centrA:
Fundación Centro de Estudios Andaluces

Documento de trabajo
Serie Economía E2003/30

The Construction of input-output Coefficients Matrices in an Axiomatic Context: Some Further Considerations

Thijs ten Raa[*]
Tilburg University

José Manuel Rueda Cantuche[*]
Universidad Pablo de Olavide

RESUMEN

Kop Jansen y ten Raa (1990) establecen una solución puramente teórica al problema de seleccionar un modelo para la construcción de una matriz de coeficientes técnicos a partir de tablas de origen y destino. Dentro de un contexto axiomático, estos autores demuestran que la hipótesis de tecnología de producto es la más apropiada en función de un conjunto de propiedades deseables de contenido económico. El objetivo de este documento de trabajo consiste en especificar aquellas restricciones que nos permitan asegurar que otros tipos de hipótesis utilizadas por institutos de estadística oficiales, puedan cumplir dichas propiedades.

Palabras clave: Análisis input-output, coeficientes técnicos

ABSTRACT

Kop Jansen and ten Raa (1990) established a purely theoretical solution to the problem of selecting a model for the construction of coefficients on the basis of make and use tables. In an axiomatic context, they singled out the so-called commodity technology model as the best one according to some desirable properties. The aim of this paper is to delineate the restrictions on the relevant data sets that ensure fulfillment of the desirable properties by other models used by statistical offices.

Keywords: Input-output analysis, technical coefficients

JEL classification: C 67, D 57

---

[*] The authors thank to F.M. Guerrero, E. Fedriani, E. Romero and an anonymous referee for valuable comments.

1. Introduction

An input-output matrix of technical coefficients, $\mathbf{A} = (a_{ij})_{i,j\,=\,1,\ldots,n}$ (where n is the number of commodities), represents the direct requirements of commodity i needed to produce a physical unit of commodity j. For instance, if industry 1 corresponds to agriculture and industry 2 corresponds to chemicals, then a_{21} will be the amount of chemical products consumed by agriculture per physical unit of peach, apple and so on. In more general terms, the standard reference is Leontief (1986).

The matrix of technical coefficients **A** has been used for economic analysis by means of the so-called quantity equation or material balance (supply and demand) and the value equation or financial balance (costs and revenues),

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}$$
$$\mathbf{p} = \mathbf{p}\mathbf{A} + \mathbf{v}$$

Here **x** is a column vector of gross outputs, **y** is another column vector of final demand, **p** is a row vector of prices, and lastly, **v** is a row vector of value-added coefficients.

The quantity equation is used for national or regional economic planning; for instance, the output requirements to satisfy a certain final demand level could be analyzed. Final demand could be influenced by an exports or investments policy. Thus, there will be a direct effect over the output levels, which will depend on the final demand variations ($\Delta\mathbf{y}$) and additional indirect effects that will be determined by the **A**-matrix, in accordance with the material balance equation. The value equation can be used to assess the price effects resulting from an energy shock, which surely will bring about variations in the value-added shares of a product, to mention an example.

National and Regional Statistical Offices have concentrated almost exclusively on industry input-output tables instead of commodity tables and set up so-called transactions tables $\mathbf{T} = (t_{ij})_{i,j=1,\ldots,n+1}$ (where n is the number of sectors or industries) (ten Raa, 1994). In such a table, each element displays the inputs requirements of sector i per unit of sector j's production, as well as the final demand compartments (household and government consumption, investment and net exports).

Ten Raa (1994) noted that an input-output transactions table **T** reduces the construction of a matrix of technical coefficients **A** to a matter of divisions:

$$a_{ij} = \frac{t_{ij}}{\sum_{s=1}^{n+1} t_{js}}$$

However, there are three problems. Firstly, commodities and sectors cannot always be classified in the same way. Secondly, in addition to a multitude of inputs, sectors may also have a multitude of outputs. In other words, secondary products must be accommodated. Thirdly, commodities contained in each row and column of an industry-by-industry table are not homogeneous in terms of production (see Rainer, 1989; Braibant, 2002).

To address these complications, the System of National Accounts proposed by the United Nations (1967, 1993), first established the concepts of use and make matrices within an input-output framework[1]. Demand and supply of commodities are described by industries. Thus, let us define a use table, $\mathbf{U} = (u_{ij})_{i,j = 1,\dots,n}$ of commodities i consumed by sector j, and a make table $\mathbf{V} = (v_{ij})_{i,j = 1,\dots,n}$ where sector i will produce commodity j (UN, 1967; ten Raa, Chakraborty and Small, 1984; Kop Jansen and ten Raa, 1990). Notice that, though several attempts have been made to deal with rectangular use and make matrices (see Konijn, 1994), the numbers of commodities and of industries are presumed equal. Following Kop Jansen and ten Raa (1990) industry tables and mixed tables are not considered either.

This new framework provided a more accurate description of commodity flows and at the same time, made economists to face the new problem of constructing technical coefficients matrices, according to some mathematical method based on use and make matrices and which did not always make sense economically (Viet, 1994).

---

[1] The derivation of use and make matrices was first given by van Rijckeghem (1967) and a noteworthy precursor is Edmonston (1952).

Basically, the construction of a technical coefficients matrix **A** is a matter of treatment of secondary products. Many establishments produce only one group of commodities, which are the primary products of the industry to which they are classified. However, some establishments produce commodities that are not among the primary products of the industry to which they belong. As a result, non-zero off-diagonal elements appear in the make matrix. Alternative treatments of secondary products rest upon the separation of outputs and inputs associated with secondary products so that they can be added to the outputs and inputs of the industry in which the secondary product is a characteristic output. Assumptions on these inputs structures imply an **A**-matrix of technical coefficients as a function of the use and make matrices.

According to the System of National Accounts (UN, 1968), secondary products take three different forms: subsidiary products, by-products and joint products.

A subsidiary product is produced in an establishment where another primary product is produced with an entirely different technology. The common administrative costs for both products are negligible and the lack of more detailed statistics causes that subsidiary products become a problem of secondary outputs. By-products generally arise in the course of producing a primary commodity. Not only vary their outputs proportionally with the output of the primary product but also very few additional inputs are required for their production (e.g. molasses by sugar industry). Joint products are those that are produced simultaneously. They share a significant common cost and the output of the secondary product does not vary proportionally to the output of the primary commodity. Each output has separate costs structures and their production depends on their respective demand.

In what follows, **e** will denote a column vector with all entries equal to one, ^{T} will denote transposition and $^{-1}$ inversion of a matrix. Since the latter two operations commute, their composition may be denoted $^{-T}$. Also, $\hat{}$ will denote diagonalization, whether by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector. $\check{}$ will denote a matrix with all the diagonal elements set null. Table 1 shows the general input-output accounting framework from the SNA (see UN, 1967; Gigantes, 1970; UN, 1973; Armstrong, 1975).

Table 1 General input-output accounting framework

	Commodities	**Industries**	**Final demand**	**Total**
Commodities		$\mathbf{U}$	$\mathbf{Y}$	$\mathbf{q} = \mathbf{e}^T \mathbf{V}$
Industries	$\mathbf{V}$			$\mathbf{g} = \mathbf{Ve}$
Primary inputs		$\mathbf{W}^T$		
Total	$\mathbf{V}^T \mathbf{e}$	$\mathbf{e}^T \mathbf{V}^T$		

Section 2 reviews the literature on the different methods for the treatment of secondary products. In Section 3, we show the theoretical solution given by Kop Jansen and ten Raa (1990) in order to select the best method for constructing a technical coefficients matrix $\mathbf{A} = (a_{ij})_{i,j=1,\dots,n}$ (of commodities i needed for the production of one physical unit of commodity j). Further considerations will be taken into account with respect to hybrid models. The aim of this paper is to analyze how bad or good are alternative methods of constructing technical coefficients **A**-matrices in the presence of data restrictions; this is done in Section 4. Lastly, Section 5 draws some conclusions.

2. Description of the Models

Table 2 describes the literature on the treatment of secondary products. The methods can be divided into two groups: those, which transfer outputs only and those, which transfer both inputs and outputs.

