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### **Classical and Modern Business Cycle Measurement: The European Case**

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#### **RESUMEN**

Este artículo intenta armonizar dos visiones sobre el análisis del ciclo de negocio y derivar importantes hechos estilizados de éste en Europa. El artículo parte de la definición clásica sobre detección y análisis del ciclo de negocios propuesta en Burns and Mitchell (1946). En una segunda parte adoptamos una alternativa más moderna: un vector autorregresivo con cambios de estado en la media regidos por una cadena de Markov. Las probabilidades de estar en cada uno de los estados (recesión/expansión) provee una inferencia estadística sobre los puntos de inflexión del ciclo de negocios europeo. Un ejercicio de simulación valora la capacidad del modelo paramétrico de generar hechos estilizados propios de una definición clásica del ciclo de negocios.

**Palabras clave:** Ciclo de negocios internacional, Unión Europea.

#### **ABSTRACT**

This paper intends to harmonize two different approaches to the analysis of the business cycle and in doing so it retrieves the stylized facts of the business cycle in Europe. We start with the "classical" approach proposed in Burns and Mitchell (1946) of dating and analyzing the business cycle; we then adopt the "modern" alternative: the Markov-switching vector autoregression (MS-VAR). The model's regime probabilities provide an optimal statistical inference of the turning point of the European business cycle. For assessing the capacity of the parametric approach to generate the stylized facts of the *classical cycle* in Europe, the stylized facts of the original data are compared to those of simulated data. The MS-VAR model is shown to be a good candidate for use as an statistical instrument to improve the understanding of the business cycle.

**JEL Classification:** E32, F43, F47, C32.

**Keywords:** International business cycles, European Union, Markov Switching, Structural breaks, Time series analysis.

# Classical and Modern Business Cycle Measurement: The European Case

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## Abstract

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# 1 Introduction

The constitution of the European Monetary Union has raised several interesting issues. Among them, one of paramount relevance concerns the existence of a common cycle among the member countries. On the one hand, a lack of business cycle synchronization could complicate the operation of monetary policy in the Union. On the other hand it has been recently argued that the formation of a monetary union in itself would create a tendency for business cycle symmetry to emerge. If this condition holds for the European Monetary Union and the quasi-union of the Exchange Rate Mechanism of the European Monetary System, then we might expect already to be able to find an emergent 'European cycle' which will become more dominant in future years.

This paper addresses the issues of identification and dating of an European business cycle using two different approaches and analyzes the consistency of the stylized facts obtained from both approaches. We first use the classical approach proposed in Burns and Mitchell (1946). The stylized facts which are retrieved for the individual economies can shed light on the search for a reference 'European Business Cycle'. On a second stage the modern parametric approach of Markov-switching (vector) autoregressions in extension of Hamilton (1989) is used. We then investigate whether both approaches are consistent, and whether they give the same picture for the stylized facts of the European economies. That is, we will evaluate our statistical model (the MS- VAR) in terms of its ability to reproduce certain features of the classical cycle.

The first aim of the paper is to improve the understanding of the stylized facts of the business cycle in a group of European economies. By doing so we investigate whether the European Business Cycle is an intelligible concept. The analysis of turning points in individual countries can give a measure of the existence of a business cycle in the aggregate (the European business cycle) by looking at the way in which the turning points of individual countries cluster together. That is, we can use a wide set of series describing the output in individual countries to come up with a single reference cycle. However, we go beyond the analyses that assess the optimality of a optimal currency area only on the basis of a statistic, such as the cross correlation of some measures of macroeconomic activity (as in Artis and Zhang (1997)) or the coincidence of turning points of the business cycles in the countries of interest. We suggest that a rigorous analysis of the similarities in the features characterizing the business cycle provide the understanding of the interdependence in macroeconomic activity necessary to devise economic policies.

A number of recent papers, Pagan (1997a), Pagan (1997b) and Harding and Pagan (1999), have questioned the interest shown by academics in the *growth* cycle at the expense of the *classical* cycle, with the latter claimed to be more relevant to policy makers and the business community. We argue that just focusing on a set of moments of the detrended data can hardly help to understand the characteristic features of the business cycle. For example, a policy maker that was given the mean, variance and cross covariance of a measure of activity on two countries would be unable to devise a set of policies that could stabilize the interdependent economies. A better assessment of, say an optimal currency area, should look at a wide set of business cycle features.

A second aim of the paper is to specify a parametric econometric model that can replicate the stylized facts to which we referred before. We demonstrate the ability of a MS-VAR to generate the stylized facts of the *classical cycle* in Europe. Pagan (1997a), Pagan (1997b) and Harding and Pagan (1999) claim that the durations and amplitudes of expansions and contractions of the classical cycle can be reasonably well reproduced by simple random walk with drift models, where the ratio of the drift to the variance of the disturbance term is the crucial quantity. According to Pagan (1997a), Pagan (1997b) and Harding and Pagan (1999) non-linear models appear to add little over and above that which can be explained by the random walk with drift. Hess and Iwata (1997) adopt a dating of cycle turning points based on global peaks and troughs of activity, rather than the notions of local maxima and minima that underpin the NBER chronology, but nevertheless similarly conclude that non-linear models add little to reproducing the amplitudes and durations of expansions and contractions so defined. This paper finds that the MS-VAR does not only give coincident datings of recessions and expansions as the classical approach does, but is also able to replicate almost all of the stylized facts obtained from the classical analysis. Thus the MS-VAR is shown to be a good candidate to be used as instruments to improve the understanding of the business cycle.

