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WAGE DYNAMICS IN A STRUCTURAL TIME SERIES MODEL FOR LUXEMBOURG

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Abstract

This paper examines the relationships between monetary wage and its theoretical explanatory variables using a Structural Time Series (STS) model in order to take into account the unobserved components (trend, cycle, seasonal and irregular) of wage. Theoretically, the monetary wage is negatively related to labor productivity and unemployment rate but positively to the consumer price index and foreign prices. Our empirical results for a small open economy as Luxembourg indicate that the wage is positively related to the consumer price index and foreign prices as predicted by the theory, but the labor productivity and unemployment rate are not significant in the explanation of wages dynamics in the Luxembourg economy.

Keywords: Wage Bargaining, Labor Unions, Unobserved Components Models, Structural Time Series

JEL Classification: C22, E31

1. Introduction

Wage formation derives from a theoretical model of wage bargaining both on an atomistic labor market and an imperfect market of goods (see Layard and Nickell 1986, Blanchard and Kiyotaki 1987, Manning 1993, Lockwood and Manning 1993). Aggregate prices and wages are thus given or exogenous for economic agents.

The formulation of the wage bargaining is based first on firms profit maximization program and second on consumers utility

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maximization subject to their budget constraint. This process has been analyzed by several authors including Holden (1999), Pétursson and Slok (2001), Pétursson (2002), Nymoen and Rodseth (2003), Nymoen (2003).

The bargaining of wage proceeds from the consumers' behavior through their labor unions trying to maximize the utility of their members by expecting the highest wage, and from the behavior of firms, which try to get the lowest wage. Wage bargaining between labor union and firms is being established as the common process of wage setting in most European countries. Various approaches are possible but we follow here the line of Pétursson and Slok (2001), and Pétursson (2002), which allows determining both the wage rate and the level of employment in the economy.

Our main empirical findings in the case of a very small open economy like Luxembourg, where the bargaining process applies, are that the wage is positively related to the consumer price index and foreign prices, but labor productivity and unemployment rate are not significant at explaining the wage dynamics. While a 10% increase in foreign prices results in a 1.10% increase in wage, a 10% increase the consumer price index results in a 2.90% increase in the wage. It clearly appears that although the consumer price index and foreign prices are both significant in the explanation of the wage dynamics, the consumer price index and most importantly historical wages are the main variables impacting current wages in Luxembourg economy.

In the rest of the paper Section 2 presents the model of wage formation extended in section 3 with structural time series components. Section 4 presents the estimation results for Luxembourg and section 5 concludes.

2. A Model of Wage Formation

The Behavior of Consumers

The economy is composed of k consumers and in the case of a small open economy most of consumers are non-resident. Each household h has a utility function U expressed as:

$$U_h = (C_h)^{\sigma} / \sigma, \quad h = 1,\ldots,k \qquad \sigma < 1 \tag{1}$$

with C_{ih} the level of the household's consumption being a CES consumption function of all available goods in the economy with the form:

$$C_h = \left(\frac{1}{m}\right)^{\frac{1}{1-\varepsilon}} \left\{ \sum_{i=1}^{m} (C_{ih})^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where $\varepsilon > 1$ represents the elasticity of substitution between consumption goods, mostly imported. The consumer price index P is:

$$P = \left(\frac{1}{m} \sum_{i=1}^{m} P_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{3}$$

The consumers' budget constraint is given by $\sum_{i=1}^{k} P_i C_{ih} \leq W_h$, with the variable W_h indicating the nominal revenue of the consumer.

The maximization of consumers' utility subject to (s.t.) the budget constraint

$$\begin{cases} \max\ U \\ st \\ \sum_{i=1}^{k} P_i C_{ih} = W_h \end{cases} \tag{4}$$

gives the demand of product C_i to the domestic exporter firm:

$$C_i = \frac{W_h}{mP} \left(\frac{P_i}{P} \right)^{-\varepsilon} \tag{5}$$

Plugging the demand of goods C_i in the utility function U_h we get the indirect utility function J_h of the consumer *h*:

$$J_h = \frac{1}{\sigma} \left[\frac{W_h}{mP} \left(\frac{P_i}{P} \right)^{-\varepsilon} \right]^{\sigma} = \frac{1}{\sigma} \left(\frac{W_h}{mP} \right)^{\sigma} \left(\frac{P_i}{P} \right)^{-\frac{\varepsilon}{\sigma}}, \Rightarrow J_h = \frac{1}{\sigma} \left(\frac{W_h}{P} \right)^{\sigma} \tag{6}$$

An individual does decide to work if the indirect utility $J_h > 0$, and doesn't work if $J_h < 0$. He will be indifferent when $J_h = 0$.

