. If both firms choose the same timing in one market, they have to set their quantities in the second stage simultaneously. Otherwise, the player who has chosen "early" is acting as a leader, whereas the other player having chosen late acts like a follower.

13 Obviously, increasing the number of the players involved, pre-pay communication will get more and more difficult. For this argument, see Fudenberg/ Tirole (1995), p.21 - 22.

14 Aumann (1990) doubts, that pre-play communcation results in the elimination of pareto-dominated equilibria, if the loss when the other player chooses the other equilibrium strategy is too high. He assumes a game, where each player can be sure of a moderate payoff, when playing the strategy of the dominated equilibrium, regardless of the strategy of the other player, whereas he incurs a loss playing the strategy of the dominant equilibrium and the other plays the strategy of the dominated equilibrium. However in the games analized here, there is no "safe" strategy. Regardless of the equilibrium strategy chosen, players incur a loss, if one erroneously plays the other equilibrium strategy.

The strategy-spaces and the resulting competitive situations are shown in fig. 3.1:

$$\begin{array}{lllll} {}_1\backslash^2 & e,e & e,l & l,e & l,l \\ e,e & N^e,N^e & N^e,S^1 & S^1,N^e & S^1,S^1 \\ e,l & N^e,S^2 & N^e,N^l & S^1,S^2 & S^1,N^l \\ l,e & S^2,N^e & S^2,S^1 & N^l,N^e & N^l,S^1 \\ l,l & S^2,S^2 & S^2,N^l & N^l,S^2 & N^l,N^l \end{array}.$$

The first entrance indicates market A, the second market B. N stands for simultaneous play (early or late) in this market. S^i indicates that player i acts as a Stackelberg leader in the respective market. When firms choose their timing strategy, they have to take into account optimal profits resulting from each situation of the timing game ($\Pi_i(t_1^A \times t_2^A, t_1^B \times t_2^B)$). The subgame-perfect equilibrium of the timing game describes the situation where each firm's time choice is the best answer to the competitor's strategy - given the solution of the basic game.

Note that maximized profits combined with (N^e, N^e) are the same as with (N^l, N^l), thus

$$\Pi_i(N^e, N^e) = \Pi_i(N^l, N^l) = \Pi_i(N, N).$$

Also, because of the symmetry of the game,

$$\begin{aligned} \Pi_i(N^l, N^e) &= \Pi_i(N^e, N^l), \quad \Pi_1(S^1, S^1) = \Pi_2(S^2, S^2), \\ \Pi_2(S^1, S^1) &= \Pi_1(S^2, S^2), \quad \Pi_i(S^1, S^2) = \Pi_i(S^2, S^1), \quad i = 1,\ 2 \end{aligned}$$

Without the assumed link of the two markets through a joint cost-function, early simultaneous play would result in both markets: Each firm prefers Stackelberg-leadership to simultaneous play, but prefers the Nash-outcome to being a follower,

$$\Pi_i(S^i, S^i) > \Pi_i(N, N) > \Pi_i(S^j, S^j).$$

Consequently, each player would choose "early" in the timing game and - after observing the timing of the other player - choose the Cournot quantity of $a/3$ in each market in the basic game.

Nevertheless, when there are diseconomies of scope, firms could do better specializing in one of the markets, such that different outcomes of the endogenous timing game can be expected.

Proposition 1 *Whenever $g < g^* \approx 0.461$, early simultaneous play is the only equilibrium in pure strategies.*

Proof $(For\ precise\ calculations\ see\ Appendix)$:

Because of the symmetry of markets and firms, it suffices to consider the situation, where one firm (say firm 1) is leading in any situation in which only one firm is leading. With respect to markets, when there is sequential play in only one market, it is assumed to be market A.

After calculating the profit-maximizing quantities for each timing situation, it can be shown, that

$$\begin{aligned} \Pi_1(S^1, S^1) &> \Pi_1(S^1, N^e) > \Pi_1(N^e, N^l) > \Pi_1(S^1, S^2),\ \Pi_1(N, N) > \Pi_1(S^1, N^l) \quad and \\ \Pi_2(N^e, N^l) &> \Pi_2(N, N),\ \Pi_2(S^1, S^2) > \Pi_2(S^1, N^l) > \Pi_2(S^1, N^e) > \Pi_2(S^1, S^1) \end{aligned}$$

(see fig.3.2).

First, simultaneous play (N^e, N^e) with both firms choosing "early" in both markets and selecting their Cournot quantity afterwards is an equilibrium: neither firm 1 nor firm 2 has an incentive to deviate from this time choice. By moving late in one market a firm would have to act like a follower in this market in the second stage and would obtain lower profits.

Second, there are no other equilibria in pure strategies: Firm 2 strictly prefers $\Pi_2(N^e, N^e)$ to any outcomes resulting from being follower in one or two markets without being the leader in at least one market. Thus, firm 1 leading in both markets (S^1, S^1) can not be an equilibrium: firm 2 has an incentive to move early in both markets in the first stage. The same argument applies to the situation, where firm 1 is leading in market A and there is early Nash in market B (S^1, N^e) (firm 2 would deviate in market A). When there is (S^1, N^l), both firms have an incentive to deviate in the timing stage: firm 1 prefers being leader in both markets and would thus choose "early" in market B, whereas firm 2 could do better by choosing "early" in A and B.

Also, both firms playing late in both markets cannot be an equilibrium. As leading in one market is preferred to playing simultaneously in both markets, each firm has an incentive to move early in one of the markets. This argument also holds for both firms playing early in one market (say market A) but late in the other market (say market B): each has an incentive to move early in the latter market. As (N^e, N^l) yields higher profits than (N^e, N^e), firms again face a prisoner's dilemma situation.

Finally, firm 1 moving early in market A and late in market B, while firm 2 chooses late in A and early in B cannot be an equilibrium as both firms can achieve higher profits by moving early also in their follower-market. This would yield $\Pi_1(S^1, N^e)$ (resp. $\Pi_2(N^e, S^2)$), which is preferred to $\Pi_i(S^1, S^2)$. ■

If one considers only the outcomes where firms choose the same timing in both markets (i.e. firm i choosing either (e,e) or (l,l)), one gets the condition on preferences necessary for early simultaneous play stated by Hamilton/Slutzky: The leader-position in both markets is preferred to simultaneous play, which is preferred to the follower-position in both markets. The possibility to "split strategies" (choosing "early" in one market and "late" in the other) thus seems to have no significance for the resulting equilibrium in the above stated range of g.

As long as $g <\approx 0,448,$ the achieved equilibrium leads to higher profits than a situation where each firm is the leader in one market and follower in the other. This is due to the fact that the Stackelberg equilibrium involves higher outputs in each market and thus lower prices. When g is small, the positive effect of specialization through producing asymmetrically cannot outweigh the negative price-effect.

Joint profits would be maximized in a two-market monopoly situation. Nevertheless, this outcome cannot be achieved, as the timing game provides only an instrument to commit to the timing of an action, not to the action itself. A second mover is thus not able to commit to produce zero output, after the first mover had chosen its quantity. Thus, if each firm moved early in one market and produced the monopoly quantity, both would enter the second market in the late stage. Moving first thus implies offering the Stackelberg-leader quantity and lower joint profits in both markets.

Proposition 2 *When $g^* \leq g < g^{**}$ $(= 2 - \sqrt{2}\,)$, there are two pure-strategy-equilibria of the timing game, describing the situation where each firm is leading in one market (i. e. (S^1, S^2) and (S^2, S^1)). There is another potential equilibrium with both firms choosing "early" in both markets, which is dominated by (S^1, S^2) and (S^2, S^1).*

Corollary 3 *When $g \geq 0,5$, each firm is able to monopolize the market, where it moves first.*

Proof *(see also Appendix)*:
When $g^* \leq g < g^{**}$ it can be shown, that

$$\begin{aligned} \Pi_1(S^1,S^2), \Pi_1(S^1,S^1) &\geq \Pi_1(S^1,N^e) \geq \Pi_1(N^e,N^l) > \Pi_1(N,N) > \Pi_1(S^1,N^l) \quad \textit{and} \\ \Pi_2(S^1,S^2) &> \Pi_2(N^e,N^l) > \Pi_2(N,N) > \Pi_2(S^1,N^e),\ \Pi_2(S^1,N^l) > \Pi_2(S^1,S^1) \end{aligned}$$

(see fig. 3.2).

Obviously, (S^1, S^2) must be an equilibrium of the timing game: Given the timing strategy of the other firm, deviation can only lead to simultaneous play in the market, where the deviation occurs. For both firms, this would yield lower profits.

But playing early Nash in both markets is also a potential equilibrium of the timing game. Neither of the firms is able to do better by choosing another timing strategy. The only possibility to deviate would be to choose late in one or two of the markets. But this would yield a profit $\Pi_i(S^j, N^e)$, $i \neq j$, that is lower than $\Pi_i(N, N)$. Nevertheless, this equilibrium is dominated by the other two equilibria for both players. Thus, if players were able to coordinate, the outcome of the game would be (S^1, S^2) or (S^2, S^1).

It is easy to see that there is no other equilibrium of the timing game. In any of the other situations, at least one of the players can improve its profit by choosing "early" in one of the markets. ■

When each firm is leading in one market (S^1, S^2), the interior solution ceases to be valid as soon as g is higher than 0.5.[15] The optimal quantity choice for the leading firm is then

$$x_1 = y_2 = \frac{a}{1+g}$$

This quantity is just high enough to give the second-mover no incentive to enter the market where it is following: the marginal gain from entering- the price - equals the marginal loss the competitor would incur ($= gx_1 = gy_2$). Consequently

$$x_2 = y_1 = 0$$

[15] Second-order-conditions are only satisfied with $g < 0.5$. See also Appendix.

This boundary solution leads to a profit of

$$\Pi_i^b(S^1, S^2) = \frac{a^2 g}{(1+g)^2}.$$

Each firm monopolizes one market and is able to avoid inefficiencies induced by diseconomies of scope.

When $g^* \leq g < g^{**}$, profits applying to being the leader in both markets are still greater than profits applying to playing simultaneously in both markets, which again is preferred to following in both markets. Nevertheless, simultaneous play is not the only equilibrium of the timing game: there are the two additional sequential-play equilibria, which lead to higher profits than simultaneous play in both markets. This is due to the increasing impact of g: producing symmetrically in two markets causes production costs which can be avoided by specializing in one market. This specialization can be achieved when each firm moves first in one market. First movers commit to a quantity high enough to prevent the other firm from entering. This quantity must be higher than the monopoly-output to prevent the follower from entering. It is even higher than the output resulting from simultaneous play in two markets. But the negative price-effect caused by higher production is offset by the efficiency gain caused by specialization.

Proposition 4 *When $g^{**} \leq g < 1$, any time choice leading to sequential play in both markets ((S^1, S^2) and (S^2, S^1), but also (S^1, S^1) and (S^2, S^2)) can be an equilibrium of the timing game.*

Corollary 5 *In the equilibria with one firm leading in both markets, the second mover obtains higher profits than the leading firm.*

Proof $(see\ also\ Appendix)$:

When $g^{**} \leq g < 1$

$$\begin{aligned}
\Pi_1(S^1, S^2) &> \Pi_1(S^1, S^1) > \Pi_1(S^1, N^l) \geq \Pi_1(S^1, N^e)) > \Pi_1(N^e, N^l) > \Pi_1(N, N) \\
\Pi_2(S^1, S^1) &> \Pi_2(S^1, S^2) > \Pi_2(S^1, N^l),\ \Pi_2(S^1, N^e)),\ \Pi_2(N^e, N^l),\ \Pi_2(N, N)
\end{aligned}$$

(see fig. 3.2).