The paper proceeds as follows. Section 2 analyzes some stylized facts of the *classical* cycle in a set of European economies. Section 3 gives a statistical characterization of the process of output growth employing univariate Markov-switching models. The results suggest the existence of a common cycle driving macroeconomic activity in Europe. We then move to the multivariate case and present the results for a Markov-switching vector autoregression (MS-VAR), which exhibits a common cycle consisting of three phases of the business cycle. Section 4 investigates the consistency of the stylized facts of the classical cycle and those obtained with our parametric model, the MS-VAR. Section 5 concludes.

## 2 The Classical Approach of Analyzing Business Cycles in Europe

### 2.1 Detection of the Business Cycle Turning Points

We here consider a definition of the cycle phrased in terms of the turning points of a series as proposed in Burns and Mitchell (1946). In order to detect and assess the specific features of a cycle we first need to identify a set of turning points that define periods of expansion and contraction. Detection of a business cycle is thus based on an algorithm capable of pinpointing relevant turning points. We here use a version of the algorithm proposed by Bry and Boschan (1971) and that has been associated with the NBER. Bry and Boschan (1971)'s original algorithm was devised for monthly series. Because of the quarterly nature of our data we use a derived version proposed by Harding and Pagan (1999). We define a peak as having occurred at  $t$  if  $y_t$  is  $\max\{y_{t-2}, \dots, y_{t+2}\}$  and a trough if  $y_t$  is  $\min\{y_{t-2}, \dots, y_{t+2}\}$ , which is a fairly popular sequencing rule. We also impose the natural requirement that these peaks and troughs, which define the periods of expansion and contraction, alternate. Whenever this condition fails, the least pronounced of adjacent turning points is deleted.<sup>1</sup>

Burns and Mitchell (1946) suggested that a deep analysis of the business cycle should consider:

- The durations and amplitudes of the cycle.
- The durations and amplitudes of its phases.
- Specific cycle patterns.
  1. Cumulative movements between phases.
  2. A measure of 'excess cumulative movements', which captures the shape of the phase.
  3. A *take-off* measure.

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<sup>1</sup>There is a problem when the sequence terminates with two or more peaks. The 'most pronounced' will often be the last, and adopting this may over-estimate the average duration of expansions. Instead, we use the peak immediately following the last trough (and the next trough, when the sequence of peaks and troughs ends with two or more troughs). This may lead to the average durations being under-estimated.

- A measure of conformity.

The duration of the full cycle is just the interval from the initial trough to the final trough. The duration of an expansion is just the interval from the date of the initial trough to the date of the peak. The duration of a contraction is the interval from the date of the peak to the date of the final trough. The peak-trough amplitude is the difference between the level of the time series at adjacent peaks and troughs.

In order to study the specific cycle pattern Burns and Mitchell (1946) divide the full cycle in nine stages and record the average standing during these nine segments. Instead of analyzing the nine stages suggested in Burns and Mitchell (1946), we use the more parsimonious measures suggested by Pagan (1997a), Pagan (1997b) and Harding and Pagan (1999). Let  $D_i$  be the duration of the  $i^{th}$  phase, and  $A_i$  the amplitude, and the consecutive turning points defining the  $i^{th}$  phase fall in periods  $t$  and  $t + d$ , then  $D_i = d$ , then  $A_i = y_{t+d} - y_t = \Delta_d y_{t+d}$ . Imagine the amplitude and duration forming a triangle, then the area of the triangle can be seen as the welfare loss (gain) of a recession (contraction). That is the cumulated loss of output when compared to the value of output just before the turn. The ‘triangle approximation to the cumulative movements’ is given by  $C_{Ti} = \frac{1}{2}D_i \times A_i$ , the actual cumulative movements by  $C_i$ ,<sup>2</sup> and an index of ‘excess cumulated movements measure’,  $E_i$ , by  $E_i = (C_{Ti} - C_i) / C_i$ , which measures departures from the triangular approximation. Measured in this way, the average excess Trough to Peak (TP) measure (by averaging  $E_i$  over all TPs) will exceed unity if, for example, there is on average rapid growth coming out of recession which levels off at around  $A_i$ . Sichel (1993) claims that US recessions are typically followed by a period of rapid growth early on in the recovery. These effects might be measured more directly by recording the ratio of the  $n$ -period growth rate immediately following a turning point to the average growth during that

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<sup>2</sup> $C_i$  can be calculated as:

$$\frac{1}{2}A_i + \sum_{s=1}^{d-1} \Delta_s y_{t+s}.$$

This ensures that the actual and triangular measures coincide when the growth rates within a phase are equal. To see this, suppose  $\Delta y_{t+s} = g_i$ ,  $s = 1, \dots, d$ , so  $\Delta_s y_{t+s} = s g_i$ . Then

$$\begin{aligned} C_i - C_{Ti} &= \frac{1}{2}d_i g_i + \sum_{s=1}^{d-1} s g_i - \frac{1}{2}d_i^2 g_i \\ &= \frac{1}{2}d_i g_i + \frac{1}{2}g_i d_i (d_i - 1) - \frac{1}{2}d_i^2 g_i = 0. \end{aligned}$$

expansionary phase ( $A_i/D_i$ ), where sensible values of  $n$  would appear to be 2 and 4 (based on an examination of Sichel (1993), fig. 1, [p.270]).

