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RESUMEN

Analizamos el problema económico de "asignación de tareas para finalizar un proyecto complejo" cuando la información sobre la duración de dichas tareas y las secuencias precedentes constituye información privada de los agentes que llevan a cabo cada una de ellas. Para conseguir la asignación eficiente de tareas -usando el conocido método del camino crítico en la literatura de Investigación Operativa-, el planificador debe diseñar los incentivos apropiados y las compensaciones a los agentes basadas en la información transmitida por ellos. Mostramos la existencia de mecanismos que generan estrategias dominantes para la asignación eficiente de tareas. Cuando, además, añadimos nuevas propiedades deseables, como la individualidad racional, obtenemos un resultado de imposibilidad.

Palabras clave: camino crítico, PERT, estrategias dominantes, implementación, asignación de tareas, a prueba de estrategias, racionalidad individual.

ABSTRACT
In this paper we propose to analyze the economic problem of allocating tasks on time in order to finish a complex project when information about tasks' duration and predating sequences of tasks is privately owned by the agents that undertake each task. In order to achieve the efficient allocation of tasks -using the well-known Critical path method in the Operations Research literature-, the planner must design the appropriate incentives and compensations to the agents based on the reported information. We show the existence of mechanisms that implement in dominant strategies the efficient allocation of tasks on time. When we further add new desirable properties like individual rationality, an impossibility result emerges.

Keywords: Critical path, PERT, Dominant strategies, implementation, tasks allocations, strategy-proofness, individual rationality.
JEL classification: D78 y C60.

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Dominant Strategies Implementation of the Critical Path Allocation in the Project Planning Problem*

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*This paper is part of my Ph.D. dissertation in the Departament d'Economía i Història Económica de la Universitat Autònoma de Barcelona. I wish to acknowledge to my research supervisor, Prof. Salvador Barberà for his guidance and support and the Fundación Centro de Estudios Andaluces (CentrA).

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Abstract

In this paper we propose to analyze the economic problem of allocating tasks on time in order to finish a complex project when information about tasks' duration and predating sequences of tasks is privately owned by the agents that undertake each task. In order to achieve the efficient allocation of tasks -using the well-known Critical path method in the Operations Research literature-, the planner must design the appropriate incentives and compensations to the agents based on the reported information. We show the existence of mechanisms that implement in dominant strategies the efficient allocation of tasks on time. When we further add new desirable properties like individual rationality, an impossibility result emerges.

Keywords: Critical path, PERT, Dominant strategies, implementation, tasks allocations, strategy-proofness, individual rationality.

JEL classification numbers: D78, C60.

Resumen

En este artículo proponemos analizar el problema económico de asignación de tareas en el tiempo para finalizar un proyecto complejo cuando la información sobre la duración de las tareas y secuencias de tareas precedentes es información privada de los agentes que llevan a cabo cada una de ellas. Para conseguir la asignación eficiente de tareas -usando el conocido método del camino crítico en la literatura de Investigación Operativa-, el planificador debe diseñar los incentivos apropiados y las compensaciones a los agentes basadas en la información transmitida por ellos. Mostramos la existencia de mecanismos que implementan en estrategias dominantes la asignación eficiente de tareas en el tiempo. Cuando además añadimos nuevas propiedades deseables como la individualidad racional, obtenemos un resultado de imposibilidad.

1 Introduction

In this paper we study the existence of dominant strategies mechanisms in an incomplete information version of the *Critical Path Method* (CPM) or *Program Evaluation and Review Technique* (PERT), a well-known solution to a standard production network problem in the Operations Research literature -see, for example, Bazawa and Jarvis [1], Derigs [2] and Deo and Pang [3] as comprehensive introductions to the topic.

The PERT was first developed and used, with great success, during the late 1950s by the US Navy to control the progress of the construction of the Polaris missiles, an extraordinary complex project carried out by different production units. The CPM technique was found independently and applied to virtually the same kind of problems, although the PERT was a little bit more general since it allowed for some degree of uncertainty.

The simplest *production network problem* at which these techniques are applied is very simple: A project consisting in a number of different elementary tasks has to be carried out. Each task constitutes a time-consuming production activity -abstracting from any other economic resource employed- necessary for the completion of the project. Tasks cannot generally be allocated arbitrarily, since a particular task may need some others to be finished before it starts, and maybe some of its preceding tasks are also preceded by others, and this network structure is what generates the complexity of the problem. Of course, for the problem to make sense some technological restrictions must be introduced: cycles and loops of precedence are technologically unfeasible. The CPM and the PERT are equivalent methods to analyze the sequences of tasks such that the total amount of time needed to finish all the tasks is minimized.[1] A *critical path* is a sequence of tasks that are undertaken one after the other and such that the completion of the sequence requires exactly the minimum amount of time needed to terminate all tasks. The CPM and the PERT define algorithms to identify the critical paths and those tasks with some roominess -the tasks outside the critical paths that can therefore be allocated at different starting times without affecting the total (minimum) duration of the project-. It can be proven that the whole allocation problem can be transformed into a linear programming one and the operations research analysis is essentially one of computability. The CPM and the PERT also assume that the planner knows all relevant data about the technologies: tasks, time needed by each one and which are the immediate preceding tasks of each one. The PERT admits some uncer-

[1] There is an underlying assumption that the total cost of the completion of the project to the planner or principal is increasing in the total amount of time needed.

tainty about the duration of each task, but the probability distribution is always known to the planner.

This paper explores the same task allocation problem but assumes that each task is carried out by an economic agent;[2] as in many production problems, the agent responsible for each task can be a worker, supervisor, firm or the people in charge of a sub-project or division within a firm, but the key assumption is that she is better informed about the technological characteristics of her particular task than the planner herself. In the limit, we assume that the planner has to rely on the information reported by the agents in each task to allocate the times and sequence of the tasks and the rest of the agents do not need to know anything about a different task but their own. The agents are rational and will exploit their informational advantage if given the opportunity. Nevertheless, the planner is not completely uninformed: if some agents lie about the duration or precedence of their tasks and they are allocated in a technologically unfeasible way, they will be caught and punished hard enough to discourage any agent to lie in a potentially detectable way. Notice, however, that there is still room enough for safe lies: reporting longer duration times and declaring as preceding tasks more tasks that those really needed are always undetectable when the planner is allocating tasks by using the CPM or the PERT methods, and they are actually the only lies that we will allow in this paper. We also assume that the agents, if not compensated in other way, are interested in delaying the completion of their own task as much as possible. This is justified because each hour employed in carrying out the task requires a costly effort to be made by the agent and agents prefer the same disutility to take place later rather than sooner. There is, therefore, a fundamental conflict of interests between the planner or principal -who is allocating tasks by using the PERT with the information reported by the agents and who tries to minimize the total cost of the project- and her agents, who would prefer the project to be delayed forever. A simplifying and important assumption we impose is that disutility of effort is known and the same for everybody and is normalized to unity, but we still allow for differences in the agents' relative impatience -their time discounts-. Of course, planning the production network and the starting-finishing time of each task is not the only way the planner has to influence the behavior of the agents. We assume that the planner can design a *transfer scheme* depending on the reported technologies that specifies the monetary payments that each agent receives or has to pay -taxes in this case- to the planner.[3] This rule is

[2] When it is the case that the tasks are undertaken by machines or automata alone, the original task-machine allocation problem still applies, but it seems unlikely that no worker (but the planner herself) is involved in any task in real life problems.