First, (S^1, S^2) and (S^2, S^1) must be equilibria of the timing game. Even though each firm prefers following in both markets, it is not able to induce this situation by choosing another timing strategy in one or both markets. It can only reach simultaneous play in the market where it deviates, which leads to lower profits than (S^1, S^2) and (S^2, S^1).

But also (S^1, S^1) and (S^2, S^2) are equilibria of the game: Deviation results in simultaneous play in one or both markets and in lower profits for both firms.

Early simultaneous play in both markets cannot be an equilibrium any more. Each firm can do better by moving late in both markets, thus obtaining $\Pi_i(S^j, S^j)$, $j \neq i$, which is preferred to $\Pi_i(N, N)$. Also, late simultaneous play is not an equilibrium: the best answer to (l, l) is moving early in both markets. The same arguments apply to (S^i, N^e) and (S^i, N^l). ■

When one firm is leading in two markets, the result of the quantity game is again a boundary solution.[16] The leading firm (say 1) prefers to leave one market (say B). Taking into account the reaction of firm 2, it maximizes its profits in the other market. The following firm concentrates its production on market B. Depending on g, it produces only little ($g < \frac{\sqrt{5}-1}{2} \approx 0,618$) or nothing ($g \geq \frac{\sqrt{5}-1}{2} \approx 0,618$) in the second market. The leader sets quantities higher than the monopoly quantity in its chosen "home-market" A to induce the follower to leave this market, whereas the follower is able to react optimally and to supply a quantity less or equal to the monopoly quantity in market B. Consequently, it obtains higher profits than the leader.

This result differs from the result when $g < g^{**}$. There, $\Pi_i(S^i, S^i) > \Pi_i(N, N) > \Pi_i(S^j, S^j)$ $(i \neq j)$. When $g \geq g^{**}$, leading in both markets is still preferred to playing simultaneously in both markets. But firms do even better when they are in the follower-position in the two markets: the high level of diseconomies of scope induces the leader to specialize in one market, such that the follower is also able to establish a "home-market". Additionally, its strategies are not distorted by strategic considerations, whereas the leader has to take into account the reactions of the follower.

Again, if one only considers strategy-"bundles", where firms choose the same timing in both markets, the results of Hamilton/Slutzky can be applied. Following in both markets is preferred to leading in both markets, which is preferred to simultaneous play in both markets. The result is thus either firm 1 or firm 2 leading in both markets. But the fact that firms can split strategies leads to the other two sequential play equilibria, where each firm is leading in one market. This is due to the fact that leading in one while following in the other market is preferred to leading/ following in one and playing simultaneously in the other market.

In all timing situations there is a critical value, for which a boundary solution appears. This boundary solution implies, that one or both players leave one and concentrate on the other market. Quantities in each firms "home-market" are higher than the monopoly output for $g < 1$, but are falling in g, whereas in the other market they equal or approach zero with increasing g. When $g \geq 1$, the optimal quantity in each firms home market is $a/2$, whereas in the other market nothing is produced by that firm. This observation leads to

Proposition 6 *When $g \geq 1$, the timing game ceases to have an impact on the outcome of the basic game. Players are able to divide up markets and produce their optimal monopoly-output regardless of their time choices.*

Proof $(see\ also\ Appendix)$**:**

Consider first the situation (S^1, S^2). When the boundary solution is reached ($g = 0.5$), the leader (Player 1 in A, Player 2 in B) sets a quantity high enough to prevent the follower from entering the market in the late stage: $x_1 = y_2 = \frac{a}{1+g}$. This quantity is falling in g and reaches $x_1^m = y_2^m = \frac{a}{2}$ at $g = 1$. When g is higher than one, less output is necessary to avoid competition of the follower. But the leader will not supply less than $\frac{a}{2}$, as this is yet the individually optimal output in monopoly.

[16] Second-order conditions are satisfied for firm 1 as long as $g < g^{**}$. See also Appendix.

When there is one player leading in both markets, the mechanism is similar. In one market, the leader produces zero, in the second market he sets a preemptive quantity, which equals $x_1 = \frac{a(2-g)}{2}$ for $g \geq \frac{\sqrt{5}-1}{2}$.[17]

Again, this quantity is falling in g and reaches x^m, when $g = 1$. The leader has no incentive to reduce his supply any more, as x^m is his profit-maximizing quantity. The follower reacts with $x_2 = 0$ and $y_2 = \frac{a}{2}$ for any $x_1 \geq \frac{a(2-g)}{2}$.

When there is one player leading in one market and early Nash in the other market, the boundary solution is reached when $g \geq\approx 0,619$ (see Appendix). The leader then supplies a preemptive quantity $x_1 = \frac{a(3-g)}{3+g^2}$ in the market with sequential play, and sets $y_1 = \frac{a(g-1)^2}{3+g^2}$ in the market with early simultaneous play. x_1 and y_1 are falling in g and reach $a/2$ and 0 respectively at $g = 1$. As before, it is not rational to reduce output in the leader market even more, whereas output cannot become negative in the Nash market. The follower reacts with $x_2 = 0$ and plays $y_2 = \frac{a(g+1)}{3+g^2}$ in the Nash market. y_2 is increasing in g and equals the monopoly-output at $g = 1$. But for $g > 1$ it is not rational for player 2 to supply more than y^m: the optimal reaction of player 1 in the Nash-market remains $y_1 = 0$ for any $y_2 \geq \frac{a}{2}$, given $x_1 = \frac{a}{2}$ in the other market, $y_2 = \frac{a}{2}$ for $g \geq 1$.

The same arguments apply to (S^1, N^e). The boundary-solution becomes valid for $g \geq \frac{1}{\sqrt{2}}$. Optimal strategies are the same as in the situation with one player leading and early Nash in the other market. At $g = 1$, both players produce $a/2$ in one market and zero in the other market.

When there is early (late) simultaneous play in both markets, the boundary solution appears in addition to the interior solution for $g \geq 1$:[18] $x_i = \frac{a}{2}$, $y_i = 0$ and $x_j = 0$, $y_j = \frac{a}{2}$ is a stable solution of the basic game as it is each player's strategy is the best response to the strategy of the competitor. As the interior solution is dominated by the boundary solution, the outcome again is the same as the outcome of an institutionalized monopoly in each market.

When one market clears before the other, the mechanism is the same: there exists the boundary solution ($x_i = \frac{a}{2}$, $y_i = 0$ and $x_j = 0$, $y_j = \frac{a}{2}$) in addition to the interior solution as soon as $g \geq 1$.[19] But the boundary solution is preferred to the interior solution by both players and would thus be the outcome of the basic game if pre-play communication was allowed.

Consequently, regardless of the timing situation, for $g \geq 1$ players are able to achieve their maximal profit by specializing each in one market. The solution of the basic game is also the same as the solution of the one-stage-game, such that the timing stage does not have an impact on the outcome of the quantity setting game. ■

[17] For $2 - \sqrt{2} \leq g < \frac{\sqrt{5}-1}{2}$, the preemptive quantity is $\frac{a(2-g)(1+g)}{2(2-g^2)}$ and the follower remains active in both markets.

[18] The interior solution ceases to be an equilibrium for $g \geq 2$: given the Nash-strategy of player i, $x_i = y_i = \frac{a}{3+g}$, it pays to react with a boundary strategy $x_j = 0$ and $y_j = \frac{2a}{3+g}$. This can also be seen by computing the second-order-conditions.

[19] The interior solution ceases to be valid when $g \geq \frac{3}{2}$.

4. Welfare Implications

To estimate the welfare effects of the timing-game, two effects are to be considered: The quantity effect is the welfare effect resulting from the impact of the timing game on total output. It influences profits as well as consumer surplus. The cost or efficiency effect is the effect of endogenous timing on production costs, induced by the impact on the level of specialization of the firms. This effect influences mainly profits. But also consumer surplus is influenced by the indirect effect of (lower) costs on output.

As a benchmark I consider the outcome of a one-stage game with two firms being (potentially) active in two markets (all quantities are set at the same time). In this case total social surplus is

$$W(N,N) = 2 \cdot \int_0^{x(N,N)} (a-x)dx - \sum_i C_i(N,N) = \begin{cases} \frac{2a^2(4+g)}{(3+g)^2} & if \quad g < 1 \\ \frac{3a^2}{4} & if \quad g \geq 1 \end{cases},$$

divided between consumer surplus

$$CS(N,N) = 2 \cdot \int_0^{x(N,N)} (a-x)dx - 2\Pi_i(N,N) - \sum_i C_i(N,N) = \begin{cases} \frac{4a^2}{(3+g)^2} & if \quad g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{cases}$$

and summed profits

$$\begin{aligned} PS(N,N) &= 2\Pi_i(N,N) = p_A \sum_i x_i(N,N) + p_B \sum_i y_i(N,N) - \sum_i C_i(N,N) = \\ &= \begin{cases} \frac{2a^2(2+g)}{(3+g)^2} & if \quad g < 1 \\ \frac{a^2}{2} & if \quad g \geq 1 \end{cases}. \end{aligned}$$

As the timing game may lead to a monopolization of the markets, I will also consider welfare ensuing from an institutionalized monopoly without (potential) competition, which equals welfare of two-market competition in a one-stage game when $g \geq 1$:

$$W(M^1, M^2) = W^b(N,N) = 2 \cdot \int_0^{\frac{a}{2}} (a-x)dx = \frac{3a^2}{4}$$

divided between consumer surplus

$$CS(M^1, M^2) = 2 \cdot \int_0^{\frac{a}{2}} (a-x)dx - 2\Pi_i(M^1, M^2) = \frac{a^2}{4}$$

and summed profits

$$PS(M^1, M^2) = 2\Pi_i(M^1, M^2) = \frac{a^2}{2}$$

The maximum welfare that could be achieved either with a social planner maximizing total welfare, or with perfect price discrimination of two monopolies in both markets equals

$$W^{sp} = 2 \cdot \int_0^a (a-x)dx = a^2.$$

It implies total specialization and supply of $\sum_i x_i = \sum_i y_i = a$ in each market. Whereas with perfect price discrimination, profits equal summed welfare, in the case of a social planner it is assumed that prices are zero and consumer welfare is maximized.

Proposition 7 *Whenever the timing game leads to equilibria different from a one-stage game without timing opportunity, the effects on welfare are positive.*

Proof *(see also Appendix)*:

As long as $g < g^*$, the opportunity to time actions in the quantity-setting game does not have an impact on the equilibrium of the basic game. The result of the timing stage is early simultaneous play in both markets, such that quantities, prices and welfare are the same as in a one-stage-game without timing-possibility.

Note, that in comparison to a two-market-monopoly, diversification of both firms yields higher social surplus as long as $g <\approx 0.441$. Only when $\approx 0.441 \leq g < g^*$, a monopoly would be socially preferred to two-market competition with or without timing possibility. The dead-weight-loss that can be attributed to a socially suboptimal output in monopoly is outweighed by the efficiency-gain caused by specialization.[20]

When $g^* \leq g < g^{**}$ and coordination between players is allowed, the two possible equilibria are the sequential-play equilibria with each firm leading in one market. In this case, equilibrium strategies are

$$x_1 = y_2 = \begin{cases} \frac{a(g-1)}{g^2+2g-2} & for \quad g < 0.5 \\ \frac{a}{1+g} & for \quad 0.5 \leq g < g^{**} \end{cases}$$

and

$$x_2 = y_1 = \begin{cases} \frac{a(2g-1)}{2(g^2+2g-2)} & for \quad g < 0.5 \\ 0 & for \quad 0.5 \leq g < g^{**} \end{cases} {}^{21}.$$

Comparing this equilibrium output with the output resulting from the one-stage-game,

$$x_i = y_i = \frac{a}{3+g}, \quad i = 1, 2$$

one finds that total supply ($\sum_i x_i + \sum_i y_i$) is higher with sequential play than with two-market competition. Additionally, because of asymmetric production, diseconomies of scope are avoided. Consequently, the welfare effect (ΔW^1) must be positive.[22] ■

With respect to the division of the welfare increase on CS and PS, one finds that both - consumers as well as firms - gain:

$$\begin{aligned} \Delta CS^1 &= CS(S^i, S^j) - CS(N, N) > 0 \\ \Delta PS^1 &= PS(S^i, S^j) - PS(N, N) > 0 \end{aligned}$$

[20] Nevertheless, monopoly does not Pareto-dominate the two-market-competition: whereas producers gain, consumers incur a reduction of consumer-surplus, as supply is smaller.

