Implementing the 35 Hour Workweek by Means of Overtime Taxation*

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Summary: In this paper we study the implications of taxing overtime work in order to reduce the workweek. To this purpose we study the roles played by team work, commuting costs and idiosyncratic output risk in determining the choice of the workweek. In order to obtain reliable estimates of the consequences of our policy experiment, we calibrate our model economy to the substitutability between overtime and employment using business cycle information. We find that a tax-rate of 12% of overtime wages implements the desired reduction of the workweek from 40 to 35 hours (12.5%). We also find that this tax change increases employment by 7% and reduces output and productivity by 10.2% and 4.2%, respectively. We also study a model economy with cross-sectional variations in the workweek that arise from plant-specific output risk and we find that in this model economy the tax-rates needed to achieve the same workweek reduction are significantly larger. Finally, we find that taxing overtime dampens the business cycle fluctuations and that its welfare costs seem to be very large.

Keywords: Workweek; Overtime; 35 Hours week; Labor Policy.

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1 Introduction

In some European countries trade unions, economic analysts and policymakers alike have proposed to shorten the workweek as a means to increase employment —France, for instance, has recently enacted a 35 hour workweek and other countries like Germany are giving similar measures serious consideration. In the academic literature Fitzgerald (1996), Fitzgerald (1998), and Marimon and Zilibotti (2000) amongst others have studied the implications of imposing legal restrictions on the number of hours worked. In this paper we study this problem from a different angle and we look for the tax-rate on overtime that results in a 35 hour workweek as part of the equilibrium of a fully explicit model economy.

Our analysis shows that the size of the overtime tax-rate that achieves this objective depends crucially on the degree of substitutability between the duration of the workweek and the size of the labor force (hereinafter referred to as the substitutability between the workweek and employment). Consequently, we are very careful in providing a tight measurement of this degree of substitutability, for which we use business cycle fluctuations of hours and employment. More explicitly, by the substitutability between the workweek and employment we mean the following: When a firm wants to change the labor input of a plant, the firm can change either the workweek of its existing labor force, or the size of its labor force while keeping the workweek constant. In this paper these two ways of increasing the labor input are imperfect substitutes for the following two reasons: *(i)* team-work; and *(ii)* additional frictions that we have modelled as congestion based commuting costs.

By team-work, we mean that a plant can only be operated when all its workers are present, and hence, that the length of the workweek is common to every worker in the plant. Consequently, when a plant changes its workweek, the amount of capital available to each worker does not change. On the other hand, when a plant changes the size of its labor force, the amount of capital available to each worker also changes. This implies that workweek length and employment are imperfect substitutes, inducing a form of decreasing returns to employment that do not apply to the workweek. Moreover, it also implies that the wage-rate is a non-linear function of the number of hours worked.

Undoubtedly, one could think of many frictions that make the workweek and employment less than perfect substitutes. In this paper we have chosen to model these frictions solely as the result of congestion based commuting costs. Commuting implies that workers have to use a certain amount of time before they provide any labor services. Furthermore, in this paper we assume that commuting creates an externality and that, consequently, commuting costs are increasing in employment. We make this assumption to impose further restrictions on the substitutability between the workweek and employment. The imperfect substitutability between employment and hours per worker introduces a non convexity in the choice set that we deal with, following Hansen (1985) and Rogerson (1988), by assuming that agents have access to employment lotteries. It goes without saying that we do not intend these commuting costs to be taken literally. Instead, we think of them as a stand in for all the frictions that limit the substitutability between the workweek and employment. What gives commuting costs a modelling advantage over alternative mechanisms, such as the existence of firm-specific human capital, internal commuting costs, or adjustment costs to move in and out of the labor force, are the serious technical difficulties that these other frictions pose that prevent their use in models that aggregate nicely into the representative agent construct.

In all other respects our model economy resembles the standard business cycle model when overtime is not taxed. We calibrate a baseline model economy so that the average duration of its workweek is 40 hours and so that it mimics the main features of the U.S. economy. In addition to the standard steady–state properties, we also target some business cycle statistics (the relative volatility of employment and hours per worker) to get a tight measurement of the parameters that govern the frictions. We have discussed why our calibration seems reasonable to us by comparing our estimates with those in direct empirical studies. We have also compared with Cho and Cooley (1994) results (that use PSID data at a steady state frequency), and showed that they are extremely similar, what lends support to our measurements.

Next, we look for the tax-rate on overtime that reduces the workweek from 40 to 35 hours in steady-state. We find that a 12% tax-rate on overtime wages achieves this end. We also find that this policy brings about a 7% increase in steady-state employment, a 10.2% decrease in steady-state output and a 4.2% decrease in steady-state productivity. In addition, we compute the transition between the steady-states of the two model economies in order to measure the welfare costs implied by overtime taxes. We find that this welfare costs are very significant. Specifically, they are at least 0.6% of average consumption on a flow basis, which is a large number as far as welfare calculations go.

We also study the effects of overtime taxation in an model economy where plants face idiosyncratic output shocks and, consequently, the workweek varies across plants. In this model economy we find that the overtime tax-rate that achieves the desired five-hour reduction in the workweek is significantly larger than the 12% that obtains in the benchmark model economy (in which the workweek is the same in the entire economy), but the findings concerning the effect on employment are quite similar.

We also compare the business cycle behavior of the benchmark model economy with that of the model economy with overtime taxes and we find that this tax reduces the volatility, both of output and of the labor input, rather considerably. However, since in our model economy business cycles are the endogenous responses to productivity changes, this reduced volatility should not necessarily be interpreted as a good thing.

Finally, we explore the robustness of our findings with respect to our identifying assumptions. We have explored the fact that not all workers may be affected by the restrictions on overtime, different values of the degree of team work versus fatigue (the properties of the production function), different estimates of the size of the workforce, different values of commuting times, and we have found that the results are very similar to those reported above, as long as the model economies are calibrated to the same degree of substitutability between the workweek and employment.

Henceforth, our findings should be taken with care, given that our strategy to measure the substitutability of hours and employment is based on business cycle information. It is conceivable that what shows as a business cycle friction is not a low frequency friction, and that the actual substitutability between hours and bodies is larger than what we describe in this paper. As we describe below, the exploration of this issue (that there are costs in moving in and out of employment) requires some future technical developments. An additional caveat that we ignore in this paper is the existence of changes in *work practices*, that go together with the reduction of the workweek, that may increase firm's productivity. We think that to account for this issue similar techniques to the one developed in this paper are needed, together with a detailed description of what those productivity gains may be. We leave this issue outside the current study. Finally, we state another major caveat which has to do with the fact that there are not distributional effects in our study. That is, every agent, workers and non-workers, is worst after the policy, not only ex-ante because everybody has a higher probability of ending up working, but also ex-post. The lack of distributional issues in the model makes the implementation of the policy that we study at least puzzling, since it is welfare reducing. The inclusion of some form of market incompleteness would have given an insurance role to this work sharing policy and a rationale to its implementation. Obviously, the lack of distributional concerns is another limitation of our analysis, but as we have already said, our primary interest in this paper is to quantify the trade-offs between employment and productivity, not to explain why a government would put in place such a policy. All these considerations show that our findings should be considered as a first approximation to the study of the implications of work sharing policies. Moreover, we see our contribution as a methodological benchmark, that can be extended to include additional aspects that are currently missing, of the type of explicit modelling that is needed to assess work sharing policies.

On the technical side, in this paper we develop the methods needed to compute the equilibria of non–convex business cycle economies where the Second Welfare Theorem does not hold because of distortionary taxation. This feature of our model economies forces us to compute equilibria directly. This turned out to be a relatively daunting task because households must know the wage function in order to compute their decisions and, as we have already mentioned, in our model economy wages are non-linear functions of hours. These wage functions are part of the fixed-point problem that must be solved to compute the equilibrium.[1] Our techniques are not easily applied to the characterization of equilibria with all kinds of frictions. We would have liked to use labor adjustment costs, as in Kydland and Prescott (1991), to introduce the friction in the substitutability between employment and hours. Unfortunately, doing so imposes an additional level of extreme complexity, since lagged employment status would enter as an additional state variable in the household problem, implying the existence of an endogenous distribution of households indexed both by their lagged employment status and by their asset holdings. In general, even with complete markets and with all households starting being alike, this would imply that, as time goes by, households would become heterogeneous with respect to their asset holdings and their lagged employment status, precluding the use of the representative household abstraction that we have used here.