[3] We assume that the transfers are implemented before the task allocation starts. There-

known by the agents, who have committed to work in the project and cannot quit if they think that they will not receive enough money and money enters additively on the agents' payoffs. Our main aim is to analyze the possibility of designing *anonymous, strategy-proof, balanced and invariant to the project size transfer schemes*, i.e., a payoff structure such that: (i) The agents are treated the same regardless of their names, (ii) no agent has an incentive to lie about her technology regardless of what other agents report to the planner and whatever her own technology is, (iii) the sum of all the agents' transfers amount to the same fixed amount -the total budget for the project- whatever the revealed technologies and (iv) the time units used in the problem do not affect the agent's relative transfers. After proving the existence of such mechanisms, we study the possibility of designing *individually rational payments* -transfer schemes such that for every admissible reported technology, the agents can always guarantee for themselves at least a certain utility. We prove the impossibility of finding mechanisms with this additional property.

The paper proceeds as follows: In *Section 2* the formal model is introduced and the definitions properly stated. *Section 3* deals with the results and we conclude with some comments.

2 The model

Let $\overline{N}$ be a potential finite set of productive agents and $N = \{1, ..., n\} \subseteq \overline{N}$ be a subset of *agents* indexed by $i, j, k, l, z \in N$. The total number of agents in set N is $n \geq 2$. Each agent has to perform a *task* for the completion of a *project*. We do not allow in this model for multi-task agents to simplify the problem and we are not concerned with the matching allocation problem between agents and tasks. Hence, we assume that each agent either is the only -or the best- agent capable to perform a particular task or that the matching or allocation of tasks to agents took place in an earlier stage and is given. Therefore, there is no reason to define a separate set of tasks and we identify the set of tasks with the set of agents N. Each task -or alternatively from now on, each agent- is characterized by its belonging to a network such that a given task cannot be undertaken before some other tasks are finished. Moreover, carrying out each task is a time-consuming process and some work effort -or maybe some cost of capital- has to be invested in order to be successfully completed. Given any task $i \in N$, we denote as $P_i \in 2^N$ the set of *preceding tasks* of task i. For the problem to make sense, we need to impose some minimal structure to the admissible sets of preceding tasks for all tasks. In particular, the following constraints should hold in any

fore, in period "0" the agents report their technologies and the transfers are made.

well-defined problem: we say that a project is *technologically feasible* if the following conditions hold:

(i). Temporal irreflexivity: $\forall i \in N,\ i \notin P_i$.

(ii). Temporal asymmetry: $\forall i, j \in N,\ i \neq j,\ i \in P_j \longrightarrow j \notin P_i$.

(iii). Temporal acyclicity: $\forall i, j, k, ..., l, z \in N$, (all distinct), $i \in P_j$ & $j \in P_k$ &...& $l \in P_z \longrightarrow z \notin P_i$.

Condition (i) establish that no task is ever preceded by itself. Condition (ii) prevents two different tasks to precede each other and (iii) rules out cycles of precedence. Given the linear structure of time, the meaning of the above properties becomes obvious.

Furthermore, technology requires the use of costly time for undertaking each task. For simplicity, we assume time to be discrete -measured in any relevant unit-. Henceforth, time intervals might be hours, minutes or days, but let us call them hours-. Let E be the real line and Z_+ be the set of non-negative integers -time structure considering that 0 stands for *now*- and Z_{++} be the set of positive integers. $T_i \in Z_{++}$ is the minimum number of hours that task i needs to be terminated, given the optimal use of the resources available and given that preceding tasks -and preceding tasks of its preceding tasks and so on- have been done before. We assume that each hour employed by the agent to the completion of his task entails a disutility of effort -or some depreciation of the use of capital-. Moreover, agents discount future effort with respect to effort now at time 0 -when the allocation has to be decided-, but they will not have any cost until their own task has to be performed. For instance, if agent i's task lasts -efficiently done- T_i hours and it is allocated to start at time $t_i^0 \in Z_+$, the disutility of the agent performing the task is -measured in some monetary unit-:

$$-\sum_{t=t_i^0}^{t=t_i^0+T_i-1} \beta_i^t$$

where the time discount applied by each agent (β_i, with $0 < \beta_i < 1$) is a function of the intrinsic difficulty of the task and of the laziness of the workers, which is part of their private information.[4]We further assume that disutility of effort done in period 0 (now) of any agent is known to the planner and equal to 1.[5] We call a *project planning economy* to a tuple

[4]Note that the discount used by task i, β_i may also be interpreted as the cost -depreciation, funding cost, etc.- of the capital used in that task.

[5]This is partly a simplifying assumption. For our results to hold it is needed that the planner either knows the (possibly different) agents' disutility of effort or at least know that there is some common upper bound on every agent's disutility of effort, which does not seem to be an unreasonable assumption. In that case, the mechanism we propose should be properly re-scaled.

of the form: $e = \langle N, P_i, T_i, \beta_i \ \forall i \rangle = \{N, e_1, ..., e_i, ..., e_n\}$, provided that the project is technologically feasible.[6] Let PPE be the set of all project planning economies. Given an economy $e \in PPE$, a feasible allocation -or simply, an *allocation*-, denoted by $x, y, z \in Z_+^{2(\# N)}$ is a vector that assigns a pair of *times* to each task or agent, $x = (t_1^0, t_1^1, t_2^0, t_2^1, ..., t_i^0, t_i^1, ..., t_n^0, t_n^1)$ with the following properties:

(a). $\exists i \in N$ such that $t_i^0 = 0$.

(b). $\forall i \in N, \ t_i^1 - t_i^0 \geq T_i$.

(c). $\forall i, j \in N, \ i \in P_j \longrightarrow t_j^0 \geq t_i^1$.