[21] See Appendix .

[22] This can also be seen by comparing directly $W(N, N) = \frac{2a^2(4+g)}{(3+g)^2}$ and
$$W(S^1, S^2) = \begin{cases} \frac{a^2(8g^3+16g^2-36g+15)}{4(g^2+2g-2)^2} & for \quad g < 0,5 \\ \frac{a^2(1+2g)}{(1+g)^2} & for \quad 0,5 \leq g < g^{**}. \end{cases}$$

The positive impact on consumer surplus increases in g for $g^* \leq g < 0.5$ (supply decreases less with sequential, than with simultaneous play) but it decreases in g for $0.5 \leq g < g^{**}$ (boundary solution where supply approaches $a/2$ with g approaching 1). In comparison, the impact on profits increases in g in the whole range between $g^* \leq g < g^{**}$, due to the efficiency gain resulting from specialization.

Social welfare is also higher than in a two-market monopoly:

When $g < 0.5$, both firms are active in both markets. The positive effect of (higher) output on welfare is

$$2 \cdot \int_0^{x(S^1,S^2)} (a-x)dx - W(M,M) = \frac{a^2(g-1)^2(3g^2-2g+3)}{4(g^2+2g-2)^2}.$$

It outweighs the negative cost-effect

$$C_i(S^1,S^2) = 2\; g\; x_i\; y_i = \frac{ga^2(g-1)(2g-1)}{(g^2+2g-2)^2}.$$

When $g \geq 0.5$, markets are monopolized. Nevertheless, supply is higher than in a two-market-monopoly: the leader has to set a quantity just high enough to make it unprofitable for the follower to enter. As there is no negative cost-effect, welfare must also be higher:[23]

$$\Delta W^2 = W(S^i,S^j) - W(M^i,M^j) > 0.$$

Due to higher supply, consumers are better off, but prices and total profits are lower than in a monopoly situation:

$$\begin{aligned} \Delta CS^2 &= CS(S^i,S^j) - CS(M^i,M^j) > 0 \\ \Delta PS^2 &= PS(S^i,S^j) - PS(M^i,M^j) < 0 \end{aligned}$$

The (positive) difference in consumer surplus is decreasing in g as output is decreasing continuously when firms play (S^i, S^j), whereas it is independent from g in monopoly. Likewise, the (negative) profit- difference is decreasing in g, as specialization with sequential play leads to a an elimination of costs whereas supply approaches the monopoly-supply with increasing diseconomies of scope.

When $g^{**} \leq g < 1$, there are four equilibria of the timing game: the two equilibria with each firm leading in one market ((S^1,S^2) and (S^2,S^1) resp.) and the two equilibria where one player is leading in both markets ((S^1,S^1) and (S^2,S^2) resp.). All of them yield higher social surplus than two-market competition in a one stage game. Both - consumers and firms- gain, due to the quantity and efficiency effect:

$$\Delta CS^{31} = CS(S^i,S^j) - CS(N,N) > 0$$

[23] Again, this result can also be derived by comparing directly the welfare resulting from the two situations:
$$W(S^i,S^j) = \begin{cases} \frac{a^2(8g^3+16g^2-36g+15)}{4(g^2+2g-2)^2} & for \quad g < 0.5 \\ \frac{a^2(1+2g)}{(1+g)^2} & for \quad 0.5 \leq g < g^{**}. \end{cases} > W(M^1,M^2) = \frac{3a^2}{4}$$

$$\begin{aligned}
\Delta CS^{32} &= CS(S^i,S^i) - CS(N,N) > 0 \\
\Delta PS^{31} &= PS(S^i,S^j) - PS(N,N) > 0 \\
\Delta PS^{32} &= PS(S^i,S^i) - PS(N,N) > 0
\end{aligned}$$

It can be seen that the equilibria with each player leading in one market (S^i, S^j) imply higher total output and welfare than equilibria with one player leading in both markets (S^i, S^i). This is due to the fact, that - when one firm is leading in both markets - the follower is able to react optimally in the market that the leader leaves and acts there as a monopolist without potential rivalry. At the same time, output in the market where the leader remains active is only slightly higher than if there was (S^i, S^j). Consequently, total profits are greater and consumer surplus is lower with (S^i, S^i) than with (S^i, S^j). This again implies lower total welfare, as supply is further away from the first-best solution.

Comparing the resulting equilibria of the timing game with the outcome of an institutionalized monopoly, the same qualitative result can be drawn as with $g^* \leq g < g^{**}$: Whereas consumer welfare is higher with endogenous timing, profits are lower. The higher g, the lower the difference to monopoly:

$$\begin{aligned}
\Delta CS^{41} &= CS(S^i,S^j) - CS(M^1,M^2) > 0 \\
\Delta CS^{42} &= CS(S^i,S^i) - CS(M^1,M^2) > 0 \\
\Delta PS^{41} &= PS(S^i,S^j) - PS(M^1,M^2) < 0 \\
\Delta PS^{42} &= PS(S^i,S^i) - PS(M^1,M^2) < 0
\end{aligned}$$

With $1 \leq g < 2$, any equilibrium of the timing game yields the same outcome as an institutionalized two-market monopoly. Provided that players are able to coordinate, this result can also be achieved in a one-stage game without timing possibility, such that the timing game ceases to have an impact on costs, prices and quantities supplied.

5. Concluding Remarks

In the above two-market model with diseconomies of scope in production, it could be shown that the extension of a basic Cournot game with a timing game may lead to outcomes of the basic game, which differ from the outcomes of a one-stage Cournot-game with simultaneous quantity choice. The timing game allows firms to move sequentially and thus to commit to specialize in the basic game. It supports sequential play equilibria, where firms concentrate on one market. Thus, inefficiencies through socially undesirable diversification are avoided.

Two-market competition with timing possibility yields higher welfare than a monopoly in each market when diseconomies of scope are not too high. With very low diseconomies, firms compete in both markets, but the negative cost-effect is surpassed by a positive quantity effect. When diseconomies of scope are in a middle range, firms monopolize each one market, but offer higher quantities than in monopoly to deter the competitor from entering. This result is due to the timing-possibility, which allows firms to commit to quantities before the other firm moves. Only when diseconomies of scope are very high, the threat to enter the competitors home market is not credible any more and the outcome resembles the monopoly outcome.

In the long run, multiproduct firms might be able to divide themselves up into distinct units with separated production and marketing facilities, thus eliminating the negative impact of the two-market activity.[24] Firms would then play simultaneously in both markets and gain Cournot-Nash profits. This would favor consumers and would lead to higher welfare, as output is higher and inefficiencies are avoided. It can be shown, that, if firms could first choose production facilities (joint or separated) before they choose timing and quantities, both players would choose the technology without diseconomies of scope - even if they end up with lower profits.[25] Consequently, the above derived results only apply temporarily or when fix-costs of building up a distinct unit are too high.

Whereas diseconomies of scope are a realistic assumption for the production sphere when learning effects can not be fully exploited, multiproduct firms often gain because fixed assets can be shared (e.g. sharing marketing facilities), such that average costs decline. A complete evaluation of conglomerate firms would also have to take into account these potential gains.

Very often, the results obtained with sequential play can also be obtained if firms were able to delegate marketing decisions to managers. It would therefore be interesting to analyze the impact of delegation decisions in the context described above, when owners are able to under- or overallocate costs to managers of each division. We will leave this question for future research.

[24] Alternatively, one could think of a new technology allowing for two-product production without any additional costs or of potential one-market competitors not facing negative cost- spillovers.

[25] This is the case when diseconomies of scope are in a middle range.

