Identifying endogenous fiscal policy rules for macroeconomic models*

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Abstract

In this paper, we present a model-based method for identifying fiscal closure rules in stochastic macroeconomic models. The methodology is based on the stability analysis of the model at hand, with an endogenous derivation of a reaction on the part of the fiscal authority to state variables in the model. The rule achieves the dual aim of imposing solvency on the fiscal sector and generating a state-contingent dynamic adjustment in a framework consistent with the properties of the model. Up to now, fiscal rules in leading large-scale macroeconomic forecasting models have been imposed exogenously, and in this sense are not necessarily compatible with the formulation of other sectors of these models. An example of the derivation procedure, including some illustrative results, is provided using a small calibrated macro model.

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1 Introduction

In large-scale macroeconomic forecasting models, including those used by leading international institutions, the modelling of the fiscal sector involves some type of fiscal closure rule. This rule, or fiscal reaction function, serves a dual purpose. First, and most importantly, its inclusion is used to generate solvency for the fiscal sector, guaranteeing that the intertemporal budget constraint of the government is satisfied and generating model closure. That is, the possibility of an unstable or explosive path for the government debt ratio is ruled out, and as a result agents in the model are willing to hold public debt. Second, the rule embodies some behavioural elements regarding the intertemporal behaviour of governments – and how adjustments of fiscal variables are made vis-à-vis their steady state values in the face of shocks or policy changes. In this sense, the time path of adjustments in fiscal and other variables in the model are influenced by the formulation of the fiscal rule.

The formulation of these rules can entail significant economic consequences for the results of model simulations and forecasts. The specification and calibration of a fiscal rule can affect not only fiscal policy variables, but also key economic and financial variables, given that agents' decisions (that determine the allocation of output) depend to some extent on the time paths of fiscal variables. The forecasts and policy simulations based on these models are used in many instances as an important input into policy decision-making. Consequently, the formulation and implementation of fiscal rules has wide-ranging implications. Yet, the literature surrounding the formulation of fiscal rules has thus far received considerably less attention than the literature on monetary policy rules.

In general, the existing fiscal rules employed in leading macroeconomic forecasting models are imposed exogenously, and tend to involve backward-looking governments even when households, firms and the monetary authority are now very often modelled in an optimising forward-looking framework. In models with optimising forward-looking agents, the intertemporal fiscal rule should, in principle, also be modelled in a forward-looking way. As noted in Woodford (1999), for instance, the forward-looking behaviour that follows automatically from an optimising private sector implies that the evolution of its goal variables depends not only upon its current actions, but also upon how the private sector expects policy to be conducted in the future. In addition, a multitude of different formulations of a reaction function can rule out unstable debt paths in model simulations, and the criteria used in choosing among them is not entirely clear-cut. Although several rules used in practice to close macroeconomic forecasting models tend to share similar features, they remain quite diverse in their specification and calibration. Some recent studies have found through standardised simulations that changing the specification and calibration of these rules can significantly affect model simulation results, for example, see Mitchell *et al.* (2000), Bryant and Zhang (1996a and 1996b) and Barrell *et al.* (1994).

Ideally, a fiscal rule would be developed optimally. Such an alternative, presumably involving the strategy pursued in the optimal taxation literature of deriving optimal rules endogenously, would entail some desirable theoretical characteristics in terms of economic content. This, however, would likely involve the use of a "Ramsey"-type of government, which is generally limited to the analysis of fairly simplified economies. Consequently, such a strategy would be infeasible for large-scale macro models, given the level of complexity of the economy in these models and their level of disaggregation. Several papers highlight the complexities surrounding the solution of Ramsey problems, such as Chari and Kehoe (1999) and Chari *et al.* (1994). In particular, solving a dynamic optimisation model in which the government maximises agents' utility subject to all Euler conditions in agents' problems would be cumbersome, if not impossible, with the level of disaggregation in large-scale macroeconomic forecasting models. Moreover, notwithstanding the usefulness of this framework in deriving theoretical predictions, it is not clear that it would be useful for simulation and forecasting of actual economies.

In this paper, after reviewing the standard practice for guaranteeing debt stability via fiscal adjustment, we show that for model closure, the relevant fiscal rule can be specified and calibrated endogenously by means of standard stability analysis theory for rational expectations models as presented in Sims (2000) or Blanchard and Khan (1980). The rule is consistent with the optimising behaviour already inherent in the model. It embodies the pertinent information contained in the economic model, including expectations of future values of relevant variables and, additionally, policy reactions related to the different stochastic shocks affecting the economy. We show how a state-contingent policy rule relating fiscal instruments to state variables, different shocks affecting the economy and expectations of future developments of the economy can be derived on the basis of the existing setup of a model. Moreover, its formulation and implied behaviour can be rationalised in the framework of a model in which the policy authority internalises the need for debt to be valued – see, for instance, Barro (1979). In the absence of a monetary authority monetising shocks to debt, which is the case in most industrialised countries, a fiscal authority can be thought of as reacting to innovations affecting debt through the adjustment of budgetary items in order to guarantee debt sustainability. Indeed, some empirical evidence supports this notion (see, for instance, Bohn, 1998 and Kilpatrick, 2001).[1]

Normally the stable solution to dynamic stochastic rational expectations models, for instance, those presented in Cooley and Prescott (1995) or Marimon and Scott (1999), takes the form of a saddle-path equilibrium. The saddle-path equilibrium is selected by pinning down the stability conditions attached to unstable eigenvalues or imposed by means of appropriate final conditions as in Julliard *et al.* (1998). Based on the stability conditions, the state-contingent nature of the resulting rule would imply an adjustment profile consistent with the dynamic adjustment process of other agents in the model, and the rule would be forward-looking insofar as the model is constructed in this way. The coefficients of the fiscal policy rule would be a function of the *deep* parameters of the model, so that the Lucas critique would not apply directly to the fiscal rule when performing policy experiments. Although we illustrate the methodology using a simple standard macroeconomic model, the proposed identification methodology is general enough to be applied to larger-scale macroeconomic models. Once the fiscal closure rule is identified, and in order to solve a large-scale model, one should move to any of the solution methods available in the literature, for example, the ones in Juillard *et al.* (1998), McAdam and Hughes Hallett (1999), Gaspar and Judd (1997) or Marimon and Scott (1999).

The paper is organised as follows. In Section 2, we discuss the need to include fiscal closure rules in macroeconomic models, and present a general outline of the standard strategy to derive existing fiscal rules. In Section 3 we offer the rationale for an alternative specification in the form of an endogenous fiscal rule. In order to elucidate our identification procedure clearly, we proceed in this Section from a specific to a general framework, initially using a standard business cycle model as guidance. We then illustrate the derivation of such a rule

[1] We interpret evidence of this type purely in a positive sense, that is, we do not enter here into normative issues regarding rules versus discretion in the actual implementation of fiscal policy.

in Section 4 on the basis of this model. Finally, in Section 5 we summarise the properties of the proposed rules and discuss some avenues for further research.

2 The government budget constraint and the need of fiscal rules

In establishing the basis for including a fiscal rule in a macroeconomic model, one must look first to the government budget constraint, which takes the following standard form in discrete time:

$$\frac{B_t}{P_t} + \tau_t = g_t + R_{t-1}\frac{B_{t-1}}{P_t} \tag{1}$$

where B_t stands for time–t nominal debt, g_t is real primary spending, τ_t tax collection in real terms, and $\mathtt{I_t} \equiv 1 + i_t$ the nominal interest rate on bonds. Simply put, this condition states that the government has to issue debt to pay for spending in excess of tax collection.[2] The aggregate fiscal variables defined above might well of course be broken down into their subcategories in macroeconomic models used in practice.