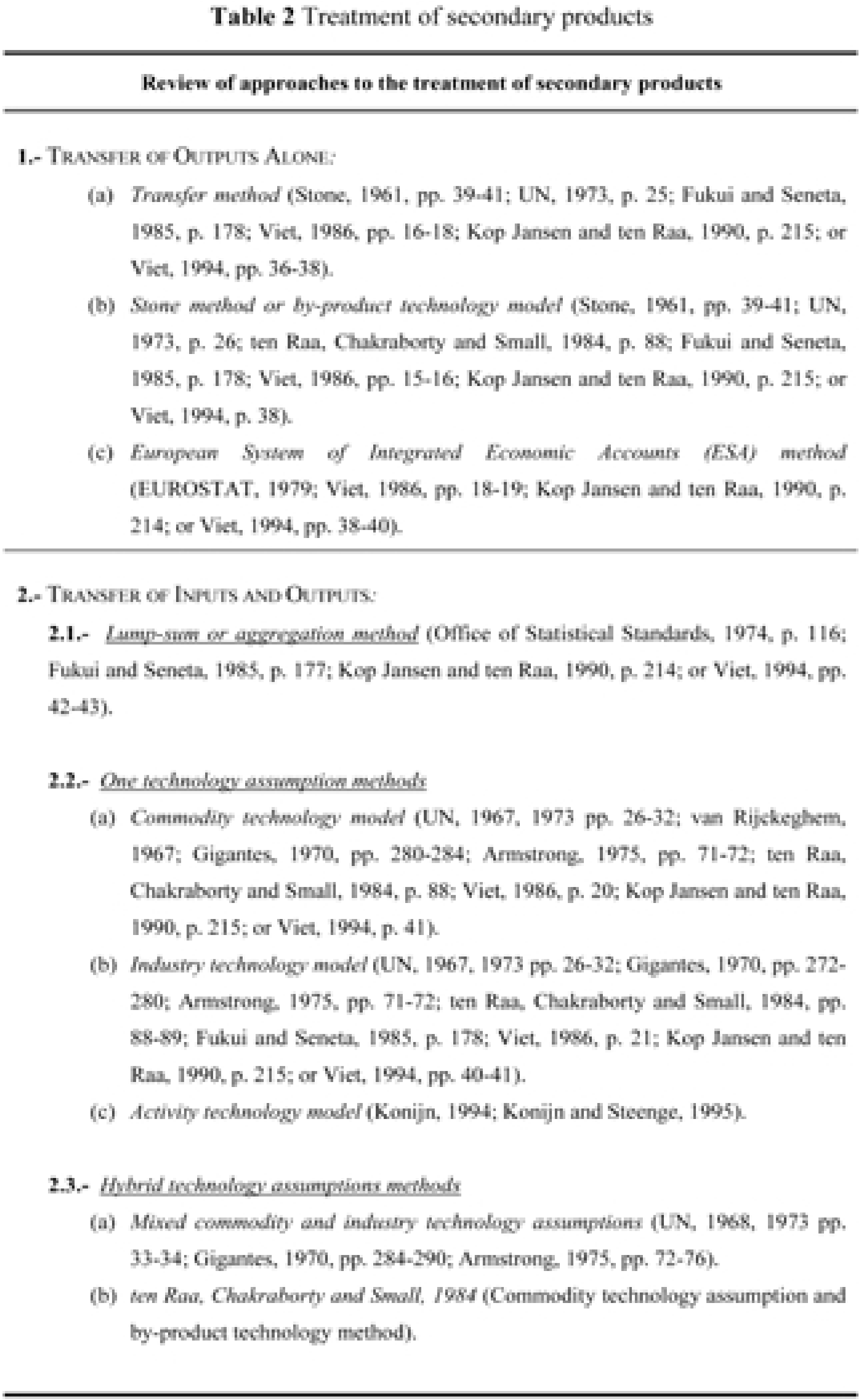

Table 2 Treatment of secondary products

Review of approaches to the treatment of secondary products

1.- TRANSFER OF OUTPUTS ALONE:

- (a) *Transfer method* (Stone, 1961, pp. 39-41; UN, 1973, p. 25; Fukui and Seneta, 1985, p. 178; Viet, 1986, pp. 16-18; Kop Jansen and ten Raa, 1990, p. 215; or Viet, 1994, pp. 36-38).
- (b) *Stone method or by-product technology model* (Stone, 1961, pp. 39-41; UN, 1973, p. 26; ten Raa, Chakraborty and Small, 1984, p. 88; Fukui and Seneta, 1985, p. 178; Viet, 1986, pp. 15-16; Kop Jansen and ten Raa, 1990, p. 215; or Viet, 1994, p. 38).
- (c) *European System of Integrated Economic Accounts (ESA) method* (EUROSTAT, 1979; Viet, 1986, pp. 18-19; Kop Jansen and ten Raa, 1990, p. 214; or Viet, 1994, pp. 38-40).

2.- TRANSFER OF INPUTS AND OUTPUTS:

2.1.- *Lump-sum or aggregation method* (Office of Statistical Standards, 1974, p. 116; Fukui and Seneta, 1985, p. 177; Kop Jansen and ten Raa, 1990, p. 214; or Viet, 1994, pp. 42-43).

2.2.- *One technology assumption methods*

- (a) *Commodity technology model* (UN, 1967, 1973 pp. 26-32; van Rijckeghem, 1967; Gigantes, 1970, pp. 280-284; Armstrong, 1975, pp. 71-72; ten Raa, Chakraborty and Small, 1984, p. 88; Viet, 1986, p. 20; Kop Jansen and ten Raa, 1990, p. 215; or Viet, 1994, p. 41).
- (b) *Industry technology model* (UN, 1967, 1973 pp. 26-32; Gigantes, 1970, pp. 272-280; Armstrong, 1975, pp. 71-72; ten Raa, Chakraborty and Small, 1984, pp. 88-89; Fukui and Seneta, 1985, p. 178; Viet, 1986, p. 21; Kop Jansen and ten Raa, 1990, p. 215; or Viet, 1994, pp. 40-41).
- (c) *Activity technology model* (Konijn, 1994; Konijn and Steenge, 1995).

2.3.- *Hybrid technology assumptions methods*

- (a) *Mixed commodity and industry technology assumptions* (UN, 1968, 1973 pp. 33-34; Gigantes, 1970, pp. 284-290; Armstrong, 1975, pp. 72-76).
- (b) *ten Raa, Chakraborty and Small, 1984* (Commodity technology assumption and by-product technology method).

2.1. Methods Based on the Transfer of Outputs Only

Methods based on the transfer of outputs only are not based on economic assumptions, but are mainly statistical devices to remove secondary products from the make table. Viet (1994) suggests that the Stone and ESA methods should be used only for by-products[2].

2.1.1. Transfer Method

The transfer method treats a secondary product as if it is sold by the industry that actually produces it to the industry to which it is a primary product. Mathematically, the technical coefficients matrix **A** is derived by the following formula, where $\mathbf{g} = \mathbf{V}\mathbf{e}$ and $\mathbf{q} = \mathbf{e}^{T}\,\mathbf{V}$ (see Table 1):

$$\mathbf{A}_{T}(\mathbf{U},\mathbf{V}) = (\mathbf{U} + \check{\mathbf{V}})(\hat{\mathbf{g}} + \hat{\mathbf{q}} - \check{\mathbf{V}})^{-1}$$

The input structure of the industry to which the secondary products are primary outputs, is distorted by the inclusion of the transfer. As a result, an increase in the final demand of those secondary products would lead to an increase in the demand for the primary outputs of the industry that actually produces them, which need not be true. Besides, the input structures of industries that produce secondary products can be altered if the proportion in which they are produced changes. Lastly, sectors outputs can either be industry or commodity outputs.

2.1.2 Stone Method (or By-Product Method)

By the Stone method, all secondary products are considered by-products. Therefore, they can be treated as a negative input in the industry where it is actually produced. Mathematically, we can obtain the technical coefficients matrix **A** by the following formula:

---

[2] For a more detailed explanation of the consequences of each one of these methods on the construction of input-output tables, see also Viet (1994).

$$a_{ij}^{B}(\mathbf{U},\mathbf{V})=\begin{cases}\dfrac{u_{jj}}{v_{jj}} & \text{if } i=j\\[2ex] \dfrac{u_{ij}-v_{ji}}{v_{jj}} & \text{if } i\neq j\end{cases}$$

or, in matrix notation,

$$\mathbf{A}_{B}(\mathbf{U},\mathbf{V})=(\mathbf{U}-\breve{\mathbf{V}}^{T})\hat{\mathbf{V}}^{-1}$$

Negative values of technical coefficients are obtained as a result of applying the Stone method. They appear when the actual use of a commodity i by an industry j is smaller than its secondary output of commodity i. Industry j would need a net amount $u_{ij}-v_{ji}$ of commodity i, which is actually a secondary product of industry j, for the production of v_{jj} units of its primary output. Note that, necessarily $i \neq j$. Besides, the input structure of the industry that produces the secondary product is distorted according to this method for the treatment of secondary outputs.