Burns and Mitchell (1946) offer two different methods of measuring conformity. The first method takes into consideration the direction and rate of movement of a series during successive expansions and contractions.<sup>3</sup> The second index of the conformity of a series to a reference expansion is obtained by assigning +100 for each rise, -100 for each fall and 0 when there is no change. An arithmetic mean of all entries delivers an expansion index. A contraction index can be built in the same vein. A problem of both indices is that they only consider the net difference between three months averages centered on reference (or specific) trough and peaks. They do not consider the intermediate values of output from peak to trough or trough to peak. An alternative measure has been suggested by Harding and Pagan (1999): the degree of *concordance*. This is defined as the fraction of times the *reference* cycle ( $y_{rt}$ ) and the *specific* cycle ( $y_{jt}$ ) are in the same state,

$$DC_{jr} = T^{-1} \sum_{t=1}^T \{I(S_{jt} = 1, S_{rt} = 1) + I(S_{jt} = 0, S_{rt} = 0)\}.$$

$$DC_{jr} = T^{-1} [\sharp(S_{jt} = 1, S_{rt} = 1)] + \frac{1}{n} [\sharp(S_{jt} = 0, S_{rt} = 0)]$$

where the symbol  $\sharp(S_{jt} = k, S_{rt} = k)$  indicates the number of times in which both two series,  $j$  and  $r$ , are in the same state  $k$ . The above equation can also be written as;

$$DC_{jr} = T^{-1} \left( \sum_{t=1}^T S_{jt}S_{rt} + \sum_{t=1}^T (1 - S_{jt})(1 - S_{rt}) \right)$$

where  $j$  and  $r$  are the indices for the different time series and  $S_{jt}$  is the state of the time series  $j$  at time  $t$  ( $S_{jt} = 1, 0$ , where 1 could denote an expansion phase and 0 a contraction phase). The index should be between 1 and 0, where 1 indicates maximum concordance. We will use this statistic to evaluate the pairwise conformity of two country specific cycles.

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<sup>3</sup>In our case we do not work with reference expansions and contractions but with specific expansions and contractions.

## 2.2 Empirical Results

### 2.2.1 Cycle Dating:

The data consists of seasonally adjusted quarterly Gross Domestic Product (GDP) for a set of European economies from 1970:1 to 1996:12 and were drawn from the OECD database. Our analysis is restricted to a subset of six European countries which are a representative sample of the countries that are part of the EMU: Germany, UK, France, Italy, Austria, and Spain.

Table 1 offers a summary of the most relevant features characterizing the set of European economies under consideration. The first striking result is that the average duration of the expansion or contraction phase differs widely across countries. Germany and France record similar amplitudes of expansions and recessions. Spain and Germany have similar expansion duration but the duration of contraction in Spain is almost half the value of Germany and France. The UK and Austria seem to have similar expansion duration, with values that are half of those recorded for France and Germany. Italy on the other hand has contractions with duration values similar to Austria, Spain and the UK, but the expansion duration is well below that of Spain. If we consider the full cycle, on average France tends to have the longest cycles followed by Germany and then Spain. Whereas Austria, Italy and UK have cycles of duration almost half those of France or Germany.

### 2.2.2 Stylized Facts:

In terms of amplitude there are striking differences. The amplitude of contractions is highest in Germany (almost 3 %). Contractions are very mild in France with almost no loss of output with respect to reference output (0.21 %). The amplitude of Austria and Spain is half of that of Germany. And Italy's contraction amplitude is just a third of that of Germany. The amplitude of expansions also reveals some asymmetries. Expansions in France and Germany record on average amplitudes of 18 %. Spain has a similar pattern with a value of 14 %. On the other hand Austria, Italy and UK experience much weaker expansions.

Differences across countries can also be found in the measures of cumulated movement in expansions and contractions. The most striking case is that of France which has a positive cumulated measure on average across contractions. This is consistent with the small value of the amplitude of contractions. The UK has a very high value (-9.8 %) relative to the other countries, if we consider the short duration of contractions. On the other hand France



Table 1: Empirical Business Cycle Characteristics

		DE	UK	FR	IT	AT	ES
Duration (quarters)	PT	6.333	4.333	6.667	3.400	3.500	3.250
Duration (quarters)	TP	22.000	9.833	27.000	15.250	10.667	20.667
Amplitude (p.c.)	PT	-2.911	-2.838	-0.210	-1.749	-1.026	-1.016
Amplitude (p.c.)	TP	18.131	9.907	19.285	12.473	8.926	14.609
Cumulated (p.c.)	PT	-8.080	-9.823	5.416	-3.455	-2.217	-1.600
Cumulated (p.c.)	TP	235.633	101.357	267.673	144.534	77.077	243.243
Excess	PT	-21.035	-14.550	-21.609	-10.181	-6.839	-13.764
Excess	TP	-7.644	-0.817	0.203	2.404	2.742	-2.515
Trough+2/Expansion		1.066	0.822	1.157	0.659	1.074	0.798
Trough+4/Expansion		0.950	0.773	0.844	1.077	0.804	0.882

and Spain record the highest values of the cumulated measure in expansion (267 % and 243 % respectively).

The excess measure of cumulated output in recessions shows that the recessionary pattern is more similar across countries. The negative excess PT measure indicates that the cumulative movements are larger than the triangular approximation, indicating more rapid subsequent decline in growth. On the other hand the excess TP statistic indicates that for Germany, Spain and UK (7.6 %, -2.5 % and -0.8 % respectively) there is a much larger gain in output during expansions than measured by the triangular representation. The excess TP statistic for Austria, France and Italy indicates that the triangular approximation exceeds the actual cumulative movement of output. This indicates that there is an average rapid growth coming out of recessions which levels off at around  $A_i$ . The previous results are consistent with the *take-off* measures. Output in Austria and France record growth in the half-year following a trough that is on average 15 % and 7 % higher than the rest of the expansion.

