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Abstract

This article investigates the consequences of population aging for long-run economic growth perspectives. We introduce age specific heterogeneity of households into a model of research and development (R&D) based technological change. We show that the framework incorporates two standard specifications as special cases: endogenous growth models with scale effects and semi-endogenous growth models without scale effects. The introduction of an age structured population implies that aggregate laws of motion for capital and consumption have to be obtained by integrating over different cohorts. It is analytically shown that these laws of motion depend on the underlying demographic assumptions. Our results are that (i) increases in longevity have positive effects on per capita output growth, (ii) decreases in fertility have negative effects on per capita output growth, (iii) the longevity effect dominates the fertility effect in case of endogenous growth models and (iv) population aging fosters long-run growth in endogenous growth models, while the converse holds true in semi-endogenous growth frameworks.

JEL classification: O41, J10, C61

Keywords: population aging, endogenous technological change, long-run economic growth

1 Introduction

Most recently, population aging in industrialized countries has been identified as one central issue regarding future economic development (see for example Bloom et al., 2008; The Economist, 2009). While declining fertility – even far below the replacement level – triggers increases in the mean age of a certain population and slows down population growth, decreasing old age mortality allows individuals to enjoy the benefits of retirement for longer time periods (cf. United Nations, 2007; Eurostat, 2009). The consequences of these developments are expected to be huge. To mention only the most well known examples: support ratios will decline such that fewer and fewer workers will have to carry the burden of financing more and more retirees (see for example Gertler, 1999; Gruescu, 2007); overall productivity levels will change because individual workers have age specific productivity profiles and age decompositions of western societies will shift (see Skirbekk, 2008, for an overview); the savings behavior of individuals will change because they expect to live longer (see for example Heijdra and Ligthart, 2006; Heijdra and Romp, 2008). However, as regards the implications of population aging for per capita output growth in a setting with diminishing marginal products of capital, there are only transient effects of changing support ratios, changing saving behavior of households and changing aggregate productivity profiles. The reason is that a shift from high to low fertility cannot lead to a *permanently changing* age decomposition of a certain population (cf. Preston et al., 2001) and the induced change in the savings behavior of households has only level effects on per capita output (cf. Ramsey, 1928; Solow, 1956).

In this paper we concentrate on the implications of population aging for per capita output *growth* over a long time horizon. Since technological progress has been identified as the main driving force behind economic prosperity (see for example Romer, 1990), we are particularly interested in the effects of changing age decompositions on research and development (R&D) intensity. Therefore the natural model class to examine our research question are endogenous and semi-endogenous growth frameworks, where the R&D effort is determined as the outcome of market forces within a general equilibrium framework assuming utility maximizing households and profit maximizing firms.

Two other branches of the literature closely relate to our efforts. The first one (see for example Reinhart, 1999; Futagami and Nakajima, 2001; Petrucci, 2002) basically follows the Romer (1986) assumption that there are knowledge spillovers in the production process and hence there are no diminishing returns of capital in the aggregate production function. This assumption allows them to even draw conclusions on the effects of demographically induced changes in individual savings behavior on long-run economic growth performance. A very interesting recent contribution (Schneider and Winkler, 2010) uses this framework to endogenize the rate of mortality and and to analyze the welfare implications of individual health investments. However, the knowledge spillover model of Romer (1986) has been criticized because empirical evidence points towards diminishing marginal products of capital (cf. Mankiw et al., 1992). Furthermore, one cannot analyze the effects of aging on purposeful R&D within such a framework and, as we will see later on, the transmission mechanism of the effects of aging on economic growth differs to our approach because we allow for an age dependent interest rate.

The second related branch to our work (see for example Kalemli-Ozcan et al., 2000; Cervellati and Sunde, 2005; Hazan and Zoabi, 2006) focuses on the implications of population aging on human capital accumulation and basically states that an increase in the life expectancy of individuals renders investments into human capital more profitable. Consequently, human capital accumulation increases which fosters economic growth via the particular link these models establish between human capital accumulation and economic development[1]. However, also these models do not consider the effects of aging on purposeful R&D and therefore the transmission mechanism of aging on economic growth is by its very nature different to ours.

Endogenous growth models with purposeful R&D investments (see for example Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) state that, aside from other influences, the population size of a certain country is crucial for long-run economic development. Larger countries are able to grow faster because there are more scientists to employ and these countries have larger markets such that profit opportunities of firms engaging in R&D are larger. The corresponding effect is called the scale effect

[1]There are various channels by which human capital accumulation can foster economic growth (see for example Lucas, 1988; Galor and Weil, 2000).

which was questioned by Jones (1995) because it had not been supported by empirical evidence. In setting up a scale-free model of technological change, Jones (1995) paved the way for semi-endogenous growth models (see also for example Kortum, 1997; Segerström, 1999), where long-run economic performance is affected by population *growth* rather than population *size*. The basic idea of semi-endogenous growth models is that developing a constant share of new technologies becomes more and more complex with an expanding technological frontier. Consequently, ever more resources have to be devoted to R&D activities in order to sustain a certain pace of technological progress.

Although the described models examine the effects of changes in demographic patterns as represented by population size and population growth, they remain silent when it comes to the consequences of population aging because they assume that economies are populated by representative individuals who live forever. We introduce age dependent heterogeneity of individuals by generalizing these frameworks to account for finite individual planning horizons and overlapping generations in the spirit of Blanchard (1985) in case of the endogenous growth paradigm and in the spirit of Buiter (1988) in case of the semi-endogenous growth paradigm. In doing so we assume that individuals do not live forever, instead they have to face a certain probability of death at each instant. The standard endogenous and semi-endogenous growth models are then special cases with the probability of death being equal to zero.

Our results show that allowing for a more realistic demographic structure in traditional endogenous and semi-endogenous growth models is desirable because we can disentangle the growth effects of a changing population size from those of a changing population age structure and thereby show that the effects of population aging differ between the endogenous growth paradigm and the semi-endogenous growth paradigm. Furthermore we can show that the population age structure has a crucial impact on the interest rate and therefore on the R&D intensity within the Romer (1990) framework.

