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Dynamic Inefficiency in an Overlapping Generation Economy with Pollution and Health Costs¹

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RESUMEN

Se analiza un modelo de generaciones sucesivas en el que la producción provoca polución que se acumula en la atmósfera. Los hogares no se preocupan directamente del medio ambiente, sin embargo la existencia de polución conlleva gastos en salud cuando se llega al periodo de la vejez. Mostramos cómo la presencia de polución hace más posible la existencia de ineficiencia dinámica en un contexto competitivo. Para estas situaciones se diseñan dos tipos de impuestos: uno basado en gravar a la producción y otro basado en el gravamen de las rentas de trabajo y capital. Mostramos cómo diseñar ambos regímenes de manera que el equilibrio competitivo alcance la regla de oro. También mostramos que gravando la producción, tanto viejos como jóvenes, pagan menos impuestos que gravando a las rentas de trabajo y capital.

Palabras clave: Ineficiencia dinámica, Externalidades, Costes de salud, Generaciones sucesivas, Polución, Impuestos.

ABSTRACT

We analyze an overlapping generations model in which pollution arises, in an accumulative way, from production. Households do not care directly about the environment, however pollution makes them incur health costs when they are elderly. We show that the presence of pollution makes the economy more likely to be dynamically inefficient. For these cases we analyze two kinds of tax scheme: one based on production taxes and the other based on capital and wage taxes. We show how to design both schemes in order to put the economy into the golden rule allocation. We also show that under the production tax scheme young and elderly agents pay less taxes (or receive more transfers) than under the capital and wage tax system.

Keywords: Dynamic inefficiency, Externalities, Health costs, Overlapping generations, Pollution, Taxes.

JEL Classification: D62, D91, H21, Q25

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1 Introduction

The World Bank (1992, p. 44) considers that the environment affects the economy through amenity, productivity and health channels. Most of the economic literature that has addressed this issue considers only the first two channels. The amenity effect is incorporated into economic models by assuming that utility agents depend negatively on pollution, which is a side product of the production process (e.g. Balcao (2001)). The productivity channel considers that pollution may directly affect production technology (e.g. Grimaud (1999) and Chao and Peck (2000)) or the productivity of any input (Ono (2002)). The aim of this paper is to analyze the effects that pollution can produce on the economy when it affects the health of agents that are forced to spend on medical care.

The health effect has not often been considered in theoretical environmental models. However, it is well established from the empirical point of view that quality environment and health are positively related. For instance, Xu *et al.* (1998), using epidemiological studies carried out in Beijing, observe that long-term exposure to air pollution is associated with increased respiratory symptoms or bronchitis in adults. Boyce *et al.* (1999) find that environmental stress is associated with lower scores on a composite public health index¹, in a cross-sectional analysis of the 50 US states. Gangadharan and Valenzuela (2001) show, using 1998 data from 51 countries, that environmental stress has a significant negative effect on health status when the environmental variable is considered endogenous in the estimation procedure. Moreover, it is well known that more protective environmental policies may generate considerable health benefits in quantitative terms.²

As far as we know only Williams (2002) develops a theoretical model to examine the economic effects of pollution when it affects consumers' health. He uses a static general equilibrium model in which pollution deteriorates

¹The public health index used is a composite measure based on 23 indicators such as overall death rate, infant mortality, percentage of low birthweight babies and other variables not directly influenced by environmental stress such as the percentage of population covered by health insurance or the extent of alcohol consumption and smoking.

²Alberini *et al.* (1997) estimate that the morbidity value associated with the reduction of the Pollution Standard Index in Taiwan is between \$109 and \$262 millions. Quah and Boon (2003) estimate that the total economic cost of PM₁₀ in Singapore is about 4.31% of its GDP in 1999. Ostro and Chestnut (1998) show that the annual health benefit for the USA of attaining the new standards for PM_{2.5} relative to 1994-1996 ambient concentration is estimated at \$32 billion. In similar research, Zaim (1999) finds that a decrease in PM₁₀ and sulphur dioxide levels to the World Health Organization guideline in the Turkish economy would have resulted in a saving of 0.12% and 0.08% of Turkey's 1990 and 1993 GDP's, respectively.

health (reducing the time available for leisure) or reduces consumers' productivity (which forces expenditure on medical care). He finds that the more pollution affects consumers' health, the lower the benefits associated with pollution abatement are. We also consider that pollution deteriorates consumers' health, forcing consumers to spend on medical care. We depart from Williams (2000) analysis by considering a dynamic general equilibrium model in which pollution is a stock that accumulates over time. These features are relevant since most pollutants remain in the atmosphere for many periods (for instance the greenhouse gases causing global warming, aldicarb affecting groundwater, pollutants causing acid rain, etc).

Dynamic general equilibrium models have been considered in the literature to examine environmental issues. However, most of the relevant studies assume dynamic models where agents live for many periods (possibly for ever) and, therefore, restrict their analyses to intragenerational problems. Following this approach all future impacts are treated as if they happened to current agents, ignoring that society is composed of mortal individuals of different generations whose actions have consequences that outlive them. Authors such as Solow (1986) and Padilla (2002) suggest that it is useful to capture these intergenerational aspects in the economic analysis of environment and natural resource. Moreover, there exists strong empirical evidence rejecting that members of extended families are altruistically linked in the way postulated by standard infinitely lived agents models (see Altonji *et al.* (1992)).

Recent articles have addressed intergenerational environmental issues using overlapping generation models (OGM). For instance, John and Pecchenino (1994) analyze the potential conflict between economic growth and the maintenance of environmental quality in a context where consumption degrades the environment. Using the same model John *et al.* (1995) and Ono (1996, 2002) examine the optimal tax policies that must be implemented by a long-lived government which lasts longer than a typical agent of the economy, in order to internalize the intergenerational externalities produced by competitive behavior. Guruswamy *et al.* (1997) analyze the relationship between resource exhaustion and pollution using an OGM framework and show that Pareto inefficient outcomes arise due to the lack of intragenerational coordination and is propagated across generations. Bovenberg and Heijdra (1998) study the effects of environmental taxation within an OGM in which the quality of the environment is considered as a durable consumption good. They show that if attention is focused on Pareto improving policies, it is harder to introduce pollution taxes than to increase those taxes when they already exist. Howarth and Norgaard (1992) and Howarth (1998) analyze climate change in an OGM framework. Jouvét *et al.* (2000) construct an

OGM model that includes a pollution externality in which individuals are altruistically linked to their offspring. They find that competitive steady-state consumption may be a decreasing function of the intergenerational degree of altruism, and moreover it is not optimal. Ono and Maeda (2001, 2002) analyze the effects of population aging on economic growth and the environment. They analyze the conditions under which the aging is beneficial to the environment.

Our paper is closely related to this literature. We also focus on intergenerational environmental issues and use an OGM to examine the effects that pollution can produce on the economy when it affects the health of agents that are forced to expend on medical care when they are elderly. Our main finding is that the presence of pollution makes more probable that competitive equilibrium will be dynamically inefficient. In particular, we show that if the economy accumulates capital above what we define as the *super golden rule* ratio, there are other allocations where no generation is worse off and some are better off. For those cases, we analyze two kinds of tax schemes. One scheme is based on production taxes and the other on capital and wage taxes. We show that if both schemes are designed to place the economy in the golden rule allocation, then young agents pay less taxes (or receive more transfers) under the production scheme than under the capital-wage tax system. The elderly population is indifferent as to which tax system is used. This implies that the production tax scheme is superior in an electoral context.³

The article is organized as follows. Section 2 presents the model. The stationary competitive equilibrium is characterized in Section 3. Section 4 focuses on optimal allocations. In section 5 we design optimal tax schemes. Finally, Section 6 concludes.

2 The model

Consider a two-period OGM with production. A new generation of N_t agents is born at each period $t = 1, 2, \dots$. Population grows at a constant rate, n , i.e. $N_t = N_{t-1}(1 + n)$. The preferences of an agent born at period t are represented by the following utility function $u(c_{1,t}) + \frac{1}{1+\theta}u(c_{2,t+1})$, where $c_{1,t}$ and $c_{2,t+1}$, represent consumption in young and old age, respectively; and θ is the subjective discount rate of the agent. We assume that $u' > 0$ and $u'' < 0$.

³In an electoral scenario in which the citizens of the economy can choose between the two tax schemes, the system preferred by young agents will be elected provided the population grows each generation and the electoral system grants one vote to each citizen.

Each agent is endowed with one unit of labor when she is young and supplies it to the firms inelastically. The agent receives the wage, w_t , which is used to consume in the first period, $c_{1,t}$, and to save, s_t . Since no income is earned in old age, all agents are savers, i.e. $s_t > 0$. This saving is inelastically supplied to firms which pay $(1 + r_{t+1})s_t$ to the agent when she is old. The agent divides her saving in old age between consumption, $c_{2,t+1}$, and pollution costs.