An allocation establishes a technologically feasible plan for the tasks to be carried out on time: t_i^0 stands for the planned starting time of task i and t_i^1 denotes the date after the termination date of task i. (a) means that some task should be initiated in period 0 (when the allocation is decided), (b) establish that no task should be allocated a working time smaller than the minimum time required to be done and (c) requires that no task can be started before all its preceding tasks have been completed. Let $FA(e)$ be the set of feasible allocations for economy e. Given an economy $e \in PPE$, a *critical path allocation* (CPA) is an allocation $\overline{x} = (\overline{t}_1^0, \overline{t}_1^1, ..., \overline{t}_i^0, \overline{t}_i^1, ..., \overline{t}_n^0, \overline{t}_n^1) \in FA(e)$ such that $\max_{i \in N} \overline{t}_i^1 \leq \max_{i \in N} t_i^1 \ \forall x = (t_1^0, t_1^1, ..., t_i^0, t_i^1, ..., t_n^0, t_n^1) \in FA(e)$, i.e., an allocation such that the period of time until the last task is finished is as small as possible within all the feasible allocations. Let us denote by $CPA(e)$ the set of $CPAs$ for an economy e. An *efficient* CPA or $CPA^+(e)$ is a critical path allocation such that no agent can be better off in any other critical path, i.e., $\overline{x} = (\overline{t}_1^0, \overline{t}_1^1, ..., \overline{t}_i^0, \overline{t}_i^1, ..., \overline{t}_n^0, \overline{t}_n^1) \in CPA(e)$ is efficient if $\forall i \in N, \ -\sum_{t=\overline{t}_i^1 - T_i}^{t=\overline{t}_i^1 - 1} \beta_i^t$ $\geq -\sum_{t=t_i^1 - T_i}^{t=t_i^1 - 1} \beta_i^t \ \forall \ x = (t_1^0, t_1^1, ..., t_i^0, t_i^1, ..., t_n^0, t_n^1) \in CPA(e)$.[7] Notice that under our assumptions, $CPA^+(e)$ is a singleton for each economy and the only efficient $CPA(e)$ corresponds to the allocation such that tasks are started as late as possible when there is some roominess.

There are several ways to find the efficient CPA for a given economy. The optimization problem can be formulated as one of linear programming or different algorithms can be applied to reach the solution. In what follows, we will use the following *strings* CPA^+ *algorithm:* given any economy e, the set of $CPA^+(e)$ comes from following the steps:

[6] Our definition of an economy includes the set of agents, that is allowed to vary within the range of admissible economies. Mechanisms that work for some set of PPE have to take into account that the number of agents could be different. The identity of agents and tasks makes the problems consistent.

[7] We assume that every task is efficiently performed and whenever more time than needed is provided in the allocation, only the last T_i hours will be used, and hence achieving the smaller possible disutility.

Step 1: Take tasks 1 to n. Assign the negative integers z_i^0 and $z_i^1 \in Z_-$: $z_i^1 = 0, z_i^0 = -T_i$.

Step 2: Take task $i = 1, ...n$ successively. In each sub-step i, do: $\forall j \in P_i$, redefine $z_j^1 = z_i^0$ and $z_j^0 = z_j^1 - T_j$.

Step 3: Repeat Step 2 until no new change emerges.

Step 4: $\forall i \in N$, redefine $\overline{t}_i^0 = z_i^0 + \min_{i \in N} z_i^0$ and $\overline{t}_i^1 = z_i^1 + \min_{i \in N} z_i^0$. $\overline{x} = (\overline{t}_1^0, \overline{t}_1^1, ..., \overline{t}_i^0, \overline{t}_i^1, ..., \overline{t}_n^0, \overline{t}_n^1) = CPA^+(e)$.

STOP.

Given any economy $e \in PPE$, we are interested in those tasks that are critical. We call a *critical string associated to the* $CPA^+(e)$ to a sequence of subsets of tasks $\{S^1(e), S^2(e), ..., S^k(e)\}$, with $S^h(e) \subseteq N$ and $S^h(e) \cap S^l(e) = \emptyset \; \forall h, l$ that can be found with the following algorithm using the $CPA^+(e)$ allocation:

Step 1: Take any $i \in N$ such that $\overline{t}_i^0 = 0$. Task i belongs to the first set of tasks in the sequence: $i \in S^1$.

Step 2: Take any $j \in N$ such that $\overline{t}_j^0 = \overline{t}_i^1$ for any $i \in S^1$ and $i \in P_j$. Then, j belongs to the second: $j \in S^2$.

....

Step k: Take any $i \in N$ such that $\overline{t}_i^0 = \overline{t}_j^1$ for any $j \in S^{k-1}$ and $j \in P_i$. Then, i belongs to the kth set (we assume that there are $k(e) \in \{1, ..., n\}$ subsets). Let us call $S(e)$ the union of the tasks belonging to any critical string associated to the $CPA^+(e)$, i.e., $S(e) = \bigcup_{h=1}^{k} S^h(e)$.

Notice that under our assumptions both algorithms work in selecting the unique $CPA^+(e)$ and the critical strings that define the minimum period of time needed for all the tasks -the project- to be finished. In what follows, we focus on implementing the $CPA^+(e)$. This is justified because that selection is the only one that maximizes the total welfare of the agents subject to the planner achieving her total time-minimizing objective -assuming that the planner receives a payoff strictly decreasing in the total duration of the project-. Moreover, other selections from the $CPA(e)$ like that which allocates tasks not belonging to the critical path to start as early as possible are even more difficult to implement because that rules defy more directly the agents' incentives by imposing an inefficient cost on them -starting early-.

Let us illustrate the former concepts with a simple example. Let us consider the following economy $e = (e_1, e_2, e_3) \in PPE$ such that $N = \{1, 2, 3\}$, $e_1 = (T_1, P_1, \beta_1) = (2, \{\emptyset\}, 0.5)$, $e_2 = (T_2, P_2, \beta_2) = (1, \{\emptyset\}, 0.7)$ and $e_3 = (T_3, P_3, \beta_3) = (4, \{1, 2\}, 0.1)$. Therefore, task 1 lasts for two hours, does not need to be preceded by any other and working one hour later entails half the disutility of effort now. Agent 2 is the less impatient and agent 3 is the most impatient of all three. The efficient CPA of $e \in PPE$ is

$$(\bar{t}_1^0, \bar{t}_1^1, \bar{t}_2^0, \bar{t}_2^1, \bar{t}_3^0, \bar{t}_3^1) = (0, 2, 1, 2, 2, 6).$$

The project's minimum duration is seven periods and the $CPA^+(e)$ allocation is represented in *Figure 1.*

[Insert *Figure 1* about here]

Moreover, there also exists a CPA allocation that is not efficient:
$(t_1^0, t_1^1, t_2^0, t_2^1, t_3^0, t_3^1) = (0, 2, 0, 1, 2, 6)$, depicted in *Figure 2*.

[Insert *Figure 2* about here]

A *Project Planning function (PPF)* is a function that assigns a feasible allocation to every admissible economy, i.e., $\varphi : PPE \longrightarrow FA(e)$. We say that a PPF is a *critical path PPF* if and only if $\forall e \in PPE,\ \varphi(e) \in CPA(e)$. An *efficient critical path PPF* is a PPF such that $\forall e \in PPE,\ \varphi(e) = CPA^+(e)$. Given an economy $e \in PPE$ and a PPF φ, $\forall i \in N$, $\varphi_i^0(e)$ will denote the component function relative to the starting time of agent $i's$ task and $\varphi_i^1(e)$ denotes the component function giving the allocation of the end of agent i's task.

