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Juan Perote Peña

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Fundación Centro de Estudios Andaluces (**centrA**)
Bailén, 50 - 41001 Sevilla

Tel: 955 055 210, Fax: 955 055 211

e-mail: centra@fundacion-centra.org
http://www.fundacion-centra.org

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Dominant Strategies Implementation when Compensations are Allowed: a Characterization*

Juan Perote Peña
Universidad Pablo de Olavide de Sevilla

RESUMEN
La implementación honesta en estrategias dominantes de objetivos sociales flexibles conlleva la posibilidad de que el planificador pueda alterar los incentivos individuales de tal forma que la externalidad impuesta sobre la sociedad por cada agente cuando informa sobre un tipo dado se internaliza completamente en el pago final del agente. En otras palabras, la función objetivo de los agentes debe imitar o replicar a los objetivos sociales. Nuestro resultado principal es lo suficientemente robusto como para explicar por qué ciertos mecanismos muy conocidos como las transferencias de Groves funcionan en algunos contextos mientras que algunos otros objetivos sociales no son implementables en estrategias dominantes.

Palabras clave: Decisividad individual, mecanismos de compensación, estrategias dominantes.

ABSTRACT
Dominant strategies truthful implementation of flexible social objectives involves the ability of the planner to alter the individual incentives in such a way that the externality imposed on society by each agent reporting a given type is fully internalized in the agent's final payoff. In other words, the agents' objective function must mimic the social objectives. We find that our main result is robust enough to explain why well-known mechanisms like Groves's transfers work in some contexts while some other social objectives are not implementable in dominant strategies.

Keywords: Individual decisiveness, compensation mechanisms, dominant strategies.

JEL classification: D78, D71.

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1 Introduction

Since the early 70's, the problem of designing incentive mechanisms to achieve socially desirable outcomes has been a major concern in economics. The initial negative results due to Gibbard [4] and Satterthwaite [14] in the context of unrestricted domains of preferences proved the need to impose domain restrictions to find some possibility results -see Dasgupta et al. [3] for a survey-. The first successful attempt to find a possibility result in mixed economies -those combining some public good with a private one- were due to Groves [6], [7] and [8], Clarke [2] and Green & Laffont [5].

In this paper, we aim at testing a regularity that emerges in many results regarding implementability in dominant strategies of social choice rules. Most well-known theorems regarding implementation in dominant strategies in "large" domains seem to imply the following: the kind of incentives that must be given to the agents in order to induce truth-revealing behavior are such that the payoff function that the agents maximize when choosing their optimal messages to be sent to the planner have the same structure than the social objectives the planner himself is interested in maximizing. In other words, dominant truth-telling strategies involve the planner's ability to design payoff functions for the agents that fully internalize the externality caused by each agent on the objectives of society as a whole -i.e., the planner's objectives-. After formulating our conjecture more precisely, we find a characterization result that shows that when the social objectives are flexible enough -a property that we call "*individual decisiveness*", the coincidence of private and social objectives in the design of incentives is unavoidable. Since a wide range of well-known social objectives in economic environments fall in this category, our main result applies and different applications easily emerge from the theorem. In particular, *Groves' mechanisms* in public good provision environments and the impossibility of implementing the Pareto-optimal social choice rule in mixed public and private goods economies with preferences that are not quasi-linear emerge as particular applications of our main result.

Explaining the motivation of the paper and being precise in what we exactly mean by "coincidence" between different "objectives" requires detailed explanations of well-known theorems that will be proven to have important points in common that not always have been sufficiently highlighted in the literature. Therefore, in *Section 2* we recall these results in order to formulate our conjecture. In *Section 3* we introduce the compensation mechanisms and *Section 4* is devoted to our main result. The main applications regarding our approach are studied in *Section 5* and finally, we conclude with some comments.

2 Conjecture

In order to motivate our approach, let us consider three well-known results:

Case 1 The Gibbard-Satterthwaite Theorem *(Gibbard* [4], *Satterthwaite* [14]*)*

Let us consider a society in which a finite number $n \geq 2$ of agents or individuals, ordered in a set $N = \{1, ..., n\}$ and indexed by $i, j \in N$ choose **alternatives**, social states or objects from some set K, $\#K > 1$. These objects may be levels of provision of some public or private goods, allocations of indivisible goods, candidates for being a boss, etc.-. Let $k \in K$ denote any alternative from that set. Each individual is endowed with some private characteristic or **type** θ_i from a set Θ_i. A **profile** is an element in the Cartesian product of the sets Θ_i $\forall i \in N$. Society can be described by a possible profile $\theta = (\theta_{1,}..., \theta_n) = (\theta_i, \theta_{-i}) \in \prod_{i=1}^{n} \Theta_i$. We call here an **economy** to the tuple $e = \langle N, K, \Theta_i \forall i \in N \rangle$. Given an economy, society -or the social planner- would like to select alternatives depending on the individual characteristics, so social desirability is summarized by a choice rule denoted by $K^* : \prod_{i=1}^{n} \Theta_i \rightarrow\rightarrow K$, called **social choice correspondence** (SCC) that assigns a set of social states for each possible profile. A single-valued SCC is a **social choice function** (SCF) and will be denoted by $f : \prod_{i=1}^{n} \Theta_i \rightarrow K$. A **social welfare function** (SWF) is a real-valued function of the type: $W : K \times \prod_{i=1}^{n} \Theta_i \rightarrow E$, where E is the real line. We will say that SCC K^* is **generated** or represented by SWF W iff $W(K^*(\theta), \theta) \geq W(k, \theta)$ $\forall k \in K$, $\forall \theta \in \prod_{i=1}^{n} \Theta_i$, and $\overline{W}(K^*)$ will be the set of SWFs representing a given SCC K^*.[1].Given an economy e, suppose now that individual objectives given the type take the form of a **preference relation** on the set of alternatives, denoted by $R_i(\theta_i)$, $\forall \theta_i \in \Theta_i$, that is, R_i $\forall i \in N$ is a mapping from the possible types to the set of all ordered pairs of alternatives: $R_i : \Theta_i \rightarrow\rightarrow K \times K$.[2] We say that the domain is **unrestricted** iff every complete weak pre-ordering is admissible as a preference relation. Assuming that each agent's type is his own private information and society - or the social planner - cannot directly observe the true individual types, the rule has to be based on the revealed types rather than on the true individual types. We are interested in SCFs such that each agent has no any incentive to lie about his true type in any case - whatever types the rest of agents

[1]Notice that for every SCC K^*, the set $\overline{W}(K^*)$ is non-empty: the constant SWF trivially represents every SCC.

[2]We are implicitly assuming that the economy is such that allow for the definition of individual preferences.

report to the planner and whatever be the agent's true type -. We say that a SCF is **strategy-proof** iff

$$\forall i \in N,\ \forall \theta \in \prod\nolimits_{i=1}^{n} \Theta_i,\ \forall \theta'_i \in \Theta_i,\ f(\theta) R_i(\theta_i) f(\theta'_i, \theta_{-i}).$$

Gibbard [4] and Satterthwaite [14] proved that whenever the domain is unrestricted and $\#range(f) \geq 3$, *the only strategy-proof social choice functions are dictatorial, i.e.,* $\exists i \in N$ *such that* $\forall \theta \in \prod_{i=1}^{n} \Theta_i,\ f(\theta) \in \begin{array}{l} \arg\max\ R_i(\theta) \\ \text{s.t. } k \in range(f) \end{array}$

Case 2 Roberts' [13] Theorem.

Let us consider an economy e such that K is a finite set. We furthermore assume that the agents' objectives are not defined by the preference ordering on K associated to each type, but by a real-valued payoff of the kind:

$$\forall i \in N, \quad P_i : K \times E \times \Theta_i \rightarrow \quad E$$

Where P_i is quasi-linear with respect to the second argument, intended to represent some out-the-model way to compensate the agent, so that we can write agent $i's$ payoff function in the form: $P_i(k, q_i, \theta_i) = v_i(k, \theta_i) + q_i,\ \forall k \in K,\ \forall \theta_i \in \Theta_i,\ \forall q_i \in E$, where for all i, $v_i : K \times \Theta_i \rightarrow E$ is a real-valued function called the **valuation function** admitting every possible cardinal utility scale on the set of (finite) alternatives. We say now that f is strategy-proof iff there exist bounded **compensation functions** $q_i : \prod_{i=1}^{n} \Theta_i \ \rightarrow E$, $\forall i \in N$, such that[3] $\forall i \in N,\ \forall \widehat{\theta} \in \prod_{i=1}^{n} \Theta_i,\ \forall \theta_i \in \Theta_i$,

$$v_i(f(\theta_i, \widehat{\theta}_{-i}), \theta_i) + q_i(\theta_i, \widehat{\theta}_{-i}) \geq v_i(f(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i) + q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) \tag{1}$$

Roberts proved that whenever $range(f) = K$, *the only strategy-proof social choice functions come from maximizing some weighted sum of the agents'* $v'_i s$, i.e.,

$\exists a_1, ..., a_n \in E_+,\ \sum_{i=1}^{n} a_i = 1$, such that $\forall \theta \in \prod_{i=1}^{n} \Theta_i$,

$f(\theta) \in \begin{array}{l} \arg\max \\ k \in K \end{array} \sum_{i=1}^{n} a_i v_i(k, \theta_i) + F(k)$ where $F : K \rightarrow E$ is any bounded real-valued function.

[3] There is no difference in the interpretation of Robert's definition of strategy-proofness from the definition above provided that we realise that there are now two ways in which the planner can affect the agents' final payoff (and hence their incentives): via the chosen alternative (affecting the $v'_i s$) and by varying the compensations (that only depend on the revealed profile), that is the direct way of changing the final payoffs.

Case 3 Groves-Clarke's mechanisms. *(Groves* [6], [7] *and* [8], *Clarke* [2], *Green & Laffont* [5]*).*

Let us consider an economy such that K is now some compact set in a topological space. Let the real-valued bounded $v_i's$ in the above framework for all $\theta_i \in \Theta_i$ be the set of all bounded upper-semi-continuous or continuous functions. The list of compensation functions (**mechanism**) $\{q_1, ..., q_n\}$ *implement by revelation* the SCC K^* iff for every selection f from K^*, i.e., $f(\widehat{\theta}) \in K^*(\widehat{\theta})\ \forall \widehat{\theta} \in \prod_{i=1}^n \Theta_i$, (1) holds.[4]

Green & Laffont [5] proved that the only mechanisms that implement the SCC $K^*(\theta) = \underset{s.t.\ k \in K}{\arg\max} \sum_{i=1}^n v_i(k, \theta_i)$ are the **Groves' mechanisms**[5], i.e., those mechanisms in which all the compensation functions take the form:

$$\forall i \in N,\ q_i(\widehat{\theta}) = \sum\nolimits_{j \neq i} v_j(f(\widehat{\theta}), \widehat{\theta}_j) + h_i(\widehat{\theta}_{-i}),\ \forall f(\widehat{\theta}) \in K^*(\widehat{\theta}) \qquad (2)$$

where $h_i : \prod_{j \neq i} \Theta_j \to E$ is any real-valued function independent of $\widehat{\theta}_i$.

Later research (Green & Laffont [5], Walker [15] and Hurwicz & Walker [11]) proved that Groves' mechanisms cannot generically *balance the budget*, that is, $\forall \{q_1, ..., q_n\}$ implementing by revelation the SCC above, $\forall \overline{k} \in E$, $\sum_{i=1}^n q_i(\widehat{\theta}) \neq \overline{k}\ \forall \widehat{\theta} \in \prod_{i=1}^n \Theta_i$.

The three implementation scenarios described above share some common properties:

(i). The domains of private characteristics are quite *large* in each model.

(ii). The incentive compatibility requirement is actually the same in all cases and amounts to the existence of truth-revealing dominant strategies.

(iii). The implementable social choice rules or functions in the three cases require that individuals' objectives are made somehow *similar* to social objectives. Gibbard does not allow for extra-model compensations that can affect the agents' objectives but not the planner's utility -any SWF representing the SCC-, so he obtains that implementability should make the social objectives be identified with those of some fixed agent -the dictator-. Roberts, in his turn, allows for *monetary* or quasi-linear compensations in the mechanism design, but the only social choice functions that are implementable come from the maximization of some linear combination of the individuals' valuation functions -some *quasi-linear* or utilitarian social objectives-. Green

[4] Again, there is no difference between the notions of strategy-proofness in Roberts' setup and the notion of mechanisms implementing by revelation a SCF.

[5] We use the names "Groves-Clarke's mechanisms" and "Groves' mechanisms" as synonims.

& Laffont actually work in the same framework of Roberts, but they are interested in implementing the utilitarian social welfare function -the sum of all the agents' valuation functions- and using quasi-linear individual objectives. Notice that the Groves' compensation functions -the only mechanisms working in that domain- somehow replicate the social objectives in the sense that if the planner delegate the outcome selection in any agent, he would actually pick up the same social alternatives that would be chosen by the planner himself.

The analogies showed above give rise to a conjecture that might be formalized as follows:

Conjecture 1 *Truthful implementation in* large *domains of SCC K^* requires that $\forall i \in N$, for all* admissible *functions $P_i : E^2 \to E$, the following holds: $\forall \theta_i, \widehat{\theta}_i \in \Theta_i, \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,*

$$P_i(v_i(f(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = W(f(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i}). \tag{3}$$

for some $W \in \overline{W}(K^*)$.

It is not difficult to check that the three results summarized above are particular cases of the above statement -or can be written in these terms-.

We shall then raise the following questions: Is that a common feature of all truthful implementation problems in large domains?. How far can this conjecture be extended? Is it valid for every compensation scheme, even for those not restricted to quasi-linear compensations?

We find that there exists a requirement on the social choice rule called **individual decisiveness**, under which truthful implementation demands social and individual objectives to coincide in the above sense. This property assumes a strong responsiveness of the social choices to changes on the individual types. Examples will be provided later, but let us point out now that individual decisiveness holds for the usual social choice rules when allowing sufficiently rich domains. In particular, it holds and plays a crucial role in environments admitting Groves' type mechanisms.

The main result in this paper shows the strong connection between the specific agents' payoff function structure and the individually decisive social choice rules that can be truthfully implemented in dominant strategies. In other words, the compensations allowed in any mechanism should be such that they have to allow that the payoff function structure replicates some social welfare function representing the social choice rule. In other words, we should give to the agents exactly the same incentive scheme that the one implied in the social welfare function; the objective function of agents and that of society should somehow coincide.