We assume that the pollution costs faced by the older generation depend on the current level of the stock of pollution. Formally, we define $H(E_t)$ as the total amount paid by elderly agents living in period t as pollution costs, where E_t is the stock of pollution.⁴ We assume that H is a linear homogeneous function, therefore the pollution costs paid by an elderly agent born at period $t - 1$ can be expressed as $(1 + n)h(e_t)$, where $e_t = E_t/N_t$. This formalization implies that pollution costs paid by each elderly agent depends upon the stock of pollution and the number of old agents. One possible interpretation of this assumption is that pollution results in illness that requires new medical investigations, treatments and hospital equipment; this may imply that pollution costs in each period act as a fixed cost for society.⁵ From now on we refer to pollution costs as health costs.

Firms behave perfectly competitively and maximize profits. They use capital, K , and labor, N , as inputs with a standard production function $F(K_t, N_t)$ that exhibits constant returns to scale. This implies that production can be expressed in *per* worker terms as $f(k_t)$, where $k_t = K_t/N_t$. We assume that $f' > 0$ and $f'' < 0$. Capital stock depreciates at a constant rate $0 \leq \lambda \leq 1$.

The stock of pollution in the current period, E_t , is determined by a proportion of past pollutant stock, E_{t-1} , plus the new emissions which are proportional to current production⁶. Therefore, the dynamics of the stock of pollution can be expressed as

$$E_t = (1 - \delta) E_{t-1} + \alpha F(K_t, N_t), \quad (1)$$

where the parameter $0 \leq \delta \leq 1$ denotes the rate of decay. This parameter

⁴Williams (2002) in a static general equilibrium model considers that pollution may affect the health of the population either by causing the representative agent to spend time sick (reducing the time available for labor) or by forcing expenditure on medical care. We following this second approach.

⁵One may think that most of the health damage from air pollution comes in the form of premature mortality. We do not consider this possibility in this article; in fact, we could think that pollution costs paid by elderly agents are enough to guarantee that agents live two periods.

⁶The results of the model do not change qualitatively if current production directly affects the future stock of pollution instead of the current one.

can be interpreted as the rate of natural purification of pollutants from the environment per period. Note that $\delta = 1$ means that pollution is not an accumulative process. In this case the health costs of an elderly agent depend exclusively on the production of that period. Parameter $\alpha > 0$ represents the cleanness of the technology used to produce; the lower α is the cleaner the technology is⁷.

3 Competitive Equilibrium

In the model described above, a dynamic competitive equilibrium is a sequence of $\{k_t, c_{1,t}, c_{2,t}, w_t, r_t, s_t, e_t\}_{t=0}^{\infty}$ such that, for given k_0 and e_0 , agents maximize utility, firms maximize profits and markets clear.

The representative agent born at period t , maximizes her utility function with respect to young and old consumption taking wage, interest rate and stock of pollution as given. This problem can be set out as follows

$$\underset{\{c_{1,t}, c_{2,t+1}\}}{\text{Max}} \quad u(c_{1,t}) + \frac{1}{1 + \theta} u(c_{2,t+1}), \quad (2)$$

$$\text{s.t.} \quad \begin{cases} c_{1,t} + s_t = w_t, \\ c_{2,t+1} + (1 + n) h(e_{t+1}) = (1 + r_{t+1}) s_t. \end{cases} \quad (3)$$

The first-order condition associated with the agent's optimization problem, (2)-(3), is

$$(1 + \theta) u'(c_{1,t}) = (1 + r_{t+1}) u'(c_{2,t+1}). \quad (4)$$

This equation states that the representative agent chooses consumption such that the marginal rate of substitution between current and future consumption is equal to the marginal rate of transformation. This optimal condition and the restrictions, (3), implicitly define saving as a function of wage, interest rate and pollution; it also depends upon the subjective discount rate parameter and the marginal private costs of pollution⁸,

$$s_t = s(w_t, r_{t+1}, e_{t+1} \mid \theta, h').$$

⁷This equation differs from John and Pecchenino (1994) and John *et al.* (1995) who consider a model where consumption (instead of production) degrades the environment.

⁸We define the marginal private cost of pollution as $h' \equiv \partial h(e_{t+1}) / \partial e_{t+1}$, which is a proportion of the marginal cost of pollution faced by an elderly agent (the proportion factor is $1/(1+n)$). Notice that we are assuming a linear homogeneous health cost function so its derivative, h' , is a constant parameter.

It easy to show that the saving function is characterized as follows,

$$\begin{aligned}
s_w &\equiv \frac{\partial s_t}{\partial w_t} = \frac{(1 + \theta) u''(c_{1,t})}{\Delta} \in (0, 1), \\
s_e &\equiv \frac{\partial s_t}{\partial e_{t+1}} = \frac{(1 + r_{t+1})(1 + n) h'(e_{t+1}) u''(c_{2,t+1})}{\Delta} > 0, \\
s_r &\equiv \frac{\partial s_t}{\partial r_{t+1}} = -\frac{u'(c_{2,t+1}) + (1 + r_{t+1}) s_t u''(c_{2,t+1})}{\Delta} \gtrless 0, \\
s_\theta &\equiv \frac{\partial s_t}{\partial \theta} = -\frac{u'(c_{1,t})}{\Delta} < 0, \\
s_{h'} &\equiv \frac{\partial s_t}{\partial h'} = \frac{(1 + r_{t+1})(1 + n) e_{t+1} u''(c_{2,t+1})}{\Delta} > 0.
\end{aligned}$$

where $\Delta = (1 + \theta) u''(c_{1,t}) + (1 + r_{t+1})^2 u''(c_{2,t+1}) < 0$.

We obtain the standard OGM results for wages and interest rates. If wages increase, the agent saves part of the increase for future purchases. On the other hand, an increase in interest rate may increase or decrease saving depending on the importance of the substitution, income and wealth effects. If the intertemporal elasticity of substitution for consumption is high enough then $s_r > 0$.⁹ We also obtain that an increase in the next period stock of pollution implies that agents save more because they will face higher health costs when elderly. With respect to the parameters affecting the saving function, we obtain the expected results. The more patient the agent is (i.e. the lower θ is) and the larger her marginal private cost of pollution is, the larger savings are.

Firms maximize profits and hire labor and capital until their marginal products equal their factor prices. Given the constant return to scale production function assumed, these conditions can be expressed as

$$w_t = f(k_t) - f'(k_t) k_t, \quad (5)$$

$$r_t = f'(k_t) - \lambda. \quad (6)$$

The market clears when total investment equals total savings. In *per worker* terms, this means

$$(1 + n) k_{t+1} = s(w_t, r_{t+1}, e_{t+1}).$$

⁹In the standard OGM without health costs, an intertemporal elasticity of substitution, σ , larger than one is a necessary and sufficient condition for $s_r > 0$. However, in this version of the model,

$$\sigma = -\frac{u'(c_{2,t+1})}{u''(c_{2,t+1}) c_{2,t+1}} \gtrless 1 + \frac{(1 + n) h(e_{t+1})}{c_{2,t+1}} \iff s_r \gtrless 0.$$

Substituting equations (5) and (6) into this last equation, we obtain

$$(1+n)k_{t+1} = s[f(k_t) - f'(k_t)k_t, f'(k_{t+1}) - \lambda, e_{t+1}], \quad (7)$$

which is a nonlinear dynamic equation relating the two state variables of the model, k and e . The other relationship between the state variables is given by the environmental equation, (1), which in *per* worker terms can be expressed as

$$e_{t+1} = \frac{1-\delta}{1+n}e_t + \alpha f(k_{t+1}). \quad (8)$$

Equations (7) and (8) form a system of first-order nonlinear difference equations in k and e . The solution of this system characterizes the competitive equilibrium paths for capital and pollution, $\{k_t, e_t\}_{t=1}^{\infty}$, given the initial values of the state variables, k_0 and e_0 ¹⁰. Once those paths are known $\{c_{1,t}, c_{2,t}, s_t, w_t, r_t\}_{t=1}^{\infty}$ can be obtained using (3)-(6).

3.1 The Steady State

A steady state equilibrium is an allocation where capital and pollution *per* worker ratios are stationary¹¹, i.e. \bar{k} and \bar{e} are such that

$$(1+n)\bar{k} = s[f(\bar{k}) - f'(\bar{k})\bar{k}, f'(\bar{k}) - \lambda, \bar{e}], \quad (9)$$

$$\bar{e} = \frac{1+n}{n+\delta}\alpha f(\bar{k}). \quad (10)$$

The following proposition characterizes the comparative static behavior of the steady state of this model.

