Dynamic Common Agency, Vertical Integration, and Investment: The Economics of Movie Distribution*

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Abstract

This paper analyzes the impact of vertical integration on investment and other strategies in a dynamic common agency framework. Movie distribution is used as a motivating example. The model matches several facts about movie distribution; distributors avoid head-to-head new hit releases, hits have longer runs than flops, and distributors receive the lion's share of value generated by hits. Welfare comparisons show that integration is privately profitable and may improve social welfare even though it reduces industry profits. The effects of integration on strategies and welfare depend critically on how integration affects the bargaining power of the non-integrated firm.

JEL Codes: L14: Transactional Relationships and Contracts; **L22**: Firm Organization and Market Structure: Markets vs. Hierarchies, Vertical Integration; **L82**: Industry Studies: Entertainment; **C61:** Dynamic Analysis.

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1. Introduction

The management of vertical relationships is one of the fundamental issues addressed by contract and organization theorists. How should suppliers and retailers design contracts? What are the consequences if suppliers forward integrate? How do choices about contracts and organizational form interact with other strategic choices? The answers to these questions inform management strategy and public policy. This paper provides insight into vertical relationships by analyzing relationships between movie distributors (studios or independent distributors) and exhibitors (theaters). Vertical integration in the movie industry is particularly interesting because the *Paramount* decrees of the late 1940s and early 1950s barred studios from owning theaters. Then in the 1980s the U.S. Department of Justice gave studios permission to forward integrate into exhibition again (De Vany and Eckert 1991). Several distributors did so, including some of the largest ones.

This paper introduces an infinite-horizon common agency model with two distributors and one exhibitor where distributors can invest each period before contracting with the exhibitor, who owns a theater with multiple screens. A distributor's investment stochastically improves the movies it holds in inventory; the probability of having "hits" as opposed to "flops" rises. Vertical integration occurs when one distributor owns the exhibitor.

Static common agency models have been analyzed by Bernheim and Whinston (1986) and many others but authors have only recently begun to explore dynamic models (Bergemann and Valimaki 1998, Prat and Rustichini 1998). Bergemann and Valimaki (1998) provide results for infinite-horizon models. They show that the Bernheim-Whinston marginal contribution equilibrium (where principal i's value each period is the difference between the value all players can generate and the value the others can generate by contracting without i in the period and playing optimally thereafter) is efficient in the sense of maximizing the joint value of the players. However, Bergemann and Valimaki (1998) do not consider investment or vertical integration.

I begin by providing intuition using a two-period model where distributors invest in period one and contract with the exhibitor in period two. I compare three settings: a monopolist who owns all three firms, two independent distributors who receive their marginal contributions, and vertical integration where the independent distributor receives a portion of its marginal contribution that depends on its bargaining power. For the infinite-horizon model, computational results show how strategies and welfare vary across these settings.

The infinite-horizon model explains several facts about movie distribution.[1] Distributors avoid simultaneous new hit releases, and hits stay in the theater longer than flops. In reality, release timing and run lengths are critical choices. Distributors often adjust release dates to avoid competing head-to-head (Reardon 1992, Krider and Weinberg 1998, Chisholm 1999) and run lengths depend on early performance (De Vany and Eckert 1991, De Vany and Walls 1996). Further, in the marginal contribution equilibrium distributors appropriate the lion's share of the marginal value created by their hits. This occurs because hits are rare and the exhibitor's next best alternative is to show worse movies. This is consistent with real-world exhibition contracts, which use non-linear revenue sharing rules that give distributors higher shares of ticket revenue when revenue is high (Filson et al. 2002).

Non-integration yields the same choices and welfare as monopoly. Thus, as in Bergemann and Valimaki (1998), non-integration is efficient in the sense of maximizing firm value. If investment is sufficiently costly, investment never occurs and vertical integration yields the same choices and welfare as monopoly. Otherwise, vertical integration is privately profitable but lowers industry profits: the combined value of the two firms that merge rises because they appropriate value from the remaining distributor, but sub-optimal investment occurs so the combined value of all three firms falls.

Additional results depend on how integration affects the bargaining power of the independent distributor. Suppose that the independent distributor maintains its bargaining power after integration and continues to receive its marginal contribution. In this case, integration causes the integrated distributor to increase its investment and delay releasing its hits. The integrated distributor uses its hits as threats to appropriate value from the independent distributor. In response, the independent distributor reduces its investment. However, total

[1] Other authors provide complementary explanations for various features of movie exhibition contracts and distribution arrangements. De Vany and Eckert (1991) and De Vany and Walls (1996) emphasize that difficulties with forecasting demand necessitate the use of short-term contingency-rich contracts. Filson et al. (2002) show how revenue sharing allows distributors to share risk with exhibitors. Corts (2001) considers how vertical structures affect release date scheduling. Kenney and Klein (1983) and Hanssen (2000) analyze block booking. Borcherding and Filson (2001) review the literature on contracts in the movie business.

industry investment never falls, and it rises for a wide range of investment costs. Higher industry investment causes more hits to reach the theater, and consumer surplus rises. As a result, integration may improve welfare. Beginning from a high cost of investment, the welfare comparison favors integration more as the cost of investing falls. However, when the cost of investing is sufficiently low, industry investment is already high in the non-integrated setting and does not increase when integration occurs. In this case, integration has no impact on industry value or consumer surplus and thus is welfare neutral (although it is still privately profitable).

Now suppose that the independent distributor loses all of its bargaining power after integration and gives its entire marginal contribution to the integrated firm. In this case, integration causes the independent distributor to reduce its investment to zero. The integrated distributor still increases its investment, but industry investment falls. Lower industry investment causes fewer hits to reach the theater than in the non-integrated setting, and welfare unambiguously falls. Other strategies also change. The integrated distributor does not need to delay releasing its hits in order to appropriate value from the independent distributor. As a result, it tends to show its own new hits in preference to the independent distributor's. In cases between these two extremes (where the distributors share the independent distributor's marginal contribution) the welfare effects are between these two extremes, and the critical level of consumer surplus per ticket required for integration to improve welfare rises.

This paper contributes to the growing literature on vertical relationships. In this literature, most of the emphasis is on exclusive dealing. Recent papers along this line include Martimort (1996) and Bernheim and Whinston (1998). In other related literature, Aghion and Bolton (1987) analyze how an incumbent seller facing a threat of entry can sign a long-term contract with a buyer to attempt to deter entry, and O'Brien and Shaffer (1997) and Kahn and Mookherjee (1998) analyze how multiple principals can use non-exclusive contracts to provide incentives to a common agent. The results here on the effects of vertical integration on investment and welfare are similar to those obtained by Bolton and Whinston (1993) in a model of supply assurance with multiple downstream firms. This paper contributes to the literature by combining contract choice with investment and other strategic choices and considering dynamic competition, which allows for richer strategic interaction.

2. The Multiplex

The multiplex (a theater with multiple screens) provides a simple environment with multiple upstream firms and a single downstream firm. A Multiplex deals with multiple distributors, and competing multiplexes are typically located far away.[2] Consider a simple model of movie allocation in a multiplex based on Bernheim and Whinston (1985, 1998). There are two distributors, D1 and D2, and one exhibitor EX. Each distributor has two movies: D1 has 1a and 1b and D2 has 2a and 2b. EX has a single theater with two auditoriums, so it can show only two of the four movies. Both auditoriums are identical. If a monopolist owns D1, D2, and EX, it chooses an allocation of movies to auditoriums to maximize total profit. Denote the resulting total profit by π^{all}.

Now suppose that D1 and D2 are independent from EX. Each distributor submits six bids, one for every possible allocation ({1a,2a}, {1a,1b}, {1a,2b}, {2a,1b}, {2a,2b}, and {1b,2b}). Then EX chooses the allocation that maximizes the total bid. A distributor can attempt to avoid undesirable pairings by conditioning its bid for a movie on which other movie its movie will be paired with. Following Bernheim and Whinston (1998), focus on subgame perfect Nash equilibria that are Pareto-undominated (within the set of equilibria) for the distributors. The equilibrium allocation of movies to auditoriums maximizes total profit and payoffs of each player are:

$$\begin{aligned} D1: &\quad \pi^{all} - \pi^2 \\ D2: &\quad \pi^{all} - \pi^1 \\ EX: &\quad \pi^1 + \pi^2 - \pi^{all} \end{aligned}$$

where π^1 is the profit D1 and EX could obtain by showing only D1's movies and π^2 is defined similarly. Each distributor receives its marginal contribution to π^{all}. In order for this to be a proper equilibrium EX's payoff must be positive; otherwise EX would refuse to show any

[2] Several reasons for constructing multiplexes are not explored here. Multiplexes absorb idiosyncratic demand shocks better because negative shocks on one movie can be balanced against positive shocks on others. Reallocating movies to auditoriums of different sizes occurs as demand is observed. Showing times are staggered to keep staff and other inputs continuously employed. Multiplexes are pervasive. The Motion Picture Association of America (MPAA) reports that domestically in 2001 there were 7,070 theaters; 2,280 single screens, 2,901 two to seven screens, 1,458 eight to fifteen screens, and 431 sixteen or more screens.

movies and obtain a payoff of 0. The distributors' payoffs are always positive because π^{all} must be at least as large as π^1 and π^2.

