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Creating Capitalism:

Using Growth Models to Assess Transition

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*Eldon Smith Professor of Economics, Pomona College. This paper modifies Appendix A of Creating Capitalism: Reforms and Growth in Post-Soviet Europe, with Patricia Dillon, Edward Elgar Press, forthcoming, so that it may be read as a stand alone article. It is the first of two papers of an econometric analysis of the interactions between politics, reforms, and economic progress. In the second paper, Appendix B, the model is applied to ten years of experience in six transition economies. Here the precise linkages between each of the five key generic economic reforms from socialism to capitalism and choices and outcomes derived from neoclassical and endogenous growth models are developed.

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Abstract

Five generic reforms, price liberalization, property privatization, macroeconomic stabilization, microeconomic restructuring and trade liberalization, are integrated into both exogenous and endogenous growth models. This integration allows one to assess the implications of each reform for a representative consumer.

If one assumes that in assessing a prospective reform each voter, given his unique characteristics and circumstances, acts as if he were the representative consumer, then this framework allows one to evaluate quantitatively the prospects of each reform for each distinct group. This model can be used to forecast how different voters, young-old, flexible-rigid, working-retired, taxpayer-transfer recipient, will respond to each proposal. This can in turn be used to determine the likelihood of success of a democratic polity in transition to capitalism.

Creating Capitalism: Using Growth Models to Assess Transition

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Western economists have identified five generic economic reforms that are necessary for former socialist states to adopt in order to transform their economies into private market economic systems.[1] Table I lists these reforms. Much of the research on transition economies has dealt with various aspects of these five reforms,[2] with most of the emphasis on fiscal and monetary policies, exchange rate regimes and other monetary and financial issues.[3] Researchers have also focused on privatization methods adopted in different countries that have had significantly varying degrees of success.[4] Industry deregulation is often referred to in transition research as "restructuring."[5] It means how the economies disentangle centralized organization and disengage central power from more microeconomic economic decision-making processes. Our focus here is on long-run consequences of reforms for the real economy rather than on financial issues.[6]

Part of the reason that different countries have adopted reforms in varying degrees is that there has been considerable variance in the virulence of political resistance. In some countries voters through democratic processes have at times ejected reform governments in favor of parties of former communists.[7] In others, resistance has taken the form of subversion by insiders and of

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[1] Blanchard et al. (1992) and Lazear (1995) are two good examples.

[2] The burgeoning literature on this subject cannot be done justice here, but see Chavance (1994), Prust (1990), Wijnbergon (1992), Frydman and Rapaczynski (1994), Gotting (1993), Portes (1994), Hall and Koparanova (1995), Roemer (2000), Svejnar (1993) and Willett et al. (1995).

[3] See, for some examples, Oesterreichische Bank (1996, 1997, 1998, 1999, 2000), Bonin and Szekely (1992), Willett et al. (1995), Behrman and Srinivasan (1995), Hochreiter (2000), Kutan and Brada (2000), Lainela and Sutela (1995), Miller and Petrov (1992) and Nuti (1994).

[4] See Brady (1999), Boyco, Shleifer and Vishny (1995), Kortba (1993), Rondinelli (1994), Torok (1992), Vince (1993), Murrell and Wang (1993), Yamada and Braguinsky (1999).

[5] Hrncir (1992).

[6] See Brown and Earle (2000), Gustafson (1999) and Gylfason (1994).

[7] Bulgaria elected socialists (former communists) in July 1990 and in December 1994. Hungary elected former communists (MSZP) in June 1994. In May 1998 communists received 32 percent of the vote in Hungary. In the Russian Duma (the lower house of parliament that to some extent reflected popular sentiment), communists held pluralities in the elections of March 1993 and December 1995 with over 20 percent of the vote. Slovakia split from the Czechs and was largely under the control of former communist leader Merciar from December 1994 until October 1998.

corruption and violence.[8] But what can explain the variance in the pace of reform and economic performance among countries? More importantly, does reform imply better performance?

Table 1. Five generic economic reforms
1. Price liberalization
2. Property privatization
3. Macroeconomic stabilization
4. International trade liberalization
5. Industry deregulation/restructuring

We pose some specific questions: (1) Can we explain why some countries adopt reforms faster and more completely than others? (2) Can we show why these reforms matter—what exactly do they accomplish? (3) Do reforms actually lead to higher living standards? (3) As these countries evolve simultaneously toward democracy and markets, how do political choices about reforms interact with voting behavior?

To answer these types of questions we develop a set of models that links improvements in consumer well being to reforms. We then tie voting behavior to prospects for improved well being. Next we establish dual-direction linkages between economic performance and the politics of reforms. Elections are central to the linkage. We begin the process by linking the five generic economic reforms to models of economic growth. We subsequently link the reform-growth framework to voting behavior. We then design an econometric model and test the models with evidence from six countries.

Here we begin with a simple corn economy that at each moment in time produces one homogenous output (corn) Y_t with two inputs, labor N_t and capital (corn) K_t. Capital depreciates

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[8] Russia is of course the most prominent case of criminal conduct in economic affairs, but other transition economies were not immune. See Black, Kraakman and Tarassova (1999), Brady (1999), Gaddy and Ickes (1998), Gustafson (1999) and Frye and Shleifer (1997). See also Wissels (1996), Winiecki (1989, 1990) Murrell and Wang (1993), Brainerd (1998) and Yamada and Braguinsky (1999).

(corn rots) at constant rate δ.[9] Harrod neutral technological change, A_t, falls to earth like "manna from heaven"[10] at the fixed exogenous rate γ. We assume formation of human capital occurs at a constant geometric rate that depends on two parameters: the proportion of time spent in education or skill development, φ, and on the "efficiency" of education, μ, as measured by its impact on labor productivity.[11] Output may either be consumed (eaten), C_t, or saved and invested (as seed corn) for additions to the stock of capital (corn plants), I_t.

We assume that private individuals acting in a competitive environment make all production and consumption decisions. In section 1, we develop this growth model in three stages. First, we develop a Solow production function with the above features and derive the solution system with exogenously determined savings at rate s. Second, we develop consumer behavior; and third, we develop producer behavior. We show how price liberalization and property privatization operate directly through parameters of this model.

In section 2 we introduce a government's budget, beginning with a balanced budget model. Revenues derive from head taxes X_t. Government expenditures contain two items: defense spending (corn given to potential enemies as bribes for good behavior), G_t, and transfer payments (corn simply re-allocated among persons), Q_t. Net head taxes are $T_t \equiv X_t - Q_t$. We next allow a richer fiscal policy, taxes on capital income at rate z. We allude briefly to deficit spending. Government in the model provides new parameters through which macroeconomic stabilization and industry deregulation (restructuring) influence living standards.

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[9] This model follows the famous Solow (1956, 1970) model, which has formed the basis of growth analysis for over forty years.

[10] If new technology augments the labor input, it is called Harrod neutral technological change. We discuss below alternative models of technological change which include Hicks neutral, capital augmenting and labor and capital embodied.

[11] 'Jones (1998) suggests that development of human capital may be the transmission mechanism through which free trade assists in the transfer of technology from advanced to developing economies. The idea of human capital is used extensively by Becker (1964, 1993).

In section 3, we introduce endogenous technological change.[12] This allows us to model the causes of technological change, A_t. Endogenous technological change models either assume that new capital spawns externalities or assume that scale economies characterize new technology. Assuming scale economies, a noncompetitive (possibly patent-protected) sector uses labor input as researchers to produce new "ideas" or "designs" or, in the corn economy, genetically improved strains of corn. Thus, the rate of technological change will depend on three parameters, the rate of discovery, θ; the effect of existing knowledge A on the discovery rate, ϕ; and the elasticity of discoveries by researchers, ε.

In section 4 we open the economy so that trade can transfer technology from advanced societies where it is most likely to occur to transition economies. We assume technology is transferred through trade at rate κ to transition economies. This model provides a channel for free trade reform to influence living standards in transition.