Proposition 1 *If $(1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k})\bar{k} - s_e \frac{1+n}{n+\delta} \alpha f'(\bar{k}) > 0$ then the steady state associated with the dynamic system, (7)-(8), is a sink. Under this condition changes in the parameters of the model affect the steady state value of \bar{k} and \bar{e} as follows:*

$$\begin{aligned} \partial \bar{k} / \partial \theta < 0, \quad \partial \bar{k} / \partial n < 0, \quad \partial \bar{k} / \partial \delta < 0, \quad \partial \bar{k} / \partial \alpha > 0, \quad \partial \bar{k} / \partial h' > 0, \\ \partial \bar{e} / \partial \theta < 0, \quad \partial \bar{e} / \partial n < 0, \quad \partial \bar{e} / \partial \delta < 0, \quad \partial \bar{e} / \partial \alpha > 0, \quad \partial \bar{e} / \partial h' > 0. \end{aligned}$$

Furthermore, if the saving function is an increasing (decreasing) function of the interest rate, then, $\partial \bar{k} / \partial \lambda < 0$ (> 0), and $\partial \bar{e} / \partial \lambda < 0$ (> 0).

¹⁰Since capital is essential for production, k_0 must be positive. If $k_0 = 0$ young agents of the initial generation would have no income and consumption would remain zero forever. However e_0 may be zero.

¹¹Stationary paths are called golden aged paths by some authors (Diamond (1965), Phelps(1965)). We focus on stationary paths because this allows for direct comparison between our results and others based on OGM (for instance John *et al.* (1995) or Ono (1996)).

Proof. See Appendix. ■

Notice that in our set up we have two initial conditions; the characteristics of our problem lead us to assume that the initial stocks of capital and pollution, k_0 and e_0 , are known. Therefore, we are interested in having a sink steady state in such a way that whatever the initial conditions are, the steady state can be reached. If the steady state were a saddle-path¹², only one of the two state variables could be predetermined at the initial period; the other has to be determined to be sure that the solution is along the unique saddle path.

We can see that the results of the standard Diamond's model (1965) still arise when we have a pollution externality such as the one presented in this model. On the one hand, we observe that economies with high population growth rates have lower levels, in *per* worker terms, of capital and pollution. The intuition is clear, if the population grows faster, a greater part of output has to be destined to consumption, and therefore less to investment in future capital and pollution (in *per* worker terms). This result about pollution seems counterintuitive at first glance. Notice however that the stock of pollution, $E_t = e_t N_t$, in the steady state always grows at the same rate as the population, n . This means that higher population growth rates imply that the total stock of pollution grows faster. On the other hand, when saving is an increasing function of the interest rate¹³, economies with a high capital depreciation rate accumulate less capital and pollution than those with a low depreciation rate. This is so because a higher capital depreciation rate results in a lower interest rate, and therefore lower savings, capital and pollution.

Relating to the parameters associated with the externality introduced in the model, our characterization of the steady state shows that the more severe the pollution problem is for the economy the greater the capital and pollution ratios are. This can be observed through the parameters, δ , α and h' .

For instance, a lower rate of natural purification of pollutants from the atmosphere, δ , implies a higher future stock of pollution for any given stock of capital, and therefore a higher health cost faced by elderly agents. This larger future costs makes agents save more for old age and this implies that society accumulates more capital and degrades the environment more. Likewise if the technology used to produce becomes dirtier (i.e. α higher), new emissions of pollutants are higher for any stock of pollution, and so are health costs.

¹²If $(1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k}) \bar{k} - s_e \frac{1+n}{n+\delta} \alpha f'(\bar{k}) < 0$ and $\Psi > 0$, then the steady state associated with the dynamic system, (7)-(8), is a saddle path instead of a sink.

¹³Savings depend positively on interest rate when the substitution effect is greater than the sum of the income and wealth effects. A high enough intertemporal elasticity of substitution for consumption guarantees $s_r > 0$ (See footnote 9).

This results in higher savings, capital and pollution. The intuition is the same for the marginal private cost of pollution, h' ; a larger value for this parameter induces higher savings because it makes agents pay higher health costs for any stock of pollutant. This leads capital and pollution stocks to increase.

These results differ in some terms from those obtained by John *et al.* (1995), who analyze an OGM in which consumption degrades the environment and the government may improve environmental quality by levying taxes on young generation. One of their results is that economies in which consumption causes greater environmental degradation accumulate less capital. Our result goes in the opposite direction, showing that economies in which production causes higher environmental degradation accumulate more capital. This is so because in John *et al.*'s model agents pay taxes for maintaining environmental quality when they are young and therefore an increase in degradation reduces their savings for the future; however, in our model, higher environmental degradation increases health costs, which are paid in the old age, and therefore agents have to increase savings and therefore capital.

4 Efficiency

It is well known that in the standard OGM without externalities the competitive equilibrium is, in general, not Pareto optimal (Diamond (1965)). Phelps (1965) defines the golden rule capital ratio as the capital ratio that maximizes consumption among all paths in which the capital grows at a constant rate (i.e. in which capital ratio is constant). He proved that any growth economy whose stationary capital ratio exceeds the golden rule level is dynamically inefficient in the sense that there exists another stationary allocation (with a lower capital ratio) where no generation is worse off and at least some generations are better off. This result was also pointed out by Diamond (1965) in the standard OGM. We expect the introduction of any negative externality in the OGM to extend the scope of this feature.

We start by solving the problem of a central planner who cares about current and future generations. If the current period is $t = 0$, this problem can be set out as follows:

$$\underset{\{k_{t+1}, c_{1,t}, c_{2,t}\}_{t=0}^{\infty}}{\text{Max}} \frac{1}{1+\theta} u(c_{2,0}) + \sum_{t=0}^{\infty} \frac{1}{(1+R)^{t+1}} \left[u(c_{1,t}) + \frac{1}{1+\theta} u(c_{2,t+1}) \right], \quad (11)$$

$$s.t. \begin{cases} F(K_t, N_t) = N_t c_{1,t} + N_{t-1} c_{2,t} + K_{t+1} - (1 - \lambda) K_t + H(E_t), \\ E_t = (1 - \delta) E_{t-1} + \alpha F(K_t, N_t), \end{cases}$$

where $R \geq 0$ is the subjective discount rate of the planner. If the planner cares less about future generations, R is strictly positive and if she cares equally about all generations, R is zero¹⁴. The first restriction of problem (11) is the resource constraint of the economy in period t , which means that the total supply of goods is allocated to the consumption of young and old, health expenditures and providing for the next period's capital stock. The second restriction indicates the dynamics of the stock of pollution.

The first order condition of the planner's maximization problem can be summarized in the following equations,

$$(1 + \theta) u'(c_{1,t}) = (1 + R)(1 + n) u'(c_{2,t}), \quad (12)$$

$$(1 + \theta) \frac{u'(c_{1,t})}{u'(c_{2,t+1})} = [(1 - \alpha h') f'(k_{t+1}) + (1 - \lambda) + (1 - \delta)] - \frac{u'(c_{2,t+2})}{u'(c_{1,t+1})} \frac{1 - \delta f'(k_{t+1})}{1 + \theta f'(k_{t+2})} [f'(k_{t+2}) + (1 - \lambda)], \quad (13)$$

Equation (12) represents *intergenerational* optimal consumption allocations and states that the marginal rate of substitution, from the point of view of the central planner, between consumption of the young and consumption of the elderly must be equal to the rate of transformation, $(1 + n)$. Equation (13) is *intragenerational* optimal consumption condition and indicates how consumption for each generation is chosen.

Evaluating these conditions and the restrictions of the problem in the steady state, we have the system that defines the efficient steady state,

¹⁴Note that if R is non-positive the sum of the utilities does not converge. However, we consider the borderline case, $R = 0$, because it is possible in this case to discuss optimality using the *overtaking criterion* (Burmeister, 1980), which essentially states that path A overtakes path B if there exists a finite t^* such that the present value of the future utilities associated with path A up to time t^* exceeds that associated with path B up to t^* , and that inequality remains in the same direction for all $t > t^*$. A path is optimal if it overtakes all other paths.

$(\bar{k}^E, \bar{c}_1^E, \bar{c}_2^E, \bar{e}^E)$, for a given planner's discount rate,

$$(1 + \theta) u'(\bar{c}_1^E) = (1 + R)(1 + n) u'(\bar{c}_2^E), \quad (14)$$

$$\left[1 - \frac{\alpha h'(1+n)(1+R)}{(1+n)(1+R) - (1-\delta)} \right] f'(\bar{k}^E) = (1+n)(1+R) - (1-\lambda), \quad (15)$$

$$f(\bar{k}^E) = \bar{c}_1^E + \frac{\bar{c}_2^E}{1+n} + (n+\lambda)\bar{k}^E + h(\bar{e}^E), \quad (16)$$

$$\bar{e}^E = \frac{1+n}{n+\delta} \alpha f(\bar{k}^E), \quad (17)$$

where superscript E stands for efficient.

Condition (14) shows how the planner allocates consumption intergenerationally. Notice that when all generations are treated equally by the planner (i.e. $R = 0$), this is the allocation that would arise if consumption decisions were made individually based on an interest rate equal to the growth rate of the population (compare (14) with (4)). The intuition is clear. In stationary equilibria, a switch of one unit of consumption of an agent from her youth to her old age is equivalent to removing one unit of consumption from each of the young agents of the living generation and giving the total amount to the contemporary older generation, of whom there are n percent fewer members. Equation (15) indicates that the central planner recognizes that current production degrades the future environment, affecting future health costs faced by agents; therefore, the discounted marginal net benefit of an extra unit of capital (left-hand side) equates to its discounted marginal cost (right-hand side)¹⁵. Equations (16) and (17) are the resource constraint of the economy and the stock of pollution in the steady state, respectively.