The overall interest of the organization is modeled as the objectives of the *planner or principal.* We are assuming all along the paper, following the traditional PERT literature, that the cost of the project to the principal is proportional to the maximum amount of time spent for its completion. Let us assume that the planner wants to minimize the length of the project by selecting always CPA allocations. If the planner is perfectly informed about the relevant economy e, it should not be difficult for him to apply the PERT techniques or the linear programming version of them to find the CPA allocations. But we are not concerned in this paper about how to find these allocations, but about the possibility for the planner to achieve those outcomes when she is not informed about the technologies. We assume that each agent is better informed about the characteristics of his own task than the planner -or even any other agent-, so both the minimum duration of the task, T_i, the set of preceding tasks, P_i, and the time discount β_i are agent i's private information ($e_i = (T_i, P_i, \beta_i)$). The planner can only decide the final allocation based on the *reported technologies*, denoted as $\widehat{e}_i = (\widehat{T}_i, \widehat{P}_i, \widehat{\beta}_i)$ of each agent. Therefore, the planner is interested in designing a *direct revelation mechanism* such that the agents will have no incentive to lie about their true technologies.[8] However, the planner still knows some information about the relevant set of agents or tasks involved in the project, N, and the consistency of the whole project -$e \in PPE$-. Hence, we will assume that, given an economy e, the final allocation given the agents' reported technologies has to be technologically feasible, i.e., the planner can always find a lie if when the project is not technologically feasible, some task cannot be undertaken by an agent given his reported $\widehat{e}_i$. We assume that any detected lie can be so heavily punished that no agent is ever interested in reporting a set $\widehat{P}_i \subset P_i$.[9]Using an identical reasoning, no agent can ever use a lie such that $\widehat{T}_i < T_i$.[10] Notice that the agents can still lie by using $\widehat{P}_i \supset P_i$ and $\widehat{T}_i > T_i$ -trying to delay the completion of the project in order to avoid early costs- if they are not given other additional incentives. We allow for monetary transfers to the agents based on the agents' reported technologies, but we assume that the total amount to be transferred to the agents is a fixed quantity -the price of the project-. Now, we define the concept of an incentive compatible mechanism in this setting. Given two economies $e = \langle N, P_i, T_i, \beta_i \; \forall i \rangle = \{N, e_1, ..., e_i, ..., e_n\}$ and $e' = \langle N, P'_i, T'_i, \beta'_i \; \forall i \rangle = \{N, e'_1, ..., e'_i, ..., e'_n\}$, we write $e'_i \subset e_i$ whenever $P'_i \subseteq P_i$ and $T'_i \leq T_i$ and $e' \subset e$ when $e'_i \subset e_i \; \forall i \in N$. We shall also make use of the following well-known notation to avoid large expressions: $e = (N, e) = (N, e_S, e_{-S})$, $\forall S \subseteq N$, and in particular, for $S = \{i\}$, $e = (N, e_i, e_{-i})$. [11]

Definition 1 *A* ***mechanism*** M *is a set of transfer functions* $\{w_i \; \forall i \in N\}$ *of the kind* $w_i : PPE \longrightarrow E$ *for every set of agents* $N \subseteq \overline{N}$.

Notice that the above definition entails that mechanisms are direct: the only information used by the planner to allocate transfers are the agents' revealed technologies -their types-.

Definition 2 *A mechanism* $M = \{w_i \; \forall i \in N\}$ ***implements an efficient critical path*** PPF φ *if the following holds:*
$\forall e \in PPE, \; \forall i \in N, \; \forall e'_i \supset e_i,$

$$w_i(e) - \sum_{t=\varphi_i^1(e)-T_i}^{t=\varphi_i^1(e)-1} \beta_i^t \geq w_i(N, e'_i, e_{-i}) - \sum_{t=\varphi_i^1(N,e'_i,e_{-i})-T_i}^{t=\varphi_i^1(N,e'_i,e_{-i})-1} \beta_i^t$$

We also say that these mechanisms are ***strategy-proof***.

[8] A direct revelation mechanism asks the agents about their *types*.

[9] Notice that actually any agent that reports a narrower set of preceding tasks, either leads to her detection or cannot change the allocation, so it is not individually rational to do so and we can therefore eliminate all those irrelevant strategies.

[10] Basically, we are assuming that the planner can monitor when the agents start their tasks and when they finish.

[11] This definitions hold irrespective of $\beta_i \; \forall i \in N$.

A mechanism implements the efficient critical path PPF if any agent, by reporting a different technology cannot improve her net payoff -the transfer received minus the disutility of effort-, and this whatever her true technology is and regardless the others' reported technologies. Therefore, we are interested in a strong incentive compatibility property to hold.

Definition 3 *Given any positive number* $C > 0$, *a mechanism* $M^C = \{w_i \; \forall i \in N\}$ *is* ***balanced*** *if* $\forall e \in PPE$, $\sum_{i=1}^{n} w_i(e) = C$.

This property imposes that the transfer or reward scheme designed by the planner has to be balanced -the whole budget funding the project should be distributed among the agents involved in it-.

Definition 4 *A balanced mechanism* $M^C = \{w_i \; \forall i \in N\}$ *is* ***invariant to the project size*** *if* $\forall e = (N, P_i, T_i, \beta_i) \in PPE, \forall \lambda > 0$, *if* $e' = (N, P_i, \lambda T_i, \beta_i)$, $M^{\lambda C} = \{w_i(e') \; \forall i \in N\} = \{\lambda w_i(e) \; \forall i \in N\}$.

A balanced mechanism is invariant to the project size if the transfers are proportionally affected by a proportional re-scaling of the project; for example, doubling the tasks minimum durations joint with the project value C should double every agent transfers. This property may be desirable because it introduces some fairness criterion to the sharing rule when the project is re-scaled: the agent's relative payoffs do not change when we measure the resource "time" in hours, minutes, days or months.

Definition 5 *Given any reservation utility* $\overline{U} \in Z$, *a mechanism* $M = \{w_i \; \forall i \in N\}$ *implementing* PPF φ *is* ***individually rational*** *if* $\forall e \in PPE$, $\forall i \in N$,

$$w_i(e) - \sum_{t=\varphi_i^1(e)-T_i}^{t=\varphi_i^1(e)-1} \beta_i^t \geq \overline{U}$$

Individual rationality requires the payoffs to be designed such that for every economy no agent gets such a small payoff that might lead the agent to quit the project if possible. An implicit simplifying assumption is that a net utility of $\overline{U}$ constitutes the common agents' reservation utility threshold for project acceptance.