The corn economy

From growth models one can determine paths over time for the flows of income, consumption expenditures, saving and the rate of capital accumulation. The steady state conditions will be shown to depend upon the values of certain parameters: initial conditions, consumer tastes, the technology of production, savings rates, population growth, capital replacement rates, and relative costs of capital and labor inputs.

Growth models traditionally are employed in cross-country studies in two ways. First, the models explain why different economies have different steady states. We show how reforms improve the steady state by altering specific steady-state conditions. Second, "transition dynamics" are used to explore the evolution of a transition economy toward a new reform-induced steady state. We suggest a third application of growth models, as inputs into the decision-making

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[12] This material is based on Jones (1998) as well as on earlier sources.

process of voters in elections.[13] We use the model to predict how people will vote when offered choices between various degrees of reform by different political parties or candidates in an election.

Specifically, we assume that each citizen views herself as the representative consumer in a growth model. She plugs the effects of each choice (implicitly or explicitly associated with each candidate or party) into her growth model, which is defined by the model that reflects her unique characteristics and tastes (which we develop more fully below). She then votes for the candidate whose policies regarding reforms make her better off in the calculus of her optimization problem. This integration of growth and political modeling formalizes the notion that voters act like economic agents, they are forward-looking, self-interested optimizers.

The steady state is determined and described first, then we will see how certain parameter changes cause the steady state to change, leading to a different path or outcome. This model informs how we integrate economic performance, reform measures and political outcome data for subsequent econometric analysis.

Production

Let Y_t, K_t, and N_t, be the time t quantities of output, capital, and labor respectively. We assume that the labor force grows at the constant rate η:

(1) $N_t = N_0e^{\eta t}$.

$N_0 > 0$ is the initial quantity of labor. The labor input is augmented by technological change and evolves over time as a result of the evolution of human capital. First, consider technological change. Let A_t be the level of technological change at time t and define $E_t \equiv A_tN_t$. We have assumed the A_t grows at rate γ. Thus, $A_t = A_0e^{\gamma t}$. Combining this with (1), $E_t = A_0e^{\gamma t}N_0e^{\eta t}$. E_t is

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[13] Hall and Jones (1997), Jones (1998), Gylfason (1994), Easterly and Levine (1997) all suggest that institutions are key to explaining why some economies are less successful than others. We take this one step farther by linking specific economic reforms to voting behavior, which is in turn linked to growth. This helps to bridge the gap between growth theory and public choice analysis.

the quantity of labor input at time t measured in efficiency units.[14] Now consider development of human capital. Let $H_t \equiv e^{\varphi\mu}E_t$ where φ is time spent in skill development and education and μ is the impact of this learning on labor productivity. Notice that human capital acquisition does not depend on time per se. Combining human capital and efficiency improvements from technology, we have

$$(2)\ H_t = e^{\varphi\mu}A_0e^{\gamma t}N_0e^{\eta t}.$$

The Solow aggregate production function in Cobb-Douglas form is[15]

$$(3)\ Y_t = K_t^{\alpha}H_t^{\beta}.$$

We assume that production is increasing in both arguments, and that marginal products are declining.[16] Both α and β are positive, constant, and less than one. Taking natural logs and partial derivatives of (3) shows that α and β are the respective output elasticities of the inputs. In the corn economy Y_t is either consumed, C_t, or invested, I_t. Given δ, gross investment equals the sum of net new investment (growth in capital at time t) and replacement requirements, δK_t; thus, the growth path of capital obeys the differential equation

$$(4)\ K\bullet_t = I_t - \delta K_t$$

where $K\bullet_t \equiv dK_t/dt$. If we assume constant returns to scale, then $\alpha+\beta = 1$, and if $y_{ht} \equiv Y_t/H_t$ and $k_{ht} \equiv K_t/H_t$, equation (3) may be rewritten as

$$(3')\ y_{ht} = k_{ht}^{\alpha}.$$

Taking natural log derivatives of the definition of k_{ht} we have $k\bullet_{ht}/k_{nt} = K\bullet_t/K_t - H\bullet_t/H_t$. Under constant returns to scale with constant growth of labor, the choice variable is the level of capital per H_t, k_{ht}. All the analysis is now done only in terms of equation (3'), i.e., in units of

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[14] Suppose, for instance, we start with 100 efficiency units of labor at time t. Let the number of workers increase by 1 percent and technology augments the efficiency of all workers (not just the new ones) at a .05 percent rate. Then E at time t+1 is 101.05.

[15] We begin here the practice of presenting only the Cobb-Douglas version of the models. In general all results that we exploit hold for the general case in which production satisfies constant returns to scale and diminishing marginal product of inputs.

output per worker measured in efficiency units of an effective human. Efficiency of an effective human allows both for technological change at rate γ and for evolution of human capital at rate $\varphi\mu$. Once the quantities of capital and output per efficiency unit of an effective human are determined, these same ratios will hold for all scale levels. Using the time path of capital, equation (4), we have

(5) $k\bullet_{ht} = i_{ht} - (\gamma + \eta + \delta)k_{ht}$

where $i_{ht} \equiv I_t/H_t$. If consumption is proportional to income (output) with marginal propensity to save of s,[17] then

(5') $k\bullet_{ht} = sk_{ht}^{\alpha} - (\gamma + \eta + \delta)k_{ht}$.

Figure 1 illustrates the situation. The top curve depicts equation (3'); output per efficiency unit of an effective human increases with capital per efficiency unit of an effective human at a declining rate (positive diminishing marginal product). The lower curve is the fraction of output per efficiency unit of an effective human not consumed, sy_{ht}, and thus invested, $sy_{ht} = i_{ht}$.

The ray from the origin is the sum of the growth rate of labor-augmenting technological change, labor growth rate and replacement requirements, $(\gamma+\eta+\delta)k_{ht}$. Consider points k_{h1} and k_{h2}. At k_{h1} investment exceeds the sum of the growth rate of technology, population and the rate of replacement requirements. At k_{h1}, then, K/H is increasing (the economy is in disequilibrium); it is moving toward k_h* from the left. At k_{h2}, the rate of capital formation is less than capital replacement plus population growth plus technological change so that capital per efficiency unit of an effective human is falling - the economy is moving toward k_h* from the right. Only at k_h* does gross investment exactly offset the per-efficiency human capital requirements needed to accommodate growth in technology plus population plus capital depreciation, so that capital per

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[16] This means that $\partial Y/\partial K>0$, $\partial Y/\partial N>0$, $\partial^2Y/\partial K^2 <0$, and $\partial^2Y/\partial N^2 <0$. In the Cobb-Douglas case $0<\alpha<1$ and $0<\beta<1$. Below we add the assumption that $\alpha+\beta=1$, constant returns to scale.

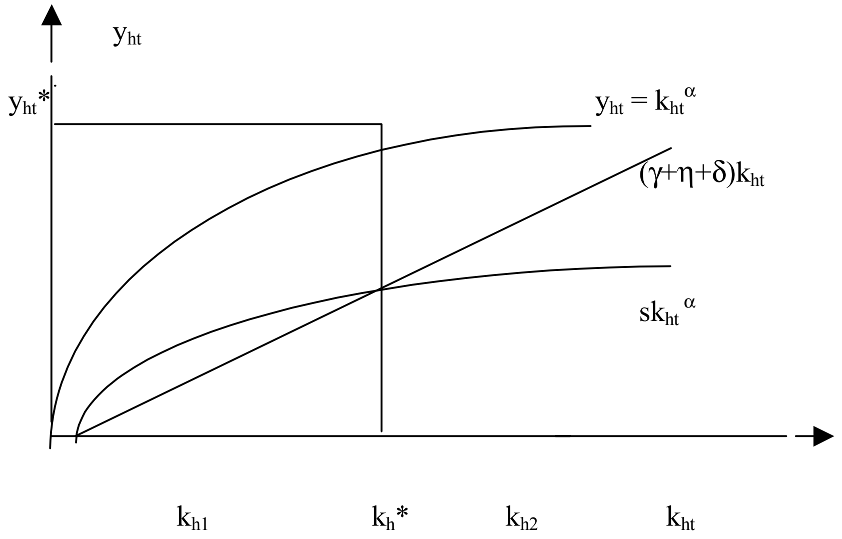
[17] We allow the consumer to choose s later; for now it is not necessary.

efficiency unit of an effective human is constant. Thus, at k_{ht}* the economy satisfies the steady state condition that $k\bullet_{ht} = 0$.