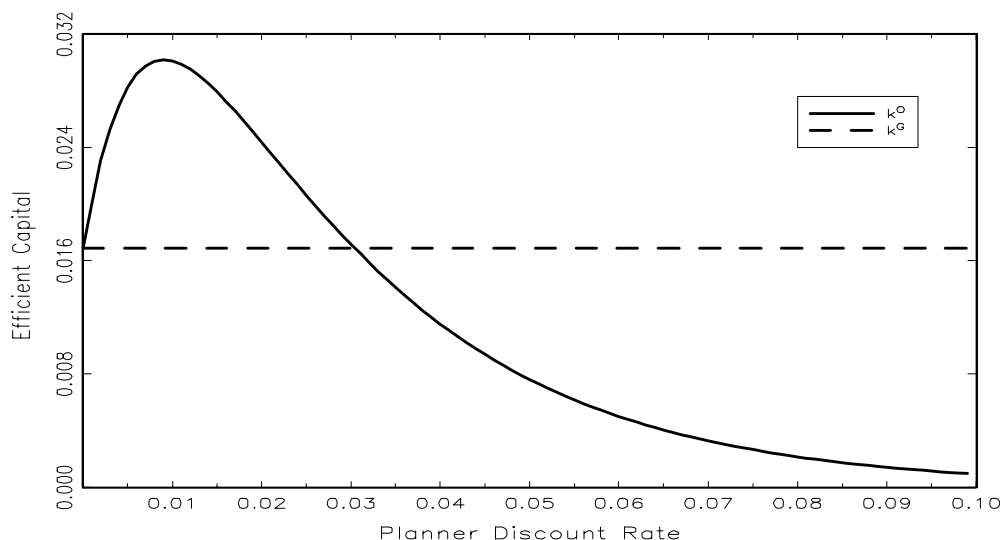
The golden rule capital ratio is the central planner's solution when she treats all generations symmetrically, i.e. when her discount rate is equal to zero. Considering equation (15), we can see that the optimal capital ratio for the golden rule case is given by,

$$\left[1 - \frac{\alpha h'(1+n)}{n+\delta} \right] f'(\bar{k}^G) = n + \lambda,$$

where superscript G stands for golden rule allocation. Diamond (1965) shows that in the standard OGM without pollution externalities any economy whose capital ratio exceeds the golden rule level is dynamically inefficient in the

¹⁵Notice that in order to have a positive \bar{k}^E it is necessary that $1 - \frac{(1+n)(1+R)}{(1+n)(1+R) - (1-\delta)} \alpha h' > 0$. This condition establishes that, from the central planner's point of view, the discounted health costs associated with one unit of production should be less than that unit; otherwise net output will be negative.

Figure 1: Efficient capital and planner's discount rate



sense that there exists another stationary allocation where no generation is worse off and at least some generations are better off. The following example shows that, in general, this statement is not true when there are pollution externalities.

Example 1 Consider an economy where each period covers 30 years with the following annual parameter values¹⁶: $n = 0.01$, $\lambda = 0.05$, $\delta = 0.005$, $\alpha = 1$, $h' = 0.01$. The production function is $f(k_t) = k_t^\beta$ with $\beta = 0.35$. Then, when the central planner's annual discount rate is lower than 3%, the efficient capital ratio is larger than that associated with the golden rule.

The result of this example can be seen in Figure 1 where the efficient capital ratio, k^E , is plotted as a function of the subjective annual discount rate of the planner, R . It is clear that the efficient steady state capital stock is not monotonically decreasing in the central planner's discount rate. In particular, when the central planner's annual discount rate is lower than 3%, the efficient capital ratio is larger than that associated with the golden rule. This can be generalized in the following lemma.

¹⁶It is easy to show that if each period covers 30 years, the parameter values for each period are such that $(1 + n^*) = (1 + n)^p$, $(1 - \lambda^*) = (1 - \lambda)^p$, $(1 - \delta^*) = (1 - \delta)^p$, $\alpha^* = \alpha\delta^*/\delta$ and $(1 + R^*) = (1 + R)^p$ where variables without * are the annual parameter values and $p = 30$.

Lemma 1 *If the pollution externality is such that $\alpha h' > \frac{(n+\delta)^2}{(1+n)^2+(1-\delta)(1-\lambda)}$, then there always exist efficient capital ratios that exceed the golden rule capital ratio. Otherwise, the maximum efficient capital ratio is given by the golden rule allocation.*

Proof. See Appendix. ■

The intuition for this result is as follows. The discount rate affects the steady state capital stock through two channels. The first, classic, channel says that a lower planner discount rate implies higher weights attached to the welfare of future generations, and thereby higher savings that transfer higher capital stock to the future. The second channel is based on the pollution externality and contrasts with the effects of the first channel. A lower planner discount rate implies lower pollution levels transferred to future periods, but since pollution is linked to the capital stock, it implies lower capital stock levels transferred as well. The relative strength of the two channels determines whether a decrease in the planner's discount rate implies higher or lower capital stock. In particular, we have shown that if the pollution externality is weak (i.e. if $\alpha h'$ is small), or if the pollution decay rate is large (δ close to one), then the classic conclusion holds that a decrease in the planner's discount rate implies an increase in the capital stock.¹⁷

It is well known that without the pollution externality all generations of an economy whose capital ratio exceeds the golden rule capital ratio may be better off if the stock of capital is reduced once and forever and the consumption of current elderly agents increases by the same amount (Blanchard and Fischer (1989), p. 102-102). In general, this kind of reallocation of capital over time only works in the presence of pollution externalities for economies whose capital ratio is high enough. In particular, we can find a capital ratio benchmark such that any economy exceeding it is dynamically inefficient for any δ . We call this benchmark the *super golden rule capital ratio*, and it is defined such that

$$f'(k^{SG}) = \frac{n + \lambda}{1 - \alpha h'},$$

where superscript SG stands for super golden rule allocation. It is easy to show that $k^{SG} > k^G$ for $\delta < 1$ and $k^{SG} = k^G$ for $\delta = 1$. The following proposition formalizes this result.

¹⁷Observe that when $\delta = 1$, the condition for having efficient capital ratios larger than the golden rule capital ratio becomes $\alpha h' > 1$, which is not possible because then \bar{k}^E becomes negative (see footnote 15).

Proposition 2 *Any economy with pollution externalities whose stationary capital ratio exceeds the super golden rule capital ratio, k^{SG} , is dynamically inefficient.*

Proof. See Appendix. ■

The main reason why the Blanchard and Fischer reallocation of capital does not work in our model is the fact that pollution is an accumulative process. Observe that we have two stock variables in our model (capital and pollution) and this makes the model work in some peculiar ways. Suppose that we are in a stationary equilibrium and at some period τ , the capital ratio is changed once and for ever. Then the economy will not be in stationary equilibrium from this period on. The economy will need some periods to make the pollution stock stationary; the number will depend on the rate of natural purification of the pollutant from the environment, δ . Only when $\delta = 1$, i.e. when emissions stay in the environment for just one period, does a change in the capital ratio of the once and for ever type makes the economy pass immediately from one stationary equilibrium to another. Because of this, Diamond's result (1965) still holds in the presence of a negative production externality if the pollution is not an accumulative process ($\delta = 1$). Notice that Proposition 2 also shows this case because the super golden rule capital ratio coincides with the golden rule in the case of $\delta = 1$.

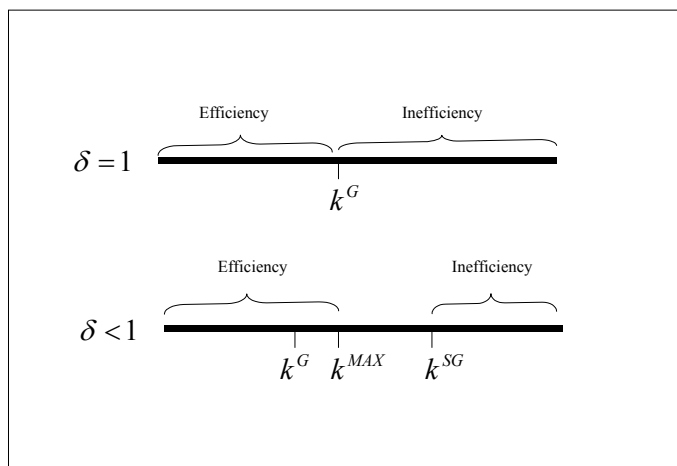
Figure 2 summarizes these results. Proposition 2 shows that any economy with a capital ratio greater than k^{SG} is dynamically inefficient. Let k^{MAX} denote the maximum efficient capital ratio among all the solutions of the central planner's optimization problem, (11),¹⁸ then those economies whose capital ratio is lower than k^{MAX} are dynamically efficient. Lemma 1 states the conditions under which $k^{MAX} = k^G$. We are not able to classify economies with capital ratios in (k^{MAX}, k^{SG}) in terms of efficiency. On the one hand, those economies could be dynamically efficient because the solution set of the central planner problem's, (11), may not give us the whole set of optimal allocations. For example, we may think of a planner with a subjective discount rate, R , that changes over time; the optimization problem (11) does not take this possibility into account. On the other hand, we cannot rule out the possibility that those economies may be dynamically inefficient. The proof of

¹⁸In the proof of Lemma 1, we show that k^{MAX} is the capital ratio associated with the central planner's discount rate R^+ . It is easy to prove that k^{MAX} satisfies

$$f'(k^{MAX}) = \chi^{1/2} \left[\frac{(1-\delta) + \chi^{1/2}}{(1-\alpha h')} - (1-\lambda) \right] \left[\frac{(1-\delta) + \chi^{1/2}}{(1-\alpha h')} - (1-\delta) \right],$$

where $\chi = (1-\delta)\alpha h' [(1-\delta) - (1-\lambda)(1-\alpha h')]$ and $k^{MAX} < k^{SG}$.