2.1.3 ESA Method

The European System of Integrated Economic Accounts (ESA) published in 1979 recommends that secondary products should be treated as if they were produced by the industry for which these secondary outputs are primary products. Mathematically, the technical coefficients matrix **A** is calculated as follows:

$$a_{ij}^{E}(\mathrm{U},\mathrm{V})=\frac{u_{ij}}{\sum_{j=1}^{n}v_{ji}}\qquad i,j=1,2,\ldots,n$$

or, in matrix notation,

$$\mathbf{A}_{E}(\mathbf{U},\mathbf{V})=\mathbf{U}\hat{\mathbf{q}}^{-1}$$

The technical coefficients are constructed by division of all the entries of the use table by the total output of the commodity corresponding to the column in the make table. This total output is not necessarily produced by a single sector.

The shortcoming of this treatment is the distortion of input structures of industries with no secondary products but with primary products, which are also produced by other industries. Input structures of industries with secondary production are distorted similarly. Moreover, the sum of intermediate uses of an industry *j* can be larger than the total output of commodity *j* due to the subtraction of their secondary outputs. This will lead to sums of input coefficients greater than one, and therefore, to the non-existence of the Leontief inverse or the negativity of some of its cells.

2.2 Methods Based on the Transfer of Inputs and Outputs

Other methods transfer secondary products and their inputs to the outputs and inputs of the industries where the secondary products constitute a primary output. Since data on inputs associated with secondary products are rarely available, assumptions are made, such as the commodity technology hypothesis, by which the input structure of a secondary commodity is independent of the sector of production, or the industry technology hypothesis, by which the input structure of a secondary output is the same as that of the industry to which it is a primary output. In other terms, production processes substitute commodities when the activity technology model is applied (Konijn, 1994). In fact, this model borrows the mathematical structure of the commodity technology model. Also, we will present hybrid methods based on mixed technology assumptions.

2.2.1 Lump-Sum or Aggregation Method

This method treats a secondary product as if it were actually produced as a primary output. Mathematically, the matrix of technical coefficients **A** is given by:

$$a_{ij}^{L}(\mathrm{U},\mathrm{V}) = \frac{u_{ij}}{\sum_{j=1}^{n} v_{ji}} \qquad i, j = 1, 2, \ldots, n$$

or, in matrix notation,

$$\mathbf{A}_{L}(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{g}^{-1}$$

Technical coefficients are obtained by dividing all the entries of each of the columns from the use table by the total output of industry *j*, specified in row *j* of the make table. This total output includes not only primary products, but also secondary products and by-products.

2.2.2 Methods with a Single Technology Assumption

Three methods rely on a single technology assumption, namely the commodity, industry and activity technology models.

The commodity technology model assumes that each commodity has its own input structure, irrespective the industry of production[3]. Hence, if a_{ik} represents the direct requirements of commodity *i* needed by industry *j* for the production of one physical unit of commodity *k* and also, v_{jk} stands for the total secondary output of commodity *k* produced by industry *j*, it can be derived that the amount $a_{ik}v_{jk}$ is the total inputs requirements of commodity *i* needed for the production of v_{jk} units of output *k*. Then, if we also assume that industry *j* has multiple outputs and all different from commodity *k*, we finally could sum over outputs to obtain the industry *j*'s total demand for input *i*. Thus, u_{ij} can be written as:

$$u_{ij} = \sum_{k=1}^{n} a_{ik} v_{jk} \qquad i, j = 1,\ldots,n$$

If industry *j* produces v_{jk} outputs of commodity *k*, a_{ik} inputs of commodity *i* per physical unit of output *k* will be required. Furthermore, another industry *t* produces v_{tk} outputs of commodity *k*, the direct requirements of input *i* per physical unit of output

---

[3] Actually, under the commodity technology model, not only are inputs structures of industries proportional to their outputs, but also the outputs structures of the commodities they produce.

k, results again a_{ik}. In matrix terms, the commodity technology assumption is given by:

$$\mathbf{U} = \mathbf{A}_C\,(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}}$$

And therefore,

$$\mathbf{A}_C\,(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{V}^{-\mathrm{T}}$$

This method requires the same number of commodities as of industries due to the inverse of the make table. However, its main shortcoming is the negativity of some technical coefficients. According to Viet (1994), negative elements arise when the input structure of the secondary output is not the same as that of the primary product produced elsewhere, and the input, which is transferred out, is greater than the input that is actually consumed. Negative values in the **A**-matrix have prompted a huge literature on the possible solutions to overcome this shortcoming (Armstrong, 1975; Almon, 1972; ten Raa, Chakraborty and Small, 1984; ten Raa, 1988; ten Raa and van der Ploeg, 1989; Rainer, 1989; Steenge, 1990; Rainer and Richter, 1992; Mattey, 1993; Konijn, 1994; Konijn and Steenge, 1995; Mattey and ten Raa, 1997; Avonds and Gilot, 2002).

By the industry technology assumption, each industry has the same inputs requirements for any unit of output (this time, measured in values). This implies that every commodity has different technologies depending on what industry produces it. Actually, the industry technology model assumes that:

(1) Inputs structures of industries are proportional to their outputs (as the commodity technology model does).
(2) Market shares of industries are fixed and independent of the level of commodity or industry outputs.

Mathematically, the **A**-matrix of technical coefficients is given by:

$$a_{ij}^{I}(\mathbf{U},\mathbf{V}) = \sum_{k=1}^{n}\left(\frac{u_{ik}}{v_k}\right)\left(\frac{v_{kj}}{v'_j}\right)$$

where v_k is the total output of industry k and v'_j is the total output of commodity j. In matrix notation,

$$\mathbf{A}_I(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{g}^{-1}\,\mathbf{V}\mathbf{q}^{-1}$$

Let us examine in more detail the above expression in order to cast light on the economic foundations of the industry technology model. u_{ik} / v_k represents the direct requirements of commodity i needed for the production of one physical unit of commodity k. On the contrary, v_{kj} / v'_j denotes the proportion of the commodity j output produced by industry k to the total output of such commodity. In short, the result is called market share. Hence, according to this model, technical coefficients result from a (market share) weighted average over industries k.

Though this assumption has been widely used in many countries, its popularity stemming from the non-negativity of the resulting technical coefficients matrix as well as the fact that the number of commodities need not be equal to the number of industries, is economically unacceptable. As Viet (1994) pointed out, the resulting **A**-matrix is obtained assuming that costs associated with either primary or secondary products are the same, while prices of these products are obviously different. Since the financial balance in input-output analysis states that for each commodity unit, revenue equals to material cost plus value added, applying the industry technology model implies that this meaningful input-output economics assumption does not hold. Moreover, a change in the base year prices and also proportional variations in input requirements and outputs alone will affect the internal structure of technical coefficients.

With respect to the activity technology model, Konijn (1994) redefines the use and make matrices in such a way that application of the commodity technology formula prompts no negatives. Instead of the commodity unit, Konijn assumes that industries can produce commodities according to several production processes and that the same production process can be used by other industries. Moreover, one commodity can be produced by several production processes as well as one production process can generate different goods. Unfortunately, the resulting activity-by-activity input-output table does not rid the negatives. Konijn (1994) and Konijn and Steenge (1995) argue that remaining negatives indicate that some classification adjustments must be made or some further research on errors data must be developed. Although the need of further information on the use and make system is required to apply this activity technology model, Statistics Netherlands adopts this way of removing negatives. In conclusion, Konijn (1994) proposes that we explicitly look at production processes instead of commodities and that we consider the commodity classification of use and make matrices an instrument instead of an exogenous scheme.

2.2.3 Mixed Technology Assumptions Methods

Hybrid technology methods assume that subsidiary production fits either the commodity technology or the industry technology assumptions. Following Armstrong (1975), one would expect that most commodities have the same input structure wherever they are produced, but when secondary products are obtained as a result of industrial processes (i.e. by-products), the assumption of an industry technology assumption may be more appropriate. Hybrid methods require that the make matrix is split in two matrices, $\mathbf{V}_1$ and $\mathbf{V}_2$, where the first one includes outputs for which the commodity technology assumption is made and the second includes those, which are to be treated on an industry technology assumption.