3 Compensation mechanisms

We shall propose a general model, including the setups for the examples of the preceding section as particular cases. *Case 1* (the Gibbard-Satterthwaite Theorem) occurs in a setup which is traditional in social choice theory: alternatives are defined as those objects over which agents are assumed to have preferences. *Cases 2* and *3* (Robert's and Groves-Clarke's models) refer to setups where social states are described through two sets of variables: the levels of variables in the first set are interpreted as the result of a *public decision*; the levels of others are interpreted as transfers of goods or compensations to agents. Individual payoffs depend on the overall levels of both variables. If we want to keep the conventions of social choice we should reserve the term alternative to denote these combinations of levels, since it is on them that agents have preferences. But it is more useful to keep the distinction found in the other two models, and eventually to generalize it. Hence, we distinguish between those parts of a decision which involve a public decision (K) and those (q) which can be interpreted as compensations for the agents. The agent's valuations of the public decisions will be assumed to be well defined as functions of their types, and the overall preferences of agents over public decisions and compensation levels are assumed to take a quite flexible form. This will make our setup more general than any of those mentioned above.

Let us consider **any economy** e. Given e, we define the following:

Definition 1 *A **compensation mechanism** $\{P, q\}$ is the tuple defined by the following sets:*

*(i) $P = \{P_i, \forall i \in N\}$ is the set of **payoff functions**[6], where $\forall i \in N$, $P_i : E^2 \to E$ is a continuous upper-bounded real-valued function* monotonic *in both arguments -and strictly monotonic in the second-, i.e.,*

$\forall x, x', y, y' \in E, \;\; y > y' \Rightarrow P_i(x, y) > P_i(x, y')$ *and* $x > x' \Rightarrow P_i(x, y) \geq P_i(x', y)$.

*(ii) $q = \{q_i, \forall i \in N\}$ is the list of **compensation functions**: $\forall i \in N$, $q_i : \prod_{i=1}^{n} \Theta_i \to E$, upper-bounded real-valued functions that serve the planner to distribute utility among the agents based in the information contained in the strategies.*

Given a compensation mechanism, the **final payoff** that any individual

[6]These functions stand for the payoffs that the agents obtain, given their types -be it a production function, a utility function or a general arbitrary type of agent-, and some individual real argument which can be used within the mechanism in order to compensate the agent.

gets from any strategy vector is always given by the following expression:

$$\forall i \in N,\ \pi_i(\widehat{\theta}, \theta_i) = P_i(v_i(g(\widehat{\theta}), \theta_i), q_i(\widehat{\theta})).$$

Therefore the functional form of the final payoff is partially given by the mechanism and does not necessarily coincide with the valuation function, except in the limit case of a mechanism such that $\forall i \in N,\ \forall x, y_i \in E,\ P_i(x, y_i) = x$, which is not strictly monotonic in the second argument. The specific payoff function structure will allow us to classify every compensation mechanism. In order to illustrate this point, it will be useful to think about a production economy: e is such that N is a set of firms, divisions within a firm or productive agents that produce a single homogeneous good, K represents either feasible levels of a public input used by the agents or distributions of a private input; let us assume that the set of types determines the feasible technologies available such that the valuation function will be a production function. Consider the following examples of particular payoff functions: $\forall x, y_i \in E,\ P_i(x, y_i) = x,\ P_i(x, y_i) = x + y_i,\ P_i(x, y_i) = xy_i$, and $P_i(x, y_i) = y_i$. Those payoff functions imply final payoffs of the form: $\pi_i = v_i(g(\widehat{\theta}), \theta_i),\ \pi_i = v_i(g(\widehat{\theta}), \theta_i) + q_i(\widehat{\theta}),\ \pi_i = v_i(g(\widehat{\theta}), \theta_i)q_i(\widehat{\theta}),\ \pi_i = q_i(\widehat{\theta})$ respectively. The first one represents the impossibility of compensating agents. Agent $i's$ objective function is fully determined given by his own private characteristic -or his produced output for given levels of the input-, so it coincides with the usual implementation framework, where every possible compensation is modelled *inside* the set of feasible alternatives. We will refer to this case as the *compensation free payoff functions*. In the second example above, the productive agent sells his output at some given (unitary) price and gets the profit, but the planner or principal can only set some tax or subsidy to provide an incentive for truthful behavior. This will be called the *compensation by transfers* case. The third example assumes the ability of the planner to set the final price of the produced good according to some pre-specified rule -*compensation by prices*-. Finally, in the last example the agent has no property rights on the good produced, and he only receives a wage that can depend on the information reported by all the agents. This will be the *full compensation* case. Notice that all the above examples allow for different compensation or surplus-sharing schemes and the possibility of one or another may be discretional to the planner in some contexts or given by nature in others. Furthermore, the monotonicity property imposed on the payoff functions establishes a specific restriction on the functional form of compensations, so that for any given allocation, the higher the compensation, the higher the final payoff. A non-monotonic payoff function might be, for example, the following one: $P_i(x, y) = xy_i^2$. This condition does not seem

to be too restrictive, since the specific nature of the compensation requires a clear guide to reward the agents.

Finally, notice that the compensation functions can also be viewed as part of the *real alternatives*, and we could define an extended set of alternatives as: $K' = K \times E^n$, where compensations for all the agents are included within the set of alternatives. Once the payoff function structure is fixed, it amounts exactly to the usual framework of compensation free payoff functions with the agents' valuations re-defined to be: $\forall i \in N,\ v_i'(k', \theta_i) = P_i(v_i(k, \theta_i), y_i)$ and $k' = (k, y_1, .., y_n)$, so the approach followed in most of the implementation literature -see, for example, Green and Laffont [5]- is a particular case of our compensation mechanisms too.

Now we define the notion of truthful implementation that we shall use, together with some additional definitions related to SCCs and mechanisms.

Definition 2 *Given an economy e and a SCC K^*, we say that a compensation mechanism $\{P, q\}$ is an* ***incentive compatible mechanism for*** *i if the following condition holds for $i : \forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,*

$$P_i(v_i(f(\theta_i, \widehat{\theta}_{-i}), \theta_i), q_i(\theta_i, \widehat{\theta}_{-i})) \geq P_i(v_i(f(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})), \qquad (4)$$

and this for any selection $f(\widehat{\theta})$ from $K^*(\widehat{\theta})$.

A compensation mechanism which is incentive compatible for all $i \in N$ is said to be *incentive compatible.* If an incentive compatible mechanism exists for some SCC we say that the mechanism *implements by revelation* that SCC.

The last definitions are natural generalizations of those in Green and Laffont [5].

Definition 3 *A SCC $K^* : \prod_{i \in N} \Theta_i \to K.$ is called* ***individually decisive for*** *i iff*

$$\forall \theta_{-i} \in \Theta_{-i},\ \ \forall k \in K,\ \exists \widehat{\theta}_i \in \Theta_i\ \ni\ k \in K^*(\widehat{\theta}_i, \theta_{-i}).$$

A SCC that is individually decisive for all i is individually decisive and any SWF which represents some individually decisive SCC will be individually decisive.

This property means that individual i can force any alternative under some circumstances to be in the choice set by declaring an appropriate characteristic for any others' types.

It may be useful to illustrate the above definitions with some examples of both individually decisive and non-decisive SCCs.

Example 1 *Let the economy e be such that K is a compact set defined in a topological space and $\forall i \in N$, Θ_i includes all the bounded, upper-semi-continuous functions on K. Let us consider the utilitarian welfare function: $W = \sum_{i=1}^{n} v_i(k, \theta_i)$. The SCC that maximizes W on K, i.e., $K^*(\theta_1, ..., \theta_n) = \underset{k \in K}{\arg\max} \ \sum_{i=1}^{n} v_i(k, \theta_i)$, can be proved (see* Proposition 2 *later) to be individually decisive.*

Example 2 *The Pareto SCC is individually decisive when defined on a large set of rich domains; $\forall \theta \in \prod_{i \in N} \Theta_i$,*

$$PO(\theta) = \\ = \left\{ \begin{array}{c} k \in K \text{ such that } \nexists \overline{k} \in K \text{ such that } \ \forall i \in N, \ v_i(\overline{k}, \theta_i) \geq v_i(k, \theta_i) \\ \text{and } v_i(\overline{k}, \theta_i) > v_i(k, \theta_i) \text{ for some } i \in N \end{array} \right\}.$$

Consider the unrestricted domain on K : $\forall i \in N$, Θ_i is such that $\forall k, l \in K$ $(l \neq k)$, $\exists \widehat{\theta}_i \in \Theta_i$ such that $v_i(k, \widehat{\theta}_i) > v_i(l, \widehat{\theta}_i) \ \Rightarrow \forall k \in K$, $\exists \widehat{\theta}_i \in \Theta_i$ such that: $k \in PO(\widehat{\theta}_i, \theta_{-i}) = K^*(\widehat{\theta}_i, \theta_{-i})$.

Example 3 *Consider now the Pareto SCC in an economy such that $N = \{1, 2\}$ and the domain of all continuous, strictly monotonic and convex preference orderings over the 2-good commodity space E_+^2 when the set K is the set of all feasible allocations of the 2 goods available in fixed finite amounts between the two agents (the Edgeworth Box). Take any $\theta_2 \in \Theta_2$ and any feasible allocation $z \in K$. Construct the agent 2's upper contour set on z : $C_2(z, \theta_2) = \{y \in K \text{ s.t. } v_2(y, \theta_2) \geq v_2(z, \theta_2)\}$, and find, for example, the value "a" $\in E_+$ s.t. $z \in \underset{x}{\arg\max} \ ax_1^1 + x_2^1$. This value will al-*

$$\begin{array}{ll} \underset{x}{\arg\max} & ax_1^1 + x_2^1 \\ s.t. & x \in C_2(z, \theta_2) \\ & x \in K \end{array}$$

ways exist because of the convexity and monotonicity assumptions on Θ_1 and Θ_2. Now, set $\widehat{\theta}_1 = a$ and define $v_1(k, \widehat{\theta}_1) = ax_1^1 + x_2^1 \ \forall x_1^1, x_2^1$. This is a convex, strictly monotonic and continuous function, so $\widehat{\theta}_1 \in \Theta_1$, and by construction, $z \in PO(\widehat{\theta}_1, \theta_2) = K^*(\widehat{\theta}_1, \theta_2)$.

Example 4 *In an economy like that in* Example 3*, The Walrasian correspondence with respect to some vector of initial endowments can be proved to be an individually decisive SCC by using a similar argument.*

Example 5 *Let us assume an economy such that $n \geq 3$ is odd, K is a closed interval of the real line $K = [0, 1] \subset E$ and the agents' types are such that the agents may have every continuous single-peaked valuation function, i.e., $\forall i \in N$, Θ_i is such that $\forall \theta_i \in \Theta_i$, $\exists \overline{k}(\theta_i) \in K$ s.t. $v_i(\overline{k}(\theta_i), \theta_i) > v_i(k, \theta_i) \ \forall k \in K \backslash \overline{k}(\theta_i)$ & $\forall k', k'' \in K$ $(k' > k'')$,*

$\overline{k}(\theta_i) \geq k' \Rightarrow v_i(k', \theta_i) \geq v_i(k'', \theta_i)$ & $\overline{k}(\theta_i) \leq k'' \Rightarrow v_i(k', \theta_i) \leq v_i(k'', \theta_i)$. *Consider the following SCC (the* median voter *SCC):* $\forall \theta \in \prod_{i=1}^{n} \Theta_i,\ K^*(\theta) = med_{i \in N} \left\{ \overline{k}(\theta_i) \right\}$, *where the function "med" stands for the median of the revealed* peaks *of the agents, i.e., the peak such that leaves the same number of other peaks on the right and on the left. This is a well-known selection of the Pareto correspondence in this economy -see, for example, Moulin* [12]*-. It is easy to see that there exist situations where some individuals cannot individually change the decision, for example, individual i cannot even affect the decision for* $\theta_{-i} \in \Theta_{-i}$ *such that* $\overline{k}(\theta_j) = \overline{k}(\theta_h)\ \forall j, h \neq i$, *so* K^* *is not individually decisive.*

Example 6 *Let us consider an economy e such that* $K = [0, 1] \subset E_+$ *and* $\forall i \in N,\ \Theta_i = E_+$, *and* $v_i(k, \theta_i) = a_i k - \frac{1}{2}k^2\ \forall k \in K, \forall a_i \in \Theta_i$. *The utilitarian SWF in* Example 1 *above is individually non-decisive, but it will be individually decisive if the economy is enlarged to allow for types such that* $\Theta_i = E$ *(actually, allowing for convex valuation functions). Notice that in this cases,* $K^*(a_1, ..., a_n) = \frac{1}{n} \sum_{i=1}^{n} a_i$.

We can now address the main question we face: what kind of compensation mechanisms, if they exist, should we use in order to implement by revelation any individually decisive SCC?. We provide a complete answer to this question.

4 Main result

The results presented below hold true for every given economy
$e = \langle N, K, \Theta_i\ \forall i \in N \rangle$.

Theorem 1 *The only incentive compatible compensation mechanisms* $\{P, q\}$ *that implement by revelation an individually decisive SCC* K^* *are such that:* $\forall i \in N,\ \forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = W(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i}),$$

for every selection g from K^*.

In order to prove the theorem, we make use of the following intermediate results:

Lemma 1 *Let* $\{P, q\}$ *be an incentive compatible for i compensation mechanism implementing the SCC* K^*, *and take two -possibly the same- selections from* K^* : g *and* $\widehat{g}$. *Then,* $\forall \theta_i, \theta_i' \in \Theta_i,\ \forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$, *s.t.* $g(\theta_i, \theta_{-i}) = \widehat{g}(\theta_i', \theta_{-i})$, *it holds that* $q_i(\theta_i, \theta_{-i}) = q_i(\theta_i', \theta_{-i})$.