Figure 2: Rank and capital ratios



Proposition 2 only shows that a reduction of current investment, increasing current consumption by the same amount and keeping future capital ratio constant over time, dominates, in the Pareto sense, any allocation where the capital ratio is greater than the super golden rule level. However we cannot exclude the possibility that other kinds of reallocation of capital over time may improve some generations without worsening others for economies with capital ratios in the rank (k^{MAX}, k^{SG}) .

We have shown in Proposition 2 that any economy with a capital ratio greater than k^{SG} is dynamically inefficient. It is easy to see that k^{SG} is decreasing in the pollution parameters. Therefore economies with more environmental problems have a larger rank of dynamically inefficient allocations. On the other hand, we have shown in Proposition 1 that economies with more environmental problems accumulate more capital and degrade the environment more when they act in a competitive manner. These two results allow us to formalize the following proposition.

Proposition 3 *The higher the pollution externalities are, the larger the competitive stationary capital ratio is and the lower the super golden rule capital ratio is.*

Proof. Pollution externalities are the result of three factors: a lower rate of natural purification of the pollutant from the atmosphere, δ , a dirtier production technology (i.e. a higher α) and/or a higher marginal private cost of pollution, h' .

It is easy to see the $\partial k^{SG}/\partial\delta = 0$, $\partial k^{SG}/\partial\alpha < 0$ and $\partial k^{SG}/\partial h' < 0$. Moreover, in Proposition 1, we prove that $\partial\bar{k}/\partial\delta < 0$, $\partial\bar{k}/\partial\alpha > 0$ and $\partial\bar{k}/\partial h' > 0$. Therefore, $\frac{\partial(\bar{k}-k^{SG})}{\partial\delta} < 0$, $\frac{\partial(\bar{k}-k^{SG})}{\partial\alpha} > 0$, and $\frac{\partial(\bar{k}-k^{SG})}{\partial h'} > 0$. ■

5 Tax schemes

In a competitive equilibrium, no generation has any incentive to consider the successive generations. Therefore, future generations suffer from past production which has degraded the environment. We have seen that a competitive stationary equilibrium is dynamically inefficient when the capital ratio is greater than the super golden rule allocation and also that the larger the pollution externality is the higher the likelihood of dynamic inefficiency is. This section studies how to implement tax schemes in order to achieve efficient allocations for economies whose competitive equilibrium is dynamically inefficient. In particular, we find the optimal tax schemes that place stationary competitive economies in the golden rule allocation¹⁹.

5.1 Production Taxes

We introduce a Pigouvian production tax and a lump-sum tax system into our model. Let τ_p be a production tax paid by firms *per* unit of output produced and let τ_y and τ_o be lump-sum taxes levied on young and elderly agents, respectively, in any period. The government budget constraint implies that the system should be set in such way that the following condition is satisfied

$$\tau_y + \frac{\tau_o}{1+n} + \tau_p f(k) = 0.$$

This means that at least one of the lump-sum taxes must be a transfer.

In the presence of such a tax scheme, the problem that firms must solve is

$$\text{Max}_{\{k_t\}_{t=0}^{\infty}} (1 - \tau_p) f(k_t) - w_t - (r_t + \lambda) k_t,$$

for k_o given.

¹⁹As indicated above, the golden rule allocation is the efficient allocation when the central planner treats all generations symmetrically. It would be possible to consider alternative efficient allocations with higher capital ratios than that associated with the golden rule allocation. However, as Lemma 1 points out, under some conditions the golden rule capital ratio is the maximum efficient capital ratio.

And the problem faced by the representative consumer is

$$\begin{aligned} & \underset{\{c_{1,t}, c_{2,t+1}\}_{t=0}^{\infty}}{Max} && u(c_{1,t}) + \frac{1}{1+\theta} u(c_{2,t+1}), \\ & s.t. && \begin{cases} c_{1,t} + s_t = w_t - \tau_y, \\ c_{2,t+1} + (1+n)h(e_{t+1}) = (1+r_{t+1})s_t - \tau_o. \end{cases} \end{aligned}$$

In this scenario, a stationary competitive equilibrium is an allocation $\{\bar{k}^C, \bar{e}^C, \bar{c}_1^C, \bar{c}_2^C, \bar{w}^C, \bar{r}^C, \bar{s}^C\}$ such that for a given tax scheme, $\{\tau_p, \tau_y, \tau_o\}$, satisfies

$$(1+\theta)u'(\bar{c}_1^C) = (1+\bar{r}^C)u'(\bar{c}_2^C), \quad (18)$$

$$\bar{c}_1^C + \bar{s}^C = \bar{w}^C - \tau_y^C, \quad (19)$$

$$\bar{c}_2^C + (1+n)h(\bar{e}^C) = (1+\bar{r}^C)\bar{s}^C - \tau_o, \quad (20)$$

$$\bar{w}^C = (1-\tau_p) \left[f(\bar{k}^C) - f'(\bar{k}^C)\bar{k}^C \right], \quad (21)$$

$$\bar{r}^C = (1-\tau_p)f'(\bar{k}^C) - \lambda, \quad (22)$$

$$\bar{s}^C = (1+n)\bar{k}^C, \quad (23)$$

$$\bar{e}^C = \frac{1+n}{n+\delta}\alpha f(\bar{k}^C), \quad (24)$$

where equations (18), (19) and (20) are from the consumers maximization problem, equations (21) and (22) correspond to the wage and interest rate that maximize firms' profits, respectively, equation (23) comes from market clearance condition and equation (24) indicates the relationship between the environment and the capital ratio in the steady state.

Comparing these equations with the optimal conditions defined by the golden rule allocation, we can set the optimal production tax scheme that satisfies the government budget constraint. The following proposition shows the results.

Proposition 4 *Economies whose stationary competitive equilibrium over-accumulates capital can achieve the optimal golden rule allocation by the following combination of production and lump-sum taxes:*

$$\begin{aligned} \tau_p^* &= \frac{1+n}{n+\delta}\alpha h', \\ \tau_y^* &= (1-\tau_p^*) \left[f(k^G) - f'(k^G)k^G \right] - c_1^G - (1+n)k^G, \\ \tau_o^* &= - \left[\tau_p^* f(k^G) + \tau_y^* \right] (1+n), \end{aligned}$$

where $\tau_o^* < 0$ and $\tau_y^* \leq 0$ (> 0) when the presence of pollution is (is not) the only cause of capital overaccumulation.

Proof. See Appendix. ■

Notice that with this optimal scheme elderly agents always receive transfers. However young agents can be characterized both as tax payers and transfer receivers. Those economies in which the presence of pollution makes the capital accumulation problem even worse will make young agents transfer resources to elderly agents (who also receive the production tax revenue). Alternatively, economies in which only the presence of the environmental problem makes excess capital appear, will implement the scheme in such way that both young and elderly agents share out the tax production revenue.

The following example illustrates an economy that may be in either of the two situations depending on the labor share parameter.

Example 2 Consider an economy where each period covers 30 years with the following annual parameter values²⁰: $n = 0.02$, $\lambda = 0.04$, $\delta = 0.005$. The utility function is logarithmic, $u(c_{i,t}) = \ln(c_{i,t})$, with an annual subjective discount rate $\theta = 0.02$. The production function is given by $f(k_t) = Ak_t^\beta$ with $A = 20$.

- i) If the labor share is such that $\beta > 0.2637$ then dynamic inefficiency only appears when pollution externalities are high enough. In this case young and elderly agents are transfer receivers with the production tax scheme.*
- ii) If the labor share is such that $\beta < 0.2637$ then dynamic inefficiency appears even in absence of pollution externalities. In this case elderly agents are transfer receivers and young agents are tax payers under the production tax scheme.*

Table 1 shows the optimal production taxes under the two situations for the economy described in Example 2. On the one hand, we can see that when the labor share is low, the market economy overaccumulates capital even when the economy does not exhibit pollution externalities ($k^C > k^{SG}$). This inefficiency disappears with an adequate transfer from young people to elderly agents without taxing firms. If the economy also suffers pollution, the overaccumulation of capital is even larger. In this state, the optimal production tax implies taxing firms; this revenue goes to increase the transfer received by elderly agents and to lower the taxes paid by young agents (although they are still net tax payers). On the other hand, if the labor share is high, the competitive equilibrium is efficient provided there are not pollution

²⁰See footnote 16.