If we look at the concordance indices presented in Table 2 we can see that there is a high degree of concordance among countries. However, despite the high degree of concordance, the most striking conclusion from the results just reviewed is that the stylized facts of the business cycle for the countries under analysis differ notably.

Table 2: Empirical Business Cycle Concordance

	DE	UK	FR	IT	AT	ES
DE	1.000					
UK	0.686	1.000				
FR	0.911	0.833	1.000			
IT	0.676	0.921	0.843	1.000		
AT	0.862	0.647	0.843	0.833	1.000	
ES	0.686	0.852	0.705	0.862	0.803	1.000

### 3 A Modern Approach to Model the European Business Cycle

#### 3.1 Univariate Markov-Switching Models of the European Business Cycles

Recent theoretical and empirical business cycle research has revived interest in the co-movement of macroeconomic time series and the regime-switching nature of macroeconomic activity. For the statistical measurement of macroeconomic fluctuations, the Markov-switching autoregressive time series model has become increasingly popular since Hamilton's (1989) application of this technique to measuring the US business cycle. Contractions and expansions are modeled as switching regimes of the stochastic process generating output growth:

$$\Delta y_t - \mu(s_t) = \alpha_1 (\Delta y_{t-1} - \mu(s_{t-1})) + \dots + \alpha_4 (\Delta y_{t-4} - \mu(s_{t-4})) + u_t. \quad (1)$$

The regimes are associated with different conditional distributions of the growth rate of real GNP, where the mean  $\mu_1$  is positive in the first regime ('expansion') and negative in the second regime ('contraction'),  $\mu_2 < 0$ . The variance of the disturbance term,  $u_t \sim \text{NID}(0, \sigma^2)$ , is assumed to be the same in both regimes.

There have been a number of subsequent extensions and refinements. The general idea behind this class of regime-switching models is that the parameters of a VAR depend upon a stochastic, unobservable regime variable  $s_t \in \{1, \dots, M\}$ . The stochastic process generating the unobservable regimes is an ergodic Markov chain defined by the transition

Table 3: Univariate MS-AR Models of the Business Cycle

	Germany	UK	France	Italy	Austria	Spain
<i>Regime-dependent intercepts (<math>10^{-2}</math>)</i>						
$\nu_1$	-0.39	-0.3	0.01	-0.4	0.1	-0.1
$\nu_2$	0.83	0.9	0.8	0.5	1	0.6
$\nu_3$				0.01		0.01
<i>Autoregressive parameters</i>						
$\alpha_1$	-0.23	-0.15	0.028		-0.30	
$\alpha_2$	-0.04	-0.15				
$\alpha_3$	0.11	0.04				
$\alpha_4$	0.24	-0.23				
<i>Variances (<math>10^{-5}</math>)</i>						
$\sigma_1^2$	7.49	8.10	2.59	3.62	6.26	0.96
<i>Persistence of Recessions (Regime 1)</i>						
Erg. Prob	0.22	0.29	0.33	0.13	0.48	0.20
Duration	3.05	7.12	4.08	3.32	5.61	3.57
Log Lik.	314.42	311.45	363.31	345.32	321.86	403.42
LR Test	3.36	6.3717	4.15	31.0	5.02	91.86

probabilities:

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}. \quad (2)$$

Maximum likelihood (ML) estimation of the model is based on a version of the Expectation-Maximization (EM) algorithm discussed in Hamilton (1990).<sup>4</sup> By inferring the probabilities of the unobserved regimes conditional on an available information set, it is then possible to reconstruct the regimes.

Important issues that arise in our analysis are: (i) the convergence process of Spain and Italy and (ii) the secular decline of the mean growth rates of most OECD countries in the post-Bretton Woods era (see also Lumsdaine and Prasad (1997)). A two-regime model representing contractions and expansions is unable to reflect these two stylized facts of the postwar economic history of Italy and Spain. Therefore we extend the Markov-switching process for a third regime. The choice of the final model was based on the Akaike Information Criterion (AIC) and the Hannan-Quinn criterion (HQ). For Germany, UK, France and Austria two regimes were sufficient on the basis of likelihood criteria.<sup>5</sup>

<sup>4</sup>All the computations reported in this paper were carried out with the MSVAR class for Ox, see Krolzig (1998).

<sup>5</sup>Conventional testing approaches are not applicable due to the presence of unidentified nuisance para-

The Maximum likelihood estimations are given in Table 3 which also reports measures of the persistence of recession: the expected number of months a recession prevails (duration) and the unconditional (ergodic) probability of recessions.

