Cheap talk comparisons in multi-issue bargaining

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Abstract

Bargaining over two issues as a bundle permits credible cheap talk about their relative importance even when interests are directly opposed on each issue. The resulting communication gains can exceed the gains from bundling previously identified in the monopoly pricing literature.

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1 Introduction

In multi-issue bargaining both sides can benefit by compromising on the issues they care least about in exchange for a better deal on the issues they care most about. But in an asymmetric information environment there is no assurance that the parties have either the incentive or the credibility to communicate which issues to compromise on.

We analyze this problem in a "take it or leave it" bargaining game where an offerer proposes concessions on two issues to an offeree after listening to messages sent by the offeree. When the issues are bargained over separately we find that the offeree will lie about which issue is of greater importance so communication is not credible. This communication problem can be solved by bundling the two issues together in a single offer that must be accepted or rejected in its entirety.

The communication in our model is non-verifiable "cheap talk." The cheap-talk literature shows that some signals can be credible if sender and receiver interests are partly aligned (Crawford and Sobel, 1982), but in our model the two sides' interests are directly opposed on each issue. Considered separately, the offeree has an incentive to lie about the importance of each issue. But if both issues must be accepted or rejected together, a comparative statement about which issue is better can be credible because it simultaneously reveals favorable information about one issue and unfavorable information about the other issue.

A standard result of the monopoly pricing literature is that bundling multiple goods together can increase a monopolist's profits because buyer valuations of a bundle are more predictable than buyer valuations of individual goods (Stigler, 1963; Adams and Yellen, 1976; McAfee, McMillan, and Whinston, 1989). This same logic clearly applies to our model of multi-issue bargaining. The communication gains we identify are in addition to the standard benefits of bundling previously identified in the monopoly pricing literature.

2 The Model

We consider a game between two players, A and B, bargaining over two issues, 1 and 2. Player B has private information $v_k \in [0,1]$ relevant to his value for each issue k. Let $v = (v_1, v_2)$. We suppose that v_k has continuous density f, and distribution F, i.i.d. across $k \in \{1,2\}$.

We model the potential for communication under two different bargaining protocols. Under the first protocol, each offer can be accepted or rejected separately. Under the second protocol, the players either reach agreement on both issues or on none. We use the term "no bundling" to refer to the first protocol and "bundling" to refer to the second.

For each bargaining protocol, communication is modelled as a cheap talk game. Player B first chooses a message m from some set M, possibly as a function of his private information v. Upon hearing the message m, player A will make a "take it or leave it" offer $x_k \in [0,1]$ on each issue k. The game ends when B accepts (agreement) or rejects (disagreement). Let $x = (x_1, x_2)$.

Notice that, for a fixed protocol, the timing structure states that the offer x has to be optimal *given player A's inference* about v upon hearing m. This distinguishes our cheap talk model from a screening problem where player A first *commits* to a menu of offers and player B chooses among them, with or without sending messages.

Given a realization v of player B's private information and concessions x by player A, the payoffs for each player from reaching agreement on issue k are equal to

$$U_{A,k} = 1 - x_k$$

for player A and

$$U_{B,k} = g(v_k, x_k)$$

for player B.[1] We assume that g is twice differentiable and strictly increasing in each argument. Payoffs are additive across issues and we denote by $U_i = U_{i,1} + U_{i,2}$ the total payoffs for $i \in \{A, B\}$.

For each player there is a common outside opportunity equal to 0 for each issue k. We assume that $g(1,1) > 0$ so that an agreement can be reached with positive probability, and $g(1,0) \leq 0$ so that an agreement with no concession is worse than B's outside option with probability 1.

Under these assumptions, there exists $\underline{x} \in [0,1)$ such that $g(1,\underline{x}) = 0$. The offer $\underline{x}$ corresponds to the highest offer that will give B a payoff less than his outside option with probability 1.

Let $\overline{x} = \max\{x \in [0,1] | g(0,x) \leq 0\}$. When $\overline{x} < 1$, it corresponds to the lowest offer that gives B a payoff (weakly) more than his outside option regardless of v. Note that $\underline{x} < \overline{x}$. Note also that, from the assumed properties of g, for each $x \in [\underline{x}, \overline{x}]$ there exists a cutoff value $v(x) \in [0,1]$, strictly decreasing and continuously differentiable in x, such that $g(v(x), x) = 0$.