Proof. Suppose the contrary, i.e., $\exists\theta_{-i} \in \prod_{j\neq i}\Theta_j,\ \exists\theta_i, \theta'_i \in \Theta_i$ such that both yield the same outcome with both selections: $g(\theta_i, \theta_{-i}) = \widehat{g}(\theta'_i, \theta_{-i}) = \overline{k}$, and one of them leads to a bigger compensation: θ_i w.l.g., then: $q_i(\theta_i, \theta_{-i}) >$ $q_i(\theta'_i, \theta_{-i})$. Then, take type θ'_i, and consider the payoffs:

$P_i(v_i(g(\theta_i, \theta_{-i}), \theta'_i), q_i(\theta_i, \theta_{-i}))$ and $P_i(v_i(\widehat{g}(\theta'_i, \theta_{-i}), \theta'_i), q_i(\theta'_i, \theta_{-i}))$. By assumption, the outcome will be the same: $g(\theta_i, \theta_{-i}) = \widehat{g}(\theta'_i, \theta_{-i}) = \overline{k} \Rightarrow$

$\Rightarrow v_i(g(\theta_i, \theta_{-i}), \theta'_i) = v_i(\widehat{g}(\theta'_i, \theta_{-i}), \theta'_i)$. Then, by monotonicity of the payoff functions structure, we should have:

$P_i(v_i(g(\theta_i, \theta_{-i}), \theta'_i), q_i(\theta_i, \theta_{-i})) > P_i(v_i(\widehat{g}(\theta'_i, \theta_{-i}), \theta'_i), q_i(\theta'_i, \theta_{-i})). \Rightarrow$

For θ'_i, $\exists\theta_i \in \Theta_i$, $\exists\theta_{-i} \in \prod_{j\neq i}\Theta_j$, $\exists\overline{g}(\theta_i, \theta_{-i}) \in K^*(\theta_i, \theta_{-i})$ $\forall\theta_i \in \Theta_i$, $\forall\theta_{-i} \in \prod_{j\neq i}\Theta_j$, - a selection from K^*- defined as:

$$\overline{g}(\widehat{\theta}_i, \theta_{-i}) = \begin{cases} g(\widehat{\theta}_i, \theta_{-i}) & iff\ \widehat{\theta}_i \neq \theta'_i \\ \widehat{g}(\theta'_i, \theta_{-i}) & iff\ \widehat{\theta}_i = \theta'_i \end{cases}$$

such that by declaring the last one we are better than reporting the true characteristic, i.e.,

$P_i(v_i(\overline{g}(\theta_i, \theta_{-i}), \theta'_i), q_i(\theta_i, \theta_{-i})) > P_i(v_i(\overline{g}(\theta'_i, \theta_{-i}), \theta'_i), q_i(\theta'_i, \theta_{-i}))$, so the mec anism cannot be incentive compatible for i.[7] ■

Lemma 2 *The only incentive compatible for i compensation mechanisms $\{P, q\}$ that implement by revelation any individually decisive for i SCC K^* are such that: $\forall\theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall\widehat{\theta}_{-i} \in \prod_{j\neq i}\Theta_j$,*

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = W(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i}),$$

for any selection g from K^*.

Proof. Necessity $\Rightarrow$) We suppose that $\{P, q\}$ is an incentive compatible for i compensation mechanism implementing by revelation some individually decisive for i SCC K^*. Since K^* is individually decisive for i, it is true that $\forall\theta_{-i} \in \Theta_{-i}$, $\forall k \in K$, $\exists\widehat{\theta}_i(k, \theta_{-i}) \in \Theta_i \ \ni\ k \in K^*(\widehat{\theta}_i, \theta_{-i})$. Hence, the mapping $\widehat{\theta}_i : K \times \prod_{j\neq i}\Theta_j \rightarrow\rightarrow \Theta_i$ is well-defined and for any selection $\overline{\theta}_i$ from that mapping, it holds that $\forall k \in K, \forall\theta_{-i} \in \prod_{j\neq i}\Theta_j,\ k \in K^*(\overline{\theta}_i(k, \theta_{-i}), \theta_{-i})$. Now, let us define the following mapping for individual $i \in N$: $\widehat{q}_i : K \times \prod_{j\neq i}\Theta_j \rightarrow\rightarrow E$, defined as follows: $\forall k \in K$, $\forall\theta_{-i} \in \prod_{j\neq i}\Theta_j$,

$\widehat{q}_i(k, \theta_{-i}) = \left\{q_i(\overline{\theta}_i(k, \theta_{-i}), \theta_{-i}),\ \text{ for any selection } \overline{\theta}_i \text{ from } \widehat{\theta}_i\right\}$. This mapping is well defined and has the following properties:

i) $range(\widehat{q}_i) = range(q_i)$.

ii) $dom(\widehat{q}_i) = K \times \prod_{j\neq i}\Theta_j$.

[7] This lemma is a generalization of part of Green and Laffont's [5] Theorem 3.

iii) $\widehat{q}_i$ is a real-valued *function.*

iv) $\forall\theta_i \in \Theta_i,\ \forall\theta_{-i} \in \prod_{j\neq i}\Theta_j,\quad \widehat{q}_i(g(\theta_i,\theta_{-i}),\theta_{-i}) = q_i(\theta_i,\theta_{-i}),$ for any selection g from K^*.

Property (i) is obvious by definition: for any $\theta_i \in \Theta_i$, $\exists k = g(\theta_i,\theta_{-i})$, so $q_i(\theta_i,\theta_{-i}) \in \widehat{q}_i(k,\theta_{-i})$. (ii) holds because $\widehat{\theta}_i$ is a well-defined mapping from $K\times\prod_{j\neq i}\Theta_j$. Property (iii) holds because we are in the conditions of applying *Lemma 1*: Since $\{P,q\}$ is an incentive compatible for i compensation mechanism implementing the SCC K^* by assumption, $\forall\theta_{-i} \in \prod_{j\neq i}\Theta_j,\ \forall\theta_i,\theta_i' \in \Theta_i$ s.t. $g(\theta_i,\theta_{-i}) = \widehat{g}(\theta_i',\theta_{-i}) = k$ for two -possibly the same- selections from $K^* \Rightarrow q_i(\theta_i,\theta_{-i}) = q_i(\theta_i',\theta_{-i})$. Thus, $\forall\theta_{-i} \in \prod_{j\neq i}\Theta_j$, $\forall k \in K$, $q_i(\overline{\theta}_i(k,\theta_{-i}),\theta_{-i}) = q_i(\widetilde{\theta}_i(k,\theta_{-i}),\theta_{-i})$ for any two arbitrary elections $\overline{\theta}_i$ and $\widetilde{\theta}_i$ from $\widehat{\theta}_i$ and hence a single real number is associated to each $k \in K$ in the function $\widehat{q}_i(k,\theta_{-i})$.

Finally, property (iv) is straightforward by definition and (iii).

Now, we know by assumption that $\{P,q\}$ is an incentive compatible compensation mechanism for i, so that it holds that:

$P_i(v_i(g(\theta_i,\theta_{-i}),\theta_i),q_i(\theta_i,\theta_{-i})) \geq P_i(v_i(g(\theta_i',\theta_{-i}),\theta_i),q_i(\theta_i',\theta_{-i}))$

$\forall\theta_i,\theta_i' \in \Theta_i,\ \forall\theta_{-i} \in \prod_{j\neq i}\Theta_j$ and this for any selection $g(\theta_i,\theta_{-i}) \in K^*(\theta_i,\theta_{-i})$.

Now, using iv), we can state the following:

$P_i(v_i(g(\theta_i,\theta_{-i}),\theta_i),\widehat{q}_i(g(\theta_i,\theta_{-i}),\theta_{-i})) \geq$

$\geq P_i(v_i(g(\theta_i',\theta_{-i}),\theta_i),\widehat{q}_i(g(\theta_i',\theta_{-i}),\theta_{-i}))\quad \forall\theta_i,\theta_i' \in \Theta_i,\ \forall\theta_{-i} \in \prod_{j\neq i}\Theta_j$ and for any selection $g(\theta_i,\theta_{-i}) \in K^*(\theta_i,\theta_{-i})$.

Then, since K^* is individually decisive by hypothesis, we can choose the following selection $\widetilde{g}(\theta_i',\theta_{-i})$ from K^*: Given any $\theta_{-i} \in \prod_{j\neq i}\Theta_j$ and any $\theta_i \in \Theta_i,\ \forall\theta_i' \in \Theta_i$,

$$\widetilde{g}(\theta_i',\theta_{-i}) = \begin{cases} k & iff\ \theta_i' = \overline{\theta}_i(k,\theta_{-i})\ \&\ \theta_i' \neq \theta_i \\ g(\theta_i',\theta_{-i}) & otherwise \end{cases},\quad (2).$$

where $\overline{\theta}_i(k)$ is any selection from $\widehat{\theta}_i$ and $g(\theta_i',\theta_{-i})$ is any arbitrary selection from K^*. It holds for this selection that: $\forall\theta_i,\theta_i' \in \Theta_i$,

$P_i(v_i(\widetilde{g}(\theta_i,\theta_{-i}),\theta_i),\widehat{q}_i(\widetilde{g}(\theta_i,\theta_{-i}),\theta_{-i})) \geq$

$P_i(v_i(\widetilde{g}(\theta_i',\theta_{-i}),\theta_i),\widehat{q}_i(\widetilde{g}(\theta_i',\theta_{-i}),\theta_{-i}))$. (3). But this is true for all $\theta_i' \in \Theta_i$, which only affects the right hand side of the above inequality and, by definition of $\widetilde{g}$, $\{\widetilde{g}(\theta_i',\theta_{-i}),\ \forall\theta_i' \in \Theta_i\} = K$, so for each $\theta_i \in \Theta_i$,we can write (3) in the following way: given any θ_i and θ_{-i}, we can construct a selection $\widetilde{g}$ defined above and obtain:

$P_i(v_i(g(\theta_i,\theta_{-i}),\theta_i),\widehat{q}_i(g(\theta_i,\theta_{-i}),\theta_{-i})) \geq P_i(v_i(k,\theta_i),\widehat{q}_i(k,\theta_{-i}))\ \ \forall k \in K.$ (4

But $g(\theta_i,\theta_{-i})$ for each $\theta_i \in \Theta_i$ is selected arbitrary from $K^*(\theta_i,\theta_{-i})$, while the right hand side of the above inequality is the same for each selection

given θ_i and θ_{-i}, so statement (4) holds for every selection from $K^*(\theta_i, \theta_{-i})$. Abusing notation, we can write (4) as follows:

$\forall \theta_i \in \Theta_i$,

$P_i(v_i(K^*(\theta_i, \theta_{-i}), \theta_i), \widehat{q}_i(K^*(\theta_i, \theta_{-i}), \theta_{-i})) \geq P_i(v_i(k, \theta_i), \widehat{q}_i(k, \theta_{-i})) \quad \forall k \in K$. (5).

Now, let us consider the following composite function $\widehat{P}_i : K \times \prod_{i=1}^{n} \Theta_i \to E$ defined as: $\forall k \in K,\ \forall \theta \in \prod_{j=1}^{n} \Theta_j,\ \ \widehat{P}_i(k, \theta_i, \theta_{-i}) = P_i(v_i(k, \theta_i), \widehat{q}_i(k, \theta_{-i}))$, (6), which is well-defined when K^* is individually decisive. Notice that (5) can be written -slightly abusing notation again- as:

$\forall k \in K,\ \forall \theta_i \in \Theta_i,\ \forall \theta_{-i} \in \prod_{j \neq i} \Theta_j,\ \widehat{P}_i(K^*(\theta_i, \theta_{-i}), \theta_i, \theta_{-i}) \geq \widehat{P}_i(k, \theta_i, \theta_{-i})$.

But this last expression is the definition of some SWF representing SCC K^*. In other words, let us suppose that $\widehat{P}_i \notin \overline{W}(K^*)$. This can only be true when $\exists \theta_i \in \Theta_i,\ \exists \widetilde{\theta}_{-i} \in \prod_{j \neq i} \Theta_j,\ \exists \widetilde{k} \in K$ such that $\widehat{P}_i(\widetilde{k}, \theta_i, \widetilde{\theta}_{-i}) > \widehat{P}_i(K^*(\theta_i, \widetilde{\theta}_{-i}), \theta_i, \widetilde{\theta}_{-i})$ (7). But since K^* is individually decisive for i, for $\widetilde{\theta}_{-i}$ there exist $\exists \widetilde{\theta}_i \in \Theta_i$ such that $\widetilde{k} \in K^*(\widetilde{\theta}_i, \widetilde{\theta}_{-i})$. Substituting this into (7), we have found a selection of K^* such that, slightly abusing notation again, $\exists \widetilde{\theta}_i \in \Theta_i\ (\widetilde{\theta}_i \neq \theta_i),\ \exists \theta_i \in \Theta_i,\ \exists \widetilde{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, such that

$\widehat{P}_i(K^*(\widetilde{\theta}_i, \widetilde{\theta}_{-i}), \theta_i, \widetilde{\theta}_{-i}) > \widehat{P}_i(K^*(\theta_i, \widetilde{\theta}_{-i}), \theta_i, \widetilde{\theta}_{-i})$, and, which by (6) can be written again as: $P_i(v_i(K^*(\widetilde{\theta}_i, \widetilde{\theta}_{-i}), \theta_i), \widetilde{\theta}_{-i}) > P_i(v_i(K^*(\theta_i, \widetilde{\theta}_{-i}), \theta_i), \widetilde{\theta}_{-i})$, and this clearly contradicts mechanism $\{P, q\}$ to be incentive compatible for i. Hence, it has to be that $\widehat{P}_i \in \overline{W}(K^*)$ and the existence is proved.

Sufficiency $\Leftarrow$) Now we have to prove that every mechanism such that $\forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, can be written as $P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = W(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i})$, (1) is an incentive compatible for i compensation mechanism. Suppose, on the contrary, that $\exists g \in K^*,\ \exists \widetilde{\theta}_i \in \Theta_i\ ,\ \exists \widetilde{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$ and $\exists \theta'_i \in \Theta_i$ such that $W(g(\theta'_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i}) > W(g(\widetilde{\theta}_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i})$ (3). But, since W represents K^*, it must be that $\forall \theta_i \in \Theta_i,\ \forall \theta_{-i} \in \prod_{j \neq i} \Theta_j,\ \forall k \in K,\ W(g(\theta_i, \theta_{-i}), \theta_i, \theta_{-i}) \geq W(k, \theta_i, \theta_{-i})$, and, in particular,

$W(g(\widetilde{\theta}_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i}) \geq W(k, \widetilde{\theta}_i, \widetilde{\theta}_{-i})$ (4). take any $k = g(\theta'_i, \widetilde{\theta}_{-i}) \in K$, and (3) and (4) imply:

$W(g(\theta'_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i}) > W(g(\widetilde{\theta}_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i}) \geq W(g(\theta'_i, \widetilde{\theta}_{-i}), \widetilde{\theta}_i, \widetilde{\theta}_{-i})$, a contradiction, so $\{P, q\}$ is an incentive compatible compensation mechanism for i and the lemma is proved. ■

Proof of Theorem 1:

Using *Lemma 2* and applying it for all i, it holds trivially.