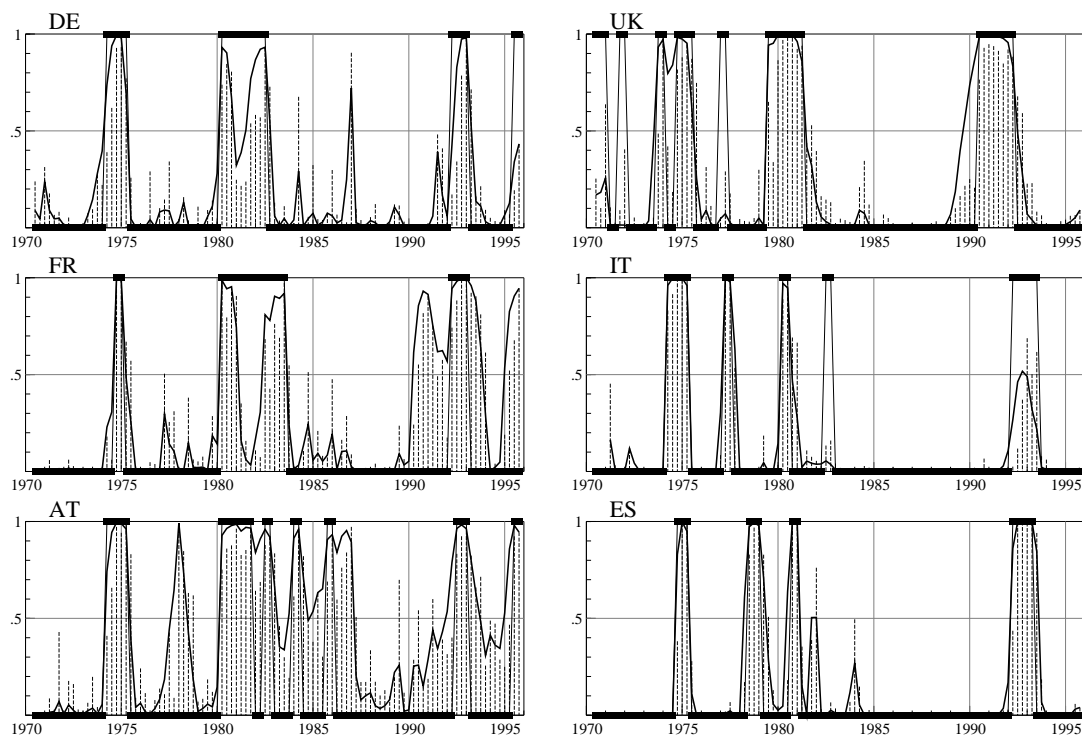


Figure 1: Smoothed and filtered probability of being in a recession for the univariate MS-AR with the classical dating superimposed

The time paths of the smoothed and filtered probabilities are presented in Figure 1. The filtered probability can be understood as an optimal inference on the state variable (whether we are in boom or recession) at time  $t$  using only the information up to time  $t$ , *i.e.*  $\Pr(s_t = m | Y_t)$ , where  $m$  stands for a given regime. The smoothed probability meters under the null of linearity (that is, the transition probabilities) and because the scores associated with parameters of interest under the alternative may be identically zero under the null. Formal tests of the Markov-switching model against linear alternatives employing the standardized LR test designed to deliver (asymptotically) valid inference have been proposed by Hansen (1992), Hansen (1996) and Garcia (1993). The extension of Hansen's approach to our model seems to be impossible to implement computationally (see Ang and Bekaert (1995)) and is certainly beyond the scope of this paper. Furthermore it delivers only a bound on the asymptotic distribution of the standardized LR test. The test is conservative, tending to be under-sized in practice and of low power.

stands for the optimal inference on the regime at time  $t$  using the full sample information,  $\Pr(s_t = m | Y_T)$ . The filtered probabilities are shown with a dashed line and the smooth probabilities are shown with a thick line. For Germany, the UK, Italy, France and Spain the univariate MS-AR models detect recessions fairly well. In the case of Austria the MS-AR probably delivers the worst fit, with difficulties distinguishing clearly the recessionary periods.

In Figure 1 we have overimposed the contraction periods dated with the classical method (as discussed in Section 2) to the smoothed and filtered probabilities of the recessionary regime. Either the smoothed or the filtered regime probabilities can be used as instrument to date recessions. Due to the different information sets used to calculate the smoothed and filtered probabilities of recessions, the former can be seen as an ex-post dating algorithm whereas the latter can be used as a real-time dating statistic. Despite the limitations mentioned above, Figure 1 demonstrate the consistency of both methods as an instrument to date the business cycle .

It is worthwhile stressing that, by definition, univariate MS-AR models as proposed by Hamilton (1989) are only able to capture some of the stylized facts of business cycle fluctuations. They can capture the non-linearity or asymmetry stressed in some part of the literature but, obviously, they are unable to reflect the idea of comovement among economic time series. Hence modelling a vector of time series does not only correspond to the definition of the business cycle, but does also improve the inferences of the Markov process if the business cycle is a common feature of the variables. The contemporaneity of the regime shifts in the growth process of the six European countries suggests a system approach to the investigation of the common cycle of these countries which constitutes the European business cycle.

### 3.2 The Markov-Switching Vector Autoregression

Despite the importance of the transmission of shocks across countries, the identification of common cycles and the recent appreciation of empirical business cycle research, there has been little attempt to investigate cross-country effects with modern non-linear time series models. Still, most studies consider business cycle phenomena for individual countries only. First attempts at the analysis of international business cycles with Markov-switching models have been undertaken by Phillips (1991), Filardo and Gordon (1994) and Krolzig (1997). Phillips's study of a two-country two-regime models was the very first multivariate

Markov-switching analysis of all. Filardo and Gordon (1994) have extended this analysis to a trivariate two-regime model by using leading indicators for the prediction of turning points.