Solving this equation forward we can express debt as a function of its determinants,

$$\frac{B_t}{P_t} = \sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\pi_{t+j}}{R_{t+j}}\right)[\tau_{t+i+1} - g_{t+i+1}] + \lim_{i\to\infty}\left(\prod_{j=0}^{i}\frac{\pi_{t+i+1}}{R_{t+i+1}}\right)\frac{B_{t+i+1}}{P_{t+i+1}} \tag{2}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ is the time–t inflation rate. For the government to be solvent, the second term of the right-hand side of the previous expression has to be equal to zero. Effectively, this can be considered as a no-Ponzi game condition, whereby in the limit, either agents are holding a zero amount of assets in the aggregate, or assets receive no valuation (their price is zero). In any standard model with optimising debt holders, this is the transversality condition attached to bond holdings that has to be verified. The remaining terms in the above equation imply that for the current level of debt to be consistent with the current status of the economy, any deviation from tax or spending plans at any point in the future – due to any shocks affecting revenue, spending, interest rates or inflation – has to be backed by changes in policy instruments.

The use of fiscal closure rules for model economies approximates the actual reaction to shocks by a fiscal authority. Nevertheless, some empirical evidence supports the notion of capturing actual government behaviour via a rule. For example, Bohn (1998) has provided evidence that governments take corrective measures in response to disturbances to avert an unstable or explosive path for debt. Specifically, based on the analysis of time series data for the United States, he finds evidence that the government has historically reacted to increases in the debt-to-GDP ratio by either reducing its primary deficit or improving its primary surplus.

[2]Seigniorage revenues are neglected for simplicity. In this framework, the monetary authority is active in the sense of Leeper (1991) insofar as it sets its control variables independently of tax collection and debt issuance, in the manner it considers more convenient.

2.1 Standard practice to obtain fiscal closure in large-scale macroeconomic models

In order to rule out unstable debt paths, it is necessary to *close* any given model by fully specifying the behaviour of the fiscal authority to prevent explosive deviations of debt from its normal (steady-state) value. This is normally done by positing a reaction function (rule) for the fiscal authority. In defining this rule, two decisions have to be taken by the modeller. The first one is the selection of the budget item that would eventually adjust in reaction to unforeseen events. The second decision consists of specifying a functional form for the reaction.

2.1.1 The adjusting budgetary item

As regards the first decision outlined above, the fiscal rules used in existing macroeconomic models normally involve adjustment of a budgetary item on the revenue side of the government budget. For clarity of exposition, the revenue side of the model can be thought of as being composed of two distinct components,

$$\tau_t = \tau\left(c_t, y_t, ...\right) + \tau_t^{rule} \tag{3}$$

where the first part of the equation embodies the normal tax system of the economy (income taxes, consumption taxes, etc.), embodying, when applicable, automatic stabilisation properties of budgets, while the second component represents the selected revenue adjustment by the government in the face of extra-budgetary shocks to guarantee solvency. In practice, the second component is a tax rate of some sort, and in this way the strict separation implicit in (3) is blurred somewhat. The specific adjusting tax item can vary from one model to other, ranging from the adjustment of the aggregate "basic tax rate", defined as the ratio of total tax revenues to GDP (e.g. in IMF's MULTIMOD), to more specific revenue items, such as direct taxes paid by households (e.g. in ECB's Area Wide Model, Bank of Canada's QPM). For the moment, there is no clear consensus on the appropriate adjusting item – this remains model-specific. Moreover, there may be the lack of a sound theoretical or empirical criterion for this selection, although this may be of limited concern if taxes are lump sum (i.e. changes in tax rates have no real effects) and depends on the focus of the model at hand. Any attempt to model distortionary elements of taxation, however, could be complicated by the behaviour induced by such a rule. Were a modeller to introduce such elements, the choice of revenue item reacting to budgetary variability would no longer have neutral effects, and as such could have important consequences for aspects of agents' behaviour. Accordingly, modellers have generally opted to model tax revenues accruing from the rule as lump-sum.

2.1.2 The functional form and the calibration

As for the chosen form of the rule – although in principle fiscal closure rules can take various functional forms – including several types of variables, in practice most modellers have opted for a broadly similar specification of the fiscal closure rule in the tradition of partial adjustment equations to deviations of targets, as pioneered in the economic literature by Phillips (1958). In most cases, budgetary adjustment is a function of the distance of a variable such as deficit or debt from its target value. A tax-difference rule is used in models such as MULTIMOD (IMF) and NiGEM (National Research Institute) – whereby the change in a tax rate is a function of an objective variable. Alternatively, some models specify their rules in terms of levels – for instance, in MSG2 (a model developed by McKibbin and Sachs) – whereby the tax rate itself is adjusted in reference to the objective variable.[3] To illustrate, a tax-difference rule would be some variant of the following generic form:

$$\tau_t^{rule} = \tau_{t-1}^{rule} + a\left(x_{t-1} - x_{t-1}^*\right) + b\Delta\left(x_{t-1} - x_{t-1}^*\right) \tag{4}$$

where x is the objective variable governing budgetary adjustment, with an asterisk denoting the steady state value, and $\Delta \equiv 1 - L$ denoting the first difference operator. The coefficients a and b are the speed of adjustment parameters.

As a first input, the calibration of this type of rule requires the identification of *target* values for the objective variable in the above equation (usually deficit or debt). This is relatively straightforward, given the long-run properties of the model. The more difficult elements to calibrate are the speed of adjustment parameters. The feasible calibration set for a and b can be obtained on the basis of the stability analysis of the system formed by the budget constraint, (1), and the equation for the fiscal closure rule, in the form of, for example, (4).

The government budget constraint (1) is an unstable difference equation in real debt insofar as $\tau_t - g_t$ is stationary and R_{t-1} is greater than one. As shown in Mitchell *et al.* (2000), the standard practice for building exogenous fiscal rules can be thought of as specifying a given element of τ_t –what we call τ_t^{rule}– as a function of past debt so that the resulting coefficient attached to past debt when solving for the two equations happens to be lower than one. This two-equation analysis is helpful because it constrains the possible values of the coefficients in the fiscal rule. It does not result in a unique calibration, but a range of feasible values for stability. Despite the appealing intuitive nature of this type of presentation, it leaves open the question about the specification of the rule itself, and in addition, in models not displaying separability of the government block the above two-equation analysis would not be valid.

Within the range of values for a and b resulting from the preceding analysis, the specific values of a and b in (4) can then be fine-tuned by means of some a priori desired properties of the model solution and responses to shocks. In this sense, the calibration process has the potential to involve the considerable use of judgement. Various exercises have been pursued by modellers in the derivation and calibration of fiscal rules. For instance, the parameters of the fiscal reaction function in MULTIMOD are set at levels that produce stable outcomes and tend to induce the ratio of government debt in GDP to revert to its target value over the typical simulation horizon. Other approaches include a derivation based on a cost of adjustment model to highlight the short-term payoffs facing a government choosing a solvency rule (Barrell *et al.*, 1994). In this vein, an alternative derivation pursued particularly in the monetary policy rules literature could be to derive an optimal policy rule on the basis of the minimisation of a loss function. This, however, would remain an exogenous derivation procedure, as to fully account for all aspects of a large-scale model in such a derivation would be cumbersome if not impossible. In general, although these approaches to calibrating the rule constitute a practical approximation, they do not guarantee that the dynamic adjustment process resulting from the operation of the fiscal rule is fully consistent with the properties of the model it is closing.