The hybrid technology methods suggested by Gigantes (1970), and incorporated in the United Nations System of National Accounts (SNA, 1968), are based on the following assumptions over the use and make matrices. Firstly, inputs requirements of commodities are proportional to the total output (either primary or secondary) of each industry. This is generally denoted as (Gigantes, 1970; SNA, 1968; Armstrong, 1975):

$$\mathbf{U} = \mathbf{B}\hat{\mathbf{g}} \tag{1}$$

Secondly, industries' outputs of commodities for which a commodity technology assumption is made, are proportional to the output of each industry. This is implicitly assumed when the commodity technology model is applied. It is denoted as:

$$\mathbf{V}_1^T = \mathbf{C}_1\hat{\mathbf{g}}_1 \tag{2}$$

where $\mathbf{g}_1 = \mathbf{V}_1\mathbf{e}$.

Thirdly, industries' outputs of commodities for which an industry technology assumption is made, are proportional to these commodity outputs. The proportions are the market shares of each industry's products in the total output of each one of these kind of commodities. This is implicitly assumed when the industry technology model is applied:

$$\mathbf{V}_2 = \mathbf{D}_2^*\hat{\mathbf{q}}_2 \tag{3}$$

where $\mathbf{q}_2 = \mathbf{V}_2^T\mathbf{e}$.

Fourthly, the production of commodities for which an industry technology assumption is made follows fixed market shares. That is, industries' commodity outputs for which an industry technology assumption is made, are proportional to the total commodity outputs produced in the economy. It is denoted as:

$$\mathbf{g}_2 = \mathbf{D}_2\mathbf{q} \tag{4}$$

where $\mathbf{g}_2 = \mathbf{V}_2\mathbf{e}$.

Following Armstrong (1975), after some transformations the resulting matrix of technical coefficients is given by:

$$\mathbf{A}_H(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\left(\mathbf{I}-\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2\right)+\mathbf{V}_2\hat{\mathbf{q}}^{-1}\right)$$

It can be seen that if $\mathbf{V}_2 = 0$ then $\mathbf{V} = \mathbf{V}_1$, $\mathbf{g}_1 = \mathbf{g}$ and $\mathbf{q}_2 = 0$; hence, the solution is given by:

$$\mathbf{A}_H(\mathbf{U},\mathbf{V}) = \mathbf{U}\mathbf{V}^{-T} = \mathbf{A}_C(\mathbf{U},\mathbf{V})$$

which is the commodity technology model. Analogously, if $\mathbf{V}_1 = 0$ then $\mathbf{V} = \mathbf{V}_2$, $\mathbf{q}_2 = \mathbf{q}$ and $\mathbf{q}_1 = 0$; hence the solution is given by:

$$\mathbf{A}_H(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1} = \mathbf{A}_I(\mathbf{U},\mathbf{V})$$

which is the industry technology model.

The hybrid technology model generates negative values of technical coefficients, just like the commodity technology model. A different solution can be obtained if we assume that the outputs of the products for which an industry technology assumption is made, are proportional to the outputs of the producing industries instead of to the outputs of each commodity, whatever industry produced them. A more detailed explanation is shown in Armstrong (1975, pp. 74-76). The resulting **A**-matrix is given by:

$$\mathbf{A}_Y(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\left(\mathbf{I}-\mathbf{V}_2^{T}\hat{\mathbf{g}}^{-1}\mathbf{H}\right)+\mathbf{V}_2\hat{\mathbf{q}}^{-1}\right)$$

with $\mathbf{H}$ such that $\mathbf{g} = \mathbf{H}\mathbf{q}$.

Ten Raa, Chakraborty and Small (1984) elaborated a new hybrid technology model where the industry technology assumption was replaced by the Stone (or by-product) method. In this case, the make table is split into two matrices, $\mathbf{V}_1$ and $\mathbf{V}_2$. $\mathbf{V}_1$ includes those outputs for which a commodity technology assumption is made (primary and ordinary secondary outputs) and $\mathbf{V}_2$ includes those, which are to be treated on by-product technology assumption (by-products). Since by-products are treated as negative inputs, the total requirements of commodity *i* by industry *j* for the production of all primary and ordinary secondary products of industry *j*, are given by a net amount of requirements. Mathematically,

$$\mathbf{A}_{\mathrm{CB}}(\mathbf{U},\mathbf{V}) = (\mathbf{U} - \mathbf{V}_2^{\mathrm{T}})\,\mathbf{V}_1^{-\mathrm{T}}$$

Notice that if all secondary products are ordinary ($\mathbf{V}_1 = \mathbf{V}$), we will have the commodity technology model and that if all secondary products are by-products ($\mathbf{V}_1 = \hat{\mathbf{V}}, \mathbf{V}_2 = \tilde{\mathbf{V}}$), we will have the Stone method (or by-product technology model). As discussed in ten Raa, Chakraborty and Small (1984), negative elements in the technical coefficients matrix also arise when this hybrid model is applied.

3. The Choice of Model

Kop Jansen and ten Raa (1990) developed and examined axiomatically how well various methods for treatment secondary products fulfill four desirable properties of input-output coefficients $\mathbf{A}(\mathbf{U}, \mathbf{V})$, namely:

(1) Axiom *M* refers to the material balance or the quantity equation and is denoted by

$$\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} = \mathbf{U}\mathbf{e}$$

Economically, this axiom implies that total supply must meet total demand (intermediate consumption plus final demand compartments). In other words, the total input requirements must be equal to the observed total input.

(2) Axiom *F* refers to the financial balance or the value equation and is denoted by

$$\mathbf{e}^{T}\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{T} = \mathbf{e}^{T}\mathbf{U}$$

That is to say, for each commodity unit, revenue equals to material cost plus value added. In words, the input cost of output must match the observed value of input.

(3) Axiom *P* refers to the invariance of the resulting **A**-matrix with respect to units of measurement or, in other terms, to prices. It is called the price invariance axiom and denoted by

$$\mathbf{A}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \hat{\mathbf{p}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{p}}^{-1} \quad \forall \mathbf{p} > 0$$

Evidently, this property tries to avoid that a change in the base year prices could affect technical coefficients. Variations in the internal structure of **A**(**U**,**V**) should be caused by real economic phenomena.

(4) Axiom *S* is the so-called scale invariance axiom:

$$\mathbf{A}(\mathbf{U}\hat{\mathbf{s}},\hat{\mathbf{s}}\mathbf{V}) = \mathbf{A}(\mathbf{U},\mathbf{V}) \quad \forall \mathbf{s} > 0$$

It stipulates that technical coefficients do not change when input requirements and outputs vary proportionally.

Kop Jansen and ten Raa (1990) developed and examined how well various methods for treatment secondary products fulfill axioms *M*, *F*, *P* and *S*. More precisely, the authors proved that the just described structure of input-output analysis, involving the four axioms, not only imposes restrictions on the choice of model of construction, but determines it uniquely, namely the commodity technology model. Their theorem in the real sphere states that the combination of axioms *M* and *S* implies that the commodity technology model specifies the **A**-matrix and their theorem in the nominal sphere that the combination of axioms *F* and *P* has the same implication.

Table 3 illustrates to how the other methods fare in the light of the axioms. In addition to the results obtained by Kop Jansen and ten Raa (1990), Table 3 includes the performance of the mixed commodity and industry technology models (UN, 1968; Gigantes, 1970); the proofs are in the Appendix. Lastly, since the activity technology model (Konijn, 1994) borrows the mathematical structure from the commodity technology model, it requires no separate performance report.

Table 3 Axioms fulfillment of input-output coefficients constructs

Model	Axiom *M*	Axiom *F*	Axiom *P*	Axiom *S*
Transfer				
Stone method			√	√
ESA method	√		√	
Lump-sum				√
Commodity technology	√	√	√	√
Industry technology	√			
ten Raa et al. method			√	√
United Nations hybrid model	√			

Source: Kop Jansen and ten Raa (1990) and own elaboration.

Our main result is a closer examination of Table 3, answering the question what restrictions on the data restore the desirable properties for the models. In other words, for each input-output construct we delineate regions in data space where axioms are fulfilled.

4. Equivalent Conditions for Axioms *M* and *F*

In this section, we will prove that under certain data limitations, some methods for the treatment of secondary products other than the commodity technology model fulfill the material and financial balance. Unfortunately, the price and scale invariance axioms admit no such results.

Two theorems express the material and financial balance axioms in terms of the commodity technology model.