The behavior of Firms

For a given wage rate, the domestic producer's problem is to maximize its profit noted:

$$\prod_i = PR_i Y_i - W_i N_i \tag{7}$$

taking into account the technology used for production, the demand of goods to its firm and the market conditions $C_i = Y_i$. The variable *Yi* represents the quantity of goods *i*, and *PRi* is the producer price of the good *i*.

Setting the production function here to a Cobb-Douglas type:

$$Y_i = AK_i^{\alpha} N_i^{1-\alpha} \tag{8}$$

the first order conditions of the profit maximization problem give the marginal productivity relative to the real wage *w/p* (see detail in appendix A1)

$$\frac{(1-\alpha)AK_i^{\alpha}}{N_i^{\alpha}} = \frac{W_i}{PR_i} \tag{9}$$

Solving Eq. (9) for *N* gives the total labor demand:

$$N^d = \left(\frac{W_i}{PR_i}\right)^{-\frac{1}{\alpha}} \left((1-\alpha)AK_i^{\alpha}\right)^{\frac{1}{\alpha}} \tag{10}$$

which is, assuming symmetrically that *Yi=Y*, *Ni=N*, *Wi=W* and *PRi=P*, a function of real wage and capital as follows:

$$N^d = f\left(\frac{W}{P}, K\right) \tag{11}$$

When wages have been fixed by profit maximization, the firm fixes unilaterally the level of employment.

The Behavior of Labor Unions

There are *m* labor unions composed of *l* workers in the economy. The objective of a labor union is to maximize the expected utility of each of its members. The utility of a worker who is member of a union is given by the indirect utility function of the consumer:

$$J_h = \frac{1}{\sigma}\left(\frac{W_h}{P}\right)^{\sigma} \tag{12}$$

while the utility function of a member loosing its job is $\overline{U_j}$. The expected utility of a member is therefore:

$$\left(\frac{N_j}{l_j}\right)J_j + \left(\frac{l_j - N_j}{l_j}\right)\overline{U_j} \tag{13}$$

with N_j representing the number of employees in the sector *j*, and the ratio N_j / l_j indicates thus the proportion of employees in the sector *j*.

The objective function T_j of labour unions is obtained by (i) replacing Eq. (12) in the expected utility function Eq. (13), (ii) subtracting $\overline{U_j}$ and (iii) multiplying the whole by the number of workers *l*. It gives:

$$T_j = \left(\frac{N_j}{l_j}\right)\frac{1}{\sigma}\left(\frac{W_h}{P}\right)^{\sigma} + \left(\frac{l_j - N_j}{l_j}\right)\overline{U_j} - \overline{U_j} \tag{14}$$

$$T_j = N_j\left\{\frac{1}{\sigma}\left(\frac{W_j}{P}\right)^{\sigma} - \overline{U_j}\right\} \tag{15}$$

Nominal wages are fixed as solution of the Nash bargaining between profit maximizing firms (trying to get the lowest wage) and labour unions trying to maximize the utility of their members (by getting the highest wage). Here wage is the unique object of negotiation (« Right To Manage » model)[3]. The solution of the process is obtained by maximizing the following Nash program:

$$\begin{cases} Nash = \max\limits_{W_i} \; T_i^{\mu}\pi_j \\ st \\ N^d = f\left(\frac{W}{P}, K\right) \end{cases} \tag{16}$$

[3] In «Efficient Negotiation» models firms and unions determine wage and employment together. The constraint in Eq. (16) is then removed.

where π_j indicates the profit of each firm and the parameter μ indicates the negotiation power of unions. Solving this program gives the wage rate that maximizes the preceding Nash negotiation (see appendix A2).

The log linear solution can be written finally (see Péturson, 2002):

$$w = \alpha p_r + (1-\alpha) p + \delta z - \theta u + \xi_w \quad 0 \le \alpha, \delta \le 1, \theta \ge 0, \tag{17}$$

where *w* is the nominal wage rate, p_r the producer price, p the consumer price index, *z=(y-n)* labour productivity, *u* the unemployment rate, and ξ_w representing all other terms affecting the result of the negotiation (tax rate, level of skilled labor, labor unions). Replacing the producer price by its value $p_r = \frac{1}{1-\eta} p + \frac{\eta}{1-\eta} q$ (with q indicating foreign prices) we get the following wage equation:

$$w = \frac{\alpha + (1-\alpha)(1-\eta)}{1-\eta} p + \frac{\alpha\eta}{1-\eta} q + \delta z - \theta u + \xi_w \tag{18}$$

This equation of the general form:

$$w = \beta_1 p + \beta_2 q + \beta_3 z + \beta_4 u + \beta_0 \tag{19}$$

can be found in various empirical works and has been used to describe wage formation in several sectors of the economy.