We provide three different examples of the function g below. In example 1 there is no interaction between the realized uncertainty and the offeree's marginal benefit of a concession on the issue. It corresponds to the standard linear and additively separable utility function that is usually considered in the literature on bundling by a multi–product monopolist. In example

[1]Our results depend only $U_{A,k}$ being strictly decreasing in x, not on its linearity.

2, player B's utility is supermodular in the unknown parameter v and the concession x. In example 3, player B's utility is submodular in v and x.

1. $g(v,x) = v + x - u$, with $u \in (1,2)$. Then $\underline{x} = u - 1$, $\overline{x} = 1$ and $v(x) = u - x$.
2. $g(v,x) = vx - u$, with $u \in (0,1)$. Then $\underline{x} = u$, $\overline{x} = 1$ and $v(x) = \frac{u}{x}$.
3. $g(v,x) = \ln(v + x) + u$ with $u \geq 0$. Then $\underline{x} = 0$, $\overline{x} = e^{-u}$, and $v(x) = e^{-u} - x$.

2.1 Babbling

Since we model communication as cheap talk, there is always a babbling equilibrium under either bargaining protocol where A refuses to ascribe any meaning to B's message m and makes her offers accordingly.

In a babbling equilibrium under no bundling, for each issue $k \in \{1,2\}$, player A chooses the concession x_k such that

$$x_k \in \arg\max_{x\in[0,1]} (1-x)\Pr[g(v,x) \geq 0]. \tag{1}$$

In contrast, in a babbling equilibrium under bundling, player A chooses the concessions x to solve

$$x \in \arg\max_{x\in[0,1]^2} (2 - x_1 - x_2)\Pr[g(v_1,x_1) + g(v_2,x_2) \geq 0\,]. \tag{2}$$

We do not provide a general characterization of babbling equilibria in this note but turn now to the existence of an informative equilibrium.

2.2 Rank-Revealing Equilibrium

We consider the possibility that B might credibly disclose his ordinal ranking of the different issues, i.e., whether $v_1 > v_2$ or vice versa, without disclosing anything about the magnitude of either v_1 or v_2. We call such an informative equilibrium a *rank–revealing equilibrium* (RRE). We show below that, under a fairly general set of conditions, there does not exist a RRE unless bundling is allowed. We assume without loss of generality that $v_1 \geq v_2$ so that issue 1 is more valuable to B than issue 2.

2.2.1 No Bundling

In a RRE (given a message from player B that $v_1 \geq v_2$), for each $k \in \{1,2\}$ player A chooses x_k to solve

$$x_k \in \arg\max_{x \in [0,1]} \ (1-x)(1-F_k(v(x))) \tag{3}$$

where $F_k(\cdot)$ is the distribution of v_k given $v_1 \geq v_2$. Let $f_k(\cdot)$ be the density associated with $F_k(\cdot)$. Note that $F_1(v) = \{F(v)\}^2$ and $F_2(v) = 1 - \{1-F(v)\}^2$.

Since the objective function is continuous, problem (3) must have a solution x_k. Moreover, the solution must be interior, i.e., $x_k \in (\underline{x}, \overline{x})$, $k \in \{1,2\}$. To see this note that, if $x_k \leq \underline{x}$ then the offer is rejected with probability 1 and A can do strictly better by making an offer in $(\underline{x}, \overline{x})$. Regarding the upper bound, when $\overline{x} = 1$ then an offer of $x_k = \overline{x}$ earns A an expected payoff of 0 so it is strictly dominated by an offer in $(\underline{x}, \overline{x})$. And when $\overline{x} < 1$, an offer of $x_k > \overline{x}$ is strictly dominated by the offer $\overline{x}$. Moreover, since $v(\overline{x}) = 0$ when $\overline{x} < 1$, the derivative of the objective function at $x_k = \overline{x}$ is $-(1-\overline{x})v'(\overline{x}) < 0$ so that an offer slightly less than $\overline{x}$ dominates an offer of $\overline{x}$.