It goes without saying that this is not the only paper that has studied the implications of policies designed to reduce the workweek. Perhaps the work that shares the most the spirit of ours is Fitzgerald (1996) and Fitzgerald (1998). In those papers Fitzgerald studies economies in which two types of agents are needed for production, and he evaluates the consequences of policies that impose quantitative legal restrictions on the number of hours worked. He finds that the increases in employment that result from this type of policies are substantial. However, as we discuss below, his calibration strategy is likely to over-predict the response of employment to this type of policies. Not surprisingly, he is aware of this fact and he explicitly declares that he does not attempt to obtain a precise quantitative assessment of these responses. Marimon and Zilibotti (2000) also study this issue. They are interested in the implications of the restrictions in the duration of the workweek in a model of job search and matching frictions, in which job creation entails fixed costs, and in which there are decreasing returns to the workweek. In their model economy the main reason that makes the quantity restriction operative is that it affects the bargaining power of workers and firms asymmetrically. They find that, in general, restricting the duration of the

[1] In the appendix we describe in some detail the computational methods that allow us to solve this problem.

workweek benefits both the employed and the unemployed workers —even in the cases when it decreases the wage-rate— and that it reduces both profits and output. They also find that, in general, the effects on employment of these quantity restrictions are very sensitive to the degree of substitutability between consumption and leisure. Their work characterizes qualitatively the outcomes for special cases of functional forms of the utility and production functions.

Our paper builds on several different strands of the literature. On the one hand, we use the distinction between hours and bodies used in Ehrenberg (1971), and later in Kydland and Prescott (1991), Fitzgerald (1995) and Fitzgerald (1998). We also build on the work of Prescott and Townsend (1984), Hansen (1985), Rogerson (1988), Prescott and Ríos-Rull (1992), Kydland and Prescott (1991) and, especially, on Hornstein and Prescott (1993). All these papers use lotteries to get around non-convexities in general equilibrium macroeconomic models.

There are some papers that have used different mechanisms to get adjustment along both margins, hours and bodies.[2] The closest to ours is Cho and Cooley (1994). They also introduce fixed costs associated with labor supply that depend on employment and relate them to household production, although they acknowledge that they could also be interpreted as costs of commuting or getting ready to work. The difference, though, is that they depend on individual employment and, therefore, there is no externality. They are interested in studying the implications of this model for the elasticity of substitution of labor supply, and the volatility of hours, productivity and employment. They find that, for the model to be consistent with the empirical elasticities of labor supply, fixed costs must be a function of employment, what lends support to the mechanism that we use in this paper.

In many senses the paper that we are closest to is Kydland and Prescott (1991). In that paper the calibration of the model to the relative volatilities of hours and employment also plays a central role, even though the data are matched through a different modelling

[2]See Cho and Cooley (1994), Bils and Cho (1994), Cho and Rogerson (1988) and Card (1989).

device than the one that we use here. Kydland and Prescott (1991) use a version of our benchmark model economy to study the role played by productivity shocks in accounting for the business cycle volatility of U.S. output. Their economy satisfies the welfare theorems and, therefore, Kydland and Prescott use the social planner's problem as a shortcut to calculate the competitive equilibrium allocation of their economy. They show that an economy that differs from a standard real business cycle model only on the existence of team-work has the counterfactual property that all the variation in the labor input occurs along the employment margin, while hours per worker remain constant. They then add labor adjustment costs to their team-work model economy and show that this feature allows them to replicate the observed substitutability between the workweek and employment. The model economy with team-work and labor adjustment costs also satisfies the welfare theorems, but in this case the lagged employment status becomes a part of the individual households' state variables.

Section 2 describes the model and the use of lotteries to implement its recursive competitive equilibrium. We also adapt the model to include overtime taxation and idiosyncratic shocks. Section 3 describes how to map the model to data. Section 4 reports the results for the baseline model, the transition, the model where plants face idiosyncratic output shocks and the business cycle properties implied by the policy. We perform some robustness checks in Section 5. Finally, Section 6 offers some concluding comments. Appendix A briefly describes the computational methods that we have used, while Appendix B describes the tax function.

2 The model

We start in Section 2.1 with the description of households and preferences and then we move to technology in Section 2.2. Next, in Sections 2.3– 2.3.3 we describe the contracts that agents use and the problems that they solve. In Section 2.4 we go on to define equilibrium recursively in a way that is suited for computation. We then expand the economy to include taxes in Sections 2.5. Section 2.6 extends the economy to have shocks to firm specific productivity. In response to those shocks, firms can adjust the number of hours per worker but not their employment. This fact implies that in the cross-section there are more than one type of workweeks.

2.1 Households and Preferences

There is a continuum of *ex-ante* identical agents of measure one, with preferences given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t \; u(c_t, \ell_t)\right\} \tag{1}$$

where c_t is consumption and ℓ_t is leisure in period t. The instantaneous utility function is strictly concave and satisfies the Inada conditions. Finally, $\beta \in (0,1)$ is the subjective time discount factor.

An individual's time endowment in each period is one. The amount of time that can be allocated to work is $1-\ell-\eta(N)$, where $\eta(N)>0$ measures the amount of time required for commuting to work every period that the individual is employed, and where N is aggregate employment. We are assuming that there is congestion which creates a negative externality in employment. In particular, we assume that $\eta(N) > 0$, and $\eta'(N) > 0$. Notice that $\ell(h,N)$ is not a continuous function since if hours worked are zero, no commuting is needed. This introduces a non-convexity. We will see in detail below how agents deal with the non-convexity. For technical reasons we define an underlying consumption possibility set $C = \{[0,\bar{c}],[0,1]\}$, where $\bar{c}$ is an upper bound in consumption that is non-binding.[3]

[3]See Prescott and Ríos-Rull (1992) for a detailed explanation of why we impose this upper bound and for why it is irrelevant.

2.2 Plant's Technology

Production takes place in plants of which there may be a large number. Moreover, new plants can be opened at zero costs. Plants are operated during a number of hours that is *the same* for all workers. Plants also use capital and they are restricted to have one shift.[4]

We write the plant's production function f as

$$f(z, h, k, n) = z\, h^{\xi}\, k^{1-\theta}\, n^{\theta} \tag{2}$$

where h denotes the workweek, n employment in the plant and k the amount of capital in the plant.[5] Variable z represents total factor productivity and we use it to incorporate shocks to productivity. When we describe the stochastic version of the economy we assume that z follows a first order Markov process. Capital depreciates geometrically at rate δ.

Note that given the workweek, plants are subject to constant returns to scale.

When $\xi = \theta$, we have the standard Cobb-Douglas technology where total hours and not its decomposition in hours per worker and employment is what matters. When $\xi = 1$, the technology is linear in hours and we have an extreme case of team production, where workers are not subject to fatigue (an increase in the workers' workweek results in an increase of output of the same proportion).

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[4] While some advocates of workweek reduction policies might have in mind gains in employment due to increases in the number of shifts, we think that team–work is a more important feature in actual plants. Accordingly, we chose this technology to capture how this policy affects firms that need to increase their labor force for pick demand reasons (see Beers (2000) for a discussion of both shift work and flexible schedules in recent U.S. data).

[5] In a sense, this form of writing the plant production function is a reduced form. We could redefine h to be the hours worked by the worker that works the least. However, given constant returns in bodies and capital, the plants can split at no costs into units where all workers work the same number of hours.

2.3 Contracts

We assume complete markets. In this economy with non-convexities, there are efficiency gains from the introduction of lotteries.[6] The non-convexities apply only to the labor market, so we only need to specify lotteries for the labor contract. Moreover, rather than having plants buy measures of agents, as for example in Prescott and Ríos-Rull (1992), we find that it is easier to describe the economy by posing an, otherwise irrelevant, insurance firm that contracts in measures with households and that contracts in real quantities with the operators of plants. In the absence of distortionary taxation and the commuting externality, the equilibrium would have been optimal under this market structure.[7]

2.3.1 The firms' problem

To see the nonlinearity of the wage as a function of the workweek, we start fixing the workweek, h, and we use w_h to denote the salary paid to a worker who works for h hours. Then the problem of a firm with an h hour workweek is the following

$$\max_{k,n} \; z\, h^{\xi}\, k^{1-\theta}\, n^{\theta} - k(r+\delta) - n\; w_h \tag{3}$$

where r is the rental rate of capital (the interest rate). There is free entry, which implies that firms have zero profits. Moreover, there are constant returns to scale, so we can normalize the size of a firm to have one employee that works h hours. This means that for any given r, we can solve

$$\max_{k} \; z\, h^{\xi}\, k^{1-\theta} - k(r+\delta) - w_h \tag{4}$$

with solution given by $k(z, h, r)$.