3. An Extended Structural Time Series Model of the Wage Equation

The stochastic formulation of a structural time series (STS) model for the logarithm of wage denoted w_t with explanatory variables is the following:

$$
\begin{aligned}
w_t &= \mu_t + \psi_t + \varepsilon_t \\
\mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\
\beta_t &= \beta_{t-1} + \zeta_t
\end{aligned}
\tag{20}
$$

where μ_t is the trend, ψ_t is the cyclical component, and ε_t the irregular component, all assumed to be stochastic. The parameter β_t is the slope of the trend component. The stochastic properties of the level and the slope are driven by η_t and ζ_t .

The seasonal (cyclical) component in trigonometric form, see Harvey (1989), may be expressed as follows:

$$
\psi_t = \sum_{i=1}^{s/2} \psi_{jt} \quad t = 1, \cdots, T \quad , \tag{21}
$$

with ψ_{jt} determined by:

$$
\begin{bmatrix} \psi_{jt} \\ \psi^*_{jt} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \psi_{jt-1} \\ \psi^*_{jt-1} \end{bmatrix} + \begin{bmatrix} \omega_{jt} \\ \omega^*_{jt} \end{bmatrix}, \tag{22}
$$

(for $j = 1, 2, \cdots, s/2, t = 1, \cdots, T$) where $\lambda_j = 2\pi j / s$ is the frequency in radians and ρ is the damping factor ($0 < \rho \leq 1$).

Combining Eq. (18) or (19) with Eq. (20), we get the following extended stochastic specification:

$$
w_t = \mu_t + \psi_t + \gamma_1 p + \gamma_2 q + \gamma_3 z + \gamma_4 u + \lambda w_{t-1} + \varepsilon_t \tag{23}
$$

where γ_1 , γ_2 , γ_3 , γ_4 and λ are real coefficients. Eq. (23) can be estimated to see which of these components explains much the wage dynamics. The results of the estimated model for Luxembourg are presented in the next section.

4. Empirical Results

The model in Eq. (23) is estimated using STAMP 6.02 (see Koopman, Harvey, Doornik and Shepard, 2000). The quarterly data used are provided by the STATEC[4] (Luxembourg) and cover the period 1995:1-2005:4. Figure 1 shows the time series in level (left panel) and first difference (right panel). We can notice that the data show a non-constant upward trend over time.

We tested the various models for combination of level and slope characteristics for seasonal dummies and for trigonometric dummies. The results not furnished here have shown that the seasonal dummies models with no level and no slope are better than trigonometric dummies models in terms of Log-likelihood. The synthetic results for seasonal dummies models (see Table 1) indicate that models 3 overcome others. In effect the log-Likelihood of model 3 is the highest and we therefore choose this model in the rest of the paper.

In fact examining the Figure of residuals of Model 1 we see that there are outliers. We therefore included irregular components in Model 1 giving Model 2 and estimate again. The Log-likelihood of Model 2 appears lower than Model 1.

In fact we could notice that among the explanatory variables the coefficient of unemployment is not significant. Thus we removed this variable from the model 2 and estimated it again. The final result (model 3) with higher log-Likelihood (**158.848**, see Table 1) indicates that all remaining explanatory variables are significant in terms of t-value (see Table 4). The final model includes impulse dummies at appropriate outlier dates. This model is the most parsimonious estimate of the wage function in Luxembourg.

[4] We are grateful to S. Allegrezza (Director of Statec, Luxembourg) and F. Adam (Statec) for providing the quarterly data (Project “MODEL”). Special thanks to F. Adam for collaboration during the project “MODEL”.