The paper proceeds as follows: Section 2 describes a model that nests the Romer (1990) and the Jones (1995) frameworks as special cases and features a richer demographic structure. Section 3 examines the effects of demographic change for long-run economic growth in both types of models. Finally, section 4 draws conclusions and highlights scope for further research.

2 The model

This section characterizes the basic model of endogenous R&D which relies on horizontal innovations, i.e. on the development of new product varieties[2]. It nests the Romer (1990) framework with strong spillovers in the research sector and a constant population size as well as the Jones (1995) framework with weaker spillovers in the research sector and a growing population size as special cases (cf. Strulik, 2009).

2.1 Basic assumptions

The basic structure of our model economy is that there are three sectors: final goods production, intermediate goods production and R&D. Altogether the economy has two productive factors at its disposal: capital and labor. Labor and machines are used to produce final goods in a perfectly competitive market, capital and blueprints are used in the Dixit and Stiglitz (1977) monopolistically competitive intermediate goods sector to produce machines and labor is used to produce blueprints in the perfectly competitive R&D sector.

In contrast to the representative agent assumption, we introduce overlapping generations in the spirit of Blanchard (1985) to the Romer (1990) case, since there the population size has to stay constant, and in the spirit of Buiter (1988) to the Jones (1995) case, since there the population size has to grow. First of all we assume that the total population of an economy consists of different cohorts that are distinguishable by their date of birth denoted as t_0. Each cohort consists of a measure $N(t_0, t)$ of individuals at a certain point in time $t > t_0$. In addition, we assume that individuals have to face a constant risk of death at each instant which we denote as μ. Due to the law of large numbers, this expression also denotes the fraction of individuals dying at each instant. In the Romer (1990) case the population does not grow and therefore the period fertility rate[3] is also equal to μ, whereas in the Jones (1995) case the population grows at rate $n = \beta - \mu$, where $\beta > \mu$ denotes the period fertility rate. Note that demographic change can then be analyzed by changing mortality and fertility separately in the Jones (1995)

[2]Using a model with vertical innovations would not change the results.

[3]In our demographic setting the period fertility rate is equivalent to the birth rate (cf. Preston et al., 2001)

case, while in the Romer (1990) case only the impacts of contemporaneous proportional changes in both demographic parameters can be analyzed. In the Jones (1995) framework decreases in fertility lead to both a slowdown of population growth *and* to population aging, while decreases in mortality *only* increase the population growth rate and have *no effect* on the aggregate age decomposition (cf. Preston et al., 2001). In the Romer (1990) framework contemporaneous proportional decreases in both mortality and fertility lead to population aging, while leaving the population size constant.

2.2 Consumption side

Suppressing time subscripts, a certain individual maximizes its discounted stream of lifetime utility

$$U = \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau-t_0)} \log(c) d\tau, \tag{1}$$

where $\rho > 0$ is the subjective time discount rate, the mortality rate $\mu > 0$ augments the subjective time discount rate because individuals who face the risk of death are less likely to postpone consumption into the future to the same extent as in case of no lifetime uncertainty and c refers to individual consumption of the final good. Note that we restrict our attention to the case of logarithmic utility which simplifies the aggregation procedure considerably and allows us to focus on the demographic aspects. Furthermore we implement the assumption of Yaari (1965) that individuals insure themselves against the risk of dying with positive assets by using their whole amount of savings to buy actuarial notes of a fair life-insurance company. A fair life-insurance company basically redistributes wealth of individuals who died among those who survived and therefore the real rate of return on capital is augmented by the mortality rate. Consequently, the wealth constraint of individuals reads

$$\dot{k} = (r + \mu - \delta)k + \hat{w} - c, \tag{2}$$

where k refers to the individual capital stock, r is the rental rate of capital, $\delta > 0$ is the rate at which machines depreciate and $\hat{w}$ represents non-interest income consisting of wage payments and possible lump-sum redistributions. Note that we assume an inelastic labor supply, i.e. each individual supplies all her available labor disregarding the wage rate. The left hand side of the constraint denotes the change in the individual's capital stock, while the right hand side comprises total individual savings, i.e. capital income and non-interest income net of consumption expenditures. Note that this formulation implies that we refer to final goods as numéraire. Carrying out utility maximization subject to the wealth constraint yields the familiar individual Euler equation

$$\frac{\dot{c}}{c} = r - \delta - \rho, \tag{3}$$

stating that consumption expenditure growth is positive if and only if the interest rate, $r-\delta$, exceeds the time discount rate, ρ.[4] However, our economy does not feature only one single representative individual in this setting and we have to use certain aggregation rules to come up with expressions for aggregate consumption expenditure growth as well as laws of motion for aggregate capital. This is done in subsection 2.2.1 for the Romer (1990) case of a constant population and in subsection 2.2.2 for the Jones (1995) case of a growing population.

2.2.1 Aggregation in case of a constant population

In our framework, agents are heterogeneous with respect to age and therefore also with respect to accumulated wealth because older agents have had more time to build up positive assets. In order to get to the law of motion for aggregate capital and to the economy-wide ("aggregate") Euler equation, we have to apply the following rules to integrate over all cohorts alive at time t (cf. Heijdra and van der Ploeg, 2002):

$$K(t) \equiv \int_{-\infty}^{t} k(t_0,t)N(t_0,t)dt_0, \tag{4}$$

$$C(t) \equiv \int_{-\infty}^{t} c(t_0,t)N(t_0,t)dt_0. \tag{5}$$

[4]Note that this Euler equation applies to all individuals disregarding their age. Therefore either all individuals borrow or all individuals save. If all individuals were to borrow, the capital stock would decrease and therefore the capital rental rate would increase until $r-\delta \geq \rho$.