Bernheim and Whinston (1998) show that the equilibrium allocation can be implemented by making EX the residual claimant. Each distributor receives a fixed fee from EX equal to its equilibrium payoff, and EX chooses the two movies that maximize profit. Profit maximization is roughly equivalent to revenue maximization because when a movie is placed in the theater most of the distributor's costs are sunk and most of the exhibitor's costs are fixed. Thus, the model suggests one explanation for why real-world exhibition contracts involve revenue sharing: the contracts encourage exhibitors to consider cross effects on demand.[3] The infinite-horizon model does not allow this interpretation of the marginal contribution equilibrium because future values must be considered as well as current revenue. However, as discussed below, the distribution of payoffs in the infinite-horizon model roughly matches that of real-world exhibition contracts.

Now suppose that D2 owns EX and that D1 receives some fraction of its marginal contribution that depends on its bargaining power. If we continue to focus on Pareto-undominated equilibria, the equilibrium allocation continues to maximize total profit and payoffs are:

$$\begin{aligned} D1: &\quad \alpha(\pi^{all} - \pi^2) \\ D2: &\quad \pi^2 + (1-\alpha)(\pi^{all} - \pi^2) \end{aligned}$$

where $0 \leq \alpha \leq 1$.

Note that in every setting the allocation is efficient in the sense of maximizing total profit. Thus, a welfare comparison across monopoly, non-integration, and integration yields no differences. However, the distribution of profit changes. Because of this, introducing investment decisions that precede the allocation of movies to auditoriums yields differences across the settings. Consider a two-period game where D1 and D2 can invest in improving their movies in period 1 before playing the "allocation game" in period 2. Suppose that investing in a movie increases the profit that can be obtained from it but that the amount of the increase may depend on which other movie it is paired with.

[3] De Vany and Eckert (1991) and Filson et al. (2001) note that revenue sharing evolved prior to multiplexes becoming wide-spread. Therefore, there are clearly other reasons for revenue sharing, such as risk sharing.

Consider subgame perfect equilibria: in period 1 the distributors look ahead, realize what their payoffs will be in the allocation game, and choose their investments accordingly. If a monopolist owns D1 and D2, it maximizes π^{all} net of the cost of investment. In this sense, investment is efficient. If D1 and D2 are both independent, they still tend to make efficient investments as long as they cannot affect each other's movies. For example, D1 looks ahead and knows that its payoff in the allocation game will be $\pi^{all} - \pi^2$. If D1 cannot affect π^2 (because π^2 depends only on D2's movies), then D1 invests if and only if doing so improves π^{all} net of the cost of investment. There is a potential coordination problem if distributors' optimal choices are interdependent, but selecting the Pareto-superior equilibrium is enough to eliminate any difference between the monopoly and non-integrated settings.

However, under integration the incentives change. Consider two extreme cases. If $\alpha = 1$, D2 invests as long as doing so increases π^2 net of the cost of investment, regardless of the effect on π^{all}. Thus, D2 may invest too much or too little compared to the efficient level. As a result, even though D1 has the incentive to maximize π^{all} net of the cost of investment, industry investment is likely to be inefficient. Alternatively, if $\alpha = 0$, D1 has no incentive to invest at all. Thus, even though D2 has the incentive to maximize π^{all} net of the cost of investment, industry investment is likely to be inefficient. Note that if the distributors' optimal investments are interdependent, then both are likely to depart from the monopoly choices in these cases, even though one has the incentive to invest efficiently. For intermediate values of α, both distributors may invest inefficiently.

The results in this section are not specific to the movie industry; the "movies" could be any products. The results provide intuition for the results in the next section, where I compute a stationary infinite-horizon model. Computing requires specifying additional structure specific to the movie industry, but many of the general features hold in other industries. As in this section, the issue of efficiency revolves around the question of whether investment decisions are efficient; if investments do not differ across settings then efficiency does not differ across settings. Investments and allocations in the non-integrated setting maximize industry value. However, these investments and allocations may not maximize consumer surplus, and as a result vertical integration may improve total welfare even though it lowers industry value.

3. A Stationary Infinite-Horizon Model of the Multiplex

The model in Section 2 has some implications that are at odds with real-world observations. Multiple hits would be shown simultaneously, because such an allocation would maximize single-period profit. In reality, release dates of hits are often staggered.[4] Krider and Weinberg (1998) and Chisholm (1999) provide evidence on this, and Reardon (1992), president of Warner Brothers Distributing Corporation at the time of writing, writes: "Competition is a crucial issue in targeting a release. Will we be up against a movie with similar audience-segment appeal? Sometimes this leads to moving dates up or back." Another fact is that the length of the theatrical run varies across movies depending on whether they are hits or flops. Distributor choices about release dates and run lengths interact with their choices of contract terms and investment. In order to allow for these phenomena I develop a dynamic version of the model of the multiplex.

In the model, two distributors, D1 and D2, and a single exhibitor with two auditoriums, EX, interact over an infinite number of periods. Over time, movies are released, play out their runs, and are replaced with new movies. At the beginning of each period each distributor has two movies in its inventory: D1 has 1a and 1b and D2 has 2a and 2b. Each movie is of one of five types: new hit, new flop, old hit, old flop, and finished. New movies have not been shown in the theater, old movies were shown in the theater in the previous period, and finished movies were old in the previous period. The state space is all of the possible combinations of movie types of the four movies {1a, 1b} and {2a, 2b}. For example, one state is {new hit, old flop} {finished, new flop}: D1 has a new hit and an old flop and D2 has a movie that has finished its run and a new flop.

Each period, before movies are allocated to auditoriums distributors play a "replacement game," a simultaneous-move game where each distributor may replace one or both of its movies with new movies. Each new movie is a hit with probability p and a flop with probability $1 - p$, and all players observe whether each new movie is a hit or a flop as soon

[4] That this occurs shows that avoiding head-to-head competition is important in the movie business. In other industries other concerns might dominate. For example, Doyle and Snyder (1999) show that when an automobile manufacturer announces an increase in production other automobile manufacturers respond by increasing their production. Announcements reveal private information about common demand parameters; this effect dominates the desire to avoid head-to-head competition.

as it is added to inventory.[5] Movies that are finished must be replaced and there is no cost to doing so.[6] Movies that are new or old can be replaced by paying a cost c. This decision is an investment of the type described in Section 2; flops may be replaced early in an attempt to obtain hits. The assumption that replacing movies before they are finished is costly reflects the fact that production schedules are set years in advance along with tentative release windows, and it is costly to alter the schedule or obtain additional movies for distribution. This investment decision has alternative interpretations. Essentially, by paying c a distributor has a chance of improving the movie types held in inventory.[7]

The replacement game can be depicted using a simple payoff matrix. For example, Figure 1 describes the replacement game for the non-integrated setting when the state at the beginning of the period is {new hit, old flop} {finished, new flop}. Since D2 must replace its finished movie, its only choice is whether to replace 2b or not. D1 can replace neither of its movies, one or the other, or both. The $W^1_{i,,j}, W^2_{i,j}, W^e_{i.j}$ terms represent each player's expected value associated with each possible pair of strategies, where the superscripts index the player and the subscripts index which movies are replaced. Note that in the non-integrated setting,

[5]Note that a movie's type is not observed until the distributor adds it to inventory. This keeps the state space manageable because the only state variables are the types of movies held in inventory rather than all those in production. Evidence suggests the assumption is reasonable. De Vany and Walls (1996, 1999) show that the distribution of cumulative box office revenues has an infinite variance. Thus, forecasting is impossible without movie-specific information, and when the production schedule is set there is very little movie-specific information available (at best, the script and main actors are known). Chisholm (1999) asserts that pre-production information is sufficient for studios to plan to release hits around holiday weekends. While this happens, studio forecasts are often wrong (see Borcherding and Filson 2001 and the sources cited there). Some of the most successful movies in history were initially turned down by studios, including *Star Wars* and *Titanic,* and big bets have been made on flops, too, like *Howard the Duck* and *Ishtar.* Thus, assuming that pre-production forecasting is impossible is reasonable. Once a movie has been made, pre-release surveys yield signals of its potential. Even then errors are possible; the signals are noisy. Allowing for noisy public signals of new movie types instead of perfect observation of types does not affect the conclusions below.