Theorem 1

A technical coefficients matrix **A(U,V)** fulfills axiom *M* for all **U** and non-singular **V** if and only if,

$$\sum_{j=1}^{n} a_{ij} q_j = \sum_{j=1}^{n} a_{ij}^{C} q_j \quad \forall i = 1,\ldots,n$$

or, in matrix terms,

$$\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}\mathbf{e} = \mathbf{A}_{C}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}\mathbf{e}$$

Proof

By definition of $\mathbf{A}_C$, the right hand side of axiom *M* reads

$$\mathbf{U}\mathbf{e} = \mathbf{U}\mathbf{V}^{-T}\mathbf{V}^{T}\mathbf{e} = \mathbf{A}_{C}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}\mathbf{e}$$

□

Theorem 2

A technical coefficients matrix **A(U,V)** fulfills axiom F if and only if the sum of each column of **A(U,V)** matches the sum of the respective column of $\mathbf{A}_C$ **(U,V)** for all **U** and non-singular **V**.

$$\sum_{i=1}^{n} a_{ij} = \sum_{i=1}^{n} a_{ij}^{C} \quad \forall j = 1,\ldots,n$$

or, in matrix terms,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\mathbf{A}_{C}(\mathbf{U},\mathbf{V})$$

Proof

Sufficiency is proved as follows. Suppose

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\mathbf{A}_{C}(\mathbf{U},\mathbf{V}),$$

then, by definition of $\mathbf{A}_C$,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\,\mathbf{U}\mathbf{V}^{-\mathrm{T}}$$

and, therefore,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}\mathbf{V}^{-\mathrm{T}}\,\mathbf{V}^{\mathrm{T}}$$

or

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

which is axiom *F* indeed. All steps can be reversed, proving necessity.

□

For the lump-sum (or aggregation) method we have the following result.

Corollary 1

The lump-sum method fulfills the material balance axiom if the total output of industry *j* matches the total output of commodity *j*, for all **U** and **V** with $\mathbf{q}_j = \mathbf{g}_j$, $\forall j$.

Proof

Under the lump-sum method the **A**-matrix is defined as:

$$\mathbf{A}_L(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\hat{\mathbf{g}}^{-1}$$

and, therefore, if we assume that $\mathbf{V}\mathbf{e} = \mathbf{V}^{T}\mathbf{e}$, that is all $\mathbf{g}_j = \mathbf{q}_j$, we obtain:

$$\mathbf{A}_L(\mathbf{U},\mathbf{V})\mathbf{q} = \mathbf{U}\,\hat{\mathbf{g}}^{-1}\mathbf{q} = \mathbf{U}\,\hat{\mathbf{q}}^{-1}\mathbf{q} = \mathbf{U}\,\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{e} = \mathbf{U}\mathbf{e}$$

since $\mathbf{q} = \hat{\mathbf{q}}\mathbf{e}$.

□

We conclude that the material balance axiom will be fulfilled in the lump-sum model if, for all *j*, total industry output is equal to total commodity output. The reverse does not hold; axiom *M* does not imply that $\mathbf{g}_j = \mathbf{q}_j$.

The Stone (or by-product) technology model also admits a specific result.

Corollary 2

The Stone (or by-product) technology model fulfills the financial balance axiom for all **U** and **V** if and only if value-added is zero.

$$\mathbf{e}^{T}\mathbf{V}^{T} = \mathbf{e}^{T}\mathbf{U}$$

Proof

Under the financial balance axiom, the by-product technology model should verify,

$$\mathbf{e}^{T}\,\mathbf{A}_{B}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{T} = \mathbf{e}^{T}\mathbf{U}$$

with the left-hand side of this equality such as,

$$\mathbf{e}^{T}(\mathbf{U} - \tilde{\mathbf{V}}^{T})\hat{\mathbf{V}}^{-1}\mathbf{V}^{T} = (\mathbf{e}^{T}\mathbf{U} - \mathbf{e}^{T}\tilde{\mathbf{V}}^{T})\hat{\mathbf{V}}^{-1}\mathbf{V}^{T} = \mathbf{e}^{T}\mathbf{U}\,\hat{\mathbf{V}}^{-1}\mathbf{V}^{T} - \mathbf{e}^{T}\tilde{\mathbf{V}}^{T}\hat{\mathbf{V}}^{-1}\mathbf{V}^{T}$$

Moreover, since $\tilde{\mathbf{V}}^{\mathrm{T}} = \mathbf{V}^{\mathrm{T}} - \hat{\mathbf{V}}^{\mathrm{T}}$ and $\hat{\mathbf{V}}^{\mathrm{T}} = \hat{\mathbf{V}}$ it yields,

$$\mathbf{e}^{\mathrm{T}}\mathbf{U}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} - \mathbf{e}^{\mathrm{T}}\tilde{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} - \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} + \mathbf{e}^{\mathrm{T}}\hat{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}}$$

which is the same as,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{B}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}} = (\mathbf{e}^{\mathrm{T}}\mathbf{U} - \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}})\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} + \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

So, let us assume now that $\mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$, then

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{B}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

□

All steps can be reversed, proving necessity.

The last model we consider is the mixed technology model presented by ten Raa, Chakraborty and Small (1984), where the make matrix is split into a table $\mathbf{V}_1$ of primary products and ordinary secondary products, i.e. those products which involve an alternative activity and which are not being generated automatically by the primary productive process, and a table $\mathbf{V}_2$ of by-products.

Kop Jansen and ten Raa (1990) demonstrate that both axioms *M* and *F* hold if and only if the mixed technology model proposed by ten Raa, Chakraborty and Small (1984) reduces to the commodity technology model. In other words, both axioms hold only when the $\mathbf{V}_2$ table is null, i.e. when there are no by-products, since according to the authors, the so-called ordinary secondary products are included in table $\mathbf{V}_1$.

But, what happens in the presence of by-products? Under what restrictions on the data axioms *M* and *F* still hold? We will take some preliminary results from Kop Jansen and ten Raa (1990) as our point of departure in order to cast light on this issue.

Corollary 3

The CB-Mixed (ten Raa, Chakraborty and Small, 1984) technology model fulfills the material balance axiom for all $\mathbf{U}$ and non-singular $\mathbf{V}_1$ when $\mathbf{U}=\mathbf{V}^{T}$.

Proof

Under the CB-Mixed technology model construct, the material balance axiom should verify that,

$$\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}\mathbf{e}=(\mathbf{U}-\mathbf{V}_2^{T})\mathbf{V}_1^{-T}\mathbf{V}^{T}\mathbf{e}=\mathbf{U}\mathbf{e}$$

where $\mathbf{V}_1$ stands for the primary outputs and those secondary products considered as "ordinary" according to ten Raa, Chakraborty and Small (1984) definition, and $\mathbf{V}_2$, for the by-products. Since we are assuming that $\mathbf{U}=\mathbf{V}^{T}=\mathbf{V}_1^{T}+\mathbf{V}_2^{T}$ it can be shown that,

$$(\mathbf{U}-\mathbf{V}_2^{T})\mathbf{V}_1^{-T}\mathbf{V}^{T}\mathbf{e}=\mathbf{V}_1^{T}\mathbf{V}_1^{-T}\mathbf{V}^{T}\mathbf{e}=\mathbf{U}\mathbf{e}$$

□

Corollary 4

The CB-Mixed (ten Raa, Chakraborty and Small, 1984) technology model fulfills the financial balance axiom if and only if value-added is zero.

$$\mathbf{e}^{T}\mathbf{V}^{T}=\mathbf{e}^{T}\mathbf{U}$$

Proof

As Kop Jansen and ten Raa (1990) demonstrate, under the CB-Mixed technology model construct the financial balance axiom should verify that,

$$\mathbf{e}^{T}\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}=\mathbf{e}^{T}(\mathbf{U}-\mathbf{V}_2^{T})\mathbf{V}_1^{-T}\mathbf{V}^{T}=\mathbf{e}^{T}\mathbf{U}$$

This can also be expressed as,

$$\mathbf{e}^{T}\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}=\mathbf{e}^{T}(\mathbf{U}-\mathbf{V}_{2}^{T})\mathbf{V}_{1}^{-T}\mathbf{V}^{T}=(\mathbf{e}^{T}\mathbf{U}-\mathbf{e}^{T}\mathbf{V}_{2}^{T})\mathbf{V}_{1}^{-T}\mathbf{V}^{T}$$

and substituting $\mathbf{e}^{T}\mathbf{V}^{T}=\mathbf{e}^{T}\mathbf{U}$, it yields,

$$\mathbf{e}^{T}\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^{T}=(\mathbf{e}^{T}\mathbf{V}^{T}-\mathbf{e}^{T}\mathbf{V}_{2}^{T})\mathbf{V}_{1}^{-T}\mathbf{V}^{T}=\mathbf{e}^{T}(\mathbf{V}^{T}-\mathbf{V}_{2}^{T})\mathbf{V}_{1}^{-T}\mathbf{V}^{T}$$

$$=\mathbf{e}^{T}\mathbf{V}_{1}^{T}\mathbf{V}_{1}^{-T}\mathbf{V}^{T}=\mathbf{e}^{T}\mathbf{V}^{T}=\mathbf{e}^{T}\mathbf{U}$$

□

All steps can be reversed, proving necessity.