Table 4: Estimation Results: The MSIH(3)-VAR(1) Model of the European GDP Growth Rates

	Germany	UK	France	Italy	Austria	Spain		
<i>Regime-dependent intercepts (<math>10^{-2}</math>)</i>								
Regime 1	-0.448	-0.033	0.078	-0.261	-0.194	-0.086		
Regime 2	0.884	0.463	0.332	0.436	0.843	0.117		
Regime 3	0.921	0.109	0.694	0.991	1.667	0.351		
<i>Autoregressive parameters at lag 1</i>								
Germany	-0.268	-0.272	0.021	-0.038	-0.189	-0.025		
UK	0.082	0.108	0.152	0.124	-0.034	0.021		
France	-0.141	0.017	-0.106	-0.054	0.132	0.040		
Italy	0.237	0.217	0.106	0.181	0.093	-0.018		
Austria	0.101	0.244	0.067	0.119	-0.456	0.017		
Spain	-0.069	0.061	0.159	-0.032	0.227	0.760		
log-likelihood 2311.37 (vs. linear 2227.19)								
AIC	-42.44	(-41.96)	HQ	-40.91	(-41.06)	SC	-38.66	(-39.73)
	$p_{1i}$	$p_{2i}$	$p_{3i}$	Duration	Ergodic Prob.	Observations		
Regime 1	0.842	0.019	0.077	6.3	0.166	19.6		
Regime 2	0.104	0.962	0.041	26.3	0.651	57.1		
Regime 3	0.05	0.019	0.883	8.54	0.118	25.3		

For the reasons discussed earlier we consider a three-regime Markov-switching vector autoregression with regime-dependent covariances where the variables are modeled in first differences<sup>6</sup>:

$$\Delta y_t = \nu(s_t) + A_1 \Delta y_{t-1} + u_t, \quad u_t | s_t \sim \text{NID}(\mathbf{0}, \Sigma(s_t)), \quad (3)$$

where  $\Delta y_t$  is the vector of growth rates. A three-regime model which allows for changes in contemporaneous correlation structure, was chosen based on the Akaike Information Criterion (AIC) and the Hannan-Quinn criterion (HQ). As there are three regimes shifting the (I)ntercept of the VAR(1) and regime-dependent (H)eteroskedasticity, we call this model an MSIH(3)-VAR(1). The estimation results for the period from 1970:3 – 1995:4 are given in Table 4.

<sup>6</sup>The cointegration analysis gave no clear indication of the presence of cointegrating vectors. Therefore we work with differenced data. Though the trace test supported one cointegrating relationships, graphical inspection of the recursively calculated eigenvalues suggested that this long-run relationship broke down at some point within the sample of our analysis. This could be explained by the convergence process in Europe that took place in that period (see Artis, Krolzig and Toro (1998)).

Major differences in the mean growth rate across regimes and the contemporaneous correlation structure in the data are evident. We found this model to pass all specification tests. The different persistence of the regimes can be observed by analyzing the transition probabilities. Note from the transition matrix given in Table 4, that the ‘high growth regime’ can only be reached through the ‘normal growth regime’ and not directly from a recessionary period. This is in contrast with the evidence presented by Sichel (1994) for the US. Sichel (1994) documents that for the US during the post-war period, contractions have typically been followed by a high-growth recovery phase that quickly boosts output back to its prerecession level. The transition matrix allows us to observe the asymmetry of the business cycle in terms of the duration of recessions and the two types of growth periods. Whereas recessions have a duration of approximately 6.3 quarters, the ‘normal growth’ state has a average duration of three times the recession states (26.3 quarters) and the ‘high growth’ state tends to last 8.54 quarters.