Figure 3.2.a: Profits Player 1

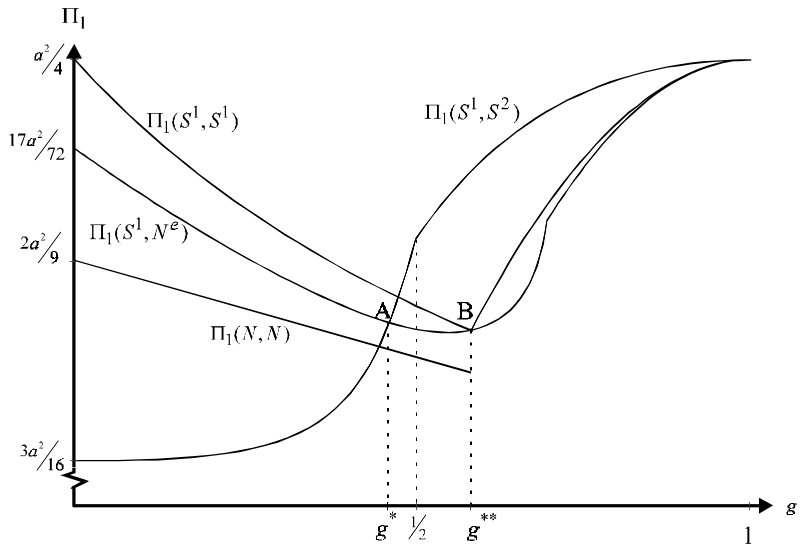


Figure 3.2.b: Profits Player 2

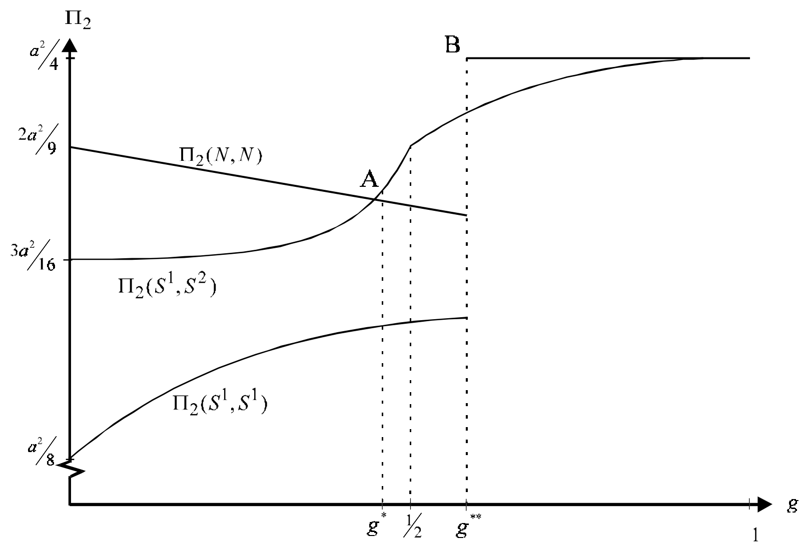


Figure 3.3.a: Equilibrium Profits Player 1

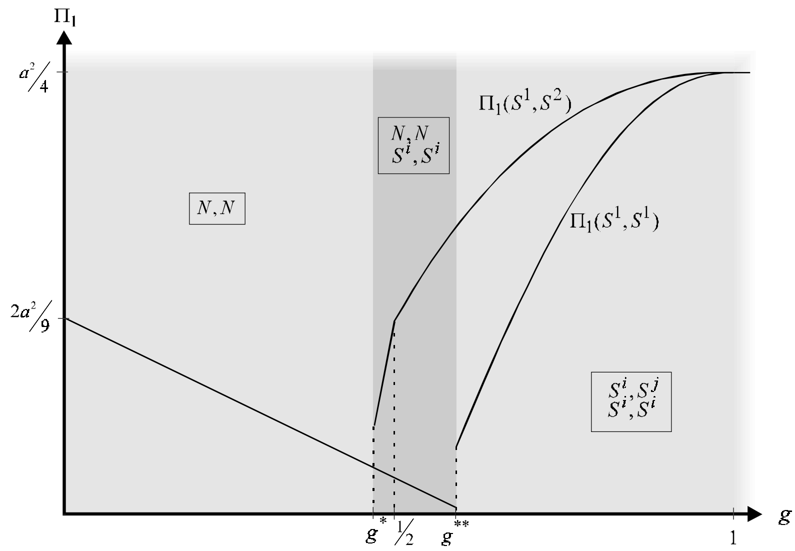


Figure 3.3.b: Equilibrium profits Player 2

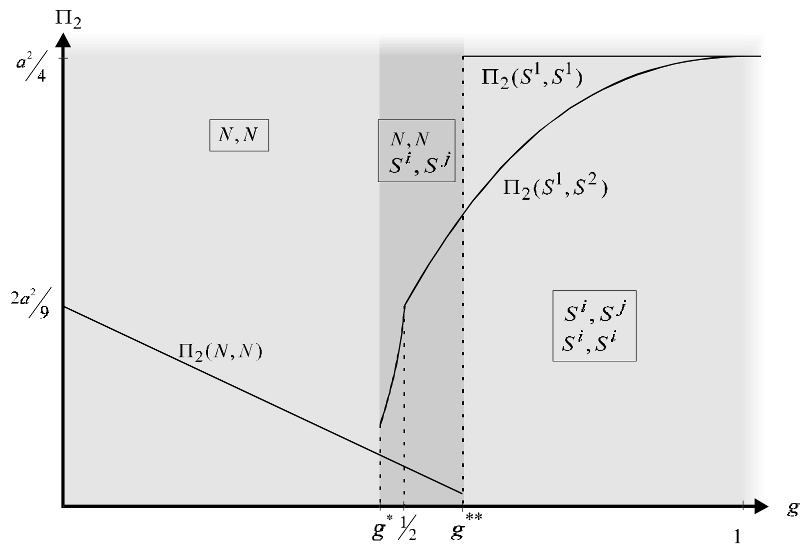
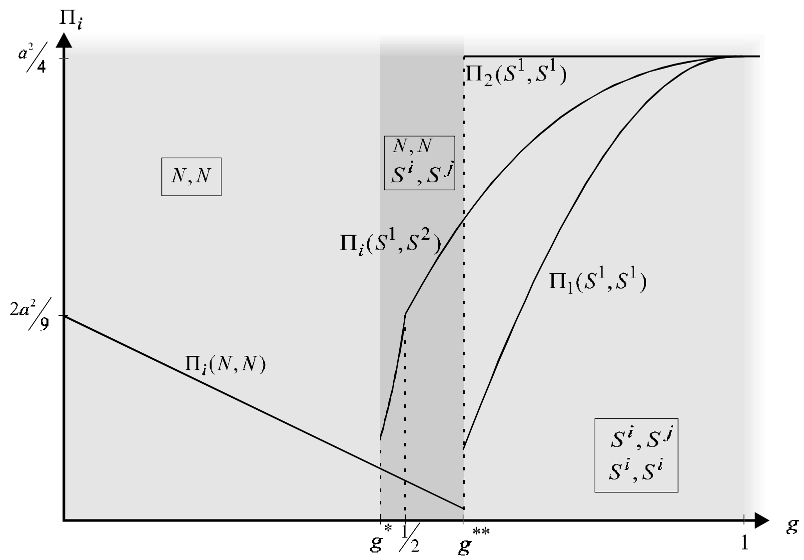


Figure 3.3.c: Both Players' Equilibrium Profits



Appendix A: Solution of the two stage game

The extended game is to be solved by backwards induction. First the solution of the quantity-setting game for each timing situation and for different levels of diseconomies of scope is to be calculated. In the first stage firms compare profits pertaining to each timing situation and find their best time choice.

A.1 Solution of the second stage

In the second stage, firms maximize

$$\begin{aligned}\Pi_1 &= x_1(a - x_1 - x_2) + y_1(a - y_1 - y_2) - gx_1y_1 \\ \Pi_2 &= x_2(a - x_1 - x_2) + y_2(a - y_1 - y_2) - gx_2y_2\end{aligned}$$

over x and y, subject to

$$x,\ y \geq 0.$$

It is also reasonable to assume, that firms would not supply more than demand would allow:

$$\begin{aligned}x_1 + x_2 &\leq a \\ y_1 + y_2 &\leq a\end{aligned}$$

In simultaneous play, each players' quantities must be the optimal response to the chosen quantities of the other. When firms move sequentially, the leader takes into account the reaction of the follower to his strategies in market(s) he is leading.

A.1.1 Profits applying to $(N^e, N^e) = (N^l, N^l)$:

$\mathbf{g < 1}$:

In this case profits are maximized for

$$x_i = y_i = \frac{a}{3+g}.$$

This yields

$$\Pi_i = \frac{a^2(2+g)}{(3+g)^2} \qquad i = 1, 2 \quad and \quad p_k = \frac{a(1+g)}{3+g} \qquad k = A, B.$$

$\mathbf{1 \leq g < 2}$:

In addition to the (interior) solution there is another equilibrium implying

$$x_i = y_j = \frac{a}{2} \quad and \quad x_j = y_i = 0, \qquad j \neq i.$$

To see this, given $x_1 = \frac{a}{2}$ and $y_1 = 0$, maximize

$$\Pi_2 \;=\; x_2(\frac{a}{2} - x_2) + y_2(a - y_2) - gx_2y_2 \quad with$$

$$
\begin{aligned}
0 &\leq x_2 \leq \frac{a}{2} \\
0 &\leq y_2 \leq a
\end{aligned}
$$

Taking the partial derivatives with respect to x_2 and y_2 yields

$$
\begin{aligned}
\frac{\partial \Pi_2}{\partial x_2} &= \frac{a}{2} - 2x_2 - gy_2 \\
\frac{\partial \Pi_2}{\partial y_2} &= \frac{a}{2} - 2y_2 - gx_2.
\end{aligned}
$$

As the interior solution yields negative output in market A, the following conditions (Kuhn-Tucker-conditions) are checked for the boundary solutions ($x_2 = 0$ and $y_2 > 0$, $x_2 > 0$ and $y_2 = 0$ or $x_2 = y_2 = 0$):

$$
\begin{aligned}
x_2 \frac{\partial \Pi_2}{dx_2}(\frac{a}{2} - x_2) &= 0, \qquad \frac{\partial \Pi_2}{\partial x_2} < 0 \quad if \quad x_2 = 0 \\
y_2 \frac{\partial \Pi_2}{dy_2}(a - y_2) &= 0, \qquad \frac{\partial \Pi_2}{\partial y_2} < 0 \quad if \quad y_2 = 0.
\end{aligned}
$$

Furthermore, the above restrictions for the two variables are to be taken into account. It is easy to see that the only solution fulfilling these conditions is $x_2 = 0$ and $y_2 = \frac{a}{2}$.

When $g \geq 2$, the second-order-conditions are not fulfilled any more:

$$
\frac{\partial^2 \Pi_i}{\partial x_i^2} \frac{\partial^2 \Pi_i}{\partial y_i^2} - \frac{\partial^2 \Pi_i}{\partial x_i \partial y_i} \frac{\partial^2 \Pi_i}{\partial y_i d x_i} = 4 - g^2 \geq 0 \quad for \quad g \leq 2.
$$

The interior solution ceases to be valid: Players have an incentive to deviate even when the other is playing the Nash-strategy proposed by the interior solution. The only equilibrium is the boundary solution.

A.1.2 Profits applying to $(N^e, N^l) = (N^l, N^e)$:

In this case, one market clears before the other. Firms have to take into account their optimal y_i, over which they decide late, when choosing x_i early.

Late: Maximization of Π_i over y_i yields

$$
\begin{aligned}
y_1(N^e, N^l) &= \frac{a - 2gx_1 + gx_2}{3} \\
y_2(N^e, N^l) &= \frac{a - 2gx_2 + gx_1}{3}
\end{aligned}
$$

Early: Maximization of Π_i over x_i yields

$$
\begin{aligned}
x_i(N^e, N^l) &= \frac{a(9 - 4g)}{27 - 4g^2} \qquad and\ thus \\
y_i(N^e, N^l) &= \frac{3a(3 - g)}{27 - 4g^2} \\
\Pi_i(N^e, N^l) &= \frac{a^2(16g^3 - 59g^2 - 18g + 162)}{(4g^2 - 27)^2} \qquad for\ i = 1, 2
\end{aligned}
$$

$$p^A(N^e,N^l) = \frac{a(4g^2-8g-9)}{4g^2-27} \qquad and \qquad p^B(N^e,N^l) = \frac{a(4g^2-6g-9)}{4g^2-27}.$$

Checking the second-order-conditions, one sees, that the interior solution ceases to be a maximum, when $g \geq \frac{2}{3}$:

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = -2 + \frac{8}{9}g^2 < 0 \quad for \quad g < \frac{3}{2}.$$

Firms then move to a boundary solution with

$$x_i = y_j = \frac{a}{2} \quad and \quad x_i = y_i = 0, \qquad j \neq i.$$

But as in the case with early (late) simultaneous play in both markets, this boundary solution exists and replaces the interior solution as soon as $g \geq 1$, as it is preferred to the interior solution for both players.

To see this, first assume, that $x_2 = 0$ and check the optimal response of player 1. Player 1 has to take into account the reaction of player 2 in market B. He also knows his optimal strategy in B depending on the outcome in A:

$$y_1(N^e,N^l) = \begin{cases} \frac{a-2x_1}{3} & if \quad x_1 < \frac{a}{2} \\ 0 & if \quad x_1 \geq \frac{a}{2} \end{cases} \qquad y_2(N^e,N^l) = \begin{cases} \frac{a+gx_1}{3} & if \quad x_1 < \frac{a}{2} \\ \frac{a}{2} & if \quad x_1 \geq \frac{a}{2} \end{cases}$$

Obviously, $x_1 = \frac{a}{2}$ is the optimal response to $x_2 = 0$. Neither $x_1 > \frac{a}{2}$, nor $x_1 < \frac{a}{2}$ can increase 1's profits. Supplying more reduces revenue in market A without changing revenue in market B, whereas supplying less changes 1's strategy also in market B and thus influences production costs (as y_1 is not zero any more). Marginally,

$$\frac{\partial \Pi_1}{\partial x_1}(x_1 = \frac{a}{2}, x_2 = 0) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C_1}{\partial x_1} - \frac{\partial C_1}{\partial y_1}\frac{dy_1}{dx_1} = 0$$

$\frac{d\Pi_1}{dx_1}{}_{-0}(x_1 = \frac{a}{2}, x_2 = 0) = -[\frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C_1}{\partial x_1} - \frac{\partial C_1}{\partial y_1}\frac{dy_1}{dx_1}] = -[\frac{4a(g-1)}{9}] < 0$ for $g \geq 1$.[26]

$x_2 = 0$ is also the optimal response to $x_1 = \frac{a}{2}$. Given $x_1 = \frac{a}{2}$, the reactions in market B are

$$y_1 = \begin{cases} \frac{a(1-g)+x_2}{3} & if \quad x_2 > \frac{a(g-1)}{g} \\ 0 & if \quad x_2 \leq \frac{a(g-1)}{g} \end{cases} \qquad y_2 = \begin{cases} \frac{a(2+g)-4gx_2}{6} & if \quad x_2 > \frac{a(g-1)}{g} \\ \frac{a-gx_2}{2} & if \quad x_2 \leq \frac{a(g-1)}{g} \end{cases}.$$

If player 2 chooses $x_2 = 0$, the optimal reaction of player 1 is $y_1 = 0$ whereas $y_2 = \frac{a-gx_2}{2} = \frac{a}{2}$. Player 2 can change his strategy by increasing x_2. But marginal profit at $x_2 = 0$ is

$$\frac{\partial \Pi_2}{\partial x_2}(x_1 = \frac{a}{2}, x_2 = 0) = \frac{\partial R_2}{\partial x_2} + \frac{\partial R_2}{\partial y_1}\frac{dy_1}{dx_2} + \frac{\partial R_2}{\partial y_2}\frac{dy_2}{dx_2} - \frac{\partial C_2}{\partial x_2} = \frac{a(9-32g)}{18} < 0.$$

[26] Note, that one has to take into account the different reactions in the late stage, depending on the direction of the derivative. Thus,
$y_1 = \frac{a-2x_1}{3}$, if one considers a neagtive deviation (less supply), whereas $y_1 = 0$, if one considers a positive deviation (more supply).