Definition 6 *Given a* PPF φ, *a mechanism* $M = \{w_i\}$ *is* ***innovation-monotonic*** *if the following holds:*
$\forall e \in PPE$, $\forall i \in N$, $\forall e_i' \supset e_i$,

$$w_i(e) - \sum_{t=\varphi_i^1(e)-T_i}^{t=\varphi_i^1(e)-1} \beta_i^t \geq w_i(N, e_i', e_{-i}) - \sum_{t=\varphi_i^1(N,e_i',e_{-i})-T_i'}^{t=\varphi_i^1(N,e_i',e_{-i})-1} \beta_i^t$$

A mechanism is innovation-monotonic if an innovation that makes any agent more "productive" cannot make him be worse off. Notice that every mechanism that implements the critical path PPF has to be innovation monotonic given the efficient critical path PPF. This property constitutes an additional justification for both implementing the efficient critical path PPF and using strategy-proof mechanisms.

Definition 7 *A PPF φ is **anonymous** if for all $N \subseteq \overline{N}$, $e \in PPE$ and any permutation $\sigma(N)$ of the agents, the following holds: $\varphi_i^k(N, e_i, e_{-i}) = \varphi_{\sigma(i)}^k(N, e_{\sigma(i)}, e_{-\sigma(i)})$, for $k = \{0, 1\}$ and for all $i \in N$.*

This requirement is an obvious fairness property that excludes $PPFs$ that take into account the agents' names and not only their technology. Notice that the efficient critical path PPF is always anonymous.

Definition 8 *A mechanism $M = \{w_i\}$ is **anonymous** if for all $N \subseteq \overline{N}$, $e \in PPE$ and any permutation $\sigma(N)$ of the agents, the following holds: $w_i(N, e_i, e_{-i}) = w_{\sigma(i)}(N, e_{\sigma(i)}, e_{-\sigma(i)})$ for all $i \in N$.*

Again, anonymity establish that the information about the agents' names is not used to allocate the transfers.

Now, we will proceed with the main results in the paper.

3 Results

Our first possibility result proves the existence of anonymous and balanced mechanisms implementing the efficient critical path PPF. To prove the theorem, we still need some definitions.

Let us call a *string* associated to agent $i \in N$ and economy $\widehat{e} \in PPE$, denoted as $R^i(\widehat{e})$, to the set of agents obtained with the following algorithm:

Step 1: take a single $j \in \widehat{P}_i$. $j \in R^i(\widehat{e})$. If there is no such an agent, $R^i(\widehat{e}) = \emptyset$ and the process stops.

Step 2: take a single $k \in \widehat{P}_j$. $k \in R^i(\widehat{e})$. If there is no such an agent, the process stops.

Step 3: take any $h \in \widehat{P}_k$. $h \in R^i(\widehat{e})$. If there is no such an agent, the process stops.

....

Eventually, since N is finite and technologies are feasible, the algorithm stops and it is clear that any string is such that $R^i(\widehat{e}) \subseteq N \backslash \{i\}$. Now, take the union of all the strings associated to agent $i \in N$ and declared technology $\widehat{e} \in PPE$, i.e., the set of agents with tasks that should necessarily precede i. Let us denote as $r^i(\widehat{e})$ the total number of agents belonging to any string associated to agent $i \in N$ and technology $\widehat{e} \in PPE$, i.e., $r^i(\widehat{e}) = \# \bigcup R^i(\widehat{e})$ $\forall i \in N$, $\forall \widehat{e} \in PPE$. Notice that $r^i(\widehat{e}) = 0$ if and only if $\widehat{P}_i = \emptyset$. Let us call *terminal agents* to the set of agents such that have not declared to have any preceding task and let us denote them as $S(\widehat{e})$, i.e., $\forall \widehat{e} \in PPE$, $S(\widehat{e}) = \left\{ i \in N \text{ s.t. } \widehat{P}_i = \emptyset \right\}$, or identically, $S(\widehat{e}) = \{ i \in N \text{ s.t. } r^i(\widehat{e}) = 0 \}$. Notice also that there exists always at least one terminal agent for every feasible technology. Now, we state our main result in this chapter:

Theorem 1 *There exist anonymous, balanced and invariant to the project size mechanisms implementing the efficient critical path PPF.*

Proof. Let us consider the following mechanism: $\forall \widehat{e} \in PPE$, $\forall N \in \overline{N}$, $\forall i \in N$, $w_i(N, \widehat{e}_1, ..., \widehat{e}_n) = \dfrac{C}{n} +$

$$\begin{cases} -(r^i(\widehat{e})+1)\widehat{T}_i & \textit{if } i \notin S(\widehat{e}) \textit{ and } \#S(\widehat{e}) > 1 \\ \dfrac{\sum_{j \notin S(\widehat{e})} (r^j(\widehat{e})+1)\widehat{T}_j}{\#S(\widehat{e})} - \widehat{T}_i + \dfrac{\sum_{j \in S(\widehat{e}) \setminus \{i\}} \widehat{T}_j}{\#S(\widehat{e}) - 1} & \textit{if } i \in S(\widehat{e}) \textit{ and } \#S(\widehat{e}) > 1 \\ -(r^i(\widehat{e})+1)\widehat{T}_i + \dfrac{\sum_{j \in S(\widehat{e})} \widehat{T}_j}{n-1} & \textit{if } i \notin S(\widehat{e}) \textit{ and } \#S(\widehat{e}) = 1 \\ \sum_{j \notin S(\widehat{e})} (r^j(\widehat{e})+1)\widehat{T}_j - \widehat{T}_i & \textit{if } i \in S(\widehat{e}) \textit{ and } \#S(\widehat{e}) = 1 \end{cases}$$

In words, starting from an equal sharing of C, this mechanism tax every agent that has declared to have at least one preceding task to pay her own declared duration $(r^i(\widehat{e}) + 1)$ times. The agents that have declared not to have any preceding task share equally the total tax paid by the formers, pay a quantity equal to their total declared duration time and receive a positive transfer equal to the total tax paid by her partners in $S(\widehat{e})$ divided by $(\#S(\widehat{e}) - 1)$. If just one terminal agent exist, her $\widehat{T}_i$ tax is distributed evenly among the non-terminal agents.