The implications of *Theorem 1* are wide: It shows, for example, that when the social objectives are flexible enough, like the set of all continuous

preferences on some compact set of alternatives, and we are trying to implement selections of the Pareto-optimal correspondence, which is clearly an individually decisive SCC, we must make coincide social interest with every individual's to achieve a positive result. Notice that this is a generalization of the well-known Groves' mechanisms: Green & Laffont's [5] result can be seen as a corollary of this one, and, moreover, it shows that the only restriction on preferences that allow for efficient and strong incentive compatible implementation are the quasi-linear domain.

5 Applications

In what follows, we will be concerned with different applications of *Theorem 1* in different contexts that fit their assumptions. We will show that a wide range of interesting economic environments match in our general model and our result will be very useful to characterize the mechanisms and social choice rules that are implementable. We proceed to classify the different applications by the kind of compensation mechanism allowed.

5.1 Compensation-free schemes

Corollary 1 *The only individually decisive for i SCCs that can be implemented by means of compensation mechanisms when the payoff function structure is compensation-free is the dictatorial one.*

Proof. The compensation-free payoff function structure is not actually monotonic, but we do not need monotonicity to hold in this special case. Following the reasoning in *Theorem 1*, it is easy to see that *Lemma 1* is not necessary to prove that *Theorem 1* holds in this particular context, so the only compensation mechanism that implements by revelation any individually decisive for i SCC is such that the following holds: $\forall \theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i) = W(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i})$$

for any $W \in \overline{W}(K^*)$. So we can only implement the SWF that represents the characteristic of individual i, and whatever any other individual reports, the SCC will be: $K^*(\theta_i, \theta_{-i}) = \underset{k}{argmax} \;\; v_i(k, \theta_i) \;\; \forall \theta_{-i} \in \Theta_{-i}$, i.e., individual i is a dictator. ■

Corollary 1 is a stronger version of Gibbard-Satterthwaite famous Theorem since we are imposing additional restrictions on the SCC to get the result -the SCC or SCF should be individually decisive for i-, which is a

much stronger assumption that Gibbard's condition -the range of the function contains at least three elements-. It is also related with Barberà-Peleg's [1] version when considering continuous preferences and with *Theorem 3.2* in Roberts [13], but the case of compensation-free payoff functions becomes trivial in our framework[8] and the reason for considering this case here is to compare the gains from the possibilities of different compensation schemes below with the radical result of not allowing any kind of compensation.

Corollary 2 *Consider any economy where* Θ_i, $\forall i \in N$ *are such that* $\exists i, j \in N$, $\exists \theta_i \in \Theta_i, \exists \theta_j \in \Theta_j$ *such that* $\underset{k}{argmax} \; v_i(k, \theta_i) \neq \underset{k}{argmax} \; v_j(k, \theta_j)$.
Then, there does not exist any individually decisive SCC that can be implemented by revelation by a compensation-free mechanism.

Proof. Trivial: There cannot be more than one dictator when extending *Corollary 1* to more agents. ■

The compensation-free scheme is also interesting because usual mechanisms in the literature do not allow for compensations that are not included in the set of feasible alternatives. If this is the case and the planner cares about the whole payoff that the agents receive, our approach remains valid if we restrict the analysis to the compensation-free scheme. Notice that *Theorem 1* hold for the compensation-free case, and this will allow us to deal with the problem of balance -omitted until now- and provide a partial answer to an important question posed by Hurwicz &Walker [11]: Does an incentive compatible mechanism implementing the Pareto-optimal SCC exist for the mixed economy -public and private goods- when we drop the quasi-linear payoffs assumption?. In their own words, "*First, there is no reason to believe that* [their result] *depends upon the quasi-linearity of the individual's preferences; however, it is not clear how to obtain the result without the quasi-linearity assumption.*" The former authors could not directly face that problem because they used a characterization theorem due to Holmstrom which only holds for those payoffs, but we will show in what follows that our previous results are actually a powerful tool to deal with the general problem.

Let us consider a particular and simplified 2-agents mixed economy. There are two goods: one public and other private. Let $Y = [\underline{y}, \overline{y}]$, $0 < \underline{y} < \overline{y} < \infty$, be some compact interval on the real line representing the quantity provided of the public good and let $X = Y \times X_1 \times X_2$ be the consumption

[8]Note that Barberá & Peleg's [1] proof of Gibbard-Satterthwaite Theorem consists in proving that strategy-proofness alone in an unrestricted domain of preferences makes the SCF either individually decisive for each agent or invariant to changes of his type.

space in the economy, standing for the public good and the private good each individual can get. A particular element from X will be denoted by $x = (y, x_1, x_2)$. We will identify $X_1 \equiv X_2 \equiv E_+$ for simplicity and consider a fixed finite quantity $\overline{x} > 0$ of the private good to be distributed among the individuals. Let $T = \{(x_1, x_2) \in X_1 \times X_2 \; s.t. \; x_1 + x_2 \leq \overline{x}\}$ be the feasibility constraint on the private good, so assuming that both goods are technologically independent, the set of feasible alternatives will be: $FA \equiv X \cap (Y \times T)$. Both agents are endowed with preferences representable by a continuous utility function: $u_i : Y \times X_i \to E$, for $i = 1, 2$, defined on his affective space - in Hurwicz &Walker's [11] terminology -. We suppose the functions to be strictly increasing in the quantity of the private good and not quasi-linear: $\forall i = 1, 2, \; \forall y, \widehat{y} \in Y, \; u_i(y, x_i) - u_i(\widehat{y}, x_i) = u_i(y, \widetilde{x}_i) - u_i(\widehat{y}, \widetilde{x}_i) \Rightarrow x_i = \widetilde{x}_i$. The set of these admissible preferences will be $\Theta_i, \; i = 1, 2$. The reason why we completely exclude quasi-linear preferences on the domain is that Hurwicz &Walker [11] proved a similar theorem only for this kind of preferences. If we admit them into our new domain, the impossibility result becomes trivial and has no interest.

Proposition 1 *Consider the above economy. There does not exist any incentive compatible compensation-free mechanism implementing the Pareto-optimal correspondence.*

Proof. First of all, notice that we do not need to focus on the whole Pareto-optimal correspondence since incentive compatibility requires that every selection correspondence must satisfy it. Let us take the following selection correspondence of the Pareto-optimal rule:

$\widehat{K}^*(\theta_1, \theta_2) = \underset{x \in FA}{\arg\max} \; v_1(y, x_1, \theta_1) + v_2(y, x_2, \theta_2)$. Now we will consider a quite narrower subdomain of characteristics belonging to $\Theta_i \; \forall i = 1, 2$, which will be denoted by $\widehat{\Theta}_i \; \forall i = 1, 2$. $\widehat{\Theta}_i = \{a_i, b_i \in E, \; a_i > 0\}$ and $v_i(y, x_i, \theta_i) = a_i x_i^2 y - \dfrac{b_i}{2} y^2$. Notice that $\widehat{\Theta}_i \subset \Theta_i \; \forall i = 1, 2$, since all of them are continuous, strictly increasing on the private good and no quasi-linear. Hence, incentive compatibility holds within this subdomain too. But we can prove that $\widehat{K}^*$ is individually decisive for both agents. First, notice that since the utility functions are strictly increasing in the private good, every allocation in the whole Pareto-optimal SCC -and, of course, any selection from this- distributes the total amount available of the good among the agents, so if agent 1 can secure any $y \in Y$ and any amount $0 < x_1 < \overline{x}$ for himself by declaring an appropriate type, he can actually select some Pareto-optimal alternative, since $x_2 = \overline{x} - x_1$ in any efficient allocation. Thus, we prove the following, i.e., for individual 1, $\forall y \in Y, \; x_1 \in X_1 \; \& \; x_1 \leq \overline{x}$, $\forall \theta_2 \in$

$\widehat{\Theta}_2$, $\exists\widehat{\theta}_1 \in \widehat{\Theta}_1 \ni (y, x_1, \overline{x} - x_1) \in \widehat{K}^*(\widehat{\theta}_1, \theta_2) \in FA$. We can easily find the $\widehat{K}^*$ correspondence[9] $\forall(\theta_1, \theta_2) \in \widehat{\Theta}_1 \times \widehat{\Theta}_2$: $\widehat{x}_1^* = \left(\dfrac{a_2}{a_1 + a_2}\right)\overline{x}$, $\widehat{x}_2^* = \left(\dfrac{a_1}{a_1 + a_2}\right)\overline{x}$, $\widehat{y}^* = \begin{cases} \max\left\{\dfrac{a_1 a_2 \overline{x}^2}{(a_1 + a_2)(b_1 + b_2)}, \underline{y}\right\} & if\ (b_1 + b_2) > 0 \\ \overline{y} & otherwise \end{cases}$

Now, take individual 1: $\forall a_2 > 0$, $\forall b_2 \in E$, $\forall x_1$ $s.t.$ $\overline{x} > x_1 > 0$, $\forall y \in Y$, $\exists\widehat{a}_1 = a_2\left(\dfrac{\overline{x} - x_1}{x_1}\right) > 0$,

$\exists\widehat{b}_1 = \dfrac{\widehat{a}_1 a_2 \overline{x} - (\widehat{a}_1 + a_2) b_2 y}{(\widehat{a}_1 + a_2)} \in E$, $s.t.$ $\widehat{x}_1^*(\widehat{a}_1, a_2) = x_1$ & $\widehat{y}^*(\widehat{a}_1, a_2, \widehat{b}_1, b_2) = y$. The fact that each agent cannot achieve the extremes - everything or nothing - of the total endowment of the private good prevent it to be individually decisive in the whole K, but it is clear that he can get quantities of the private good as close as desired to both extremes.

Henceforth, applying *Theorem 1* and taking into account that we only allow for compensation-free mechanisms, we have that agent $1's$ valuation function should be of the form: $v_1(g(\widehat{\theta}_1, \widehat{\theta}_2), \theta_1) = W(g(\widehat{\theta}_1, \widehat{\theta}_2), \theta_1, \widehat{\theta}_2)$, $\forall\theta_1, \widehat{\theta}_1 \in \widehat{\Theta}_1$, $\forall\widehat{\theta}_2 \in \widehat{\Theta}_2$, for any selection g from $\widehat{K}^*$. Since $v_1(y, x_1, \theta_1) + v_2(y, x_2, \theta_2)$ is a twice differentiable function -and concave for a large range of parameters-, the following equation has to hold for any $W \in \overline{W}(\widehat{K}^*)$ within that range of parameters:

$$\frac{\partial W(x_1, y, a_1, b_1, \widehat{b}_1, \widehat{b}_2)}{\partial y}(x_1^*, y^*) = 0 = \frac{\partial v_1(x_1, y, a_1, b_1)}{\partial y}(x_1^*, y^*).$$

And some simple calculations show that $\dfrac{\partial W(x_1, y, a_1, b_1, \widehat{b}_1, \widehat{b}_2)}{\partial y}(x_1^*, y^*) = a_1 x_1^{*2} - b_1 y^* - \widehat{b}_2 y^*$

and $\dfrac{\partial v_1(x_1, y, a_1, b_1)}{\partial y}(x_1^*, y^*) = a_1 x_1^{*2} - b_1 y^* \neq a_1 x_1^{*2} - b_1 y^* - \widehat{b}_2 y^*$ for all $\widehat{b}_2 \neq 0$, so the impossibility is proved. ■

The former proposition provides strong evidence about the non-existence of truthful revelation mechanisms for mixed economies that both generically provide the efficient quantities of the public and private goods and balance the budget when we limit ourselves to domains of preferences or characteristics with some income effect, so the income effect cannot be generically used

[9]Note that $PO(\theta)$ in this case requires some positive amount of the private good to be given to both agents, so that this SCC can be shown to be individually decisive for the set K restricted to $x_i > 0\ \forall i \in \{1, 2\}$. To apply our result to the whole closed set K we need some continuity assumptions, which is the problem we shall tackle in the Appendix.

to enforce strong implementation. Nevertheless, slightly different domains might lead to different implementation results, so the general question of existence is still open. Notice, for example, that the former proof is not valid with the additional restriction of concavity imposed on the preferences for the public good.

Moreover, when more than 2 agents are present, we may find that in some contexts, the SCC is not individually decisive, but it is for some fixed subset of the feasible alternatives -the individual consumption spaces, but not the others' consumptions-. In those cases, although we cannot directly apply the results in the former section, we can refine them to get useful tools to deal with those problems. For example, the following straightforward lemma might be applied:

Lemma 3 *Let K_i, $\forall i \in N$, be a family of compact and convex sets. Let $v_i : K_i \times \Theta_i \to E$ be the continuous valuations for each individual and characteristic and suppose $K_i^*(\theta_1, .., \theta_n)$ be the restriction of $K^* : \prod_{i=1}^n \Theta_i \to \cup_{i=1}^n K_i$ on K_i. If K_i^* is individually decisive on K_i for some i, the only compensation-free mechanism that implements K^* should be such that: $\forall \theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, $v_i(g_i(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i) = W(g_i(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i})$, and for any selection g from K^*. (where g_i stands for the restriction of any g on K_i).*

The proof of this lemma is omitted since it follows strictly the same reasoning of that in *Theorem 1* with the difference of considering K_i as the whole set K in the proof.