### 3.3 Dating the European Business Cycle

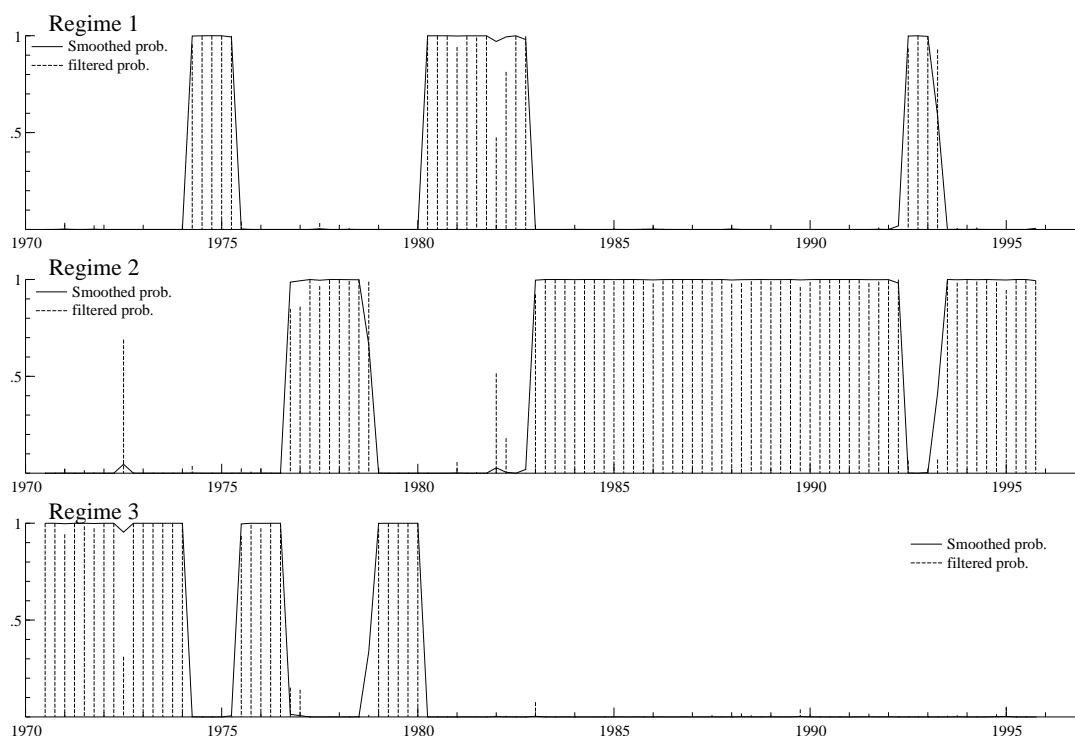


Figure 2: Regime probabilities for the European Business Cycle

The regime probabilities plotted in Figure 2 can again be used to date the European business cycle. The classification of the regimes and the dating of the business cycle amounts to assigning every observation  $y_t$  to one of the regimes. The rule applied here is to assign the observation at time  $t$  to the regime  $m \in \{1, 2, 3\}$  with the highest smoothed probability:

$$m^* = \arg \max_m \Pr(s_t = m \mid Y_T)$$

At every point in time, a smoothed probability of being in an given regime is calculated (the inference is made using the whole data set), and we will assign that observation to a given regime according to the highest smoothed probability. For the simplest case of two regimes, the rule reduces to assigning the observation to the first regime if  $\Pr(s_t = 1 \mid Y_T) > 0.5$  and assigning it to the second regime if  $\Pr(s_t = 1 \mid Y_T) < 0.5$ .

The latter procedure allows a corresponding dating of the European Business Cycle which is given in the left hand side of Table 5. The peak date denotes the period  $t$  just before the beginning of a recession, i.e.  $\Pr(s_t = 1 \mid Y_T) < 0.5$  and  $\Pr(s_{t+1} = 1 \mid Y_T) > 0.5$ ; the trough is the last period of the recession.

## 4 Do the Classical Approach and the MS-Model Deliver the Same Stylized Facts?

The MSIH(3)-VAR(1) model estimated in Section 3 seems to be a good model in terms of specification tests. It remains to show that it does not miss its main goal: the representation of the European Business Cycle. We assess its performance on two grounds. First, the reference European Business Cycle obtained with the MSIH(3)-VAR(1) should be consistent with the reference cycle obtained using the classical methodology in the tradition of Burns and Mitchell (1946). In other words we would like the model to deliver datings of expansions and recessions similar to those obtained in Section 2. Secondly, the statistical model should be able to replicate the stylized facts retrieved with the classical methodology. In order to perform this cross-examination we simulate our statistical model, the MSIH(3)-VAR(1), and the generated data (in levels) are subject to the same classical business cycle analysis the empirically observed data have been.



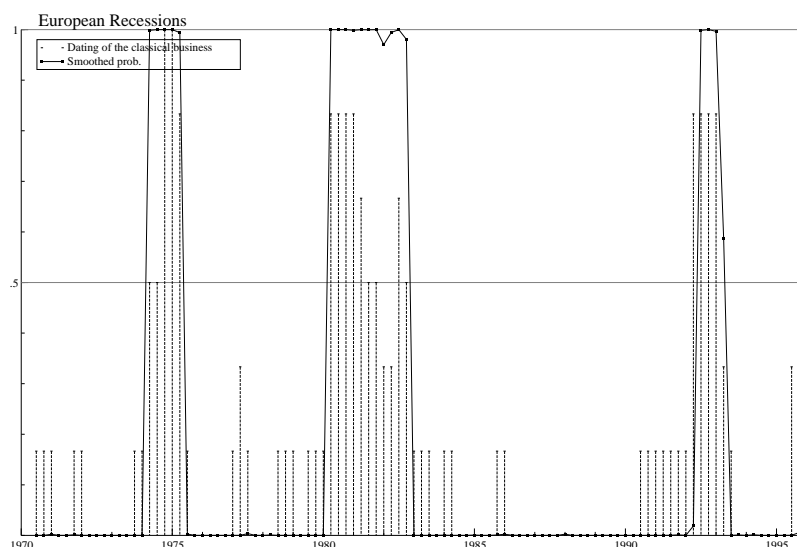


Figure 3: Comparison of the dating of the classical business cycle and the probability of recession in the MS-VAR

#### 4.1 The Reference Cycle

We start by comparing the dating of recessions of the classical business cycle to the dating of recessions based on the regime probabilities of the MS-VAR. Figure 3 plots the dating of the classical business cycle and the alternative dating given by the probabilities of recession in the MSIH(3)-VAR(1). The vertical lines reflect the percentage of countries that are in a recession phase, where the recession phases correspond to those found in Section 2. According to this criterion the definition of the European business cycle depends on the number of countries that we take as a benchmark in order to define an European business cycle and the weight we give to each individual country.

Table 5: Dating of the European Business Cycle

MSVAR for GDP Growth <sup>1</sup>			Classical Cycle Dating		
Peak	Trough	Duration <sup>2</sup>	Peak	Trough	Duration
1974Q1	1975Q2	1.25	1974Q1	1975Q2	1.25
1980Q1	1982Q4	2.75	1980Q1	1982Q4	2.75
1992Q2	1993Q2	1.00	1992Q1	1993Q1	1.25

<sup>1</sup> Using quarterly GDP data for Germany, UK, France, Italy, Austria, and Spain.