[6] See Hansen (1985), Rogerson (1988) Prescott and Ríos-Rull (1992), for earlier applications of lotteries to labor contracts.

[7] Obviously, the actual details of what types of firms buy and sell these lotteries do not matter. We could have chosen other arrangements without changing the equilibrium allocations (see Prescott and Ríos-Rull (1992)).

We then determine the salary for workweek h as the value of w_h that satisfies

$$0 = z\, h^{\xi}\, [k(z, h, r)]^{1-\theta} - k(z, h, r)(r + \delta) - w_h \tag{5}$$

using the zero-profit condition.

In fact, we can do this for all h and we obtain the wage rate w_h as a function of the interest rate. As we can see from equation (5) this is a non-linear function of h. In sum, plants can be indexed by their workweek and their capital per worker.

To describe the number of existing one-worker plants at any point in time we can use a measure Ψ defined not over C, but just over its second component, the workweeks. Let $\Psi(B)$ be the measure of plants of size one worker that operates a workweek of size $h \in B \subset [0, 1]$ for any Borel set B.

Aggregating over firms we get that aggregate output can be written as

$$\int_{[0,1]} z\, h^{\xi}\, k(z, r, h)^{1-\theta}\ \Psi(dh) \tag{6}$$

and aggregate employment, N, can be denoted as

$$N = \int_{[0,1]} \Psi(dh) \tag{7}$$

while aggregate total hours (not per worker) is given by

$$H\, N = \int_{[0,1]} h\ \Psi(dh) \tag{8}$$

2.3.2 Households choices

Let $\mathcal{C}$ be an appropriate family of subsets of C, say, its Borel σ–algebra. Households choose probabilities over $\mathcal{C}$. Let $\mathcal{M}$ be the space of signed measures that includes the probabilities.

The per-period consumption possibility set of households is indexed by aggregate employment in the period:

$$X(N) = \{x \in \mathcal{M} : \quad x \text{ is a probability}, \;\; i.e. \; x \geq 0, \quad \text{and} \quad x(C) = 1, \tag{9}$$
$$\text{If} \quad h \in (0,1], \quad \text{and } x\left([0,\bar{c}],[h,1]\right) > 0, \; \text{then } h \leq 1 - \eta(N)\} \, .$$

where the last condition is the requirement that in no case working hours plus commuting time is greater than the time endowment.

A household that chooses x has indirect instantaneous utility function given by

$$U(x, N) = \int_C u[c, \ell(h, N)] \;\; dx. \tag{10}$$

Note that function U is linear in x.

The price of x is given by a linear function, which we write as $\int_C q(c,h) \; dx$, where $q(c,h)$ gives the value of consuming c units and working h hours with probability one. A detailed discussion of q can be found in Section 2.3.3.

Moreover, households own assets a, and choose savings that we denote by a'. Since working does not have dynamic implications (a period later agents with wealth a' are identical regardless of what was the labor situation today) all agents with the same assets choose the same savings independently of the outcome of the lottery. These considerations imply that the budget constraint of the household is

$$\int_C q(c,h) \;\; dx + a' = (1+r) \;\; a.$$

We can define an indirect current return function R that takes as given the saving behavior of the household and solves for the optimal x. The static household problem given its saving behavior is to solve

$$R(a,N,q,r,a') = \max_{x\in X(N)} \quad U(x,N) \tag{11}$$

$$\text{s.t.} \quad \int_C q(c,h) \;\; dx + a' = (1+r) \;\; a \tag{12}$$

where both the objective and the constraint are linear.

We write $x(a,N,q,r,a')$ as the optimal choice for a household with a assets, that saves a', when aggregate employment is N, and prices are given by function q and by r.

2.3.3 The intermediate insurance companies

These companies have zero profits and their only role is to insure the households. They deal with both households and firms and they choose signed measures $y \in \mathcal{M}$. In exchange of y that sells at price q, these firms acquire the rights to working time that they sell to plants at price w_h and provide to consumers the consumption implied by the lottery. These insurance firms contract with a large number of households which allows a law of large numbers to hold (see Uhlig (1996)), which precludes any aggregate uncertainty over the realizations of the lotteries.

Their problem is to

$$\max_y \int_C \; q(c,h) \; dy + \int_C \; w_h \; dy - \int_C \; c \; dy \tag{13}$$

This problem only has a solution if the pricing function is such that its consumption component only depends on the first moment of the measure y and if the wage is given by the function w_h. This implies that the pricing function q satisfies

$$\int_C q(c,h) \; dx = \int_C c \; x(dc,[0,1]) \; - \int_C w_h \; x([0,\bar{c}],dh) \tag{14}$$

and, hence, we can rewrite the households' budget constraint as

$$\int_C c \; x(dc, [0,1]) \; + a' = (1+r) \; a + \int_C w_h \; x([0,\bar{c}], dh) \tag{15}$$

Accordingly, we redefine the current return of the household after the static optimization problem described in equation (11) as $R(a, N, w_h, r, a')$ and the lottery choice as $x(a, N, w_h, r, a')$.

An important property of $x(a, N, w_h, r, a')$ given the properties of w_h and q[8] is that it has positive mass in at most two points, one of which is $\{c, 0\}$ where $c \in [0, \bar{c}]$. It is easy to see why this is the case. Strict concavity of function u, and convexity of the choice set for $h > 0$ shows that households prefer to get a point with mass on only one h than another point with mass in more than one point with positive h and the same mean.

So, the solution to the household problem can only be one of the following three possibilities. One, there is positive mass in only one point with $h > 0$; two, there is positive mass only in $h = 0$; and three, there is positive mass in two points, one with $h = 0$ and one with $h > 0$. The use of a production function that satisfies the Inada conditions guarantees that in our economies there is always mass at some $h > 0$. We denote by $h(a, N, w_h, r, a')$ the point with positive mass in $h > 0$ and by $n(a, N, w_h, r, a')$ the mass at that point.

2.4 The recursive problem

Once we have set the within periods contracting problems, we can turn to define equilibrium for the economy. We will do so recursively and we start by defining the aggregate state variables of the economy. These are those variables that are needed to evaluate and forecast all prices. For this economy, they are total factor productivity z and aggregate capital K. The household's individual asset level a is also part of the individual state vector. In order for a household to solve its maximization problem it has to be able to compute the values

[8] Actually this is a property derived from a standard result in linear programming, see Hornstein and Prescott (1993).

for $\{r, w_h, K', N\}$. We assume that the household uses functions $\{\phi_r, \phi_{w_h}, G_K, G_N\}$ to do so. These functions map the aggregate state variables into the variables that the household needs to know to solve its maximization problem. The value function in expression (16) is indexed by these functions for clarity. In later expressions we omit the indices. We have

$$v(z, K, a; \phi, G) = \max_{a'} \; R(a, N, w_h, r, a') + \beta \; E\{v(z', k', a'; \phi, G)|z\} \tag{16}$$

$$\begin{aligned} &\text{s.t.} && (17)\\ r &= \phi_r(z, K) && (18)\\ w_h &= \phi_w(z, K, h) && (19)\\ K' &= G_K(z, K) && (20)\\ N &= G_N(z, K) && (21)\\ H &= G_H(z, K) && (22) \end{aligned}$$

Let $a' = g_a(z, K, a; \phi, G)$ denote the solution to this problem. Substitution of this solution in (11) yields $x(z, K, a; \phi, G)$, and given that our problem puts mass in at most two points, it also yields $h = g_h(z, K, a; \phi, G)$ and $n = g_n(z, K, a; \phi, G)$. Note that equation (22) is not really necessary to solve the problem of the household. We include it just for completeness sake and to state that this function is needed to define an equilibrium. We are now ready to do so.

Definition 1 *A recursive competitive equilibrium is a set of decision rules for households* $\{g_a, g_h, g_n\}$*, a value function* v*, functions for aggregate variables* $\{G_K, G_H, G_N\}$*, for the interest rate* $\phi_r(z, K)$*, a wage schedule function* $\phi_w(z, K, h)$*, a measure of firms* $\Psi(z, K)$*, and a capital renting policy of the plants* $k(z, r, h)$ *such that* i) *the decision rules and value function satisfy* (16), ii) *the agent is representative, i.e.* $g_a(z, K, K; \phi, G) = G_K(z, K)$, $g_h(z, K, K; \phi, G) = G_H(z, K)$ *and* $g_n(z, K, K; \phi, G) = G_N(z, K)$, iii) *plants choose capital optimally and have zero profits,* i.e. *they solve* (4) *and* (5), iv) *the labor market clears,* i.e., Ψ *has mass in only one point with positive hours worked which is given by* $G_H(z, K)$ *and* $\Psi[z, K, G_H(z, K)] = G_N(z, K)$, *and* v) *the market for capital clears,* $K = \Psi[z, K, G_H(z, K)] = \dot{k}[z, \phi_r(z, K), G_H(z, K)]$.