We can explore now the efficient implementation problem with compensatio free mechanisms in the extreme cases of pure public goods and private goods. Consider the public good case: depending on the admissible domain of characteristic we may choose, we can easily check if the Pareto-optimal SCC is individually decisive. For example, if we focus on the unrestricted domain of characteristics -see *Example 2* in the first section-, we can apply *Corollary 2* to get a strong version of Gibbard-Satterthwaite impossibility result. For the case of private goods, the strategy is similar: Let us consider, for example, the 2-agents, 2-goods general environment in *Example 3*, which is the classical Edgeworth Box economy, where the admissible characteristics are all continuous, strictly monotonic and convex utility functions over the 2-goods commodity space E_+^2. If we want to implement the Pareto-optimal SCC by means of compensation-free mechanisms, and it has been shown to be an individually decisive SCC, we can apply *Theorem 1* and obtain a similar impossibility to that in *Proposition 1* for the mixed economy close to that of Hurwicz [10]. Nevertheless, there is a difference that we should point out: Our work always assumes **full implementation**, since every selection

of the SCC has to yield every individual's highest payoff, so when the set of possible social choices is large -as in the case of private goods along the contract curve, it is easier to get an impossibility result and there might be selections of the SCC that can be implemented by revelation in a partial implementation framework-. As an example, consider the Pareto-optimal SCC when individuals are endowed with single-peaked preferences in *Example 4*'s economy in *Section 3*: The whole Pareto-optimal SCC with that domain is individually decisive, so it is easy to prove that there do not exist compensation-free mechanisms implementing it, but there are selections from this characterized by Moulin [12] -the SCF in the example is one of them- such that are not individually decisive and can be implemented by revelation.

Finally, we should note that the compensation mechanisms reproducing social objectives can be useful even in the absence of individual decisiveness. For some quasi-linear preferences in public goods environments, it is possible to achieve complete efficiency in the mixed economy using some particular Groves' mechanism if we restrict the domain even more. Groves and Loeb [9] found that the quadratic family of valuation functions on some public good in *Example 6* in *Section 3*, joint with quasi-linear preferences on the private good can be balanced with an appropriate Groves' mechanism, so that the efficient choice of the public good can be implemented by revelation for that domain. Notice, however, that this result does not enters into contradiction with *Corollary 1*, since balanced implementation requires some appropriate compensation functions (transfers) and only this selection of the Pareto-optimal rule is implemented. Since a part of these transfers only depends on the others' reported types, no individual can get any transfer irrespective of the others' strategies, so the implementable Pareto-optimal selection is not individually decisive in the part of the private good so *Corollary 1* does not apply.

5.2 Full compensation scheme

Corollary 3 *When the payoff function structure is that of* full compensation*, i.e.,* $\forall x, y_i \in E,\ P_i(x, y_i) = y_i,$ *every SCC can be trivially implemented by revelation by means of some compensation mechanism.*

Proof. Consider an incentive compatible compensation mechanism with full compensation payoff functions for every agent; it is easy to check that these payoff functions are monotonic, so applying *Theorem 1*, every individually decisive SCC implementable by revelation is such that: $\forall i \in N$, $\forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j,$

$W(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i})) = P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})$. Notic that the implementable SWF admitted in the full compensation case cannot depend on the real individual characteristics, so every SWF allowed is such that: $W(k, \theta) = f(k)\ \forall \theta \in \prod_{i=1}^{n} \Theta_i$, where $f : K \to E$ is any function. But, what kind of SCCs are represented by such SWFs? The only class is the following: $\exists \overline{K} \subseteq K$ such that $\forall \theta \in \prod_{i=1}^{n} \Theta_i,\ K^*(\theta) = \overline{K}$. Even though many members in that class are trivial undesirable SCCs like all the cases where $\overline{K}$ is a singleton, the case $\overline{K} \equiv K$ includes every possible social choice function as a selection from K^*. Moreover, we can say more about the form of the compensation functions: since $q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) \in E\ \forall \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, incentive compatibility implies the following: $\forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j,\ q_i(\theta_i, \widehat{\theta}_{-i}) \geq q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) \Rightarrow q_i(\theta_i, \widehat{\theta}_{-i}) \geq range \left\{ q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) \right\} \Rightarrow$

$\Rightarrow q_i(\theta_i, \widehat{\theta}_{-i}) \geq \max\limits_{\substack{(\widehat{\theta}_i, \widehat{\theta}_{-i}) \\ \in \prod_{i=1}^{n} \Theta_i}} q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) \quad \forall \theta_i \in \Theta_i, \Rightarrow \forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$q_i(\theta_i, \widehat{\theta}_{-i}) = q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})$, or, in other words, $\forall i \in N,\ \forall \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j,\ q_i(\widehat{\theta}_i, \widehat{\theta}_{-i}) = q_i(\widehat{\theta}_{-i})$. The compensation mechanisms associated to those compensation functions with the full compensation payoff function structure are independent of each own's reported type, so they are always incentive compatible and (trivially) implement every SCC -not only the individually decisive SCCs-. Notice that for this trivial compensation functions, *Theorem 1* assigns the constant SWF, which trivially represents any SCC. ■

Notice that both the compensation-free and full compensation schemes are extreme or polar compensation mechanisms with opposite implementation properties: The impossibility of compensations makes the agents' payoff fully depending on their characteristics, so they are strongly interested in exploiting their private information, while if the agents' payoff can be designed independently of their types, any compensation scheme for an agent such that makes no use of his reported private information works. The reason is that in the full compensation case, the planner (or principal) owns the total power to modify the agent's payoff against changes in the characteristics, while in the no compensation scheme he can only use his discretion about the selected alternative, and finally there will only be strong implementation possibilities if the planner himself behaves as a dictatorial agent. Nevertheless, this trivial case has a clearly undesirable property: the agents have no incentives to lie, but they have not an incentive to tell the truth either. For a discussion of a similar setup, see Groves [7].

The remainder of this section studies some specific intermediate cases between the full compensation and the no compensation possibilities.

5.3 Compensations with transfers

Proposition 2 *Let K be some compact set in a topological space and Θ_i be the set of all upper semi-continuous functions $\forall i \in N$, the only incentive compatible compensation mechanisms that implement the SWF $W = \sum_{i=1}^{n} v_i(k, \theta_i)$ are the following:*

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) =$$

$$= W \left[\begin{array}{l} \arg\max \quad v_i(k, \theta_i) + \sum_{j \neq i} v_j(k, \widehat{\theta}_j) \\ k \in K \end{array} \right].$$

$\forall i \in N$, $\forall \theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, *where W is any SWF representing the SCC in brackets.*

Proof. It suffices to prove that under the above condition, the SWF W is individually decisive.

Take any $i \in N$ and any $\theta'_{-i} \in \prod_{j \neq i} \Theta_j$; as every $\theta'_j \in \Theta_j$ $\forall j \in N$, is bounded above by assumption, then, for any $\overline{k} \in K$, take the following type for individual i:

$v_i(k, \widehat{\theta}_i) = \left\{ \begin{array}{ll} -\sum_{j \neq i} v_j(k, \theta'_j) + 1 & if \;\; k = \overline{k} \\ -\sum_{j \neq i} v_j(k, \theta'_j) & if \;\; k \neq \overline{k} \end{array} \right.$ which is clearly upper semi-continuous and it will be true that:

$\forall \theta'_{-i} \in \prod_{j \neq i} \Theta_j$, $W(\overline{k}, \widehat{\theta}_i, \theta'_{-i}) = v_i(\overline{k}, \widehat{\theta}_i) + \sum_{j \neq i} v_j(\overline{k}, \theta'_j) > v_i(k, \widehat{\theta}_i) + \sum_{j \neq i} v_j(k, \theta'_j) = W(k, \widehat{\theta}_i, \theta'_{-i})$

$\forall k \neq \overline{k} \in K$. This clearly implies that $\overline{k} \in K^*(\widehat{\theta}_i, \theta'_{-i})$, so for any $\forall \overline{k} \in K$ $\forall \theta'_{-i} \in \Theta_{-i}$ $\exists \widehat{\theta}_i \in \Theta_i$ s.t. $\overline{k} \in K^*(\widehat{\theta}_i, \theta'_{-i})$, so the SWF is individually decisive, because we can do the same for any $i \in N$.

Then, we are under the conditions of *Theorem 1*, so the only compensation mechanisms implementing by revelation W is of the form described above.

Proposition 3 *Let K be some compact set in a topological space and the set Θ_i be the set of all continuous functions $\forall i \in N$, the only incentive compatible compensation mechanisms that implement the SWF: $W = \sum_{i=1}^{n} v_i(k, \theta_i)$, are the same of those in* Proposition 2.

Proof. We must prove that the SWF W is individually decisive even when we restrict the domain of types to be continuous. Consider, then, for any $i \in N$, and for any $\overline{k} \in K$, given any $\theta'_{-i} \in \prod_{j \neq i} \Theta_j$, the following characteristic: $v_i(k, \widehat{\theta}_i) = -z \left\| k - \overline{k} \right\| - \sum_{j \neq i} v_j(k, \theta'_j) - \widehat{\varepsilon}(k)$. $z \in E_+$, & $\widehat{\varepsilon}(k)$ being any continuous function such that: $\widehat{\varepsilon}(k) \geq 0$ $\forall k \in K$ & $\widehat{\varepsilon}(\overline{k}) = 0$. We can prove that the SWF. with the profile $(\widehat{\theta}_i, \theta'_{-i})$ get a maximum on $k = \overline{k}$:

$W(\overline{k},\widehat{\theta}_i,\theta'_{-i}) = -\sum_{j\neq i} v_j(\overline{k},\theta'_j)+\sum_{j\neq i} v_j(\overline{k},\theta_j)-\widehat{\varepsilon}(\overline{k}) = -\widehat{\varepsilon}(\overline{k}) > -\widehat{\varepsilon}(k)-z\left\|k-\overline{k}\right\| =$

$-\sum_{j\neq i} v_j(k,\theta'_j)+\sum_{j\neq i} v_j(k,\theta'_j)-\widehat{\varepsilon}(k)-z\left\|k-\overline{k}\right\| = W(k,\widehat{\theta}_i,\theta'_{-i})\ \ \forall k\in K.$

Notice that $v_i(k,\widehat{\theta}_i)$ is continuous since every $v_j(k,\theta'_j)\ \forall j\neq i$ are continuous, so we conclude as before, that W is individually decisive. Applying *Theorem 1*, we can get the Generalized Groves' mechanisms again, being this a second generalization of Green & Laffont's results. ■

Proposition 4 *Let $K\subset E^m$ be an open set endowed with the euclidean metric for $m\geq 2$ and the set Θ_i be the set containing all concave or strictly concave and differentiable functions $\forall i\in N$; the only incentive compatible compensation mechanisms that implement the SWF $W=\sum_{i=1}^n v_i(k,\theta_i)$ are the same that those in* Proposition 2.

Proof. We show that W has to be individually decisive even when the domain of characteristics is restricted to be every concave -or strictly concave- and differentiable function: First, consider any $i\in N$, and for any $\overline{k}\in K$, given any $\theta_{-i}\in\prod_{j\neq i}\Theta_j$, since every $v_j(k,\theta_j)\ \forall j\in N$ is a differentiable function, the expression: $\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))$ is differentiable, so we know that there exists a vector $l(\overline{k},\theta_{-i})\in E^m$, $-\infty<l(\overline{k},\theta_{-i})<\infty$, such that:

$$lim_{\|k-\overline{k}\|\to 0}\frac{\left|\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))-l(\overline{k},\theta_{-i})'(k-\overline{k})\right|}{\left\|k-\overline{k}\right\|}=0.$$

Now, let us construct a real number $\widehat{h}(\overline{k},\theta_{-i})\in E$ defined as follows:

$$\widehat{h}(\overline{k},\theta_{-i}) = \\ = \max_{k\in K}\begin{cases}\dfrac{\left|\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))-l(\overline{k},\theta_{-i})'(k-\overline{k})\right|}{\left\|k-\overline{k}\right\|} & if\ k\neq\overline{k}.\\ 0 & if\ k=\overline{k}\end{cases}$$

Again, this number exists because all the $v_j(k,\theta_j)$ are always bounded above and the denominator is positive, so $0<\widehat{h}(\overline{k},\theta_{-i})<\infty$. Finally, we can write the following by construction:

$$\widehat{h}(\overline{k},\theta_{-i})\geq\frac{\left|\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))-l(\overline{k},\theta_{-i})'(k-\overline{k})\right|}{\left\|k-\overline{k}\right\|}\quad\forall k\in K.$$

Rearranging the above inequality, we have:

$\widehat{h}(\overline{k},\theta_{-i})\left\|k-\overline{k}\right\|\geq\left|\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))-l(\overline{k},\theta_{-i})'(k-\overline{k})\right|\geq$

$\geq\sum_{j\neq i}(v_j(k,\theta_j)-v_j(\overline{k},\theta_j))-l(\overline{k},\theta_{-i})'(k-\overline{k}).\ \Rightarrow$

$\widehat{h}(\overline{k},\theta_{-i})\left\|k-\overline{k}\right\| - \sum_{j\neq i} v_j(k,\theta_j) \geq -\sum_{j\neq i} v_j(\overline{k},\theta_j) - l(\overline{k},\theta_{-i})'(k-\overline{k})$

$\forall k \in K$, and finally, multiplying the inequality by -1, and rearranging terms, we have: $\sum_{j\neq i} v_j(\overline{k},\theta_j) \geq -\widehat{h}(\overline{k},\theta_{-i})\left\|k-\overline{k}\right\| - l(\overline{k},\theta_{-i})'(k-\overline{k}) + \sum_{j\neq i} v_j(k,\theta_j)$

$\forall k \in K$. (1). So, we proved that $\forall i \in N$, $\forall \overline{k} \in K$, $\forall\ \theta_{-i} \in \prod_{j\neq i}\Theta_j$,

$\exists \widehat{h}(\overline{k},\theta_{-i}) < \infty$ such that the last expression holds for all $k \in K$. Now, define the following characteristic for individual i:

$v_i(k,\widetilde{\theta}_i) = -\widetilde{h}(\overline{k},\theta_{-i})\left\|k-\overline{k}\right\| - l(\overline{k},\theta_{-i})'(k-\overline{k})$. Notice that $v_i(\overline{k},\widetilde{\theta}_i) = 0$ and it is easy to see that $\widetilde{\theta}_i \in \Theta_i$, since $\widetilde{h}(\overline{k},\theta_{-i})$ and $l(\overline{k},\theta_{-i})$ exist and it is a differentiable function - it is the sum of two differentiable functions - and is concave since the euclidean norm is strictly convex and the second term is convex. ($\|\lambda x + (1-\lambda)y\| \leq \|\lambda x\| + \|(1-\lambda)y\| = |\lambda|\,\|x\| + |(1-\lambda)|\,\|y\|$ $\forall \lambda \in [0,1]$, $\forall x,y \in K$.). Now, the only thing to do is interpreting expression (1) as follows:

$\forall \overline{k} \in K$, $\forall \theta_{-i} \in \prod_{j\neq i}\Theta_j$, $\exists \widetilde{h}(\overline{k},\theta_{-i}) < \infty$, $\exists l(\overline{k},\theta_{-i}) \in E^n$, $\exists \widetilde{\theta}_i \in \Theta_i$ such that: $v_i(\overline{k},\widetilde{\theta}_i) + \sum_{j\neq i} v_j(\overline{k},\theta_j) = 0 + \sum_{j\neq i} v_j(\overline{k},\theta_j) \geq$

$\geq -\widetilde{h}(\overline{k},\theta_{-i})\left\|k-\overline{k}\right\| - l(\overline{k},\theta_{-i})'(k-\overline{k}) + \sum_{j\neq i} v_j(k,\theta_j) =$

$= v_i(k,\widetilde{\theta}_i) + \sum_{j\neq i} v_j(k,\theta_j)$

$\forall k \in K \Rightarrow v_i(\overline{k},\widetilde{\theta}_i) + \sum_{j\neq i} v_j(\overline{k},\theta_j) \geq v_i(k,\widetilde{\theta}_i) + \sum_{j\neq i} v_j(k,\theta_j)$ $\forall k \in K \Rightarrow$

$\forall i \in N$, $\forall \overline{k} \in K$, $\forall \theta_{-i} \in \prod_{j\neq i}\Theta_j$, $\exists \widetilde{\theta}_i \in \Theta_i$ such that $W(\overline{k},\widetilde{\theta}_i,\theta_{-i}) \geq W(k,\widetilde{\theta}_i,\theta_{-i}) \Rightarrow \overline{k} \in K^*(\widetilde{\theta}_i,\theta_{-i})$.