[6]This implies that a movie can be in the theater at most two periods and these periods must be consecutive. This captures the main features of real-world runs: when a movie is released an initial contract is signed for a four to eight week run and there is an option for extending the run. In the model new contracts are negotiated each period, whereas real-world contracts may include holdover clauses that achieve the same result: hit movies are kept in the theater longer. Holdover clauses reduce transaction costs by removing the need for new negotiations.

[7]For example, it is possible to model investment as an advertising or promotional expenditure that can improve a movie's type. For convenience and to minimize notation, I assume that spending c allows a distributor to draw a new movie at random. Slight variations in this assumption, such as allowing for a different value of p to apply for these cases, or allowing the "replacement" to occur only if the new movie is better than the existing one (which matters only if the existing movie is an old hit), or assuming that c is not movie specific (the movies to be replaced are chosen only after the new movies' types are revealed) do not change the conclusions. I discuss these and other variations of the model in Subsection 3.7.

EX is a passive player in the replacement game. Its value is affected by the decisions but it has no role in making the decisions.

Suppose that the Nash equilibrium[8] of the game in Figure 1 is unique and that in equilibrium D1 replaces its old flop and D2 replaces only its finished movie.[9] Then the expected values of each distributor are given by $W^1_{b,a}$ and $W^2_{b,a}$. As shown in Figure 1, given the choices the distributors make, the state that results from the replacement game is randomly determined because it depends on the probability that each newly drawn movie is a hit.

After the replacement game is over all players observe each new movie's type. Then the players play the "allocation game." Clearly, a monopolist allocates movies to maximize the total value. I consider each other setting in turn.

3.1. Non-Integration

In the allocation game with two independent distributors, each distributor submits six allocation bids, one for each possible allocation {1a,2a}, {1a,1b}, {1a,2b}, {2a,1b}, {2a,2b}, and {1b,2b}.[10] EX then chooses the allocation to maximize its value, which takes into account the current bids and EX's discounted expected future value.

The equilibrium of the allocation game is Markov perfect - each player makes its choices to maximize its value given the choices of the other players, and choices depend only on the current state. As in Section 2, I focus on equilibria that are Pareto-undominated (within the set of equilibria) for the distributors. Given this, the equilibrium of the allocation game is similar to that described by Bernheim and Whinston (1985, 1998) for static games and Bergemann and Valimaki (1998) for dynamic games: 1) The allocation maximizes the sum of the players' expected values; and 2) Each distributor receives the value of its contribution as its payoff. For example, D1's expected value is the total expected value minus the total

[8] By assumption, choices depend only on the current state, so this Nash equilibrium is Markov perfect.

[9] In general, it is possible that the replacement game has multiple equilibria, in which case any Pareto dominated equilibria are eliminated and each remaining equilibrium is selected with equal probability. It is also possible that no pure strategy equilibrium exists. Dealing with this latter possibility proved to be unecessary because in every case a pure strategy equilibrium existed.

[10] Thus, as in Section 2 each distributor can condition its bid for a movie on which other movie its movie will be paired with. In reality, contracts do not do this explicitly. However, two features of real-world contracting may produce this effect. First, because exhibition contracts are typically the result of negotiations, there may be some scope for conditioning payments on the exhibitor's other choices. Second, because real-world contracts use revenue sharing, the exhibitor has an incentive to avoid pairings with undesirable cross effects.

expected value that D2 and EX could generate if only D2's movies were shown in the theater in the current period.

To continue the example, suppose that each new movie drawn in the replacement game is a flop. Then at the beginning of the allocation game the state is {new hit, new flop, new flop, new flop}. Suppose that the allocation that maximizes the total expected value includes 1a (the new hit) and either 2a or 2b. Denote the total expected value this allocation generates by $\overline{V}^{all}$, the expected value D1 and EX can generate if only D1's movies are shown in the current period and all players play optimally thereafter by $\overline{V}^{1e}$, and the expected value D2 and EX can generate if only D2's movies are shown in the current period and all players play optimally thereafter by $\overline{V}^{2e}$. Then

$$\begin{aligned} W^1(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{all} - \overline{V}^{2e} \\ W^2(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{all} - \overline{V}^{1e} \\ W^e(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{1e} + \overline{V}^{2e} - \overline{V}^{all} \end{aligned} \quad (3.1)$$

Appendix A describes explicit formulas for $\overline{V}^{all}$, $\overline{V}^{1e}$, and $\overline{V}^{2e}$.[11] To minimize notation, I use the notation $\overline{V}^{all}$, $\overline{V}^{1e}$, and $\overline{V}^{2e}$ in the following subsection where I consider vertical integration. However, it should be understood that the numerical values associated with $\overline{V}^{all}$, $\overline{V}^{1e}$, and $\overline{V}^{2e}$ differ there. Under integration with different levels of bargaining power, equilibrium investment changes. As a result, the value functions of every player change. This makes it difficult to obtain analytical results when comparing the various settings. As a result, I describe numerical results for a variety of parameter values.

[11] I assume that EX must show two movies each period. This can be rationalized by assuming that if EX shows no movies or only one, it loses substantial customer goodwill and incurs a large cost as a result. This assumption is reasonable in the context of the movie business, where a contracting period is typically four to eight weeks in length, and no exhibitor could credibly threaten to leave auditoriums empty for that long. However, as a matter of theory it would be desirable to explore the properties of an equilibrium where the exhibitor could show less than two movies, perhaps in an attempt to influence replacement decisions the following period. I discuss this possibility in Subsection 3.7.

3.2. Vertical Integration

Suppose D2 owns EX and the two units operate as one.[12] Timing remains the same: each period both distributors make replacement decisions simultaneously, then new movies' types are observed, and then D1 contracts with D2 and D2 chooses which two movies to show in the theater. I consider three cases. First, suppose that D1 has sufficient bargaining power to obtain its marginal contribution. Expression (3.1) changes as follows:

$$\begin{aligned} W^1(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{all} - \overline{V}^{2e} \\ W^2(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{2e} \end{aligned} \tag{3.2}$$

Expression (3.2) implies that D2 always obtains the full value of any improvement in its movies *whether its movies are placed in the theater or not*. As noted in Section 2, this may encourage inefficient investment decisions. D2 invests if doing so increases $\overline{V}^{2e}$ sufficiently, regardless of the effect on $\overline{V}^{all}$.

Now consider the opposite extreme: suppose D2's ownership of EX allows it to obtain D1's entire marginal contribution. Expression (3.1) changes as follows:

$$\begin{aligned} W^1(\text{new hit, new flop, new flop, new flop}) &= 0 \\ W^2(\text{new hit, new flop, new flop, new flop}) &= \overline{V}^{all} \end{aligned} \tag{3.3}$$

Now D1 never invests, whereas D2 invests whenever doing so increases $\overline{V}^{all}$ sufficiently.

Finally, suppose that the two players have equal bargaining power. Then they share D1's marginal contribution equally and expression (3.1) changes as follows:

$$W^1(\text{new hit, new flop, new flop, new flop}) = \overline{V}^{1,2e} + \frac{\overline{V}^{all} - (\overline{V}^{1,2e} + \overline{V}^{2e})}{2} \tag{3.4}$$

[12] Several industry practitioners claim that integrated units attempt to operate at arm's length (Friedberg 1992, Murphy 1992, Reardon 1992). While this is possible, it is still useful to examine the incentives integration creates.

$$W^2(\text{new hit, new flop, new flop, new flop}) = \overline{V}^{2e} + \frac{\overline{V}^{all} - (\overline{V}^{1,2e} + \overline{V}^{2e})}{2}$$

where $\overline{V}^{1,2e}$ is the value D1 gets if only D2's movies are shown in the current period.