Hence, both player's strategies are optimal responses to each other and $x_i = y_j = \frac{a}{2}$ and $x_i = y_i = 0$, $i \neq j$, is an equilibrium for $g \geq 1$.

A.1.3 Profits applying to $S^1, N^e (= N^e, S^2 = S^1, N^e = N^e, S^1)$:

In this situation, only x_2 is chosen late, whereas firms decide early over $x_1,\ y_1$ and y_2.

Firm 2's maximization in the **late stage** yields

$$x_2(S^1, N^e) = \begin{cases} \frac{a - x_1 - gy_2}{2} & if \quad x_1 < a - gy_2 \\ 0 & if \quad x_1 \geq a - gy_2 \end{cases}.$$

Firm 1 is thus able to induce firm 2 to leave the market without having to satisfy the whole demand.

In the **early stage**, players maximize over $x_1,\ y_1$ and y_2, taking into account player 2's reaction in A. This leads to an interior solution whenever $g < \frac{1}{\sqrt{2}}$ (≈ 0.707). At this point, the optimal x_1 proposed by the interior solution equals $a - gy_2$, to which 2 reacts $x_2 = 0$ (the marginal gain from entering equals the marginal negative impact on production costs). For any x_1 smaller than this, firm 2 would enter the market again. The supply needed to keep player 2 out of the market is falling in g. Thus, for $g \geq \frac{1}{\sqrt{2}}$, player 1 offers the maximum of $a - gy_2$ and $x^m = a/2$ and is thus able to monopolize his market. Firm 2 reacts by concentrating on the market with simultaneous play.

The reaction-functions of the early stage are[27]

$$\begin{aligned} x_1(S^1, N^e) &= \begin{cases} \frac{a - gy_1 + gy_2}{2} & if \quad x_1 < a - gy_2 \\ \max\{\frac{a}{2}, a - gy_2\} & if \quad x_1 \geq a - gy_2 \end{cases} \\ y_1(S^1, N^e) &= \begin{cases} \frac{a - gx_1 - y_2}{2} & if \quad x_1 < a - gy_2 \\ \frac{a(g-1) - y_2(1-g^2)}{2} & if \quad a - gy_2 \leq x_1 < \frac{a}{2} \\ 0 & if \quad x_1 \geq \frac{a}{2} \end{cases} \\ y_2(S^1, N^e) &= \begin{cases} \frac{a - gx_2 - y_1}{2} & if \quad x_1 < a - gy_2 \\ 0 & if \quad x_1 \geq a - gy_2 \end{cases} \end{aligned}$$

This leads to the solution of the game[28]

$$\begin{aligned} x_1(S^1, N^e) &= \begin{cases} \frac{a(g^3 - 3g^2 - g + 3)}{g^4 - 9g^2 + 6} & if \quad g < \frac{1}{\sqrt{2}} \\ \frac{a(3-g)}{3+g^2} & if \quad \frac{1}{\sqrt{2}} \leq g < 1 \\ \frac{a}{2} & if \quad g \geq 1 \end{cases} \\ x_2(S^1, N^e) &= \begin{cases} \frac{a(2g^3 - 6g^2 - g + 3)}{2(g^4 - 9g^2 + 6)} & if \quad g < \frac{1}{\sqrt{2}} \\ 0 & if \quad g \geq \frac{1}{\sqrt{2}} \end{cases} \\ y_1(S^1, N^e) &= \begin{cases} \frac{a(2g^3 - 5g^2 - 3g + 4)}{2(g^4 - 9g^2 + 6)} & if \quad g < \frac{1}{\sqrt{2}} \\ \frac{a(g-1)^2}{3+g^2} & if \quad \frac{1}{\sqrt{2}} \leq g < 1 \\ 0 & if \quad g \geq 1 \end{cases} \end{aligned}$$

---

27 y_1 would become negative when $g > 1$. At the same time, for any $g \geq 1$, $x_1 = a/2$ (which is bigger than $a - gy_2$) is the optimal strategy for player 1. It is thus possible to rewrite the conditions for y_1 depending on x_1.

28 The critical value of $g = \frac{1}{\sqrt{2}} \approx 0,707$ is achieved by inserting the values of the interior solution in $x_1 = a - gy_2$ and solving for g.

$$y_2(S^1,N^e) = \{ \begin{array}{ll} \frac{a(g^3-3g^2-2)}{g^4-9g^2+6} & if \quad g < \frac{1}{\sqrt{2}} \\ \frac{a(g+1)}{3+g^2} & if \quad \frac{1}{\sqrt{2}} \leq g < 1 \\ \frac{a}{2} & if \quad g \geq 1 \end{array}$$

Profits are

$$\Pi_1(S^1,N^e) = \{ \begin{array}{ll} \frac{(a^2(1+g)^2(4g^5-24g^4+32g^3+35g^2-80g+34))}{4(g^4-9g^2+6)^2} & if \quad g < \frac{1}{\sqrt{2}} \\ \frac{a^2(1-g+8g^2-5g^3+g^4}{(3+g^2)^2} & if \quad \frac{1}{\sqrt{2}} \leq g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{array}$$

$$\Pi_2(S^1,N^e) = \{ \begin{array}{ll} \frac{a^2(4g^7-16g^6-14g^5+88g^4-2g^3-87g^2+6g+25)}{4(g^4-9g^2+6)^2} & if \quad g < \frac{1}{\sqrt{2}} \\ \frac{a^2(1+g)^2}{(3+g^2)^2} & if \quad \frac{1}{\sqrt{2}} \leq g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{array}$$

To see that the boundary solution ($x_1,\ y_1 = 0$ if $\frac{1}{\sqrt{2}} \leq g < 1$) is an equilibrium of the basic game, I first check player 1's incentive to deviate, given player 2's reaction in the late stage and given $y_2 = \frac{a(g+1)}{3+g^2}$:

$\frac{\partial \Pi_1}{\partial x_1}{}_{-0}(x_1 = \frac{a(3-g)}{3+g^2}, y_1 = \frac{a(g-1)^2}{3+g^2}) = -[\frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} - \frac{\partial C}{\partial x_1}] =$
$-\frac{a(-2g^3+6g2-37g)}{2(3+g^2)} < 0$ for $g > \frac{1}{\sqrt{2}}$.

$\frac{\partial \Pi_1}{\partial y_1}{}_{-0}(x_1 = \frac{a(3-g)}{3+g^2}, y_1 = \frac{a(g-1)^2}{3+g^2}) = -[\frac{\partial R_1}{\partial y_1} + \frac{\partial R_1}{\partial x_1}\frac{dx_1}{dy_1} + \frac{\partial R_1}{\partial x_2}\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dy_1} - \frac{\partial C}{\partial y_1}] = 0.$

Consequently, player 1 would not want to decrease $x_1$and is happy with $y_{1.}$

But he does not want to increase x_1 and $y_1$neither:

$\frac{d\Pi_1}{dx_1}(x_1 = \frac{a(3-g)}{3+g^2},\ y_1 = \frac{a(g-1)^2}{3+g^2}) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} - \frac{\partial C}{\partial x_1} =$
$= \frac{a(-g^3+3g^2+g-3)}{3+g^2} < 0$

$\frac{d\Pi_1}{dy_1}(x_1 = \frac{a(3-g)}{3+g^2}, y_1 = \frac{a(g-1)^2}{3+g^2}) = \frac{\partial R_1}{\partial y_1} + \frac{\partial R_1}{\partial x_1}\frac{dx_1}{dy_1} + \frac{\partial R_1}{\partial x_2}\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dy_1} - \frac{\partial C}{\partial y_1} = 0.$

In market A the marginal impact of x_1 on profits is always negative for $g < 1$, because player 1 already produces more than his optimal monopoly-quantity and $x_2 = 0$ for any $x_1 > \frac{a(3-g)}{3+g^2}$. In market B the marginal impact on revenue is zero: Being at the optimum in market B, the marginal decrease of y_2 resulting from a marginal increase of x_1 does not influence revenue there. The marginal impact on production-costs is negative, such that the total effect must also be negative. The proposed solution for y_1 is the optimal solution for player 1, as $\frac{\partial \Pi_1}{\partial y_1} = 0$.

Player 2 does not want to deviate, if $y_2 = \frac{a(g+1)}{3+g^2}$ is the optimal reaction to $x_1 = \frac{a(3-g)}{3+g^2}$ and $y_1 = \frac{a(g-1)^2}{3+g^2}$. As

$\frac{\partial \Pi_2}{\partial y_2}(x_1 = \frac{a(3-g)}{3+g^2},\ y_1 = \frac{a(g-1)^2}{3+g^2},\ y_2 = \frac{a(g+1)}{3+g^2}) = \frac{\partial R_2}{\partial y_2} + \frac{\partial R_2}{\partial x_2}\frac{dx_2}{dy_2} - \frac{\partial C_2}{\partial y_2} = 0$

$\frac{\partial \Pi_2}{\partial y_2}{}_{-0}(x_1 = \frac{a(3-g)}{3+g^2},\ y_1 = \frac{a(g-1)^2}{3+g^2},\ y_2 = \frac{a(g+1)}{3+g^2}) = -[\frac{\partial R_2}{\partial y_2} + \frac{\partial R_2}{\partial x_2}\frac{dx_2}{dy_2} - \frac{\partial C_2}{\partial y_2}] = 0,$

the proposed boundary solution is an equilibrium of the basic game.

It is obvious, that $x_1 = \frac{a}{2}$, $x_2 = 0$, $y_1 = 0$, $y_2 = \frac{a}{2}$, is the outcome of the game as soon as $g \geq 1$. At $g = 1$, $\frac{a(3-g)}{3+g^2} = \frac{a}{2}$. For $g > 1$, the output needed to keep player 2 out of the market is even lower than $\frac{a}{2}$. But player 1 is not interested to reduce supply in his leading market any more, because this does not have an impact on player 2's reaction in this market ($x_2 = 0$) and his own his optimal strategy in market B ($y_1 = 0$), but reduces total profits in his monopolized market. The same reasoning applies to player 2 in market B.

A.1.4 Profits applying to $N^l, S^1 (= N^l, S^2 = S^1, N^l = S^1, N^l)$:

In this case, firms decide late over $x_2,\ y_1$ and y_2. Firm 1 chooses only x_1 at the early stage and takes into account the optimal reaction of firm 2 in the late stage.

Again, there is a critical value $g \approx 0.619$, at which the interior solution leads to $x_2 = 0$ (the follower's output equals zero) whereas $x_1 = a - gy_2$. As x_2 cannot become negative, $x_1 = \max\{a - gy_2, \frac{a}{2}\}$ is the optimal strategy for player 1 in his leading market for any $\approx 0.619 \leq g < 1$. Output and prices in both markets are then the same as with S^1, N^e.