Figure 1 **The corn economy**

K/H and Y/H are constant:

$sy_{ht} = (\gamma + \eta + \delta)k_{ht}$.



Imposing the steady state condition, $k_h\bullet_t = 0$, on equation (5’), the capital-output ratio is

(6) $k_{ht}/y_{ht} = s/(\gamma+\eta+\delta)$.

The capital-output ratio is a constant determined by the savings rate and the rates of technological change, population growth and replacement requirements. In the Cobb-Douglas case, steady state k and y are

(6’) $k_h^* = [s/(\gamma+\eta+\delta)]^{1/(1-\alpha)}$ $\qquad y_h^* = [s/(\gamma+\eta+\delta)]^{\alpha/(1-\alpha)}$.

This model with a constant savings rate reveals important economic forces that determine different living standards in different countries.[18] If the savings rate (in the sense of the proportion of output devoted to productive capital), s, is high, then living standards will be high. Low savings rates could reflect extreme poverty or institutions, customs, and policies that discourage acquisitiveness. Ceteris paribus, rapid population growth, η, or high replacement requirements, δ, imply lower living standards. Rapid population growth could reflect institutions, customs and policies that encourage large families or rapid reproduction rates and longer life spans, and high replacement requirements could reflect shoddy production methods. However, because (6') indicates growth of capital and output per efficiency unit of labor, an increase in the rate of labor-augmenting technological change will cause growth in output per worker. Also, increases in the formation of human capital $e^{\varphi\mu}$ will raise output per worker. Finally, as α, the output elasticity of capital, converges on one, $\alpha \rightarrow 1$, living standards rise for each level of capital per worker.

The consumer

We now drop the assumption of an exogenous savings rate, and model consumer behavior. Assume the representative consumer is forward looking, self-interested, and infinitely lived.[19] Utility, u, at each moment in time depends only on consumption at that time, c_{ht}.[20] The present value of the consumer's future stream of utility is the continuous weighted sum of utility received at each moment in the future. Since she is evaluating future utility from today's perspective, she may choose to discount future consumption relative to current consumption. Let ρ be her subjective discount rate.[21] At time zero, utility U_0 is the present value of the discounted sum of the future stream of utility; each future moment's consumption is discounted by subjective rate ρ:

$$(7)\ U_0 = \int_{t=0}^{t=\infty} u(c_{ht})e^{-\rho t}dt.$$

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[18] We use the phrase "living standards" loosely here to refer to output per worker. In growth models, population and labor force growth are assumed to be the same.

[19] Since N is growing over time at rate η, we can think of the representative consumer as a family.

[20] We maintain the assumption of constant returns to scale and measure all quantity variables in units per efficiency unit of an effective human: $c_h \equiv C/H$. The consumer maximizes c_h. Recall that $H \equiv e^{\varphi\mu}EN$ where E represents Harrod neutral technological change and $e^{\varphi\mu}$ is human capital. One could assume that the consumer maximizes utility from consumption per family member, not per efficiency unit of an effective human; however, this results in an unreasonable result: consumption does not grow as a result of new technology, because the consumer discounts utility to offset future gains from technological change. This may be the case, but it suggests current generations can anticipate the rate of technological change.

[21] In Aesop's fable the cricket played all summer while the ant worked. Crickets have high values for ρ and ants have low values, perhaps even zero, for ρ.

If ρ is zero, then she does not prefer consumption now relative to consumption in the future.[22] The consumer receives income, y_{ht}, from two sources - labor (measured in efficiency units of effective humans) supplied inelastically earns wage w_t, and the capital stock, k_{ht}, which she owns, yields gross return v_t per unit.[23] The consumer selects the consumption-savings path that maximizes utility, subject to her income stream. She may either consume income or save it; thus income not consumed is saved and, therefore, available for investment, i_{ht}:

(8) $w_t + v_t k_{ht} = y_{ht} = c_{ht} + s_{ht} = c_{ht} + i_{ht}$.

The sources of income, $w_t + v_t k_{ht}$, equal the uses of income, $c_{ht} + i_{ht}$.

To solve the consumer optimization problem, we augment the time-t utility function to allow for the budget constraint. We introduce the budget constraint as imposed by the rate of growth of capital. The augmented optimization function, the Hamiltonian, is[24]

(9) $H_t = u(c_{ht})e^{-\rho t} + \lambda_t\{y_{ht} - [c_{ht} + (\gamma+\eta+\delta)k_{ht}]\}$.

The variable λ_t, the costate variable, is the value at t=0 of a time-t increment to capital. The optimum requires that the Hamiltonian satisfy three conditions:[25]

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[22] As with the production function, we will consider special cases of utility functions. At this time, though, this detail is not elucidating. An obvious candidate is the Cobb-Douglas function with constant returns to scale:

$u(c_{ht}) = c_{ht}^{\xi}$.

Unfortunately, with only one argument in the utility function, constant returns to scale restricts $\xi = 1$. The Cobb-Douglas form forces the elasticity of substitution parameter to one. A more general form is the constant elasticity of substitution form:

$$u(c_{ht}) = \begin{cases} (c_{tht}^{1-\xi} - 1)/1-\xi & \text{for } \xi > 0 \text{ and } \xi \# 1 \\ \text{In } c_{ht}, & \text{for } \xi = 1. \end{cases}$$

Here, the substitution, $\sigma = -\xi^{-1}$, is constant Other specific functional forms have been employed in growth modeling, including quadratic and log linear. In general, σ is not constant and depends on c_{ht}. With technological change the parameters of the utility function will enter into the steady state condition.

[23] The consumer is taking input prices as given. The steady state values for these terms will only be known after optimizing by the producer and the consumer.

[24] Here we follow Blanchard and Fischer (1989). See also Intriligator (1971).

[25] Condition (i) yields $\lambda_t = u'(c)e^{-\rho t}$ where $u' \equiv du(c_{ht})/dc_{ht}$. Solving this expression for $\lambda\bullet_t$, the left-hand side of condition (ii) is

$\lambda\bullet_t = [u''(c_{ht})c\bullet_{ht} - \rho u'(c_{ht})]e^{-\rho t}$.

If we assume the simple Cobb-Douglas technology then the right-hand side of condition (ii) is

$-H_k = \lambda_t\,[(\gamma+\eta+\delta) - \alpha k_{ht}^{(\alpha-1)})]$

Equating the two sides of condition (ii) yields

$[u''(c_{ht})c\bullet_{ht} - \rho u'(c_{ht})]\, e^{-\rho t} = \lambda_t[(\gamma+\eta+\delta) - \alpha k_{ht}^{(\alpha-1)}]$. Using (i) to remove λ_t, we have equation (A.10).

(i) $\partial H/\partial c = 0$

(ii) $\lambda\bullet_t = -\partial H/\partial k$

(iii) $\lim_{t\to\infty} k_t\lambda_t = 0$.

Imposing (i) and (ii) for utility maximization yields the Euler condition:

(10) $[c_{ht}u''/u'] [c\bullet_{ht}/c_{ht}] = \rho + \gamma + \delta + \eta - (\partial y_{ht}/\partial k_{ht})$.