[3] A more detailed analysis of the underpinnings of these rules is contained in Mitchell *et al.* (2000) and Bryant and Zhang (1996a).

The functional form and calibration of the fiscal closure rule in a model can be of considerable importance in influencing the output of that model in simulation exercises. For instance, Bryant and Zhang (1996b) find that the response of variables can differ quite substantially on the basis of alternative standardised fiscal rules. They point out that the values of the feedback coefficients in monetary and fiscal reaction functions can strongly influence the dynamic behaviour of a model, sometimes in unwanted or implausible ways. It is concluded on the basis of this evidence that generally, there is a particularly imprecise understanding of how economies respond to fiscal policy actions. Barrell *et al.* (1994) also find that the implementation of the fiscal policy rule has a significant effect on model properties in comparing the tax rules of NIGEM and MULTIMOD. Church *et al.* (2000, 1997) have the similar finding for models of the UK economy that different simulation results may be obtained simply given different assumptions about the way in which fiscal policy reacts to shocks. Mitchell *et al.* (2000), when comparing the behaviour of the fiscal rules of NIGEM, MSG2 and MULTIMOD, show that the rules can be adjusted to produce similar outcomes. They point out that the rules and, more importantly, their calibration, are model-specific and not directly transferable to other models. In performing the two-equation – (1) and some variants of (4) – stability analysis referred to above, they stress the theoretical equivalencies between debt and deficit target rules. Nonetheless, their simulation analysis reveals that the impulse response functions to shocks can differ considerably depending on the form and calibration of the actually implemented fiscal closure rule. A similar finding is yielded by the analysis of Barrell *et al.* (1994), who compare the tax rules of MULTIMOD and NiGEM within the same model.

Despite the variation in results, little consensus exists in the literature on the proper formulation of fiscal closure rules. The usual two-equation stability analysis only imposes some general restrictions on the coefficients of a fiscal rule of type (4). It is not surprising, based on the reasonably wide scope of possibilities, that in practice, varying specifications are evident for the formulation of fiscal rules in macroeconomic models. This lack of standardisation highlights the lack of agreement amongst modellers regarding the appropriate functional form for these rules, while it also highlights that a multitude of fiscal rules can generate the same desired outcome of model closure. In other words, the rules used in practice all produce stable outcomes (i.e. result in transversality holding), although several different formulations of a rule could generate the same desired mean-reverting behaviour of the debt ratio.

The main shortcoming of the preceding analysis is that when calibrating the key parameters and designing the rules, the specification and calibration are considered outside the auspices of the model, and in this way, the structure and specific characteristics of the model under analysis are only considered indirectly. This type of lack of internal consistency in modelling has been criticised by many for its lack of microfoundations starting with Lucas (1976). A more fundamental criticism of exogenously imposed fiscal rules is their inherent vulnerability to the points raised by Lucas, as changes in the deep parameters of the model (representing preferences, technology, etc.) may not directly lead to a change in the form or calibration of the fiscal rule.

In addition, while the preceding simple analysis serves to illustrate the need for a fiscal rule, it is too simplistic to be generalised to a more complex setup in which the fiscal block cannot be considered in isolation. The presence of distortionary taxes, for instance, requires an integrated analysis of the fiscal block in the larger context of the model as changes in taxation have wider repercussions for key economic variables such as output determination and capital accumulation. Moreover, the interaction with a monetary authority in this context may entail that changes in fiscal variables induce changes in interest rates, in turn affecting debt service and, accordingly, the government budget constraint.[4]

3 Identifying endogenous fiscal rules

In modelling fiscal policy one may wish to consider the formulation and calibration of a fiscal rule in an integrated framework, where the government sector is considered in conjunction with the economic sector and monetary authority. In this section, we employ stability analysis leading to debt stabilisation in a given model in an alternative manner to the two-equation approach discussed in the previous section. We also discuss some standard results of stability analysis theory that allow us to restrict the set of potential candidate instruments for fiscal closure given a model. We do so in the context of the basic macro model of the next subsection to develop some intuition, and we move in Section 3.2 to a more general setting.

3.1 Developing intuition via a basic macro model

3.1.1 The model

In order to develop intuition in this Section, we employ the model in Leeper (1993, 1991). This model can be characterised as a standard neoclassical model where money is motivated via its presence in the utility function, and the supply side is governed by a microfounded Lucas supply curve. The model can be summarised by the following set of core equations, augmented by a tax rule,

$$
\begin{aligned}
y_t &= \lambda_0 (1-\lambda_2) + \lambda_1 (\pi_t - E_{t-1}\pi_t) + \lambda_2 y_{t-1} + \epsilon_t && (5)\\
m_t^d &= \delta_0 + \delta_1 R_t + \delta_2 c_t && (6)\\
\frac{u'(c_t)}{P_t} &= \beta R_t E_t \left(\frac{u'(c_{t+1})}{P_{t+1}} \right) && (7)\\
y_t &= c_t + g_t && (8)\\
R_t &= \alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t && (9)\\
b_t + m_t - m_{t-1}\frac{1}{\pi_t} + \tau_t &= g_t + \frac{R_{t-1} b_{t-1}}{\pi_t} && (10)\\
\tau_t &= \tau^0 + \tau^y y_t + \tau_t^{rule} && (11)
\end{aligned}
$$

Equation (5) is the aggregate supply function in this model, where production (y_t) is driven by surprise changes in inflation (π_t) and stochastic shocks to productivity (ϵ_t). Some persistence is present, the degree of which is captured by $0 < \lambda_2 < 1$, while λ_0 and λ_1 are additional parameters. The operator $E_t(\bullet)$ denotes the expectations operator condition on information up to time t. As regards equation (6), it denotes the demand for real money balances (m_t^d) depending on the nominal rate of interest (R_t) and consumption (c_t). The demand for government debt is captured by means of equation (7), where utility is $u(c_t) = \log(c_t)$. $0 < \beta < 1$ is the discount factor in the model so that $1/\beta$ is the steady state real interest rate in this model with no growth. The model implies a simplified form of the standard national income identity (Equation (8)), which abstracts from an investment and

[4]For a discussion of the need to examine monetary and fiscal policy in a holistic framework, see Leeper (1993).

external sector, so that total demand is equal to the sum of private consumption and public consumption (g_t).

Equations (9) to (11) contain the monetary and fiscal policy actions. Equation (9) is the monetary policy rule, where the monetary authority is assumed to follow an interest rate policy adjusting high-powered money to change the interest rate in response to changes in inflation. The specification also embodies countercyclical behaviour ($\alpha_2 > 0$) with respect to output. The government budget constraint (10) is analogous to that described in Section 2 (note that $b_t \equiv \frac{B_t}{P_t}$). Finally, (11) is the tax system of this economy that is assumed to be composed of some fixed amount of revenue given by τ_0, and revenue accruing from a proportional tax rate on income given by τ^y. In addition, the fiscal authority will be assumed to collect a lump-sum tax earmarked to stabilise debt, τ_t^{rule}.

The random environment in the economy is captured by two exogenous shocks: one to aggregate supply and one to government spending (i.e. a component of aggregate demand). Both are assumed to follow stationary first-order autoregressive processes. The shock to aggregate supply ($\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \eta_{\epsilon_t}$) has a zero mean, while the shock to government spending ($g_t = (1-\rho_g)g + \rho_g g_{t-1} + \eta_{g_t}$) has mean g.