Maximization over $x_2,\ y_1$and y_2 in the late stage yields[29]

$$
\begin{aligned}
x_2(S^1, N^l) &= \begin{cases} \frac{a(3-g)-x_1(3+g^2)}{2(3-g^2)} & if \quad x_1 < \frac{a(3-g)}{3+g^2} \\ 0 & if \quad x_1 \geq \frac{a(3-g)}{3+g^2} \end{cases} \\
y_1(S^1, N^l) &= \begin{cases} \frac{a(2-g^2+g)+x_1(g^3-5g)}{2(3-g^2)} & if \quad x_1 < \frac{a(2-g^2+g)}{g^3-5g} \\ 0 & if \quad x_1 \geq \frac{a(2-g^2+g)}{g^3-5g} \end{cases} \\
y_2(S^1, N^l) &= \begin{cases} \frac{a(1-g)+2gx_1}{3-g^2} & if \quad x_1 < \frac{a(2-g^2+g)}{g^3-5g} \\ \frac{a}{2} & if \quad x_1 \geq \frac{a(2-g^2+g)}{g^3-5g} \end{cases}
\end{aligned}
$$

Inserting these values into the profit-function of player 1, early-stage maximization leads to[30]

$$
\begin{aligned}
x_1(S^1, N^l) &= \begin{cases} \frac{a(g^5-3g^4-6g^3+14g^2+7g-9)}{g^6-16g^4+49g^2-18} & if \quad g <\approx 0.619 \\ \frac{a(3-g)}{3+g^2} & if \quad \approx 0.619 \leq g < 1 \\ \frac{a}{2} & if \quad g \geq 1 \end{cases} \\
x_2(S^1, N^l) &= \begin{cases} \frac{a(2g^5-6g^4-13g^3+35g^2-g-9)}{2(g^6-16g^4+49g^2-18)} & if \quad g <\approx 0.619 \\ 0 & if \quad g \geq\approx 0.619 \end{cases} \\
y_1(S^1, N^l) &= \begin{cases} \frac{a(2g^5-7g^4-7g^3+23g^2+9g-12)}{2(g^6-16g^4+49g^2-18)} & if \quad g <\approx 0.619 \\ \frac{a(g-1)^2}{3+g^2} & if \quad \approx 0.619 \leq g < 1 \\ 0 & if \quad g \geq 1 \end{cases} \\
y_2(S^1, N^l) &= \begin{cases} \frac{a(g^5-3g^4-7g^3+19g^2-6)}{g^6-16g^4+49g^2-18} & if \quad g <\approx 0.619 \\ \frac{a(g+1)}{3+g^2} & if \quad \approx 0.619 \leq g < 1 \\ \frac{a}{2} & if \quad g \geq 1. \end{cases}
\end{aligned}
$$

---

29 The conditions for y_2 follow from $\frac{a(2-g^2+g)}{g^3-5g} > \frac{a(3-g)}{3+g^2}$ and the fact, that player 2's profit-maximizing quantity is $\frac{a}{2}$ when $y_1 = 0$ and $x_2 = 0$ (no competition in market B and no diseconomies of scope).

30 Again, the critical value for g in the condition for x_1 is obtained by not allowing x_2 to be negative. At this value, the interior solution for x_1 equals $\frac{a(3-g)}{3+g^2}$.

Profits are

$$\Pi_1(S^1,N^l) \;=\; \{ \begin{array}{ll} \frac{(a^2(1+g)^2(4g^3-24g^2+40g-17))}{4(g^6-16g^4+49g^2-18)^2} & if \quad g <\approx 0.619 \\ \frac{a^2(1-g+8g^2-5g^3+g^4)}{(3+g^2)^2} & if \quad \approx 0.619 \leq g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{array}$$

$$\Pi_2(S^1,N^l) \;=\; \{ \begin{array}{ll} \frac{a^2(4g^{11}-16g^{10}-66g^9+272g^8+358g^7-1535g^6}{4(g^6-16g^4+49g^2-18)^2} \\ \frac{-576g^5+3191g^4-262g^3-1529g^2+126g+255)}{4(g^6-16g^4+49g^2-18)^2} & if \quad g <\approx 0.619 \\ \frac{a^2(1+g)^2}{(3+g^2)^2} & if \quad \approx 0.619 \leq g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{array}$$

The interior solution follows from standard profit maximization. To prove, that the boundary solution is an equilibrium, I proceed as before and show, that neither of the firms has an incentive to deviate from this solution. For firm 1, one gets:

$$\frac{\partial \Pi_1}{\partial x_1}_{-0}(x_1 = \frac{a(3-g)}{3+g^2}) = -[\frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C_1}{\partial x_1}] = \frac{a(-2g^5+6g^4+13g^3-35g^2+9)}{18-2g^4} < 0 \text{ for } g >\approx 0.619$$

$$\frac{\partial \Pi_1}{\partial x_1}(x_1 = \frac{a(3-g)}{3+g^2}) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C_1}{\partial x_1} = \frac{(1-g)(a(-g^4+2g^3+9g^2-8g-18)+x_1(g^5+g^4-13g^3-13g^2+36g+36))}{2(3-g^2)} < 0 \text{ for } g < 1$$

$$\frac{\partial \Pi_1}{\partial y_1}(x_1 = \frac{a(3-g)}{3+g^2},\ x_2 = 0,\ y_1 = \frac{a(g-1)^2}{3+g^2},\ y_2 = \frac{a(g+1)}{3+g^2}) = \frac{\partial R_1}{\partial y_1} - \frac{\partial C_1}{\partial y_1} = 0.$$

Thus, player 1's strategy is the optimal answer to the strategy of player 2.

Marginal profit when changing the strategy for player 2 are

$$\frac{\partial \Pi_2}{\partial x_2}(x_1 = \frac{a(3-g)}{3+g^2},\ x_2 = 0,\ y_1 = \frac{a(g-1)^2}{3+g^2},\ y_2 = \frac{a(g+1)}{3+g^2}) = \frac{\partial R_2}{\partial x_2} - \frac{\partial C_2}{\partial x_2} = 0$$

$$\frac{d\Pi_2}{dy_2}(x_1 = \frac{a(3-g)}{3+g^2},\ x_2 = 0,\ y_1 = \frac{a(g-1)^2}{3+g^2},\ y_2 = \frac{a(g+1)}{3+g^2}) = \frac{\partial R_2}{\partial y_2} - \frac{\partial C_2}{\partial y_2} = 0.$$

Therefore, the boundary solution is an equilibrium of the game for $\approx 0.619 \leq g < 1$.

When $g \geq 1$, less (equal) supply than the optimal monopoly supply is necessary to induce the competitor to stay out of the market. Consequently, players switch to $x_1 = \frac{a}{2}$, $x_2 = 0$, $y_1 = 0$, $y_2 = \frac{a}{2}$.

A.1.5 Profits applying to S^1, S^2 $(= S^2, S^1)$:

In this situation each firm acts like a leader in one market. At $g = \frac{1}{2}$, second-order conditions do not hold any more. Firm 1 then supplies $x_1 = a - g\ y_2$ whereas firm 2 supplies $y_2 = a - g\ x_2,$ which prevents mutual entry in the late stage. When $g \geq 1$, the monopoly-output x^m is high enough to deprive the other firm of the incentive to enter the leader-market: given both produce x^m, the marginal gain of entering the others market in the late stage is $p = \frac{a}{2}$ and equals or is lower than the potential loss of $g\frac{a}{2}$.

The reaction-functions of the late stage are

$$x_2(S^1,S^2) = \{ \begin{array}{lll} \frac{a-x_1-gy_2}{2} & if & x_1 < a-gy_2 \\ 0 & if & x_1 \geq a-gy_2 \end{array} \quad y_1(S^1,S^2) = \{ \begin{array}{lll} \frac{a-y_2-gx_1}{2} & if & y_2 < a-gx_1 \\ 0 & if & y_2 \geq a-gx_1. \end{array}$$

Maximization over x_1 and y_2 in the early stage leads to

$$\begin{array}{rcl} x_1(S^1,S^2) & = & y_2(S^1,S^2) = \{ \begin{array}{lll} \frac{a(1-g)}{-g^2-2g+2} & if & g < \frac{1}{2} \\ \frac{a}{1+g} & if & \frac{1}{2} \leq g < 1 \\ \frac{a}{2} & if & g \geq 1 \end{array} \\ x_2(S^1,S^2) & = & y_1(S^1,S^2) = \{ \begin{array}{lll} \frac{a(1-2g)}{2(-g^2-2g+2)} & if & g < \frac{1}{2} \\ 0 & if & g \geq \frac{1}{2} \end{array} \qquad and \\ \Pi_i(S^1,S^2) & = & \{ \begin{array}{lll} \frac{a^2(4g^3-6g+3)}{4(g^2+2g-2)^2} & if & g < \frac{1}{2} \\ \frac{a^2 g}{(1+g)^2} & if & \frac{1}{2} \leq g < 1 \\ \frac{a^2}{4} & if & g \geq 1. \end{array} \qquad i = 1,2 \end{array}$$

The interior solution follows from standard profit-maximization. To check, if the boundary solution is an equilibrium, I calculate the partial derivatives of the profit-functions at the proposed values for x_1 and y_2:

$$\frac{\partial \Pi_1}{\partial x_1}(x_1 = \frac{a}{1+g},\ y_2 = \frac{a}{1+g}) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} - \frac{\partial C}{\partial x_1} < 0 \text{ for } g < 1$$

$$\frac{d\Pi_1}{dx_1}_{-0}(x_1 = \frac{a}{1+g},\ y_2 = \frac{a}{1+g}) = -[\frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_1}\frac{dy_1}{dx_1} - \frac{\partial C}{\partial x_1}] < 0 \text{ for } g < \frac{1}{2}.$$

By symmetry, the same result holds for player 2, such that the boundary solution is an equilibrium for $\frac{1}{2} \leq g < 1$. When $g \geq 1$, the monopoly output is bigger than the output proposed by the boundary solution and the only equilibrium is

$x_1 = \frac{a}{2}, x_2 = 0, y_1 = 0, y_2 = \frac{a}{2}$.

A.1.6 Profits applying to S^1, S^1 $(= S^2, S^2)$:

In this case, one firm (firm 1) is able to move before the other firm in both markets. It is thus able to choose its' preferred points on firm 2's reaction curves. Again, there are different equilibria, depending on the value of g. There is an interior solution, where firm 1 is active in both markets. This solution ceases to be an equilibrium when $g \geq 2-\sqrt{2} \approx 0.586$ (second-order-conditions do not hold any more). The leading firm 1 then switches to the boundary and leaves one market (say market B). Firm 2 remains active in both markets for $g < \frac{\sqrt{5}-1}{2} \approx 0.618$. But when g is equal or bigger than this value, it leaves the market where firm 1 is active and concentrates on market B.