Table 1: Optimal Production Taxes (Example 2)

$\beta = 0.25$			
No pollution ($\alpha h' = 0$)		Pollution externalities ($\alpha h' = 0.003$)	
$k^C = 4.2224$	$\tau_p = 0$	$k^C = 8.5563$	$\tau_p = 0.1596$
$k^G = 3.8722$	$\tau_y = 0.4706$	$k^G = 3.0711$	$\tau_y = 0.3733$
$k^{SG} = 3.8722$	$\tau_o = -0.8525$	$k^{SG} = 3.4458$	$\tau_o = -8.3282$
$\beta = 0.35$			
No pollution ($\alpha h' = 0$)		Pollution externalities ($\alpha h' = 0.003$)	
$k^C = 4.2285$	$\tau_p = 0$	$k^C = 7.2200$	$\tau_p = 0.1596$
$k^G = 8.0024$	$\tau_y = 0$	$k^G = 6.1246$	$\tau_y = -3.7654$
$k^{SG} = 8.0024$	$\tau_o = 0$	$k^{SG} = 6.9946$	$\tau_o = -4.0796$

externalities ($k^C < k^G$). In this case it is not necessary to implement any tax scheme. However, when pollution affects the economy, the dynamic inefficiency may appear when the externalities are high enough.²¹ In this case, the optimal production tax scheme consists of taxing firms and sharing out the revenue among elderly and young agents.

5.2 Capital and Wage Taxes

Following John *et al.* (1995), we consider an alternative tax scheme based on capital and wage taxes. Suppose that the planner levies taxes on gross capital and wages at the rates τ_k and τ_w , respectively, and let σ be a transfer that elderly agents receive, in any period. The government budget constraint implies that the tax system should be set as follows,

$$w\tau_w + \frac{(1+r)s}{1+n}\tau_k - \frac{\sigma}{1+n} = 0.$$

Since taxes are levied on consumers, firms face the same problem as without taxes. Each representative consumer solves

$$\underset{\{c_{1,t}, c_{2,t+1}\}_{t=0}^{\infty}}{\text{Max}} \quad u(c_{1,t}) + \frac{1}{1+\theta}u(c_{2,t+1}),$$

²¹In this example, if the pollution externality is such that $\alpha h' < 0.0025$, the market equilibrium is efficient because $k^C < k^G$.

$$s.t. \begin{cases} c_{1,t} + s_t = w_t(1 - \tau_w), \\ c_{2,t+1} + (1+n)h(e_{t+1}) = (1+r_{t+1})(1-\tau_k)s_t + \sigma. \end{cases}$$

Under this tax scheme, a stationary equilibrium $\{\bar{k}^C, \bar{e}^C, \bar{c}_1^C, \bar{c}_2^C, \bar{w}^C, \bar{r}^C, \bar{s}^C\}$ must satisfy

$$(1+\theta)u'(\bar{c}_1^C) = (1+\bar{r}^C)(1-\tau_k)u'(\bar{c}_2^C), \quad (25)$$

$$\bar{c}_1^C + \bar{s}^C = \bar{w}^C(1-\tau_w), \quad (26)$$

$$\bar{c}_2^C + (1+n)h(\bar{e}^C) = (1+\bar{r}^C)(1-\tau_k)\bar{s}^C + \sigma, \quad (27)$$

$$\bar{w}^C = f(\bar{k}^C) - f'(\bar{k}^C)\bar{k}^C, \quad (28)$$

$$\bar{r}^C = f'(\bar{k}^C) - \lambda, \quad (29)$$

$$\bar{s}^C = (1+n)\bar{k}^C, \quad (30)$$

$$\bar{e}^C = \frac{1+n}{n+\delta}\alpha f(\bar{k}^C). \quad (31)$$

The following proposition summarizes the features of this optimal tax scheme.

Proposition 5 *Dynamically inefficient economies can achieve the golden rule allocation by a combination of the following wage and capital taxes:*

$$\begin{aligned} \tau_k^* &= \frac{(n+\lambda)\alpha h'}{(n+\delta) - (1-\lambda)\alpha h'}, \\ \tau_w^* &= 1 - \frac{c_1^G + (1+n)k^G}{f(k^G) - f'(k^G)k^G}, \\ \sigma^* &= \left[\frac{c_2^G}{1+n} + h(e^G) - (1+n)k^G \right] (1+n), \end{aligned}$$

where $\sigma^* > 0$, $0 \leq \tau_k^* < 1$ and $\tau_w^* < 1$. Moreover $\tau_w^* > 0$ provided the economy overaccumulates capital even without the presence of pollution.

Proof. See Appendix. ■

With this tax scheme, both young and old agents can be tax payers or transfer receivers. It is easy to show that net taxes paid for each type of agent can be expressed as

$$\begin{aligned} T_y &\equiv w\tau_w = \frac{c_2^G}{1+n} + h(e^G) - (1+n)k^G + k[f'(k^{GN}) - f'(k^G)], \\ T_o &\equiv (1+r)s\tau_k - \sigma = -(1+n)T_y, \end{aligned}$$

where T_y and T_o are the net taxes paid by each young and elderly agent, respectively; and k^{GN} is defined as the golden rule of the economy in the case in which there is no pollution externality, i.e. k^{GN} satisfies $f'(k^{GN}) = n + \lambda$. Notice that $\frac{c_2^G}{1+n} + h(e^G) - (1+n)k^G > 0$, because we are analyzing economies that overaccumulate capital and $k[f'(k^{GN}) - f'(k^G)] < 0$ is always negative. Therefore economies whose competitive equilibrium is far away from the golden rule allocation and/or with a golden rule allocation that does not change much with pollution parameters will tend to have young agents as tax payers and elderly agents as transfer receivers. However, economies in which the competitive equilibrium is close to (but above) the golden rule allocation and/or where the golden rule allocation is very sensitive to pollution parameters will be characterized by young agents as transfer receivers and elderly as tax payers²².

Continuing with the example 2, Table 2 is the prolongation of Table 1 and shows the optimal capital and wage taxes that places the market equilibrium in the golden rule allocation. Two facts can be pointed out. First, in the absence of pollution externalities, if the market allocation is dynamically inefficient the optimal capital-wage tax scheme consists of levying young agents through work income ($\tau_w > 0$) and transferring the revenue to the old agents. Second, the existence of a pollution externality increases inefficiency and increases the capital and wage tax rates.

5.3 Production Taxes vs. Capital and Wages Taxes

Both tax schemes analyzed succeed in carrying the market economy to the stationary golden rule allocation. This means that from the welfare point of view, young and elderly agents are indifferent as to which tax scheme they prefer. However, we can suppose that in choosing between different tax schemes agents may prefer those systems in which the amount of taxes paid is lower.

Let us consider consumers as the only citizens from the electoral point of view. The following proposition shows the amount paid by each agent in both scheme analyzed.

Proposition 6 *Young and elderly agents pay more taxes with the capital-wage system than with the production tax schemes.*

²²In fact Proposition 5 shows that a sufficient condition for having young agents as tax payers and elderly agents as transfer receivers is that the economy overaccumulates capital even in the absence of pollution externalities. However, the reverse is not true.

Table 2: Optimal Capital and Wage Taxes (Example 2)

$\beta = 0.25$			
No pollution ($\alpha h' = 0$)		Pollution externalities ($\alpha h' = 0.003$)	
$k^C = 4.2224$	$\tau_k = 0$	$k^C = 8.5563$	$\tau_k = 0.1596$
$k^G = 3.8722$	$\tau_w = 0.0224$	$k^G = 3.0711$	$\tau_w = 0.1784$
$k^{SG} = 3.8722$	$\sigma = 0.8525$	$k^{SG} = 3.4458$	$\sigma = 8.3282$
$\beta = 0.35$			
No pollution ($\alpha h' = 0$)		Pollution externalities ($\alpha h' = 0.003$)	
$k^C = 4.2285$	$\tau_k = 0$	$k^C = 7.2200$	$\tau_k = 0.1596$
$k^G = 8.0024$	$\tau_w = 0$	$k^G = 6.1246$	$\tau_w = 0.0060$
$k^{SG} = 8.0024$	$\sigma = 0$	$k^{SG} = 6.9946$	$\sigma = 4.0796$

Proof. See Appendix. ■

This proposition shows that consumers prefers the production tax system when the amount paid as taxes is the criterion considered. Therefore the production tax scheme is superior in an electoral context in which one vote is granted to each consumer. This result can be consider a kind of “electoral” illusion because consumer do not take into account the taxes paid by firms with the production scheme

6 Conclusions

This paper analyzes the effects that pollution can produce on the economy when it affects the health of agents who are forced to spend on medical care. The health effect has not often been considered in theoretical environmental models although it is well established from the empirical point of view that quality of environment and health are positively related.