5. Summary and Conclusions

The most interesting conclusion is that the material and financial axioms will be fulfilled under some restrictions on the data, related to the commodity technology assumption (Theorems 1 and 2). A brief summary of the main results is presented in Table 4.

Table 4 Additional assumptions over axioms according to models.

Model	Axiom *M*	Axiom *F*	Axiom *P*	Axiom *S*
Transfer	$\mathbf{A}_T(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}=$ $\mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_T(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	Never
By-product technology	$\mathbf{A}_B(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}=$ $\mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_B(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$ or $\mathbf{e}^T\mathbf{V}^T=\mathbf{e}^T\mathbf{U}$	√	√
European System	√	$\mathbf{e}^T\mathbf{A}_E(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	√	Never
Lump-Sum	$\mathbf{A}_L(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}=$ $\mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$ or $\mathbf{V}\mathbf{e}=\mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_L(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	√
Commodity technology	√	√	√	√
Industry technology	√	$\mathbf{e}^T\mathbf{A}_I(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	Never
ten Raa et al. method	$\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}=$ $\mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$ or $\mathbf{U}=\mathbf{V}^T$	$\mathbf{e}^T\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})=$ $\mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$ or $\mathbf{e}^T\mathbf{V}^T=\mathbf{e}^T\mathbf{U}$	√	√

For the lump-sum method the material balance is fulfilled if the total commodity outputs match respective total industry outputs. This restriction involves somehow that commodities and industries can be classified in the same way. Since this is not unusual to find in several input-output tables published to date, it seems to be reasonable to assume this hypothesis in order to improve the fulfillment of the material balance axiom.

Other theoretically correct restrictions can be found but with little real and economic sense: null value-added, constant proportionality of scale returns, equality of all commodity prices and absence of secondary products or by-products. The transfer and the industry technology model need restrictive conditions in order to fulfill all axioms.

References

Almon, C. (1972) Investment in input-output models and the treatment of secondary products, in: A. P. Carter & A. Bródy (eds.) *Applications of Input-Output Analysis* (Amsterdam, North-Holland).

Armstrong, A.G. (1975) Technology assumptions in the construction of United Kingdom input-output tables, in: R. I. G. Allen & W. F. Gossling (eds.) *Estimating and Updating Input-Output Coefficients* (London, Input-Output Publishing Co).

Avonds, L. & Gilot, A. (2002) The new Belgian Input-Output Table. General Principles, XIV*th International Conference on Input-Output Techniques*, Montreal (Canada).

Braibant, M. (2002) Transformation of supply and use tables to symmetric input-output tables, XIV*th International Conference on Input-Output Techniques*, Montreal (Canada).

Edmonston, J. H. (1952) A treatment of multiple-process industries, *Quarterly Journal of Economics*, pp. 557-571.

EUROSTAT (1979) *European System of Integrated Economic Accounts – ESA* (Luxembourg, EUROSTAT).

Fukui, Y. & Seneta, E. (1985) A theoretical approach to the conventional treatment of joint product in input-output tables, *Economic Letters*, 18 , pp. 175-179.

Gigantes, T. (1970) The representation of technology in input-output systems, in: A. P. Carter & A. Bródy (eds.) *Contributions to input-output analysis* (Amsterdam, North-Holland).

Konijn, P. J. A. (1994) The make and use of commodities by industries, PhD Thesis, University of Twente, The Netherlands.

Konijn, P. J. A. & Steenge, A. E. (1995) Compilation of input-output data from the National Accounts, *Economic Systems Research*, 7, pp. 31-45.

Kop Jansen, P. S. M. & ten Raa, Th. (1990) The choice of model in the construction of input-output coefficients matrices, *International Economic Review*, 31, pp. 213-227.

Leontief, W. (1986) *Input-Output Economics* (New York, Oxford University Press).

Mattey, J. P. (1993) Evidence on Input-Output technology assumptions from the Longitudinal Research Database, *Discussion Paper Center for Economic Studies*, 93-8, (Washington DC, US Bureau of the Census).

Mattey, J. P. & ten Raa, Th. (1997) Primary versus secondary production techniques in US manufacturing, *Review of Income and Wealth*, 43, pp. 449-464.

Office of Statistical Standards (1974) *Input-Output tables for 1970* (Tokyo, Institute for Dissemination of Government Data).

Rainer, N. (1989) Descriptive versus analytical make-use systems: some Austrian experiences, in: R. Miller, K. Polenske & A.Z. Rose (eds.) *Frontiers of input-output analysis* (New York, Oxford University Press).

Rainer, N. & Richter, J. (1992) Some aspects of the analytical use of descriptive make and absorption tables, *Economic Systems Research*, 4, pp. 159-172.

Steenge, A. E. (1990) The commodity technology revisited: theoretical basis and an application to error location in the make-use framework, *Economic Modelling*, 7 , pp. 376-387.

Stone, R. (1961) *Input-Output and National Accounts* (Paris, OECD).

ten Raa, Th. (1988) An alternative treatment of secondary products in input-output analysis: frustration, *Review of Economics and Statistics*, 70, pp. 535-540.

ten Raa, Th. (1994) On the methodology of input-output analysis, *Regional Science and Urban Economics*, 24, pp. 3-27.

ten Raa, Th., Chakraborty, D. & Small, J. A. (1984) An alternative treatment of secondary products in input-output analysis, *Review of Economics and Statistics*, 66, pp. 88-97.

ten Raa, Th. & van der Ploeg, R. (1989) A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, 6, pp. 2-19.

United Nations (1968) *A system of national accounts*, Studies in Methods Series F, no. 2, rev. 3 (New York, United Nations).

United Nations (1973) *Input-output tables and analysis*, Studies in Methods Series F, no. 14, rev. 1 (New York, United Nations).

United Nations (1993) *Revised system of national accounts*, Studies in Methods Series F, no. 2, rev. 4 (New York, United Nations).

United Nations Statistical Commission (1967) *Proposals for the revision of SNA, 1952*, Document E/CN.3/356 (New York, United Nations).

van Rijckeghem, W. (1967) An exact method for determining the technology matrix in a situation with secondary products, *Review of Economics and Statistics*, 49, pp. 607-608.

Viet, V.Q. (1986) *Study of Input-Output Tables: 1970-1980* (New York, UN Statistical Office).

Viet, V.Q. (1994) Practices in input-output table compilation, *Regional Science and Urban Economics*, 24, pp. 27-54.

Appendix

This Appendix proves that the established hybrid constructs on the basis of commodity and industry technology assumptions fulfill only the material balance. As in Kop Jansen and ten Raa (1990) and using their same imaginary use and make matrices, it also presents counterexamples that violate the financial balance, price or scale invariance axioms.

Let us define the following use and make matrices to generate counterexamples

$$\mathbf{U} = \begin{pmatrix} 1/2 & 0 \\ 1 & 1/2 \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{p} = \mathbf{s} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Also, the make table is split into the following $\mathbf{V}_1$ and $\mathbf{V}_2$ matrices,

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}, \quad \mathbf{V}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

A straightforward computation shows that,

$$\mathbf{g} = \mathbf{V}\mathbf{e} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{g}_1 = \mathbf{V}_1\mathbf{e} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{q} = \mathbf{V}^T\mathbf{e} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{q}_2 = \mathbf{V}_2^T\mathbf{e} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and therefore,

$$\mathbf{A}_H(\mathbf{U},\mathbf{V}) = \begin{pmatrix} 1/4 & 1/8 \\ 1/2 & 1/2 \end{pmatrix} \quad \text{and} \quad \mathbf{A}_Y(\mathbf{U},\mathbf{V}) = \begin{pmatrix} 1/4 & 1/8 \\ 0 & 3/4 \end{pmatrix}$$

with $\mathbf{H} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$.