The first term in square brackets geometrically represents the degree of curvature of the utility function that reflects the degree of flexibility of consumer tastes in shifting consumption over time. As a consumer's tastes in terms of indifference between consumption over different periods become more rigid, $u''\to\infty$. Right-angle Leontief indifference curves illustrate the extreme version of this case.[26] As a consumer's tastes become more flexible in terms of preferences for consumption among periods, $u''\to 0$ and the indifference curves become flatter, approaching linearity.

If we assume constant elasticity of substitution form among consumption between periods, then $[-u'/cu''] \equiv \sigma$; where σ is the elasticity of intertemporal substitution.[27] In this case, (10) becomes

(10') $c\bullet_{ht}/c_{ht} = \sigma\{[\partial y_{ht}/\partial k_{ht} - (\eta + \delta)] - [\gamma + \rho]\}$.

The Euler condition for the steady state suggests an intuitive explanation of the forward-looking consumer's optimal policy behavior.

This rule governs the optimal time path of consumption and depends on four concepts: the intertemporal rate of substitution (flexibility of tastes), σ; the subjective discount rate (degree of patience), ρ; the rate of technological change, γ; and the term in the first set of square brackets on the right had side of (10'). As with any elasticity concept, σ ranges from zero to infinity. If $\sigma = 0$ then the consumer is unwilling to substitute consumption between periods and $c\bullet_{ht} = 0$. If

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[26] A Leontief utility function illustrates the extreme case: $u(c_1 + c_2) = \text{Min}\,[c_1, c_2]$.

[27] See note 373. The CES function satisfies this condition.

$0<\sigma<1$ then she has an inelastic rate of substitution between periods—she is reluctant to substitute one period's utility for another—she is comparatively inflexible. If $\sigma = 1$, her intertemporal elasticity is unitary and she will substitute between periods if conditions warrant it. As $u'' \to 0$, $\sigma \to \infty$ and she is becoming more flexible between consumption at different times.

The term in the first set of square brackets on the right-hand side of (10') is the net marginal product of an increment of capital, i.e., the net increment is the marginal product of capital minus replacement requirements for depreciation and population growth.[28] For $\sigma \neq 0$, consumption per efficiency unit of an effective human will grow over time when this net marginal product of capital exceeds the sum of ρ, the consumer's rate of time preference, and γ, the rate of technological change. If, however, the net marginal product of capital is less than the rate of time preference plus the rate of technological change, then consumption per efficiency unit of an effective human will be declining over time.

Thus, the savings-consumption choice that determines how much current output the consumer is willing to put aside for capital formation depends on σ, ρ, $\partial y/\partial k$, η, δ, and γ. Ceteris paribus, flexible consumers (easygoing people with high σ) will be more willing to substitute consumption between periods in order to accommodate capital acquisition. ρ reflects the consumer's degree of impatience; impatient grasshoppers have a large ρ, so that ceteris paribus their savings rate is low. Farsighted ants have small ρ, so that ceteris paribus they will save more. A large value for η or δ discourages savings, because more of the gross marginal product of capital has to compensate for population growth or for more rapid depreciation. These forces can each reduce the net benefits from sacrificing consumption now.

Assuming that the parameters σ and ρ are fixed when the consumer is optimizing reflects the idea that historical, social, political, and economic forces have already determined tastes.

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[28] Since the consumer is optimizing consumption per efficiency unit of human capital the rate of technological change enters into the Euler condition.

These historical forces influence behavior even of forward-looking consumers. This serves as a proof that initial conditions in transition economies will influence growth patterns. It furthermore indicates precisely how initial conditions enter the optimization calculus. This allows us to identify behavior of various different economic agents.

Older, inflexible and impatient consumers and myopic, carefree grasshoppers will have a low value of σ and a high value for ρ. They will resist policies with short-run costs and long-range benefits. Young, flexible consumers and industrious ants with foresight will have large σ and low ρ and will tolerate current sacrifice for future consumption.[29] Finally, if the marginal product of capital net of population growth and depreciation is large relative to the subjective discount rate, then the person will forgo current consumption for future gains from capital formation. This means that economic efficiency, population growth and the quality of capital goods will also influence the proportion of output devoted to savings and investment. The rate of technological change enhances the growth rate of per-person consumption growth, because $c_t = c_{ht}e^{(\varphi\mu + \gamma t)}$. Thus, c_t grows over time at rate γ faster than c_{ht}.

The producer

The producer maximizes profits subject to input prices and the constraints of contemporary technology. Given exogenous labor growth η, exogenous evolution of human capital $e^{\varphi\mu}$ and exogenous technological change γ, the choice variable for the producer is the amount of capital per efficiency unit of an effective human, k_{ht}. Once this is determined for a steady state, the marginal product of capital, $\partial y_{ht}/\partial k_{ht}$, will be locked in and we can solve (10') for consumption and saving.

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[29] This is not to say that all young consumers are flexible and all old consumers are inflexible. These are illustrative extreme examples.

The gross return on one unit of capital, v_t, is the sum of r_t, the net return on capital, and δ, the rate of depreciation.[30] The time path of capital can be derived by employing the fact that $s_{ht} = y_{ht} - c_{ht}$. The growth rate of capital per efficiency unit of an effective human equals income minus the sum of consumption expenditures and the amount of new investment goods needed to accommodate labor force growth and improvements in worker efficiency (via technological change) and to replace depreciated capital:

$$(11)\ k\bullet_{ht} = s_{ht} - (\gamma+\eta+\delta)k_{ht} = y_{ht} - [c_{ht} + (\gamma+\eta+\delta)k_{ht}].$$

Equation (11) is the time path of capital condition. Consumer behavior is more complicated now so this equation is actually more complex than it may appear. Now consumption and therefore saving depends on utility maximization rather than being determined exogenously.

The producer maximizes the present value of his future profit stream. Profit at time t is output minus current labor and capital costs. Product price is normalized to one. In present value terms, profits are discounted from the future to the present at the net rate of return on capital, r:

$$(12)\ \Pi_t = {}_{t=0}\int^{t=\infty} [k_{ht}^{\alpha} - (w_t + v_t k_{ht})]e^{-rt}dt.$$

The producer selects the quantity of capital that maximizes profit, yielding

$$(13)\ \partial y_{ht}/\partial k_{ht} = \alpha k_{ht}^{\alpha-1} = v_t = r_t + \delta.$$

Thus, the producer's optimal decision rule is to set the marginal product of capital equal to the gross user-cost of capital at each moment in time.[31] This condition is called the marginal product of capital condition. Under constant returns to scale, the residual after payments to capital is equal to wage income. (Recall that one unit of labor is measured by an efficiency unit of an effective human):

$$w_t = (1-\alpha)k_{ht}^{\alpha}.$$

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[30] The relationship between capital prices, say q_t, and the service prices v_t, rate of return r and depreciation δ may be derived from the dual to the producer optimization problem; see Hulten (1992).

[31] The last equality is for the constant returns Cobb-Douglas case.

At the steady state, both inputs are paid the value of their marginal product; and under constant returns to scale, final product is exhausted by these payments.[32] The consumer's income is

$y_{ht} = w_t + vk_{ht}$.

This completes our derivation of the three steady state conditions:[33] The Euler condition for optimal consumption (10), the time path of capital (11), and the marginal product of capital (13). The conditions depend upon the parameters σ, ρ, δ, η, γ, α, and on factor prices w and r.