The model has a well-defined steady state. For a given variable X_t, the steady state solution is that particular solution in which $X_t = X_{t-1} = X$, $\forall t$. All shocks are set to zero in the steady state, so that this particular solution reads,

$$\begin{array}{rclrcl} y & = & \lambda_0, & R & = & \frac{\alpha_0+\alpha_2\lambda_0}{1-\alpha_1\beta} \\ c & = & \lambda_0 - g & m & = & \delta_0 + \delta_1 R + \delta_2(\lambda_0 - g) \\ \pi & = & \beta R, & \tau & = & \tau^0 + \tau^y \lambda_0 \end{array}$$

and

$$b = \left(\frac{\beta}{\beta-1}\right)\left[m\left(\frac{1}{\pi}-1\right) + g - \tau^0 - \tau^y\lambda_0\right]$$

for all variables y, R, c, m, π, τ and b.

3.1.2 Characterising the stable manifold of the model

Solving the model developed above in the absence of a fiscal closure rule, we can explicitly identify the conditions under which the model would produce stable outcomes. This underpins the rationale for the inclusion of a fiscal rule. Linearising the system around the well-defined steady state, we can express it as follows:

$$\Gamma_0\ Y_t = \Gamma_1\ Y_{t-1} + \Psi\ \varepsilon_t + \Pi\ \eta_t \tag{12}$$

plus the transversality condition on debt,

$$\lim_{j\to\infty}\left(\beta\frac{u'(c_t)}{P_t}\right)^j b_{t+j} = 0 \tag{13}$$

where $Y_t \equiv \left[\widetilde{y}_t, \widetilde{\pi}_t, \widetilde{b}_t, E_t(\widetilde{\pi}_{t+1}), \widetilde{\varepsilon}_t, \widetilde{g}_t\right]'$, and a tilde (the notation $\widetilde{X}$) denotes the deviation of a variable from its steady-state value. $\varepsilon_t = [\eta_{\epsilon_t},\ \eta_{g_t}]'$ is the vector of shocks in the model, and η_t is the vector of expectational errors, in this case comprised of a single element, $\eta_{\pi_{t+1}} \equiv \pi_{t+1} - E_t\left(\pi_{t+1}\right)$.

In order to develop the intuition, we set in this section all sources of stochastic disturbances equal to zero, so that $\eta_{\epsilon_t} = 0$ and $\eta_{g_t} = 0$, thereby simplifying the model to exclude the exogenous processes. This yields, after further working out the remaining equations,

$$\begin{bmatrix} \widetilde{y}_t \\ \widetilde{\pi}_t \\ \widetilde{b}_t \end{bmatrix} = \begin{bmatrix} \lambda_2 & 0 & 0 \\ a_{12} & \alpha_1\beta & 0 \\ a_{13} & a_{23} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \widetilde{y}_{t-1} \\ \widetilde{\pi}_{t-1} \\ \widetilde{b}_{t-1} \end{bmatrix} \tag{14}$$

where:

$$a_{12} = \frac{\beta}{c}\left(\alpha_2 c + (1-\lambda_2)R\right),$$

$$a_{13} = -\lambda_2\left(\delta_1\alpha_2 + \delta_2 + \tau^y\right) - \left(\alpha_1\delta_1 + \frac{b}{\beta\pi} + \frac{m}{\pi^2}\right)\left[\frac{\beta}{c}\left(\alpha_2 c + (1-\lambda_2)R\right)\right] + \frac{\alpha_2\left(\delta_1 + b\right) + \delta_2}{\pi}$$

and

$$a_{23} = -\alpha_1\beta\left[\alpha_1\delta_1 + \frac{b}{\beta\pi} + \frac{m}{\pi^2}\right] + \frac{\alpha_1\left(\delta_1 + b\right)}{\pi}$$

Given the lower-triangular structure of the matrix premultiplying the lagged values of the variables, the eigenvalues of the system lie on the main diagonal of the coefficient matrix in (14): λ_2, $\alpha_1\beta$, and $1/\beta$. The first one lies within the unit circle by assumption. As regards the second one we assume that the monetary authority reacts when facing deviations from its target level of inflation. The third eigenvalue, however, is located outside the unit circle for any sensible parameterisation of the discount factor, implying an unstable solution to the system (14). This eigenvalue, $1/\beta$, is the one associated with fiscal policy. This is easy to see if one proceeds with the full eigenvalue/eigenvector analysis of (14). Indeed, solving recursively, the solution for debt turns out to be unstable, according to the following expression,

$$\tilde{b}_t = (\lambda_2)^t\ w_1\ \tilde{y}_0 + (\alpha_1\beta)^t\ w_2\ \left[w_3\ \tilde{y}_0 + \ \tilde{\pi}_0\right] + (1/\beta)^t\ \left[w_4\ \tilde{y}_0 + w_5\ \tilde{\pi}_0 + \tilde{b}_0\right] \tag{15}$$

where the weights w_1 to w_5 are known and computable functions of the deep parameters. The solutions for output and inflation are stable, starting from any appropriate initial condition. For the solution for debt to be stable it has to be that either: (i) $\mid 1/\beta \mid < 1$, which is not the case for reasons outlined above; or (ii) a weighted sum of the initial values of the state variables is equal to zero, $w_4\tilde{y}_0 + w_5\tilde{\pi}_0 + \tilde{b}_0 = 0$.

In order to achieve stability, the standard practice in macroeconomic modelling usually rectifies instability via imposing (i) – thereby directly eliminating the unstable root from the solution. For example, if we posit a fiscal closure rule along the lines of those typically used in macroeconomic models of the form $\tau_t^{rule} = \tau^b\tilde{b}_{t-1}$ and we repeat the preceding eigenvalue/eigenvector analysis, then the unstable eigenvalue turns out to be $1/\beta - \tau^b$, so that calibrating τ^b appropriately one can make $\mid 1/\beta - \tau^b \mid < 1$, and as a consequence stabilise the solution for debt. Under this approach, the solution is globally stable in the sense that all eigenvalues lie within the unit circle, and any deviation from the steady state will be automatically corrected.

Keeping in mind the shortcomings of this approach outlined at the end of Section 2.1.2, an alternative approach is based on forcing the linear combination of the state and control variables $w_4\tilde{y}_0 + w_5\tilde{\pi}_0 + \tilde{b}_0$ to be zero. In analytical terms, if we get back to the solution for debt (15), this means imposing the period by period condition,

$$\tilde{b}_t = [-\delta_1\alpha_2 - \delta_2\tau^y] \ \tilde{y}_t + \left[-\alpha_1\delta_1 - \frac{b}{\beta\pi} - \frac{m}{\pi^2}\right] \ \tilde{\pi}_t \tag{16}$$

This relation is the stable manifold of the system, and it is usually denoted as the *stability condition*. In this case the model would display a saddle path structure, in line with the solution to standard rational expectations models. The stability condition would force the system to be on the stable arm of the saddle in every period, and would let the dynamics of the system determine the dynamics of the transition back to the steady state. Appending this stability condition to the core equations of the model therefore generates stability, as agents internalise the transversality condition and adjust their optimal decisions so that the real value of debt is bounded, and the deviation from the target in real terms (or, in models with economic growth, as a percentage of nominal income) is prevented. In other words, once agents internalise that the government commits itself to be solvent, they behave in such a way that indeed the resulting equilibrium is stationary and the government debt is valued and held by the agents. This reasoning is close to the one used by advocates of the *Fiscal Theory of the Price Level*. Rather than the government budget equation constraining the behaviour of the policymaker, changes in the equilibrium price level would force the fulfilment of the budget constraint.