Williams (2002) examines the economic effects of pollution when it affects consumers’ health in the context of an static general equilibrium model in which pollution deteriorates the health (reducing the time available for leisure) or productivity of consumers (which forces expenditure on medical care). Our model, unlike Williams (2000), is a dynamic general equilibrium model in which pollution is an stock that accumulates over time. This is relevant if we take into account that in most real environmental prob-

lems, pollutants remain in the environment for many periods (for instance the greenhouse gases causing global warming, aldicarb affecting groundwater, pollutants causing acid rain).

We use an overlapping generation framework in which different generations of agents cohabit at any time. This model feature allows us to study the relationship between pollution and health in an intergenerational context. Two main alternative features are introduced with respect to the OGM existing contributions (John and Pecchenino (1994), John *et al.* (1995), Ono (1996), Bovenberg and Heijdra (1998), Jouvet *et al.* (2000), Ono and Maeda (2001,2002)). We assume that pollution rises with production in an accumulative way and that households do not care directly about the environment, but pollution makes them incur health costs when elderly.

Our characterization of the steady state shows that the more severe the pollution problem is for the economy the greater the capital and the pollution ratios are. These results differ in some terms from those obtained by John *et al.* (1995), who find that economies in which consumption causes greater environmental degradation accumulate less capital. This is so because in John *et al.*'s model agents pay taxes for maintaining environmental quality when they are young and therefore an increase in degradation reduces their savings for the future. Contrariwise, higher environmental degradation increases health costs in our model, which are paid in the old age and thus agents have to increase savings and capital.

We show that the introduction of pollution externalities in the standard OGM yields two new results with regard to the efficiency. On the one hand, there are capital ratios above the golden rule capital ratio which may be efficient in the Pareto sense. Due to the existence of health costs associated with pollution, it may appear a bell shape relationship between planner's discount rate and capital ratio. The planner's discount rate affects the steady state capital stock through a new channel. Namely. a lower planner's discount rate implies lower pollution levels transferred to future periods, but since pollution is linked to the capital stock, it implies lower capital stock levels transferred as well. This effect is opposite to the classic one according to which a lower planner's discount rate implies higher weights to future generations and thereby higher savings that transfer higher capital to the future. The relative strength of the two channels determines whether a decrease in the planner's discount rate implies higher or lower capital stock.

On the other hand, we show that the presence of pollution makes it more probable that competitive equilibrium will be dynamically inefficient. In particular, we show that if the economy accumulates more capital than the one above what we defined by the *super golden rule* ratio, there are other allocations where no generation is worse off and some of them are better off.

For those cases, we analyze two types of tax scheme. One type is based on production taxes and the other on capital and wage taxes. We show that if the two schemes are designed to place the economy in the golden rule allocation, then young and elderly agents pay less taxes (or receive more transfers) under the production scheme than under the capital-wage tax system. This implies that the production tax scheme is superior in an electoral context that grants one vote to each consumer.

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Appendix

Proof of Proposition 1:

To prove the first part of the proposition, about **stability**, we linearize the dynamic system (7)-(8) around the steady state. The linear approximation can be written in matrix form as

$$\begin{pmatrix} k_{t+1} - \bar{k} \\ e_{t+1} - \bar{e} \end{pmatrix} = \Phi \begin{pmatrix} k_t - \bar{k} \\ e_t - \bar{e} \end{pmatrix},$$

where $\Phi = \begin{pmatrix} -\frac{s_w f''(\bar{k}) \bar{k}}{\Psi} & \frac{s_e(1-\delta)}{\Psi(1+n)} \\ -\frac{s_w f''(\bar{k}) \bar{k} \alpha f'(\bar{k})}{\Psi} & \frac{[1+n-s_r f''(\bar{k})](1-\delta)}{\Psi(1+n)} \end{pmatrix}$ and $\Psi = 1+n-s_r f''(\bar{k})-s_e \alpha f'(\bar{k})$.

Following Azariadis (1993, pg 62-67), a steady state (\bar{k}, \bar{e}) , is a sink if the following inequalities hold:

- i) $|\Phi| < 1$,
- ii) $|\Phi| + T + 1 > 0$,
- iii) $|\Phi| - T + 1 > 0$,

where T is the trace of Φ .

The strategy of the proof is to show that the stability condition imposed in Proposition 1 implies i), ii) and iii).

Proof of i): Assume $(1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k}) \bar{k} - s_e \frac{1+n}{n+\delta} \alpha f'(\bar{k}) > 0$ holds. Since s_w and s_e are positive and $0 < \delta < 1$, this condition implies

$$(1+n) - s_r f''(\bar{k}) + \frac{1-\delta}{1+n} s_w f''(\bar{k}) \bar{k} - s_e \alpha f'(\bar{k}) > 0,$$

and

$$\Psi = (1+n) - s_r f''(\bar{k}) - s_e \alpha f'(\bar{k}) > 0.$$

Both inequalities imply

$$|\Phi| = -\frac{1-\delta}{1+n} \frac{s_w f''(\bar{k}) \bar{k}}{\Psi} < 1.$$

Proof of ii): Since $0 < |\Phi| < 1$, to prove ii) it suffices to prove that $T > 0$. Notice that

$$T = -\frac{s_w f''(\bar{k}) \bar{k}}{\Psi} + \frac{(1+n) - s_r f''(\bar{k})}{\Psi} \frac{1-\delta}{1+n} > 0,$$

because $\Psi = (1+n) - s_r f''(\bar{k}) - s_e \alpha f'(\bar{k}) > 0$ and therefore $(1+n) - s_r f''(\bar{k}) > 0$.

Proof of iii): It is easy to show that

$$|\Phi| - T + 1 = \frac{1}{\Psi} \frac{n+\delta}{1+n} \left[(1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k}) \bar{k} - s_e \frac{1+n}{n+\delta} \alpha f'(\bar{k}) \right],$$

which is positive under the stability condition imposed.

To prove **comparative static** behavior, we totally differentiate equations (9) and (10)

$$\Gamma \begin{pmatrix} d\bar{k} \\ d\bar{e} \end{pmatrix} = \Delta \begin{pmatrix} d\theta \\ dh' \\ d\lambda \\ d\alpha \\ dn \\ d\delta \end{pmatrix},$$

where

$$\Gamma = \begin{pmatrix} (1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k}) \bar{k} & -s_e \\ -\frac{1+n}{n+\delta} \alpha f'(\bar{k}) & 1 \end{pmatrix},$$

$$\Delta = \begin{pmatrix} s_\theta & s_h & -s_r & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1+n}{n+\delta} f(\bar{k}) & -\frac{1-\delta}{(n+\delta)^2} \alpha f(\bar{k}) & -\frac{1+n}{(n+\delta)^2} \alpha f(\bar{k}) \end{pmatrix}.$$

Notice that $|\Gamma|$ is positive under the stability condition imposed. It is straightforward to show that

$$\begin{aligned} \frac{\partial \bar{k}}{\partial \theta} &= \frac{s_\theta}{|\Gamma|} < 0, & \frac{\partial \bar{k}}{\partial h'} &= \frac{s_h}{|\Gamma|} > 0, \\ \frac{\partial \bar{k}}{\partial \lambda} &= -\frac{s_r}{|\Gamma|} \geq 0, & \frac{\partial \bar{k}}{\partial \alpha} &= \frac{1+n}{n+\delta} f(\bar{k}) \frac{s_e}{|\Gamma|} > 0, \\ \frac{\partial \bar{k}}{\partial n} &= -\frac{1-\delta}{(n+\delta)^2} \alpha f(\bar{k}) \frac{s_e}{|\Gamma|} < 0, & \frac{\partial \bar{k}}{\partial \delta} &= -\frac{1+n}{(n+\delta)^2} \alpha f(\bar{k}) \frac{s_e}{|\Gamma|} < 0, \end{aligned}$$

and

$$\frac{\partial \bar{e}}{\partial i} = \begin{cases} \frac{1+n}{n+\delta} \alpha f'(\bar{k}) \frac{\partial \bar{k}}{\partial i} & \text{for } i = \theta, h', \lambda, \\ \frac{(1+n) - s_r f''(\bar{k}) + s_w f''(\bar{k}) \bar{k}}{s_e} \frac{\partial \bar{k}}{\partial i} & \text{for } i = \alpha, n, \delta. \end{cases} \quad \blacksquare$$

Proof of Lemma 1:

The efficient capital ratio is defined as

$$\left[1 - \frac{\alpha h' (1+n)(1+R)}{(1+n)(1+R) - (1-\delta)} \right] f'(\bar{k}^E) = (1+n)(1+R) - (1-\lambda),$$

or, alternatively

$$f'(\bar{k}^E) = \frac{[(1+n)(1+R) - (1-\lambda)][(1+n)(1+R) - (1-\delta)]}{(1+n)(1+R)(1-\alpha h') - (1-\delta)}.$$