The modified golden rule

We can solve the steady state for the per-worker values of capital, output, consumption and utility. Let y*, c*, k* and u* be steady state values for per-worker income, consumption, capital and utility. We begin with the Euler condition, (10'). At the steady state $c\bullet_t = 0$ so that

(14) $y_k \equiv \partial y_{ht}/\partial k_{ht} = \alpha k_{ht}^{\alpha-1} = \alpha y_h^*/k_h^* = \rho + \gamma + \eta + \delta \qquad \text{for } \sigma>0.$

The steady state marginal product of capital equals the sum of four terms, the subjective discount rate, the rate of technological change, the growth rate of labor and the rate of replacement. The capital-output ratio under constant returns Cobb-Douglas technology is the constant $\alpha/(\rho + \gamma + \eta + \delta)$. The firm hires capital up to the point at which the marginal product equals the gross cost of capital, v. Thus, from equation (14), $r+\delta = \rho+\gamma+\eta+\delta$. The net rate of return on capital, r, in the steady state is determined by the sum of the rate of time preference, ρ, the rate of technological change, γ, and the rate of labor force growth, η.

The marginal product condition for capital (14) may be solved for the quantity of capital at the steady state, k_h^*.[34] The optimization problem brings ρ into the solution system. Higher discount rates usually lead to smaller steady state capital stocks, because $\rho>0$ indicates that

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[32] In the Cobb-Douglas case with constant returns, $w + vk = (1-\alpha)k^{\alpha} + \alpha k^{\alpha-1}k = 1$.

[33] Blanchard and Fischer (1989) show that an additional condition in this type of problem, called the transversality condition, must also be satisfied to prevent a solution that explodes and in which consumers could always accumulate capital without lack of benefits. This condition is satisfied by our system:

$\lim_{t\to\infty} k_t u'(c_{ht})e^{-\rho t} = 0.$

This condition rules out Ponzi-scheme financing.

consumers are less willing to sacrifice current consumption for accumulation and capital formation. In the Cobb-Douglas case,

$$
\begin{aligned}
(15) \qquad & k_h^* = [\alpha/(\rho + \gamma + \eta + \delta)]^{1/1-\alpha} \\
& y_h^* = k_h^{*\alpha} \\
& c_h^* = y_h^* - (\gamma + \eta + \delta)\, k_h^* \\
& u^* = u(c_t^*).
\end{aligned}
$$

Recall that $y_t = y_{ht} e^{\varphi\mu} A_0 e^{\gamma t} = Y_t e^{\varphi\mu} A_0 e^{(\gamma+\eta)t}$ so that the growth rate of income exceeds the growth rate of income per effective unit of human capital by $\eta + \gamma$. Thus per capita income is growing at the steady state at rate γ. Furthermore per capita income is increased at the steady state by higher human capital, $e^{\varphi\mu}$. Recall also that $c_t^* = c_{ht}^* e^{\varphi\mu} A_0 e^{\gamma t}$ and that $u[c_{ht}^*]$, so that at the steady state when $c\bullet_{ht} = 0$, consumption per person will be higher by the level of human capital and rising at the rate of technological change, γ. The second equation in (15) follows directly from the production function, $y_h = k_h^{\alpha}$. In general, the steady state capital output ratio, k_h^*/y_h^*, will be constant. At the steady state, k_h^* is constant. Thus, from equation (11), $i_h^* = (\gamma+\eta+\delta)k_h^*$ and savings equals investment, and so the third equality in (15) follows; namely, consumption c_h^* is y_h^* minus $(\gamma+\eta+\delta)k_h^*$. Steady-state utility simply depends on c_{ht}^*, given the functional form of utility.

If we set the derivative of c_{ht}^* with respect to k_h^* to zero, we have the maximum steady state value of consumption, $c_{ht}^{\ m}$. The consumer maximizes consumption when the marginal product of capital, y_k, equals $\gamma+\delta+\eta$. However, by equation (14), the steady-state marginal product of capital equals $\rho+\gamma+\delta+\eta$; thus, socially optimal consumption, c^g, will be less than c^m by ρ. Intuitively, this means that if consumers discount future utility then they will maximize utility at a lower c^*. Condition (14) modifies the golden rule*,* the famous rule for long-run maximization of

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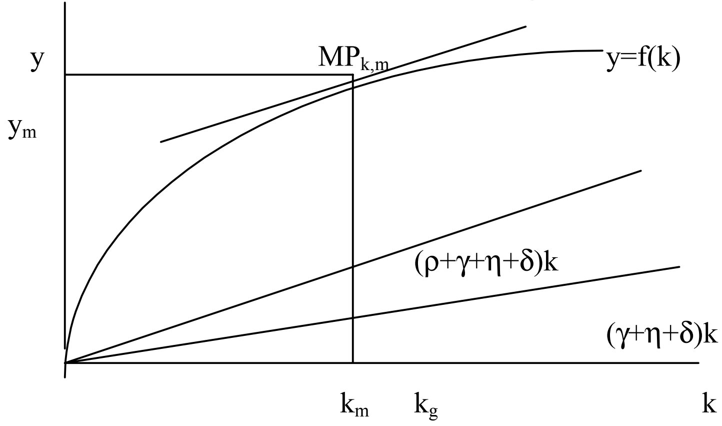
[34] This solution follows from the assumption of diminishing marginal product, because in this case f' is a monotonically decreasing function of k. Since y is increasing in k, we can also solve for y.

consumption modified because the steady state increment of output from capital must cover the subjective discounted rate ρ as well as γ+η+δ. The marginal product of capital is larger at the modified golden rule if ρ>0, so the modified golden rule level of capital and income are also less than the maximum values: $(c^{mg}, k^{mg}, y^{mg}) < (c^{m}, k^{m}, y^{m})$ where mg represents the modified golden rule and m the maximum consumption levels.[35]

Figure 2 illustrates the modified golden rule. The steady state occurs where the slope of the ray ρ +γ + η + δ is the same as the slope of the tangent to the production function. This point determines k* and y*. For ρ>0, this ray is steeper than the replacement requirements for capital per efficiency unit of an effective human. This result in turn implies that k* is lower than it would be in the case of the golden rule. Large discount rates mean that crickets are unwilling to sacrifice current consumption for capital accumulation and higher c* at the steady state.

Figure 2 The modified golden rule with ρ>γ



Reforms 1 and 2: Price liberalization and property privatization

With this model we can show how two of the generic economic reforms influence the long-run growth path. Price liberalization essentially replaces a system of administered prices with a

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[35] Blanchard and Fischer (1989) show that such a steady state is stable. All points in (c, k) space are characterized by pressures moving the economy toward a saddle path that leads to the modified golden rule steady state.

system in which prices are market determined. Thus prices now reflect consumer sovereignty. This means in turn that the mix of consumer goods will improve from the point of view of the consumer. Econometric interpretation of the formal model requires that the consumer good, c_t, be represented by an index of consumer goods. Thus, we model price liberalization as an increase in utility associated with each level of consumption:

$$\mathtt{u}(c_t) > u(c_t) \qquad \forall\, c_t,$$

where $\mathtt{u}$ is the new, post price liberalization utility function.[36] Price liberalization also applies to the prices of factor inputs so that the rate of return on capital, r, will rise too. One would also expect proper relative capital prices to lead to a lower depreciation rate δ in that those paying the appropriate price for capital would acquire less shoddy capital.

Property privatization also influences the steady state. The essence of property privatization is to foster improved production methods by creating private individuals as residual claimants of profits.[37] This reform has several important consequences on the steady state as represented by the solutions in equation (15). Private property owners are likely to develop better production methods. In our model this means an improvement in $f(k_{ht})$. This could be viewed as an upward shift in the production curve in Figure A.2. This is like a one shot increase in Hicks neutral technological change:

$$y_{ht} = a_t f(k_{ht})$$

where a_t is growing at an exponential rate and represents the shift that results from a Hicks neutral technological change shock. Since the production process itself may improve, we represent the new, post property privatization production technology as $\mathbf{f}$, assuming $\mathbf{f}(k_{ht}) > f(k_{ht})$ for all k_{ht}. In the model with Harrod neutral technological change developed above, privatization

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[36] Since utility here is ordinal, this is simply a rescaling, but the fact is that a better mix of consumer goods implies a happier consumer.