A steady state for a deterministic version of this economy (a fixed value of total factor productivity $\bar{z}$) is a just a number K^* such that, when substituted in the above general definition of recursive equilibrium, satisfies

$$K^* = G_K(\bar{z}, K^*), \tag{23}$$

in addition to all the requirements above.

2.5 The economy with overtime taxation

An overtime tax is a policy $\tau(\bar{h}, h)$ such that if $h > \bar{h}$ then firms have to pay $\tau(\bar{h}, h) \cdot \hat{w}_h$ to the government, where $\hat{w}_h$ is the total payment that the firm has to make, $\hat{w}_h = w_h + \tau(\bar{h}, h) \cdot \hat{w}_h$. Equation (3) becomes

$$\max_{k,n} \; z\, h^{\xi}\, k^{1-\theta}\, n^{\theta} - k(r + \delta) - n[\; w_h + \tau(\bar{h}, h) \cdot \hat{w}_h] \tag{24}$$

Equations (4) and (5) also change in a similar fashion. An important feature of our computational procedure is that all the relevant objects that the agent face are differentiable. Therefore, we can use the first derivatives to help characterize the solution. To this end, we use a function τ that is differentiable at $h = \bar{h}$. The properties of this function are that $\tau(\bar{h}, h) = 0$ if $h \leq \bar{h}$, $\tau(\bar{h}, h) > 0$ if $h > \bar{h}$, $\lim_{h \to 1} = \bar{\tau}$, $\frac{\partial \tau(\bar{h}, h)}{\partial h}$ is non decreasing. Note that given these assumptions, for the tax to have any effects, $\bar{h}$ has to be lower than the target of hours worked.[9]

[9] See Appendix B for details.

All the proceeds of the overtime tax are redistributed lump sum to the households. This changes equation (12) to

$$\int_C q(c,h) \;\; dx + a' = (1+r) \;\; a + T \tag{25}$$

where T are the transfers.

Equation (15) also has to be adjusted, specifying the transfers not as a number T, but as a function $T(z, K)$.

In addition to the changes in the profit function of firms and in the budget constraint of the household, we have to add a balanced budget condition for the government to the definition of equilibrium. This condition is simply that

$$T(z,K) = \tau[\bar{h}, G_H(z,K)] \tag{26}$$

2.6 Shocks to Plant level productivity: the heterogeneous workweek model

Next, we introduce a new element in our model economy to induce cross–sectional variation of workweeks across plants, as a result of plant specific shocks. This new element implies that taxing overtime adversely affects the flexibility of firms to adjust their labor input to temporary changes in productivity (or demand). In the calibration stage we discuss the extent to which this may be the case.

To model the importance of plant level flexibility, we assume *i.i.d.* transitory shocks to plant level productivity, revealed after the workers have been hired but before production takes place, and independent from the economy wide productivity shock.[10] Consequently, the only margin that can be used to exploit this additional productivity change is to vary

[10] A more theoretically consistent way to describe this would be to say that it exists a time to hire restriction. The nature of the timing is not dissimilar to that in Burnside, Eichenbaum, and Rebelo (1990), although, in that paper hours cannot adjust. This shock could be interpreted as a demand shock, but it is simpler to specify it as a plant specific productivity shock.

the plant's workweek. The new plant level production function is given by

$$z\, s\, h^{\xi}\, k^{1-\theta}\, n^{\theta} \tag{27}$$

where all variables are as before except for the plant specific shock, s. The shock takes only finitely many values $s \in \{s_1, \cdots, s_{m_s}\}$ and is drawn from probability distribution γ_s.

This change requires the indexation of agents' choices by the possible realizations of the shock, resulting in

$$\begin{aligned} X(N) = \{x_1, \cdots, x_{m_s} :\ x_s \in \mathcal{M} :\quad & x_s \text{ is a probability, } i.e.\ x_s \geq 0,\ x_s\,(C) = 1, \\ & x_s([0,\bar{c}], \{0\}) \text{ is the same for all } s, \\ & \text{if } \ h \in (0,1], \ \text{ and } x_s\,([0,\bar{c}],[h,1]) > 0, \text{ then } h \leq 1 - \eta(N)\}\,. \end{aligned} \tag{28}$$

Note that all we are adding is that the employment probability cannot depend on the idiosyncratic shock of the plant. From the point of view of the firm, equation (4) has to be rewritten. We arbitrarily specify a workweek of $h(s)$ hours for $s \in \{s_1, \cdots, s_{m_s}\}$, and we get

$$\max_{k} \ z\, k^{1-\theta} \sum_{s} \ \gamma_s \ s \ h(s)^{\xi} \ - \ k(r+\delta) - w_{\{h(s)\}} \tag{29}$$

with solution given by $k(z, \{h(s)\}, r)$. The zero profit condition requires that for each vector $\{h(s)\}$, the salary $w_{\{h(s)\}}$ satisfies

$$0 = z\,[k(z,\{h(s)\},r)]^{1-\theta} \sum_{s} \ \gamma_s \ s \ h(s)^{\xi} \ - \ k(z,\{h(s)\},r)(r+\delta) - w_{\{h(s)\}} \tag{30}$$

The rest of the changes to adapt the model to the case with idiosyncratic shocks to firms is a tedious minor variation of the equations described above and we omit them for brevity.

3 Mapping the Model to Data

In order to calibrate the model we have to be aware that this model is standard in all dimensions except for the existence of team work and an externality based commuting costs.

Team work is described by the parameter ξ. This feature is hard to measure. We choose the economy to be midway between the standard Cobb-Douglas case where $\xi = \theta$, and the strict fatigue-less case where $\xi = 1$, and we set $\xi = .85$. As part of our robustness tests we will look at economies that are closer to these extremes. The choice of this parameter affects the choice of the other parameters. Therefore, when we look at economies without fatigue, we recalibrate the economy.

Time spent in commuting is described by function $\eta(N)$, and we assume that it takes the following form:

$$\eta(N) = A_N N^{\lambda} \tag{31}$$

With regard to the rest of the model, we choose the time period to be a quarter and we assume that household preferences can be described by the following standard Cobb-Douglas function in consumption and leisure

$$U(c_t, \ell_t) = \frac{\left[c_t^{\alpha}\ \ell_t^{1-\alpha}\right]^{1-\sigma} - 1}{1-\sigma} \tag{32}$$

where $0 < \alpha < 1$ and $\sigma > 0$.

Therefore, the model has 10 parameters. Two of those parameters characterize the process for the Solow residual, the auto–regressive coefficient ρ and the variance of the shock σ_ϵ. We take these measurements from Prescott (1986). Another parameter is the already mentioned parameter ξ (in Section 5 we explore alternative choices for this parameter as a robustness check). The model has 7 additional parameters: θ, δ, β, α, σ, A_N and λ. We need to impose seven conditions to set these 7 parameters. The conditions that we impose are

1. A labor share of 64%.
2. A steady state yearly interest rate of 4%.
3. A steady state consumption to output ratio of .75.
4. A steady state fraction of the working-age population who work of 75%. See Kydland and Prescott (1991) for a discussion of this choice.
5. We impose a 40 hour workweek. We assume that out of the 168 hours in each week, 68 of them are devoted either to sleeping or personal care. This implies that workers work 40% of their time. See Cooley and Prescott (1995) for a discussion of this choice.
6. The relative volatility of hours and bodies is .5 as in the U.S. data. See Kydland and Prescott (1991) or Cooley and Prescott (1995).
7. Average commuting time of 5 hours a week (30 minutes each way).

   This target requires some further comments. The actual value of this target is not very important. What is important is the role that it plays in creating a friction between adjusting the workweek and adjusting employment. This role depends more on the derivative of $\eta(N)$ than on its value. In section 5 we increase the commuting time to almost ten hours per week (fifty minutes each way) as a robustness exercise.[11]

Imposing those 7 conditions results in the values of the parameters reported in Table 1. Note that the coefficient of risk aversion is determined as part of the calibration process unlike many other studies where it is exogenously set at some level.