Therefore,

$$\frac{\partial \bar{k}^E}{\partial R} = f''(\bar{k}^E) \frac{\partial f'(\bar{k}^E)}{\partial R}.$$

It is easy to prove that $\frac{\partial f'(\bar{k}^E)}{\partial R} = 0$ has the following two roots:

$$\begin{aligned} R^- &= -1 + \frac{(1-\delta) - \chi^{1/2}}{(1+n)(1-\alpha h')}, \\ R^+ &= -1 + \frac{(1-\delta) + \chi^{1/2}}{(1+n)(1-\alpha h')}, \end{aligned}$$

where $\chi = (1-\delta)\alpha h'[(1-\delta) - (1-\lambda)(1-\alpha h')]$ and $R^- < R^+$. Therefore,

$$\frac{\partial f'(\bar{k}^E)}{\partial R} = 0 = \frac{(1+n)^3(1-\alpha h')}{[(1+n)(1+R)(1-\alpha h') - (1-\delta)]^2} (1+R-R^-)(1+R-R^+),$$

and given that the economy feasibility condition guarantees that $\alpha h' < \frac{n+\delta}{1+n} < 1$, it can immediately be seen that

$$\frac{\partial \bar{k}^E}{\partial R} = \begin{cases} < 0 & \forall R < R^- < R^+, \\ > 0 & \forall R^- < R < R^+, \\ < 0 & \forall R^- < R^+ < R. \end{cases}$$

This means that R^+ is the planner's discount rate which leads to the maximum optimal capital ratio. On the other hand,

$$\frac{\partial R^+}{\partial \alpha h'} = \frac{\frac{1}{2}\chi^{-1/2}(1-\alpha h')(1-\delta)[(1-\delta) - (1-\lambda)(1-2\alpha h')] + (1-\delta) + \chi^{1/2}}{(1+n)(1-\alpha h')^2},$$

which is positive provided R^+ is a real number. Therefore, R^+ is a continuous and increasing function of $\alpha h'$ with the following fixed point,

$$\frac{\partial R^+}{\partial \alpha h'} = 0, \quad \implies \quad \alpha h'^* = \frac{(n + \delta)^2}{(1 + n)^2 - (1 - \delta)(1 - \lambda)}.$$

This means that $\forall \alpha h' \in (\alpha h'^*, 1)$, the maximum efficient capital ratio is achieved for $R^+ > 0$. And $\forall \alpha h' \in (0, \alpha h'^*)$, the maximum efficient capital ratio is achieved for $R^+ < 0$; since the central planner's discount factor is only defined in the rank $[0, \infty)$, for all values of $\alpha h' \in (0, \alpha h'^*)$, the maximum efficient capital ratio is reached for $R = 0$. ■

Proof of Proposition 2:

Suppose that the economy is initially on a stationary equilibrium path $(\bar{k}, \bar{e}, \bar{c}_1, \bar{c}_2)$. Consider the following disturbance. At time τ , the capital ratio is reduced by a small amount ε and consumption by old and young agents is increased by the same amount. Finally, let the capital from τ on be given by $k = \bar{k} - \varepsilon$. The utility of all generations born up to $\tau - 1$ is unchanged. The generation born at $\tau - 1$ is better off because the new allocation has increased its consumption in old age.

Generations born at τ and later are better off because their (current or/and future) consumption increases. This is easy to see, since from τ on the capital ratio remains constant, the stock of pollution in any future period $\tau + i$, $\forall i \geq 0$, is given by

$$\begin{aligned} e_{\tau+i} &= \left(\frac{1-\delta}{1+n}\right)^{i+1} \bar{e} + \alpha \sum_{j=0}^i \left(\frac{1-\delta}{1+n}\right)^j f(k_{\tau+i-j}) \\ &= \left(\frac{1-\delta}{1+n}\right)^{i+1} \bar{e} + \alpha f(k) \frac{1+n}{n+\delta} \left[1 - \left(\frac{1-\delta}{1+n}\right)^{i+1}\right]. \end{aligned}$$

Substituting this in the feasibility condition for a period $\tau + i$, $\forall i \geq 0$,

$$\begin{aligned} f(k) &= c_{1,\tau+i} + \frac{c_{2,\tau+i}}{1+n} + (n + \lambda)k + h(e_{\tau+i}) \\ &= c_{1,\tau+i} + \frac{c_{2,\tau+i}}{1+n} + (n + \lambda)k + h' \left(\frac{1-\delta}{1+n}\right)^{i+1} \bar{e} + \alpha h' f(k) \frac{1+n}{n+\delta} \left[1 - \left(\frac{1-\delta}{1+n}\right)^{i+1}\right]. \end{aligned}$$

Therefore, any change in k will affect $c_{\tau+i} = c_{1,\tau+i} + \frac{c_{2,\tau+i}}{1+n}$ in the following manner,

$$\frac{\partial c_{\tau+i}}{\partial k} = f'(k) \left[1 - \frac{1+n}{n+\delta} \alpha h' \left(1 - \left(\frac{1-\delta}{1+n}\right)^{i+1}\right)\right] - (n + \lambda).$$

This reallocation of capital is a Pareto improvement over the initial allocation if

$$f'(k) < \frac{n + \lambda}{1 - \frac{1+n}{n+\delta} \alpha h' \left[1 - \left(\frac{1-\delta}{1+n} \right)^{i+1} \right]}, \quad \forall i \geq 0.$$

Given that the right hand side is an increasing function with i , this condition holds whenever it is satisfied by $i = 0$, i.e. if

$$f'(k) < \frac{n + \lambda}{1 - \alpha h'} = f'(k^{SG}).$$

Since $f'' > 0$, this expression means that whenever the stock of capital is larger than the super golden rule capital ratio, a reallocation of capital such as the one proposed increases the utility of the generations born at τ and later. Therefore this new allocation is a Pareto improvement over the initial allocation. ■

Proof of Proposition 4:

The optimal scheme follows directly on comparing equations (18)-(24) with equations (14)-(15) for the case in which $R = 0$, and considering the government budget constraint. Moreover, it is easy to show that

$$\begin{aligned} \tau_y^* &= \left[\frac{c_2^G}{1+n} - (1+n)k^G \right], \\ \tau_o^* &= \left[(1+n)k^G - \frac{c_2^G}{1+n} - h(e^G) \right] (1+n). \end{aligned}$$

Since we are considering economies with capital overaccumulation, in the golden rule allocation it must be true that savings of young agents cannot cover the consumption and health expenses of elderly agents. Therefore τ_o must be a transfer (negative). However τ_y only is negative if the pollution is the only cause the overaccumulation, i.e. if $\frac{c_2^G}{1+n} < (1+n)k^G < \frac{c_2^G}{1+n} + h(e^G)$. ■

Proof of Proposition 5:

The optimal scheme follows directly on comparing equations (25)-(31) to equations (14)-(17) for the case in which $R = 0$ and considering the government budget constraint.

Since we are considering economies with capital overaccumulation, in the golden rule allocation it must be true that savings of young agents cannot cover the consumption and health expenses of elderly agents. Therefore σ must be positive

On the other hand, since $n + \delta > (1 + n) \alpha h'$ (see footnote 15), $0 < \tau_k < 1$. In regard to the tax rate on wages, τ_w , it can immediately be seen that it must be less than one, since concavity of the production function guarantees $f < f'k$. Moreover it is easy to see that

$$\tau_w = \frac{1 + n}{n + \delta} h' \alpha + \frac{\frac{c_2^G}{1+n} - (1 + n) k^G}{f(k^G) - f'(k^G) k^G},$$

which is positive if $\frac{c_2^G}{1+n} > (1 + n) k^G$. ■

Proof of Proposition 6:

It is easy to prove that the amount of taxes paid by a young agent is

$$\begin{aligned} T_y^P &= \left[\frac{c_2^G}{1+n} - (1+n) k^G \right], \\ T_y^{C-W} &= \left[\frac{c_2^G}{1+n} - (1+n) k^G + \frac{1+n}{n+\delta} h' \alpha [f(k^G) - k^G f'(k^G)] \right], \end{aligned}$$

with the production and capital-wage schemes, respectively. Since the production function is concave, $f(k^G) - k^G f'(k^G) > 0$. Therefore, $T_y^P < T_y^{C-W}$.

The amount of taxes paid by an elderly agent is

$$\begin{aligned} T_o^P &= \left[\frac{c_2^G}{1+n} - (1+n) k^G + h(k^G) \right] (1+n), \\ T_o^{C-W} &= -(1+n) T_y^{C-W} = - \left[\frac{c_2^G}{1+n} - (1+n) k^G + h(k^G) + k^G [f'(k^{GN}) - f'(k^G)] \right] (1+n) \end{aligned}$$

with the production and capital-wage schemes, respectively. $T_o^P < T_o^{C-W}$ because $f'(k^{GN}) < f'(k^G)$. ■