[37] Intertemporal optimization implies that accumulated capital lasts over time. To induce private agents to acquire such capital requires some degree of certainty that ownership rights will not be eradicated by nationalization. The wide political swings from promarket reform regimes to socialist (former communist) regimes certainly causes concern for potential investors, especially foreign investors.

has a sustained effect by increasing the rate of technological change, γ. Since property privatization improves production, it also reduces the rate of depreciation: $\boldsymbol{\delta} < \delta$ where $\boldsymbol{\delta}$ is the new depreciation rate. In terms of Figure A.2, slower depreciation means lower replacement requirements which rotates the rays from the origin downward, thus raising the steady state capital/income pair.

Third, property privatization, by creating private residual claimants, will foster acquisitive behavior. More acquisitive people are ceteris paribus more thrifty. Thus, ρ should be reduced and steady state savings s^* increased:

$\boldsymbol{\rho} < \rho$, $\boldsymbol{s}^* > s^*$.

The model with a specific functional form provides more insight into the growth consequences of property privatization. The utility function enters into the steady state in this model through two symbols, ρ and σ. As noted above, property privatization is likely to lower ρ because residual claimants are more foresighted. Private property owners are likely to more flexible and involved in their economic choices as well so we would expect σ to rise.

The human capital aspects of the models provide two new channels through which property privatization can raise the steady state. By fostering acquisitiveness, private property also encourages human capital acquisition via more education (φ rises) and via the transmission of this improved knowledge to production (μ rises). We turn now to vehicles through which macroeconomic stabilization reforms can enhance growth.

2. The public fisc and private budgets

Assume the government spends amount G_t on goods and services. The government also spends Q_t on transfer payments such as retirement benefits, welfare payments, veterans' benefits and other social spending. These outlays are financed in part by either direct taxes on the consumer, X_t, or taxes on capital income, v_tK_t, at rate z. We can simplify the derivations by defining all government budget magnitudes in the same units as output, capital and consumption; namely, amounts per efficiency-unit of effective human capital. Defining $g_t \equiv G_t/N_t$ and $g_{ht} \equiv G_t/H_t$, recall the definition of H as

(16) $H_t \equiv e^{\varphi\mu}A_tN_t$.

Therefore, $g_t = g_{ht}e^{\varphi\mu}A_t$ and $g\bullet_t/g_t = g\bullet_{ht}/g_{ht} + \gamma$.

These results show that growth of government spending per capita will exceed growth of government spending per efficiency unit of effective human capital by the rate of technological change γ. These formulations illustrate the point that once we solve the system in H units, we can employ (16) to solve for per capita values.

Defining net taxes to be head taxes net of transfers, $\tau_{ht} \equiv x_{ht} - q_{ht}$, then a balanced budget policy implies that $g_{ht} = \tau_{ht} + zv_tk_{ht}$. If $g_{ht} > \tau_{ht} + zvk_{ht}$ then the government issues bonds, $b\bullet_{ht}$. The public, indifferent between holding bonds and capital, purchases the bonds. The government must pay the competitive bond rate r the same net rate of return as capital. Table 2 summarizes the growth model with human capital, exogenous technological change and a government budget.

Consequences of fiscal policy

Fiscal policies affect the private sector through the consumer's budget constraint and the producer's cost of capital. The after-tax cost of capital becomes $(1- z)v_t = r + \delta$. This means that the productivity of the private capital stock must not only cover a return to the owner of capital, but also the proportion of tax revenues paid out of capital income.

Recall that the consumer's income derives from her ownership of the means of production as well as from her wage income. Her budget constraint is changed in two ways by taxes. First, she must pay τ_{ht} taxes net of transfers (she will be a net recipient if transfers exceed head taxes). Second, her income (say, in the form of dividend income) from capital is based on the after-tax cost of capital.

Table 2. Growth model assumptions

1. Technology: Labor growth rate η
production linear homogeneous $\alpha+\beta=1$
capital depreciation rate δ
diminishing marginal product $y_k = (\alpha-1)\alpha k_{ht}^{\alpha-2} < 0$
harrod neutral technological change rate γ
human capital from education $e^{\varphi\mu}$
$\Rightarrow$ growth rate rule of capital per human efficiency unit or
$k\bullet_{ht} = i_{ht} - (\gamma + \eta + \delta)\, k_{ht}$

2. Producer : rational, self-interested, forward looking
price taker w, r
pays capital tax rate z
labor (H) supply elasticity = 0
$H_t = e^{\varphi\mu}A_0e^{\gamma t}N_0e^{\eta t}$
chooses quantity of capital k_{ht}
maximizes profit
$\Rightarrow$ max present discounted value (PDV) of output minus costs or
$\text{Max} \int [k_{ht}^{\alpha} - \{w_t + [(r+\delta)/(1-z)]k_{ht}\}]e^{-rt}dt$

3. Consumer: rational, self-interested, forward looking
utility additive over time $U_0 = {}_t\!\int u_t\, dt$
utility depends on consumption per human $u(c_{ht})$
(efficiency unit of an effective human)
maximizes PDV of utility
owns capital stock k_{ht}

$c_{ht} \equiv C_t/H_t = C_t/e^{\varphi\mu}E_t = c_t/e^{\varphi\mu}A_te^{\gamma t}$

$\Rightarrow$ max PDV of future utility stream (subject to the budget constraint) or
$\max \int U(c_{ht})e^{-\rho t}dt$ subject to $y_{ht} - \tau_{ht} = w_t + (1-z)v_tk_{ht}$

4. Government: spends on goods and services g_{ht}
transfers payments q_{ht}
collects head taxes x_{ht}
net head taxes are τ_{ht}
taxes capital income rate z
finances deficits by issuing bonds b_t
$\Rightarrow$ sources and uses of GDP
$y_{ht} = c_{ht} + i_{ht} + g_{ht} = c_{ht} + s_{ht} + \tau_{ht} + zvk_{ht}$

Balanced budget

If the government balances its budget, then $g_{ht} = \tau_{ht} + zv_tk_{ht}$; and $b\bullet_{ht} = 0$, then $s_{ht} = i_{ht}$, private uses of income (consumption plus investment) equal private sources of income (wage income plus after tax-capital income minus taxes net of transfers):

$$(17)\ c_{ht} + i_{ht} = w_{ht} + (1 - z)v_tk_{ht} - \tau_{ht}.$$

The new time path of capital is

$$(18)\ k\bullet_{ht} = y_{ht} - g_{ht} - [c_{ht} + (\gamma + \delta + \eta)k_{ht}].$$

We assume that government expenditures do not alter marginal utility, so the consumer maximizes the same subjectively discounted future utility stream subject to the new budget constraint.[38] The new Hamiltonian is

$$(19)\ H_t = u(c_t)\, e^{-\rho t} + \lambda_t\{y_{ht} - g_{ht} - [c_{ht} + (\gamma + \eta + \delta)k_{ht}]\}.$$

The tax on capital income alters private market performance. The gross cost of capital, v, now equals $(r+\delta)/(1-z)$; i.e., v is deflated by one minus the marginal tax rate on capital income. Thus, the cost of capital is higher since it has to yield taxes before the producer can earn capital income. Given diminishing marginal product, this implies a smaller equilibrium capital stock. Wage income is the residual from total earnings minus gross income earned by capital. As above, this result follows from constant returns to scale technology. Recall that if each factor is paid the value of its marginal product, then all income is exhausted.[39]

More demands are placed on aggregate income now that the government uses a portion of output. Income, y_{ht}, must now accommodate government spending, g_{ht}, as well as private consumption and investment for replacement, labor force growth, and Harrod neutral technological

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[38] It is important to note that g may enter the utility function, and consumers may be willing to accept lower consumption and growth if the government spending is worth the costs. In the case of European transition economies, we now expect little utility from Warsaw Pact military spending and continued soft-budget constraints that prop up failing state owned enterprises. Productive infrastructure investment is in the investment term. This implies it has to meet the same rate of return requirement as private investment expenditures.