3.3. Demand

Before computing the model, demand must be described. The main fact driving strategic delay in the movie industry is that if two new hits are released at the same time not all consumers will see both of them. To incorporate this into the model, assume that some fraction γ of consumers face time constraints each period and can see at most one movie. Denote the movies in the theater by i and j. Each consumer wants to see movie i with probability λ_i and movie j with probability λ_j. Assume that if a time constrained consumer wants to see both movies he chooses each with probability $\frac{1}{2}$. This implies that the probability that each consumer sees movie i is

$$\lambda_i(1 - \lambda_j) + \gamma\frac{\lambda_i\lambda_j}{2} + (1 - \gamma)\lambda_i\lambda_j = \lambda_i(1 - \gamma\frac{\lambda_j}{2}). \tag{3.5}$$

Expression (3.5) shows that hits steal more demand from other movies: if λ_j is higher then movie i gets fewer customers. In the computations, I normalize monetary units so that the number of consumers served by the theater multiplied by the ticket price is 1. Given this, the expected revenue of movie i is given by expression (3.5). The probability λ_i depends on the movie's type: For a new hit, $\lambda_i = \lambda_h$, for a new flop, $\lambda_i = \lambda_l$, for an old hit, $\lambda_i = \alpha\lambda_h$, for an old flop, $\lambda_i = \alpha\lambda_l$, and for a finished movie, $\lambda_i = 0$, where λ_h, λ_l, and α are parameters.

3.4. Parameterization

I use industry data to estimate the model's parameters. The National Association of Theater Owners (NATO) (1998) lists the domestic theatrical grosses for every movie released from 1985 to 1997 by the major studios: Buena Vista, Columbia, MGM/UA, New Line, Orion, Paramount, Sony, TriStar, Twentieth Century Fox, Universal, and Warner Brothers. There are 2004 movies on the list. I deflate the grosses using the Consumer Price Index and define a "hit" as a movie that achieved a gross of at least \$50 million 1990 dollars. Using this definition, 15% of the movies are hits, which implies that $p = .15$.[13] The average gross of a hit is seven times the average gross of a flop ($95.95 million vs. $13.68 million). In the model's equilibrium flops leave the theater after one period while hits typically remain for two. Thus, to approximately match the data, the ratio $\lambda_h + \alpha\lambda_h : \lambda_l$ should be seven to one. I set $\lambda_l = .1$ (its scale does not matter), which implies that $\lambda_h + \alpha\lambda_h = .7$.

To proceed further, I use weekly data on revenue per screen and the number of screens each movie is showing on nationwide for a sample of 33 hits identified in the NATO data. The source for this information is www.the-numbers.com. A typical contract period is either four, six, or eight weeks in length. For each of these period lengths, I compute the percentage of box office revenue earned in the second period on a per screen basis. Revenue in the second period is approximately 30% of revenue in the first period, which implies that $\alpha = .3$.[14] Given that $\lambda_h + \alpha\lambda_h = .7$, this implies that $\lambda_h = .54$. The values of λ_h, λ_l, and α imply that to maximize current revenue all new hits should be allocated, then old hits, then new flops, and then old flops.

The discount factor $\delta = .985$. Given that a typical contract covers a six week period, this discount factor implies an annual discount rate of 14%, which is reasonable in a high-risk industry. The remaining parameters are the percentage of the population that is time constrained each period γ and the cost of replacing a movie early c. I lack data to estimate these parameters. I set $\gamma = .5$, which implies that every period, half of the consumers are time constrained.[15] Because the welfare comparisons and predictions depend critically on the values of c, I report results for different levels of c. The algorithm used to compute the value functions is described in Appendix B.

[13] This is consistent with De Vany and Walls (1996, 1999), who show that hits are rare: the distribution of cumulative box office revenues is best-approximated by the Pareto distribution, which is skewed to the right. The high cutoff point ($50 milllion 1990 dollars) reflects the high cost of making and marketing movies. The MPAA reports that in 2001 the average production and marketing costs of an MPAA member movie exceeded $70 million.

[14] This is consistent with De Vany and Eckert (1991), De Vany and Walls (1996, 1999) and Sawhney and Eliashberg (1996), who examine time series of ticket revenue: revenue per screen tends to fall over time.

[15] The MPAA reports that the average person in the U.S. and Canada age 12 and over sees 5.2 movies in a theater per year. Further, 26% of consumers age 12 and over never go to the theater, 10% go less than once in six months, 34% go at least once in six months, and only 30% go at least once a month. While this is not direct evidence of time constraints, it shows that assuming that many of a theater's potential customers see at most one movie in any given six week period is reasonable.

3.5. General Characteristics of the Equilibrium

One result that appears to be robust to a wide range of parameter values and modeling assumptions is that the monopoly setting (where one firm owns D1, D2, and EX) and the non-integrated setting yield identical choices and welfare. Thus, non-integration is efficient in the sense of maximizing total firm value. Because these two settings yield identical outcomes, I do not discuss monopoly in what follows.

As anticipated in Section 2, there are no differences between vertical integration and non-integration if c is sufficiently high. At values of c in excess of .085 (roughly $11.5 million 1990 dollars) early replacements never occur and all settings are identical in all respects. Distributors replace only finished movies. Equilibrium allocations maximize current revenue except in two respects. First, distributors avoid head-to-head competition with new hits. Second, new flops may bump old hits from the theater. This latter strategy may be optimal when no investment occurs because releasing a new flop is the only way to eventually remove it from inventory and obtain a chance of drawing a new hit. The rest of the discussion in this paper concerns values of c that are below .085.

Tables 1-6 provide results for increasingly higher values of c. The values of c approximate 1990 dollar amounts. Results are presented for two extreme values of c, $1 million and $8 million, and four intermediate values: $3 million, $4 million, $5 million, and $6 million. Each table lists firm value and the present values of revenue and replacement costs for the state where the players begin the allocation game with four new flops. Considering other states yields identical conclusions, so it is sufficient to examine this state. The tables also report simulation results of 100,000 periods of play (the simulation is described in Appendix B).

There are several general characteristics of the equilibrium that hold across settings and values of c. As in reality, hits have longer runs than flops: old flops are never shown in the theater, while old hits often are. Also as in reality, distributors avoid head-to-head competition with new hits. When two new hits are available, the release of one is typically delayed. Delay allows the distributor to avoid a large adverse cross effect on demand. One hit can play out in the theater and then the other one can be released. The percentage of periods in which at least one distributor delays releasing a new hit varies across settings and values of c.

The equilibrium contract terms affect firm value and thus influence strategic decisions. For example, in the non-integrated setting, taking D1's inventory as given, D1's value tends to be higher, the lower the quality of D2's movies. This follows from expression (3.1), which shows that D1's value is decreasing in the value D2 and EX can generate if only D2's movies are shown in the theater. Thus, D1 can increase its value by having hits, but its value increases even more if *it has hits while its competitor has flops.* Thus, delaying the release of new hits until the other distributor has flops is value-maximizing.

The payoffs from the marginal contribution equilibria roughly match those of real-world exhibition contracts. Real-world exhibition contracts use non-linear revenue sharing rules; distributors receive a higher share of revenue when revenue is high. The marginal contribution equilibria yield this pattern because hits are rare and the exhibitor's next best alternative is to show worse movies. For example, Table 7 compares allocation game payoffs for various states in the non-integrated setting when $c = .0365$. The only difference between the first and second rows is that D1 has a new hit in the second row. Although EX's payoff rises slightly, D1's payoff rises dramatically. Thus, D1 obtains the lion's share of the marginal value generated by its hit. The third row confirms that this pattern holds for old hits as well, and the fourth row shows that introducing a competing new hit does not change this conclusion. Comparing the fourth row to the first, each distributor's value goes up by .36 but EX's value goes up by only .16. The fifth through seventh rows confirm that similar patterns hold when D1 has two new hits, even if D2 also has one or more new hits.

3.6. Effects of Vertical Integration

When c is low enough that investment occurs, some general conclusions hold for every value of c. First, integration is always privately profitable no matter what the impact on D1's bargaining power. Since the non-integrated setting produces the same industry value as monopoly, integration cannot improve industry value; the best that can be achieved is no change. Second, D2's value is always higher when it appropriates more of D1's marginal contribution, and industry value is lowest in this case. This suggests that D2 will profitably appropriate D1's contribution if it can, but doing so harms the industry.