Material balance (*Axiom* M)

The material balance equation for $\mathbf{A}_H$ is verified as follows:

$$\mathbf{A}_H(\mathbf{U},\mathbf{V})\,\mathbf{V}^T\mathbf{e} = \mathbf{A}_H(\mathbf{U},\mathbf{V})\,\mathbf{q} = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\left(\mathbf{I} - \hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2\right) + \mathbf{V}_2\hat{\mathbf{q}}^{-1}\right)\mathbf{q} \tag{5}$$

Since it is true that $\hat{\mathbf{q}}^{-1}\mathbf{q} = \mathbf{e}$, $\mathbf{g}_2 = \mathbf{V}_2\mathbf{e}$ and $\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2\mathbf{q} = \mathbf{q}_2$ equation (5) can be written as:

$$\begin{aligned} \mathbf{A}_H(\mathbf{U},\mathbf{V})\,\mathbf{V}^T\mathbf{e} &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\mathbf{q} - \hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\mathbf{q}_2 + \mathbf{g}_2\right) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}(\mathbf{q} - \mathbf{q}_2) + \mathbf{g}_2\right) \\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\mathbf{q}_1 + \mathbf{g}_2\right) \end{aligned} \tag{6}$$

and if we substitute $\mathbf{q}_1 = \mathbf{V}_1^T\mathbf{e}$ in equation (6) bearing in mind that $\mathbf{g}_1 = \hat{\mathbf{g}}_1\mathbf{e}$,

$$\begin{aligned} \mathbf{A}_H(\mathbf{U},\mathbf{V})\,\mathbf{V}^T\mathbf{e} &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-T}\mathbf{q}_1 + \mathbf{g}_2\right) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{e} + \mathbf{g}_2) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\mathbf{g}_1 + \mathbf{g}_2) \\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{g} = \mathbf{U}\mathbf{e} \end{aligned}$$

For $\mathbf{A}_Y$, axiom *M* is verified as follows:

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}}\mathbf{e} = \mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\,\mathbf{q} &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\left(\mathbf{I}-\mathbf{V}_2^{\mathrm{T}}\hat{\mathbf{g}}^{-1}\mathbf{H}\right)+\mathbf{V}_2\hat{\mathbf{q}}^{-1}\right)\mathbf{q} \\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}-\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{V}_2^{\mathrm{T}}\hat{\mathbf{g}}^{-1}\mathbf{H}\mathbf{q}+\mathbf{V}_2\hat{\mathbf{q}}^{-1}\mathbf{q}\right)\end{aligned} \tag{7}$$

Since $\hat{\mathbf{q}}^{-1}\mathbf{q} = \hat{\mathbf{g}}^{-1}\mathbf{g} = \mathbf{e}$, $\mathbf{g}_2 = \mathbf{V}_2\mathbf{e}$ and $\mathbf{g} = \mathbf{H}\mathbf{q}$ (see Armstrong, 1975, p. 75) equation (7) can be written as:

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}}\mathbf{e} &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}-\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{V}_2^{\mathrm{T}}\mathbf{e}+\mathbf{g}_2\right) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{q}-\mathbf{q}_2)+\mathbf{g}_2\right) \\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1+\mathbf{g}_2\right)\end{aligned} \tag{8}$$

for $\mathbf{V}_2^{\mathrm{T}}\mathbf{e} = \mathbf{q}_2$.

If we substitute $\mathbf{q}_1 = \mathbf{V}_1^{\mathrm{T}}\mathbf{e}$ in equation (8), bearing in mind that $\mathbf{g}_1 = \hat{\mathbf{g}}_1\mathbf{e}$,

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}}\mathbf{e} &= \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1+\mathbf{g}_2\right) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\hat{\mathbf{g}}_1\mathbf{e}+\mathbf{g}_2\right) = \mathbf{U}\hat{\mathbf{g}}^{-1}\left(\mathbf{g}_1+\mathbf{g}_2\right) \\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{g} = \mathbf{U}\mathbf{e}\end{aligned}$$

Financial balance (*Axiom* F)

For both hybrid technology assumptions, the financial balance equation is not fulfilled since,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1/4 & 1/8 \\ 1/2 & 1/2\end{pmatrix}\begin{pmatrix}1 & 0 \\ 1 & 1\end{pmatrix} = \begin{pmatrix}11/8 & 5/8\end{pmatrix},$$

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1/4 & 1/8 \\ 0 & 3/4\end{pmatrix}\begin{pmatrix}1 & 0 \\ 1 & 1\end{pmatrix} = \begin{pmatrix}9/8 & 7/8\end{pmatrix}$$

and

$$\mathbf{e}^{T}\mathbf{U} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1 & 1/2 \end{pmatrix} = \begin{pmatrix} 3/2 & 1/2 \end{pmatrix}$$

Price invariance (*Axiom* P)

The price invariance axiom is violated since,

$$\mathbf{A}_{H}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \begin{pmatrix} 1/3 & 0 \\ 1/3 & 1/2 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} + \begin{pmatrix} 0 & 1/2 \\ 0 & 0 \end{pmatrix}\right] = \begin{pmatrix} 1/3 & 1/6 \\ 1/3 & 5/12 \end{pmatrix},$$

$$\mathbf{A}_{V}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \begin{pmatrix} 1/3 & 0 \\ 1/3 & 1/2 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1/2 \\ 0 & 0 \end{pmatrix}\right] = \begin{pmatrix} 1/3 & -1/6 \\ 1/3 & 1/3 \end{pmatrix}$$

but

$$\hat{\mathbf{p}}\mathbf{A}_{H}(\mathbf{U},\mathbf{V})\hat{\mathbf{p}}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1/4 & 1/8 \\ 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}$$

and

$$\hat{\mathbf{p}}\mathbf{A}_{V}(\mathbf{U},\mathbf{V})\hat{\mathbf{p}}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1/4 & 1/8 \\ 0 & 3/4 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 0 & 3/4 \end{pmatrix}$$

Scale invariance (*Axiom* S)

According to the real sphere theorem in Kop Jansen and ten Raa (1990) the material balance and scale invariance axioms characterize the commodity technology model. Hence, if it has been proved that material balance holds under hybrid technology assumptions, necessarily scale invariance axiom must not hold. Otherwise, commodity technology model must be imposed. Actually, scale invariance axiom holds when the hybrid technology model is reduced to the commodity technology model.

centrA:

Fundación Centro de Estudios Andaluces

Documentos de Trabajo

Serie Economía

E2001/01 “The nineties in Spain: so much flexibility in the labor market?’’, J. Ignacio García Pérez y Fernando Muñoz Bullón.

E2001/02 “A Log-linear Homotopy Approach to Initialize the Parameterized Expectations Algorithm’’, Javier J. Pérez.

E2001/03 “Computing Robust Stylized Facts on Comovement’’, Francisco J. André, Javier J. Pérez, y Ricardo Martín.

E2001/04 “Linking public investment to private investment. The case of the Spanish regions”, Diego Martínez López.

E2001/05 “Price Wars and Collusion in the Spanish Electricity Market”, Juan Toro y Natalia Fabra.

E2001/06 “Expedient and Monotone Learning Rules”, Tilman Börgers, Antonio J. Morales y Rajiv Sarin.

E2001/07 “A Generalized Production Set. The Production and Recycling Function”, Francisco J. André y Emilio Cerdá.

E2002/01 “Flujos Migratorios entre provincias andaluzas y entre éstas y el resto de España’’, J. Ignacio García Pérez y Consuelo Gámez Amián.

E2002/02 “Flujos de trabajadores en el mercado de trabajo andaluz’’, J. Ignacio García Pérez y Consuelo Gámez Amián.

E2002/03 “Absolute Expediency and Imitative Behaviour”, Antonio J. Morales Siles.

E2002/04 “Implementing the 35 Hour Workweek by means of Overtime Taxation”, Victoria Osuna Padilla y José-Víctor Ríos-Rull.

E2002/05 “Landfilling, Set-Up costs and Optimal Capacity”, Francisco J. André y Emilio Cerdá.

E2002/06 “Identifying endogenous fiscal policy rules for macroeconomic models”, Javier J. Pérez y Paul Hiebert.

E2002/07 “Análisis dinámico de la relación entre ciclo económico y ciclo del desempleo en Andalucía en comparación con el resto de España”, Javier J. Pérez, Jesús Rodríguez López y Carlos Usabiaga.