Solving the game backwards yields

$$x_2(S^1,S^1) = \{ \begin{array}{lll} \frac{a}{2} & if & x_1 = 0 \; and \; y_1 \geq \frac{ag(2-g)}{2g} \\ \frac{a(2-g)-2x_1+gy_1}{4-g^2} & if & x_1 < \frac{a(2-g)+gy_1}{2} \\ 0 & if & x_1 \geq \frac{a(2-g)+gy_1}{2} \end{array}$$

$$y_2(S^1,S^1) = \{ \begin{array}{lll} \frac{a}{2} & if & y_1 = 0 \; and \; x_1 \geq \frac{ag(2-g)}{2g} \\ \frac{a(2-g)-2y_1+gx_1}{4-g^2} & if & x_1 > \frac{a(g-2)+2y_1}{g} \\ 0 & & x_1 \leq \frac{a(g-2)+2y_1}{g}. \end{array}$$

Assuming, that player 1 concentrates on market B for $g \geq 2-\sqrt{2}$, one gets

$$x_1(S^1,S^1) = \{ \begin{array}{lll} \frac{a(1+g)}{g^2+4g+2} & if & g < 2-\sqrt{2} \\ \frac{a(2-g)(g+1)}{2(2-g^2)} & if & 2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a(2-g)}{2} & if & \frac{\sqrt{5}-1}{2} \leq g < 1 \\ \frac{a}{2} & if & g \geq 1 \end{array}$$

$$y_1(S^1,S^1) = \{ \begin{array}{lll} \frac{a(1+g)}{g^2+4g+2} & if & g < 2-\sqrt{2} \\ 0 & if & g \geq 2-\sqrt{2} \end{array}$$

$$x_2(S^1,S^1) = \{ \begin{array}{lll} \frac{a(g^2+3g+1)}{g^3+6g^3+10g+4} & if & g < 2-\sqrt{2} \\ \frac{a(1-g^2-g)}{4-g^3-2g^2+2g} & if & 2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2} \\ 0 & if & g \geq \frac{\sqrt{5}-1}{2} \end{array}$$

$$y_2(S^1,S^1) = \{ \begin{array}{lll} \frac{a(g^2+3g+1)}{g^3+6g^3+10g+4} & if & g < 2-\sqrt{2} \\ \frac{a(4-g^2+g)}{2(4-g^3-2g^2+2g)} & if & 2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a}{2} & if & g \geq \frac{\sqrt{5}-1}{2} \end{array}$$

and

$$\Pi_1(S^1,S^1) = \{ \begin{array}{lll} \frac{a^2(1+g)^2}{4+10g+6g^2+g^3} & if & g < 2-\sqrt{2} \\ \frac{a^2(2-g)(1+g)^2}{4(2+g)2-g^2)} & if & 2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2 g(2-g)}{4} & if & \frac{\sqrt{5}-1}{2} \leq g < 1 \\ \frac{a^2}{4} & if & g \geq 1 \end{array}$$

$$\Pi_2(S^1,S^1) = \{ \begin{array}{lll} \frac{a^2(1+3g+g^2)^2}{(2+g)(2+4g+g^2)^2} & if & g < 2-\sqrt{2} \\ \frac{a^2(2g^4+g^3-8g^2-g+10)}{4(2+g)2-g^2)} & if & 2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2}{4} & if & g \geq \frac{\sqrt{5}-1}{2}. \end{array}$$

The equilibrium for $g < 2-\sqrt{2}$ follows from standard profit-maximization. When $2-\sqrt{2} \leq g < \frac{\sqrt{5}-1}{2}$, the above proposed solution (firm 1 leaving one market and firm 2 remaining active in both markets) is an equilibrium, as

$$\frac{\partial \Pi_1}{\partial x_1}(x_1 = \frac{a(2-g)(g+1)}{2(2-g^2)},\; y_1 = 0) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C}{\partial x_1} = 0$$

$\frac{\partial \Pi_1}{\partial y_1}(x_1 = \frac{a(2-g)(g+1)}{2(2-g^2)},\; y_1 = 0) = \frac{\partial R_1}{\partial y_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dy_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dy_1} - \frac{\partial C}{\partial y_1} < 0$ for $g > 2-\sqrt{2}$.[31]

Firm 2's strategy is derived from its reaction-function and must therefore be the optimal strategy given x_1 and y_1 :

$$\frac{\partial \Pi_2}{\partial x_2}(x_1 = \frac{a(2-g)(g+1)}{2(2-g^2)},\; y_1 = 0,\; x_2 = \frac{a(1-g^2-g)}{4-g^3-2g^2+2g},\; y_2 = \frac{a(4-g^2+g)}{2(4-g^3-2g^2+2g)}) = \frac{\partial R_2}{\partial x_2} - \frac{\partial C}{\partial x_2} = 0$$

[31] To derive this result, one has to insert the optimal reaction for firm 2, irrespective of $y_1 = 0$, that is $x_2(x_1, y_1) = \frac{a(2-g)-2x_1+gy_1}{4-g^2}$ and $y_2 = \frac{a(2-g)-2y_1+gx_1}{4-g^2}$.

$\frac{\partial \Pi_2}{\partial y_2}(x_1 = \frac{a(2-g)(g+1)}{2(2-g^2)},\ y_1 = 0,\ x_2 = \frac{a(1-g^2-g)}{4-g^3-2g^2+2g},\ y_2 = \frac{a(4-g^2+g)}{2(4-g^3-2g^2+2g)}) = \frac{\partial R_2}{\partial y_2} - \frac{\partial C}{\partial y_2} = 0.$

For $\frac{\sqrt{5}-1}{2} \leq g < 1$ one gets[32]

$$\frac{\partial \Pi_1}{\partial x_1}(x_1 = \frac{a(2-g)}{2},\ y_1 = 0) = \frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C}{\partial x_1} = a(g-1) < 0 \text{ for } g < 1$$

$\frac{\partial \Pi_1}{\partial x_{1\,-0}}(x_1 = \frac{a(2-g)}{2},\ y_1 = 0) = -[\frac{\partial R_1}{\partial x_1} + \frac{\partial R_1}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial R_1}{\partial y_2}\frac{dy_2}{dx_1} - \frac{\partial C}{\partial x_1}] = -[\frac{a(g^2+g-1)}{2+g}] < 0$ for $g < \frac{\sqrt{5}-1}{2}$.

Again, the solution for player 2 is derived from his reaction-function, such that the derivatives of Π_2 at $x_2 = 0$ and $y_2 = \frac{a}{2}$ must equal zero.

Finally, when $g > 1$, the same argument applies as in the former cases: The x_1 necessary to prevent firm 2 from entering the market is smaller than x^m, such that the optimal solution is $x_1 = x^m = \frac{a}{2}$, $x_2 = 0$, $y_1 = 0$ and $y_2 = y^m = \frac{a}{2}$.

A.2 Solution of the first stage

In the timing stage of the extended game, players choose the optimal timing, comparing the profits pertaining to each timing situation and taking into account the strategy space of the competitor. The chosen timing in the two markets (t_1^A,t_1^B,t_2^A,t_2^B), t = e, l, constitute an equilibrium, when for both firms its own timing is the best answer to the competitors timing choice.

Optimal profits for the different timing situations depending on g are plotted in fig. 3.3 c) (profits are calculated for $a = 1$. Whereas the level of profits changes with different a, the critical values for g remain the same).

As long as $g < g^*$, $\Pi_1(S^1, N^e)$ is higher than $\Pi_i(S^1, S^2)$. It is also higher than $\Pi_i(N, N)$ and $\Pi_i(S^j, S^j)$, $j \neq i$. Therefore, both firms choosing "early" in both markets ((e_1^A,e_1^B,e_2^A,e_2^B)) must be the only equilibrium. In any other situation one of the firms having chosen "late" is able to do better by choosing early.

When g gets bigger than g*, $\Pi_1(S^1, N^e) = \Pi_2(N^e, S^2) < \Pi_i(S^1, S^2)$. The situation dominates any timing-situation achievable for each firm given the other firm chooses (e, l) or (l, e). Thus, neither firm can do better and $(e_1^A, l_1^B, l_2^A, e_2^B)((l_1^A, e_1^B, e_2^A, l_2^B)$ resp.) must be equilibria of the timing-game.

Profits applying to (S^i, S^j) are preferred to profits resulting from (N, N) by both firms when g gets bigger than ≈ 0.41. Nevertheless, $(e_1^A, e_1^B, e_2^A, e_2^B)$ leading to early simultaneous play in both markets is a (dominated) equilibrium of the timing game as long as $g < g^{**}$. At this point, when there was (N^e, S^2), firm 2 would choose the boundary solution as a leader in market B. $\Pi_1(N^e, S^2)$ then becomes higher than $\Pi_1(N, N)$,such that (e_1^A, e_1^B) is not any more the optimal

[32] As before, there is to be made a difference in the direction of deviation. The optimal reaction of firm 2 in market A, when firm 1 produces marginally more, remains $x_2 = 0$, whereas it is $x_2 = \frac{a(2-g)-2x_1+gy_1}{4-g^2}$, if firm 1 produces marginally less.

response to (e_2^A,e_2^B). The choices $(e_1^A, l_1^B, l_2^A, e_2^B)$ and $(l_1^A, e_1^B, e_2^A, l_2^B)$ remain equilibria of the timing game, as for both firms they are preferred to any other achievable situation. But two other equilibria appear, being described by one firm leading in two markets. Both firms prefer the followership-role in both markets. Nevertheless, as the leader can only achieve simultaneous play by changing his timing, $(e_1^A, e_1^B, l_2^A, l_2^B)$ and $(l_1^A, l_1^B, e_2^A, e_2^B)$ are equilibria of the timing game.

Appendix B: Welfare Effects

B.1 Welfare applying to equilibrium timing situations

(1) Welfare, consumer surplus (CS) and summed profits (PS) in an institutionalized monopoly:

$$\begin{aligned} W(M^1, M^2) &= W^b(N,N) = 2 \cdot \int_0^{\frac{a}{2}} (a-x)dx = \frac{3a^2}{4} \\ CS(M^1, M^2) &= 2 \cdot \int_0^{\frac{a}{2}} (a-x)dx - 2\Pi_i(M^1, M^2) = \frac{a^2}{4} \\ PS(M^1, M^2) &= 2\Pi_i(M^1, M^2) = \frac{a^2}{2} \end{aligned}$$

(2) Welfare, CS and PS in a two-market duopoly with diseconomies of scope when $g < 1$:

$$\begin{aligned} W(N,N) &= 2 \cdot \int_0^{x(N,N)} (a-x)dx - \sum_i C_i(N,N) = \{ \begin{array}{ll} \frac{2a^2(4+g)}{(3+g)^2} & if \quad g < 1 \\ \frac{3a^2}{4} & if \quad g \geq 1 \end{array} \\ CS(N,N) &= 2 \cdot \int_0^{x(N,N)} (a-x)dx - 2\Pi_i(N,N) - \sum_i C_i(N,N) \\ &= \{ \begin{array}{ll} \frac{4a^2}{(3+g)^2} & if \quad g < 1 \\ \frac{a^2}{4} & if \quad g \geq 1 \end{array} \\ PS(N,N) &= 2\Pi_i(N,N) = p_A \sum_i x_i(N,N) + p_B \sum_i y_i\,(N,N) - \sum_i C_i(N,N) = \\ &= \{ \begin{array}{ll} \frac{2a^2(2+g)}{(3+g)^2} & if \quad g < 1 \\ \frac{a^2}{2} & if \quad g \geq 1 \end{array} . \end{aligned}$$

(3) Welfare, CS and PS when each firm is leading in one market and $g^* \leq g < 1$:

$$\begin{aligned} W(S^i, S^j) &= 2 \int_0^{x(S^i,S^j)} (a-x)dx - \sum_i C_i(S^i, S^j) = \\ &= \{ \begin{array}{ll} \frac{a^2(8g^3+16g^2-36g+15)}{4(g^2+2g-2)^2} & for \quad g^* \leq g < 0,5 \\ \frac{a^2(1+2g)}{(1+g)^2} & for \quad 0,5 \leq g < g^{**} \end{array} \\ CS(S^i, S^j) &= W(S^i, S^j) - 2\Pi_i(S^i, S^j) - \sum_i C_i(S^i, S^j) = \\ &= \{ \begin{array}{ll} \frac{a^2(4g-3)^2}{4(g^2+2g-2)^2} & for \quad g^* \leq g < 0,5 \\ \frac{a^2}{(1+g)^2} & for \quad 0,5 \leq g < g^{**} \end{array} \\ PS(S^i, S^j) &= 2\Pi_i(S^i, S^j) = \\ &= \{ \begin{array}{ll} \frac{a^2(4g^3-6g+3)}{2(g^2+2g-2)^2} & for \quad g^* \leq g < 0,5 \\ \frac{2a^2g}{(1+g)^2} & for \quad 0,5 \leq g < g^{**} \end{array} \end{aligned}$$

(4) Welfare, CS and PS when one firm is leading in both markets and $g^{**} \leq g < 1$:

$$W(S^i, S^i) = 2\int_0^{x(S^i,S^i)} (a-x)dx - \sum_i C_i(S^i, S^i) =$$
$$= \begin{cases} \frac{a^2(3g^6+15g^5+6g^4-62g^3-69g^2+80g+108)}{8(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(7-g^2)}{8} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases}$$

$$CS(S^i, S^i) = W(S^i, S^i) - \sum_i \Pi_i(S^i, S^i) - \sum_i C_i(S^i, S^i) =$$
$$= \begin{cases} \frac{a^2(g^6+6g^5+6g^4-26g^3-39g^2+32g+52)}{8(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(5-4g+g^2)}{8} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases}$$

$$PS(S^i, S^i) = \sum_i \Pi_i(S^i, S^i) =$$
$$= \begin{cases} \frac{a^2(g^5+2g^4-4g^3-10g^2+5g+14)}{4(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(1+2g-g^2)}{4} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases}$$

B.2 Welfare differences

(1) $g < g^*$

When $g < g^*$, there is no difference between the extended game and the basic game. The welfare difference between the extended / basic game and a two-market monopoly equals

$$W(N, N) - W(M^i, M^j) = \frac{2a^2(2+g)}{(3+g)^2} - \frac{3a^2}{4} \begin{matrix} < 0 & if & g <\approx 0.441 \\ \geq 0 & if & g \geq\approx 0.441 \end{matrix}.$$

Consumers are better off with two-market competition whereas firms incur losses in comparison to a monopolistic setting.