3.1.3 Computing an explicit model-based rule

From an economic point of view and for the purposes of policy analysis, the intuition behind the formulation in (16) might be a bit obscure. Although imposing such conditions to solve for the variables in the model is technically correct, it is somewhat more difficult to give some economic meaning in the framework of the model being analysed. Specifically, when imposing the transversality conditions applying to government bonds, one may wonder which instrument the government would be moving on the event of, for instance, a recession. Moreover, the "implicit" endogenous fiscal policy rule in (16) relies on an assumption that governments are credible, thus generating the outcome that agents internalise the government budget constraint completely. This form of the rule, however, suffers from the drawback that this assumption of credibility is maintained despite a lack of systematic stabilisation on the part of the government in response to imbalances in the economy.

From the fiscal policy point of view we would be interested in knowing what amount of revenue given by τ_t^{rule} would stabilise debt and make the transversality condition hold, while being fully based on the model itself. If we want to assign the role of reacting to deviations in targets explicitly to the fiscal authority, we can do so by exploiting the information in the structure of the saddle-path solution. Specifically, this can be used to measure the government reaction needed to keep the system on the saddle-path equilibrium. In developing this rule, we address the two basic decisions of a modeller as developed in Section 2.1: (i) the adjusting budgetary item and (ii) the functional form and calibration of the rule.

The adjusting budgetary item

First, we sidestep issues related to the decision regarding which tax item to adjust by creating a tax adjustment *at the margin*, to cater for transitory shocks. For reasons outlined earlier, this separation of "core" taxes (income taxes, consumption taxes, etc.) from marginal adjustments gives the modeller the full flexibility to maintain a practical specification for the behaviour of regular taxation in the model in whatever format seems most appropriate. Modelling the reaction separately as a marginal adjustment then allows us to avoid sacrificing a tax item for this purpose, while the lump-sum nature of this adjusting tax item avoids any distortionary effects on other variables in the model.

Therefore, we maintain a strict separation of the tax system in the form of (3). The first part involves the standard tax items of budgets (and, accordingly, the applicable automatic stabilisation properties), and the second component represents the incremental revenue adjustment by the government required to guarantee solvency. The strict separation of the lump-sum term equipped to handle "extra-budgetary operations" from the "core" tax system means that adjustment to these sources of variability do not focus on one particular tax category. This is based on the premise that clear theoretical or empirical evidence of which budgetary item should be adjusting to imbalances generated by stochastic disturbances is scant. In the case where compelling evidence is available, the rule can obviously be adapted accordingly. This could include, if appropriate, the modelling of a fiscal policy response in the form of a distortionary tax.

The functional form and calibration

The second basic decision of a modeller in building a tax rule is the selection of the functional form. Regarding this point, our rule is based on the selection of the stable manifold outlined above. With this additional equation in the system, we can solve for one additional variable, τ_t^{rule} – which represents the fiscal rule. The tax item is the "jumping" variable - moving the system back to the stable manifold.

This way, and in terms of the example model economy we are using, one would need to solve for τ_t^{rule}, τ_t, and $\tilde{b}_t$ by means of the system made up of three basic ingredients: the stable manifold equation in (16), the government budget constraint (10), and the tax system represented by (11). Substituting τ_t from (11) in (10), and eliminating $\tilde{b}_t$ from (10) using (16), we get the following expression for τ_t^{rule},

$$\tau_t^{rule} = \left[\frac{1}{\beta}\right] \tilde{b}_{t-1} + \left[\frac{\alpha_2(\delta_1 + b) + \delta_2}{\pi}\right] \tilde{y}_{t-1} + \left[\frac{\alpha_1(\delta_1 + b)}{\pi}\right] \tilde{\pi}_{t-1} \qquad (17)$$

This rule governs the period-by-period reaction of the government in order to guarantee debt stability in this model. From (17) it is apparent that the proposed type of rule, while not deviating substantially from the simplicity of standard rules, displays the following desirable properties: (i) the debt path is not explosive and the intertemporal budget constraint of the government is fulfilled; (ii) it assigns an explicit role to τ_t^{rule} while being fully based on the stable manifold of the model; and (iii) the coefficients of the rule depend explicitly on the deep parameters of the model under consideration, thus alleviating susceptibility of this sector to the Lucas critique.

For simplicity we have excluded all stochastic elements in the preceding analysis. Nevertheless, as we will see below, the shocks present in the solution would appear in a fiscal rule of the type of (17). In fact, the fiscal rule item τ_t^{rule} could also be expressed solely as a reaction to shocks, as we will see in Section 4.

3.2 Generalisation of the discussion in linear models

3.2.1 Standard stability analysis theory

For the purpose of developing clear intuition, we have restricted the discussion to the analysis of a simplified model thus far. In this Section, we generalise the discussion to models with a linear/log-linear structure, including stochastic components.[5] The linearised version of the system around the deterministic steady-state can be expressed as the general representation in (12), plus a set of transversality conditions,

$$\lim_{j \to \infty} \left(\phi^j \; Y_{t+j} \right) = 0 \tag{18}$$

where now the vector Y_t contains all the endogenous and exogenous variables in the model, as well as the conditional expectations in the model, and ϕ is the appropriate discount rate for the variables in the model. In particular, in the cases of interest to us, Y_t will contain the variable debt, b_t, and taxes, τ_t. Taking an arbitrary set of initial conditions Y_0 and simulating Y_t using solely (12), conditional on sample realisations for ε_t, will normally lead to unstable trajectories for Y_t that would eventually violate the transversality conditions given by (18). For the transversality conditions to hold, we need to add a set of stability conditions to the system in (12).

These stability conditions are defined by the eigenvectors associated with the unstable eigenvalues of the system (12). When Γ_0 is non-singular the stability analysis is based on the eigenvalues of $\Gamma_0^{-1}\Gamma_1$. If Γ_0 is singular we have to analyse the generalised eigenvalues of the pair (Γ_0, Γ_1), by means of, for example, QZ-decompositions (see Sims, 2000). In what follows, without lack of generality and for the sake of exposition, we will assume that the matrix $\Gamma_0^{-1}\Gamma_1$ can be decomposed as $P\Lambda P^{-1}$, where Λ is a diagonal matrix containing the eigenvalues of $\Gamma_0^{-1}\Gamma_1$ and P^{-1} is the matrix of left eigenvectors. Let P^s denotes the rows of P^{-1} associated with the unstable eigenvalues. Then a unique stationary equilibrium must satisfy the conditions,

$$P^s \; Y_t = 0, \;\; \forall t \tag{19}$$

or alternatively,

$$P^s \; \Gamma_0^{-1} \; \Pi \; \eta_t = P^s \; \Gamma_0^{-1} \; \Psi \varepsilon_t, \;\; \forall t \tag{20}$$

The above is obtained simply by solving forward the system in (12), and substituting the transversality conditions in (18). The set of equations in (19) are the stability conditions of the system, amounting to a particular linear (or log-linear) combination of the endogenous and exogenous variables in the model. There is one such condition per unstable eigenvalue (the one attached to each transversality condition). In models with a stochastic structure, the conditions are needed to solve for all expectational errors in the model. As discussed in Sims (2000), for the equilibrium to be uniquely determined one such condition should be present for each expectational error in the model.

[5] For this general analysis, we only explicitly consider models characterised by complete information. For a recent examination of issues related to learning on the part of agents in the context of monetary rules for models with incomplete information, see Tetlow and von zur Muelen (2001).