Figure 1: Level and first difference of variables

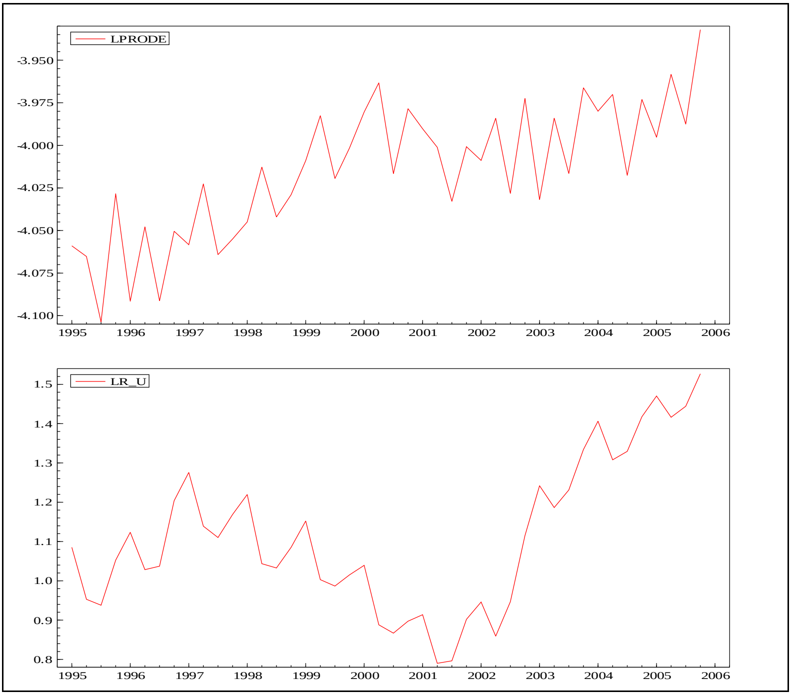


Table 1: Summary Statistics (Model selection) Wage

Models	**Components**				**Log-Likelihood**
	Level	**Slope**	**Variables**	**Irregular**	
			With Dummy Seasonal		
1:	*No*	*No*	**Expl.**	**Irregular**	156.648
2:	*No*	*No*	Expl.	**Irregular**	152.156
3:	*No*	*No*	Expl.	**Irregular**	**158.848**

Table 2: Estimated variances of disturbances Wage (Sample: 1995. 2 - 2005. 4) (T = 43)

Components	**LSALM**	**(q-ratio)**
	Model 3	
Irregular	0.0079877	(1.0000)
Seasonal	0.0010811	(0.1353)

Table 3: Estimated coefficients of final state vector (Sample: 1995. 2 - 2005. 4) (T = 43)

Variables	Coefficient	R.m.s.e.	t-value
	Model 3		
Sea_1	0.15129	0.0070320	21.515
Sea_2	-0.050327	0.0032566	-15.454
Sea_3	0.01615	0.0041544	3.8875

Table 4: Estimated coefficients of explanatory variables (Sample: 1995. 2 - 2005. 4) (T = 43)

Variables	Coefficient	R.m.s.e.	t-value
	Model 3		
LSALM_1	0.7844	0.088795	8.8338
LP_CFIN	0.29066	0.11933	2.4358
LP_XBSEU12	0.11029	0.044874	2.4578
Irr 1999. 4	0.028035	0.0087195	3.2152
Irr 2002. 1	0.019035	0.0085898	2.216
Irr 2004. 4	0.036676	0.0089126	4.1151

Note: LSALM (log of wage); LP_CFIN (log of CPI); LP_XBSEU12 (log of foreign prices)

Figure 2: Residual of Wage Model 3

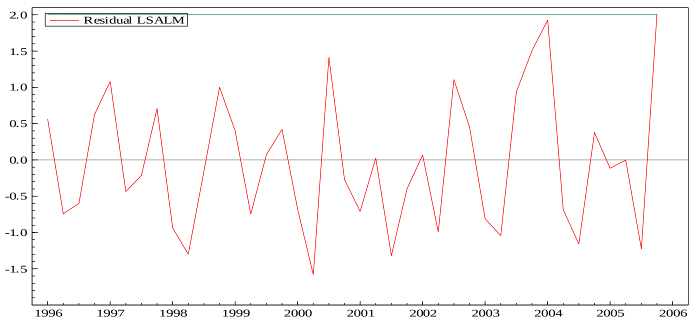
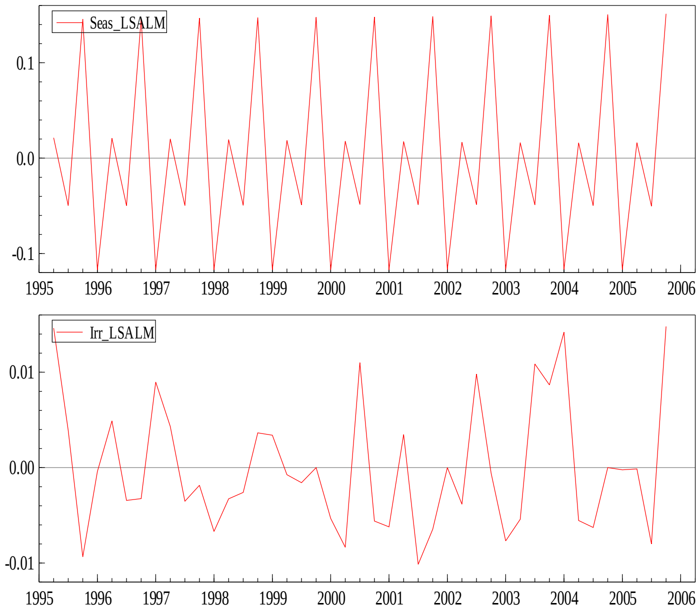


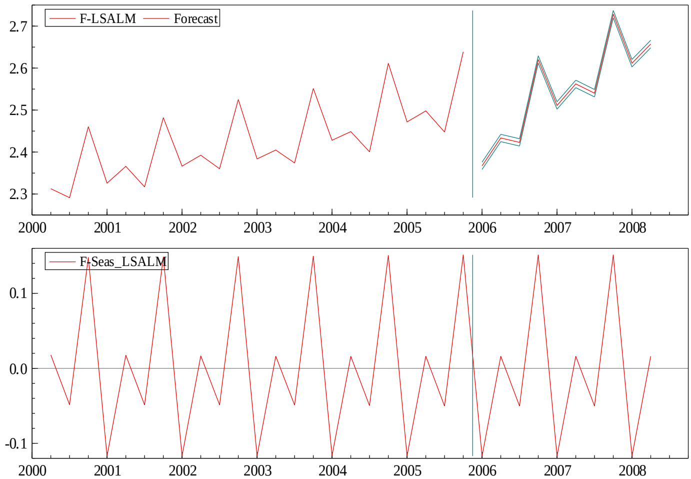
Figure 3: Estimated Component of the Wage



Estimates of explanatory variables in the final state vector show that the one-lag wage, the consumer price index and foreign prices are significant in the explanation of the wage behaviour (Table 4). The wage increases with the consumer price index and with foreign prices. An increase of 10% in the consumer price index results in 2.90% increase in the wage rate, while a 10% increase in foreign prices results in 1.10% increase in the wage rate.

Wage dynamics in Luxembourg are mainly explained by the historical wage level, the consumer price index and foreign prices.

Figure 4: Post-Sample Prediction of CPI (Model 10)



5. Conclusion

This paper analyzes the wage dynamics in Luxembourg along with its unobserved components using a structural time series (STS) model. Theoretically the open economy model of wage formation where unions and firms bargain wage on an atomistic labour market and an imperfect market of goods show that an increase in the consumer price index, in foreign prices and in the labour productivity leads to an increase in wages. But an increase in unemployment leads to a decrease in wage rate.

We find empirically that monetary wage is positively related to the consumer price index and historical wages and to foreign prices, unemployment rate and real labour productivity are not significant in the behaviour of monetary wages in Luxembourg. A 10% increase in foreign prices results in a 1.10% increase in the wage, but a 10% increase in the consumer price index results in 2.9% increase of wages. More importantly, historical wages account for 78% of the evolution of current wages.

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Annex on line at the journal Website: http://www.usc.es/economet/ijaeqs.htm

APPENDICES

A1: The first order condition is:

$$\frac{\partial \Pi}{\partial N} = PR_i (1-\alpha) N_i^{-\alpha} AK_i^{\alpha} - W_i = 0 \quad \text{(A1.1)}$$

$$(1-\alpha) N_i^{-\alpha} AK_i^{\alpha} PR_i = W_i \quad \text{(A1.2)}$$

$$(1-\alpha) N_i^{-\alpha} AK_i^{\alpha} = \frac{W_i}{PR_i} \quad \text{(A1.3)}$$

This gives the labor demand:

$$N_i = \left(\frac{W_i}{PR_i}\right)^{-1/\alpha} \left((1-\alpha) AK_i^{\alpha}\right)^{1/\alpha} \text{ i.e. } N^d = f\left(\frac{W}{P}, K\right) \quad \text{(A1.4)}$$

A2: To solve the following program:

$$\begin{cases} Nash = \max\limits_{W_i} T_i^{\mu} \pi_j \\ st \\ N^d = f\left(\dfrac{W}{P}, K\right) \end{cases} \quad \text{(A2.1)}$$

we form the Lagrangian:

$$L = T_i^{\mu} \pi_j + \lambda N^d \quad \text{(A2.2)}$$

in log: $\log L = \mu \log T + \log \pi + \log \lambda + \log N$ (A2.3)

The first order conditions of the negotiation problem give (in derivative log):

$$\frac{\partial \log L}{\partial w} = \mu \frac{\partial T}{\partial w} / T + \frac{\partial \pi}{\partial w} / \pi + \frac{\partial N}{\partial w} / N = 0 \quad \text{(A2.4)}$$

Each derivative element gives:

1- $$\mu \frac{\partial T / \partial w}{T} = \mu \frac{(W_i / P)^{\sigma - 1}}{P (W_i / P)^{\sigma} - \overline{\overline{U}}} \quad \text{(A2.5)}$$

2- $$\frac{\partial \pi / \partial w}{\pi} = \frac{-N_i}{P_i Y_i - N_i W_i} \quad \text{(A2.6)}$$

3- $$\frac{\partial N^d / \partial w}{N^d} = -\frac{1}{\alpha} \frac{(W_i / P_i)^{-\frac{1}{\alpha} - 1}}{P_i} \quad \text{(A2.7)}$$

Replacing each derivative with its value in (A2.4) we get:

$$\mu \frac{(W_i / P)^{\sigma - 1}}{P(W_i / P)^{\sigma} - \overline{\overline{U}}} = - \frac{\partial \pi / \partial w}{\pi} - \frac{\partial N^d / \partial w}{N^d} \qquad \text{(A2.8)}$$

$$\mu \frac{\dfrac{(W_i / P)^{\sigma}}{(W_i / P)}}{P(W_i / P)^{\sigma} - \overline{\overline{U}}} = - \frac{\partial \pi / \partial w}{\pi} - \frac{\partial N^d / \partial w}{N^d} \qquad \text{(A2.9)}$$

$$\frac{(W_i / P)^{\sigma}}{W_i((W_i / P)^{\sigma} - \overline{\overline{U}})} = \frac{1}{\mu}\left[- \frac{\partial \pi / \partial w}{\pi} - \frac{\partial N^d / \partial w}{N^d} \right] \qquad \text{(A2.10)}$$

We multiply each member by W

$$\frac{W_i(W_i / P)^{\sigma}}{W_i((W_i / P)^{\sigma} - \overline{\overline{U}})} = \frac{W_i}{\mu}\left[- \frac{\partial \pi}{\partial w}\frac{1}{\pi} - \frac{\partial N^d}{\partial w}\frac{1}{N^d} \right] \qquad \text{(A2.11)}$$

$$\frac{(W_i / P)^{\sigma}}{((W_i / P)^{\sigma} - \overline{\overline{U}})} = \frac{1}{\mu}\left[- \frac{\partial \pi}{\partial w}\frac{W_i}{\pi} - \frac{\partial N^d}{\partial w}\frac{W_i}{N^d} \right] \qquad \text{(A2.12)}$$

Next setting:

$$\in_{\pi} \equiv -\left(\frac{\delta \pi_i}{\delta W_i} \cdot \frac{W_i}{\pi_i} \right) \qquad \text{(A2.12)}$$

$$\in_{N} \equiv -\left(\frac{\delta N^d}{\delta W} \cdot \frac{W_i}{N_i^d} \right) \qquad \text{(A2.13)}$$

representing respectively the elasticity of profit and labor demand with respect to wage gives

$$\frac{(W_i / P)^{\sigma}}{(W_i / P)^{\sigma} / \sigma - \overline{\overline{U}}_i} = \frac{\in_{\pi} + \in_{N}}{\mu} \qquad \text{(A2.14)}$$

We set $\kappa = \left(\dfrac{\in_{\pi} + \in_{N}}{\mu} \right)^{-1}$, and the wage rate which maximizes the Nash negotiation is: $\dfrac{(W_i / P)^{\sigma}}{\dfrac{(W_i / P)^{\sigma}}{\sigma} - \overline{U}_i} = \dfrac{1}{\kappa}$ or $\kappa \left(\dfrac{W_i}{P} \right)^{\sigma} = \dfrac{(W / P)^{\sigma}}{\sigma} - \overline{U}_i$ giving $\sigma\kappa \left(\dfrac{W_i}{P} \right)^{\sigma} - \left(\dfrac{W_i}{P} \right)^{\sigma} = -\sigma \overline{U}_i$ and finally $\left(\dfrac{W_i}{P} \right)^{\sigma} (\sigma\kappa - 1) = -\sigma \overline{U}_i$ or $\left(\dfrac{W_i}{P} \right)^{\sigma} = \dfrac{\sigma}{(1 - \sigma\kappa)} \overline{U}_i$ or

$$\frac{1}{\sigma}\left(\frac{W_i}{P}\right)^{\sigma} = (1-\sigma\kappa_i)^{-1}\overline{U_i} \tag{A2.15}$$

The wage is a markup over the various wages $\overline{U_j}$. For the chosen production function, setting the share of wage in the profit as

$$S_i = \frac{W_i / P}{Y_i / N_i} = \frac{W_i / P}{(PR/P)(Y_i / N_i)} \tag{A2.16}$$

with $Y/N=Z$ the productivity of labour and $PR/P=V$ the gap between the producer price and the consumer price, elasticities of the profit function and the labour demand become respectively:

$$\in_{\pi} = \left(\frac{S_i}{1-S_i}\right) \tag{A2.17}$$

$$\in_N = \left[(1-\rho) - \left\{1 - \frac{\rho}{\gamma(1-\varepsilon)}\right\} s_i\right]^{-1} \tag{A2.18}$$

The equilibrium of the model is obtained by finding the value of $\overline{U_j}$. At the equilibrium $\overline{U_j} = \overline{U}$, and

$$\overline{U} = \frac{1}{\sigma}\left[\{1-\varphi(u)\}\left(\frac{W}{P}\right)^{\sigma} + \varphi(u)\left(\frac{B}{P}\right)^{\sigma}\right] \tag{A2.19}$$

i.e. a weighted mean of the probability to find a job in another sector, given by the real wage (*W/P*) and the probability to remain unemployed given by the real benefit of unemployment (*B/P*), i.e. the compensation received by an employee which looses its job. The weights are given by the probability of not being re-employed $\varphi(u)$ which depends on the unemployment rate u in the economy and on the real advantages related to unemployment (*B/P*). Replacing $\overline{U}$ with its value in Eq. (A2.15) we get:

$$\left(\frac{W_i}{P}\right)^{\sigma} = \frac{1-\varphi(u)}{(1-\sigma\kappa_i)}\left(\frac{W}{P}\right)^{\sigma} + \frac{\varphi(u)}{(1-\sigma\kappa_i)}\left(\frac{B}{P}\right)^{\sigma} \tag{A2.20}$$

$$\left(\frac{W_i}{P}\right)^{\sigma} - \frac{1-\varphi(u)}{(1-\sigma\kappa_i)}\left(\frac{W}{P}\right)^{\sigma} = \frac{\varphi(u)}{(1-\sigma\kappa_i)}\left(\frac{B}{P}\right)^{\sigma} \tag{A2.21}$$

$$\left[1 - \frac{1-\varphi(u)}{(1-\sigma\kappa_i)}\right]\left(\frac{W}{P}\right)^{\sigma} = \frac{\varphi(u)}{(1-\sigma\kappa_i)}\left(\frac{B}{P}\right)^{\sigma} \tag{A2.22}$$

$$\left[-\sigma\kappa_i+\varphi(u)\right]\left(\frac{W}{P}\right)^{\sigma}=\varphi(u)\left(\frac{B}{P}\right)^{\sigma} \quad \text{(A2.23)}$$

$$\frac{\left(\frac{W}{P}\right)^{\sigma}}{\left(\frac{B}{P}\right)^{\sigma}}=\left(1-\frac{\sigma\kappa_i}{\varphi(u)}\right)^{-1} \quad \text{(A2.24)}$$

$$\left(\frac{B}{W}\right)^{\sigma}=1-\frac{\sigma\kappa_i}{\varphi(u)} \quad \text{(A2.25)}$$

Setting $R=B/W$ we have

$$R^{\sigma}=1-\frac{\sigma\kappa_i}{\varphi(u)} \quad \text{(A2.26)}$$

We set $\kappa=\kappa(S)$ and we get

$$\kappa(S)=\frac{\varphi(u)}{\sigma}(1-R^{\sigma}) \quad \text{(A2.27)}$$

Given the values of $\kappa_i=\left(\frac{\in_\pi}{\mu}+\in_N\right)^{-1}$, $\in_\pi=\left(\frac{S_i}{1-S_i}\right)$ and $\in_N=\left[(1-\rho)-\left\{1-\frac{\rho}{\gamma(1-\varepsilon)}\right\}s_i\right]^{-1}$. We have:

$$\kappa=\frac{\mu(1-S_i)}{S_i}+(1-\rho)-\left(1-\frac{\rho}{\gamma(1-\varepsilon)}\right)S_i \quad \text{(A2.28)}$$

Knowing that $S=\frac{W/P}{VZ}$ we get after few manipulations:

$$\kappa=\lambda VZ-\left(\mu+\frac{\rho-1}{VZ}\right)\frac{W}{P}+(1-\rho) \quad \text{(A2.29)}$$

$$\lambda VZ-\left(\mu+\frac{\rho-1}{VZ}\right)\frac{W}{P}+(1-\rho)=\frac{\varphi(u)}{\sigma}(1-R^{\sigma}) \quad \text{(A2.30)}$$

From where we extract the following equation:

$$\frac{W}{P}=\frac{\frac{\varphi(u)}{\sigma}(1-R^{\sigma})-\lambda VZ-(1-\rho)}{\left[\frac{(1-\rho)-\lambda VZ}{VZ}\right]} \quad \text{(A2.31)}$$

The employment equation in obtained by log linearization of

$$N^{d} = \left(\frac{W_i}{P_i}\right)^{-1/\alpha} \left((1-\alpha)AK_i^{\alpha}\right)^{1/\alpha} \tag{A2.32}$$

i.e.: $\log N = -\frac{1}{\alpha}\log W + \frac{1}{\alpha}\log P + \frac{1}{\alpha}\log(1-\alpha) + \frac{1}{\alpha}\log A + \alpha \log K$ (A2.33)

$$n = -\frac{1}{\alpha}w + \frac{1}{\alpha}p + \frac{1}{\alpha}\log(1-\alpha) + \frac{1}{\alpha}a + \alpha k \tag{A2.34}$$

where $n = -\frac{1}{\alpha}(w-p) + \alpha k + \frac{1}{\alpha}a + \frac{1}{\alpha}\log(1-\alpha)$. This equation is of the form $n = \beta_3(w-p) + \beta_2 k + \beta_1 a + \beta_0$. The wage equation in which we are interested is obtained by taking the log of:

$$\frac{W}{P} = \frac{\frac{\varphi(u)}{\sigma}(1-R^{\sigma}) - \lambda VZ - (1-\rho)}{\left[\frac{(1-\rho)-\lambda VZ}{VZ}\right]} \tag{A2.35}$$

$$\log W - \log P = \log\left[\frac{\varphi(u)}{\sigma}(1-R^{\sigma}) - \lambda VZ - (1-\rho)\right] - \log\left[\frac{(1-\rho)-\lambda VZ}{VZ}\right] \tag{A2.36}$$

$$(w-q)_t = \log\left[\frac{\varphi(u)}{\sigma}(1-R^{\sigma})\right] + v_t + z_t \tag{A2.37}$$

$$(w-q)_t = v_t + z_t + \psi \log u_t + \theta r_t + C \tag{A2.38}$$

with $\psi = \frac{\varphi'(u^*)u^*\{1-(R^*\}^{\sigma}}{\sigma\mu}$ and $\theta = \frac{\varphi(u^*)(R^*)^{\sigma}}{\mu}$

with $\mu = S^*(\kappa^*)^2\left[\frac{\{1-\frac{\rho}{\gamma(\varepsilon-1)}\}}{\left[(1-\rho)-\{1-\frac{\rho}{\gamma(\varepsilon-1)}\}S^*\right]^2} + \frac{1}{\lambda(1-S^*)^2}\right]$, variables with asterix representing the equilibrium values. The log linear solution can be written:

$$w = \alpha p_r + (1-\alpha)p + \delta z - \theta u + \xi_w \quad 0 \le \alpha, \delta \le 1, \theta \ge 0, \tag{A2.39}$$

Replacing the price $p_r = \frac{1}{1-\eta}p + \frac{\eta}{1-\eta}q$ with its value we get:

$$w = \frac{\alpha}{1-\eta}p + \frac{\alpha\eta}{1-\eta}q + (1-\alpha)p + \delta z - \theta u + \xi_w \tag{A2.40}$$

$$w = \frac{\alpha + (1-\alpha)(1-\eta)}{1-\eta}p + \frac{\alpha\eta}{1-\eta}q + \delta z - \theta u + \xi_w \tag{A2.41}$$