So there exists some characteristic in the admissible domain such that any individual can get any alternative for any others' characteristics, which is the definition of an individually decisive SCC. Applying *Theorem 1*, we find the same class of mechanisms as above. ■

Corollary 4 *(Green & Laffont* [5]*). Let K be a compact set in a topological space and Θ_i $\forall i \in N$ contain any upper semi-continuous or continuous or concave and differentiable valuation functions. The only compensation by transfers mechanisms that can implement by revelation the utilitarian SWF are the Groves' mechanisms.*

In any of the possible domains considered, the only compensation mechanisms that can implement by revelation the utilitarian SWF are of the same form using *Propositions 2* to 4, i.e.,

$$P_i(v_i(g(\widehat{\theta}_i,\widehat{\theta}_{-i}),\theta_i), q_i(\widehat{\theta}_i,\widehat{\theta}_{-i})) = W\left[\begin{array}{l} \arg\max \quad v_i(k,\theta_i) + \sum_{j\neq i} v_j(k,\widehat{\theta}_j) \\ k \in K \end{array}\right].$$

$\forall i \in N,\ \forall \theta_i, \widehat{\theta}_i \in \Theta_i,\ \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, where W is any SWF representing the SCC in brackets.

Now, we are imposing an additional restriction to the compensation mechanisms allowed: we are only interested in compensations by transfers, i.e., the payoff functions structure have the form: $\forall x, y_i \in E,\ P_i(x, y_i) = x + y_i$, so, applying *Propositions 2* to 4:

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i) + \widehat{q}_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_{-i}) =$$

$$= W \left[\begin{array}{l} \arg\max \quad v_i(k, \theta_i) + \sum_{j \neq i} v_j(k, \widehat{\theta}_j) \\ k \in K \end{array} \right] =$$

$$= v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i) + f(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_{-i})),$$

$\forall i \in N,\quad \forall \theta_i, \widehat{\theta}_i \in \Theta_i,\quad \forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, where f is some adequate function, so it suffices to prove that $\sum_{j \neq i} v_j(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_j) + h_i(\theta_{-i}) =$

$= f(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_{-i}))\ \forall i \in N,\ \forall \widehat{\theta} \in \prod_{i=1}^{n} \Theta_i$, where $h_i(\theta_{-i})$ is any real valued function, which is the only functional form allowed -see Green & Laffont [5]-.

Notice that the Groves' mechanisms are a particular case of those compensation mechanisms in the case of interpreting the model as choosing some vector of public goods. Notice that in *Propositions 2* to 4, we did not assume the quasi-linearity of the final payoff function, which will be interpreted in our context as allowing only for compensations by transfers, which is a particular member of the family of payoff function structures, so this propositions are stronger than Green & Laffont's Theorem in the sense that concluding that the only form of the utility function on both private and public goods that allows for the implementability of the utilitarian SWF is exactly the domain imposed by the former authors, i.e., the quasi-linear preferences without any income effect. Notice, also, that only under this restriction on the domain of extended preferences the SWF as representing the Pareto optimal SCC makes complete sense.

An important feature of the model that should be pointed out is that Green and Laffont's results, as well as ours, are extremely dependent on the **non-existence of a common fixed bound** on the types allowed in the domain. Notice that if the planner possess the additional information that the characteristics cannot be *too high*, the utilitarian SCC will not be individually decisive. Suppose, for example, that the domain of types is restricted to be all the bounded and continuous or upper semi-continuous functions such that $\forall i \in N,\ \forall \theta_i \in \Theta_i,\ \exists \overline{c}, \underline{c} \in E\ s.t.\ \forall k \in K,\ \underline{c} \leq v_i(k, \theta_i) \leq \overline{c}$. With this new restriction, for some $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, it is impossible to find

feasible types $\theta_i \in \Theta_i$, $s.t.\ \forall k \in K$, $k \in \underset{k \in K}{\arg\max} \sum_{j=1}^{n} v_j(k, \theta_j)$. Hence, with this new domain, the utilitarian SWF is not individually decisive and *Theorem 1* cannot be applied. Take, for example, $n = 3$, $\underline{c} = 0$, $\overline{c} = 1$, and $\forall j \neq i$, $v_j(k, \theta_j) = \begin{cases} 1 & for\ k \geq \overline{k}. \\ 0 & otherwise. \end{cases}$ Notice that for any $\theta_i \in \Theta_i$, the following holds: $W(\widehat{k}, \theta_i, \theta_{-i}) \geq 2 > 1 \geq W(\widetilde{k}, \theta_i, \theta_{-i})\ \forall \widehat{k} \geq \overline{k},\ \forall \widetilde{k} < \overline{k}$.

5.4 Compensations by means of prices

Consider the following particular problem: $N = \{1, 2\}$,

$K = \left\{(k_1, k_2) \in E_+^2\ \ s.t.\ k_1 + k_2 = \overline{k}\right\}$, $\forall \theta_1 \in \Theta_1$, $v_1(k, \theta_1)$ is such that: $\forall (k_1, k_2), (k_1, \widehat{k}_2) \in K$, $v_1(k_1, k_2, \theta_1) = v_1(k_1, \widehat{k}_2, \theta_1)$ & $\forall \theta_i \in \Theta_i$, $\exists (k_1, k_2) \in K$ such that: $v_i(k_1, k_2, \theta_i) > 0$, $\forall i = 1, 2$.

$v_1(k_1, k_2, \theta_1)$ is continuous and strictly increasing in the first argument. Agent 2's characteristics will be of the same kind but permuting the arguments of the set K. We will write them $v_1(k_1, \theta_1)$ and $v_2(k_2, \theta_2)$.

Proposition 5 *The only incentive compatible compensation mechanisms that implement by revelation the Nash SWF, i.e.,* $W = v_1(k, \theta_1) v_2(k, \theta_2)$, *are such that:* $\forall i \in \{1, 2\}$, $\forall \theta_i \in \Theta_i$, $\forall \theta_j \in \Theta_j$, $(j \neq i)$,

$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_j), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_j)) =$

$= W(g(\widehat{\theta}_i, \widehat{\theta}_j)), \theta_i, \widehat{\theta}_j) = W \left[\underset{k \in K}{\arg\max}\ v_1(k, \theta_1) v_2(k, \theta_2) \right]$, *where* W *is any SWF such that* $W \in \overline{W}(K^*)$.

Proof. We are going to prove that in the above economy, the Nash bargaining SWF is, in fact, individually decisive. We show this for individual 1 and the proof for the other agent is, of course, symmetric.

Suppose any admissible type for individual 2: $\theta_2 \in \Theta_2$, take any alternative from the range of the Nash SCC, i.e., the open set:

$$\widehat{K} = \left\{(k_1, k_2) \in E_+^2\ \ s.t.\ k_1 k_2 \neq 0\ \ \&\ \ k_1 + k_2 = \overline{k}\right\}.$$

Notice that we cannot pick up alternatives with some zero component. Let us call any of this $\widehat{k} = (\widehat{k}_1, \widehat{k}_2) = (\widehat{k}_1, \overline{k} - \widehat{k}_1) \in \widehat{K}$. Now, consider the following characteristic for individual 1: for any $\theta_2 \in \Theta_2$ declared by agent 2,

$v_1(k_1, \widehat{\theta}_1) = 2 v_1(\widehat{k}_1, \widehat{\theta}_1) - \dfrac{v_1(\widehat{k}_1, \widehat{\theta}_1)}{v_2(\widehat{k}_2, \theta_2)} v_2(\overline{k} - k_1, \theta_2)$. $v_1(\widehat{k}_1, \widehat{\theta}_1)$ being any positive real number. Let us call $\widehat{\beta}(\widehat{\theta}_1, \theta_2)$ the absolute slope of the function

$\widehat{\beta}(\widehat{\theta}_1, \theta_2) = \dfrac{v_1(\widehat{k}_1, \widehat{\theta}_1)}{v_2(\widehat{k}_2, \theta_2)}$. Notice that this is an admissible type for individual 1, because it is decreasing on k_2 (increasing on k_1) when $(k_1, k_2) \in \widehat{K}$, and continuous since θ_2 is continuous. By using this strategy, the outcome of the SWF will be the set:

$$K^*(\widehat{\theta}_1, \theta_2) = \begin{array}{ll} \arg\max & W = v_1(k_1, \widehat{\theta}_1) v_2(k_2, \theta_2) \\ k_1, k_2 & \\ s.t & k_1 + k_2 \leq \overline{k} \end{array}$$

. Suppose that $(k_1^*, k_2^*) \in K^*(\widehat{\theta}_1, \theta_2)$, it always holds that, if we define $\widehat{u}_1^*(k_1) = v_1(k_1, \widehat{\theta}_1)$, and $u_2^*(k_2) = v_2(k_2, \theta_2)$

$$(\widehat{u}_1^*(k_1^*), u_2^*(k_2^*)) = \begin{array}{ll} \arg\max & W = \widehat{u}_1 u_2 \\ \widehat{u}_1, u_2 & \\ s.t & \widehat{u}_1 = 2v_1(\widehat{k}_1, \widehat{\theta}_1) - \widehat{\beta}(\widehat{\theta}_1, \theta_2) u_2 \end{array}$$

But the feasibility constraint is a linear function in the space $(\widehat{u}_1, u_2)$, so the solution will be unique and the necessary and sufficient conditions that hold in the optimum are the following:

i) $\widehat{\beta} = \dfrac{\widehat{u}_1(k_1^*)}{u_2(k_2^*)} = \dfrac{du_1}{du_2} = \widehat{\beta}(\widehat{\theta}_1, \theta_2) = \dfrac{v_1(\widehat{k}_1, \widehat{\theta}_1)}{v_2(\widehat{k}_2, \theta_2)}$ (By definition of the slope).

ii) $\widehat{u}_1(k_1^*) = 2v_1(\widehat{k}_1, \widehat{\theta}_1) - \widehat{\beta}(\widehat{\theta}_1, \theta_2) u_2(\overline{k} - k_1^*)$.

Notice that by construction, the first tangency condition can only be fulfilled in $k^* = \widehat{k}$, and the second holds for $k_1^* = \widehat{k}_1$ as well. Therefore, agent 1 has always some admissible strategy so that he can get any alternative he wants with the exception of the extremes, but he can choose alternatives as closed as desired to them (an alternative out of the range will make the slope become infinity) and always any alternative in $range(K^*)$, so the SWF is individually decisive. Since the types are continuous functions, we can now apply *Theorem 1* and the conclusion is obvious. ■

This proposition posses its own interest since the above admissible domain of characteristics can be interpreted as production sets of two firms that have to share some fixed amount of a common input. Both firms have private information about his own technology and send their revealed technologic characteristics to the planner or authority, which makes the final sharing decision. It shows that we can implement by revelation the Nash bargaining solution even with lack of information if the authority can establish the prices at which both firms sell their respective output. Therefore, we should use some price payoff structure to obtain it.

Corollary 5 *Suppose $\theta_1, \theta_2 \in \Theta_1, \Theta_2$. Every individually decisive SCC can*

be trivially implemented by means of prices, i.e., when individual's payoff functions are of the type:
$P_i(v_i(g(\theta_i, \theta_{-i}), \theta_i), q_i(\theta_i, \theta_{-i})) = v_i(g(\theta_i, \theta_{-i}), \theta_i) q_i(\theta_i, \theta_{-i})$.

Proof. Obvious: just consider the following compensation functions: $q_i(\theta_i, \theta_{-i}) = 0 \; \forall \theta_i \in \Theta_i, \; \forall \theta_{-i} \in \prod_{j \neq i} \theta_j$. ■

5.5 Other types of compensations

Now, we may wonder what other SWFs the planner could be interested to implement. Thinking on some ethical and efficient rules, we can investigate if there exists a method to implement some kind of equal welfare among the agents.