By applying our demographic assumptions for the Romer (1990) case, we can rewrite these rules as

$$C(t) \equiv \mu N \int_{-\infty}^{t} c(t_0,t)e^{\mu(t_0-t)}dt_0, \tag{6}$$

$$K(t) \equiv \mu N \int_{-\infty}^{t} k(t_0,t)e^{\mu(t_0-t)}dt_0 \tag{7}$$

because in case of a constant population size N, each cohort is of size $\mu N e^{\mu(t_0-t)}$ at a certain point in time $t > t_0$[5]. After carrying out the calculations described in the appendix, we arrive at the following expressions for the law of motion of aggregate capital and for the aggregate Euler equation

$$\dot{K} = (r-\delta)K(t) - C(t) + \hat{W}(t), \tag{8}$$

$$\frac{\dot{C}(t)}{C(t)} = r - \rho - \delta - \mu(\rho+\mu)\frac{K(t)}{C(t)}, \tag{9}$$

where we have that $(\rho+\mu)K(t)/C(t) = (C(t) - c(t,t)N)/C(t)$ which we denote by Ω. Note that average consumption in an economy is always higher than consumption of newborns because newborns do not have any accumulated financial wealth yet. Therefore aggregate consumption, $C(t)$, which can be written as the product of average consumption and the population size, is always higher than consumption of the newborns multiplied by the population size, $c(t,t)N$, and hence $\Omega \in [0,1]$. Consequently, *aggregate* consumption expenditure growth will always be lower than *individual* consumption expenditure growth. The reason is that at each instant, a fraction μ of older and therefore wealthier individuals die and they are replaced by poorer newborns. Since the latter can afford less consumption than the former, the turnover of generations slows down aggregate consumption expenditure growth as compared to individual consumption expenditure growth (cf. Heijdra and van der Ploeg, 2002). Regarding the law of motion for aggregate capital, we see that the mortality rate does not show up. The reason is that the life insurance company only redistributes capital between cohorts and does not itself create or subtract capital from the whole economy.

[5]Consequently, we have that $\int_{-\infty}^{t} \mu N e^{\mu(t_0-t)}dt_0 = N$ holds for the total population size at time t and due to our assumption of inelastic labor supply also for the size of the workforce $L \equiv N$.

2.2.2 Aggregation in case of a growing population

In case of the Jones (1995) model, population growth is allowed for. The aggregation rules in such a setting remain the same as in the previous subsection but the demographic assumptions change because the period fertility rate, i.e. the birth rate β, has to exceed the mortality rate μ. Therefore the population grows at rate $n = \beta - \mu$ and we normalize the initial population size to $N(0)$, which is again equivalent to the initial workforce $L(0)$. Altogether we can then write the size of a cohort born at $t_0 < t$ at a certain point in time as

$$N(t_0, t) \quad = \quad \beta L(0) e^{\beta t_0} e^{-\mu t}. \tag{10}$$

Integrating over all cohorts alive yields the population size, i.e. the available amount of labor at time t as

$$L(t) \quad = \quad L(0) e^{(\beta - \mu) t}. \tag{11}$$

Therefore we can define the aggregate capital stock and aggregate consumption according to

$$C(t) \quad \equiv \quad \beta L(0) e^{-\mu t} \int_{-\infty}^{t} c(t_0, t) e^{\beta t_0} dt_0, \tag{12}$$

$$K(t) \quad \equiv \quad \beta L(0) e^{-\mu t} \int_{-\infty}^{t} k(t_0, t) e^{\beta t_0} dt_0. \tag{13}$$

After carrying out the calculations described in the appendix, we arrive at the aggregate law of motion for capital and the aggregate Euler equation

$$\dot{K} \quad = \quad (r - \delta) K(t) - C(t) + \hat{W}(t), \tag{14}$$

$$\frac{\dot{C}}{C} \quad = \quad r - \rho - \delta + \beta \frac{H(t)}{K(t) + H(t)} - \mu, \tag{15}$$

where $H(t)$ refers to aggregate human wealth. If we denote $H(t)/(K(t) + H(t))$ as Ω', we can immediately conclude that $\Omega' \in [0, 1]$ holds and economy-wide consumption expenditure growth differs from individual consumption expenditure growth. Now the argument still holds that an increase in mortality means that older and richer individuals die more frequently and their replacement by newborns without financial wealth leads to a slowdown of aggregate consumption expenditure growth as compared to individual consumption expenditure growth. However, there is an additional effect arising from changes in fertility: A higher fertility rate leads to faster population growth which spurs aggregate consumption expenditure growth as compared to individual consumption expenditure growth. Note that the law of motion for aggregate capital is the same as in case of a constant population size (cf. Buiter, 1988).

2.3 Production side

Now we turn to the production side of our model economies. The final goods sector produces the consumption aggregate with labor and intermediates as inputs. To have a sensible economic interpretation, we refer to intermediate varieties as differentiated machines. The production function of the final goods sector can be written as

$$Y = L_Y^{1-\alpha} \int_0^A x_i^{\alpha} di, \tag{16}$$

where Y represents output of the consumption aggregate, i.e. the gross domestic product (GDP) of a country, L_Y refers to labor used in final goods production, A is the technological frontier, i.e. loosely speaking the "number" of differentiated machines available, x_i is the amount of a certain specific machine i used in final goods production and $\alpha \in [0, 1]$ is the intermediate input share. Profit maximization and the assumption of perfect competition in the final goods sector imply that factors are paid their marginal products

$$w_Y = (1-\alpha)\frac{Y}{L_Y}, \tag{17}$$

$$p_i = \alpha L_Y^{1-\alpha} x_i^{\alpha-1}, \tag{18}$$

where w_Y refers to the wage rate paid in the final goods sector and p_i to prices paid for intermediate inputs.

The intermediate goods sector is monopolistically competitive in the spirit of Dixit and Stiglitz (1977) such that each firm produces one of the differentiated machines. In doing so, it has to purchase one blueprint from the R&D sector and afterwards employ capital as variable input in production. The costs of blueprints represent fixed costs to each firm. Free entry ensures that operating profits equal fixed costs such that overall profits are zero[6]. After an intermediate goods producer has purchased a blueprint, it can transform one unit of capital into one unit of the intermediate good, i.e. we have that $k_i = x_i$. Thus operating profits can be written as

$$\begin{aligned} \pi_i &= p_i k_i - r k_i \\ &= \alpha L_y^{1-\alpha} k_i^{\alpha} - r k_i. \end{aligned} \tag{19}$$

Profit maximization of firms yields prices of machines

$$p_i = \frac{r}{\alpha}, \tag{20}$$

where $1/\alpha$ is the markup over marginal cost (cf. Dixit and Stiglitz, 1977). Note that this holds for all firms, so we can drop the index i from now on due to symmetry. The aggregate capital stock is equal to the amount of all intermediates produced, i.e. $K = Ax$, such that equation (16) becomes

$$Y = (AL_Y)^{1-\alpha} K^{\alpha} \tag{21}$$

and we immediately see that technological progress is labor augmenting.

The R&D sector employs scientists to discover new blueprints. Depending on the productivity of scientists, λ, and the size of technology spillovers, ϕ, the number of blueprints evolves according to

$$\dot{A} = \lambda A^{\phi} L_A, \tag{22}$$

where L_A denotes the amount of scientists employed. Consequently, the technological frontier expands faster if scientists are more productive or technological spillovers are higher[7]. If $\phi = 1$, spillovers are strong enough such that developing a constant fraction of new blueprints does not become ever more difficult as the technological frontier expands. If, in contrast, $\phi < 1$, the spillovers are insufficiently low and developing a constant fraction of new blueprints becomes more and more difficult with an expanding

[6]If positive overall profits were present, new firms would enter the market until these profits had vanished.

[7]Note that we do not allow for the possibility of duplication in the research process for the sake of comparability between the Romer (1990) model and the Jones (1995) model. However, allowing different researchers to develop the same blueprint would not change our results.

technological frontier[8]. In the former case our economy behaves like in the Romer (1990) scenario, whereas in the latter case our economy behaves like in the Jones (1995) scenario. Furthermore, there is perfect competition in the research sector such that firms maximize

$$\max_{L_A} \pi_A = p_A \lambda A^{\phi} L_A - w_A L_A, \tag{23}$$

with π_A being the profit of a firm in the R&D sector and p_A representing the price of a blueprint. The first order condition pins down wages in the research sector to

$$w_A = p_A \lambda A^{\phi}. \tag{24}$$

The interpretation of this equation is straightforward: wages of scientists increase in their productivity as well as in the prices of blueprints. If $\phi = 1$, an expanding technological frontier gradually increases wages of scientists, whereas $\phi < 1$ means that the increases in scientist's wages caused by technological progress become smaller and smaller. Since the wages of workers in the final goods sector linearly increase in A, this implies that being a scientist would become less and less attractive.

2.4 Market clearing

There is perfect labor mobility between sectors, therefore wages of final goods producers and wages of scientists equalize. The reason is that workers in the final goods sector and scientists do not differ with respect to education nor with respect to productivity. Consequently, if wages were higher in one of these two sectors, it would attract workers from the other sector until wages are equal again. Therefore we can insert (17) into (24) to get to following equilibrium condition

$$p_A \lambda A^{\phi} = (1 - \alpha) \frac{Y}{L_Y}. \tag{25}$$

Firms in the R&D sector can charge prices of blueprints that are equal to the present value of operating profits in the intermediate goods sector because there is always a potential entrant who is willing to pay that price due to

[8]This can easily be shown by dividing equation (22) by the technological frontier A.

free entry. Therefore we have

$$p_A = \int_{t_0}^{\infty} e^{-(R(\tau)-R(t_0))}\pi \; d\tau, \tag{26}$$

where $R(t_0) = \int_0^{t_0} (r(s) - \delta) \; ds$, i.e. the discount rate is the market interest rate paid for household's savings. Via the Leibniz rule and the fact that prices of blueprints do not change along a balanced growth path (BGP), we can obtain

$$p_A = \frac{\pi}{r - \delta} \tag{27}$$

such that these prices are equal to operating profits of intermediate goods producers divided by the market interest rate[9]. Next, we obtain profits by using equation (19) as

$$\pi \;\; = \;\; (1-\alpha)\alpha \frac{Y}{A} \tag{28}$$

such that equation (27) becomes

$$p_A = \frac{(1-\alpha)\alpha Y}{(r-\delta)A}. \tag{29}$$

Assuming that labor markets clear, i.e. $L = L_A + L_Y$, we can determine the amount of labor employed in the final goods sector and in the R&D sector by using equation (25):

$$\begin{aligned} L_Y &= \frac{(r-\delta)A^{1-\phi}}{\alpha\lambda}, \\ L_A &= L - \frac{(r-\delta)A^{1-\phi}}{\alpha\lambda}. \end{aligned} \tag{30}$$

The interpretation of these two equations is straightforward: the higher the market interest rate on capital, $r - \delta$, the higher are the opportunity costs of R&D investments and consequently, the lower is the number of scientists in the R&D sector and the higher is the number of workers employed in the final goods sector; the higher the productivity of researchers, λ, the more scientists in the R&D sector and the less workers in the final goods sector are employed; if knowledge spillovers ϕ are insufficiently low to prevent R&D

[9] Note that we cannot analyze transitional dynamics in this framework. Instead, as in Romer (1990), the capital stock is assumed to be on its BGP level right from the beginning.

from becoming ever more complex, an expanding technological frontier A reduces employment of scientists in the R&D sector and increases employment of workers in the final goods sector; finally, an increase in the intermediate share of final output, α, increases the number of scientists in the R&D sector and decreases the number of workers in the final goods sector because production of final output becomes less labor intensive. Inserting (30) into (22) leads to the evolution of technology:

$$\dot{A} \;=\; \lambda A^{\phi} L - \frac{(r-\delta)A}{\alpha}, \tag{31}$$

where we see that the technological frontier expends faster, the larger the population size is. All factors identified above to reduce the amount of scientists employed in the R&D sector also reduce the pace of technological progress. From now on we have to distinguish between the Romer (1990) case, where technological spillovers are strong and the population size is constant, and the Jones (1995) case, where technological spillovers are weaker and the population grows at rate n[10].

3 Effects of demographic change on economic growth

This section is devoted to deriving the per capita growth rates of output along a BGP in the Romer (1990) and the Jones (1995) case and to analyze the effects of demographic change in these different frameworks.

3.1 The BGP growth rate in the Romer (1990) case

After implementing the central assumption $\phi = 1$ of the Romer (1990) model, the growth rate of the economy can be written as

$$g = \lambda L - \frac{r-\delta}{\alpha} \tag{32}$$

because we know that along a BGP we have $\dot{A}/A = \dot{C}/C = \dot{K}/K = g$. To eliminate the endogenous market rate of return on capital we use the

[10]Note that our assumption $\dot{p}_A = 0$ implies a constant interest rate r along the BGP. Therefore we cannot analyze the equilibrium growth rate in the Jones (1995) case by using equation (31). Instead, we use a slightly different approach to calculate the BGP growth rate in the Jones (1995) case which is outlined in the appendix.

aggregate Euler equation for a constant population size to get the following expression

$$r = g + \rho + \delta + \mu(\rho + \mu)\frac{K}{C}. \tag{33}$$

However, in contrast to a setting with a representative infinitely lived agent, there is still an unknown expression to account for, namely K/C. Therefore we rewrite the law of motion of aggregate capital as $\dot{K} = Y - C - \delta K$ such that we get the additional equation

$$g = \frac{r}{\alpha^2} - \frac{C}{K} - \delta, \tag{34}$$

where we used that $Y/K = r/\alpha^2$. Altogether we therefore have three equations to solve for the three unknowns g, r and $\xi = C/K$.[11] Since we are interested in analyzing the BGP growth rate of the economy, we focus our attention on its solution which is

$$\begin{aligned} g_R^{BGP} &= \frac{\alpha(L\lambda(1+\alpha) - \rho - \delta\alpha) + \delta}{2\alpha(\alpha+1)} \\ &- \frac{\sqrt{4\mu(\mu+\rho)\alpha^3 + ((\alpha-1)(\alpha\delta + \delta + L\alpha\lambda) - \alpha\rho)^2}}{2\alpha(\alpha+1)}, \end{aligned} \tag{35}$$

where the subscript refers to the Romer (1990) case. Now we can state the first central result:

Proposition 1. *In case of endogenous growth in the spirit of Romer (1990), increasing longevity has a positive effect on the BGP growth rate of an economy.*

Proof. The derivative of equation (35) with respect to mortality is equal to

$$\frac{\partial g_R^{BGP}}{\partial \mu} = -\frac{\alpha^2(2\mu+\rho)}{(\alpha+1)\sqrt{4\mu(\mu+\rho)\alpha^3 + ((\alpha-1)(\alpha\delta + \delta + L\alpha\lambda) - \alpha\rho)^2}}$$

We know that α, μ and ρ are positive and the second term under the square root in the denominator is always nonnegative. Therefore the whole ex-

[11] We solved the system using Mathematica. The corresponding file is available upon request. Note that there are two solution triples for g, r, and ξ. However, as one of them features a negative ξ, it can be ruled out by economic arguments because neither the aggregate capital stock nor aggregate consumption can become negative. We therefore restrict our attention to the economically meaningful solution triple.

pression is negative and due to the fact that an increase in longevity is represented by a decrease in mortality μ, the proposition holds. $\square$

The intuition for this finding is that a decrease in mortality slows down the turnover of generations and so a lower market interest rate is required to sustain a given growth rate of aggregate consumption expenditures. Due to the fact that future profits of R&D investments are discounted with this market interest rate, the profitability of R&D investments rises. Consequently, R&D efforts increase which fosters long-run growth because intertemporal knowledge spillovers in the Romer (1990) case are high enough for the effect to be sustainable.

3.2 The BGP growth rate in the Jones (1995) case

To come up with the BGP growth rate in the Jones (1995) case denoted as g_J^{BGP}, we search for an expression where the growth rate of technology is constant and carry out the associated calculations in the appendix. This leads us to

$$g_J^{BGP} = \frac{\beta - \mu}{1 - \phi} \tag{36}$$

and therefore we can state the second central result:

Proposition 2. *In case of semi-endogenous growth in the spirit of Jones (1995), increasing longevity raises the BGP growth rate of an economy.*

Proof. The derivative of equation (36) with respect to mortality is equal to

$$\frac{\partial g_J^{BGP}}{\partial \mu} = -\frac{1}{1 - \phi}$$

which is unambiguously negative because $\phi < 1$ is a central assumption in the Jones (1995) case. As an increase in longevity is represented by a decrease in mortality μ, the proposition holds. $\square$

The interpretation for this finding is that a decrease in mortality, while holding fertility constant, leads to an increase in the population growth rate. This represents a permanent increase in the flow of scientists devoted to R&D and therefore a faster growth rate of the number of patents can be sustained. Of course, the same holds true for increasing fertility:

Proposition 3. *In case of semi-endogenous growth in the spirit of Jones (1995), increasing fertility raises the BGP growth rate of an economy.*

Proof. The derivative of equation (36) with respect to fertility is equal to

$$\frac{\partial g_J^{BGP}}{\partial \mu} = \frac{1}{1-\phi}$$

which is unambiguously positive because $\phi < 1$ is a central assumption in the Jones (1995) case. □

The interpretation for this finding is analogous to the interpretation of proposition 2. Increasing fertility, while holding mortality constant, leads to an increase in the population growth rate and therefore to a growing number of scientists devoted to R&D activities.

The interesting fact is that in the Romer (1990) model a decrease in mortality is accompanied by a decrease in fertility. Both effects offset each other with regards to population growth such that the population size stays constant as in the standard Romer (1990) framework. This allows us to conclude that the benefits of decreasing mortality for economic growth overcompensate the drawbacks of decreasing fertility. The reason is that decreasing mortality not only changes the population growth rate but also decreases the market interest rate by which future profits of R&D investments are discounted. This leads to a shift of resources into R&D and consequently fosters per capita output growth. We summarize this finding in the following remark:

Remark 1. *In case of endogenous growth in the spirit of Romer (1990), the benefits of decreasing mortality overcompensate the drawbacks of similar decreases in fertility for long-run economic growth perspectives.*

Furthermore, we know that population aging is described by contemporaneous proportional decreases in fertility and mortality in the Romer (1990) case, whereas population aging is described by decreases in fertility only in the Jones (1995) case. Therefore we have that population aging has a positive impact on long-run economic growth if endogenous growth models are the accurate description of underlying growth processes, whereas the converse holds true for semi-endogenous growth models. We summarize this finding in the following proposition:

Proposition 4. *In case of endogenous growth in the spirit of Romer (1990), population aging has positive impacts on the long-run economic growth rate, while in in case of semi-endogenous growth in the spirit of Jones (1995), population aging has negative impacts on the long-run economic growth rate.*

Proof. The proof follows immediately from propositions 1 and 3 and the fact that population aging is represented by a decrease in μ in the Romer (1990) model and by a decrease in β in the Jones (1995) model. □

Altogether, we have been able to describe some important impacts of demographic change on economic development. In general, decreases in fertility negatively impact upon long-run growth, whereas decreasing mortality fosters long-run growth. The effects of population aging depend on the underlying model used to describe the growth process. While population aging is beneficial in the Romer (1990) case, the converse holds true in a Jones (1995) environment.

4 Conclusions

We set up a model of endogenous technological change that nests the Romer (1990) and the Jones (1995) frameworks. We generalized this model by introducing finite individual planning horizons and thereby allowing for overlapping generations and heterogeneous individuals. Altogether we showed that the underlying demographic assumptions play a crucial role in describing the research and development (R&D) intensity and thereby the long-run growth rates of industrialized economies.

Our results regarding the impacts of demographic change on long-run economic growth perspectives have been the following: (i) decreasing mortality positively affects long-run growth, (ii) decreasing fertility negatively affects long-run growth, (iii) the negative effects of decreases in fertility are overcompensated by the positive effects of decreases in mortality in case of the Romer (1990) model, (iv) population aging is beneficial for long-run economic growth in the Romer (1990) case, whereas it hampers economic growth in the Jones (1995) case.

From an applied perspective, our conclusion is that currently ongoing demographic changes do not necessarily hamper technological progress and therefore economic prosperity. If both demographic parameters fertility and mortality decrease simultaneously, there might only be modest effects on long-run growth. There are some studies that support such a finding (cf. Bloom et al., 2008, 2010). If we believe that the Romer (1990) model is an accurate description of the growth process of western economies, demographic change induced by contemporaneous decreases in fertility and mortality could even be associated with increasing investments into knowledge creation and therefore faster economic growth.

Finally, we can state that there is scope for further research because a constant mortality rate is still at odds with reality so one could try to introduce age dependent mortality rates. Another promising field for additional investigations could be to introduce heterogeneity of researchers with respect to age. These issues are on top of our research agenda.

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Appendix

The individual Euler equation with aging: The current value Hamiltonian is

$$H = \log(c) + \lambda\left[(r+\mu-\delta)k + \hat{w} - c\right].$$

The first order conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial c} &= \frac{1}{c} - \lambda \overset{!}{=} 0 \\
\Rightarrow \frac{1}{c} &= \lambda && \text{(A.1)} \\
\frac{\partial H}{\partial k} &= (r+\mu-\delta)\lambda \overset{!}{=} (\rho+\mu)\lambda - \dot{\lambda} \\
\Rightarrow \dot{\lambda} &= (\rho+\delta-r)\lambda. && \text{(A.2)}
\end{aligned}$$

Taking the time derivative of equation (A.1) and plugging it into equation (A.2) yields

$$\frac{\dot{c}}{c} = r - \rho - \delta$$

which is the individual Euler equation.

Aggregate capital and aggregate consumption in the Romer (1990) case: Following Heijdra and van der Ploeg (2002) and differentiating equations (6) and (7) with respect to time yields

$$\begin{aligned}
\dot{C}(t) &= \mu N\left[\int_{-\infty}^{t} \dot{c}(t_0,t)e^{\mu(t_0-t)}dt_0 - \mu\int_{-\infty}^{t} c(t_0,t)e^{\mu(t_0-t)}dt_0\right] + \mu N c(t,t) - 0 \\
&= \mu N c(t,t) - \mu C(t) + \mu N\int_{-\infty}^{t} \dot{c}(t_0,t)e^{-\mu(t-t_0)}dt_0 && \text{(A.3)} \\
\dot{K}(t) &= \mu N\left[\int_{-\infty}^{t} \dot{k}(t_0,t)e^{\mu(t_0-t)}dt_0 - \mu\int_{-\infty}^{t} k(t_0,t)e^{\mu(t_0-t)}dt_0\right] + \mu N k(t,t) - 0 \\
&= \mu N \underbrace{k(t,t)}_{=0} - \mu K(t) + \mu N\int_{-\infty}^{t} \dot{k}(t_0,t)e^{-\mu(t-t_0)}dt_0. && \text{(A.4)}
\end{aligned}$$

From equation (2) it follows that

$$\begin{aligned}
\dot{K}(t) &= -\mu K(t) + \mu N \int_{-\infty}^{t} \left[(r+\mu-\delta)k(t_0,t) + \hat{w}(t) - c(t_0,t)\right] e^{-\mu(t-t_0)} dt_0 \\
&= -\mu K(t) + (r+\mu-\delta)\mu N \int_{-\infty}^{t} k(t_0,t) e^{-\mu(t-t_0)} dt_0 \\
&\quad -\mu N \int_{-\infty}^{t} c(t_0,t) e^{-\mu(t-t_0)} dt_0 + N \left(\frac{\mu \hat{w} e^{-\mu(t-t_0)}}{\mu}\right)_{-\infty}^{t} \\
&= -\mu K(t) + (r+\mu-\delta)K(t) - C(t) + \hat{W}(t) \\
&= (r-\delta)K(t) - C(t) + \hat{W}(t)
\end{aligned}$$

which is the aggregate law of motion for capital. Reformulating an agent's optimization problem subject to its lifetime budget restriction, stating that the present value of lifetime consumption expenditures have to be equal to the present value of lifetime non-interest income plus initial assets, yields the optimization problem

$$\begin{aligned}
\max_{c(t_0,\tau)} \quad U &= \int_{t}^{\infty} e^{(\rho+\mu)(t-\tau)} \log(c(t_0,\tau)) d\tau \\
s.t. \qquad & k(t_0,t) + \int_{t}^{\infty} \hat{w}(\tau) e^{-R^A(t,\tau)} d\tau = \int_{t}^{\infty} c(t_0,\tau) e^{-R^A(t,\tau)} d\tau,
\end{aligned} \tag{A.5}$$

where $R^A(\tau,t) = \int_t^\tau (r(s)+\mu-\delta)ds$. The FOC to this optimization problem is

$$\frac{1}{c(t_0,\tau)} e^{(\rho+\mu)(t-\tau)} = \lambda(t) e^{-R^A(t,\tau)}.$$

In period ($\tau = t$) we have

$$c(t_0,t) = \frac{1}{\lambda(t)}.$$

Therefore we can write

$$\begin{aligned}
\frac{1}{c(t_0,\tau)} e^{(\rho+\mu)(t-\tau)} &= \frac{1}{c(t_0,t)} e^{-R^A(t,\tau)} \\
c(t_0,t) e^{(\rho+\mu)(t-\tau)} &= c(t_0,\tau) e^{-R^A(t,\tau)}.
\end{aligned}$$

Integrating and using equation (A.5) yields

$$\begin{aligned}
\int_t^\infty c(t_0,t)e^{(\rho+\mu)(t-\tau)}d\tau &= \int_t^\infty c(t_0,\tau)e^{-R^A(t,\tau)}d\tau \\
\frac{c(t_0,t)}{\rho+\mu}\left[-e^{(\rho+\mu)(t-\tau)}\right]_t^\infty &= k(t_0,t) + \underbrace{\int_t^\infty \hat{w}(\tau)e^{-R^A(t,\tau)}d\tau}_{h(t)} \\
\Rightarrow c(t_0,t) &= (\rho+\mu)\left[k(t_0,t)+h(t)\right], \qquad \text{(A.6)}
\end{aligned}$$

where h refers to human wealth, i.e. non-interest wealth, of individuals. Human wealth does not depend on the date of birth because productivity and lump-sum transfers are age independent. The above calculations show that optimal consumption in the planning period is proportional to total wealth with a marginal propensity to consume of $\rho + \mu$. Aggregate consumption evolves according to

$$\begin{aligned}
C(t) &\equiv \mu N \int_{-\infty}^t c(t_0,t)e^{\mu(t_0-t)}dt_0 \\
&= \mu N \int_{-\infty}^t e^{\mu(t_0-t)}(\rho+\mu)\left[k(t_0,t)+h(t)\right]dt_0 \\
&= (\rho+\mu)\left[K(t)+H(t)\right]. \qquad \text{(A.7)}
\end{aligned}$$

Note that newborns do not own capital because there are no bequests. Therefore

$$c(t,t) = (\rho+\mu)h(t) \qquad \text{(A.8)}$$

holds for each newborn individual. Putting equations (3), (A.3), (A.7) and (A.8) together yields

$$\begin{aligned}
\dot{C}(t) &= \mu(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right] + \\
&\quad \mu N \int_{-\infty}^{t} (r-\rho-\delta)c(t_0,t)e^{-\mu(t-t_0)}dt_0 \\
&= \mu(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right] + (r-\rho-\delta)C(t) \\
\Rightarrow \frac{\dot{C}(t)}{C(t)} &= r-\rho-\delta + \frac{\mu(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right]}{C(t)} \\
&= r-\rho-\delta-\mu(\rho+\mu)\frac{K(t)}{C(t)} \\
&= r-\rho-\delta-\mu\,\underbrace{\frac{C(t)-c(t,t)N}{C(t)}}_{\in(0,1)}
\end{aligned}$$

which is the aggregate Euler equation that differs from the individual Euler equation by the term $-\mu\frac{C(t)-c(t,t)N}{C(t)}$.

Aggregate capital and aggregate consumption in the Jones (1995) case: Using our demographic assumptions we can write the size of a cohort born at $t_0 < t$ at time t as

$$\begin{aligned}
N(t_0,t) &= \beta L(t_0)e^{-\mu(t-t_0)} \\
&= \beta L(0)e^{nt_0}e^{-\mu(t-t_0)} \\
&= \beta L(0)e^{\beta t_0}e^{-\mu t}.
\end{aligned}$$

Integrating over all cohorts yields the population size as

$$\begin{aligned}
L(t) &= \int_{-\infty}^{t} \beta L(0)e^{\beta t_0}e^{-\mu t}dt_0 \\
&= L(0)e^{(\beta-\mu)t}.
\end{aligned}$$

Following Buiter (1988) and differentiating equations (12) and (13) with respect to time yields:

$$\begin{aligned}
\dot{C}(t) &= \left[\int_{-\infty}^{t} \beta L(0)e^{-\mu t}\dot{c}(t_0,t)e^{\beta(t_0)} - \mu\beta L(0)e^{-\mu t}c(t_0,t)e^{\beta t_0}dt_0\right] \\
&+ \beta L(0)e^{-\mu t}c(t,t)e^{\beta t} - 0 \\
&= \beta L(0)e^{-\mu t}c(t,t)e^{\beta t} - \mu C(t) + \beta L(0)e^{-\mu t}\int_{-\infty}^{t}\dot{c}(t_0,t)e^{\beta t_0}dt_0
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
\dot{K}(t) &= \left[\int_{-\infty}^{t} \beta L(0)e^{-\mu t}\dot{k}(t_0,t)e^{\beta(t_0)} - \mu\beta L(0)e^{-\mu t}k(t_0,t)e^{\beta t_0}dt_0\right] \\
&+ \beta L(0)e^{-\mu t}k(t,t)e^{\beta t} - 0 \\
&= \beta L(0)e^{-\mu t}\underbrace{k(t,t)}_{=0}e^{\beta t} - \mu K(t) + \beta L(0)e^{-\mu t}\int_{-\infty}^{t}\dot{k}(t_0,t)e^{\beta t_0}dt_0.
\end{aligned} \tag{A.10}$$