It is easy to check that this is an anonymous mechanism: any permutation of the names of the agents only permute their payoffs and no information about the agents' names is used in the mechanism. It is also an invariant to the project size mechanism, since every agent transfer in every circumstance is proportional to both any project size C and the reported duration of the agents. Moreover, it is always balanced: for any reported $\widehat{e} \in PPE$, adding up the agents' transfers yields: $\sum_{i=1}^{n} w_i(N, \widehat{e}_1, ..., \widehat{e}_n) =$

$$= \sum_{i \notin S(\widehat{e})} \left[\frac{C}{n} - (r^i(\widehat{e}) + 1)\widehat{T}_i \right] +$$

$$+\sum\nolimits_{i\in S(\widehat{e})}\left[\frac{C}{n}+\frac{\sum_{j\notin S(\widehat{e})}(r^j(\widehat{e})+1)\widehat{T}_j}{\#S(\widehat{e})}-\widehat{T}_i+\frac{\sum_{j\in S(\widehat{e})/\{i\}}\widehat{T}_j}{\#S(\widehat{e})-1}\right]=C \text{ if there}$$

are at least two terminal agents and

$$\sum\nolimits_{i=1}^{n} w_i(N,\widehat{e}_1,...,\widehat{e}_n)=\sum\nolimits_{i\notin S(\widehat{e})}\left[\frac{C}{n}-(r^i(\widehat{e})+1)\widehat{T}_i+\frac{\widehat{T}_k}{n-1}\right]+$$

$$+\left[\frac{C}{n}+\sum\nolimits_{j\notin S(\widehat{e})}(r^j(\widehat{e})+1)\widehat{T}_j-\widehat{T}_k\right]=C \text{ if just one agent -}k\text{- is terminal.}$$

To prove that it implements the efficient critical path PPF, we shall compare the payoff each agent obtains by both reporting the truth and lying for any possible $\widehat{e}_{-i}$.

Case 1: $i\notin S(e_i,\widehat{e}_{-i})$. We have to distinguish two cases:

Case 1.1.: $\#S(e_i,\widehat{e}_{-i})>1$.

In this case, reporting a technologically feasible task duration longer than the true one T_i, say $\widehat{T}_i>T_i$ will not affect her classification as $i\notin S(\widehat{e}_i,\widehat{e}_{-i})$, since $\widehat{P}_i=P_i\neq\emptyset$ and $r^i(e_i,\widehat{e}_{-i})=r^i(\widehat{e})$. Hence, agent $i\in N$, by reporting the truth T_i, would obtain the following payoff: $w_i(N,e_i\widehat{e}_{-i})-\sum\nolimits_{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-T_i}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-1}\beta_i^t=\frac{C}{n}-(r^i(\widehat{e})+1)\widehat{T}_i-\sum\nolimits_{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-T_i}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-1}\beta_i^t$

By declaring $\widehat{T}_i=T_i+1$, agent i is enlarging the critical path in 1 hour and, given its true β_i, can save

$$\sum\nolimits_{t=\varphi_i^1(N,e_i,\widehat{e}_{-i})-T_i}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-1}\left(\beta_i^t-\beta_i^{t+1}\right)=\beta_i^{\varphi_i^1(N,e_i\widehat{e}_{-i})-T_i}-\beta_i^{\varphi_i^1(N,e_i\widehat{e}_{-i})}>0 \qquad (1)$$

monetary units. The cost of obtaining that benefit is one monetary unit, but notice that the benefit can never outweigh its cost. Now, notice that by declaring $\widehat{T}_i=T_i+2$, the benefit will be $\sum\nolimits_{t=\varphi_i^1(N,e_i,\widehat{e}_{-i})-T_i}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-1}\left(\beta_i^t-\beta_i^{t+2}\right)=\sum\nolimits_{t=\varphi_i^1(N,e_i,\widehat{e}_{-i})-T_i}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})-1}\left(\beta_i^t-\beta_i^{t+1}\right)+$

$+\sum\nolimits_{t=\varphi_i^1(N,e_i,\widehat{e}_{-i})-T_i+1}^{t=\varphi_i^1(N,e_i\widehat{e}_{-i})}\left(\beta_i^t-\beta_i^{t+1}\right)<2$, which is the cost of declaring the task to be 2 hours longer, and so on, so $\forall\widehat{T}_i>T_i$, agent i -actually, any agent- can never find that lying is more profitable than saying the truth.

By reporting a technologically feasible preceding tasks set $\widehat{P}_i\supset P_i$, agent $i\in N$ cannot change neither her non-terminal category nor the fact that $S(\widehat{e}_i,\widehat{e}_{-i})>1$, so it holds that $\forall\widehat{P}_i\supset P_i,\ i\notin S(e_i,\widehat{e}_{-i})\Rightarrow i\notin S(\widehat{e}_i,\widehat{e}_{-i})$ and she can get a benefit of at most a delay such that $\sum_{j\in\widehat{P}_i}\widehat{T}_j-\sum_{j\in P_i}\widehat{T}_j\geq 0$. By reporting $\widehat{P}_i=P_i\cup\{k\}$, for any $k\in N\backslash\{i\}$, agent $i\in N$ cannot in any case get a direct benefit as large as $\widehat{T}_i$ -see the argument above-, and this only in the case of being part of the critical path after the lie -and maybe before the lie-, in which case it holds that $r^i(\widehat{e})\geq r^i(e_i,\widehat{e}_{-i})+1$. Notice that both arguments are valid for possible lies that mix both declaring longer duration time and a larger preceding tasks set.

Finally, revealed $\widehat{\beta}_i$'s do not enter into the definition of the mechanism, so there is no point in lying about them. Notice, however, that no mechanism implementing the efficient CPA can make non-trivial use of information about the revealed β's.

Case 1.2.: $\#S(e_i, \widehat{e}_{-i}) = 1$. In this case, no lie of any form: $\widehat{T}_i > T_i$ or $\widehat{P}_i \supset P_i$ can change the facts of $i \notin S(\widehat{e}_i, \widehat{e}_{-i})$ and $\#S(\widehat{e}_i, \widehat{e}_{-i}) = 1$, so a transfer of the form: $w_i(N, \widehat{e}_1, ..., \widehat{e}_n) = \dfrac{C}{n} - (r^i(\widehat{e}) + 1)\widehat{T}_i + \dfrac{\sum_{j \in S(\widehat{e})} \widehat{T}_j}{n-1}$ is unavoidable. The third term is independent of the reported lie, so an identical reasoning to that of Case 1.1. applies to this case as well and there are no incentives to lie.

Case 2: $i \in S(e_i, \widehat{e}_{-i})$.

In this case, if agent i is terminal for that technology, any lie consisting in declaring a longer duration $\widehat{T}_i > T_i$ cannot neither change her terminal status nor alter the set $S(e_i, \widehat{e}_{-i}) = S(\widehat{e}_i, \widehat{e}_{-i})$, but can delay the project at most $\tau = \widehat{T}_i - T_i$ hours. The benefit for one hour delay is given by (1) and the same reasoning ensures that no such lie can ever yield a direct benefit of min $\{\tau, T_i\}$ monetary units. In both cases of $\#S(e_i, \widehat{e}_{-i}) > 1$ or $\{i\} = S(e_i, \widehat{e}_{-i})$, since both $r^j(\widehat{e})$ and $\widehat{T}_j$ for all $j \in N \backslash \{i\}$ cannot change with any technologically admissible lie, the cost of the lie is exactly $\widehat{T}_i > T_i$ monetary units -see the payoff function-, which is always bigger than the cost.