Proposition 6 *Let K be a compact set in a topological space and Θ_i be the set of continuous or upper semi-continuous bounded functions. There does not exist any incentive compatible compensation mechanism implementing by revelation the Rawlsian SWF, i.e., $W = min\{v_1(k, \theta_1), v_2(k, \theta_2), ..., v_n(k, \theta_n)\}$*

Proof. We prove that if the set of characteristics $\Theta_i \; \forall i \in N$, are either upper semi-continuous or continuous, the Rawlsian or egalitarian SWF represent an individually decisive SCC. Suppose that any individual i, and any $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, set any $\overline{k} \in K$, and consider the real number $c_j \in E$ be the lower bound of each $j \neq i$, then, there exists the number $\widehat{c}_i = \min_{j \neq i} \{c_j\} < \infty$, and, for any $\overline{k} \in K$, take the following type for individual i:
$v_i(k, \widehat{\theta}_i) = \begin{cases} \widehat{c}_i & if \;\; k = \overline{k} \\ \widehat{c}_i - 1 & if \;\; k \neq \overline{k} \end{cases}$ Notice that this is a bounded below, upper semi-continuous function, and it holds by construction that:

$$\min\{v_1(k, \theta_1), ..., v_{i-1}(k, \theta_{i-1}), v_{i+1}(k, \theta_{i+1}), ..., v_n(k, \theta_n)\} \geq \min_{j \neq i}\{c_j\} = \widehat{c}_i,$$

because every characteristic is bounded below by c_j, so $\min_{j \neq i}\{v_j(k, \theta_j)\} \geq \widehat{c}_i \geq v_i(k, \widehat{\theta}_i) \;\; \forall k \in K$. Consider now the problem:

$$\max_{k \in K} \quad \min\left\{v_1(k, \theta_1), ..., v_i(k, \widehat{\theta}_i), ..., v_n(k, \theta_n)\right\} \;=\; \max_{k \in K} \quad v_i(k, \widehat{\theta}_i)$$

and observing the definition of $v_i(k, \widehat{\theta}_i)$, $v_i(\overline{k}, \theta_i) \geq v_i(k, \widehat{\theta}_i) \;\; \forall k \in K$, so $\overline{k} \in K^*(k, \widehat{\theta}_i, \theta_{-i})$, and we can generate some $v_i(k, \widehat{\theta}_i)$ for every $\overline{k} \in K$, and

every $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, so the Rawlsian SCC is individually decisive. The case where Θ_i contains only continuous preferences does not differ very much from this one: just consider instead of $\widehat{\theta}_i \in \Theta_i$, the following characteristic: $\widetilde{\theta}_i \in \Theta_i$ such that:

$$v_i(k, \widetilde{\theta}_i) = \begin{cases} \widehat{c}_i - \left\| k - \overline{k} \right\| & for \;\; k \;\; s.t. \;\; \left\| k - \overline{k} \right\| \leq 1 \\ \widehat{c}_i - 1 & otherwise. \end{cases}$$

And it is easy to check that the function is continuous and individual i can attain any alternative he wants just by changing $\overline{k}$, $\forall k \in K$.

Now we can apply *Theorem 1* in both cases and obtain: $\forall i \in N$, $\forall \theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) = W((\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i, \widehat{\theta}_{-i}) =$$

$f(\min \left\{ v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), v_{-i}(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_{-i}) \right\})$, for any selection g from K^* and f being some function. But notice that the payoff function structure associated with the mechanism has to be the following: $P_i(v_i(k, \theta_i), q_i(k, \widehat{\theta}_{-i})) = W \left[\arg\min_k \left\{ v_i(k, \theta_i), q_i(k, \widehat{\theta}_{-i}) \right\} \right]$, for any $W \in W(K^*)$. But all these are non-monotonic payoff functions: Suppose $\theta \in \prod_{i=1}^n \Theta_i$ such that $\exists i \in N$ such that $v_i(k, \theta_i) < v_j(k, \theta_j)$ $\forall k \in K$. Then, $K^*(\theta) = \underset{k \in K}{\arg\max} \;\; v_i(k, \theta_i)$ and the compensation cannot change. Suppose $\overline{q} \in E$, $\widehat{\theta}_i \in \Theta_i$ such that: $\overline{q} > v_i(k, \widehat{\theta}_i)$, $\forall k \in K$, then, if we consider $\widetilde{q} > \overline{q}$, we have $\widetilde{q} > \overline{q} > v_i(k, \widehat{\theta}_i)$, and the payoff remains the same: $P_i(v_i(k, \widehat{\theta}_i), \overline{\theta}) = P_i(v_i(k, \widehat{\theta}_i), \widetilde{\theta}) = v_i(k, \widehat{\theta}_i)$, a contradiction with the monotonicity assumption, so there cannot exist compensation mechanisms implementing the Rawlsian SWF under the conditions above. ■

Corollary 6 *The following compensation and non-monotonic mechanism allows for implementation of the Rawlsian SCC in the above economy:* $\forall \theta_i, \widehat{\theta}_i \in \Theta_i$, $\forall \widehat{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$$P_i(v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), q_i(\widehat{\theta}_i, \widehat{\theta}_{-i})) =$$

$$= \max \left\{ -v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), \; - \max_{j \neq i} \left\{ v_j(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_j) \right\} \right\}.$$

Proof. Notice that the expression above can be written as:

$$= - \min \left\{ v_i(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \theta_i), \; \max_{j \neq i} \left\{ v_j(g(\widehat{\theta}_i, \widehat{\theta}_{-i}), \widehat{\theta}_j) \right\} \right\} =$$

$= -\min\left\{ v_1(g(\widehat{\theta}),\theta_i),...,v_i(g(\widehat{\theta}),\theta_i),...,v_n(g(\widehat{\theta}),\widehat{\theta}_n)\right\}$. -of course the associated mechanism is not monotonic-. Abusing notation, we can write:

$P_i(v_i(g(\theta_i,\widehat{\theta}_{-i}),\theta_i),q_i(\theta_i,\widehat{\theta}_{-i})) =$

$-\min\left\{ v_i(g(\theta_i,\widehat{\theta}_{-i}),\theta_i),\ v_{-i}(g(\theta_i,\widehat{\theta}_{-i}),\widehat{\theta}_{-i})\right\} \geq$

$\geq -\min\left\{ v_i(g(\widehat{\theta}_i,\widehat{\theta}_{-i}),\widehat{\theta}_i),\ v_{-i}(g(\widehat{\theta}_i,\widehat{\theta}_{-i}),\widehat{\theta}_{-i})\right\} =$

$= P_i(v_i(g(\widehat{\theta}_i,\widehat{\theta}_{-i})),q_i(\widehat{\theta}_i,\widehat{\theta}_{-i}))$

$\forall\theta_i,\widehat{\theta}_i \in \Theta_i,\ \forall\widehat{\theta}_{-i} \in \prod_{j\neq i}\Theta_j$, and this for any selection $g(\widehat{\theta})$ from $K^*(\widehat{\theta})$, so the non-monotonic compensation mechanism implements the Rawlsian SCC. ■

6 Concluding remarks

We have proved in this paper that when we are trying to implement by revelation any SCC *too sensitive to individual preferences*, we have to rely on individual compensation mechanisms that replicate the social objectives. This is the reason why the well-known Groves' mechanism works in quasi-linear domains of preferences, which can be viewed in terms of our model. Hence, it is not by chance that the transfer any individual receives takes the same functional form that the SWF we are trying to implement, but is a general feature that can be extended to different social welfare criteria and to different compensation schemes. Therefore, there exists a strong linkage between the compensation structure we allow in each case and the social welfare functions that we can implement. Essentially, we need an additive transfer scheme -like taxes or subsidies- to implement the utilitarian SWF, some multiplicative prices scheme to achieve the Nash bargaining solution, and it is impossible within our assumptions to implement the egalitarian rule. When the planner cannot make compensations, implementation by revelation requires dictatorship or it is often impossible. In the opposite extreme, when the planner can expropriate the part of the agents' objective functions affected by the type and completely determine the final payoff, every social choice rule can be trivially implemented. It seems that diminishing the effect of the agents' types on their own payoff considerably enlarge the set of rules that can be implemented. We have abstracted thorough the paper the possible costs the planner may face in choosing one or another contract structure -the mechanism-, but if they exist, the planner might compare the implementation gains with the costs associated to each contract. Moreover, one or another social choice rule or compensation mechanism might be more

appropriate in different contexts: public good provision, production implementation, etc., but we are always constrained by the menu provided by *Theorem 1.*

7 Appendix

Although the dependence of *Theorem 1* to the individually decisiveness assumption is clear, we can slightly relax the class of admitted SCCs if we restrict attention to continuous mechanisms (when both the compensation functions and the payoff functions are continuous when considering the sup norm). The following result provides the continuous mechanisms version of *Theorem 1.*, but, before we state it, we need one more definition and a lemma.

Definition 4 *A SCC K^* is called* ***individually quasi-decisive*** *iff* $\forall i \in N,\ \forall \theta_{-i} \in \prod_{j\neq i}\Theta_j,\ \forall \tau > 0,\ \forall k \in K,\ \exists \widehat{\theta}_i(\tau, k, \theta_{-i}) \in \Theta_i$ *such that* $\overline{k} \in K^*(\widehat{\theta}_i(\tau, k, \theta_{-i}), \theta_{-i}) \in K,\ \ \overline{k} \in B_\tau(k)\ \ \&\ \ \exists \lim_{\tau\to 0}\widehat{\theta}_i(\tau, k, \theta_{-i})$. *(where $B_\tau(k)$ stands for the open ball with center in k and radius τ using the Euclidean metric).*

This property means that everybody can obtain an alternative as close as desired to any other by reporting an adequate type and is weaker than individual decisiveness -every individually decisive SCC is always quasi-decisive, but the converse is not true-.

Lemma 4 *Suppose Θ_i contains only continuous functions for each i. Let $\{P, q\}$ be a continuous, incentive compatible for i compensation mechanism implementing the SCC K^*, and take two -possibly the same- selections from $K^* : g$ and $\widehat{g}$. Then, $\forall \theta_i, \theta_i' \in \Theta_i,\ \forall \theta_{-i} \in \prod_{j\neq i}\Theta_j,\ \forall \phi > 0,\ \exists \rho > 0$ such that $g(\theta_i, \theta_{-i}) \in B_\rho(\widehat{g}(\theta_i', \theta_{-i})) \Rightarrow\ q_i(\theta_i, \theta_{-i}) \in B_\phi(q_i(\theta_i', \theta_{-i}))$.*

Proof. By contradiction, suppose that $\forall \rho > 0,\ \exists \theta_i, \theta_i' \in \Theta_i,\ \exists \theta_{-i} \in \prod_{j\neq i}\Theta_j,\ \exists \phi > 0$, such that $g(\theta_i, \theta_{-i}) \in B_\rho(\widehat{g}(\theta_i', \theta_{-i}))$ &
$q_i(\theta_i, \theta_{-i}) \notin B_\phi(q_i(\theta_i', \theta_{-i}))$. Suppose w.l.g. that
$q_i(\theta_i, \theta_{-i}) - q_i(\theta_i', \theta_{-i}) > \phi$. Then, take type θ_i', and consider the payoffs:
$P_i(v_i(g(\theta_i, \theta_{-i}), \theta_i'), q_i(\theta_i, \theta_{-i}))$ and $P_i(v_i(\widehat{g}(\theta_i', \theta_{-i}), \theta_i'), q_i(\theta_i', \theta_{-i}))$. But, by assumption, $\forall \rho > 0,\ \exists \theta_i, \theta_i' \in \Theta_i,\ \exists \theta_{-i} \in \prod_{j\neq i}\Theta_j$, such that $g(\theta_i, \theta_{-i}) \in B_\rho(\widehat{g}(\theta_i', \theta_{-i}))$, so we can choose θ_i, θ_i' such that $g(\theta_i, \theta_{-i})$ will be as close to $\widehat{g}(\theta_i', \theta_{-i})$ as we want. Since Θ_i includes only continuous functions, we can make $v_i(g(\theta_i, \theta_{-i}), \theta_i')$ as close as desired to $v_i(\widehat{g}(\theta_i', \theta_{-i}), \theta_i')$ by choosing $\theta_i, \theta_i' \in \Theta_i$. But P_i is continuous in both arguments and monotonic so, by

carefully choosing $\rho > 0$ and $\theta_i, \theta'_i \in \Theta_i$, since $q_i(\theta_i, \theta_{-i}) - q_i(\theta'_i, \theta_{-i}) > \phi$, we should have

$P_i(v_i(g(\theta_i, \theta_{-i}), \theta'_i), q_i(\theta_i, \theta_{-i})) > P_i(v_i(\widehat{g}(\theta'_i, \theta_{-i}), \theta'_i), q_i(\theta'_i, \theta_{-i})). \Rightarrow$

For θ'_i, $\exists \theta_i \in \Theta_i$, $\exists \overline{g}(\theta_i, \theta_{-i}) \in K^*(\theta_i, \theta_{-i})$ $\forall \theta_i \in \Theta_i$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$, -a selection from K^*- defined as:

$\overline{g}(\widehat{\theta}_i, \theta_{-i}) = \begin{cases} g(\widehat{\theta}_i, \theta_{-i}) & iff\ \widehat{\theta}_i \neq \theta'_i \\ \widehat{g}(\theta'_i, \theta_{-i}) & iff\ \widehat{\theta}_i = \theta'_i \end{cases}$, such that by declaring $\theta_i \in \Theta_i$, agent i will receive a higher payoff than reporting the truth (θ'_i), i.e.,

$P_i(v_i(\overline{g}(\theta_i, \theta_{-i}), \theta'_i), q_i(\theta_i, \theta_{-i})) > P_i(v_i(\overline{g}(\theta'_i, \theta_{-i}), \theta'_i), q_i(\theta'_i, \theta_{-i}))$, so the mec anism cannot be incentive compatible for i, a contradiction. ■

Theorem 2 *Suppose that $\forall i \in N$, Θ_i is restricted to contain continuous functions and let K^* be an individually quasi-decisive SCC. Then if $\{P, q\}$ is an incentive compatible and continuous compensation mechanism that implement K^*, then it must be of the form of those in* Theorem 1.

Proof. $\Rightarrow$) First of all, notice that *Lemma 1* applies to every compensation mechanism implementing any SCC, so it holds now that $\forall \theta_i, \theta'_i \in \Theta_i$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$, such that $g(\theta_i, \theta_{-i}) = g(\theta'_i, \theta_{-i})$, it holds that: $q_i(\theta_i, \theta_{-i}) = q_i(\theta'_i, \theta_{-i})$.

Moreover, since K^* is individually quasi-decisive, $\forall i \in N$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$, $\forall \tau > 0$, $\forall k \in K$, $\exists \widehat{\theta}_i : E_{++} \times K \times \prod_{j \neq i} \Theta_j \rightarrow\rightarrow \Theta_i$ such that

$\overline{k} \in K^*(\widehat{\theta}_i(\tau, k, \theta_{-i}), \theta_{-i}) \in K$ & $\overline{k} \in B_\tau(k)$. Let us now define the following correspondence: $\widehat{q}_i : K \times \Theta_{-i} \rightarrow\rightarrow E$, such that: $\widehat{q}_i(k, \theta_{-i}) =$

$= q_i(\lim_{\tau \to 0} \widehat{\theta}_i(\tau, k, \theta_{-i}), \theta_{-i})$ for all $k \in K$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$.