The stable paths of the approximated model economy can be simulated recursively given (12) just by solving for the expectations errors using (20), and rewriting so that,

$$Y_t = \left[\Gamma_0^{-1}\Gamma_1\right]\ Y_{t-1} + \left[\Gamma_0^{-1}\Psi\right]\ \varepsilon_t + \left[\Gamma_0^{-1}\Pi\right]\ \eta_t \tag{21}$$

starting from an appropriate Y_0 and given the realisations of the exogenous shocks in ε_t, or directly using,

$$\left[\begin{array}{c} Y_t \\ \eta_t \end{array}\right] = \left[\begin{array}{cc} \Gamma_0 & -\Pi \\ P^s & 0 \end{array}\right]^{-1} \left[\begin{array}{c} \Gamma_1 \\ 0 \end{array}\right] Y_{t-1} + \left[\begin{array}{cc} \Gamma_0 & -\Pi \\ P^s & 0 \end{array}\right]^{-1} \left[\begin{array}{c} \Psi \\ 0 \end{array}\right] \varepsilon_t \tag{22}$$

starting from an appropriate Y_0.

3.2.2 The identification procedure

The same set of stability conditions could also be combined to identify additional variables – for example τ_t^{rule}. The equilibrium we select by solving jointly (12) and (19) (or (20)), as we do in (21) and (22), is one in which the government is credible, and as a consequence agents adjust their decisions so that government debt is valued and the rest of transversality conditions hold, even when the policy reaction to shocks is non-existing, i.e. $\tau_t^{rule} = 0$. In assigning content to a policy reaction, τ_t^{rule} as done in the preceding Section, we first need to perform the stability analysis of the system (12). From the set of stability conditions in (19) we can isolate the stability conditions associated to the state variables representing government debt. Assuming there is only one government asset in the economy, so that the subset comprises only one condition, let $P^{s,b} \subset P^s$ be the corresponding row of P^s. The transversality condition associated with government debt then has a corresponding stability condition given by

$$P^{s,b}\ Y_t = 0, \quad \forall t \tag{23}$$

Once we have this new equation, we can solve for τ_t^{rule}. To do this, we solve for τ_t using the stability condition (23), and the (linearised) government budget constraint, so that we get $\tau_t = f(Y_t^*, Y_{t-1})$, where Y_t^* is a vector containing the same elements as Y_t with the exception of $\tilde{b}_t$, and $f(\bullet)$ denotes a known function of the variables. Then, using the obtained equation together with the specification of the tax system in the model – defining τ_t^{rule} as one element of τ_t – we can express τ_t^{rule} as a function of the variables in the information set,

$$\tau_t^{rule} = -f_0\ Y_t + f_1\ Y_{t-1} \tag{24}$$

with f_0 and f_1 being known vectors of coefficients, and the element of f_0 corresponding to $\tilde{b}_t$ being null. This condition can be considered as the fiscal rule. An example of its form is (17), in the simplified version of the model developed in a previous Section. As an additional representation one could also express the tax rule item τ_t^{rule} solely as a function of the shocks affecting the economy and the expectational errors. Thus, from

$$\left[\begin{array}{c} Y_t \\ \tau_t^{rule} \end{array}\right] = \left(\left[\begin{array}{cc} \Gamma_0 & 1 \\ f_0 & 1 \end{array}\right] - \left[\begin{array}{cc} \Gamma_1 & 0 \\ f_1 & 0 \end{array}\right] L\right)^{-1} (\ \Psi\ \varepsilon_t + \Pi\ \eta_t\ )$$

the last row – call it N – corresponding to τ_t^{rule} could then be picked up and used as the fiscal rule,

$$\tau_t^{rule} \quad = \quad \left(\begin{bmatrix} \Gamma_0 & 1 \\ f_0 & 1 \end{bmatrix} - \begin{bmatrix} \Gamma_1 & 0 \\ f_1 & 0 \end{bmatrix} \; L \right)_N^{-1} \; (\; \Psi \; \varepsilon_t + \Pi \; \eta_t \;)_N \tag{25}$$

where L denotes the lag operator. The innovations in ε_t are known at t, and the expectational errors can be simulated using the set of stability conditions in (20). This form of expressing the rule has the advantage that the coefficients of the fiscal rule can be interpreted directly as the reaction the government depending on the shock affecting the economy, and the impact on the expectational errors the agents commit.

Once a functional form for the government reaction has been identified in the form of either (24) or (25), one can solve the model to get the equilibrium values of all the variables in the economy by means of: (i) the linearised system (12); (ii) the selected tax rule; and (iii) the set of stability conditions $P^s Y_t = 0, \forall t$ minus $P^{s,b} Y_t = 0$ to identify the set of expectation errors, minus the one attached to debt, that are assumed to be white noise.

This last assumption merits some additional explanation. In both deterministic and perfect foresight models, the stability conditions simply represent the stable manifold of the system, so that one has additional equations with which to operate to identify additional variables as in the simple example developed above. Each unstable eigenvalue has a corresponding stability condition. In non-perfect foresight models where one has to identify all expectations errors, a potential indeterminacy problem has to be sorted out in a first stage. For the rational expectations equilibrium to be uniquely determined the solution has to contain as many stability conditions as elements in the vector η_t. In order to avoid this potential (but standard) problem one can assume some structure for that element of η_t. There are various alternatives. The first and most straightforward one could be to assume perfect foresight for the needed element of η_t. Another one would be to evaluate ex-post the rationality of the expectation error. A final way would be to assume a known stochastic process for an element of η_t, as in the backward solution method of Sims (1989). As all elements of η_t, being rational errors, are white noise by construction, this structure is a natural candidate.[6]

Summarising the steps a modeller would have to follow in order to compute the fiscal rule proposed in this Section,

1. Compute the steady state solution of the model. Perform a linear approximation of the model around the steady state.

2. Perform the stability analysis of the system with τ_t^{rule} set to zero. Detect the stability condition attached to debt (the one coming from the government budget constraint).

3. Solve for the a variable τ_t^{rule} using the system made up of the stability condition attached to debt, (23), the government budget constraint, and the equation defining the tax system in the model. Choose the reduced form rule (24), or the shock-based rule (25).

4. Append this new equation to the model (12), and simulate the model.

[6] Another alternative would be to develop an iterative process to solve non-linear models along the same lines of the identification procedure described in this paper, but with the possibility to refine the coefficients of the rule until the *ex-post* expectation error attached to debt is indeed white noise. This would also open the possibility of developing a polynomial function which could summarise all of the information in the rule in a sufficient set of variables. A provisional algorithm of this type is available from the authors upon request.

4 Application of the identification scheme

In this Section we implement our identification scheme on the basis of the model developed in Section 3.1.1. We solve the full model, including the entire stochastic structure left aside in Section 3.1.2.

The tax system of the type (11) is composed of a fixed amount of tax collection, τ^0, an income tax captured by $\tau^y\, y_t$, and the tax-rule-based lump-sum amount τ_t^{rule} calibrated using the endogenous identification procedure. The results from such an approach are compared with the outcome of a standard tax reaction function, proxied by a rule of the type of

$$\tau_t^{rule} = \tau^b\ \tilde{b}_{t-1} \tag{26}$$

which is fully based on lump-sum tax collection. This assumption that this tax does not have a distortionary impact on the decisions of private agents does not reflect a limitation of the identification method. It rather reflects the idea that budgetary adjustment takes place at the margin – so that the tax item designed to guarantee debt stability in the model should not unduly distort agents' allocation of output or assets.

4.1 Calibration of the model

The example in this Section is intended only to illustrate how to implement our methodology in a small-sized macroeconomic model usable for policy analysis, and a fairly standard calibration along the lines of Leeper (1993) has accordingly been chosen. A complete and quantitative interpretation of the economic meaning of the numbers produced below is not provided, rather, the focus is only on the qualitative results.