[39] Inserting the producer decision rules for hiring capital and labor into the budget constraint yields a new expression for the growth path of capital:

change. All of these uses must be accommodated before income can contribute to growth in capital per worker and thus improvements in living standards.

Although we caution interpretation of government in this model,[40] two fiscal policies reduce the steady state values of consumption and capital, (c_{ht}*, k_{ht}*). First, the marginal product of capital is higher because it equals $(\rho+\delta+\eta)/(1-z\alpha)$. As long as tax rate z is positive the gross marginal product must be greater and consequently the steady state quantity of capital, k*, lower. That is, capital taxes distort the private economy away from capital formation.

Steady state consumption is

(20) $c_{ht}^* = y_{ht}^* - [g_{ht}^* + (\gamma + \eta + \delta)k_{ht}^*]$.

The level c* at each k* is lower by the size of government - namely its spending level in the sense of its use of GDP. Since k* itself is lower, output is lower, thus c* is even smaller. We re-emphasize here that this does not imply that all government spending is bad. In fact from an econometric viewpoint it makes sense to think of infrastructure investment as part of the investment term and of government consumption goods as part of consumption. Government in our model consists of expenditures that are no longer necessary in a private market economy. This would include Warsaw Pact military expenditures and subsidies to prop up inefficient state owned enterprises and so forth. The main point is that government spending has a social cost that must be balanced against potential gains from its expenditures. Table 3 summarizes the steady state conditions.

Budget deficits

In the analysis so far we only modeled balanced budget policy. The effects of deficit spending in growth models on private sector decisions vary with the specification. In some cases deficits have no effects beyond those associated with the level of government activity itself. This is because in order to guarantee stability, constraints must be imposed on borrowing over the long run. This

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$k\bullet_t = y_{ht} - \{[z(r+\delta)/(1-z)]k_{ht} + \tau_{ht} + c_{ht} + (\gamma+\eta+\delta)k_{ht}\} = y_{ht} - [g_{ht} + c_{ht} + (\gamma+\eta+\delta)k_{ht}]$.

[40] See footnote 387 and the next paragraph of the text.

means that deficits are eventually cancelled out by surpluses – which means that the choice of taxing in the current period or running deficits has no effect on output—a case of Ricardian Equivalence.[41]

If arbitrary constraints were not imposed on long-run government borrowing, then governments would be able to postpone taxing to finance past deficits indefinitely, creating a Ponzi scheme outcome. Clearly such Ponzi schemes may well explain the behavior of state enterprise and financial officers in the endgame preceding the collapse of the Soviet system.

Table 3. - **Steady state conditions**

The Euler condition

$$\dot{c}_{ht}/c_{ht} = \sigma[(\rho + \gamma + \delta + \eta) - (1\text{-}z\alpha)y_k]$$

The modified golden rule

$$\partial y_{ht}/\partial k_{ht} \equiv y_k = (\rho + \gamma + \eta + \delta)/(1\text{-}z\alpha)$$

The dynamic path of capital

$$\dot{k}_{ht} = y_{ht} - [g_{ht} + c_{ht} + (\gamma + \eta + \delta)k_{ht}]$$

Steady state consumption

$$c_h^* = y_h^* - [g_{ht} + (\gamma + \delta + \eta)k_{ht}^*]$$

Reforms 3 and 4: Macroeconomic stabilization and industrial restructuring

We can now link two additional reforms to growth. First, consider macroeconomic stabilization. If reform consists of shrinking wasteful government spending, then we model this by a reduction in g. A lower value for g implies more private consumption at each steady state level of income. In the context of Central Europe, this reform reflects reductions in two types of government spending: Warsaw Pact spending on the military, and inefficient subsidies for state owned enterprises that could be better run by private owners.

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[41] See Blanchard and Fischer (1989) for derivations of the deficit case and an excellent discussion of the results.

Centralized governments had also established enormous systems of transfers creating various classes of wards of the state such as veterans, retired workers, ill citizens and so on. Recall that transfers in the model are just negative head taxes, $q = -x$. From a modeling perspective, reducing q has the effect of reducing taxes on some people and reducing benefits for others. While harmless in the aggregate model, this is an important result to each individual, because it suggests why some persons would endorse such a reform and some would not. This ambiguity reflects the ubiquitous welfare state debate over entitlements.

The government model includes a tax on capital income at rate z that distorts the economy away from capital formation and growth. In the model, this means that the after-tax earnings by the private sector fall and less capital is accumulated. We use this parameter as the one through which industry restructuring (deregulation) operates on the steady state. In many transition economies, nominal privatization has been easier than reducing burdensome government regulations on industry. Wage controls, restrictive hiring and firing practices, and confusing tax rules may combine with inadequate enforcement institutions for private property rights to impede effective privatization. Thus reducing inefficient government regulations on private enterprise acts like an increase in the after-tax rate of return on capital. Less regulation lowers z, the effective "tax and regulatory" burden imposed by government.

3. Endogenous technological change and technology transfer

The endogenous technological change model builds on the Harrod neutral technological change with human capital model developed in section 1. Endogenous models are based on several key insights. The first is that new technologies, rather than falling like manna from heaven, may be costly to produce, i.e., the development of new technology requires inputs that cannot be used to produce corn. Suppose a share of the labor force produces new " ideas," so that

(21) $N_t = N_{yt} + N_{At}$ and $s_A \equiv N_A/N$,

where N_{yt} is labor-producing final product (corn), and where N_{At} is labor-producing research (or ideas). The second insight is that ideas have unique characteristics that force us to treat them as public goods - they are nonrivalrous and nonexcludable. Once a new idea is discovered additional people can consume it without cost, and these additional users cannot be easily excluded from consumption of the idea once it is discovered. Put together this means that we assume that new ideas have initial setup costs in discovery, but are then free to reproduce and easy to disseminate. These features mean that technology of production in the ideas-producing sector is characterized by increasing returns to scale.[42] Increasing returns requires a non-competitive sector in order to achieve efficient allocation of new ideas. We will assume that government issues patents to researchers who discover new varieties of seed corn.

Assume that new varieties add to the supply of capital available for production. Suppose the stock of capital is the sum of A-types of seed corn, so that

(22) $K = {}_{j=1}\Sigma^{j=A} x_j$.

This formulation is awkward because the quantity of capital simply grows as new technology is brought on line.[43] Because research is noncompetitive, we need to model the demand side for new varieties of corn. Suppose we let the demand for each variety be the same, then K = xA. Under these assumptions the derivation of the steady state can be easily produced.

If we start with the human capital production function, then

(23) $Y_t = F(K_t,H_t) = K_t^{\alpha}(e^{\varphi\mu}A_tN_t)^{1-\alpha} = A_t(K_t/A_t)^{\alpha}(e^{\varphi\mu}N_t)^{1-\alpha} = A_tx^{\alpha}(e^{\varphi\mu}N_t)^{1-\alpha}$.

The human capital model is the same form as this model, because

$Ax^{\alpha} = {}_{j=1}\Sigma^{j=A} x_j^{\alpha}$.

The advantage of endogenous over exogenous models is that one can model the causes of A_t. Recall that $H_t \equiv e^{\varphi\mu}A_tN_t$; human capital is the number of effective units of human capital measured in efficiency units. We maintain the assumption that $e^{\varphi\mu}$ reflects additions to effective

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[42] The research sector in the corn economy uses genetics to produce new types of corn stalks that produce more corn per seed unit.

[43] Jones (1998) provides a lucid analysis of this model.

human capital from education and training, and we maintain the assumption of constant geometric growth of labor η.