Obviously, this is not the only plausible calibration strategy. In particular, a natural question in this context is to what extent can business cycles variation be informative about the substitutability between hours and employment, and how does it relate to alternative

[11]See French (2000) for an estimation of commuting costs.

Table 1: **Baseline Economy Parameters.**

ξ	A_N	λ	α	σ	δ	θ	β
.85	.35	6.75	.33	1.5	.025	.64	.99

measurement procedures that draw on microeconomic observations to calibrate. The latter approach has been used by Bils and Cho (1994) and Cho and Cooley (1994). Both studies use individual variation from the PSID to find the values of some preference parameters. In fact, Cho and Cooley (1994) use two different calibration strategies. The first one is quite similar to ours, in the sense that they use aggregate data. Specifically, they fix the value of one of the preference parameters and get the others to fit three observed facts for the US economy (the employment rate, the proportion of time spent in labor market activity and the ratio of fluctuations in hours per person relative to that in employment). Their second calibration strategy uses the first two facts mentioned above and draw on microeconomic observations to calibrate one the remaining preference parameters, as in Bils and Cho (1994). Both papers use first order conditions to get a relationship between employment and hours that they estimate using PSID data. Then, they use the regression coefficient to recover one of the preference parameters, while the other is arbitrarily fixed to some value as in the first approach.

We prefer to use aggregate data for our calibration for several reasons. First, there seems to be some evidence of reporting errors for the PSID regarding hours per week. In fact, Bils and Cho (1994) increase their estimate by 24%, and Cho and Cooley (1994) acknowledge that the results based on the model calibrated to microeconomic observations are not as encouraging as those obtained with the model calibrated to aggregate observations. Second, given the assumed functional forms, it is not possible to get a clean relationship relating the two relevant preference parameters, as in Cho and Cooley (1994), that allows for a simple estimation using the PSID. Third, there is not a simple answer to the issue of how to treat people in the data that move in and out of the labor force.

Table 2: **Comparison of cyclical properties of alternative calibration strategies**

Standard Deviation Relative to Output's	Cho and Cooley using Aggregate Data	Cho and Cooley using PSID Data	Baseline
Consumption	0.30	0.32	0.31
Investment	3.20	3.11	3.12
Capital	0.27	0.26	0.29
Aggregate Hours	0.60	0.43	0.43
Hours per worker	0.14	0.16	0.15
Employment	0.46	0.28	0.29
Output per worker	0.43	0.59	0.58

The source for the Cho and Cooley economies statistics is Cho and Cooley (1994).

Still, we should not ignore alternative measurements of the substitutability between the workweek and employment. In this respect, Table 2 shows the business cycle properties of our model economy and those obtained by Cho and Cooley (1994) using their two strategies. Our baseline model economy's statistics lie between the two sets of statistics obtained by Cho and Cooley, perhaps closer to the one that uses microeconomic studies.

Another justification of the validity of our calibration strategy comes from its implications for the elasticity of employment with respect to hours, a subject of numerous empirical studies. Some of these studies, Bosch (1990), Brunello (1989), DeRegt (1988) and Wadwhani (1987), find values for this elasticity in the range $[.35, .6]$. Others, such as Hunt (1996), Hunt (1997) and Cette and Taddei (1994) find slightly lower values in the range $[.1, .3]$. The elasticity of employment with respect to hours in our baseline model economy is 0.57, still in the range of plausible values. As an additional consideration, we explore a heterogeneous workweek version of the model in Section 4.3. In this economy the elasticity takes a slightly lower value of 0.51. We conclude that our approach to the measurement of the trade-offs between the workweek and employment, while being on the high side, is well within what the literature considers acceptable.

4 Main Findings

In this section we report the answers to the questions that we posed. In Section 4.1, we start reporting the steady state implications of the introduction of the policy. In Section 4.2 we report the welfare implications and the transition path of the economy once the policy is in place. In section 4.3 we show the answers for the heterogeneous workweeks model economy, and discuss the differences with the baseline. Finally, section 4.4 shows how the tax policy affects the business cycle behavior of this economy.

4.1 Steady State Findings

Table 3 shows, along the steady state of the baseline economy, the steady state of an economy where the tax policy yields the desired workweek reduction (to ease comparisons the table includes the percentage change for each relevant variable (%var), as well as the percentage change relative to the hours per worker percentage change ($\frac{\%var}{\%h}$). The table shows that a tax rate on overtime of 12% induces households to reduce hours worked by 12.5% (from 40 to 35 hours a week). The workweek reduction from 40 to 35 hours generates a 7.04% increase in employment, a 10.2% decrease in output and a 4.2% decrease in productivity. Note that due to the reduction in productivity, the reduction in the salary is larger than the reduction in hours per worker.

Table 3: **Baseline Economy Findings.** $\xi = .85$

Economy	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.0	-12.5	1
Employment	0.75	0.80	7.04	0.57
Total hours	30.0	28.0	-6.29	-0.50
Output	1.00	0.90	-10.2	-0.82
Productivity	1.00	0.96	-4.19	-0.34
Salary	0.86	0.72	-16.1	-1.29

4.2 The transition

As it is well known, an assessment of a policy cannot be carried out based on steady state comparisons since the two economies have different initial conditions. In order to assess the implications of the policy, we take the steady state of the economy without taxation and impose the workweek reduction policy.

Figure 1 shows the transition paths for the main aggregate variables that we are interested in plus aggregate capital (the aggregate state variable). To make the picture clearer, we normalize all variables so that in the steady state of an economy without taxes their value is one. The first thing to note is that the adjustment of most variables is very fast. Indeed, hours and employment immediately jump to almost their final value. Only capital adjusts slowly as there is deccumulation, and output and the salary go down slowly following the path of capital.

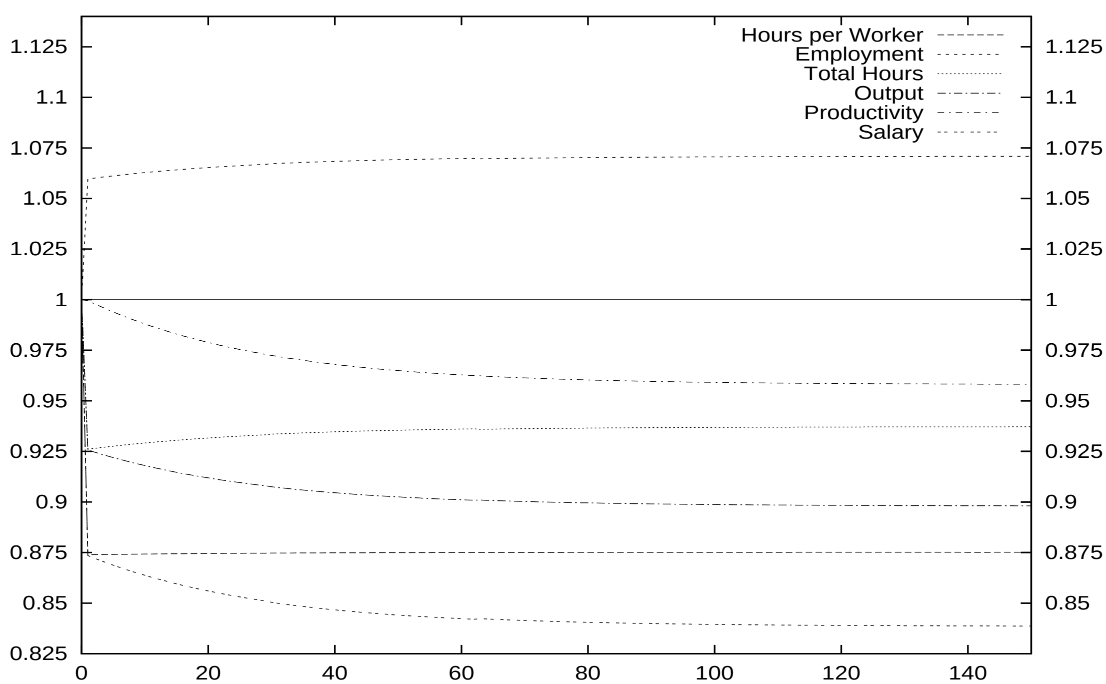

Figure 1: **Introduction of the Tax. Transition Path of the main variables.**

The computation of the transition also allows us to compute the welfare cost of the policy. We compute the welfare cost as the proportional decrease in consumption (both for the employed and the unemployed) with respect to the steady state without the policy that would leave agents indifferent to implementing the policy. Before reporting the number we state a major caveat which has to do with the type of friction that we use to measure the substitutability between hours per worker and employment. Recall that in the model there is an externality based commuting cost. As a result, the increase in employment increases the commuting time and reduces utility. If we had modelled the economy with a different type of friction, the utility cost of the policy would have been very different, and probably much smaller, as the friction does not need to have utility costs as large as those imposed by the externality. This is due in part to the fact that we calibrate the derivative of the utility to get the right relative variation of hours per worker and employment, and the differences in welfare are differences in levels of utility. Henceforth, the welfare cost that we compute is likely to overstate the actual welfare costs of the policy. The drop in average consumption that makes agents indifferent between the previous situation and the implementation of the policy is 8.1%. Because of the above considerations regarding the friction, we also computed the welfare costs that would come up if we do not take into account the changes in commuting time that are imposed by the externality. In this case, the welfare costs are of a smaller order of magnitude, .6% of average consumption. We think that the second number is a better assessment of the welfare costs of the policy, given that is likely to be more in line with what would have resulted from modelling the frictions in the labor market differently.