If agent $i \in N$ declares to have some preceding tasks, she will always change her own terminal status to non-terminal, so $\forall \widehat{P}_i \supset P_i = \emptyset, \;\; i \in S(e_i, \widehat{e}_{-i}) \Rightarrow i \notin S(\widehat{e}_i, \widehat{e}_{-i})$. There are again two possibilities here:

Case 2.1.: $\{i\} = S(e_i, \widehat{e}_{-i})$.

If $S(e_i, \widehat{e}_{-i})$ is a singleton -i.e., i is the only terminal agent for e_i and $\widehat{e}_{-i}$-, there is no technologically feasible lie available to agent i by declaring a non-empty $\widehat{P}_i$, since for any $k \in N$ such that $k \in \widehat{P}_i$, there will necessarily be a sequence of tasks $j, l, z \in N$ such that $j \in \widehat{P}_k, l \in \widehat{P}_j, ... \; z \in \widehat{P}_i$ and we get a cycle, so there is no possibility in this case of getting the non-terminal agents payoff.

Case 2.2.: $\#S(e_i, \widehat{e}_{-i}) > 1$. In this case, agent i may get a non-terminal agent status by declaring $\widehat{P}_i \neq \emptyset$ and $\widehat{P}_i \subseteq S(e_i, \widehat{e}_{-i}) \backslash \{i\}$. There are two possibilities now:

Case 2.2.1.: $\#S(\widehat{e}_i, \widehat{e}_{-i}) > 1$. In this case, observe that for every possible $\widehat{e} \in PPE$, the total payoff of any terminal agent is always bigger than that of any non-terminal agent, i.e., the following holds:

$$\frac{C}{n}+\frac{\sum_{j\notin S(\widehat{e})}(r^j(\widehat{e})+1)\widehat{T}_j}{\#S(\widehat{e})}-\widehat{T}_i+\frac{\sum_{j\in S(\widehat{e})/\{i\}}\widehat{T}_j}{\#S(\widehat{e})-1}>\frac{C}{n}-(r^i(\widehat{e})+1)\widehat{T}_i. \quad (2)$$

Notice that, although the first term is cancelled in both sides, since $r^i(\widehat{e})\geq 1$ for any non-terminal agent, the second term is always at least $\widehat{T}_i$ monetary units bigger on the right than on the left -in absolute terms-, while the third term on the left is always positive, so by lying declaring a non-empty $\widehat{P}_i$, the cost in terms of the transfer is always bigger than $r^i(\widehat{e})\widehat{T}_i$. On the other hand, the direct benefit of getting a delay of at most $\sum_{j\in\widehat{P}_i}\widehat{T}_j$ -the maximum possible for some $\widehat{e}_{-i}$- is again always bounded by $\min\left\{\sum_{j\in\widehat{P}_i}\widehat{T}_j,T_i\right\}$ monetary units, and the loss in terms of the transfer will be -as was argued above- bigger than $r^i(\widehat{e})\widehat{T}_i$, so since $r^i(\widehat{e})\widehat{T}_i>\min\left\{\sum_{j\in\widehat{P}_i}\widehat{T}_j,T_i\right\}$ for $\widehat{T}_i>T_i$, true terminal agent i has no incentive to declare to be non-terminal.

Case 2.2.2.: $\#S(\widehat{e}_i,\widehat{e}_{-i})=1$. In this case, agent i lies in the following way: there are other terminal agent initially and by reporting this agent to precede herself makes this agent the only terminal agent after the lie. Henceforth, agent i changes status from terminal to non-terminal and gets an additional bonus by getting a share of the terminal agent tax -see *Figure 3* below-. Agent i will not have an incentive to lie if, for every $\widehat{e}\in PPE$ and e_i, the transfer obtained by reporting her true technology -left hand side of inequality (3) below- exceeds the transfer obtained by lying plus the maximum possible direct gain from lying, i.e., T_i -in brackets below- as we have seen above, so if the new terminal agent is $k\in N$, $i\in N$ will not lie if:

$$\frac{C}{n}+\frac{\sum_{j\notin S(\widehat{e})}(r^j(\widehat{e})+1)\widehat{T}_j}{2}-\widehat{T}_i+\widehat{T}_k\geq\frac{C}{n}-(r^i(\widehat{e})+1)\widehat{T}_i+\frac{\widehat{T}_k}{n-1}+[T_i]\,. \quad (3)$$

If $n=2$, expression (3) is true when $-\widehat{T}_i+\widehat{T}_k\geq-2\widehat{T}_i+\widehat{T}_k+T_i\Rightarrow\widehat{T}_i\geq T_i$, which is always true by assumption. If $n>2$, for any $\widehat{e}\in PPE$, the left hand side of (3) becomes bigger as the second term is positive and the right hand side becomes smaller as the second term can only amount to either $-2\widehat{T}_i$ or less -until reaching a minimum of $-(n-1)\widehat{T}_i$-. Moreover, the third term on the right hand side becomes always smaller as the number of agents increase. Obviously, expression (3) holds for every possible economy and lie compatible with the case.

Since we have checked that in every possible economy no lie can be ever profitable, the mechanism is strategy-proof and the proof is complete.

■

[Insert *Figure 3* about here]

Given the above possibility result, the next obvious step is to refine the set of desirable mechanisms by imposing additional properties and test the robustness of the above result to the introduction of other desirable properties in this context. We opt for individual rationality since it is likely to be important in real-life situations. Our next result proves the impossibility of designing anonymous, balanced and individually rational mechanism implementing the efficient critical path PPF.

Theorem 2 *There do not exist anonymous, balanced and individually rational mechanisms implementing the efficient critical path PPF for any C and $\overline{U}$.*

Proof. Let us fix any reservation utility threshold $\overline{U} \in Z$ and any project size $C > 0$. Let us call $g(x) \in Z \ \forall x \in E$ to the function that assigns the minimum integer between the two closest integers to any real number -the smallest integer between the two closest or the *integer part* of $x \in E$-. We assume that a mechanism is balanced, anonymous and strategy-proof and shall prove that no such a mechanism can ever be individually rational as well. Now, we consider two cases:

Case 1: $\frac{C}{2} < \overline{U} + 1$. In this case, consider the following admissible economy (N, e): $N = \{1, 2\}$, $P_i = \emptyset$, and $T_i = 1 \ \forall i = 1, 2$, regardless of the agents' β's. By anonymity and balance, $w_i(N, e) = \frac{C}{2}$, so the total payoff each agent receive is $\frac{C}{2} - 1$, which is strictly smaller than $\overline{U}$ by assumption, so individual rationality is violated in this case.