At an interior solution we must have $v(x_k) \in (0,1)$ for all $k \in \{1,2\}$. Moreover, x_k must satisfy the first–order necessary condition for an interior maximum:

$$-(1-x_k)v'(x_k) - \frac{1-F_k(v(x_k))}{f_k(v(x_k))} = 0, \tag{4}$$

and, furthermore, the left–hand side of (4) must be non–increasing in x at x_k. Assume that (4) has a unique solution.[2] Then for all $x \in [x_k, \overline{x})$ we must have

$$-(1-x)v'(x) - \frac{1-F_k(v(x))}{f_k(v(x))} \leq 0.$$

This implies that $x_1 < x_2$. For if not,

$$\frac{1-F_1(v(x_1))}{f_1(v(x_1))} = -(1-x_1)v'(x_1) \leq \frac{1-F_2(v(x_1))}{f_2(v(x_1))}.$$

Since $v(x_1) \in (0,1)$ and

$$\frac{1-F_1(v)}{f_1(v)} > \frac{1-F_2(v)}{f_2(v)}$$

for all $v \in (0,1)$, we have a contradiction.

Since $x_1 < x_2$ clearly B will lie for any v_1 and v_2 if the marginal value of a concession is higher for higher v (supermodular g). The following shows that the problem is more general in that for any g there are always some realizations of v_1 and v_2 such that B will lie.

[2] Sufficient conditions are that g is quasi–concave and $\frac{1-F(v)}{f(v)}$ is monotonically decreasing in v.

Claim 1 *If there is a unique solution to (4) then there is no rank–revealing equilibrium when B can reject each offer separately.*

Proof. Suppose there is a rank-revealing equilibrium. Then, it follows from the arguments above that A will choose concessions x_1 and x_2 such that $\overline{x} > x_2 > x_1 > \underline{x}$ so that $1 > v(x_1) > v(x_2) > 0$. But if v_1 and v_2 are such that $v(x_1) > v_1 > v(x_2) > v_2$, then B has a strict incentive to lie and claim issue 2 is more important. B will then accept the larger concession on issue 1 and reject the smaller concession on issue 2. This will give B a payoff of $g(v_1, x_2)$ which is strictly greater than 0, the payoff from revealing the rank truthfully. ■

2.2.2 Bundling

In this case, given a message m from player B that $v_1 \geq v_2$, player A chooses x_1 and x_2 to solve

$$x \in \arg\max_{x\in[0,1]^2} \ (2 - x_1 - x_2) \Pr[g(v_1, x_1) + g(v_2, x_2) \geq 0 \,|\, v_1 \geq v_2]. \tag{5}$$

For B to reveal the ranking truthfully it is sufficient that whenever $v_1 \geq v_2$,

$$g(v_1, x_1) + g(v_2, x_2) \geq g(v_1, x_2) + g(v_2, x_1). \tag{6}$$

We consider two cases:

1. g is supermodular: $\frac{\partial^2 g}{\partial v \partial x} > 0$ for all v, x.

2. g is submodular: $\frac{\partial^2 g}{\partial v \partial x} < 0$ for all v, x.

Claim 2 *If g is supermodular (respectively, submodular), then there exists a RRE with $x_1 > x_2$ (resp., $x_1 < x_2$), when B can only accept or reject the bundle.*

Proof. Suppose g is supermodular. Then, for (6) to hold for all $v_1 > v_2$ it is sufficient that $x_1 > x_2$.

In problem (5) A must choose $x_1 > x_2$. For if $x_1 < x_2$ then A can simply switch the concessions, increasing the probability of agreement without increasing the total concessions in the event of agreement. And if $x_1 = x_2 = x$ then we must have $x > \underline{x}$ (otherwise agreement will be reached with zero probability) and $x < 1$ (otherwise agreement leaves A with no surplus). But then there exists $\varepsilon > 0$ such that an offer $x_1 = x + \varepsilon$ and $x_2 = x - \varepsilon$ will increase the probability of agreement without increasing the total concessions in the event of agreement.

The symmetric argument applies to the case where g is submodular. ■

With bundling A wants to increase the probability of agreement by raising the total value of the offer to B for any given amount of concessions $x_1 + x_2$. B has an incentive to help A do this by revealing the ranking. When g is supermodular, the value v and the concession x are complements from B's perspective and A concedes more on the more valuable issue to increase the probability of acceptance. On the other hand, when g is submodular, the value v and the concession x are substitutes from B's perspective and A concedes more on the less valuable issue to increase the probability of acceptance.

A is better off in an informative RRE with bundling compared to the babbling equilibrium with bundling because she can enforce the same outcome in both cases and has more information in a RRE. However, A's expected payoff in a RRE with bundling is not always higher than her expected payoff in the babbling equilibrium under no bundling. That is, the gain from communication need not be greater than the flexibility allowed by separate bargaining. Similar remarks apply to B's ex–ante expected payoffs from different protocols.