This mapping is similar to the analogous one used in the proof of *Theorem 1*. Moreover, $\widehat{q}_i(k, \theta_{-i})$ always exist and is a **continuous function on** K. Notice that $\forall k \in K$ such that $\exists \theta'_i \in \Theta_i$ such that $k = g(\theta'_i, \theta_{-i})$ for some selection g from K^*, $\widehat{q}_i(k, \theta_{-i})$ is a singleton following the same reasoning in proving (ii) in *Theorem 1* -using *Lemma 2* for all i-, and the cases where $\forall \theta'_i \in \Theta_i$, $k \neq g(\theta'_i, \theta_{-i})$, individual quasi-decisiveness assures that the limit exists. It remains to prove that the function is continuous for every $k \in K$, so we prove the following: $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$, $\forall k \in K$, $\forall \varepsilon > 0$, $\exists \delta(k, \varepsilon) > 0$ such that $\overline{k} \in B_\delta(k) \Rightarrow \widehat{q}_i(\overline{k}, \theta_{-i}) \in B_\varepsilon(\widehat{q}_i(k, \theta_{-i}))$. By contradiction, suppose that the converse is true, i.e., $\exists \theta_{-i} \in \prod_{j \neq i} \Theta_j$, $\exists k \in K$, $\exists \varepsilon > 0$, such that $\forall \delta > 0$, $\exists \overline{k} \in B_\delta(k)$ & $\widehat{q}_i(\overline{k}, \theta_{-i}) \notin B_\varepsilon(\widehat{q}_i(k, \theta_{-i}))$. Then, choose any selection $\widehat{q}'_i(k, \theta_{-i}) \in \widehat{q}_i(k, \theta_{-i})$, $\forall k \in K$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$ and set any $\theta_{-i} \in \prod_{j \neq i} \Theta_j$; Now, take any $k \in K$ such that $\exists \theta'_i \in \Theta_i$ such that $k = g(\theta'_i, \theta_{-i})$ for some selection g from K^*. Then, suppose that $\exists k \in K$ such that $\exists \varepsilon > 0$ such that $\forall \delta > 0$, $\exists \overline{k}(\delta) \in B_\delta(k)$ & $\widehat{q}'_i(\overline{k}, \theta_{-i}) \notin B_\varepsilon(\widehat{q}'_i(k, \theta_{-i}))$.

First, since K^* is individually quasi-decisive, we know that there exists some selection $g \in K^*$ such that, for $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, $k, \overline{k}(\delta) \in K$ and $\forall \delta > 0$, these two conditions hold:

(i). $\forall \tau' > 0$, $\exists \widetilde{\theta}_i \in \Theta_i$ such that $g(\widetilde{\theta}_i, \theta_{-i}) \in B_{\tau'}(k)$.

(ii). $\forall \tau > 0$, $\exists \overline{\theta}_i \in \Theta_i$ such that $g(\overline{\theta}_i, \theta_{-i}) \in B_{\tau}(\overline{k}(\delta))$.

Moreover, by definition of $\widehat{\theta}_i(\tau, k, \theta_{-i})$, we know that:

$\widetilde{\theta}_i \in B_{\tau'}(\lim_{\tau' \to 0} \widehat{\theta}_i(\tau', k, \theta_{-i}))$ and $\overline{\theta}_i \in B_{\tau}(\lim_{\tau \to 0} \widehat{\theta}_i(\tau, \overline{k}(\delta), \theta_{-i}))$ so it holds that $\forall \delta, \tau, \tau' > 0$, $\exists \widetilde{\theta}_i, \overline{\theta}_i \in \Theta_i$ such that $g(\widetilde{\theta}_i, \theta_{-i}) \in B_{\delta+\tau+\tau'}(g(\overline{\theta}_i, \theta_{-i}))$. Furthermore, by continuity of the bounded compensation functions in

$\lim_{\tau \to 0} \widehat{\theta}_i(\tau, \overline{k}(\delta), \theta_{-i})$ and $\lim_{\tau' \to 0} \widehat{\theta}_i(\tau', k, \theta_{-i})$, it is true that $\forall \widehat{\varepsilon}_1, \widehat{\varepsilon}_2 > 0$,

$\exists \widehat{\delta}_1(\widehat{\varepsilon}_1), \widehat{\delta}_2(\widehat{\varepsilon}_2) > 0$, such that

$\theta_i \in B_{\widehat{\delta}_1(\widehat{\varepsilon}_1)}(\lim_{\tau \to 0} \widehat{\theta}_i(\tau, \overline{k}(\delta), \theta_{-i}))$ and $\theta'_i \in B_{\widehat{\delta}_2(\widehat{\varepsilon}_2)}(\lim_{\tau' \to 0} \widehat{\theta}_i(\tau', k, \theta_{-i})), \Rightarrow$
$\Rightarrow q_i(\theta_i, \theta_{-i}) \in B_{\widehat{\varepsilon}_1}(q_i(\lim_{\tau \to 0} \widehat{\theta}_i(\tau, \overline{k}(\delta), \theta_{-i}), \theta_{-i}))$ and

$q_i(\theta'_i, \theta_{-i}) \in B_{\widehat{\varepsilon}_2}(q_i(\lim_{\tau' \to 0} \widehat{\theta}_i(\tau', k, \theta_{-i}), \theta_{-i}))$. Now, given any $\delta > 0$, for every $\widehat{\varepsilon}_1, \widehat{\varepsilon}_2 > 0$, we can take $\tau \leq \widehat{\delta}_1(\widehat{\varepsilon}_1)$ and $\tau' \leq \widehat{\delta}_2(\widehat{\varepsilon}_2)$ and there will exist $\widetilde{\theta}_i, \overline{\theta}_i \in \Theta_i$ with the above properties.

Finally, we can apply *Lemma 4* for $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, for $\widehat{g} = g$ and for the above $\widetilde{\theta}_i, \overline{\theta}_i \in \Theta_i$ to get: $\forall \phi > 0$, $\exists \rho > 0$ such that $g(\theta_i, \theta_{-i}) \in B_\rho(\widehat{g}(\theta'_i, \theta_{-i})) \Rightarrow q_i(\theta_i, \theta_{-i}) \in B_\phi(q_i(\theta'_i, \theta_{-i}))$, whenever we choose $\delta, \widehat{\varepsilon}_1, \widehat{\varepsilon}_2 > 0$ such that, for any given $\phi > 0$, $\rho(\phi) \geq \delta + \tau + \tau'$. Then, by (i) and (ii), it holds: $\exists \widetilde{\theta}_i, \overline{\theta}_i \in \Theta_i$ such that $g(\overline{\theta}_i, \theta_{-i}) \in B_\rho(g(\widetilde{\theta}_i, \theta_{-i}))$, so $\forall \phi, \delta, \widehat{\varepsilon}_1, \widehat{\varepsilon}_2 > 0$, $\exists \widetilde{\theta}_i, \overline{\theta}_i \in \Theta_i$ such that:

(1). $q_i(\overline{\theta}_i, \theta_{-i}) \in B_\phi(q_i(\widetilde{\theta}_i, \theta_{-i}))$,

(2). $q_i(\overline{\theta}_i, \theta_{-i}) \in B_{\widehat{\varepsilon}_1}(q_i(\lim_{\tau \to 0} \widehat{\theta}_i(\tau, \overline{k}(\delta), \theta_{-i}), \theta_{-i}))$ and

(3). $q_i(\widetilde{\theta}_i, \theta_{-i}) \in B_{\widehat{\varepsilon}_2}(q_i(\lim_{\tau' \to 0} \widehat{\theta}_i(\tau', k, \theta_{-i}), \theta_{-i}))$.

Now, choose $\phi, \widehat{\varepsilon}_1, \widehat{\varepsilon}_2 > 0$ sufficiently small such that, for example, $\phi + \widehat{\varepsilon}_1 + \widehat{\varepsilon}_2 \leq \dfrac{\varepsilon}{4}$, to observe that (1), (2) and (3) make any $\varepsilon > 0$ such that $\forall \delta > 0$, $\exists \overline{k}(\delta) \in B_\delta(k)$, $\widehat{q}'_i(\overline{k}, \theta_{-i}) \notin B_\varepsilon(\widehat{q}'_i(k, \theta_{-i}))$ impossible, so we enter into a contradiction and the function $\widehat{q}'_i(k, \theta_{-i})$ is continuous for all $k \in K$.

Now we consider the same kind of composite function of *Theorem 1*: $\widehat{P}_i : K \times \prod_{i=1}^n \Theta_i \to E$ defined as: $\forall k \in K$, $\forall \theta_i \in \Theta_i$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta_j$,

$\widehat{P}_i(k, \theta_i, \theta_{-i}) = P_i(v_i(k, \theta_i), \widehat{q}_i(k, \theta_{-i}))$. We know now that, given any $\theta_{-i} \in \prod_{j \neq i} \Theta_j$, this function is continuous in the whole K, since Θ_i are continuous by assumption and we have already proved that $\widehat{q}_i$ are continuous functions. Hence suppose, by contradiction, that the theorem is not true: $\exists i \in N$, $\exists \overline{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$, $\exists \theta_i \in \Theta_i$, $\exists k \in K$ such that: $\widehat{P}_i(g(\theta_i, \overline{\theta}_{-i}), \theta_i, \overline{\theta}_{-i}) < \widehat{P}_i(k, \theta_i, \overline{\theta}_{-i})$ for some selection g from K^*-note that in other case, $\widehat{P}_i \in$

$W(K^*)$-, or in other words, the following statements are true:

(i). $\exists\widehat{\varepsilon} > 0$ such that $\widehat{P}_i(k, \theta_i, \overline{\theta}_{-i}) - \widehat{P}_i(g(\theta_i, \overline{\theta}_{-i}), \theta_i, \overline{\theta}_{-i}) = \widehat{\varepsilon}$.

(ii). Since K^* is individually quasi-decisive, for k, $\overline{\theta}_{-i}$ and any $\tau > 0$, there exist a selection from $\widehat{\theta}_i$ such that $\exists\overline{k} \in K$ such that $\overline{k} \in K^*(\widehat{\theta}_i(\tau, \overline{k}, \overline{\theta}_{-i}), \theta_{-i}) \in K$ & $k \in B_\tau(\overline{k})$.

(iii). Since given θ_i and $\overline{\theta}_{-i}$, $\widehat{P}_i$ is continuous in K, in particular for k it is true that: $\forall\varepsilon > 0$, $\exists\delta(k, \varepsilon) > 0 \ni \underline{k} \in B_\delta(k) \Rightarrow P_i(\underline{k}, \theta_i, \overline{\theta}_{-i}) \in B_\varepsilon(P_i(k, \theta_i, \overline{\theta}_{-i}))$.

First, using (iii), for $k \in K$, for $\widehat{\varepsilon} > 0$, $\exists\delta(k, \widehat{\varepsilon}) > 0 \ni \widetilde{k} \in B_{\delta(k,\widehat{\varepsilon})}(k)$ -by (b)- $\Rightarrow \widehat{P}_i(\widetilde{k}, \theta_i, \overline{\theta}_{-i}) \in B_{\widehat{\varepsilon}}(\widehat{P}_i(k, \theta_i, \overline{\theta}_{-i}))$. Now, let us set $\tau = \delta(k, \widehat{\varepsilon})$; by (ii), for this τ there will exist $\widetilde{\theta}_i \ni \widetilde{k} \in K^*(\widetilde{\theta}_i(\delta(k, \widehat{\varepsilon}), k, \overline{\theta}_{-i}), \overline{\theta}_{-i}) \in K$ & $\widetilde{k} \in B_{\delta(k,\widehat{\varepsilon})}(k)$. Finally, this last expression can be written as: for $k \in K$ and given θ_i and $\overline{\theta}_{-i}$, $\exists\widetilde{\theta}_i(\delta(k, \widehat{\varepsilon}), k, \overline{\theta}_{-i})$, $\exists\widetilde{k} \in K^*(\widetilde{\theta}_i, \overline{\theta}_{-i}) \in K$ s.t.

$\left|\widehat{P}_i(k, \theta_i, \overline{\theta}_{-i})) - \widehat{P}_i(\widetilde{k}, \theta_i, \overline{\theta}_{-i})\right| < \widehat{\varepsilon}$ and $\widehat{P}_i(k, \theta_i, \overline{\theta}_{-i})) - \widehat{P}_i(g(\theta_i, \overline{\theta}_{-i}), \theta_i, \overline{\theta}_{-i}) = \widehat{\varepsilon}$ by definition, so it has to be that for $k \in K$, $\theta_i \in \Theta_i$ and $\overline{\theta}_{-i} \in \prod_{j \neq i} \Theta_j$,

$\exists\widetilde{\theta}_i(\delta(k, \widehat{\varepsilon}), k, \overline{\theta}_{-i})$, $\exists\widetilde{k} \in K^*(\widetilde{\theta}_i, \overline{\theta}_{-i}) \in K$ such that there exists the following selection from K^* :

$\widetilde{g}(\theta) = \begin{cases} \widetilde{k} & iff\ (\theta_i, \theta_{-i}) = (\widetilde{\theta}_i, \overline{\theta}_{-i}) \\ g(\theta_i, \theta_{-i}) & otherwise \end{cases}$ such that

$\widehat{P}_i(\widetilde{g}(\widetilde{\theta}_i, \overline{\theta}_{-i}), \theta_i, \overline{\theta}_{-i}) > \widehat{P}_i(\widetilde{g}(\theta_i, \overline{\theta}_{-i}), \theta_i, \overline{\theta}_{-i})$, which is the same that

$$P_i(v_i(\widetilde{g}(\widetilde{\theta}_i, \overline{\theta}_{-i}), \theta_i), q_i(\widetilde{\theta}_i, \overline{\theta}_i)) > P_i(v_i(\widetilde{g}(\theta_i, \overline{\theta}_{-i}), \theta_i), q_i(\theta_i, \overline{\theta}_{-i}))$$

so the compensation mechanism fails to be incentive compatible for some i: a contradiction.

$\Leftarrow$) The sufficiency part of the proof is exactly the same that the one in *Theorem 1.* ■

Theorem 2 shows that our main result is robust even if we enlarge the set of admissible social rules to individually quasi-decisive SCCs. The price to pay is assuming the continuity of the compensation mechanisms, a property that does not seem to be extremely restrictive when working with our complex set of feasible alternatives. Nevertheless, we can say nothing about the set of discontinuous mechanisms implementing individually quasi-decisive SCCs.

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