From equation (2) it follows that

$$\begin{aligned}
\dot{K}(t) &= -\mu K(t) + \beta L(0)e^{-\mu t}\int_{-\infty}^{t}\left[(r+\mu-\delta)k(t_0,t) + \hat{w}(t) - c(t_0,t)\right]e^{\beta t_0}dt_0 \\
&= -\mu K(t) + (r+\mu-\delta)\beta L(0)e^{-\mu t}\int_{-\infty}^{t}k(t_0,t)e^{\beta t_0}dt_0 \\
&\quad -\beta L(0)e^{-\mu t}\int_{-\infty}^{t}c(t_0,t)e^{\beta t_0}dt_0 + L(0)e^{-\mu t}\left(\frac{\beta\hat{w}(t)e^{\beta t_0}}{\beta}\right)_{-\infty}^{t} \\
&= -\mu K(t) + (r+\mu-\delta)K(t) - C(t) + \hat{W}(t) \\
&= (r-\delta)K(t) - C(t) + \hat{W}(t)
\end{aligned}$$

which is the aggregate law of motion for capital. Note that the definition of aggregate non-interest income is $\hat{W}(t) = L(0)\hat{w}(t)e^{\beta-\mu}$. By making use of equation (A.6), we can write aggregate consumption as

$$\begin{aligned}
C(t) &\equiv \beta L(0)e^{-\mu t}\int_{-\infty}^{t}c(t_0,t)e^{\beta t_0}dt_0 \\
&= \beta L(0)e^{-\mu t}\int_{-\infty}^{t}e^{\beta t_0}(\rho+\mu)\left[k(t_0,t)+h(t)\right]dt_0 \\
&= (\rho+\mu)K(t) + \beta L(0)e^{-\mu t}(\rho+\mu)\int_{-\infty}^{t}e^{\beta t_0}h(t)dt_0 \\
&= (\rho+\mu)\left[K(t)+H(t)\right].
\end{aligned} \tag{A.11}$$

Note that the following definitions apply: $K(t) = \beta L(0)e^{-\mu t}\int_{-\infty}^{t} e^{\beta t_0}k(t_0,t)dt_0$ and $H(t) = L(0)e^{(\beta-\mu)t}h(t)$. Newborns do not own capital because there are no bequests, therefore

$$c(t,t) = (\rho+\mu)h(t) \tag{A.12}$$

holds for each newborn individual. Putting equations (3), (A.9), (A.11) and (A.12) together yields

$$\begin{aligned}
\dot{C}(t) &= \beta(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right] + \\
&\quad \beta L(0)e^{-\mu t}\int_{-\infty}^{t}(r-\rho-\delta)c(t_0,t)e^{\beta t_0}dt_0 \\
&= \beta(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right] + (r-\rho-\delta)C(t) \\
\Rightarrow \frac{\dot{C}(t)}{C(t)} &= r-\rho-\delta + \frac{\beta(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right]}{C(t)} \\
&= r-\rho-\delta + \frac{\beta(\rho+\mu)H(t) - \mu(\rho+\mu)\left[K(t)+H(t)\right]}{(\rho+\mu)\left[K(t)+H(t)\right]} \\
&= r-\rho-\delta + \beta\underbrace{\frac{H(t)}{K(t)+H(t)}}_{\Omega'\in(0,1)} - \mu
\end{aligned}$$

which is the aggregate Euler equation that differs from the individual Euler equation by the term $\beta\frac{H(t)}{K(t)+H(t)} - \mu$.

Operating profits for intermediate goods producers: Profits of intermediate goods producers can be obtained via equation (19) as

$$\begin{aligned}
\pi &= \frac{r}{\alpha}x - rx \\
&= (1-\alpha)\alpha\frac{Y}{A}.
\end{aligned}$$

Labor input in both sectors: We determine the fraction of workers employed in the final goods sector and in the R&D sector by making use of equation (25)

$$\begin{aligned}
p^A \lambda A^{\phi} &= (1-\alpha)\frac{Y}{L_Y} \\
L_Y &= \frac{(r-\delta)A^{1-\phi}}{\alpha\lambda} \\
\Rightarrow L_A &= L - \frac{(r-\delta)A^{1-\phi}}{\alpha\lambda},
\end{aligned}$$

where the last line follows from labor market clearing, i.e. $L = L_A + L_Y$.

Rewriting production per capital unit: Production per capital unit can be written as a function of the interest rate and the intermediate share in final goods production

$$\begin{aligned}
r &= \alpha p = \alpha^2 \frac{Y}{K}, \\
\Rightarrow \frac{Y}{K} &= \frac{r}{\alpha^2} \qquad \text{(A.13)}
\end{aligned}$$

The BGP growth rate in the Jones (1995) case with demography: The growth rate of the economy is

$$g = \frac{\dot{A}}{A} = \frac{\lambda L_A}{A^{1-\phi}}.$$

Taking logarithms yields

$$\log g = \log(\lambda) + \log(L_A) - (1-\phi)\log(A).$$

Taking the derivative of this expression with respect to time and noting that along the BGP the growth rate is constant yields

$$\begin{aligned}
\frac{\partial g}{\partial t} &= n - (1-\phi)g = 0 \\
\Rightarrow g &= \frac{n}{1-\phi} \\
&= \frac{\beta - \mu}{1-\phi}.
\end{aligned}$$

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