Case 2: $\frac{C}{2} > \overline{U} + 1$. We have to prove that within that range of the parameters C and $\overline{U}$, we can find an economy such that individual rationality is violated, provided that we work within anonymous, balanced and strategy-proof mechanisms. Let us consider the following economy $(N, e) : N = \{1, 2\}$, $P_i = \emptyset$, $T_i = g(C - 2\overline{U})$ and β_i sufficiently close to 1 $\forall i = 1, 2$. Notice that since $\frac{C}{2} > \overline{U} + 1$ within the assumption, $T_i \geq 2$ and is an admissible integer duration, so $e \in PPE$. Now, let us consider any economy (N, e') such that the only change with respect to $e \in PPE$ is T_1' being very large -tending to infinity-. This economy is also feasible. Observe now that agent 1 in the true economy e will lie and declare T_1' if the total payoff obtained by agent 1 after the transfer is made in $e' \in PPE$ is bigger than her total payoff in economy e, and since we assume the mechanism to be strategy-proof, we have to impose the following condition to hold: $w_i(N, e_i', e_{-i}) - \sum_{t=\varphi_i^1(N,e_i',e_{-i})-T_i}^{t=\varphi_i^1(N,e_i',e_{-i})-1} \beta_i^t \leq w_i(e) - \sum_{t=\varphi_i^1(e)-T_i}^{t=\varphi_i^1(e)-1} \beta_i^t$. Note that since β_1 is sufficiently close to 1 and $T_1 < \infty$ but still T_1' is infinitely longer than T_1 by construction, the expression above can be written as follows for some appropriate selection of both T_1' and β_1 :

$$w_1(N, T_1', \emptyset, e_2) - \varepsilon_1 \leq \frac{C}{2} - T_1 - \varepsilon_2$$

for some $\varepsilon_1, \varepsilon_2 > 0$ but as close to 0 as desired, so there exist admissible economies such that strategy-proofness, anonymity and balance require some $w_1(N, T_1', \emptyset, e_2) - \varepsilon_1$ to be smaller than $\frac{C}{2} - T_1$. Substituting T_1, we obtain: $w_1(N, T_1', \emptyset, e_2) - \varepsilon_1 \leq \frac{C}{2} - g(C - 2\overline{U}) < \frac{C}{2} - (C - 2\overline{U})$. Finally, notice that the right hand side of the last inequality can be written as $\left[\overline{U} - \frac{C}{2}\right] + \overline{U}$, which is always smaller than $\overline{U} - 1$ under our assumptions, so the following holds: $w_1(N, T_1', \emptyset, e_2) - \varepsilon_1 < \overline{U}$. But notice that $w_1(N, T_1', \emptyset, e_2)$ still applies if the true economy was e', in which case the direct cost of undertake the true task T_1' on the left hand side of the last inequality will be much larger than the negligible ε_1, so for that true economy the net agent 1's payoff will be much smaller than $\overline{U}$, so no mechanism can be individually rational for any $\overline{U}$ and any $C > 0$. ■

Theorem 2 obliges us to either restrict ourselves to a subset of feasible technologies or to abandon some other property. However, in some circumstances individual rationality is not very important.

4 Concluding Remarks

In this paper we have explored the possibility of designing strong incentive compatible mechanisms in a particular production setup: the well-known network production problem, and to the solutions defined by methods like the CPM and the PERT when transfers to the agents are possible. Although the problem is similar to other environments like the public goods problem, the opportunities for exploiting the asymmetry on the distribution of the private information is very different in this context, and it is not surprising that we obtain different results when compared to those obtained by Groves and Clarke -see, for example, Groves [4] and Groves [5]- for the public goods provision problem. Assuming that the agents' payoffs are quasi-linear on the part of the transfers, we find simple strategy-proof, anonymous and invariant to the project size mechanisms implementing the PERT that are balanced as well, so complete efficiency is achieved provided that we include the planner in the definition of Pareto-optimality. Furthermore, if we add other plausible property like assuming individually rational payoffs, we obtain an impossibility result (*Theorem 2*). The possible ways-out in this case include imposing constraints on the domain of possible economies or relaxing the equilibrium concept used, although perhaps the most promising approach to escape from the impossibility could be imposing reasonable bounds on the technologies allowed to be considered by the planner, like some maximum time for any task to be completed. The nature of the proofs -and this also includes the Groves-Clarke mechanism in the public goods provision problem- points to this lack of bounds as the key factor behind the impossibility results.

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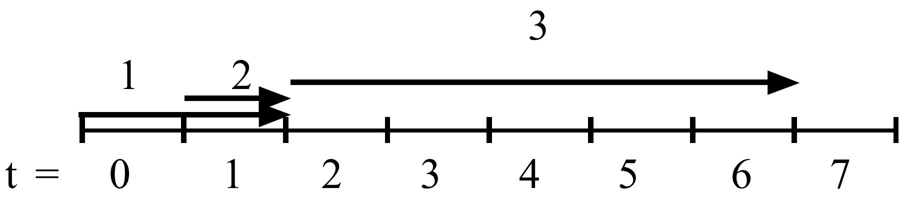

Figure 1:

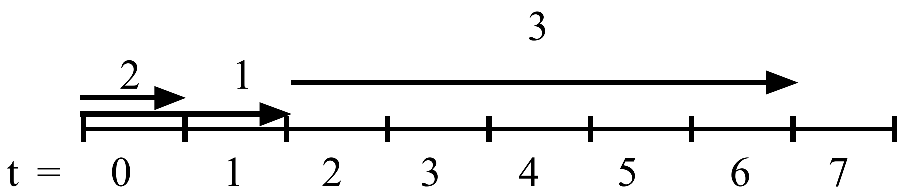

Figure 2:

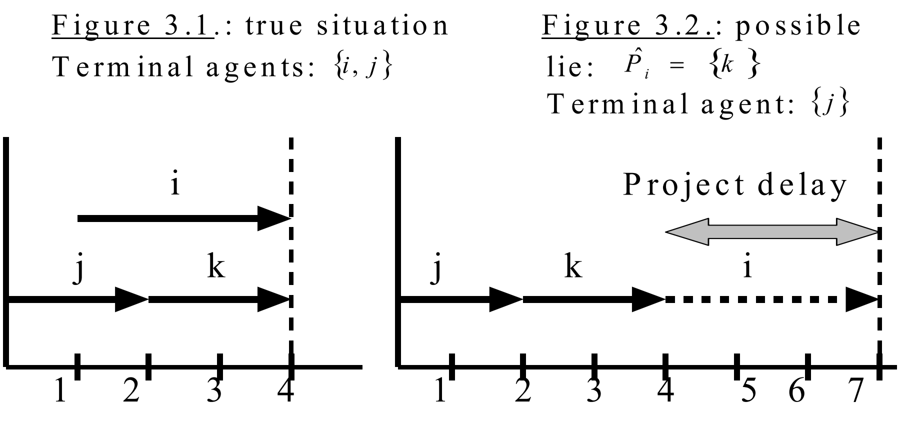

Figure 3: