

Advocacy and Dynamic Delegation*

Raphael Boleslavsky[†] Tracy R. Lewis[‡]

January 2011

Abstract

An advocate for a special interest provides information to an uninformed planner for her to consider in making a sequence of important decisions. Although the advocate may have valuable information for the planner, it is also known that the advocate is biased and will distort his advice if necessary to influence the planner's decision. Each time she repeats the problem, however, the planner learns about the accuracy of the advocate's recommendation, mitigating some of the advocate's incentive to act in a self-serving manner. We propose a theory of dynamic delegation to explain why planners do sometimes rely on information provided by advocates in making decisions. The interaction takes place in two phases, a communication phase, followed by a sequence of decisions and learning by the planner. We first establish that the capability to delegate dynamically is a necessary condition for influential communication in this setting, and characterize the optimal dynamic delegation policy. Next, we show that a planner may prefer to consult an advocate rather than a neutral adviser. Finally, we demonstrate how an advocate gains influence with a decision maker by making his preferences for actions unpredictable. Our results have implications for a variety of real world interactions including regulation, organization, and whistleblowing.

*The authors gratefully acknowledge the helpful comments of Chris Cotton, Manuel Santos, Dave Kelly, Richard Van Weelden, Silvana Krasteva, and especially Marco Ottaviani. We also thank seminar participants at the University of Miami, Northwestern, and M.I.T. A previous version of this paper circulated as "Theory of Advocates: Trading Advice for Influence."

[†]School of Business, University of Miami. r.boleslavsky@miami.edu

[‡]Fuqua School of Business, Duke University. tracy.lewis@duke.edu

1 Introduction

A representative of a polluting firm gives testimony to a regulatory agency about the economic and environmental impact of stricter emission standards. Although the exact impact of this regulation can not be known in advance, the polluting firm knows what the consequences of the regulation are likely to be. However, because stronger emissions standards act as a constraint on the firm, the firm has an incentive to make exaggerated claims about the likely costs of regulation and to downplay its likely benefits in order to manipulate the regulator into adopting weaker standards. Even though the polluter has valuable information for the regulatory agency, the polluter's incentives to make distorted claims act as a significant barrier to the influential communication of this information. Over time, however, the regulatory agency can observe the consequences of its decisions and can therefore learn about the impact of the emission standards through its own experience. In this paper, we study the consequences of this type of learning.

This situation is an example of a decision maker "relying on advice from interested parties." This pervasive practice has been much studied by economists and political scientists, but is still not completely understood. In the situation that we study, a decision maker or planner solicits a recommendation from an informed advisor. The advisor is known to have strong preferences for actions that always conflict with the planner's best interest. Just as the polluting firm benefits when emissions standards are weakened, the advisor always benefits when the planner increases her action. To reflect the extreme nature of the advisor's preferences, we refer to him as an *advocate* rather than as the more neutral "sender" or "expert." We assume that the use of message-contingent transfers are prohibited for institutional or legal reasons. To make the situation even more difficult, the accuracy of the advocate's recommendation can not be immediately established; the advocate is therefore free to distort his recommendation to his own advantage without worrying that his self-serving behavior will be immediately exposed.¹ However, if the planner faces a sequence of decisions, she will learn about the accuracy of the advocate's advice through her own experience.

Economic analysis of this type of situation can be divided into two broad categories:

¹A large literature analyzes the disclosure of information that can not be falsified (hard information). See Dewatripont and Tirole (1999), Milgrom (1981), Cotton (2009, 2010), Milgrom and Roberts (1986), Bull and Watson (2007, 2004), Che and Kartik (2010).

strategic communication and delegation. Under strategic communication, the decision maker can not commit to her response to the advisor’s message; she can only choose an action that is sequentially rational given the recommendation she receives, accounting for the motives and reporting strategy of the advisor.² In contrast, under delegation, the decision maker commits herself to a specific action associated with each report.³ In essence, in the delegation framework, the decision maker commits to a menu of actions from which the advisor can select optimally given his private information. When the planner must make a sequence of decisions, the planner can also delegate decisions to the advisor *dynamically*, by committing to an action in each period that depends on both the advocate’s recommendation and on information that the planner has learned from her own experience. In this environment, the planner can use information revealed later in the interaction to provide the advocate with incentives to issue truthful a recommendation. In this paper we characterize and analyze this type of *dynamic delegation*.

Our analysis breaks new ground by combining delegation with learning on the part of the planner in a simple and flexible framework. Our investigation centers on a model of communication that captures the salient features of advocacy relationships that we have described. In a sequential setting, a planner adjusts her action in each period to match an unknown state of the world; in the context of the previous example, the state of the world represents the optimal emission standard, given a particular period’s costs and benefits. The state in each period is stochastic, and is publicly revealed after the planner has selected an action.⁴ The planner is uncertain about the distribution of the state. An advisor (the polluting firm) knows the distribution from which the states are drawn, either ϕ_H or ϕ_L , but like the planner is unable to observe the state until after the planner selects an action. The advisor is known to be an advocate; he always prefers that the planner increase her action (reduce the standard), independent of the true state. The theory we propose addresses the fundamental problem that advocates pose for planners: the planner clearly benefits from knowing the state distribution at the beginning of the interaction,

²Crawford and Sobel (1982), Dessein (2002), Ambrus and Egorov (2009), Chakraborty and Harbaugh (2009), Chakraborty and Yilmaz (2010)

³Holmstrom (1977), Alonso and Matouschek (2008), Ambrus and Egorov (2009)

⁴For simplicity we also assume that the planner’s ability to observe the true state after she selects her action does not depend on the action that she chooses. This simplifying assumption may not be realistic in certain situations. Relaxing this assumption introduces an additional tradeoff into the problem, which we leave for future work.

but without access to payments (contingent on the advocate's recommendation), it is very difficult to align the advocate's incentives with the planner's.

We first establish that there is no influential communication in this setting under strategic communication and static delegation. The proposed framework is therefore particularly well suited to analyzing the properties of dynamic delegation, as it is the driving force behind influential communication. This result is not difficult to understand. In a static delegation framework, if the planner changes her action in response to the advocate's advice, he will always manipulate her into choosing the higher action. Absent the ability to fine the advocate for bad advice, or a future in which to punish him, the planner has no recourse other than to ignore his advice. When there is strategic communication, the problem is very similar. If the advocate's recommendation suggests to the planner that the low distribution is more likely, the planner will choose a lower action in each period, hurting the advocate's payoff. Without the ability to commit, the planner can not assure the advocate that he won't be penalized for revealing the low distribution. Without this assurance, we show that no informative communication equilibrium exists. These no-communication results are rather disappointing, but predictable, given the disparate preferences of the parties and the limited agreements that are available for governing the planner's behavior in response to the advocate's advice.

We then show that under dynamic delegation, it is possible for the parties to arrange for the advocate to advise the planner, and characterize the optimal delegation policy. Under the optimal arrangement, when the advocate reports ϕ_L , the distribution he finds unfavorable, the planner commits to a fixed long term action that exceeds the action she finds optimal under the low distribution. We interpret this as a *compromise* the planner makes in return for the advocate's report of unfavorable news. When the advocate reports that the distribution is ϕ_H , i.e. favorable, the planner commits to a sequence of actions that is contingent on the history of observed states. The actions converge towards the planner's optimal action under ϕ_H when the history of state observations supports the advocate's claim that the distribution is actually favorable. However, if the history of observed states contradicts the advocate's claim, the planner's actions are progressively reduced. We interpret this as the planner's commitment to trust the advocate's advice, as long as his advice appears *valid* given her experience. The planner's ability to distinguish the two distributions from the observed history influences the magnitude of the second

best distortions and the welfare gains of dynamic delegation.

Our analysis provides some clues as to the origins of advocacy by describing situations in which advocates are preferred to impartial advisors. Most analyses of advocacy presume that planners rely on advocates for advice because advocates must be informed about issues that they care so much about. While this is often true it doesn't explain why planners don't prefer to consult other informed but potentially less biased sources for advice.

One rationale for preferring advocates to impartial advisors, implied by our theory, is that it is easier to motivate advocates to acquire information than impartial advisors. Suppose the planner may consult a biased advocate or an objective adviser for information on what action to take. The advocate and impartial advisor must both expend resources of $c > 0$ to become informed. Once informed, the impartial advisor truthfully reports his information, whereas the advocate requires the planner to adopt certain costly actions in return for revealing his information truthfully.

We show, surprisingly, that it can be less costly for the planner to obtain information from the advocate than from the impartial adviser. Because the advocate cares about the planner's actions while the impartial advisor doesn't, the advocate can be motivated to learn through the planner's actions, while the impartial advisor will only learn if his cost is fully reimbursed. When the advocate has a strong incentive to correctly communicate the state distribution he subsidizes learning; the distorted actions necessary to induce learning generate a smaller payoff cost to the planner than the cost of learning, c . Ironically our theory shows that it is the advocate's extreme preference for high actions that sometimes makes him a more attractive source of information than an objective advisor.

Our results imply that an advocate with a known preferences for high actions has no impact on the planner's expected action choice; the planner chooses the same average action whether or not she is advised by the advocate.⁵ Although the advocate's advice influences the planner to make better decisions to the *planner's* benefit, the advocate is unable to induce the planner to select higher actions on average. Advocates are severely constrained by the causes that they are identified with; it is ironic that once an advocate becomes a known supporter of a particular issue, he relinquishes his ability to impact the planners choice of action on that issue. In contrast, when the planner is uncertain whether

⁵While this result is certainly a consequence of some assumptions of our model about the planner's preferences, we still find this result interesting, especially as it relates to some of the later results.

the advisor is an advocate or impartial, our theory shows that the advocate commands a rent to reveal his conflict of interest, leading to a higher average action. Suppose the planner has several distinct types of decisions to make. She consults with an advisor who has extreme preferences for one unknown type of decision, but is impartial with respect to the other decisions. For any given decision, the planner does not know if the advisor has an incentive to offer biased advice. An advocate thus has an incentive to claim to be impartial in order to give a manipulative report undeserved credibility. To make this possibility unattractive, the planner commits to choose actions that are more aligned on average with the advocate's preferences when he reveals his conflict of interest. By initially hiding his preferences from the planner, the advocate is able to advance his agenda more effectively.

This paper is a contribution to the literature on strategic information transmission and delegation that began with Crawford and Sobel (1982), Grossman (81), Milgrom (81), and Holmstrom (1977) who provide the first analyses of decision makers relying on the information of interested parties. More recent papers in this literature include Dessein (2002), Alonso and Matouschek (2008), and Ambrus and Egorov (2009). As in our analysis, these authors assume in any state of the world, the advisor's preferred action differs from the decision maker's. The difference between the parties' most preferred actions represents the degree of interest conflict in the relationship. For small interest conflicts, these authors show that there is influential communication under both delegation and strategic communication. In our setting, however, the conflict is "infinite": the advocate always prefers that the planner increase her action. With such an extreme conflict of interest, strategic communication and static delegation are ineffective. Our result also contrasts with those of Morris (2001) and Sobel (1985), who find that informative communication may exist in repeated settings without commitment. These authors assume that the advisor's preferences may be identical to those of the decision maker, and that the state changes in each period. By issuing truthful recommendations, biased advisors may establish a reputation for credibility, which they may profitably exploit in later periods. In our setting, the advisor's extreme preference conflict is common knowledge, so there is no scope for this type of "investment in credibility."⁶ Finally Chakraborty and Harbaugh (2010) describe the manner in which an advocate with multiple pieces of private information can influence a

⁶We discuss the possibility that the advisor is impartial in Section 6.

decision maker. In our setting, the advocate has only a single piece of private information, which precludes their construction.

The interest conflict that we analyze may appear extreme when compared with the literature. We are interested in this extreme conflict of interest for two main reasons. First, we believe that this setting is the most appropriate description of a variety of real situations in regulation, organization, political economy, and interpersonal relationships. Because the extreme conflict of interest rules out the channels of communication identified in this literature, it is not often analyzed.⁷ Second, because there is no influential communication for any of the standard reasons, we believe that this is an excellent setting in which to study the properties of *dynamic delegation*; any communication is a consequence of dynamic delegation exclusively. The optimal dynamic delegation policy shares elements of Strausz's (2005) theory of interim information in employment contracts, and Cooper and Hayes (1987) description of price discrimination in long term insurance contracts.

The process for eliciting useful, unverifiable information from biased parties has spawned a large literature in economics, organization theory and political science. Krishna and Morgan (2008) and Sobel (2008) are recent surveys of some of the more influential applications of the theory. In addition to the papers discussed above, it is important to mention the contributions of Alonso, Dessein and Matouschek (2008), Rantakari (2008), Battaglini (2002), and Gentzkow and Kamenica (2010) in economics and Austen-Smith (1994) and Grossman and Helpman (2002) in political science. This literature encompasses a wide range of topics and settings that the current paper does not address. Our theory focuses instead on explaining the combination of delegation and learning in facilitating the exchange of information for influence in long term economic and political decision processes.

The plan for the rest of the paper is as follows. Our model is presented in Section 2. In Section 3 we analyze two benchmarks, and demonstrate that no influential communication exists under static delegation and strategic communication. In Section 4 we characterize the optimal dynamic delegation policy and discuss its properties. Section 5 extends our analysis to a setting in which advocates must expend effort to learn; we also permit the planner to choose whether to consult an impartial advisor or a known advocate for advice. We illustrate that under dynamic delegation, a biased advocate may be preferred to an impartial advisor. In Section 6 we allow the advocate to hide his preferences for action by

⁷An exception is Chakraborty and Harbaugh (2009)

choosing the set of issues on which he offers advice. Section 7 concludes with a summary of results and directions for further research. Proofs of formal results are contained in the appendix.

2 The Model

A planner (e.g. lawmaker, CEO, regulator etc.) faces a decision or sequence of decisions. In each period, the planner's choice, called the action, is represented by a value $q \in R$. The planner's payoff in each period depends on the action she chooses and an unknown state of the world $x \in R$. Given state x and action q , her payoff is a quadratic loss function:

$$u_p(q, x) = -(q - x)^2$$

The planner has no inherent preferences for action; she cares only about choosing the action that is most appropriate given the state of the environment.

In each period, the true state is independently and identically distributed, but the exact distribution of the state in all periods is unknown to the planner. There are two possible distributions for the state, a high distribution $F(x|\phi_H)$ and a low distribution $F(x|\phi_L)$, with common support X .⁸ The expected value of the state under the high distribution, μ_H , is larger than the expected value of the state under the low distribution, μ_L .

$$\begin{aligned} E[x|\phi_H] &= \mu_H \\ E[x|\phi_L] &= \mu_L \\ \mu_H &\geq \mu_L \end{aligned}$$

Given her information at the beginning of the interaction, the planner believes the distribution of the state to be the low distribution with probability $\gamma \in (0, 1)$.

Faced with a choice of action in each period, the planner would clearly benefit from knowing the true distribution at the beginning of the interaction. With quadratic preferences, her ideal action in every period is the mean of the true distribution. However, if she doesn't know the true distribution, her action in any period will reflect all available information, but it will never be her ideal action. We refer to the situation in which the planner knows the true distribution as the *first best*, and her expected payoff (prior to learning the true distribution) as the *first best payoff*, denoted \bar{V}_N .⁹

⁸The assumption of common support can be easily relaxed.

⁹It is not difficult to verify that $\bar{V}_N = -(\gamma\sigma_L^2 + (1-\gamma)\sigma_H^2) \frac{1-\delta^N}{1-\delta}$.

Fortunately, an advisor knows the true distribution of the state (though he has no additional information about the realized state in each period); unfortunately the advisor is an advocate whose payoff differs from the planner's:¹⁰

$$u_a(q, x) = q$$

Unlike the planner, who would like to match the action to the state, the advocate would always like the planner to choose a higher action, regardless of the true state of the world. The advocate therefore has an incentive to claim that the distribution is ϕ_H , to induce her to choose higher actions. The preference misalignment between the planner and advocate is more extreme than the one analyzed in the cheap talk and delegation literature; the extreme misalignment of preferences acts as a significant barrier to influential communication.

Payments between the planner and the advocate are either explicitly ruled out, or are constrained to be independent of the advocate's message. These restrictions arise for legal or institutional reasons; payments between planners and advocates can lead to a variety of corrupt behaviors. Without payments, the planner can reward or punish the advocate only through her choice of action.¹¹

The planner's challenge is difficult. She would like the advocate to recommend the best action given what he knows, but it is difficult for the planner to overcome the advocate's incentive to lie without paying him. Nonetheless, the advocate does have valuable information for the planner, and the planner has the authority to select actions that benefit the advocate. There are potential gains from an exchange of advice for influence, if only the parties figure out a way to exploit their common interests without monetary payments.

¹⁰More general payoff functions of the form $u_a(q, x) = g_1(x)q + g_2(x)$ lead to identical results, provided for both possible distributions $D \in \{H, L\}$, $E[g_1(x) | \phi_D] > 0$.

¹¹There are two rationale for assuming that report-contingent payments are not possible. The first is that payments for information are difficult if not impossible to implement without the ability to objectively verify the accuracy of the information that is reported. The second is that monetary transfers between public officials and private interests may facilitate corrupt or illegal exchanges of favors or services, and is therefore prohibited.

If monetary or non monetary utility transfers can be conditioned on the advocates report, then it is possible to induce the advocate to reveal information. See Ambrus and Egorov (2009 a,b) in the context of a theory of bureaucracy and Inderst and Ottaviani (2009) in the context of commercial transactions.

2.1 Advice for Influence Game

We model the interaction between planner and advocate as a game in which advice is exchanged for influence. The game takes place over periods $k = \{0, 1, \dots, N\}$ according to the following sequence:

- 0. The advocate observes the true state distribution, ϕ_L or ϕ_H , and issues a recommendation (or sends a message) $M \in \{H, L\}$.¹²
- k=1:N. At the beginning of period k , the planner selects an action $q_k \in R$. Once the action is selected, the true state x_k is revealed, and both planner and advocate realize their period k payoffs.

The advocate's strategy is a pair of probabilities $(r_H, r_L) \in [0, 1]^2$; r_X represents the probability that the advocate reports H when the true distribution is ϕ_X . In each period the planner can base her choice of action in any period on the message she receives from the advocate and on the history of past states that she has observed. Let $X_{k-1} = (\times_{i=1}^{k-1} X)$, represents the set of possible histories at time $k - 1$, and $h_{k-1} = (x_1, \dots, x_{k-1}) \in X_{k-1}$ represents a particular history of past states,¹³ then the planner's strategy is a family of functions, $\{q_M(h_{k-1})\}_{k=1}^N$, each of which maps the Cartesian Product of M and X_{k-1} into an action $q_k \in R$:¹⁴

$$q_M(h_{k-1}) : M \times H_{k-1} \rightarrow R$$

Both planner and advocate have a common discount factor δ .

The equilibrium that we focus on for this game will depend on the setting which best describes the relationship between the advocate and planner. In situations where the planner is unable to commit ex ante to a strategy (strategic communication), we analyze Perfect Bayesian equilibrium, in which the planner and advocate select strategies that are

¹²We have assumed that the message space contains only two messages. In the case of full commitment the Revelation Principle guarantees that this assumption is without loss of generality. Without commitment, this assumption can be easily relaxed with no impact on the results. We maintain this assumption to simplify the exposition.

Furthermore, under strategic communication, if N is finite, there is no gain to allowing communication in every period. If N is infinite, we conjecture that this is still the case. We thank Alessandro Pavan and Yuk-Fai Fong for these points.

¹³By convention, h_0 is the null set and $f(h_0|\phi) = 1$.

¹⁴Throughout the paper we avoid discussing issues related to variations in the decision maker's strategy on sets of measure zero without loss of substance. Readers uncomfortable with this can assume that X is discrete with no substantive changes.

sequentially rational given the strategy of the other player, and the planner’s beliefs are Bayesian updates of her prior based on the observed history and the advocate’s strategy. In situations where the planner can commit to her strategy, (dynamic delegation), we focus on Bayesian incentive compatible mechanisms, whereby the planner chooses her action to maximize her ex ante expected payoff, subject to inducing truthful disclosure from the advocate.

The information structure of the advice for influence game differs from the standard information structure in price discrimination and employment settings in the following key respect: in standard settings the agent’s private information is about some attribute of the *agent* (e.g. his cost for executing a task, or his valuation for a good), and the only way for the principal to learn this information is to provide the agent with incentives to reveal it. In contrast, the advocate’s private information is about an exogenous feature of the *planner’s problem*. Each time she repeats the problem, the planner acquires additional information about this feature, and in the long run, she can learn the advocate’s private information on her own. This capability to learn plays a critical role in the interactions we consider.

3 No-Communication Benchmarks

With common knowledge of the severe misalignment in preferences, we would expect that in a variety of settings, the planner would ignore the advocate’s advice. The following propositions illustrate that under both static delegation and strategic communication, there is no influential communication. These no communication results provide motivation for the dynamic delegation environment that we consider in the following section. Without the ability to delegate dynamically, there is no influential communication. They also provide a context in which to view those results: the dynamics of the delegated decisions, and all welfare gains are a consequence of the combination of commitment and learning.

We say that communication is *influential* if both messages are sent in equilibrium, and for some history¹⁵ different messages induce the planner to choose different actions.

¹⁵or non-negligible set of histories

3.1 Static Delegation

In the standard static delegation problem, a decision maker must choose an action. Both the decision maker's and advisor's preferences depend on some information, privately known by the advisor. The parties have a conflict of interest: if the advisor's private information were public, the parties would choose different actions. Given a prior, the decision maker commits to a menu of actions from which the advisor can choose given what he knows.

To draw the most appropriate parallel between our model and static delegation, imagine that the planner and advocate interact for only one period. As in the standard delegation problem, the planner commits to a menu of actions $\{q_H, q_L\}$. Obviously, if $q_H \neq q_L$, then the advocate will optimally issue the recommendation that induces the higher action. In order for the menu to be incentive compatible, the planner must commit to the same action for the two reports, so that $q_H = q_L$. The advocate's report can not influence the planner's choice of action.

Proposition 1.1 *In a single period interaction with commitment, in any incentive compatible mechanism communication is non-influential.*

Proposition 1 is in stark contrast to the single encounter cheap talk literature and the static delegation literature, both of which demonstrate the possibility of influential communication in one time encounters. Here, the advocate's inability to influence the planner follows directly from the extreme misalignment of preferences. Intuitively, the state is only learned after the decision has been made, so the only information that the planner could use to help her choose an action is the report of the advocate. Because of the advocate cares only about the action, he will always choose the higher action. The planner has no choice but to ignore the recommendation.

3.2 Strategic Communication

The planner's situation seems different in a setting with multiple periods, because she can learn about the true distribution from the observed history. However, if the planner is unable to commit to her strategy (i.e. can not delegate), her actions must be sequentially rational, and only one action is sequentially rational for any combination of history and

message. Although the planner will adjust her equilibrium actions to account for her updated beliefs about the state distribution, she is still unable to elicit influential information from the advocate.

Proposition 1.2 *In every Perfect Bayesian Equilibrium of the multiperiod game without commitment the advocate's reporting strategy is independent of his information. There is no influential communication in equilibrium.*

With quadratic preferences, the sequentially rational action following a given history is the conditional expected value of the state,

$$q(\hat{\gamma}, h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

where $\gamma(h_{k-1})$ represents the updated belief that the distribution is low:

$$\gamma(h_{k-1}) = \Pr(\phi = \phi_L | h_{k-1}) = \frac{\hat{\gamma} f(h_{k-1} | \phi_L)}{\hat{\gamma} f(h_{k-1} | \phi_L) + (1 - \hat{\gamma}) f(h_{k-1} | \phi_H)}$$

The planner's updated belief $\gamma(h_{k-1})$ is monotone in $\hat{\gamma}$, her updated belief that the distribution is low following the advocate's report. By sending a message that induces a lower belief that the distribution is ϕ_L the advocate increases the planner's action following any history of states, improving his payoff. Therefore, the advocate always sends the message associated with the lowest possible posterior belief. When $\hat{\gamma}_H \neq \hat{\gamma}_L$, he sends only one message in equilibrium, and the posterior belief associated with this message is equal to the prior belief. If, $\hat{\gamma}_H = \hat{\gamma}_L$ then both messages may be sent in equilibrium, but, if the posteriors are equal, the probability of sending either messages can't depend on the true distribution; therefore, both posteriors are equal to the prior, $\hat{\gamma}_H = \hat{\gamma}_L = \gamma$. In both cases, no influential communication takes place in equilibrium.

The findings of Proposition 1.2 provide an interesting contrast to Morris (2001) and Sobel (1985) who demonstrate the possibility of influential communication in repeated settings where the planner is unable to commit. These analyses focus on what can be roughly termed, "investment in credibility;" an advisor with unknown preferences may provide a planner with useful information either because he truly shares her preferences, or because he would like to establish a reputation for credibility which he can profitably exploit later. These considerations do not apply, however, when the advisor is an advocate

with known preferences for extreme actions.¹⁶

Propositions 1.1 and 1.2 make clear that advocates can have no influence whatsoever, unless the planner repeats her problem and can commit to an action policy that depends on the advocate’s report and on the observed history. This is the *dynamic delegation* setting that analyze in section 4.

4 Dynamic Delegation

In this section we consider possibility that the planner can commit to a strategy prior to receiving the advocate’s report, relaxing the requirement that the planner’s strategy must be sequentially rational. Once she selects an action, the planner observes the true state; each time she repeats the problem, she learns about the true distribution. By committing to actions, the planner can leverage the additional information revealed by the history of states to dissuade the advocate from issuing self-serving recommendations.¹⁷

The planner commits at the beginning of the interaction to her strategy, or mechanism, m , that specifies an action in each period, conditioned on the advocate’s recommendation and the observed history:

$$m \equiv \{q_L(h_{k-1}), q_H(h_{k-1})\}_{k=1}^N$$

The advocate privately observes the true distribution and issues a recommendation. Because the advocate only has one piece of private information, the Revelation Principle implies that there is no loss of generality in restricting attention to mechanisms that induce the advocate to issue a truthful recommendation. Incentive compatibility requires that the advocate can not induce a higher discounted sum of expected actions by misreporting the distribution of the state:

$$\begin{aligned} \sum_{k=1}^N \delta^{k-1} E[q_L(h_{k-1}) | \phi_L] &\geq \sum_{k=1}^N \delta^{k-1} E[q_H(h_{k-1}) | \phi_L] & \text{(ICL)} \\ \sum_{k=1}^N \delta^{k-1} E[q_H(h_{k-1}) | \phi_H] &\geq \sum_{k=1}^N \delta^{k-1} E[q_L(h_{k-1}) | \phi_H] & \text{(ICH)} \end{aligned}$$

Because the the planner knows that the advocate’s equilibrium report is truthful; her

¹⁶We revisit the possibility that the advocate is unbiased in a later section.

¹⁷Throughout the history of states is assumed to be verifiable. Even if the true state is not verifiable but some signal correlated with each period’s state is verifiable the mechanism would exhibit similar features.

expected payoff from offering m can be written:

$$P(m) = -\gamma \sum_{k=1}^N \delta^{k-1} E [(q_L(h_{k-1}) - x)^2 | \phi_L] - (1 - \gamma) \sum_{k=1}^N \delta^{k-1} E [(q_H(h_{k-1}) - x)^2 | \phi_H]$$

Finally, to ensure that the advocate will participate in the mechanism, it is enough that the planner commit to interpret the absence of a message as a message in support of distribution ϕ_L . Constraint (ICH) then ensures that the advocate (weakly) prefers to send a message than no message at all.

The optimal mechanism maximizes (P) subject to (ICL) and (ICH). In the absence of incentive constraints, the planner would choose the first-best actions (μ_H or μ_L), but these actions are not incentive compatible. At least one of the incentive constraints must bind. An advocate who learns that the true distribution is ϕ_H would have no incentive to report L ; whereas, an advocate who knows that the true distribution is ϕ_L , might try to manipulate the planner by reporting H instead of L . In light of this intuition, in the appendix we solve a simple relaxed problem imposing only constraint (ICL).¹⁸ The solution to the relaxed problem always satisfies (ICH), and therefore characterizes the optimal mechanism.¹⁹

The optimal mechanism depends on three parameters that we introduce now in anticipation of the formal results to follow. First, the history of past states h_{k-1} influences the planner's actions through its likelihood ratio:

$$\Lambda(h_{k-1}) = \prod_{i=1}^{k-1} \frac{f(x_i | \phi_L)}{f(x_i | \phi_H)}$$

The value of the likelihood ratio indicates which of the two possible distributions are more consistent with the observed history. High values of $\Lambda(h_{k-1})$ support the inference that the true distribution is ϕ_L , while low values of $\Lambda(h_{k-1})$ support the inference that the true

¹⁸The resulting relaxed problem has a concave payoff function and a linear constraint. The solution to the relaxed problem is the unique stationary point of the Lagrangian.

¹⁹Clearly, any mechanism in which there is no influential communication (because $q_H(h_{k-1}) = q_L(h_{k-1})$ for all histories) always satisfies these constraints, and is therefore feasible; if it is not chosen, it must be sub-optimal.

distribution is ϕ_H .²⁰ Second, the optimal mechanism also depends on parameter

$$\begin{aligned}\alpha &= \int_X \frac{f(x|\phi_L)^2}{f(x|\phi_H)} dx \\ &= E[\Lambda(x)|\phi_L]\end{aligned}$$

Parameter α is a measure of the "similarity" between distribution ϕ_L and ϕ_H . This parameter plays a significant role in a variety of statistical settings.²¹; it is weakly larger than one, and larger values of α indicate that the distributions are less similar.²² As we shall see, in the optimal mechanism, α is a measure of the strength of the incentives that the planner can provide without payments. Finally, the solution also depends on parameter

$$\omega_N = \frac{(1-\alpha\delta)(1-\delta^N)}{\gamma(1-\delta)(1-\alpha\delta)^N + (1-\gamma)(1-\alpha\delta)(1-\delta^N)} (\mu_H - \mu_L)$$

which affects the optimal distortion from the first best actions, required to induce the advocate to make truthful recommendations. We refer to this parameter as the *magnitude* of the distortions. We are now ready to state our first main result.

Proposition 2 *The mechanism which maximizes (P) subject to (ICL) and (ICH) is characterized as follows:*

- (a) *If the advocate reports L, the planner's action is a constant, greater than her first best action μ_L .*

$$q_L(h_{k-1}) = \mu_L + (1 - \gamma)\omega_N$$

- (b) *If the advocate reports H, the planner's action depends on the observed history and is always less than her first best action μ_H*

$$q_H(h_{k-1}) = \mu_H - \gamma\omega_N\Lambda(h_{k-1})$$

- (c) *The planner's and advocate's payoffs are given by*

$$\begin{aligned}V_N &= \bar{V}_N - \frac{1-\delta^N}{1-\delta}\omega_N(\gamma - \gamma^2)(\mu_H - \mu_L) \\ U_N &= (\gamma\mu_L + (1 - \gamma)\mu_H) \frac{1-\delta^N}{1-\delta}\end{aligned}$$

²⁰By convention, $\Lambda(h_0) = 1$.

²¹In statistics, this parameter is one added to the value of the χ^2 -divergence of the distributions, also referred to as Pearson's ϕ^2 . See Pearson (1904), Lancaster (1969), for more information.

²²Throughout we assume that $\alpha < \infty$.

Without access to payments, the planner provides incentives for truthful reporting by committing to vary her actions in response to history. Truth-telling is optimally induced by rewarding the advocate when he issues recommendations that he personally finds unfavorable and punishing the advocate when he reports personally favorable information that appears to be inconsistent with the observed history of states. When the advocate reports L , the distortion from the efficient action is a positive constant, $(1 - \gamma)\omega_N$. By choosing an action higher than the first best action μ_L , the planner makes a report of L more attractive for the advocate, which reduces his incentive to lie. We interpret this fixed reward as a commitment to *compromise*.

The planner commits to treat seemingly self-serving advice based on her assessment of its *validity*. When the advocate reports H , the planner commits to a history-dependent distortion that contains the likelihood ratio, $\gamma\omega_N\Lambda(h_{k-1})$. If the likelihood ratio is small, the history of states supports the inference that the distribution is ϕ_H . The advocate's advice therefore appears valid, and the planner's action is close to the first best action μ_H . If the history of states suggests that the true distribution is ϕ_L , the advice appears invalid, and the planner reduces her action. This punishes the advocate but also hurts her payoff. To optimally provide incentives, the planner acts "as if" she is skeptical of the advocate's advice, cross-checking his recommendation against the observed history, even though she knows that the advocate's report is truthful. This commitment to update is a common feature of incentive compatible mechanisms in a variety of settings.²³

Dynamically delegated actions exhibit qualitatively different dynamics from the case in which the planner can not commit. As described in Section 3, without commitment, the planner's optimal action in each period is the expected value of the state, conditional on the observed history. This action will always lie between the two ideal actions μ_L and μ_H , and will adjust in every period to reflect the additional information acquired from the previous state. Under dynamic delegation, neither of these properties holds. Recall that if the advocate reports H , the optimally delegated action is μ_H adjusted down by a penalty proportional to the likelihood ratio. If certain states, unlikely under ϕ_H , are likely under ϕ_L , the associated likelihood ratio can be very large. If one of these states occurs under ϕ_H , the optimal actions associated with the report of H could be smaller than μ_L . Also observe that if the true distribution is ϕ_L , the sequence of optimally delegated actions is

²³See for example the seminal paper of Holmstorm (1977).

fixed; the action does not adjust in any way to the arrival of new information.

The less similar the distributions (as measured by α), the more the planner benefits under the mechanism. To understand why, observe that if the advocate reports H when the true distribution is ϕ_L , he experiences a negative distortion from action μ_H that is proportional to the likelihood ratio. The expected value of the period k distortion is therefore $-\gamma\omega_N\alpha^{k-1}$. Holding ω_N fixed, increases in α lead to greater punishments for an advocate who tries to manipulate the planner. If the advocate expects a greater punishment for manipulation, his incentive to do so is reduced; the planner can therefore reduce the magnitude of the distortions, ω_N , without violating incentive compatibility. If ω_N is smaller, then the distortions from the first-best actions are smaller for every combination of action and history, improving the planner's payoff. Parameter α therefore represents the strength of the incentives that the planner can provide the advocate by distorting her actions.

Although the planner benefits under this arrangement, the advocate's ex ante payoff is the same as in the absence of influential communication.²⁴ This is a curious result, in light of the fact, that standard incentive theory teaches us that privately informed agents must be paid information rents in order to divulge their information. This result points out an interesting difference between our setting and standard settings. In our setting incentive compatibility does not guarantee each type of advocate a certain share of social surplus. In some sense, the planner uses the advocate's information against him, leveraging her own capability to learn to reduce the advocate's incentive to lie. However, in a setting with costly learning, the absence of rent offered to the advocate apparently undermines the advocate's incentive to acquire information. We turn to this issue in the next section. Before doing so, we briefly consider an application, and then describe the properties of the optimal mechanism with a large number of periods.

4.1 Application: The Cassandra Effect

According to Greek myth, the god Apollo fell in love with a Trojan princess named Cassandra. As a boon to his beloved, he granted Cassandra the gift of perfect foresight, but when she spurned his advances, he cursed her. Although she retained the power to per-

²⁴In the absence of influential communication the planner's sequentially rational action given a history is just $\gamma(h_{k-1})\mu_L + (1 - \gamma(h_{k-1}))\mu_H$. By the law of iterated expectation, $E[\gamma(h_{k-1})] = \gamma$.

We would not expect this result to hold if the planner's expected payoff were not a symmetric function. Nonetheless, we find this interesting, especially as it relates to later results.

factly foretell the future, because of the curse she would never be believed. When the Greeks offered the Trojans a wooden horse, Cassandra warned of the impending disaster; her warnings were ignored, The Trojans brought the horse into the city, and the Greek force hidden inside was unleashed on the unsuspecting populace.

Imagine that an organization faces certain risks. While the organization can not affect the level risk that it faces, it can take certain steps to mitigate the impact of the risky outcomes. In addition to the risks faced by any organization, however, certain organizations in "dangerous" situations face the additional possibility of a disaster. In the model we interpret action q as steps taken to mitigate the impact of adverse events, and x is the optimal level of mitigation given that period's outcome. The advocate is a "whistle-blower" who benefits directly from these preparatory efforts, and thus has an incentive to exaggerate the chance of disaster. In the context of the model, we interpret $F(x|\phi_L)$ as uniform over support $[0, 1]$ while $F(x|\phi_H)$ is uniform over $[0, 1] \cup D$ where $\min\{x \in D\} \geq \max\{x \in X\}$. We interpret X as the set of rationalizable preparation measures that would be adopted by a firm that does not face disasters, while an action in D would be optimal only for the organization if it knew that a disaster were imminent. Thus a draw in X reveals no information about whether the situation is dangerous, but a draw in D confirms that the situation is dangerous.

Our results indicate that without the ability to engage in dynamic delegation, the whistle-blower is subject to the Cassandra effect: although he knows for certain that a disaster will occur, because of the misalignment of preferences, he will never be believed. Under dynamic delegation, following a disaster the organization's type will be revealed, and it will prepare optimally given the risk of disaster. However, if the whistle-blower claims that the organization is dangerous, preparations for disaster shrink with every period that no disaster occurs and may eventually fall below the level that would be optimal if the organization knew that it were safe. In our setting, safe organizations also over-prepare for the level of risk that they face.

4.2 Properties of the Mechanism in Long Interactions

In a long interaction ($N \rightarrow \infty$), the planner is able to learn the true distribution (with virtual certainty) from the observed history of states. This raises two interesting issues. First, to what extent does the planner's ability to learn mitigate the information asym-

metry that exists at the beginning of the interaction? Second, if the planner can learn the true distribution, in the long run does the advocate's recommendation impact the planner's action? Both of these issues are addressed by the following corollary.

Corollary (High Power Incentives) *When $\frac{1}{\delta} \leq \alpha < \infty$, as $N \rightarrow \infty$, $\omega_N \rightarrow 0$; the optimal mechanism approaches the first best mechanism, and the planner's payoff approaches the first best payoff, $V_N \rightarrow \bar{V}_\infty$*

(Low Power Incentives) *When $\alpha < \frac{1}{\delta}$, as $N \rightarrow \infty$, $\omega_N \rightarrow \omega_\infty \in (0, \mu_H - \mu_L)$. The optimal mechanism converges to an incentive compatible mechanism. The planner's payoff is bounded away from the first best payoff. When $N = \infty$ the sequence of actions $q_H(h_{k-1}) \xrightarrow{P} \mu_H$*

To understand this corollary, recall that if the advocate reports H , but the true distribution is ϕ_L , he expects a penalty in the future that grows at rate $\alpha - 1$. However the long term growth of this penalty may not be enough to completely eliminate his incentive to manipulate the planner; the advocate's lifetime penalty for manipulating the planner is the *discounted sum* of the expected distortions, which may either converge or diverge. If the incentives are sufficiently high powered, this discounted sum diverges. In the limit, the powerful incentives completely eliminate the advocate's incentive to manipulate the planner, the distortions vanish, and the optimal mechanism approaches the first best.²⁵ Thus when incentives have high power, in a long interaction, there is virtually no difference between information provided by an advocate, and information that the planner acquires herself.²⁶ If the incentives have low power, the expected penalty explodes too slowly relative to discounting, and the discounted sum of expected distortions converges. The planner can not shrink the magnitude of the distortions to zero without violating incentive compatibility and, the planner's payoffs are bounded away from the first best.

When the true distribution is ϕ_H , the planner's action converges in probability to the first best action μ_H ,²⁷ therefore, the damaging distortions associated with a report of

²⁵Although the first best is not incentive compatible, by choosing an arbitrarily large T , and repeating the optimal T period mechanism an infinite number of times, the planner can construct an infinitely repeated mechanism that is incentive compatible and approximates her payoff under the first best with arbitrary degree of accuracy. Details available upon request.

²⁶This result does not merely hold in the limit as $\delta \rightarrow 1$, but is valid for any value of $\delta \geq \frac{1}{\alpha}$.

²⁷The asymptotic behavior of the distortions is driven by the asymptotic behavior of the likelihood ratio under distribution ϕ_H , which converges in probability to zero. Therefore, $\lim_{N \rightarrow \infty} \Pr(|q_H(h_{N-1}) - \mu_H| \leq \varepsilon) = 1$, for any ε , no matter how small.

H are inherently a short run phenomenon. However, even in the absence of influential communication, the planner’s action would converge to μ_H in probability; thus, when the true distribution is ϕ_H , the impact of the advocate’s recommendation vanishes in the long run. In contrast, when the true distribution is ϕ_L , the advocate’s impact on the planner’s action never diminishes. Paradoxically, when the advocate makes the seemingly more believable report, the distortion from the first best action μ_L persists forever; when the advocate reports in a seemingly biased way, the distortion from the first best action μ_H is likely to be small after a large number of periods. In the long run, the advocate’s recommendation only impacts the planner’s behavior when he reports against his bias.

5 Motivating a Known Advocate to Learn

The theory of dynamic delegation we have developed so far explores how a planner can elicit truthful recommendations from an informed, but extremely biased advocate. In analyzing this question, we assumed that the advocate was already informed before encountering the planner. In certain settings, the advocate may need to expend resources to become informed. For example, in order to form an assessment of the likely costs and benefits of emissions standards, a polluting firm may need to analyze large amounts of data. In these situations, the planner’s reliance on the advocate seems puzzling; if the advocate is uninformed initially and has an obvious incentive to manipulate his findings, why wouldn’t the planner learn from an unbiased party or expend effort to learn herself? Furthermore, if (as in the previous section) he expects to have no impact on the planner the advocate may have weak incentives to acquire information; how can the planner motivate him to learn? From a social perspective, is it a good idea for the planner to acquire information from the advocate?

In light of these issues, we consider how the advocate might be motivated to gather the information that the planner requires to make decisions. In order to acquire information, the advocate must expend $c \geq 0$ resources, in the form of money, time and effort; throughout the section, we focus on the case of relatively small values for c .²⁸ Crucially, the advocate’s decision to become informed can not be observed by the planner. The

²⁸The cutoff between small and large costs, \tilde{c} , is defined in the appendix. While we discuss the mechanism that induces learning with high costs in a later footnote, we point out that if the cost is high enough, inducing learning is not in the planner’s interest.

advocate not only has the ability to lie when issuing an informed recommendation, he can issue a recommendation without exerting effort to acquire any information at all. We start by deriving the optimal incentive compatible mechanism that provides incentives for the advocate to acquire information. This will help us understand how the planner motivates the advocate to acquire information. We will then compare optimal dynamic delegation to consulting an impartial advisor or exerting effort, to help us understand why an advocate may be preferred to an impartial advisor.

If the planner would like to motivate the advocate to become informed, two additional constraints appear in the mechanism design problem. If an advocate decides to remain uninformed, he could just issue an uninformed recommendation, either H or L .²⁹ On the other hand, if the advocate chooses to acquire information, he anticipates that with probability γ he will learn that the distribution is ϕ_L , and with probability $1 - \gamma$ he will learn that the distribution is ϕ_H . Whatever, he learns, (ICH) and (ICL) ensure that he will report truthfully. Therefore, for the advocate to choose to learn, exerting effort and reporting truthfully should be preferred by an uninformed advocate to not exerting effort and either reporting H all the time (AICH) or reporting L all the time (AICL). Formulating and simplifying these constraints leads to the following conditions:

$$\begin{aligned} \sum_{k=1}^N \delta^{k-1} E [q_H (h_{k-1}) - q_L (h_{k-1}) | \phi_L] &\leq -\frac{c}{\gamma} & \text{(AICH)} \\ \sum_{k=1}^N \delta^{k-1} E [q_L (h_{k-1}) - q_H (h_{k-1}) | \phi_H] &\leq -\frac{c}{1-\gamma} & \text{(AICL)} \end{aligned}$$

Of course, the optimal mechanism must also satisfy the constraints for truthful reporting:

$$\begin{aligned} \sum_{k=1}^N \delta^{k-1} E [q_H (h_{k-1}) - q_L (h_{k-1}) | \phi_L] &\leq 0 & \text{(ICL)} \\ \sum_{k=1}^N \delta^{k-1} E [q_H (h_{k-1}) - q_L (h_{k-1}) | \phi_H] &\leq 0 & \text{(ICH)} \end{aligned}$$

Writing the constraints in this way makes it apparent that if $c = 0$, the constraints for reporting truthfully and the constraints for acquiring it are identical. Furthermore, because the optimal mechanism of Proposition 3 satisfies (ICL) as an equality, it violates constraint (AICH). If information acquisition is costly and subject to moral hazard, then confronted with the mechanism of Proposition 3, an advocate would never acquire information and would always report H . Finally, it is clear that the constraints for incentive compatible information acquisition imply the incentive constraints for truthful reporting;

²⁹Unless the uninformed advocate is indifferent between reporting H all the time and reporting L all the time, he would never choose to randomize. In the cases we consider in the body of the paper, an uninformed advocate always strictly prefers to report H .

constraints (ICL) and (ICH) can therefore be ignored in the planner's problem. As before, participation at both interim and ex ante stages is ensured by the planner's commitment to interpret the absence of a message as a report of L . If the planner makes such a commitment, constraint (AICL) would ensure that the advocate prefers to learn rather than report nothing to the planner.³⁰

We derive the optimal mechanism that induces information acquisition by maximizing (P) subject to (AICH) and (AICL).

Proposition 3 *For small costs, the mechanism that maximizes (P) subject to (AICH) and (AICL) is identical to the optimal mechanism of Proposition 2 with an increased magnitude of distortions:*

$$\tilde{\omega}_N = \frac{\gamma(1-\delta^N)(\mu_H - \mu_L) + c(1-\delta)}{\gamma(1-\delta^N)(\mu_H - \mu_L)} \omega_N$$

(a) *The planner's actions are*

$$\begin{aligned} q_L(h_{k-1}) &= \mu_L + (1 - \gamma) \tilde{\omega}_N \\ q_H(h_{k-1}) &= \mu_H - \gamma \tilde{\omega}_N \Lambda(h_{k-1}) \end{aligned}$$

(b) *The planner's and advocate's payoffs are given by*

$$\begin{aligned} V_N^c &= \bar{V}_N - \left(\frac{\gamma(1-\delta^N)(\mu_H - \mu_L) + c(1-\delta)}{\gamma(1-\delta^N)(\mu_H - \mu_L)} \right)^2 \frac{1-\delta^N}{1-\delta} \omega_N (\gamma - \gamma^2) (\mu_H - \mu_L) \\ U_N^c &= U_N - c \end{aligned}$$

(c) *If ω_N is not too large, the planner prefers to offer an uninformed advocate the optimal mechanism to acquire information and report truthfully, rather than pay an impartial advisor to acquire the information or exert effort to acquire the information herself*

Proposition three indicates that when the cost of acquiring information is strictly positive the planner "raises the stakes" of the mechanism, rewarding the advocate more when he reports personally unfavorable information and proportionally increasing the downward distortion (for every history) when the advocate reports a personally favorable distribution. Intuitively, there are two types that could profitably deviate by reporting the high

³⁰This may be a strong assumption in certain settings. An analysis of the alternative case is available upon request.

distribution: the type that learned that the true distribution is ϕ_L , and the uninformed advocate, who is pretending to be informed. Because a report of the low distribution can still be believed, the structure of the mechanism is similar to the zero-cost case. To motivate the advocate to learn, the magnitude of the distortions needs to be increased relative to the zero-cost case. With zero cost of information, the goal was to prevent an advocate from deliberately misleading the planner; here the mechanism also needs to motivate the advocate to exert effort to learn, rather than "gamble" by reporting H without learning anything. This result is similar to Szalay (2005) and Lewis and Sappington (1997) who find that increasing the variation in payoffs for privately informed agents is the optimal way to induce the agents to gather information.³¹

In the optimal mechanism the social cost of information acquisition exceeds c . Although the advocate bears the full cost c himself, in order to motivate the advocate to learn, the planner must increase the magnitude of the distortions inherent in the mechanism. Increased distortions harm the planner. If the planner can acquire the information herself at cost c (or hire an impartial advisor to do so on her behalf), it is socially wasteful for the planner to rely on the advocate.

Although it is socially inefficient, the planner may prefer to deal with the advocate rather than pay an impartial agent or acquire the information herself (at cost c).³² If she chooses to acquire information herself, the planner can implement the first best actions, which leads to a payoff of $\bar{V}_N - c$. If she offers the advocate the optimal mechanism, her payoff is given by V_N^c , above. Comparing these expressions it is clear that the planner would prefer to interact with the advocate if the cost of information c , were greater than

³¹In the case of large costs the optimal mechanism is quite different. When the cost of becoming informed is very high, not becoming informed at all becomes tempting for the advocate. Although no advocate would ever report L to *manipulate the decision maker*, if the cost of acquiring information is very high, the advocate may report L as a way to *avoid learning the true distribution*. The decision maker must cross-check *both* reports against the history in order to provide the advocate with incentives to exert effort in learning the true distribution.

³²The assumption that information is soft applies to information acquired by the planner as well as to information acquired by the advocate. This assumption rules out situations in which the planner commits to learn on her own with ε probability, and threatens the advocate that, if she learns and discovers that his report was wrong, she will implement extremely negative actions. By doing so, she will acquire information with arbitrarily small distortion. To implement this scheme it is crucial that a third party can verify the information, to ensure that the planner follows through on her promise to uncover information and punish lies.

her "payoff cost" dealing with the advocate:

$$c \geq \left(\frac{\gamma(1-\delta^N)(\mu_H-\mu_L)+c(1-\delta)}{\gamma(1-\delta^N)(\mu_H-\mu_L)} \right)^2 \left(\frac{1-\delta^N}{1-\delta} \right) \omega_N (\gamma - \gamma^2) (\mu_H - \mu_L)$$

A small value of ω_N ensures that this inequality is satisfied;³³ it also guarantees that the planner prefers the mechanism that induces learning to going it alone.³⁴

This result may seem quite surprising, but the rationale for the planners' preference to consult an advocate for advice is quite compelling. Imagine that the planner has access to the same information acquisition technology as the advocate. If she exerts effort to learn, the planner doesn't need to worry about truthful revelation of information, but she bears the full cost of information acquisition. On the other hand, because of his extreme bias, when dealing with an advocate, the planner bears a cost of inducing the advocate to truthfully report his information; however, because he cares about her actions, the planner can use her actions to motivate the advocate to acquire information. By increasing the magnitude of the distortions, the planner passes the direct cost of information acquisition to the advocate, hurting her own expected payoff in the process. However, when the advocate expects a large lifetime penalty from reporting H when the true distribution is ϕ_L , either deliberately or because he is uninformed, the planner does not need to increase the magnitude very much (in absolute terms) in order to induce learning. In this case the total loss of payoff to the planner from acquiring truthful information using the optimal mechanism is smaller than the cost of direct information acquisition.

This result suggests an important explanation for the prominence of advocates in many regulatory, policy making and personal decision making processes. In contrast to impartial advisors, advocates represent a particular point of view or interest that is affected by the planner's decision. Because of this, the planner has some power over the advocate; she can use her decisions to motivate the advocate to acquire information. As a result it can be easier and less costly for a planner to rely on a biased advocate rather than an objective adviser to acquire the information that is needed for decision making. Dewatripont and Tirole (1999) make a complementary argument in their explanation of why an advocacy

³³As discussed in the previous section, if $\alpha \geq \frac{1}{\delta}$, then as $N \rightarrow \infty$, $\omega_N \rightarrow 0$. Alternatively, keeping N fixed, if α is large then ω_N is close to zero.

³⁴It is worth pointing out that for sufficiently small costs, the planner prefers the mechanism that induces learning to the case of no influential communication, regardless of ω_N . Because the mechanism with $c = 0$ is always preferred by the planner to the case without influential communication, by continuity, for c small, the mechanism is also preferred to the case without influential communication.

system may be preferred to a tribunal in fact finding processes. Che and Kartik (2009) point to a related rationale to explain why planners may prefer to consult with advisers who share their preferences but have different prior beliefs about the best action to take. In both of these settings no manipulation is possible. In some sense, both of these analyses argue that interested parties have *more to gain* from establishing the truth of their preferred positions and are therefore more willing to work hard to uncover information. In contrast, we show that when manipulation is possible, an interested party has *more to lose* if it becomes clear that its report is wrong. This threat of potential loss provides the interested party with an extra incentive to acquire information.

5.1 Discovering Hidden Agendas

Known advocates can not escape from the fact that they have no inherent credibility; any influence which they have on decision making is exclusively a consequence of the planner's design. Even if the advocate for children has acquired excellent information about the likely benefits of the head start program, because it is common knowledge that he supports this cause, only the planner can provide the advocate with incentives to reveal his information truthfully, thereby giving the advice credibility and influence. Is this inevitable? Can the advocate exert a greater impact for her cause by being less transparent to the commission about the issues that he supports?

Imagine that the planner is crafting new legislation that defines the goals and responsibilities for a future health and welfare initiative.³⁵ The legislation prioritizes support for a variety of social programs, including education, health and human services, and childhood nutrition. The planner requires the advice of a knowledgeable advisor on how to draft the legislation to be most effective. The advisor's motives for advising the planner may be unknown; he may have an allegiance to a special interest group whose agenda he would like to promote, or he may have no agenda at all, issuing truthful recommendations without concern for their impact on the legislation. Alternatively, imagine that a financial advisor is offering a client investment advice and services. In offering advice, he may be pursuing his own bottom line, or he may feel bound by certain oaths or obligations to place the interests of the client ahead of his own.

To account for these possibilities, we admit the possibility that the advisor is impartial

³⁵A similar example is in Morris (2001).

into the model. An impartial advisor is indifferent over actions and is always willing to report his information truthfully. If the planner could observe that her advisor is impartial, she would optimally delegate her choice of action to him with no restrictions. However, if the advisor's conflict of interest is privately known, an advocate would have an incentive to misrepresent himself as impartial in order to give a manipulative report of H the appearance of credibility. We therefore augment the mechanism to provide incentives for an advisor to voluntarily reveal a hidden agenda, if it exists.

In analyzing this issue, we assume that the planner can not prevent the advisor from learning both the true distribution and his preferences simultaneously. If it were possible for the planner to control the advisor's access to information, the planner would require that the advisor disclose a conflict of interest before allowing him to learn the true distribution. We do not allow this type of restriction to avoid discussing the issue of whether this type of control over information is possible, how the planner could implement this type of restriction and whether such a restriction is consistent with zero cost of learning, which we assume throughout this section.³⁶

With uncertainty about the advisor's preferences, the agreement between the planner and the advisor is governed by a mechanism that is more elaborate than in earlier sections. With a hidden agenda, there are two possible types of advisors, advocates (with a hidden agenda) and impartial advisors. The advisor's private information consists of the true distribution and his agenda, and could be of four possible types. We assume that the probability that the advisor is an advocate is a ; we also assume that the advisor's agenda is statistically independent of whether the distribution is ϕ_H or ϕ_L . The mechanism offered by the planner is therefore a family of four functions, one function for each possible combination of interest conflict and information:

$$\{q_H^i(h_{k-1}), q_L^i(h_{k-1}), q_H^a(h_{k-1}), q_L^a(h_{k-1})\}_{k=1}^N$$

If the advocate reports that he has preferences $t \in \{i, a\}$ (where i denotes the impartial type), and that the true distribution is ϕ_Z , the planner commits to implement actions $q_Z^t(h_{k-1})$. The planner's expected payoff from an incentive compatible mechanism is just a weighted average of her payoffs from the "sub-mechanism" intended for the impartial

³⁶In the extended appendix we analyze the optimal mechanism assuming that the planner can restrict access to information in this way. We will discuss some of the results in a later footnote. Interested readers should consult the extended appendix for more information.

advisor, $m^i = (q_H^i(h_{k-1}), q_L^i(h_{k-1}))$, and the "sub-mechanism" intended for the advocate $m^a = (q_H^a(h_{k-1}), q_L^a(h_{k-1}))$;

$$P^d(m^i, m^a) = (1 - a)P(m^i) + aP(m^a)$$

In order for the mechanism to be incentive compatible, the advisor should be willing to disclose both the distribution and his conflict of interest truthfully. Because the impartial advisor is indifferent over all possible outcomes, he is always willing to do so; however, the advocate type needs incentives to report truthfully. This leads to a set of six incentive constraints, denoted by (ICD).

$$\begin{aligned} \sum_{k=1}^N \delta^{k-1} E[q_Z^t(h_{k-1}) - q_L^a(h_{k-1}) | \phi_L] &\leq 0 \quad (\text{ICL-Z,t}) \\ \sum_{k=1}^N \delta^{k-1} E[q_Z^t(h_{k-1}) - q_H^a(h_{k-1}) | \phi_H] &\leq 0 \quad (\text{ICH-Z,t}) \\ Z \in \{H, L\}, t \in \{i, a\} \end{aligned}$$

The optimal mechanism maximizes (PD) subject to the system (ICD). Intuitively, we would expect that several of these constraints would not be binding in the optimal mechanism.³⁷ In the appendix we formulate a relaxed problem in which we impose only constraints (ICL-H,a), (ICL-H,i). We then prove that the solution to the relaxed problem satisfies the remaining constraints, and therefore characterizes the optimal incentive compatible mechanism. Before stating the proposition, we introduce a parameter:

$$\theta = \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L)$$

This parameter is very similar to the magnitude introduced in section 3; in fact, $a = 1$ implies $\theta = \omega_N$. Furthermore, parameter θ plays a very similar role to ω_N in the optimal mechanism, but unlike ω_N which enters the planner's promised actions in a "symmetric" way in propositions 2 and 3, θ does not enter in a symmetric way in proposition 4.

³⁷The advocate has an incentive to claim to be impartial in order to give a manipulative report of H additional credibility. There is no reason for the advocate to claim to be impartial, while issuing a report of L . Furthermore, following the reasoning in section 3, we would also suspect that (ICH-L,a) is non-binding. There is no simple intuition that suggests that either of the remaining constraints are non-binding: an advocate who knows the distribution is ϕ_L could potentially benefit by reporting H (ICL-H,a), he could also potentially benefit by claiming to be impartial and reporting H (ICL-H,i). Furthermore, the advocate who knows that the true distribution is ϕ_H could try to gain additional credibility by claiming to be impartial and reporting H (ICH-H,i). Although there is no good reason to eliminate (ICH-H,i), it turns out to be active in the optimal mechanism, but with a zero Lagrange multiplier; it is active but not binding.

Proposition 4 *The mechanism that maximizes (PD) subject to (ICD) is characterized as follows:*

- (a) *If the advisor claims to be impartial and reports L , the planner's action is equal to the first best action*

$$q_L^i(h_{k-1}) = \mu_L$$

- (b) *If the advisor claims to be an advocate and reports L , the planner's action is a constant, greater than her first best action μ_L .*

$$q_L(h_{k-1}) = \mu_L + (1 - \gamma)\theta$$

- (c) *If the advisor reports H , regardless of his claim about his preferences, the planner's action depends on the observed history and is always less than her first best action μ_H*

$$q_H^i(h_{k-1}) = q_H^a(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})(a\theta)$$

- (d) *If the advisor is an advocate his expected payoff is given by*

$$U_N + \frac{1-\delta^N}{1-\delta}\gamma(1-\gamma)(1-a)\theta.$$

Proposition 4 illustrates the differences that arise when the advocate's preferences on a specific issue are not common knowledge. Because the advocate would have no reason to claim to be impartial and then report L , a report of L from an advisor who claims to be impartial can be believed at face value. A report of H is treated identically, whether the advisor is an advocate or claims to be impartial; however, the magnitude of the distortion associated with a self serving report, $a\theta_N$, is *smaller* the greater the probability that the advisor is impartial. The possibility that the advisor is impartial allows the planner to treat reports of H with greater credibility. On the other hand, the constant distortion induced when the advisor reveals a conflict of interest, but reports *against* his bias is larger when there is a possibility that the advisor may be impartial. Because the planner gives greater credibility to reports of H , these reports are more attractive to an advocate who knows that the distribution is actually ϕ_L . This increases the advocate's incentive to issue manipulative advice, and the planner must compensate the advocate for reporting L

with an additional distortion.³⁸

The advocate's world changes for the better when he becomes less transparent and his allegiance to special interests are more difficult to predict. When there is a possibility that the advocate *is not* an advocate but rather an impartial advisor, the planner benefits from a greater compromise with the *true* advocate. The advocate earns a rent for disclosing his agenda, and his ex ante payoff from the mechanism rises above U_N , his expected payoff if he were not consulted and his advice was totally ignored. Finally, the advocate is able to have a positive impact on the welfare of the special interests that he supports. It is ironic, however, that in order to help his cause the advocate must disavow and conceal his support of the cause that he cares so deeply about! In effect, just as Morris (2001) demonstrates that "bad advisors" can gain a reputation for being impartial, and Sobel finds that "unfriendly" advisors can establish a reputation for acting in the best interests of the decision maker, we find that if the advocate's ties to special interests are hidden, the advocate can credibly claim to be impartial, and is therefore better off.

6 Conclusion

In this paper we illustrate the power of dynamic delegation to mitigate extreme conflicts of interest. We consider a planner who receives advice from an advocate. The advocate's preferences are increasing in the action chosen by the planner and are independent of the true state. We first demonstrated that advocates can only influence the planner's decisions under dynamic delegation; under either strategic communication or static delegation no useful information is conveyed. These results demonstrate the negative consequences of the severe interest conflict that we consider in the paper. If the planner can delegate dynamically we find that she can elicit influential information from the advocate. The optimal delegation policy is characterized by compromise with an advocate who reports against his bias, and by a commitment to evaluate the validity of seemingly self-serving advice. Moreover, we show that planners may prefer the advice of a biased advocate to

³⁸Similar results hold if the planner can prevent the advocate from learning the true distribution until he has disclosed his interest conflict. In that setting, the planner is able to treat the advice of an impartial advisor who reports H differently from the advice of an advocate who reports H . Both of these sequences of messages lead to stochastic distortions that depend of $\Lambda(h_{k-1})$, but the impartial type generates smaller distortions on average; however the advocate does not capitalize on this because he earns a rent. See the extended appendix for more information.

the recommendation of an indifferent adviser in instances where information about the planner's problem is costly to acquire. The mechanism underlying this result is closely tied to the combination of learning and commitment at the heart of this paper. Finally our theory predicts that advocates are better served to conceal their ties to special interest as this permits them to have greater sway over the planners that they advise.

Our theory of dynamic delegation is special in a number of respects and some of the predictions of our model should be interpreted with care. One set of special assumptions is the linear-quadratic preference structure. If the advocate has preferences that are state-independent then the assumption of a linear payoff function for the advocate is without loss of generality: simply "rescale" the planner's action so that the decision variable is equal to the advocate's utility. In a more general model, however, the planner would have a concave single-peaked payoff function; virtually all of the qualitative results of the current specification carry over to such an environment. However, the result that the advocate gets the same ex ante payoff under dynamic delegation as under no communication is unlikely to generalize to more general specifications. We also make the somewhat restrictive assumption that there is just one advocate who can advise the planner. This assumption makes the planner's problem as difficult as possible and therefore underscores the power of dynamic delegation. However, in many applications, it is common for decision makers to consult two or more advocates representing opposing interests. One fruitful direction for future research would be to extend our analysis to consider how "dueling" advocates representing disparate points of view may be most effectively paired to provide informative advice to a policy maker. More generally, the optimal use of advocates with known allegiances might be analyzed as an optimal fact finding mechanisms in which informed parties with differing views are assigned to gather information and make recommendations to a common decision maker who may commit to a course of action conditional on the recommendations of the advocates as well as their relative "track" records in predicting the history of states. All of these extensions are unlikely to change the main message of this paper: the combination of commitment and learning can mitigate or overcome even the most extreme interest conflicts.

References

- [1] Aghion, P and Tirole, J (1997) "Formal and Real Authority in Organizations" *Journal of Political Economy*,
- [2] Alonso, R. and N. Matouschek (2008): "Optimal Delegation," *Review of Economic Studies*, 75, pp 259-293
- [3] Alonso, R and Dessein, W and Matouschek, N (2008) "When Does Coordination Require Centralization" *The American Economic Review*, Volume 98, Number1, March 2008 , pp. 145-179(35)
- [4] Ambrus, A and Egorov, G (2009) "Delegation and Procedural Rules" Harvard University Working Paper
- [5] Ambrus, A and Egorov, G (2009) "Delegation and Nonmonetary Incentives" Harvard University Working Paper
- [6] Ambrus, A and Azevedo, E and Kamada, Y and Takagi, Y (2010) "Legislative committees as information intermediaries: a unified theory of committee selection and amendment rules" Harvard University Working Paper
- [7] Austen-Smith, D (1994). "Strategic transmission of costly information". *Econometrica* 62, 955-963.
- [8] Battaglini, M. (2002): "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica*, 70, pp1379-1401.
- [9] Bull, J. and Watson, J. (2007) "Hard Evidence and Mechanism Design," *Games and Economic Behavior*
- [10] Bull, J. and Watson, J. (2004) "Evidence Disclosure and Verifiability," *Journal of Economic Theory*
- [11] Chakraborty, A and Harbaugh, R. (2007) "Comparative Cheap Talk," *Journal of Economic Theory*
- [12] Chakraborty, A and Harbaugh, R. (2009) "Persuasion by Cheap Talk," *American Economic Review*

- [13] Chakraborty, A. and Yilmaz, B. (2009) "Communication, Consensus, and Control in Corporate Boards" Mimeo, University of Pennsylvania
- [14] Che, Y and Kartik, N (2009) "Opinions as Incentives," *Journal of Political Economy*
- [15] Cooper, R. and Hayes B (1987), "Multi-period Insurance Contracts", *International Journal of Industrial Organization* 5, 211-231.
- [16] Crawford, V and Sobel, J (1982). "Strategic Information Transmission." *Econometrica* 50 pp1431-51.
- [17] Dessein, W. (2002): "Authority and Communication in Organizations," *Review of Economic Studies*-69, pp811-838.
- [18] Dewatripont, M and Tirole, J.(1999) "Advocates." *Journal of Political Economy*. 107 : pp1-39.
- [19] Gentzkow, M. and Kamenica E. (2010) "Bayesian Persuasion" *American Economic Review*. Forthcoming.
- [20] Grossman, S (1981) "The Informational Role of Warranties and Private Disclosure about Product Quality", *Journal of Law and Economics*, pp 461-83
- [21] Holmstrom, B. (1977): "On Incentives and Control in Organization" Ph.D. Dissertation, Stanford University
- [22] Holmstrom, B. (1984): "On the Theory of Delegation," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. Khilstrom, Amsterdam: Elsevier, 115-141
- [23] Inderst, Roman and Marco Ottaviani (2009) "Misselling through Agents" *American Economic Review*, June 2009, 99(3), 883–908
- [24] Krishna V and Morgan J (2008). "Contracting for information under imperfect commitment," *RAND Journal of Economics*, RAND Corporation, vol. 39(4), pages 905-925
- [25] Krishna V and Morgan J. (2008): "Cheap Talk," in *The New Palgrave Dictionary of Economics*, ed. by L. Blume, and S. Durlauf, 2nd edn. New York, NY: McMillan

- [26] Lancaster, H. O. (1969), *The Chi-Squared Distribution*, New York: John Wiley
- [27] Lewis, T and Sappington, D (1997) "Information Management in Incentive Problems" *Journal of Political Economy*
- [28] Milgrom, P (1981) Good News and Bad News; Representation Theorems and Applications." *Bell Journal of Economics* 12, pp 380-91
- [29] Milgrom, P and Roberts, J (1986) "Relying on the Information of Interested Parties" *Rand Journal of Economics*, 17 pp 18-32
- [30] Morris, S. (2001) "Political Correctness" *Journal of Political Economy* 109 pp. 231-261
- [31] Okuno-Fujiwara, M., and Postelwaite A and Suzumura K (1990) "Strategic Information Revelation." *Review of Economic Studies* 57 pp 25-47
- [32] Ottaviani, M and Sorenson P (2006) "Professional Advice." *Journal of Economic Theory*
- [33] Pearson, K (1904). "Mathematical Contributions to the Theory of Evolution XIII: On the Theory of Contingency and Its Relation to Association and Normal Correlation," in Draper's Company Research Memoirs, Biometric Series 1 (publisher unknown
- [34] Prat, A. (2005) "The Wrong Kind of Transparency" *American Economic Review*
- [35] Prendergast, C. (1993) "A Theory of 'Yes Men.'" *American Economic Review*. 83 pp 757-70.
- [36] Prendergast, C, and Stole, L. (1996) "Impetuous Youngsters and Jaded Old Timers: Acquiring a Reputation for Learning." *Journal of Political Economy*. 104:pp1105-34
- [37] Rantakari, H (2008) "Governing Adaptation" *Review of Economic Studies*
- [38] Shavell, S (1994) "Acquisition and Disclosure of Information Prior to Sale," *Rand Journal of Economics* 25 pp 20-36
- [39] Shin, H (1994) "News Management and the Value of Firms" *Rand Journal of Economics* 25;1 pp 58-71

- [40] Sobel, J (1985) "A Theory of Credibility." *Review of Economic Studies* 52 pp 557-73.
- [41] Sobel, J. (2009): "Signaling Games" *Meyers: Encyclopedia of Complexity and Systems Science* (ed.), Springer
- [42] Strausz, R (2005) "Interim Information in Long Term Contracts" *Journal of Economics and Management Strategy*
- [43] Szalay, Dezso (2005) "The Economics of Extreme Options and Clear Advice," *Review of Economic Studies*

7 Appendix

In the appendix we prove the results discussed in the body of the text. A more complete appendix with proofs of assertions in footnotes is available upon request.

Proposition 1.1 *In a single period interaction with commitment, the only incentive compatible mechanism is non-influential: $q_H = q_L$.*

Proof 1.1 Discussed in text

Lemma 1: *If the decision maker begins with belief $\hat{\gamma}$ that the true distribution is ϕ_L , the unique sequentially rational action in period k following history h_{k-1} is the expected value of the state, updated based on the observed history:*

$$q(\hat{\gamma}, h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

$$\gamma(h_{k-1}) = \Pr(\phi = \phi_L | h_{k-1}) = \frac{\hat{\gamma} f(h_{k-1} | \phi_L)}{\hat{\gamma} f(h_{k-1} | \phi_L) + (1 - \hat{\gamma}) f(h_{k-1} | \phi_H)}$$

Proof: If she begins her sequence of decisions with belief $\Pr(\phi = \phi_L) = \hat{\gamma}$ the sequentially rational sequence of decisions is characterized by the following optimization problem:

$$\begin{aligned} \max_{q(h_{k-1})} \quad & -\hat{\gamma} \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) \left[\int_X f(x | \phi_L) (q(h_{k-1}) - x)^2 dx \right] dh_{k-1} \\ & - (1 - \hat{\gamma}) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) \left[\int_X f(x | \phi_H) (q(h_{k-1}) - x)^2 dx \right] dh_{k-1} \end{aligned}$$

This objective function can be simply rewritten

$$\begin{aligned} & \hat{\gamma} \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) (-q(h_{k-1}))^2 + 2\mu_L q(h_{k-1}) - \eta_L dh_{k-1} \\ & + (1 - \hat{\gamma}) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (-q(h_{k-1}))^2 + 2\mu_H q(h_{k-1}) - \eta_H dh_{k-1} \end{aligned}$$

by introducing a variable to denote the decision maker's belief that the distribution is ϕ_L given the observed history of states, $\gamma(h_{k-1})$, and the conditional mean, $\mu(h_{k-1})$ the

objective function can be simplified even further

$$\gamma(h_{k-1}) = \Pr(\phi = \phi_L | h_{k-1}) = \frac{\hat{\gamma} f(h_{k-1} | \phi_L)}{\hat{\gamma} f(h_{k-1} | \phi_L) + (1 - \hat{\gamma}) f(h_{k-1} | \phi_H)}$$

$$\mu(h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

$$\eta(h_{k-1}) = \gamma(h_{k-1}) \eta_L + (1 - \gamma(h_{k-1})) \eta_H$$

Substituting this into the objective function gives

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_0) (-q(h_{k-1})^2 + 2\mu(h_{k-1}) q(h_{k-1}) - \eta(h_{k-1})) dh_{k-1}$$

The first and second order conditions imply that the unique sequentially rational decision following history h_{k-1} is given by

$$q(h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

Therefore, given an initial belief $\hat{\gamma}$, following observed history h_{k-1} the unique sequentially rational actions is

$$\tilde{q}(\hat{\gamma}, h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

QED

Proposition 1.2 *In every Perfect Bayesian Equilibrium of the multiperiod game without commitment the advocate's reporting strategy is independent of his information. There is no influential communication in equilibrium.*

Proof: We prove this result by constructing the PBE of this game. Once the planner receives a message from the advocate, she Bayesian updates her belief that the distribution is ϕ_L to either $\hat{\gamma}_H$ or $\hat{\gamma}_L$. She then faces a repeated single-player decision problem. According to Lemma 1, the planner's unique sequentially rational action following history

h_{k-1} , given that her initial belief is $\hat{\gamma}$ is the conditional expected value of the state.

$$q(\hat{\gamma}, h_{k-1}) = \gamma(h_{k-1}) \mu_L + (1 - \gamma(h_{k-1})) \mu_H$$

$$\gamma(h_{k-1}) = \Pr(\phi = \phi_L | h_{k-1}) = \frac{\hat{\gamma} f(h_{k-1} | \phi_L)}{\hat{\gamma} f(h_{k-1} | \phi_L) + (1 - \hat{\gamma}) f(h_{k-1} | \phi_H)}$$

Because $\gamma(h_{k-1})$ is monotone decreasing with respect to $\hat{\gamma}$, inducing a smaller value of $\hat{\gamma}$ increases the planner's action *following every history*, benefitting the advocate. Recall that the advocate's strategy is just $(r_H, r_L) \in [0, 1]^2$, where r_X represents the probability that the advocate reports H when the true distribution is ϕ_X . The advocate's sequentially rational strategy is therefore

$$\begin{aligned} \hat{\gamma}_H &> \hat{\gamma}_L \rightarrow r_H = 0, \quad r_L = 0 \\ \hat{\gamma}_H &< \hat{\gamma}_L \rightarrow r_H = 1, \quad r_L = 1 \\ \hat{\gamma}_H &= \hat{\gamma}_L \rightarrow r_H \in [0, 1], \quad r_L \in [0, 1] \end{aligned}$$

Consider first the case in which $\hat{\gamma}_H > \hat{\gamma}_L$ (the reverse case is identical, replacing H and L). In this case, the advocate always reports L . Upon observing L , the planner's Bayesian update is equal to the prior, $\hat{\gamma}_L = \gamma$. The off-the path-belief associated with a report of H is assigned to be any value $\hat{\gamma}_H > \gamma$. The planner's actions are $q(\gamma, h_{k-1})$ if the advocate reports L and the planner observes history h_{k-1} , and $q(\hat{\gamma}_H, h_{k-1})$ if the planner receives message H and observes history h_{k-1} (off the equilibrium path). These strategies and beliefs together constitute a PBE.

The remaining possibility is that $\hat{\gamma}_H = \hat{\gamma}_L$. In this case, the advocate can send either message with any probability, as the advocate is indifferent between the messages. If only one message is sent in equilibrium the PBE is identical to the one described previously. If both messages are sent in equilibrium, then according to Bayes Rule,

$$\begin{aligned} \hat{\gamma}_H &= \frac{\gamma r_L}{\gamma r_L + (1 - \gamma) r_H} \\ \hat{\gamma}_L &= \frac{\gamma(1 - r_L)}{\gamma(1 - r_L) + (1 - \gamma)(1 - r_H)} \end{aligned}$$

It a matter if calculation to verify that

$$\widehat{\gamma}_H = \widehat{\gamma}_L \rightarrow r_H = r_L$$

$$r_H = r_L \rightarrow \widehat{\gamma}_H = \widehat{\gamma}_L = \gamma$$

Thus, the only other possible PBE has $r_H = r_L = r \in (0, 1)$, $\widehat{\gamma}_H = \widehat{\gamma}_L = \gamma$, and an action conditional on history $q(\gamma, h_{k-1})$, regardless of the message sent. In both cases, the Bayesian update on the equilibrium path is equal to the prior, and on the equilibrium path, the planner's actions are $q(\gamma, h_{k-1})$. There is no influential communication in equilibrium.

QED

Lemma 2 *A mechanism $q_L(h_{k-1}) = \mu_L + (1 - \gamma)w$, $q_H(h_{k-1}) = \mu_H - \gamma w \Lambda(h_{k-1})$ satisfies constraint (ICH) if and only if $w \leq \mu_H - \mu_L$.*

Proof:

$$\sum_{k=1}^N \delta^{k-1} E[q_H(h_{k-1}) | \phi_H] \geq \sum_{k=1}^N \delta^{k-1} E[q_L(h_{k-1}) | \phi_H] \quad (\text{ICH})$$

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) q_H(h_{k-1}) dh_{k-1} \geq \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) q_L(h_{k-1}) dh_{k-1}$$

$$\Leftrightarrow$$

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (\mu_H - \gamma w \Lambda(h_{k-1})) dh_{k-1} \geq$$

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (\mu_L + (1 - \gamma)w) dh_{k-1}$$

Because $f(h_{k-1} | \phi_H) \Lambda(h_{k-1}) = f(h_{k-1} | \phi_L)$, this line simplifies

$$(\mu_H - \gamma w) \frac{1 - \delta^N}{1 - \delta} \geq (\mu_L + (1 - \gamma)w) \frac{1 - \delta^N}{1 - \delta}$$

$$\Leftrightarrow$$

$$w \leq \mu_H - \mu_L$$

QED

Lemma 3 *The planner's payoff under mechanism $q_L(h_{k-1}) = \mu_L + (1 - \gamma)w$, $q_H(h_{k-1}) =$*

$\mu_H - \gamma w \Lambda(h_{k-1})$ is given by.

$$\bar{V}_N - \gamma(1-\gamma)w^2 \left(\frac{1-\delta^N}{1-\delta} (1-\gamma) + \gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \right)$$

Proof:

$$\begin{aligned} & \gamma \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (-q_L(h_{k-1})^2 + 2\mu_L q_L(h_{k-1}) - \eta_L) dh_{k-1} \\ & + (1-\gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-q_H(h_{k-1})^2 + 2\mu_H q_H(h_{k-1}) - \eta_H) dh_{k-1} = \\ & \gamma \sum_{k=1}^N \delta^{k-1} (-(\mu_L + (1-\gamma)w)^2 + 2\mu_L(\mu_L + (1-\gamma)w) - \eta_L) \\ & + (1-\gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-(\mu_H - \gamma w \Lambda(h_{k-1}))^2 + 2\mu_H(\mu_H - \gamma w \Lambda(h_{k-1})) - \eta_H) dh_{k-1} = \\ & \gamma \sum_{k=1}^N \delta^{k-1} (-\sigma_L^2 - (1-\gamma)^2 w^2) \\ & + (1-\gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-\sigma_H^2 - \gamma^2 w^2 \Lambda^2(h_{k-1})) dh_{k-1} = \\ & \bar{V}_N - \gamma(1-\gamma)w^2 \left(\frac{1-\delta^N}{1-\delta} (1-\gamma) + \gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \right) = \end{aligned}$$

QED

Proposition 2 *The mechanism which maximizes (P) subject to (ICL) and (ICH) is characterized by*

$$\begin{aligned} q_L(h_{k-1}) &= \mu_L + (1-\gamma)\omega_N \\ q_H(h_{k-1}) &= \mu_H - \gamma\omega_N \Lambda(h_{k-1}) \end{aligned}$$

$$\begin{aligned} \alpha &= \int \frac{f(x|\phi_L)^2}{f(x|\phi_H)} dx \\ \omega_N &= \frac{\int_X (1-\alpha\delta)(1-\delta^N)}{\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\alpha\delta)(1-\delta^N)} (\mu_H - \mu_L) \end{aligned}$$

provided α is finite. The planner's payoff V_N , and the advocate's payoff U_N are given by

$$V_N = \bar{V}_N - \frac{1-\delta^N}{1-\delta} \omega_N (\gamma - \gamma^2) (\mu_H - \mu_L)$$

$$U_N = (\gamma \mu_L + (1 - \gamma) \mu_H) \frac{1-\delta^N}{1-\delta}$$

Proof: We first derive the optimal mechanism in a relaxed problem, imposing only constraint (ICL). Using Lemma 2, we show that the solution satisfies constraint (ICH). Finally we compute the planner's and advocate's payoffs. Therefore, consider the following relaxed problem

$$\max_{q(h_{k-1})} \quad -\hat{\gamma} \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) \left[\int_X f(x|\phi_L) (q_L(h_{k-1}) - x)^2 dx \right] dh_{k-1}$$

$$- (1 - \hat{\gamma}) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) \left[\int_X f(x|\phi_H) (q_H(h_{k-1}) - x)^2 dx \right] dh_{k-1}$$

subject to

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) q_L(h_{k-1}) dh_{k-1} = \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) q_H(h_{k-1}) dh_{k-1} \quad (\text{ICL})$$

The objective problem is strictly concave, and the constraint is linear. The solution is therefore the unique stationary point of the Lagrangian. Simplifying the planner's objective function slightly, leads to the following Lagrangian:

$$\gamma \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (-q_L(h_{k-1})^2 + 2\mu_L q_L(h_{k-1}) - \eta_L) dh_{k-1}$$

$$+ (1 - \gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-q_H(h_{k-1})^2 + 2\mu_H q_H(h_{k-1}) - \eta_H) dh_{k-1}$$

$$+ \lambda \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) [q_L(h_{k-1}) - q_H(h_{k-1})] dh_{k-1}$$

The stationarity conditions are therefore,

$$q_L(h_{k-1}) : \quad \delta^{k-1} f(h_{k-1}|\phi_L) \{2\gamma(\mu_L - q_L(h_{k-1})) + \lambda\} = 0$$

$$q_H(h_{k-1}) : \quad \delta^{k-1} f(h_{k-1}|\phi_H) \left\{ 2(1-\gamma)(\mu_H - q_H(h_{k-1})) - \lambda \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \right\} = 0$$

$$\lambda : \quad \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} = 0$$

These first order conditions imply that:

$$q_L(h_{k-1}) = \mu_L + \frac{\lambda}{2\gamma}$$

$$q_H(h_{k-1}) = \mu_H - \frac{\lambda}{2(1-\gamma)} \Lambda(h_{k-1})$$

Let

$$\omega_N = \frac{\lambda}{2\gamma(1-\gamma)}$$

Then the first order conditions simplify to

$$q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_N$$

$$q_H(h_{k-1}) = \mu_H - \gamma\omega_N \Lambda(h_{k-1})$$

To find the optimal value of ω_N , we solve the remaining condition:

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} = 0$$

Substituting

$$\frac{q_L (1 - \delta^N)}{1 - \delta} = \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) q_H(h_{k-1}) dh_{k-1}$$

$$\begin{aligned} \frac{q_L (1 - \delta^N)}{1 - \delta} &= \mu_H \frac{1 - \delta^N}{1 - \delta} - \gamma \omega_N \sum_{k=1}^N (\alpha \delta)^{k-1} \\ &= \mu_H \frac{1 - \delta^N}{1 - \delta} - \gamma \omega_N \frac{1 - (\alpha \delta)^N}{1 - \alpha \delta} \end{aligned}$$

$$\omega_N = \frac{(1 - \alpha \delta) (1 - \delta^N)}{\gamma (1 - \delta) (1 - (\alpha \delta)^N) + (1 - \gamma) (1 - \alpha \delta) (1 - \delta^N)} (\mu_H - \mu_L)$$

By Lemma 2, to establish that this mechanism satisfies (ICH) it is enough to verify that $\omega_N \leq (\mu_H - \mu_L)$.

$$\omega_N \leq \mu_H - \mu_L \quad \Leftrightarrow$$

$$\frac{(1 - \alpha \delta) (1 - \delta^N)}{\gamma (1 - \delta) (1 - (\alpha \delta)^N) + (1 - \gamma) (1 - \alpha \delta) (1 - \delta^N)} \leq 1 \quad \Leftrightarrow$$

$$\gamma (1 - \alpha \delta) (1 - \delta^N) \leq \gamma (1 - \delta) (1 - (\alpha \delta)^N) \quad \Leftrightarrow$$

$$\frac{1 - \delta^N}{1 - \delta} \leq \frac{1 - (\alpha \delta)^N}{1 - \alpha \delta} \quad \Leftrightarrow$$

$$\sum_{k=1}^N \delta^{k-1} \leq \sum_{k=1}^N (\alpha \delta)^{k-1}$$

where the last line follows because $\alpha \geq 1$. Finally, we calculate the expected payoffs. For the planner apply Lemma 3:

$$\bar{V}_N - \gamma (1 - \gamma) \omega_N^2 \left(\frac{1 - \delta^N}{1 - \delta} (1 - \gamma) + \gamma \frac{1 - (\alpha \delta)^N}{1 - \alpha \delta} \right) =$$

$$\bar{V}_N - \frac{1 - \delta^N}{1 - \delta} (\gamma - \gamma^2) \omega_N (\mu_H - \mu_L)$$

For the advocate:

$$\begin{aligned} u_H &= \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) q_H(h_{k-1}) dh_{k-1} \\ &= (\mu_H + (1 - \gamma)\omega) \frac{1 - \delta^N}{1 - \delta} \end{aligned}$$

$$\begin{aligned} u_L &= \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) q_L(h_{k-1}) dh_{k-1} \\ &= (\mu_L - \gamma\omega) \frac{1 - \delta^N}{1 - \delta} \end{aligned}$$

$$\begin{aligned} U_N &= \gamma u_L + (1 - \gamma) u_H \\ &= (\gamma\mu_L + (1 - \gamma)\mu_H) \left(\frac{1 - \delta^N}{1 - \delta} \right) \end{aligned}$$

QED

Proposition 3 *For small costs, the mechanism that maximizes (P) subject to (AICH) and (AICL) is given by*

$$\begin{aligned} q_L(h_{k-1}) &= \mu_L + (1 - \gamma)\tilde{\omega}_N \\ q_H(h_{k-1}) &= \mu_H - \gamma\tilde{\omega}_N\Lambda(h_{k-1}) \\ \tilde{\omega}_N &= \frac{\gamma(1-\delta^N)(\mu_H-\mu_L)+c(1-\delta)}{\gamma(1-\delta^N)(\mu_H-\mu_L)}\omega_N \end{aligned}$$

Constraint (AICH) is active at the optimum, and constraint (AICL) is slack. The planner's payoff under this contract is given by

$$V_N^c = \bar{V}_N - \left(\frac{\gamma(1-\delta^N)(\mu_H-\mu_L)+c(1-\delta)}{\gamma(1-\delta^N)(\mu_H-\mu_L)} \right)^2 \left(\frac{1-\delta^N}{1-\delta} \right) \omega_N (\gamma - \gamma^2) (\mu_H - \mu_L)$$

while the advocate's payoff is $U_N - c$.

Proof: The optimal mechanism is characterized by the solution to the following optimization problem:

$$\begin{aligned} \max_{q_L(\cdot), q_H(\cdot)} \quad & \gamma \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (-q_L(h_{k-1})^2 + 2\mu_L q_L(h_{k-1}) - \eta_L) dh_{k-1} \\ & + (1 - \gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-q_H(h_{k-1})^2 + 2\mu_H q_H(h_{k-1}) - \eta_H) dh_{k-1} \end{aligned}$$

subject to

$$\begin{aligned} \sum_{h_k}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} &\leq -\frac{c}{\gamma} \quad (\text{AICH}) \\ \sum_{h_k}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} &\leq -\frac{c}{1-\gamma} \quad (\text{AICL}) \end{aligned}$$

To formulate these conditions, consider the Lagrangian

$$\begin{aligned} & \gamma \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (-q_L(h_{k-1})^2 + 2\mu_L q_L(h_{k-1}) - \eta_L) dh_{k-1} \\ & + (1 - \gamma) \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (-q_H(h_{k-1})^2 + 2\mu_H q_H(h_{k-1}) - \eta_H) dh_{k-1} \\ & - \lambda_H \left[\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} + \frac{c}{\gamma} \right] \\ & - \lambda_L \left[\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} + \frac{c}{1-\gamma} \right] \end{aligned}$$

The stationarity conditions are:

$$\begin{aligned} q_L(h_{k-1}) &= \mu_L + \frac{\lambda_H}{2\gamma} - \frac{\lambda_L}{2\gamma} \frac{f(h_{k-1}|\phi_H)}{f(h_{k-1}|\phi_L)} \\ q_H(h_{k-1}) &= \mu_H - \frac{\lambda_H}{2(1-\gamma)} \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} + \frac{\lambda_L}{2(1-\gamma)} \end{aligned}$$

Making our favorite substitution:

$$\lambda_H = 2\gamma(1 - \gamma)\omega_H$$

$$\lambda_L = 2\gamma(1 - \gamma)\omega_L$$

The complete set of Kuhn Tucker conditions can be written:

$$\text{Stat: } q_L(h_{k-1}) = \mu_L + (1 - \gamma)\omega_H - (1 - \gamma)\omega_L \frac{f(h_{k-1}|\phi_H)}{f(h_{k-1}|\phi_L)}$$

$$q_H(h_{k-1}) = \mu_H - \gamma\omega_H \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} + \gamma\omega_L$$

$$\text{CS: } \begin{aligned} & \omega_H \left[\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} + \frac{c}{\gamma} \right] = 0 \\ & \omega_L \left[\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} + \frac{c}{1-\gamma} \right] = 0 \end{aligned}$$

$$\text{PF: } \begin{aligned} & \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} \leq -\frac{c}{\gamma} \\ & \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} \leq -\frac{c}{1-\gamma} \end{aligned}$$

$$\text{DF } \omega_H \geq 0, \omega_L \geq 0$$

Consider first the $\omega_H > 0, \omega_L = 0$ (AICH Active, AICL Slack). In this case, the most pressing deviation that must be prevented is that of an uninformed advocate who represents himself as informed, and learning that the true distribution is ϕ_H . This case is most similar to the zero-cost case. Under this assumption, the KT conditions become

$$\text{Stat: } \begin{aligned} q_L(h_{k-1}) &= \mu_L + (1 - \gamma)\omega_H \\ q_H(h_{k-1}) &= \mu_H - \gamma\omega_H \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \end{aligned}$$

$$\text{CS: } \sum_{h_k}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} = -\frac{c}{\gamma}$$

$$\text{PF: } \sum_{h_k}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} \leq -\frac{c}{1-\gamma}$$

$$\text{DF } \omega_H \geq 0$$

Condition (PF) reduces as follows:

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) (q_L(h_{k-1}) - q_H(h_{k-1})) dh_{k-1} \leq -\frac{c}{1-\gamma}$$

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) \left(\mu_L + (1-\gamma)\omega_H - \mu_H + \gamma\omega_H \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \right) dh_{k-1} \leq -\frac{c}{1-\gamma}$$

$$\sum_{k=1}^N \delta^{k-1} (\omega_H - (\mu_H - \mu_L)) \leq -\frac{c}{1-\gamma}$$

$$\omega_H \leq (\mu_H - \mu_L) - \frac{c}{1-\gamma} \left(\frac{1-\delta}{1-\delta^N} \right)$$

Condition (CS) reduces as follows:

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H(h_{k-1}) - q_L(h_{k-1})) dh_{k-1} = -\frac{c}{\gamma}$$

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) \left(\mu_H - \gamma \omega_H \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} - (\mu_L + (1-\gamma)\omega_H) \right) dh_{k-1} = -\frac{c}{\gamma}$$

$$\sum_{k=1}^N \delta^{k-1} (\mu_H - \gamma \omega_H \alpha^{k-1} - \mu_L - (1-\gamma)\omega_H) = -\frac{c}{\gamma}$$

$$(\mu_H - \mu_L) \frac{1 - \delta^N}{1 - \delta} - \omega_H \left(\gamma \frac{1 - (\alpha\delta)^N}{1 - (\alpha\delta)} + (1-\gamma) \frac{1 - (\delta)^N}{1 - (\delta)} \right) = -\frac{c}{\gamma}$$

$$\omega_H = \frac{(\mu_H - \mu_L) \frac{1 - \delta^N}{1 - \delta} + \frac{c}{\gamma}}{\left(\gamma \frac{1 - (\alpha\delta)^N}{1 - (\alpha\delta)} + (1-\gamma) \frac{1 - (\delta)^N}{1 - (\delta)} \right)}$$

$$\omega_H = \omega_N \left(\frac{\gamma (1 - \delta^N) (\mu_H - \mu_L) + c(1 - \delta)}{\gamma (1 - \delta^N) (\mu_H - \mu_L)} \right)$$

Obviously condition (DF) holds. Therefore, provided that

$$\omega_N \left(\frac{\gamma (1 - \delta^N) (\mu_H - \mu_L) + c(1 - \delta)}{\gamma (1 - \delta^N) (\mu_H - \mu_L)} \right) \leq (\mu_H - \mu_L) - \frac{c}{1 - \gamma} \left(\frac{1 - \delta}{1 - \delta^N} \right)$$

we have a solution. As established previously, when $c = 0$, this condition is satisfied. The left hand side of the inequality grows with c , while the right hand side shrinks. Both sides are linear in c , and therefore there is only one intersection. Call the intersection \tilde{c} . For $c \leq \tilde{c}$ the solution presented holds. Applying Lemma 3 yields:

$$\begin{aligned}
V_N^C &= \bar{V}_N - (\gamma - \gamma^2) \omega_H^2 \left(\frac{1 - \delta^N}{1 - \delta} (1 - \gamma) + \gamma \frac{1 - (\alpha\delta)^N}{1 - \alpha\delta} \right) \\
&= \bar{V}_N - \left(\frac{\gamma (1 - \delta^N) (\mu_H - \mu_L) + c(1 - \delta)}{\gamma (1 - \delta^N) (\mu_H - \mu_L)} \right)^2 \frac{1 - \delta^N}{1 - \delta} (\gamma - \gamma^2) \omega_N (\mu_H - \mu_L)
\end{aligned}$$

This proves Proposition 3

QED

Proposition 4 *The mechanism that maximizes (PD) subject to (ICD) is given by*

$$\begin{aligned}
q_L^a &= \mu_L + (1 - \gamma) \theta_N \\
q_H^a(h_{k-1}) &= \mu_H - \gamma \Lambda(h_{k-1}) (a \theta_N) \\
q_L^i(h_{k-1}) &= \mu_L \\
q_H^i(h_{k-1}) &= q_H^a(h_{k-1}) \\
\theta_N &= \frac{(1 - \delta^N)(1 - \alpha\delta)}{a\gamma(1 - \delta)(1 - (\alpha\delta)^N) + (1 - \gamma)(1 - \delta^N)(1 - \alpha\delta)} (\mu_H - \mu_L)
\end{aligned}$$

If the advisor is an advocate, his expected payoff under this mechanism is given by $U_N + \frac{1 - \delta^N}{1 - \delta} \gamma (1 - \gamma) (1 - a) \theta_N$.

Proof: We first consider a relaxed problem in which we impose only the following constraints:

$$\begin{aligned}
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) (q_H^i(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &\leq 0 \\
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) (q_H^a(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &\leq 0
\end{aligned}$$

This problem has a concave objective function and linear constraints. The KT conditions therefore characterize the solution. Making substitutions

$$\begin{aligned}
\frac{\lambda_1}{2(1 - a) a (1 - \gamma) \gamma} &= \omega_1 \\
\frac{\lambda_2}{2a\gamma(1 - \gamma)} &= \omega_2
\end{aligned}$$

and assuming both constraints are binding, the KT conditions reduce to

$$\begin{aligned}
q_L^i(h_{k-1}) &= \mu_L \\
q_H^i(h_{k-1}) &= \mu_H - a \left(\gamma \omega_1 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \right) \\
q_L^a(h_{k-1}) &= \mu_L + (1-\gamma)\omega_2 + (1-\gamma)(1-a)\omega_1 \\
q_H^a(h_{k-1}) &= \mu_H - \gamma \omega_2 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \\
\omega_1, \omega_2 &> 0
\end{aligned}$$

$$\begin{aligned}
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H^i(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &= 0 \\
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H^a(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &= 0
\end{aligned}$$

Simplifying constraint 1)

$$\begin{aligned}
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H^i(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &= 0 \\
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) \left(\mu_H - a\gamma\omega_1 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} - \mu_L - (1-\gamma)\omega_2 - (1-\gamma)(1-a)\omega_1 \right) dh_{k-1} &= 0 \\
\sum_{k=1}^N \delta^{k-1} (\mu_H - a\gamma\omega_1\alpha^{k-1} - \mu_L - (1-\gamma)\omega_2 - (1-\gamma)(1-a)\omega_1) &= 0 \\
\left(-(1-\gamma)(1-a) \frac{1-\delta^N}{1-\delta} - a\gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \right) \omega_1 - (1-\gamma) \frac{1-\delta^N}{1-\delta} \omega_2 + \frac{1-\delta^N}{1-\delta} (\mu_H - \mu_L) &= 0
\end{aligned}$$

Simplifying constraint 2)

$$\begin{aligned}
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) (q_H^a(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} &= 0 \\
\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L) \left(\mu_H - \gamma\omega_2 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} - (\mu_L + (1-\gamma)\omega_2 + (1-\gamma)(1-a)\omega_1) \right) dh_{k-1} &= 0 \\
\sum_{k=1}^N \delta^{k-1} (\mu_H - \gamma\omega_2\alpha^{k-1} + (1-a)\omega_3 - \mu_L - (1-\gamma)\omega_2 - (1-\gamma)(1-a)\omega_1) &= 0 \\
-\frac{1-\delta^N}{1-\delta} (1-\gamma)(1-a)\omega_1 - \left((1-\gamma) \frac{1-\delta^N}{1-\delta} + \gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \right) \omega_2 + \frac{1-\delta^N}{1-\delta} (\mu_H - \mu_L) &= 0
\end{aligned}$$

It is straightforward to check that:

$$\begin{aligned}
\omega_1 &= \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \\
\omega_2 &= a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L)
\end{aligned}$$

and both of these values are positive. The solution to the relaxed problem is therefore:

$$\begin{aligned}
q_L^i(h_{k-1}) &= \mu_L \\
q_H^i(h_{k-1}) &= \mu_H - \gamma \Lambda(h_{k-1}) \left(a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \right) \\
q_L^a(h_{k-1}) &= \mu_L + (1-\gamma) \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \\
q_H^a(h_{k-1}) &= \mu_H - \gamma \Lambda(h_{k-1}) a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L)
\end{aligned}$$

As in the proposition: The other constraints need to be verified. Because (H, i) is treated the same as a report of (H, a) a ϕ_H -advocate has no gain from reporting (H, i) . For the same reason, the incentive constraint

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) (q_H^a(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} = 0$$

Also ensures that

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) (q_H^i(h_{k-1}) - q_L^a(h_{k-1})) dh_{k-1} = 0$$

If a ϕ_H advocate were to report L , he would certainly prefer to report (L, a) than (L, i) .

Therefore, if

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (q_L^a - q_H^a(h_{k-1})) dh_{k-1} \leq 0$$

then it also follows that

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (q_L^i - q_H^a(h_{k-1})) dh_{k-1} \leq 0$$

What remains to check, then is

$$\sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) (q_L^a - q_H^a(h_{k-1})) dh_{k-1} \leq 0$$

To verify this condition, note

$$\begin{aligned}
& \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) q_H^a(h_{k-1}) dh_{k-1} = \\
& \sum_{k=1}^N \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) \left(\mu_H - \gamma \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \left(a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \right) \right) dh_{k-1} = \\
& \sum_{k=1}^N \delta^{k-1} \left(\mu_H - \gamma \left(a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \right) \right)
\end{aligned}$$

For the inequality to be valid,

$$\begin{aligned}
& \mu_H - \gamma \left(a \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \right) \geq \\
& \mu_L + (1-\gamma) \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} (\mu_H - \mu_L) \\
& 1 \geq (1-\gamma) \frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} + \gamma a \left(\frac{(1-\delta^N)(1-\alpha\delta)}{a\gamma(1-\delta)(1-(\alpha\delta)^N) + (1-\gamma)(1-\delta^N)(1-\alpha\delta)} \right) \\
& 1 \geq \frac{\frac{1-\delta^N}{1-\delta} (a\gamma + (1-\gamma))}{a\gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} + (1-\gamma) \frac{1-\delta^N}{1-\delta}}
\end{aligned}$$

This holds because

$$\alpha \geq 1 \rightarrow \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \geq \frac{1-\delta^N}{1-\delta}$$

To verify the advocate's payoff, note that the advocate is offered mechanism:

$$\begin{aligned}
q_L^a &= \mu_L + (1-\gamma) a\theta + (1-\gamma) (1-a)\theta \\
q_H^a(h_{k-1}) &= \mu_H - \gamma \Lambda(h_{k-1})(a\theta)
\end{aligned}$$

As previously established, a mechanism

$$\begin{aligned}
q_L^a &= \mu_L + (1-\gamma) a\theta \\
q_H^a(h_{k-1}) &= \mu_H - \gamma \Lambda(h_{k-1})(a\theta)
\end{aligned}$$

leaves the advocate with payoff U_N . Therefore, if the advisor is an advocate, and draws L , (probability γ) the planner selects a higher action in each period than under the no-rent mechanism. The advocate's expected payoff is therefore

$$U_N + \left(\frac{1-\delta^N}{1-\delta} \right) \gamma (1-\gamma) (1-a)\theta$$

QED