We want to make the point that welfare calculations usually yield very small numbers due to the concavity of the utility function, and therefore even .6% of average consumption is a large welfare cost.

4.3 Heterogeneous workweeks economy

We next perform the same experiment than in the baseline economy in a model economy where plants are subject to idiosyncratic shocks to productivity that generate a cross–

sectional distribution of workweeks across plants and households. In particular, we want to know whether our answers differ from those obtained in the baseline model economy. Recall that in this version of the model only variations in the workweek can be used to accommodate within the period the idiosyncratic shock.

4.3.1 Mapping the heterogeneous workweeks economy to data

To calibrate this version of the model, we use the cross–sectional distribution of workweeks of individuals (not of firms) as reported by the PSID. The reasons for that are simple. In the model economy plants are of size one worker, and what we really care for is the personal workweek distribution. We use a simple process to parameterize the model, an equal probability, three valued i.i.d. shock. This means that we have three parameters to set and three new statistics to match.

We select a sample of individuals that satisfy the following criteria: males, family heads, full-time workers, salaried workers, private sector jobs and a minimum of two-years experience on the main job. Some of these criteria require comments. We choose private sector workers because public firms might not adjust in the same way as private firms in response to idiosyncratic shocks to productivity. We also need those workers to have some years of experience to avoid capturing hours behavior of new entrants that might distort the results. We also eliminate some outliers who work less than 10 hours a week or more than 70. We then order this sample according to the number of weekly hours worked and divide it in three equally large groups.[12] We compute average weekly hours for each group and for the whole sample. Finally, we compute the percentage deviation of each of those averages with respect to the whole sample average. Those are the three statistics to replicate, and the associated parameters to be determined are the values of the shocks. The cross-sectional hours distribution is such that each third of workers work 1.18, 1.02 and .80 of mean hours.

[12] To compute total hours worked per week on the main job we need to combine the information provided by the following variables: total annual hours of work on main job, weekly hours on main job and total annual overtime hours.

This means that, in the model economy without the policy, one third of the plants work 45.7 hours a week, another third 39.2 hours and the remaining third 30.6 hours.

4.3.2 The workweek reduction tax in the heterogeneous workweeks economy

Table 4 shows the implications of two different long workweek taxes in the heterogeneous workweeks economy, and compares their implications with those of the baseline model economy. The first experiment imposes the same tax as in the baseline model economy. The reduction in average hours per worker is smaller than in the baseline. This is an obvious implication because the tax does not affect everybody, only the fraction of workers that are beyond the long workweek threshold. In this specification a 12% tax achieves a 7.43% reduction on hours per worker. When we normalize by the percentage change in hours per worker, we obtain an elasticity of employment to hours per worker of 0.51 (versus 0.57 in the baseline model economy). Consequently, the employment gains, are much smaller, 3.85% versus 7.05%, as are the productivity and salary losses. The second experiment poses a 19% tax rate, which is approximately the tax needed to achieve a reduction of the workweek of 10% (comparable to the 12.5% of the baseline model economy). In this experiment we obtain a larger response of employment as we expected. However, as a fraction of the reduction of the workweek, the gains in employment are smaller than those of the baseline model economy, while the productivity loses are larger. The reasons for this are straight forward. Not only the larger taxes distort more the economy, but also the distortion affects mostly the more productive sectors of the economy, so that the salary reductions are larger per hour of reduction of the workweek.

We also computed the standard deviation of hours to see the implications of the policy for the intensive margin. The reduction in the volatility of hours turns out to be 27.3%. This is also not surprising since the tax punishes in the margin only the plants with long workweeks. In fact, when the tax policy is in place, the cross-sectional distribution of hours changes from 45.7, 39.2, and 30.6 hours to 41.5, 34.9 and 30.6 hours.

Table 4: **Policy Implications in the Heterogeneous Firms Economy**

Economy	**Heterogeneous Firms**				**Heterogeneous Firms**					**Baseline**			
	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$	$\tau = 0$	$\tau = 0.19$	%var	$\frac{\%var}{\%h}$	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$	
Average hours	38.5	35.6	-7.43	1	38.5	34.4	-10.7	1		40.0	35.0	-12.5	1
Hours St.Dev.	7.56	5.49	27.3	–	6.17	3.45	–	–		0	0	–	–
Employment	0.75	0.78	3.85	0.51	0.75	0.79	5.49	0.51		0.75	0.80	7.04	0.57
Total hours	29.2	28.1	-3.87	-0.53	29.2	27.5	-5.79	-0.54		30.0	28.0	-6.29	-0.50
Output	1.00	0.93	-6.84	-0.92	1.00	0.90	-10.2	-0.95		1.00	0.90	-10.2	-0.82
Productivity	1.00	0.97	-3.08	-0.41	1.00	0.95	-4.67	-0.44		1.00	0.96	-4.19	-0.34
Salary	0.85	0.76	-10.3	-1.38	0.85	0.72	-14.9	-1.39		0.86	0.72	-16.1	-1.29

To summarize, the implications of a given tax on long workweeks are larger in the heterogeneous firms model economy than in baseline model economy. We think that these two models encompass the relevant range of values since they yield reasonable elasticities. However, these results also show that the baseline model is not a bad starting point since the implied elasticities of employment with respect to hours per worker do not differ much, both lying in the range of plausible values found in microeconometric studies.

4.4 Business Cycles Implications

We want to know how the tax policy affects the business cycles behavior of this economy. In Table 5 we report the main business cycle statistics for the baseline economy when it is subject to productivity shocks, both with and without long workweek taxes. In the first column, we show the results for the economy without taxes, and in the second, the results for the economy with a 12% tax on overtime.

Table 5: **Baseline Economy Business Cycle Properties.**

Statistics (HP filtered logged data)	Ec. $\tau = 0$	Ec. $\tau = 0.12$
Std. dev. Output	1.07	0.92
Relative std. dev. of investment to consumption	3.33	2.77
Std. dev. of hours per worker	0.16	0.03
Std. dev of employment	0.31	0.24
Relative std. dev. of hours per worker to employment	0.51	0.14

As is standard in real business cycle models, productivity shocks account for almost 2/3 of output volatility in the economy without tax policy. Recall that the model was calibrated to match the relative volatility of hours per worker and bodies, .5 for the US economy. If commuting costs were constant (no congestion), this model would have yielded zero volatility in hours per worker.

The introduction of overtime taxation not only has the effects of reducing work effort and increasing employment, but it also dampens the volatility of output by 16%. It does so by reducing the volatility of employment by one fourth but, especially, by reducing the volatility of hours per worker by four fifths. The steepness of both the tax and the friction is what accounts for this result. The relative volatility of investment to consumption also goes down after the introduction of taxation.

5 Robustness

In this section we explore whether the findings of the previous sections are specific to the baseline model economy or also hold for some variations of the model. In Section 5.1 we look at alternative values for the parameter ξ that measures the degree to which the economy is subject to team work. In Section 5.2 we calibrate the economy to larger commuting costs. Section 5.3 looks at what the findings would have been without attempting to match the hours per worker to employment relative volatility and argues why it is important to do so. In Section 5.4 we calibrate the model to a higher fraction of the working age population to have an idea of the relative margins of adjustment.

5.1 The degree of team work versus fatigue

Recall that the parameter ξ measures the extent of team work. Little is known about the margin represented by this parameter. The range of possible values is $\xi \in [\theta, 1]$. The lower bound is the labor share, and when ξ takes this value, production collapses to the standard Cobb-Douglas case. To explore the role of this parameter in shaping the findings, we pose two additional economies that are still further away from the Cobb-Douglas case. To this purpose, the whole economy has to be recalibrated to match the seven statistics described above. The required two sets of parameter values are described in the first two rows of Table 6. Notice that all parameters have to be readjusted. In particular, a larger ξ, by increasing the importance of team work, imposes an increase in the relevance of those features that prevent full employment (e.g. the parameters of the externality are larger and so is the risk aversion).

Table 6: **Other Economies Parameters.**

Economy	ξ	A_N	λ	α	σ	δ	θ	β
High ξ Ec.	.92	.83	9.75	.32	2.1	.025	.64	.99
Very High ξ Ec.	1.00	3.74	15.0	.33	2.6	.025	.64	.99
High Commuting Time	.85	.54	6.25	.34	1.9	.025	.64	.99
High Employment rate	.85	.17	7.65	.36	1.5	.025	.64	.99

We report the implications for these economies of implementing the long workweek tax in Table 7. As we can see, compared to the baseline model economy, the high ξ economies display lower employment gains than in the baseline. Moreover, reductions in productivity are larger and so are the reductions in the salary. This result is not surprising, since the higher the difference between the labor share (θ) and the degree of team work (ξ), the more attractive it is to increase the labor input by increasing the workweek instead of employment.

To summarize, the conservative assumption regarding the role of team work that we use in the baseline model economy, if anything, is likely to under predict the disruptions induced by the long workweek tax policy.[13]

5.2 Higher Commuting Costs

To explore the role of commuting costs, we include a version of the model economy calibrated to fifty minutes of commuting time each way. All the other targets remain the same. We report the parameter values of this model economy in the third row of Table 6. We then report the findings in Table 8.

[13]One might think that $\xi = 1$ is not an extreme assumption and could interpret that as being a "local" property of the technology around 40 hours per week. It is sometimes assumed that there is some warm-up costs to operating a plant, and fatigue does not have to become a dominant factor until much longer hours are worked. However, the results in Table 7 show that even around 40 hours fatigue plays a role.

Table 7: **Degree of team work versus fatigue**

$\xi = .85$ Economy	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.0	-12.5	1
Employment	0.75	0.80	7.04	0.57
Total hours	30.0	28.0	-6.29	-0.50
Output	1.00	0.90	-10.2	-0.82
Productivity	1.00	0.96	-4.19	-0.34
Salary	0.86	0.72	-16.1	-1.29
$\xi = .92$ Economy	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.0	-12.5	1
Employment	0.75	0.79	5.74	0.46
Total hours	30.0	27.6	-7.53	-0.60
Output	1.00	0.87	-12.5	-1.00
Productivity	1.00	0.95	-5.38	-0.43
Salary	0.86	0.71	-17.3	-1.38
$\xi = 1$ Economy	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.1	12.2	1
Employment	0.75	0.78	4.00	0.33
Total hours	30.0	27.4	-8.71	-0.71
Output	1.00	0.85	-15.1	-1.23
Productivity	1.00	0.93	-6.98	-0.57
Salary	0.86	0.70	-18.3	-1.50

Table 8: **High commuting costs**

Economy	High Commuting Costs				Baseline			
	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.0	-12.5	1	40.0	35.0	-12.5	1
Employment	0.75	0.79	5.25	0.42	0.75	0.80	7.04	0.57
Total hours	30.0	27.6	-7.95	-0.63	30.0	28.0	-6.29	-0.50
Output	1.00	0.88	-11.8	-0.94	1.00	0.90	-10.2	-0.82
Productivity	1.00	0.96	-4.22	-0.34	1.00	0.96	-4.19	-0.34
Salary	0.86	0.72	-16.2	-1.29	0.86	0.72	-16.1	-1.29

The results are very similar to those for the baseline economy. The gains in employment are just a little smaller. Everything else is about the same. And this happens for an economy for which average commuting costs are 80% higher. From this exercise, we learn that what matters is the derivative, which is determined by the relative hours per worker to employment volatility, and not the actual value of commuting costs.

5.3 Not Matching the Hours–Employment Relative Variation

We next proceed to ask the question of whether it is important to calibrate the economy to match the relative volatility of hours per worker and employment. We do this by postulating a constant commuting cost of the same size as in the baseline economy by imposing $\lambda = 0$, which effectively assumes no commuting externality. We recalibrate the economy to the same targets as above except for the aforementioned relative hours per worker–employment volatility. In this economy, almost all the labor input variation in response to productivity shocks is in the form of employment (not in hours).

We then put in place the long workweek tax and compare the two steady states which we report in Table 9, where we have also added a column with the findings of the baseline to ease the comparison. The results are striking. The increase in employment generated by the workweek reduction policy is 19.7%. This is larger than the decrease in hours per worker, 13.3%, so much larger as to offset the decrease in productivity leaving output basically unchanged. We see that the prediction over the increase in employment implied by the policy is three times larger than in the baseline economy.

Table 9: **Constant commuting costs**

Economy	**Constant Commuting Costs**				**Baseline**			
	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	34.7	-13.3	1	40.0	35.0	-12.5	1
Employment	0.75	0.90	19.7	1.48	0.75	0.80	7.04	0.57
Total hours	30.0	31.1	3.78	0.28	30.0	28.0	-6.29	-0.50
Output	1.00	0.99	-0.88	-0.07	1.00	0.90	-10.2	-0.82
Productivity	1.00	0.96	-4.49	-0.34	1.00	0.96	-4.19	-0.34
Salary	0.86	0.72	-17.2	-1.29	0.86	0.72	-16.1	-1.29

This, we think, is evidence of the importance of matching the hours per worker–employment relative volatility to impose discipline in the measurement. A naive approach to study the implications of the policy, that does not use tight evidence to measure the relative substitutability between the two margins of labor, may give a very different and inappropriate answer.

5.4 Higher Employment rate

We perform yet another robustness exercise. This one impose a large value for aggregate employment. In the baseline model economy we assumed that the ratio of workers to prospective workers was 0.75. This value may be too low, particularly if we think of prospective workers as household heads. Therefore, we also calibrate a model economy so that a high fraction of the working age population works ($N = .85$). All the other targets remain the same. We report the parameter values of this model economy in the fourth row of Table 6.

We then report the findings in Table 10. The results are very similar to those for the baseline

Table 10: **Economy with High Average Employment,** $N = .85$

Economy	High Employment Economy				Baseline			
	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$	$\tau = 0$	$\tau = 0.12$	%var	$\frac{\%var}{\%h}$
Hours per worker	40.0	35.0	-12.5	1	40.0	35.0	-12.5	1
Employment	0.85	0.90	6.45	0.51	0.75	0.80	7.04	0.57
Total hours	34.0	31.5	-6.90	-0.55	30.0	28.0	-6.29	-0.50
Output	1.00	0.89	-10.8	-0.86	1.00	0.90	-10.2	-0.82
Productivity	1.00	0.96	-4.22	0.34	1.00	0.96	-4.19	-0.34
Salary	0.76	0.63	-16.2	-1.29	0.86	0.72	-16.1	-1.29

economy. The ability to increase employment is a bit lower, generating a elasticity of 0.51. Everything else is about the same.

6 Conclusion

In this paper, we have looked at the implications for employment, output and productivity of a policy aimed at reducing the workweek through taxation of overtime. We have created imperfect substitutability between hours per worker and employment by means of team work in production and an externality–based commuting cost. We have shown that it is important to have a correct measurement of the degree of substitutability between hours per worker and employment in the model to give an accurate assessment of the employment gains. We have used business cycles observations to pin down this relative substitutability.

In the baseline model economy, we find that a reduction of the workweek from 40 to 35 hours per week requires a tax on overtime earnings of about 12%. The effects on employment and productivity are a 7% increase in employment and a 4.2% decrease in productivity. An alternative calibration strategy that also takes into account the cross-sectional distribution of workweeks yields slightly different answers. The taxes needed to achieve a similar workweek reduction are larger (about 20%), while the employment gains are smaller, and the productivity losses larger.

Additional findings of our work are that the implementation of the policy cushions business cycle fluctuations by about 15% thorough an almost complete shutout of the variation of hours per worker.

Technical contributions of this paper include the modelling and computation of non-optimal equilibria in economies where agents labor input varies in both the intensive and the extensive margin.

As we have pointed out, there are some caveats to our findings. The first one arises from having used commuting costs subject to congestion as the friction that stands in for a variety of adjustment costs that are difficult to model appropriately. The second caveat has to do with our use of business cycle properties to calibrate the extent of the frictions that determine the relative substitutability between hours per worker and employment. There is no doubt that our findings are affected by these assumptions even though in the paper we have explored a variety of alternative assumptions to give a sense of the range of possible values for the main variables that we are looking at. We find that the answers encountered under the alternative assumptions are not very different from those that arise in the baseline model economy. Finally, we want to restate that we see the role of this paper as an initial, not as a final formal discussion of the implications of a specific type of work sharing policy.

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Appendix

A Computation of the equilibrium

- Step 1: Get $\hat{R}$, the indirect current return function that takes as given the saving behavior of the household and solves for the optimal x given aggregate H and individual hours h. The static household problem given its saving behavior, aggregate and individual hours is to solve

$$\hat{R}(z, a, N, q, r, a', T, H, h) = \max_{x \in X(N)} \quad U(x, N) \tag{33}$$

$$\text{s.t.} \quad \int_C q(c, h) \;\; dx + a' = (1 + r) \;\; a + T \tag{34}$$

  This involves the following:

  1. Given aggregate variables, z, K, K', H, N and T and price functions q and r, compute the optimal choice of the representative household $x(z, K, N, q, r, K', T, H)$.
  2. Given z, H and price functions q and r compute firm's capital k and employment n decisions.
  3. Use equilibrium conditions to obtain aggregate variables N and T and prices q, and r.
  4. Given z and r compute the deviation wage, w_h, of a household that works h hours.
  5. Given z, a, a', H, h, N and T, and price functions w_h and r compute the optimal choice a household that works h hours $x(z, a, N, q, r, a', T, H, h)$.

- Step 2: Compute $\tilde{R}(z, a, N, q, r, a', H)$ by maximizing $\hat{R}(z, a, N, q, r, a', T, H, h)$ over h. This involves obtaining optimal hours $h(z, a, N, w_h, r, a', T, H)$.

- Step 3: Use representative agent, $g_h(z, K, K; \phi, G) = G_H(z, K)$, to get $R(z, a, N, q, r, a')$, as defined in the paper.

- Step 4: Compute $a' = g_a(z, K, a; \phi, G)$ using the previously computed indirect current return function by solving

$$v(z,K,a;\phi,G) = \max_{a'} \; R(a,N,w_h,r,a') + \beta \; E\left\{v(z',k',a';\phi,G)|z\right\} \tag{35}$$

s.t.

$$r = \phi_r(z,K) \tag{36}$$

$$w_h = \phi_w(z,K,h) \tag{37}$$

$$K' = G_K(z,K) \tag{38}$$

$$N = G_N(z,K) \tag{39}$$

$$H = G_H(z,K) \tag{40}$$

B Tax function

Let $\hat{w}_h$ be the total payment that the firm has to make and $p(\hat{w}_h)$ be the overtime payment, $p(\hat{w}_h) = \tau(\bar{h},h)\cdot\hat{w}_h$. If $h < \bar{h}$, then $\hat{w}_h = w_h$ and $p(\hat{w}_h) = 0$. Otherwise $\hat{w}_h = w_h + \tau(\bar{h},h)\cdot\hat{w}_h$, that is, the firm also has to pay the tax on overtime to the government.

As we said in section 2.5, we need the tax function to be differentiable at $h = \bar{h}$. For that purpose, we construct a function over the interval $[\bar{h},\bar{\bar{h}}]$, where $\bar{h}$ is lower than the target of hours worked, $\bar{\bar{h}}$. We can also obtain the associated total payments $[\hat{w}_{\bar{h}},\hat{w}_{\bar{\bar{h}}}]$.

The tax function has to be such that

1. $\frac{\partial p(\hat{w}_h)}{\partial \hat{w}_h} = 0$ if $\hat{w}_h \leq \hat{w}_{\bar{h}}$
2. $\frac{\partial p(\hat{w}_h)}{\partial \hat{w}_h} = \tau$ if $\hat{w}_h \geq \hat{w}_{\bar{\bar{h}}}$
3. $\frac{\partial p(\hat{w}_h)}{\partial \hat{w}_h}$ non decreasing if $\hat{w}_{\bar{h}} < \hat{w}_h < \hat{w}_{\bar{\bar{h}}}$

Let $\varphi(\hat{w}_h)$ be that function defined over $[\hat{w}_{\bar{h}},\hat{w}_{\bar{\bar{h}}}]$. To graph function $\varphi(\hat{w}_h)$ (see graph):

1. Draw the x-y axis with $\hat{w}_h$ on the horizontal axis and $p(\hat{w}_h)$ on the vertical axis, and let ($\hat{w}_{\bar{h}}$,0) be the origin.
2. Draw the line $\tau(\bar{h},h)\cdot\hat{w}_h$.
3. Fix $\hat{w}_{\bar{\bar{h}}}$.
4. Draw a parabola beginning at the origin $[\hat{w}_{\bar{h}},0]$ and ending at coordinate $[\hat{w}_{\bar{\bar{h}}},p(\hat{w}_h)free]$. Note that the value of that ordinate $p(\hat{w}_h)$ will depend on the curvature of the parabola.

5. Draw a parallel line to $\tau(\bar{h}, h) \cdot \hat{w}_h$, tangent to the parabola at $[\hat{w}_{\bar{\bar{h}}}, p(\hat{w}_h)free]$.
6. Extend this parallel line until the vertical axis to get the value of the ordinate B associated to the abscissa $\hat{w}_{\bar{h}}$.

The problem is to find the parameters that define the generic function $\varphi(\hat{w}_h) = a_0 + a_1\hat{w}_h + a_2(\hat{w}_h)^2 + a_3(\hat{w}_h)^3$ over the interval $[\hat{w}_{\bar{h}}, \hat{w}_{\bar{\bar{h}}}]$, and that satisfies the following properties:

1. $\varphi(\hat{w}_{\bar{h}}) = 0$
2. $\varphi'(\hat{w}_{\bar{h}}) = 0$
3. $\varphi(\hat{w}_{\bar{\bar{h}}}) = \tau\hat{w}_{\bar{\bar{h}}} - B$
4. $\varphi'(\hat{w}_{\bar{\bar{h}}}) = \tau$
5. $\varphi'(\hat{w}_h) > 0$.

Instead of looking for the parameters that fit the above function, we define a mapping from $[\hat{w}_{\bar{h}}, \hat{w}_{\bar{\bar{h}}}]$ to $[0, 1]$. The new function, $\varphi\left(\frac{\hat{w}_h - \hat{w}_{\bar{h}}}{\hat{w}_{\bar{\bar{h}}} - \hat{w}_{\bar{h}}}\right)$, takes values on $[0, 1]$, that is,

if $\hat{w}_h = \hat{w}_{\bar{h}}$, then $\varphi(0)$,

and if $\hat{w}_h = \hat{w}_{\bar{\bar{h}}}$, then $\varphi(1)$.

Now, we impose the above mentioned properties. Property 1 implies $a_0 = 0$. Property 2 implies $a_1 = 0$. Fix $a_3 = 0$ to end up with a quadratic function (the parabola)

$$\varphi\left(\frac{\hat{w}_h - \hat{w}_{\bar{h}}}{\hat{w}_{\bar{\bar{h}}} - \hat{w}_{\bar{h}}}\right) = a_2\left(\frac{\hat{w}_h - \hat{w}_{\bar{h}}}{\hat{w}_{\bar{\bar{h}}} - \hat{w}_{\bar{h}}}\right)^2$$

To find the values of the remaining parameters, a_2 and B, apply properties 3 and 4 to that quadratic function and choose the value of the policy parameter τ. We get two expressions in the two unknowns, a_2 and B

$$a_2 = \tau\hat{w}_{\bar{\bar{h}}} - B$$

$$2a_2 = \tau(\hat{w}_{\bar{\bar{h}}} - \hat{w}_{\bar{h}})$$

Finally, the tax function is:

$p(\hat{w}_h) = 0, \quad \text{if } \hat{w}_h \leq \hat{w}_{\bar{h}}$

$p(\hat{w}_h) = a_2 \left(\frac{\hat{w}_h - \hat{w}_{\bar{h}}}{\hat{w}_{\bar{\bar{h}}} - \hat{w}_{\bar{h}}} \right)^2, \quad \text{if } \hat{w}_{\bar{h}} < \hat{w}_h < \hat{w}_{\bar{\bar{h}}}$

$p(\hat{w}_h) = \tau \hat{w}_h - B, \quad \text{if } \hat{w}_h \geq \hat{w}_{\bar{\bar{h}}}$