E2002/08	“Provisión eficiente de inversión pública financiada con impuestos distorsionantes”, José Manuel González-Páramo y Diego Martínez López.
E2002/09	“Complete or Partial Inflation convergence in the EU?”, Consuelo Gámez y Amalia Morales-Zumaquero.
E2002/10	“On the Choice of an Exchange Regime: Target Zones Revisited”, Jesús Rodríguez López y Hugo Rodríguez Mendizábal.
E2002/11	“Should Fiscal Policy Be Different in a Non-Competitive Framework?”, Arantza Gorostiaga.
E2002/12	“Debt Reduction and Automatic Stabilisation”, Paul Hiebert, Javier J. Pérez y Massimo Rostagno.
E2002/13	“An Applied General Equilibrium Model to Assess the Impact of National Tax Changes on a Regional Economy”, M. Alejandro Cardenete y Ferran Sancho.
E2002/14	“Optimal Endowments of Public Investment: An Empirical Analysis for the Spanish Regions”, Óscar Bajo Rubio, Carmen Díaz Roldán y M. Dolores Montávez Garcés.
E2002/15	“Is it Worth Refining Linear Approximations to Non-Linear Rational Expectations Models?” , Alfonso Novales y Javier J. Pérez.
E2002/16	“Factors affecting quits and layoffs in Spain”, Antonio Caparrós Ruiz y M.ª Lucía Navarro Gómez.
E2002/17	“El problema de desempleo en la economía andaluza (1990-2001): análisis de la transición desde la educación al mercado laboral”, Emilio Congregado y J. Ignacio García Pérez.
E2002/18	“Pautas cíclicas de la economía andaluza en el período 1984-2001: un análisis comparado”, Teresa Leal, Javier J. Pérez y Jesús Rodríguez.
E2002/19	“The European Business Cycle”, Mike Artis, Hans-Martin Krolzig y Juan Toro.
E2002/20	“Classical and Modern Business Cycle Measurement: The European Case”, Hans-Martin Krolzig y Juan Toro.
E2002/21	“On the Desirability of Supply-Side Intervention in a Monetary Union”, Mª Carmen Díaz Roldán.

E2003/01	“Modelo Input-Output de agua. Análisis de las relaciones intersectoriales de agua en Andalucía”, Esther Velázquez Alonso.
E2003/02	“Robust Stylized Facts on Comovement for the Spanish Economy”, Francisco J. André y Javier Pérez.

E2003/03 “Income Distribution in a Regional Economy: A SAM Model”, Maria Llop y Antonio Manresa.

E2003/04 “Quantitative Restrictions on Clothing Imports: Impact and Determinants of the Common Trade Policy Towards Developing Countries”, Juliette Milgram.

E2003/05 “Convergencia entre Andalucía y España: una aproximación a sus causas (1965-1995). ¿Afecta la inversión pública al crecimiento?”, Javier Rodero Cosano, Diego Martínez López y Rafaela Pérez Sánchez.

E2003/06 “Human Capital Externalities: A Sectoral-Regional Application for Spain”, Lorenzo Serrano.

E2003/07 “Dominant Strategies Implementation of the Critical Path Allocation in the Project Planning Problem”, Juan Perote Peña.

E2003/08 “The Impossibility of Strategy-Proof Clustering”, Javier Perote Peña y Juan Perote Peña.

E2003/09 “Plurality Rule Works in Three-Candidate Elections”, Bernardo Moreno y M. Socorro Puy.

E2003/10 “A Social Choice Trade-off Between Alternative Fairness Concepts: Solidarity versus Flexibility”, Juan Perote Peña.

E2003/11 “Computational Errors in Guessing Games”, Pablo Brañas Garza y Antonio Morales.

E2003/12 “Dominant Strategies Implementation when Compensations are Allowed: a Characterization”, Juan Perote Peña.

E2003/13 “Filter-Design and Model-Based Analysis of Economic Cycles”, Diego J. Pedregal.

E2003/14 “Strategy-Proof Estimators for Simple Regression”, Javier Perote Peña y Juan Perote Peña.

E2003/15 “La Teoría de Grafos aplicada al estudio del consumo sectorial de agua en Andalucía", Esther Velázquez Alonso.

E2003/16 “Solidarity in Terms of Reciprocity", Juan Perote Peña.

E2003/17 “The Effects of Common Advice on One-shot Traveler’s Dilemma Games: Explaining Behavior through an Introspective Model with Errors", C. Monica Capra Susana Cabrera y Rosario Gómez.

E2003/18 “Multi-Criteria Analysis of Factors Use Level: The Case of Water for Irrigation", José A. Gómez-Limón, Laura Riesgo y Manuel Arriaza.

E2003/19 “Gender Differences in Prisoners’ Dilemma", Pablo Brañas-Garza, Antonio J. Morales-Siles.

E2003/20 “Un análisis estructural de la economía andaluza a través de matrices de contabilidad social: 1990-1999", M. Carmen Lima, M. Alejandro Cardenete y José Vallés.

E2003/21 “Análisis de multiplicadores lineales en una economía regional abierta", Maria Llop y Antonio Manresa.

E2003/22 “Testing the Fisher Effect in the Presence of Structural Change: A Case Study of the UK", Oscar Bajo-Rubio, Carmen Díaz-Roldán y Vicente Esteve.

E2003/23 "On Tests for Double Differencing: Some Extensions and the Role of Initial Values", Paulo M. M. Rodrigues y A. M. Robert Taylor.

E2003/24 "How Tight Should Central Bank’s Hands be Tied? Credibility, Volatility and the Optimal Band Width of a Target Zone", Jesús Rodríguez López y Hugo Rodríguez Mendizábal.

E2003/25 "Ethical implementation and the Creation of Moral Values", Juan Perote Peña.

E2003/26 "The Scoring Rules in an Endogenous Election", Bernardo Moreno y M. Socorro Puy.

E2003/27 "Nash Implementation and Uncertain Renegotiation", Pablo Amorós.

E2003/28 "Does Familiar Environment Affect Individual Risk Attitudes? Olive-oil Producer vs. no-producer Households", Francisca Jiménez Jiménez.

E2003/29 "Searching for Threshold Effects in the Evolution of Budget Deficits: An Application to the Spanish Case", Óscar Bajo-Rubio, Carmen Díaz-Roldán y Vicente Esteve.

E2003/30 "The Construction of input-output Coefficients Matrices in an Axiomatic Context: Some Further Considerations", Thijs ten Raa y José Manuel Rueda Cantuche.

centrA:
Fundación Centro de Estudios Andaluces

Normas de publicación de Documentos de Trabajo
centrA Economía

La Fundación Centro de Estudios Andaluces (**centrA**) tiene como uno de sus objetivos prioritarios proporcionar un marco idóneo para la discusión y difusión de resultados científicos en el ámbito de la Economía. Con esta intención pone a disposición de los investigadores interesados una colección de Documentos de Trabajo que facilita la transmisión de conocimientos. La Fundación Centro de Estudios Andaluces invita a la comunidad científica al envío de trabajos que, basados en los principios del análisis económico y/o utilizando técnicas cuantitativas rigurosas, ofrezcan resultados de investigaciones en curso.

Las normas de presentación y selección de originales son las siguientes:

1. El autor(es) interesado(s) en publicar un Documento de Trabajo en la serie de Economía de centrA debe enviar su artículo en formato PDF a la dirección de email: wpecono@fundacion-centra.org
2. Todos los trabajos que se envíen a la colección han de ser originales y no estar publicados en ningún medio de difusión. Los trabajos remitidos podrán estar redactados en castellano o en inglés.
3. Los originales recibidos serán sometidos a un breve proceso de evaluación en el que serán directamente aceptados para su publicación, aceptados sujetos a revisión o rechazados. Se valorará, asimismo, la presentación de¡ trabajo en seminarios de **centrA**.
4. En la primera página deberá aparecer el título del trabajo, nombre y filiación del autor(es), dirección postal y electrónica de referencia y agradecimientos. En esta misma página se incluirá también un resumen en castellano e inglés de no más de 100 palabras, los códigos JEL y las palabras clave de trabajo.
5. Las notas al texto deberán numerarse correlativamente al pie de página. Las ecuaciones se numerarán, cuando el autor lo considere necesario, con números arábigos entre corchetes a la derecha de las mismas.
6. La Fundación Centro de Estudios Andaluces facilitará la difusión electrónica de los documentos de trabajo. Del mismo modo, se incentivará económicamente su posterior publicación en revistas científicas de reconocido prestigio.