2.3 Example

Consider the supermodular example where $g(v, x) = vx - u$. Assume $u = 1/4$ and v is uniformly distributed in $[0, 1]$. We can think of the two issues as land in regions 1 and 2 along a common border of two countries. Country A prefers to concede less land in each region, and prefers no agreement to conceding too much. Country B prefers to receive a larger concession in each region, and prefers no agreement to insufficient concessions. Country B has private information v_1 and v_2 about the marginal value of each unit of land conceded in each region.

Figure 1 shows the zones of acceptance under the three cases of no bundling, bundling without communication, and bundling with communication. All three figures are drawn for the case where $v_1 \geq v_2$ so all of the probability mass is below the diagonal.

Without bundling Country B will not reveal which region is preferred, so each region is treated identically by Country A. Country A trades off the probability of acceptance, which is increasing in the concession offer, with the gain if the offer is accepted, which is decreasing in the offer. Solving problem (1), $x = .5$ for each region so the offer on region k is accepted when $v_k \geq .5$ as seen in figure 1(a).

With bundling but without communication, from (2) the optimal offer is $x = .564$ on each region and the bundle is accepted if $v_1 + v_2 \geq .887$ as seen in figure 1(b). This increases payoffs

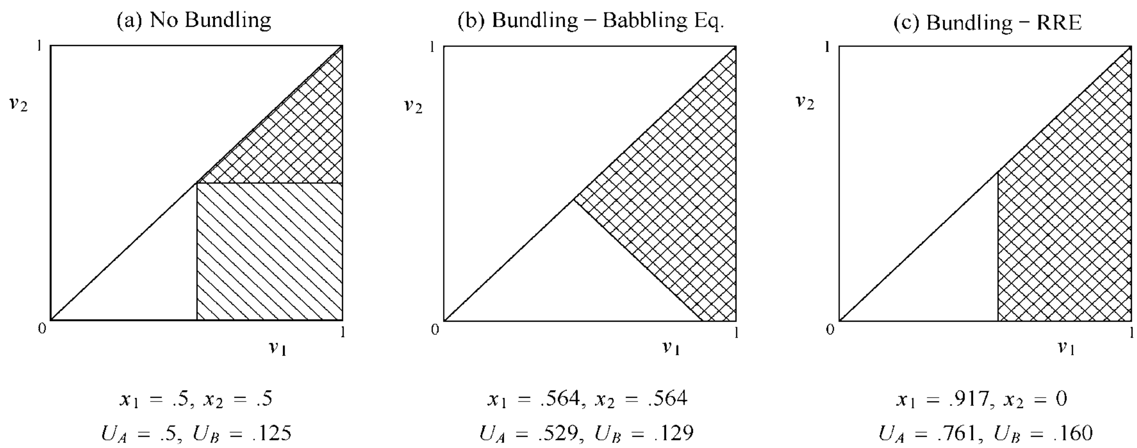

Figure 1: Acceptance regions for bargaining protocols, $v_1 \geq v_2$

for the same reason as in multi-product bundling models.[3]

Communication improves on the standard bundling outcome because the largest concession is made for the preferred region. In the RRE, Country B makes the credible statement that each unit of area in region 1 is more valued than in region 2. Solving problem (5), $x_1 = .917$, $x_2 = 0$ and agreement is reached as long as $v_1 \geq .525$ regardless of v_2. While the average concession $(x_1 + x_2)/2 = .459$ is the lowest of the three cases, the payoffs for both countries are the highest. As seen from the zones of acceptance in figure 1, the probability of agreement on both regions rises from only 25% in (a) to about 60.7% in (b) and to about 72.4% in (c).

3 Conclusion

Two areas for further research are the existence of equilibria more informative than the rank revealing equilibrium and the existence of a rank revealing equilibrium for larger numbers of issues. The latter question is of particular interest since the monopoly pricing literature has shown that the gains from bundling large numbers of products can be substantial (Armstrong, 1999; Bakos and Brynjolfsson, 1999). The rank order of issues becomes very informative as the number of issues increases so the communication gains from bundling are also likely to be significant.

[3]Bundling does not always increase payoffs. For large u it is rare that both issues are desired enough to make a bundle worthwhile so no bundling is preferred.

4 References

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