(2) $g^* \leq g < g^{**}$

The welfare difference between the one-stage-game and the extended game is

$$\Delta W^1 = W(S^i, S^j) - W(N, N) =$$
$$= \begin{cases} \frac{1}{4}a^2 \frac{68g^3+16g^4+7g^2-138g+71}{(g^2+2g-2)^2(3+g)^2} & for \quad g < 0,5 \\ a^2 \frac{5+14g+5g^2}{(1+g)^2(3+g)^2} & for \quad 0,5 \leq g < g^{**} \end{cases} > 0.$$

Both - consumers and firms gain:

$$\Delta CS^1 = CS(S^i, S^j) - CS(N, N) =$$
$$= \begin{cases} \frac{a^2(3-4g)^2}{4(2-g^2-2g)^2} - \frac{4a^2}{(3+g)^2} & if \quad g^* \leq g < 0.5 \\ \frac{a^2}{(1+g)^2} - \frac{4a^2}{(3+g)^2} = \frac{a^2(5-2g-3g^2)}{(1+g)^2(3+g)^2} & if \quad 0.5 \leq g < g^{**} \end{cases} > 0,$$

$$\Delta PS^1 = PS(S^i, S^j) - PS(N, N) = 2\Pi_i(S^1, S^2) - 2\Pi_i(N, N) =$$

$$= \left\{ \begin{array}{ll} \frac{a^2(3-6g+4g^3)}{2(-2+2g+g^2)^2} - \frac{2a^2(2+g)}{(3+g)^2} & if \quad g^* \leq g < 0.5 \\ \frac{2a^2 g}{(1+g)^2} - \frac{2a^2(2+g)}{(3+g)^2} = \frac{4a^2(g^2+2g-1)}{(1+g)^2(3+g)^2} & if \quad 0.5 \leq g < g^{**} \end{array} \right. > 0$$

Consumers' gain first increases, then decreases in g:

$$\frac{\partial(\Delta CS^1)}{\partial g} = \begin{array}{lll} \frac{a^2(17-28g+66g^2+28g^3-19g^4-6g^5)}{(3+g)^2(-2+2g+g^2)^2} & if \quad g^* \leq g < 0.5 & > 0 \\ -\frac{2a^2(23+15g+3g^2+3g^3)}{(1+g)^3(3+g)^3} & if \quad 0.5 \leq g < g^{**} & < 0 \end{array}$$

Firms' gain increases in the whole range between g* and g^{**} :

$$\frac{\partial(\Delta PS^1)}{\partial g} = \begin{array}{lll} \frac{a^2(-16+32g-57g^2-29g^3+7g^4+3g^5)}{(3+g)^3(-2+2g+g^2)^3} & if \quad g^* \leq g < 0.5 & > 0 \\ \frac{8a^2(7+g-3g^2-g^3)}{(1+g)^3(3+g)^3} & if \quad 0.5 \leq g < g^{**} & > 0 \end{array}$$

Comparison to monopoly yields

$$\begin{aligned} \Delta W^2 &= W(S^i, S^j) - W(M^1, M^2) = \\ &= \left\{ \begin{array}{ll} a^2 \frac{3-12g+16g^2-4g^3-3g^4}{4(g^2+2g-2)^2} & for \quad g < 0.5 \\ \frac{a^2(1+2g-3g^2)}{4(1+g)^2} & for \quad 0.5 \leq g < g^{**} \end{array} \right. > 0 \end{aligned}$$

$$\begin{aligned} \Delta CS^2 &= CS(S^i, S^j) - CS(M^1, M^2) = \\ &= \left\{ \begin{array}{ll} \frac{a^2(-g^4-4g^3+16g^2-16g+5)}{4(g^2+2g-2)^2} & for \quad g < 0.5 \\ \frac{a^2(-g^2-2g+3)}{4(1+g)^2} & for \quad 0.5 \leq g < g^{**} \end{array} \right. > 0 \\ & and \\ \Delta PS^2 &= PS(S^i, S^j) - PS(M^1, M^2) = \\ &= \left\{ \begin{array}{ll} a^2 \frac{2g-1-g^4}{2(g^2+2g-2)^2} & for \quad g^* \leq g < 0,5 \\ a^2 \frac{2g-1-g^2}{2(1+g)^2} & for \quad 0,5 \leq g < g^{**} \end{array} \right. < 0 \end{aligned}$$

The (positive) difference in consumer surplus as well as the (negative) difference in profits are decreasing in g:

$$\begin{aligned} \frac{\partial(\Delta CS^2)}{\partial g} &= \begin{array}{ll} \frac{a^2(3-13g+18g^2-8g^3)}{(-2+2g+g^2)^3} & if \quad g^* \leq g < 0.5 \\ \frac{-2a^2}{(1+g)^3} & if \quad 0.5 \leq g < g^{**} \end{array} < 0 \\ \frac{\partial(\Delta PS^2)}{\partial g} &= \begin{array}{ll} \frac{a^2 g^2(-3+4g-2g^2)}{(-2+2g+g^2)^3} & if \quad g^* \leq g < 0.5 \\ \frac{2a^2(1-g)}{(1+g)^3} & if \quad 0.5 \leq g < g^{**} \end{array} > 0 \end{aligned}$$

(3) $g^{**} \leq g < 1$

In this range of g there are two potential equilibria: one with each firm leading in one market and one with one firm leading in both markets. When (S^i, S^j) is the outcome of the timing game,

$$\Delta W^{31} = W(S^i, S^j) - W(N, N) = a^2 \frac{5 + 14g + 5g^2}{(1 + g^2)(3 + g)^2} > 0$$

$$\Delta CS^{31} = CS(S^i,S^j) - CS(N,N) = \frac{a^2(3-g^2-g)}{4(1+g)^2} > 0$$
$$\Delta PS^{31} = PS(S^i,S^j) - PS(N,N) = \frac{4a^2(g^2+2g-1)}{(1+g)^2(3+g)^2} > 0.$$

Differences in welfare, consumer- and producer surplus when the timing game leads to (S^i, S^i) are

$$\Delta W^{32} = W(S^i,S^i) - W(N,N)$$
$$= \begin{cases} a^2 \frac{3g^8+17g^7-5g^6-147g^5-131g^4+324g^3+479g^2+88g-52}{8(2+g)^2(-2+g^2)^2(3+g)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ a^2 \frac{-g^4-6g^3-2g^2+26g-1}{8(3+g)^2} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0$$

$$\Delta CS^{32} = CS(S^i,S^i) - CS(N,N) =$$
$$= \begin{cases} \frac{a^2(g^8+12g^7+19g^6-64g^5-141g^4+76g^3+277g^2+88g-44)}{8(2+g)^2(3+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(g^4+2g^3-10g^2-6g+13)}{8(3+g)^2} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0$$

$$\Delta PS^{32} = PS(S^i,S^i) - PS(N,N) =$$
$$= \begin{cases} \frac{a^2(g^7-15g^5-16g^4+37g^3+50g^2+g-2)}{4(2+g)(3+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(-g^4-4g^3+4g^2+16g-7)}{4(3+g)^2} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0$$

Comparison to monopoly yields:

$$\Delta W^{41} = W(S^i,S^j) - W(M^i,M^j) = \frac{a^2(1+2g-3g^2)}{4(1+g)^2} > 0$$
$$\Delta CS^{41} = CS(S^i,S^j) - CS(M^1,M^2) = \frac{a^2}{(1+g)^2} - \frac{a^2}{4} > 0$$
$$\Delta PS^{41} = PS(S^i,S^j) - PS(M^1,M^2) = \frac{a^2(g-1)^2}{2(g+1)^2} < 0$$

$$\Delta W^{42} = W(S^i,S^i) - W(M^i,M^j) =$$
$$= \begin{cases} a^2 \frac{-3g^6-9g^5+6g^4+34g^3+3g^2-16g+12}{8(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{1}{8}a^2(1-g^2) & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1. \end{cases} > 0$$

$$\Delta CS^{42} = CS(S^i,S^i) - CS(M^1,M^2) =$$
$$= \begin{cases} \frac{a^2(-g^6-2g^5+6g^4+6g^3-15g^2+20)}{8(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(g^2-4g+3)}{8} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0$$

$$\Delta PS^{42} = P(S^i,S^i) - PS(M^1,M^2) =$$
$$= \begin{cases} \frac{a^2(5g^5-2g^4+4g^3+6g^2-3g-2)}{4(2+g)(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ -\frac{a^2(g-1)^2}{4} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} < 0$$

Comparison of the sequential play equilibria (S^i, S^j) and (S^i, S^i) yields

$$\sum x_i(S^i,S^j) + \sum y_i(S^i,S^j) - (\sum x_i(S^i,S^i) + \sum y_i(S^i,S^i)$$

$$= \begin{cases} \frac{a(-6+5g+7g^2+g^3+g^4)}{2(1+g)(2+g)(-2+g^2)} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a(g-1)^2}{2(1+g)} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0$$

such that

$$\begin{aligned} \Delta CS^5 &= CS(S^i, S^j) - CS(S^i, S^i) = \\ &= \begin{cases} \frac{a^2(76-8g-173g^2-56g^3+85g^4+40g^5-11g^6-8g^7-g^8)}{8(1+g)^2(2+g)^2(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ -\frac{a^2(g-1)^2(g^2-3)}{8(1+g)^2} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0 \end{aligned}$$

but also

$$\begin{aligned} \Delta PS^5 &= PS(S^i, S^j) - PS(S^i, S^i) = \\ &= \begin{cases} \frac{a^2(-14+31g+18g^2-45g^3-16g^4+15g^5+4g^6-g^7)}{4(1+g)^2(2+g)(-2+g^2)^2} & if \quad g^{**} \leq g < \frac{\sqrt{5}-1}{2} \\ \frac{a^2(-1+4g-4g^2+g^4)}{4(1+g)^2} & if \quad \frac{\sqrt{5}-1}{2} \leq g < 1 \end{cases} > 0, \end{aligned}$$

which results in

$$\Delta W^5 = W(S^i, S^j) - W(S^i, S^i) > 0$$

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