Additional results depend on how integration affects investment and delay. In turn, the effects on investment and delay depend on how integration affects D1's bargaining power. First, suppose D1 maintains its bargaining power after integration and continues to receive its marginal contribution. In this case, integration causes D2 to increase its investment and further delay releasing its hits. D2 replaces its flops early and uses its hits as threats to appropriate value from D1. In response, D1 reduces its investment. However, industry investment never falls, and if c is sufficiently high, industry investment rises.

Consumer surplus rises if industry investment rises because more hits reach the theater. As a result, integration may improve welfare. Suppose every consumer who sees a movie receives some amount of surplus. Given this, total consumer surplus in any period is a multiple of movie revenue in that period. The present value of consumer surplus at time t is a multiple x of the present value of movie revenue, which can be computed as the value functions are computed by taking into account the current allocation, possible future states, future replacement decisions, and so on. Total welfare is higher under integration if

$$W_N + xR_N \leq W_I + xR_I, \tag{3.6}$$

where W_N and R_N are the combined value of all three firms and the present value of revenue, respectively, for the non-integrated setting, and W_I and R_I are the corresponding values for the integrated setting.

Expression (3.6) yields a critical percentage of the ticket price that consumers must obtain as surplus in order for integration to be optimal. Tables 3-6 list these percentages. As c falls, the welfare comparison becomes more favorable to integration. Each investment has a lower cost and more investment occurs, so more hits reach the theater. For example, Table 3 shows that when $c = .0292$ consumers need to receive only 3.5% of the ticket price as surplus in order for integration to improve welfare. The MPAA reports that the average price of admission in 2001 is \$5.66. This implies an average willingness to pay that exceeds the 2001 average price of admission by only 20 cents. Tables 1-2 show that as c falls further, integration has no effect on industry value or consumer surplus, and as a result is welfare neutral, although it remains privately profitable. High industry investment occurs under both non-integration and integration; the only effect of integration is to redistribute value from D1 to D2.

Now suppose D1 loses all of its bargaining power after integration and gives its entire marginal contribution to D2. In this case, integration causes D1 to reduce its investment to zero. D2 still increases its investment, but total investment falls. Fewer hits reach the theater than in the non-integrated setting, and consumer surplus and welfare unambiguously fall. In this case, the welfare-lowering effect of integration occurs at all values of c. Other strategies also differ. Less delay occurs relative to the non-integrated setting, partly because fewer hits are generated, but also because D2 tends to show its own new hits instead of delaying. D2 does not need to delay releasing its hits in order to appropriate value from D1. D2 also has a greater tendency to bump old hits from the theater with D1's new flops in order to generate more turnover from D1 (which yields more hits).

When D1 and D2 have equal bargaining power after integration, the welfare comparison yields an intermediate case between the two extremes. As D2's bargaining power after the merger rises, the profitability of merging rises, industry value falls, and the welfare comparison becomes less favorable to integration. Results are presented for all cases except $c = .0585$ (Table 6). As discussed in Subsection 3.7, the value functions did not converge in this case. In a few other cases, the incentives created by equal bargaining power caused some aspects of behavior to fall outside the bounds created by the two extremes. For example, when $c = .0439$ (Table 5), the case with equal bargaining power involves D2 carrying new hits in its inventory more than in the case where D1 has all of the bargaining power. The effect of this is that more delay occurs; the probability that new hits are shown in the theater and the welfare effects are still within those of the two extremes.

Whether the effects described here occur in reality is an interesting question that is beyond the scope of this paper. However, it is worth noting that when D2 owns EX it is better able to manage investment, release dates, and run lengths to maximize the joint value of the two operations, and De Vany and Eckert (1991) argue that this was a major advantage of the studio system. However, the results show that maximizing the joint value of the two operations does not necessarily maximize total welfare; the effect of integration on the bargaining power of independents must be considered.

3.7. Robustness

Several parameters that are constant here might differ across settings. For example, if integration generates more hits, consumers might lower their probability of attending each one (reduce λ_h) and may also see fewer flops (reduce λ_l). The optimal ticket price might differ in the two settings. More generally, the exercise of holding other things equal and changing the organizational form does not consider selection effects. Distributors may choose which exhibitors to buy on the basis of how consumers are likely to respond to the new setting, how many competing exhibitors are in the area, and other factors.

However, if one is willing to assume that the parameters are constant across settings, the results described above occur under a wide range of parameter values. Changing parameter values produces intuitive effects. Increasing p leads to less delay in releasing new hits, and this is not surprising. The main incentive to delay is to avoid head-to-head competition with another new hit, but if more new hits are likely to arrive soon then head-to-head competition cannot be avoided by delaying. Increasing λ_h, decreasing λ_l, or increasing γ leads to more delay because hits become more damaging to each other. Increasing α makes old hits more valuable relative to new flops, and it is less likely that they will be bumped from the theater.

The results are also robust to changing modeling assumptions. Footnotes 5, 7, and 11 mention some modified versions of the model that yield similar results. For example, in a version referred to in footnote 11, the exhibitor can show only one movie if it chooses. This version generates results identical to those in Tables 1-6 except in two cases. First, in the non-integrated case when $c = .0292$ the value functions did not converge. I discuss convergence further below. Second, in the non-integrated setting when $c = .0585$ some value was redistributed from the distributors to EX without changing the total industry value or anything else.

After computing several variations, some key ingredients appear necessary for generating results like those presented above. First, there must be two ways for a firm to improve its movie types: invest or wait and replace. Whether investments are replacements or other types of investments does not matter. Second, in order for integration to make a difference when D1 receives its marginal contribution, there must be some distinction between improving $\overline{V}^{all}$ and improving $\overline{V}^{2e}$. Otherwise, D2's investment choices do not change when integration occurs, and the two cases are identical.

Old flops are important for creating large differences between improving $\overline{V}^{all}$ and improving $\overline{V}^{2e}$. To see why, suppose the state at the beginning of the replacement game is {new hit, new flop, new flop, old flop}, and compare replacing 2b with waiting. Suppose that if no replacements are made, the equilibrium allocation is 1a (the new hit) with 1b or 2a (a new flop). Now note that if 2b is replaced with a new hit or new flop, the allocation will still be one new hit and one new flop because distributors avoid head-to-head competition with new hits. Thus, if the goal is improving $\overline{V}^{all}$ there is no point in investing today; it is better to wait, save the expense of investing, and replace the finished movie the following period (the same aggregate distribution of future movie types is achieved). However, if the goal is improving $\overline{V}^{2e}$ then there is a gain to investing because 2b improves. Thus, D2 has a greater incentive to invest when it owns EX and D1 receives its marginal contribution. In general, D2's investments almost always improve $\overline{V}^{2e}$ even when they do not improve $\overline{V}^{all}$.

The effects of integration when D2 receives D1's marginal contribution are robust to several modifications of the model. The effects are mainly due to D1 having no incentive to invest. Given this, D2 can show its own movies in preference to D1's without affecting any of D1's decisions. D2 also has the incentive to increase turnover from D1 by releasing D1's new flops even when it has old hits it could show instead. This increases the probability that D1 will obtain a hit, and all value from D1's hits is appropriated by D2.

Although the results are robust to several modifications, the non-integrated setting and the integrated case with equal bargaining power could not be computed at *all* parameter values in all versions of the model. As noted in the previous subsection in the discussion of integration with equal bargaining power when $c = .0585$, this is true of the main model as well. As discussed in Appendix B, the computer algorithms are based on the common technique for computing dynamic programming problems with single decision makers known as "iterating on the value function." Stokey, Lucas, and Prescott (1989) prove that this process works under general conditions in problems with a single decision maker, but there is no proof that this process works with multiple players. In practice, I have found that when firms are close to being indifferent between alternative strategies that differ substantially, the value functions may cycle instead of converging. This does not pose a problem for reaching conclusions from this class of models. Changing parameters slightly allows convergence to occur in these marginal cases.

However, in some versions of the model convergence in the non-integrated setting occurs only at extreme parameter values where investment occurs in very few states or in most states. For example, when the exhibitor can threaten to show no movies in the allocation game, the results in Tables 1 and 2 do not change, but value functions for the non-integrated settings in the other tables do not converge. Thus, I cannot be sure how these threats would affect the equilibrium at intermediate levels of c. These threats do not affect the optimal allocation because they apply to all allocations, but they may affect investment decisions if they redistribute value from distributors to the exhibitor. Future work could attempt to determine conditions under which iterating on the value function works in multi-player settings. The existing literature that deals with numerically computing Markov perfect equilibria is small and only begins to address this issue (Ericson and Pakes 1995, Pakes and McGuire 1994, 2001, Gowrisankaran 1999).