The steady-state value of output is set by assuming $\lambda_0 = 10$. The nominal rate of return on bonds is an annual 3.5% , while the subjective discount rate for the agents is fixed to 0.985, which is within the range of the standard values in the literature. The implied rate of inflation is thus about 2%. Following Leeper (1993), the coefficients in the interest rate rule are set at $\alpha_1 = 1.3$, $\alpha_2 = 0.25$, and then the implied $\alpha_0 = R - \alpha_1\beta R - \alpha_2\lambda_0$. Regarding the money demand equation, following Leeper again, we fix the elasticity of the interest rate to be $\delta_1 = -0.05$, the income coefficient to $\delta_2 = 1$, and $m = 0.77$. This way $\delta_0 = m - \delta_1 R - \delta_2 c$. As regards the supply equation $\lambda_1 = 0.25$ and $\lambda_2 = 0.7$.

The steady state level of public expenditure is set to $g = 2$, implying a 20% share of the public sector in total income, while $\tau = 2.1$, so that $\tau^0 = \tau - \tau^y\ \lambda_0$. The income tax rate is calibrated to $\tau^y = 0.25$. Finally τ^b will be equal to 0.1 or 0.8 depending on whether the assumed reaction on the part of the fiscal authority is considered to be weak or strong, as we will see below.

The persistence parameters ρ_ϵ and ρ_g, are set equal to 0.8, while the standard deviations of the shocks σ_{η_ϵ} and σ_{η_g} have been chosen (arbitrarily) to be 0.01.

4.2 Computation of the explicit model-based rule

The linear version of the full model in Section 3.1.1, after performing a linear approximation and rearranging terms and equations, can be written as,

$$
\begin{array}{rcl}
\tilde{y}_t & = & \lambda_1(\tilde{\pi} - E_{t-1}\tilde{\pi}_t) + \lambda_2 \tilde{y}_{t-1} + \tilde{\epsilon}_t \\
\frac{\beta}{c}(\alpha_2 c + (1-\lambda_2)R)\ \tilde{y}_t & = & -\alpha_1\beta\ \tilde{\pi}_t + E_t(\tilde{\pi}_{t+1}) + \frac{\beta R}{c}\ \tilde{\epsilon}_t + \frac{\beta R}{c}(1-\rho_g)\ \tilde{g}_t \\
\tilde{b}_t + \left(\alpha_1\delta_1 + \frac{b}{\beta\pi_t} + \frac{m}{{\pi_t}^2}\right)\tilde{\pi}_t & = & (-\delta_1\alpha_2 - \delta_2)\ \tilde{y}_t - \tilde{\tau}_t + (1+\delta_2)\tilde{g}_t + \left(\frac{1}{\beta}\right)\tilde{b}_{t-1} \\
 & & + \left(\frac{\alpha_1(\delta_1+b)}{\pi}\right)\tilde{\pi}_{t-1} + \left(\frac{\alpha_2(\delta_1+b)+\delta_2}{\pi}\right)\tilde{y}_{t-1} - \frac{\delta_2}{\pi}\tilde{g}_{t-1} \\
\tilde{\tau}_t & = & \tau^y \tilde{y}_t + \tau_t^{rule} \\
\tilde{\epsilon}_t & = & \rho_\epsilon \epsilon_{t-1} + \eta_{\epsilon t} \\
\tilde{g}_t & = & \rho_g \tilde{g}_{t-1} + \eta_{gt} \\
\eta_{\pi_t} & = & \tilde{\pi}_t - E_{t-1}\tilde{\pi}_t
\end{array}
\qquad (27)
$$

with τ_t^{rule} remaining to be specified. As described above, a tilda over a variable denotes absolute deviations of the variable with respect to its deterministic steady state value. The system could also be written in the general form (12), – the matrices Γ_0, Γ_1, Φ, and Π are easily obtained from (27). As a result of the eigenvalue/eigenvector analysis, the stability condition attached to debt, in the form of (23), turns out to be

$$P^{s,b}Y_t = 3.3\ \tilde{y}_t + 9.4\ \tilde{\pi}_t + \tilde{b}_t - 4.6\ E_t\left(\tilde{\pi}_{t+1}\right) + 4.5\ \tilde{\epsilon}_t + 6.4\ \tilde{g}_t = 0$$

for the chosen baseline calibration. Using this stability condition to solve for τ_t^{rule} provides an *explicit* version of the endogenous fiscal rule, as a linear function of the contemporaneous shocks and lagged state variables

$$\tau_t^{rule} = 10\ \eta_{\epsilon t} + 8.3\ \eta_{g_t} + 1.02\ \tilde{b}_{t-1} + 4.7\ \tilde{y}_{t-1} + 9.5(\tilde{\pi}_{t-1} - 0.1E_{t-1}\tilde{\pi}_t) + 8\ \tilde{\epsilon}_{t-1} + 5.7\ \tilde{g}_{t-1}$$

which can be alternatively obtained in this case (with lump-sum taxation) as already explained, using the government budget constraint, the tax system equation, and the stability condition for debt. The first two elements of the right-hand side reflect the two innovations affecting this economy, namely supply shocks and shocks to public expenditure. The rule implies a strong reaction to both of them. The third component in lagged debt enters with a coefficient of $1/\beta$. There is also a term in lagged deviations of income from its steady state value, and a term that captures a partial adjustment to the expectation error made by the agents in the previous period. Finally, the two last terms form a reaction to past shocks. Note that the above rule can be analogously expressed as a function of contemporaneous variables like $\tilde{y}_t$, $\tilde{\pi}_t$, $\tilde{\epsilon}_t$, $\tilde{g}_t$, and forward looking contemporaneous elements like $E_t(\tilde{\pi}_{t+1})$, as well as some terms in lagged variables. In the same manner, we can rewrite the rule so that it reads as a function of only contemporaneous and lagged values of the innovations hitting the economy. This provides a straightforward and intuitive interpretation of the fiscal reaction function in the sense that the nature of the perturbation and its persistence are sufficient to explain the fiscal policy response. In this case the rule would be

$$\tau_t^{rule} = 10.03\ \eta_{\epsilon t} + 8.29\ \eta_{g_t} - 0.66\ \eta_{g_{t-1}} + 0.14\ \eta_{g_{t-2}} + 0.11\ \eta_{g_{t-3}} + 0.09\ \eta_{g_{t-4}} + \ldots$$

The contemporaneous coefficients are clearly the most important ones, while there is some lagged effect in response to an innovation in public expenditure, that fades out quickly through time. It is worth noticing that any change in the parameters of the model imply a change in the coefficients of the rule. For example, a change in the income tax rate from 0.25 in the baseline calibration to 0.5 produces a change in the weight attached to $\eta_{\epsilon t}$ from 10.03 to 6.43. A 5% increase in g changes the two contemporaneous coefficients to 7.07 and 8.07, respectively.

Figures 1 and 2 illustrates the response of the model to a one standard deviation supply shock and government spending shock, respectively. The Figures demonstrate the differing impact of alternative specification of rules for closure, based on the endogenously and exogenously specified rules described above. The solid line corresponds to the behaviour under the endogenously identified rule, while the dashed and dotted lines represent responses under standard rules proxied by (26). As apparent from the figure, the dynamic response to the shock can be markedly different, both in qualitative and quantitative terms. The exogenous rule reacts to the shocks with a lag, whereas a simultaneous reaction takes place under the endogenous rule. Moreover, the behaviour of taxes and debt under the "standard" rule is clearly quite sensitive to the exogenous calibration of the speed of adjustment parameter, τ^b. The lump-sum tax of the endogenous rule is levied in such a way as to produce a smooth transition of debt back to its steady-state level, consistent with the nature and persistence of the shock and the properties of this particular model. In particular, the disequilibrium in debt immediately following the shocks under the endogenous rule is of the opposite sign of that under the "standard" rule.