<sup>2</sup> Duration denotes the length of the recession in years

In Table 5 the dating of the recessions based on the smoothed probabilities from the MSIH(3)-VAR(1) is compared to the dates based on the clustering of the turning points

obtained with the Bry and Boschan (1971) algorithm. The classical reference cycle for Europe was derived by weighting the information for every country equally. So Europe is considered to be in a recession if at least half of the countries are in a recession. As can be seen from Table 5, the dating and duration of the European Business cycles are similar whether we use the classical approach to cycle dating or the smoothed probabilities from the MSIH(3)-VAR(1) model.

## 4.2 Stylized Facts and the Adelman Test

Finally, we evaluate the proposed MSIH(3)-VAR(1) model with respect to its ability to replicate the business cycle stylized facts for the individual European economies reported in Section 2. This idea of assessing a parametric model according to its ability to replicate business cycle facts goes back to Adelman and Adelman (1954). Based on the Burns and Mitchell (1946) methodology, Adelman and Adelman (1954) developed summary statistics for time series generated by a variant of the Klein-Goldberger model. These statistics were compared with those reported for the US economy in Mitchell (1951).

Table 6 displays the simulation-based expectation of each of the stylized business cycle facts when the data generating process is the MSIH(3)-VAR(1) given in Table 4. These point estimates of the business cycle measures are the means of the empirical distribution obtained from 10000 replications of the MSIH(3)-VAR(1) process with sample sizes of 400 observations. Estimates of the uncertainty associated with the point estimates can also be easily obtained from the empirical probability distribution generated by the Monte Carlo. We consider 90%, 95% and 99% confidence intervals around the point estimates and signal instances when fewer than 10%, 5% and 1% of the simulated values are further out in the tails than the sample estimate. Table 6 shows that the ability of the MSIH(3)-VAR(1) model to replicate the stylized facts is quite remarkable. For France and Germany the empirical measures are represented very accurately. For the other countries the MSIH(3)-VAR(1) model is able to replicate most of the stylized facts. The ability to replicate the duration of expansions is rejected at the 5 % level for the UK, is rejected for Italy at 10 % and for Austria at the 1 % level. The replication of the amplitude of the expansions is only rejected for Austria and Spain. The model's cumulated movement TP measure is rejected for Italy and Spain at the 10 % level, and for Austria at the 1 % level. If we evaluate the MSIH(3)-VAR(1) in terms of its ability to replicate the degree of concordance observed in the original data, we find only two rejections at the 1 % level for the concordance between

France and Germany and between Italy and the UK. Overall the estimated MSIH(3)-VAR(1) is able to replicate most of the stylized facts that have been encountered in Section 2.

Table 6: Simulated Business Cycle Characteristics

		DE	UK	FR	IT	AT	ES
Duration (quarters)	PT	5.301	3.822	4.248	4.833	5.247	5.968 <sup>+</sup>
Duration (quarters)	TP	19.392	15.286*	29.032	26.411 <sup>+</sup>	22.312**	41.011
Amplitude (p.c.)	PT	-1.746	-1.955	-0.812	-1.864	-0.296	-2.629
Amplitude (p.c.)	TP	16.318	12.078	20.020	20.403	17.792*	34.206 <sup>+</sup>
Cumulated (p.c.)	PT	-2.365	-2.839 <sup>+</sup>	7.790	-0.116	9.381	-10.881
Cumulated (p.c.)	TP	300.46	177.37	609.19	553.59 <sup>+</sup>	390.78**	1575.6 <sup>+</sup>
Excess	PT	-13.652	-6.849	-9.839	-7.664	-6.090	-1.961
Excess	TP	-4.866	-2.675	-3.791	-4.062	-5.401	-4.307
Trough+2/Expansion		1.175	1.188*	0.993	1.005 <sup>+</sup>	1.118	0.617
Trough+4/Expansion		0.948	0.937	0.872	0.864	1.001	0.642

Based on the 1970 (3) - 1995 (4) sample and 10000 replications of samples with 400 observations.

<sup>+</sup> indicates that less than 10 p.c. of simulations were further out in the tails than the sample estimate.

\* indicates that less than 5 p.c. of simulations were further out in the tails than the sample estimate.

\*\* indicates that less than 1 p.c. of simulations were further out in the tails than the sample estimate.

## 5 Conclusions

In view of the criticisms that can be directed to conventional methods of business cycle identification, it is important to supplement those methods by others and compare their results, especially in view of the political significance of the kind of results obtained. The findings in this paper contribute to that end. Our results can be summarized as follows. First we retrieve *stylized facts* of the classical business cycle in a set of European economies. This is an essential device to understand the common behavior of economic activity and the extent to which common policies can be implemented. Secondly, we show the ability of the MS-VAR to generate the stylized facts of the *classical cycle* in Europe. Thus the statistical

Table 7: Simulated Business Cycle Concordance

	DE	UK	FR	IT	AT	ES
DE	1.000					
UK	0.729	1.000				
FR	0.777**	0.831	1.000			
IT	0.771	0.760**	0.788	1.000		
AT	0.813	0.725	0.763	0.817	1.000	
ES	0.769	0.779	0.762	0.819	0.787	1.000

Based on the 1970 (3) - 1995 (4) sample and 10000 replications of samples with 400 observations.

+ indicates that less than 10 p.c. of simulations were further out in the tails than the sample estimate.

\* indicates that less than 5 p.c. of simulations were further out in the tails than the sample estimate.

\*\* indicates that less than 1 p.c. of simulations were further out in the tails than the sample estimate.

model is tested with respect to the goal it pursues, that is being able to capture essential business cycle features. This type of model is shown to be a good device to improve the understanding of the European business cycle.

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