4. Conclusion

This paper uses a dynamic common agency model with investment to explore the impact of vertical integration on strategies and welfare in the movie industry. Contract terms and investment interact with other strategic choices, the main one being the decision to delay releasing new hits. Several general conclusions about the effects of integration emerge. First, non-integration yields the same strategies and welfare as monopoly. This implies that integration cannot improve industry value. Second, the effects of vertical integration depend on how integration affects investment. If investment does not change when integration occurs then integration yields the same strategies and welfare as non-integration.

Third, the effects of integration on investment depend critically on what happens to the bargaining power of the independent distributor when integration occurs. In general, when integration occurs the integrated distributor's investment rises and the independent distributor's investment falls; the impact on total industry investment depends on which of these two effects dominates. If the independent distributor retains its bargaining power then industry investment never falls and may rise. If industry investment rises, the average quality of movies shown in the theater rises. The resulting increase in consumer surplus may cause total welfare to rise. If the independent distributor loses too much of its bargaining power then it reduces its investment substantially, the average quality of movies shown in the theater falls, and welfare unambiguously falls.

Other types of competition and contracting, such as block booking, production scheduling, and contracting with talent, could be explored in future work.[16] Before the *Paramount* decrees, independent exhibitors complained about vertical integration in the movie industry. Thus, adding additional exhibitors would be a useful step. Several other features of the industry could also be included. For example, most theaters are now part of a large chain. Although *Paramount* requires distributors to contract with each theater separately, often boilerplate contracts are used that apply to every theater in a chain. The presence of multiple chains could also be considered. Competition with additional exhibitors and chains may allow one to endogenize the impact of integration on bargaining power.

This paper has focused on vertical relationships in the movie industry, but the model could be adapted to examine how strategies and welfare implications differ across industries that have different demand and supply conditions. The interactions between contract choice and optimal investment, product introduction, and placement strategies could be studied in further detail. Also, the model might be applied to analyze non-retail environments in which multiple decision makers attempt to influence a third party over time.

References

[1] Aghion, Philippe, and Patrick Bolton "Contracts as a Barrier to Entry" *American Economic Review* 77 no.3 (June 1987): 388-401.

[2] Bergemann, Dirk, and Juuso Valimaki "Dynamic Common Agency" Yale University Department of Economics working paper (November 1998).

[16] Chisholm (1997), Weinstein (1998), Ravid (1999) and Borcherding and Filson (2001) analyze contracts with talent.

[3] Bernheim, B.D. and M.D. Whinston "Common Agency" *Econometrica* 54 (1986): 923-42.

[4] Bernheim, B.D. and M.D. Whinston "Common Marketing Agency as a Device for Facilitating Collusion" *RAND Journal of Economics* 16 (1985): 269-81.

[5] Bernheim, B.D. and M.D. Whinston "Exclusive Dealing" *Journal of Political Economy* 106 no.1 (February 1998): 64-103.

[6] Bolton, Patrick, and Michael D. Whinston "Incomplete Contracts, Vertical Integration, and Supply Assurance" *Review of Economic Studies* 60 no.1 (January 1993): 121-48.

[7] Borcherding, Thomas E., and Darren Filson "Conflicts of Interest in the Hollywood Film Industry: Coming to America - Tales from the Casting Couch, Gross and Net, in a Risky Business" in Davis, Michael, and Andrew Stark, eds. *Conflict of Interest in the Professions* (New York: Oxford University Press, 2001).

[8] Chisholm, Darlene C. "Profit-sharing Versus Fixed-Payment Contracts: Evidence From the Motion Pictures Industry" *Journal of Law, Economics, & Organization* 13 no.1 (1997): 169-201.

[9] Chisholm, Darlene C. "The War of Attrition and Optimal Timing of Motion-Picture Releases" Lehigh University working paper, 1999.

[10] Corts, Kenneth S. "The Strategic Effects of Vertical Market Structure: Common Agency and Divisionalization in the U.S. Motion Picture Industry" *Journal of Economics & Management Strategy* 10 no.4 (Winter 2001): 509-28.

[11] De Vany, Arthur S., and Ross D. Eckert "Motion Picture Antitrust: The Paramount Cases Revisited" *Research in Law and Economics* 14 (1991): 51-112.

[12] De Vany, Arthur S., and David W. Walls "Bose-Einstein Dynamics and Adaptive Contracting in the Motion Picture Industry" *The Economic Journal* (1996): 1493-1514.

[13] De Vany, Arthur S., and David W. Walls "Uncertainty in the Movies: Does Star Power Reduce the Terror of the Box Office?" *Journal of Cultural Economics* (November 1999): 285-318.

[14] Doyle, Maura P., and Christopher M. Snyder "Information Sharing and Competition in the Motor Vehicle Industry" *Journal of Political Economy* 107 no.6 (December 1999): 1326-1364.

[15] Ericson, Richard, and Ariel Pakes "Markov-Perfect Industry Dynamics: A Framework for Empirical Work" *Review of Economic Studies* 62 no.1 (January 1995): 53-82.

[16] Filson, Darren, and Fernando Fabre, Alfredo Nava, and Paola Rodriguez "At the Movies: Risk Sharing and the Economics of Exhibition Contracts" Claremont Graduate University working paper, 2002.

[17] Friedberg, A. Alan. "The Theatrical Exhibitor" in Squire, Jason E., ed. *The Movie Business Book*, 2nd ed. (New York: Fireside. 1992).

[18] Gowrisankaran, Gautam "Efficient Representation of State Spaces for Some Dynamic Models" *Journal of Economic Dynamics & Control* 23 (1999): 1077-98.

[19] Hanssen, F. Andrew "The Block-Booking of Films Reexamined" *Journal of Law and Economics* (October 2000): 395-426.

[20] Kahn, Charles M. and Dilip Mookherjee "Competition and Incentives with Nonexclusive Contracts" *RAND Journal of Economics* 29 no.3 (Autumn 1998): 443-465.

[21] Kenny, Roy W., and Benjamin Klein "The Economics of Block Booking" *Journal of Law and Economics* (October 1983): 497-540.

[22] Krider, Robert E., and Charles B. Weinberg "Competitive Dynamics and the Introduction of New Products: The Motion Picture Timing Game" *Journal of Marketing Research* (February 1998): 1-15.

[23] Martimort, David "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory" *RAND Journal of Economics* 27 (1996): 1-31.

[24] Murphy, A.D. "Distribution and Exhibition: An Overview" in Squire, Jason E., ed. *The Movie Business Book*, 2nd ed. (New York: Fireside. 1992).

[25] National Association of Theater Owners *1998-1999 Encyclopedia of Exhibition* (North Hollywood, California: NATO. September 1998).

[26] O'Brien, Daniel P., and Greg Shaffer "Nonlinear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure" *Journal of Economics & Management Strategy* 6 no.4 (Winter 1997): 755-786.

[27] Pakes, Ariel, and Paul McGuire "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model" *RAND Journal of Economics* 25 no.4 (Winter 1994): 555-89.

[28] Pakes, Ariel, and Paul McGuire "Stochastic Algorithms, Symmetric Markov Perfect Equilibrium, and the 'Curse' of Dimensionality" *Econometrica* 69 no.5 (September 2001): 1261-81.

[29] Prat, Andrea, and Aldo Rustichini "Sequential Common Agency" Tilburg University Department of Economics working paper (May 1998).

[30] Ravid, S. A. "Information, Blockbusters, and Stars: A Study of the Film Industry" *Journal of Business* 72 (October 1999): 463-92.

[31] Reardon, D. Barry "The Studio Distributor" in Squire, Jason E., ed. *The Movie Business Book*, 2nd ed. (New York: Fireside. 1992).

[32] Sawhney, Mohanbir S., and Jehoshua Eliashberg "A Parsimonious Model for Forecasting Gross Box-Office Revenues of Motion Pictures" *Marketing Science* 15 (1996): 113-131.

[33] Stokey, Nancy L, and Robert E. Lucas, Jr., with Edward C. Prescott *Recursive Methods in Economic Dynamics* (Cambridge: Harvard University Press, 1989).

[34] Weinstein, Mark. "Profit Sharing Contracts in Hollywood: Evolution and Analysis" *Journal of Legal Studies* 27 (January 1998): 67-112.