[insert FIGURE 1 - response to a 1% supply shock]

[insert FIGURE 2 - response to a 1% government spending shock]

5 Conclusions

This paper has attempted to show that, as an alternative to the standard practice of exogenously imposing a condition to guarantee closure in the fiscal sector, an endogenous fiscal closure rule can be derived on the basis of the existing structure of a macroeconomic model. Specifically, the rule is obtained by identifying the stable manifold of the system. An illustration of the means by which a model-based fiscal rule can be derived is provided in the context of a small-scale model. In principle, however, this methodology is transferable to larger and more complex macroeconomic forecasting models. Although the complicated nature of some of the large-scale models currently in use would require a careful implementation of the methodology outlined here, the basic principles would remain: a rule can be derived in explicit form, whereby the fiscal authority reacts systematically to a variety of shocks. We show that the presence of optimising agents, combined with some other mild conditions, is sufficient to identify a unique saddlepath leading to a stable equilibrium.

The model-based rule outlined in this paper shares many of the desirable features of exogenously imposed rules. Most importantly, it guarantees solvency on the part of the government, rules out instrument instability, is consistent with some economic intuition and is relatively straightforward to interpret. In addition, it possesses some additional appealing properties not shared by exogenous fiscal rules. First, the rule is consistent with the specification of other sectors in the economy. In this respect, the rule is consistent with the setup of the model in which it is implemented – by design – meaning that a change in structural parameters will automatically be reflected in the fiscal rule. Unlike many exogenously imposed fiscal rules, the impulse response of variables is entirely consistent with the optimal time path of adjustment of agents within the model, and adjustment is not dependent on exogenously calibrated speed-of-adjustment parameters. Second, it is state-contingent. Exogenously imposed fiscal rules may involve acyclical features, where, for example, adjustment of taxes is dependent solely on the observed deviation of the deficit or debt from its target value. The endogenous fiscal rule derived here, on the other hand, is consistent with the setup of other sectors in a rational expectations-based model by construction. Consequently, it is also shock-specific in its response, which is a desirable property from an economic point of view. At a minimum, the derivation of the explicit version of a model-based rule gives some guidance regarding the functional form of a fiscal rule.

The functional form of the proposed rule – involving a state-contingent fiscal policy response aimed at offsetting/ counteracting shocks that drive the economy away from its saddle path to steady state at any point in time – represents purely a practical choice for the purposes of efficient model closure. Obviously, the rule is only intended to approximate actual fiscal reactions, as the heterogeneity of shocks affecting public finances and a reasonably wide range of possible government reactions makes it impossible to define a standard rule which can approximate actual behaviour of the fiscal authority in all cases. Accordingly, this study abstracts from normative issues concerning fiscal rules and their appropriateness for characterising actual fiscal behaviour. Nevertheless, the functional form of the rule is not without some economic interpretation. Specifically, a "rule-based" scheme for the implementation of discretionary fiscal policy – extending fiscal stabilisation of a purely automatic nature to append systematic (and symmetric) discretionary stabilisation – has also been suggested in some recent studies. Wren-Lewis (2000) argues that the main problems of discretionary fiscal policy – covering institutional lags and political economy considerations – could be mitigated by mandating an outside independent body with the role of changing certain tax rates or allowances on a temporary basis within certain limits. Similarly, Seidman (2001) discusses the possibility of "automating" certain fiscal policy changes in the United States, allowing the fiscal authority (or an independent body delegated by the fiscal authority) to change tax rates on the basis of economic conditions.

On the basis of our findings, there remain several possibilities for further work. Perhaps the most interesting avenue for further work would be to apply the analysis to a more complex framework, such as a large-scale macroeconomic model typically used for forecasting purposes. This would permit a rigorous comparison of the properties of endogenous fiscal rules against exogenously specified rules in the context of such larger-scale models. In conjunction with this, one could rigorously test the robustness of the alternative specifications of fiscal closure rules to parameter changes in these models. Moreover, one could assign the role of fiscal adjustment to a non-neutral component of the government budget (such as a distortionary tax rate) in order to gauge the implications of adjustment for the behaviour of other sectors in the model, although this may come at the cost of making the interpretation of associated simulations less straightforward. More generally, a detailed study of the interaction between various specifications of rules governing the behaviour of the fiscal and monetary authorities in these models could provide some interesting insights.

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Figure 1
Impulse response to a supply shock

Solid line: endogenous fiscal rule. Dashed line: standard fiscal rule, τ^{b}=0.8. Dotted line: standard fiscal rule, τ^{b}=0.1.

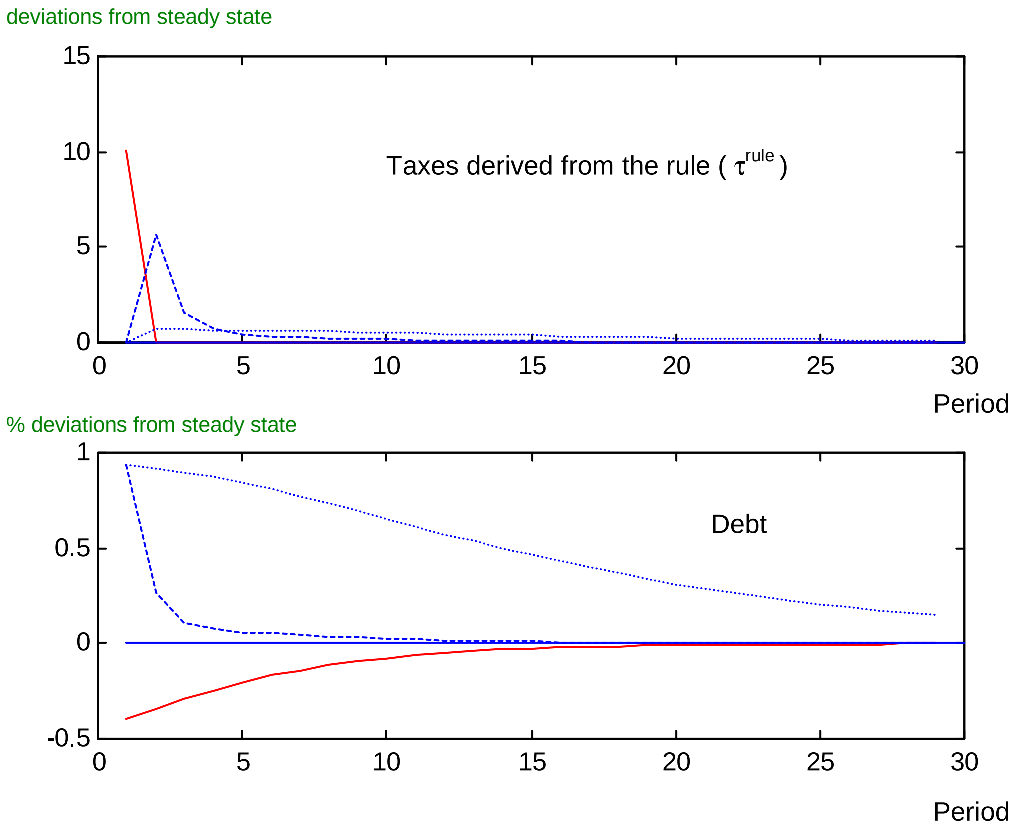

Figure 2
Impulse response to a public spending shock

Solid line: endogenous fiscal rule. Dashed line: standard fiscal rule, τ^{b}=0.8. Dotted line: standard fiscal rule, τ^{b}=0.1.