Appendix A: Value Functions

The state is {new hit, new flop, new flop, new flop} and the optimal allocation includes 1a and either 2a or 2b. The current period expected revenue is given by a function $g(.,.)$ whose arguments are the types of the two movies shown in the theater. Values are as follows:

$$\begin{aligned}\overline{V}^{all} &= g(\text{new hit, new flop}) + \delta[V^1(\text{old hit, new flop, old flop, new flop}) + \\ &\quad V^2(\text{old hit, new flop, old flop, new flop}) + V^e(\text{old hit, new flop, old flop, new flop})]\end{aligned} \tag{4.1}$$

where δ is the discount factor and V^1, V^2, and V^e are each player's expected value of beginning the following period at the given state.

$\overline{V}^{1e}$ considers only D1's and EX's value assuming that both of D1's movies are allocated to the theater. D1 has a new hit and a new flop, so

$$\begin{aligned}\overline{V}^{1e} &= g(\text{new hit, new flop}) + \delta[V^1(\text{old hit, old flop, new flop, new flop}) \\ &\quad + V^e(\text{old hit, old flop, new flop, new flop})]\end{aligned} \tag{4.2}$$

$\overline{V}^{2e}$ considers only D2's and EX's value assuming that both of D2's movies are in the theater. In this case, D2 has two new flops, so

$$\begin{aligned}\overline{V}^{2e} &= g(\text{new flop, new flop}) + \delta[V^2(\text{new hit, new flop, old flop, old flop}) \\ &\quad + V^e(\text{new hit, new flop, old flop, old flop})]\end{aligned} \tag{4.3}$$

Finally, consider how V^1, V^2 and V^e are determined. They depend on the outcome of the replacement game played at the beginning of the following period. For example, consider the state we began the analysis with: {new hit, old flop} {finished, new flop}. If in equilibrium D1 replaces 1b and D2 replaces 2a, then $V^i(\text{new hit, old flop, finished, new flop}) = W^i_{b,a}$, $i = 1, 2, e$.

Appendix B: The Computer Algorithm

The computer algorithm is based on the common technique for computing dynamic programming problems with single decision makers known as "iterating on the value function" (described by Stokey, Lucas, and Prescott, 1989). The procedure computes the value functions of each player using an iterative process. Time to convergence is minimized by performing as many calculations as possible before the iterations. First, specify the parameter values and construct a grid of all possible states. Second, compute the single-period expected revenue for each possible allocation of movies to auditoriums at each state that is a possible state of the allocation game. There are six possible allocations in each such state: {1a 2a} {1a 1b} {1a 2b} {2a 1b} {2a 2b} {1b 2b}. Third, at each such state take each possible allocation in turn and compute the state at the beginning of the next period given the allocation. For example, if the state is {new hit, new flop} {old hit, new flop} and the allocation is {2a, 1b}, then the future state is {new hit, old flop} {finished, new flop}. All of this information is stored as part of the grid.

Fourth, for each state that is a possible state of the replacement game, compute all of the possible future states that can occur given all of the possible replacement decisions that can be made, along with the probabilities that each future state occurs. For example, if the state is {new hit, old flop} {finished, new flop}, then the list of possible replacement decisions is shown in Figure 1. If the old flop and the finished movie are replaced, then the right-hand sides of the three equations below Figure 1 show the four possible outcomes and the probability that each occurs. All of this information is stored as part of the grid.

Each player has two value functions, one for its value in the replacement game and one for its value in the allocation game. To initiate the iterations all of the value functions are set equal to 0 at every state. At each iteration the algorithm takes the current value functions as inputs, computes the equilibrium at every grid point, and uses the equilibrium payoffs to compute new value functions. This process is repeated until the value functions no longer change. The convergence criteria is that the maximum difference between the new and old value functions of any player at any grid point in successive iterations must be less than 1e-10. Convergence takes 15-20 minutes using GAUSS version 5.0 on a Compaq Deskpro with a Pentium Pro processor and 128 MB of RAM.

An iteration involves two loops, one for the replacement game and one for the allocation game. In the replacement game loop, starting at the first grid point, the program uses each player's value function for the allocation game along with the probabilities and possible future states computed in advance to compute each player's expected value given every possible replacement decision. Then, the equilibrium of the replacement game is computed, and the equilibrium expected value that each player receives is stored as the value of the player if the replacement game is played from the first grid point. This procedure is repeated for every grid point until each player's value in the replacement game at every state has been computed. Then the loop for the allocation game begins.

In the allocation game loop, starting at the first grid point, the program uses the vectors of movie revenues computed in advance, each player's value function for the replacement game, and the future states computed in advance to compute each player's expected value given every possible allocation. Then, the equilibrium of the allocation game is computed, and the equilibrium expected value that each player receives is stored as the value of the player if the allocation game is played from the first grid point. This procedure is repeated for every grid point until each player's value in the allocation game has been computed. Once the last state on the grid is completed the new value functions are complete and the next iteration begins.

Once the value functions have converged, the equilibrium allocations are known for every possible state in the allocation game, and the equilibrium replacement decisions are known for every possible state in the replacement game. Given these decisions and the probabilities and possible future states computed in advance, it is straightforward to begin at a particular state and simulate an equilibrium path of play. Random numbers from the Uniform distribution are used to resolve uncertainty. When there are multiple equilibria at a particular state (as is often the case in the allocation game when some movies have the same type), each is selected with equal probability.

Figure 1. The Replacement Game when the state at the beginning of the period is {new hit, old flop} {finished, new flop}

		Distributor 2	
		Do Not Replace 2b	Replace 2b
Distributor 1	Replace Neither	$W^1_{0,a}, W^2_{0,a}, W^e_{0,a}$	$W^1_{0,ab}, W^2_{0,ab}, W^e_{0,ab}$
	Replace 1a	$W^1_{a,a}, W^2_{a,a}, W^e_{a,a}$	$W^1_{a,ab}, W^2_{a,ab}, W^e_{a,ab}$
	Replace 1b	$W^1_{b,a}, W^2_{b,a}, W^e_{b,a}$	$W^1_{b,ab}, W^2_{b,ab}, W^e_{b,ab}$
	Replace Both	$W^1_{ab,a}, W^2_{ab,a}, W^e_{ab,a}$	$W^1_{ab,ab}, W^2_{ab,ab}, W^e_{ab,ab}$

Superscripts indicate the player and subscripts indicate which movies are replaced. As an example, suppose D1 replaces 1b and D2 replaces 2a:

$$\begin{aligned} W^1_{b,a} = {} & p^2 W^1(\text{new hit, new hit, new hit, new flop}) \\ & + p(1-p)W^1(\text{new hit, new hit, new flop, new flop}) \\ & + p(1-p)W^1(\text{new hit, new flop, new hit, new flop}) \\ & + (1-p)^2 W^1(\text{new hit, new flop, new flop, new flop}) \\ & - c \end{aligned}$$

$$\begin{aligned} W^2_{b,a} = {} & p^2 W^2(\text{new hit, new hit, new hit, new flop}) \\ & + p(1-p)W^2(\text{new hit, new hit, new flop, new flop}) \\ & + p(1-p)W^2(\text{new hit, new flop, new hit, new flop}) \\ & + (1-p)^2 W^2(\text{new hit, new flop, new flop, new flop}) \end{aligned}$$

$$\begin{aligned} W^e_{b,a} = {} & p^2 W^e(\text{new hit, new hit, new hit, new flop}) \\ & + p(1-p)W^e(\text{new hit, new hit, new flop, new flop}) \\ & + p(1-p)W^e(\text{new hit, new flop, new hit, new flop}) \\ & + (1-p)^2 W^e(\text{new hit, new flop, new flop, new flop}) \end{aligned}$$

The W^1, W^2, and W^e functions on the right-hand side of these three equations represent the expected values of each player in the given states. For example, W^1(new hit, new hit, new hit, new flop) is Distributor 1's expected value if two new hits are drawn. There are four possible states because two movies are replaced, and each new movie can be a hit or a flop.

Table 1. The Non-Integrated and Integrated Settings: *c* = .00731 ($1 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: Equal bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops				
D1's Value	6.85	6.84	3.38	0
D2's Value	6.85			
EX's Value	13.45			
Value of D2 and EX Combined	20.30	20.31	23.77	24.52
Value of All Firms Combined	27.16	27.16	27.16	24.52
Value of Revenue	28.55	28.55	28.55	25.29
Effect of Integration on Welfare		No Effect	No Effect	Always Reduces Welfare
Value of Replacement Costs	1.39	1.39	1.39	.78
Value of D1's Replacement Costs	.70	.70	.70	0
Value of D2's Replacement Costs	.70	.69	.70	.78
Simulation Results: Percentage of Periods in which the Event Occurs				
D1 has a New Hit	29	29	29	20
D2 has a New Hit	29	29	29	27
Delayed Release of a New Hit Occurs	11	11	11	7.3
D1 has a Delayed New Hit Release	5.8	5.7	5.8	5.9
D2 has a Delayed New Hit Release	5.8	5.9	5.8	1.8
At Least One New Hit is in the Theater	50	50	50	41
At Least One Old Hit is in the Theater	50	50	50	29
A New Flop Bumps an Old Hit	0	0	0	4.9

Table 2. The Non-Integrated and Integrated Settings: *c* = .0219 ($3 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: Equal bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops				
D1's Value	5.52	5.48	2.61	0
D2's Value	5.52			
EX's Value	13.33			
Value of D2 and EX Combined	18.85	18.89	21.76	23.02
Value of All Firms Combined	24.37	24.37	24.36	23.02
Value of Revenue	28.53	28.53	28.49	25.19
Effect of Integration on Welfare		No Effect	Always Reduces Welfare	Always Reduces Welfare
Value of Replacement Costs	4.16	4.16	4.13	2.17
Value of D1's Replacement Costs	2.08	2.08	2.05	0
Value of D2's Replacement Costs	2.08	2.08	2.08	2.17
Simulation Results: Percentage of Periods in which the Event Occurs				
D1 has a New Hit	29	29	29	19
D2 has a New Hit	29	29	29	26
A Delayed Release of a New Hit Occurs	12	12	12	6.7
D1 has a Delayed New Hit Release	6.3	6.3	6.1	5.1
D2 has a Delayed New Hit Release	6.2	6.2	6.2	1.7
At Least One New Hit is in the Theater	50	50	50	40
At Least One Old Hit is in the Theater	50	50	50	32
A New Flop Bumps an Old Hit	0	0	0	6.6

Table 3. The Non-Integrated and Integrated Settings: c = .0292 ($4 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: Equal bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops				
D1's Value	4.87	4.82	2.23	0
D2's Value	4.87			
EX's Value	13.25			
Value of D2 and EX Combined	18.12	18.17	20.76	22.31
Value of All Firms Combined	22.99	22.99	22.99	22.31
Value of Revenue	28.36	28.40	28.40	25.07
Effect of Integration on Welfare (Percentage of the Ticket Price Consumers Must Receive as Surplus for Integration to be Optimal)		May Improve Welfare (3.5%)	May Improve Welfare (3.5%)	Always Reduces Welfare
Value of Replacement Costs	5.38	5.41	5.41	2.77
Value of D1's Replacement Costs	2.69	2.63	2.63	0
Value of D2's Replacement Costs	2.69	2.78	2.78	2.77
Simulation Results: Percentage of Periods in which the Event Occurs				
D1 has a New Hit	29	29	29	19
D2 has a New Hit	29	29	29	25
A Delayed Release of a New Hit Occurs	11	11	11	6.3
D1 has a Delayed New Hit Release	5.8	5.7	5.7	4.8
D2 has a Delayed New Hit Release	5.8	6.0	6.0	1.5
At Least One New Hit is in the Theater	50	50	50	40
At Least One Old Hit is in the Theater	50	50	50	32
A New Flop Bumps an Old Hit	0	0	0	7.4

Table 4. The Non-Integrated and Integrated Settings: $c = .0365$ ($5 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: Equal bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops				
D1's Value	7.96	4.40	2.07	0
D2's Value	7.96			
EX's Value	6.08			
Value of D2 and EX Combined	14.04	17.46	19.53	21.81
Value of All Firms Combined	22.00	21.85	21.60	21.81
Value of Revenue	23.57	25.85	25.14	23.04
Effect of Integration on Welfare (Percentage of the Ticket Price Consumers Must Receive as Surplus for Integration to be Optimal)		May Improve Welfare (6.6%)	May Improve Welfare (25%)	Always Reduces Welfare
Value of Replacement Costs	1.57	4.00	3.55	1.23
Value of D1's Replacement Costs	.79	.50	0	0
Value of D2's Replacement Costs	.79	3.50	3.55	1.23
Simulation Results: Percentage of Periods in which the Event Occurs				
D1 has a New Hit	19	19	19	17
D2 has a New Hit	19	28	26	19
A Delayed Release of a New Hit Occurs	3.6	7.4	6.7	2.9
D1 has a Delayed New Hit Release	1.9	2.9	5.2	2.4
D2 has a Delayed New Hit Release	1.8	4.6	1.5	.58
At Least One New Hit is in the Theater	34	42	40	32
At Least One Old Hit is in the Theater	34	42	32	27
A New Flop Bumps an Old Hit	0	0	7.7	4.9

Table 5. The Non-Integrated and Integrated Settings: *c* = .0439 ($6 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: Equal bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops				
D1's Value	7.69	4.32	2.10	0
D2's Value	7.69			
EX's Value	6.31			
Value of D2 and EX Combined	14.00	16.74	18.84	21.57
Value of All Firms Combined	21.68	21.06	20.94	21.57
Value of Revenue	23.57	25.83	24.79	23.04
Effect of Integration on Welfare (Percentage of the Ticket Price Consumers Must Receive as Surplus for Integration to be Optimal)		May Improve Welfare (27%)	May Improve Welfare (61%)	Always Reduces Welfare
Value of Replacement Costs	1.89	4.77	3.85	1.47
Value of D1's Replacement Costs	.94	.58	0	0
Value of D2's Replacement Costs	.94	4.20	3.85	1.47
Simulation Results: Percentage of Periods in which the Event Occurs				
D1 has a New Hit	18	19	18	17
D2 has a New Hit	19	28	36	19
A Delayed Release of a New Hit Occurs	3.7	7.4	19	2.9
D1 has a Delayed New Hit Release	1.9	2.9	3.4	2.4
D2 has a Delayed New Hit Release	1.9	4.6	15	.55
At Least One New Hit is in the Theater	34	42	38	32
At Least One Old Hit is in the Theater	34	42	37	27
A New Flop Bumps an Old Hit	0	0	.49	4.9

Table 6. The Non-Integrated and Integrated Settings: c = .0585 ($8 million 1990 dollars)

	Non-Integrated	Integrated: D1 has all of the bargaining power	Integrated: D2 has all of the bargaining power
Allocation Game with Four New Flops			
D1's Value	7.32	4.40	0
D2's Value	7.32		
EX's Value	6.48		
Value of D2 and EX Combined	13.80	15.31	21.11
Value of All Firms Combined	21.11	19.71	21.11
Value of Revenue	22.83	24.61	22.70
Effect of Integration on Welfare (Percentage of the Ticket Price Consumers Must Receive as Surplus for Integration to be Optimal)		May Improve Welfare (79%)	Always Reduces Welfare
Value of Replacement Costs	1.72	4.89	1.59
Value of D1's Replacement Costs	.86	.16	0
Value of D2's Replacement Costs	.86	4.74	1.59
Simulation Results: Percentage of Periods in which the Event Occurs			
D1 has a New Hit	17	15	15
D2 has a New Hit	17	40	20
A Delayed Release of a New Hit Occurs	2.5	21	2.4
D1 has a Delayed New Hit Release	1.2	.14	.024
D2 has a Delayed New Hit Release	1.3	21	2.4
At Least One New Hit is in the Theater	32	37	32
At Least One Old Hit is in the Theater	22	36	22
A New Flop Bumps an Old Hit	9.8	.97	10

Table 7. Equilibrium Values for Selected States in the Non-Integrated Setting: *c* = .0365 ($5 million 1990 dollars)

The State at the Beginning of the Allocation Game	D1's Value	D2's Value	EX's Value
{new flop, new flop}{new flop, new flop}	7.96	7.96	6.08
{new hit, new flop} {new flop, new flop}	8.45	7.90	6.11
{old hit, new flop} {new flop, new flop}	8.05	7.95	6.09
{new hit, new flop} {new hit, new flop}	8.32	8.32	6.24
{new hit, new hit} {new flop, new flop}	8.83	7.87	6.17
{new hit, new hit} {new hit, new flop}	8.67	8.25	6.33
{new hit, new hit} {new hit, new hit}	8.55	8.55	6.46