Before we model A_t we show that the solution system for final product is of the same form as the Solow model with exogenous technological change and human capital. Now, however, we require a fraction of the labor force to be tied up in research. Only a fraction of workers are engaged in production of final product, corn; and thus, we define the proportion of workers in the competitive final output sector in efficiency units of effective human capital:

(24) $H_{yt} \equiv e^{\varphi\mu}A_tN_t = e^{\varphi\mu}A_t(1\text{-}s_A)N_t$.

From the point of view of modeling production in the final goods sector, the setup is exactly the same as the previous model, except that we will measure output and capital in efficiency units of effective human capital working in the final product sector. Defining $y_{\tau t} \equiv Y_t/H_{yt}$ and $k_{\tau t} \equiv K_t/H_t$, the new solution system with constant savings rate is

(25) $k_{\tau t}^* = [s/(\gamma+\eta+)]^{1/(1\alpha)}$ and $y_{\tau t}^* = k_{\tau t}^{*\alpha}$

$y_t^* = y_{\tau t}^* e^{\varphi\mu}(1\text{-}s_A)A_t$.

We now model technological change. This research originates with Romer (1986). Much of this modeling has been controversial, because some of these models suggest some rapid growth of output at the steady state resulting from increments of the labor force doing research. Jones develops a model that includes Romer's insights but avoids some of the problematic conclusions. The rate of change of new technology depends on two variables, the number of workers in the research sector, N_A, and the existing stock of technology (i.e., the number of ideas already discovered or simply the state of knowledge), A_t:

(26) $A\bullet_t = \theta A^{\phi}N^{\varepsilon}$.

θA^{ϕ} is the rate of discovery with ϕ representing the effect on the rate of discovery of the state of knowledge; ε is the researcher elasticity of discoveries. Jones discusses these parameters for advanced societies in which new research and development is taking place. He argues that $0<\varepsilon<1$, where values less than one mean that researchers are less productive at the margin.

Values of ϕ depend upon one's view of the implications for research of existing knowledge. Romer had implicitly assumed $\phi=1$. A case can be made for $\phi<0$ – the early researcher fishes out the biggest ideas first and only smaller ones remain to be discovered later. If $\phi>0$ then the idea is that today's researchers “stand on the shoulders of giants.” Today's researchers produce more varieties per unit of labor input than their predecessors did, because their predecessors laid the groundwork. If $\phi<1$, we can divide (26) by A_t, so that the left-hand side is the growth rate of discoveries. If discoveries occur at the constant geometric rate y, then the left-hand side of (26) over A is the constant rate γ, and we have

(26') $A\bullet_t/A_t = \theta A^{\phi-1}N^{\varepsilon} = \gamma$.

Taking natural log derivatives of (A.26'), we have

$0 = \varepsilon(N\bullet_t/N_t) - (1-\phi)\,(A\bullet_t/A_t) = \varepsilon\eta - (1-\phi)\gamma$.

The rate of technological change, should it reach a constant rate, is

(27) $\gamma = \varepsilon\eta/(1-\phi)$.

Thus, the rate of technical change, in this model, depends on the growth of the labor force, η, (assuming a fixed proportion do research), the effect on the discovery rate of new researchers, ε, and the effect on discoveries of the state of knowledge, ϕ.

We can also solve (26') for A_t:

(28) $A_t= \theta[s_AN_t]^{\varepsilon}\, A_t^{\phi}/\gamma$.

Replacing A_t in the (25) solution for the steady state income per person (living standards) with (28) yields

(25') $y_t^* = y_{\tau t}^*[e^{\varphi\mu}\,(1-s_A)]\{\theta[s_AN_t]^{\varepsilon}A_t^{\phi}/\gamma\}$

Equation (25') tells us that steady state per capita growth (which is, in effect, living standards) reflects three types of forces. First, $y_{\tau t}^*$, from equation (25), says that higher savings rates will mean a higher level of income, but rapid population growth and shoddy construction will vitiate this effect. We have been developing these forces throughout the modeling process; a high rate of accumulation of quality capital relative to population growth implies higher living standards. The development of consumer behavior in equation (10') introduced ρ and σ, two parameters through which reforms that benefit consumers are channeled.

The second set of forces in equation (25') involves the level of human capital in the society that is working in the final product sector, $e^{\varphi\mu}(1-S_A)$. This includes education and skill development and the implementation of that education on producing output. The third set of forces, $\theta[s_A N_t]^{\varepsilon} A_t^{\phi}/\gamma$, reflects the underlying causes of technological change. In this model, the level of technology depends on the proportion of workers involved in research, $[s_A N_t]^{\varepsilon}$, and the extent to which incremental researchers influence the discovery rate. The growth rate of technology rises with both N and A; however, their effects depend upon unknown parameters, θ and ϕ. These parameters reflect the nature of the research process, the evolution of discovery, and its effective impact on output.

In order to illustrate the model's contribution to understanding growth in transition economies, we now model how technology may be transferred to transition economies from the advanced countries like Germany, the US and Japan.

4. Technology transfer

We attribute pure technological change to advanced societies only and treat the growth rate of technology in these advanced countries as exogenous to emerging economies. We begin with the model in which magnitudes were measured in efficiency units of labor, but we replace A_t with L_t, which represents the learning level of workers in transition economies at adopting new technologies. Thus, we define $E_t = L_t N_t$, so the production function is

$$(29)\ Y_t = F(K_t, E_t) = K_t^{\alpha}(E_t)^{1-\alpha} = L_t(K_t/L_t)^{\alpha}(N_t)^{1-\alpha}.$$

We now define the stock of capital for a transition economy as the sum of the number of strains of technologically different corn that can be used in production by the workers in this economy. The equation for the stock of capital will be similar to (22). The number of strains will depend upon how much the workers have learned, L, up to the level of the most advanced world technology from industrial countries, A:

(30) $K = {}_{j=1}\sum^{j=L} x_j = Lx.$

The second equality follows from the same reasoning as in the endogenous technology models. It also follows that $y_t^* = y_t^* E_t$. Now we define the learning process in the transition economy, just as we had defined the evolution of technological change in advanced economies in earlier models:

(31) $L\bullet_t = \kappa e^{\varphi\mu} A_t^{\gamma} L_t^{-\gamma}.$

Directly from the earlier models, we now have

(32) $y_t^* = y_{et}^* H_t^* = y_{et}^* \kappa e^{\varphi\mu} (A_t/L_t)^{\gamma} N_t.$

Equation (32) differs from equation (25'). For a transition economy, we have integrated domestic learning (skill and education development) and the accumulation of new capital as a result of technological change. The idea behind this model is that as a transition economy's work force learns more about new technologies, workers can employ more advanced technologies via capital. However, because L enters in the denominator of the right-hand side of (32), as $L \rightarrow A$ increments to L become less able to advance income per capita. This means that it is simply harder to accumulate knowledge as one gets closer to the frontier of knowledge. Recall that L is the level of knowledge in the developing (emerging) economy and A is the level at the world frontier.

Reform 5: Trade liberalization

The key new parameter in equation (32) is κ. This is the parameter through which advanced world technology is transferred from developed economies to emerging economies. One can think of κ as the elasticity of human capital in an emerging society as a result of adoption of new technologies from advanced societies. Income is increased because domestic human capital in the emerging society rises. Since these new technologies are transferred from developed economies, we assume they do so through the degree of free trade between the emerging economy in question and advanced economies of the world.

We see this new parameter κ as the vehicle through which the free trade reform influences the steady state. The argument is that a consequence of free trade is that domestic industries are forced to compete on open markets. Another consequence is that multinational corporations with new technologies enter the emerging economy. These new firms bring with them new ideas. The domestic firms, in order to compete on international markets, also adopt new (cost saving) technologies. These advances in technology in emerging markets occur only if free trade is allowed between the emerging and advanced economies. Thus, free trade increases κ. We would expect free trade to alter other aspects of the steady state as well. For instance, free trade lowers costs and improves the mix of consumer goods. It also improves, via comparative advantage, the quality of capital and thus lowers depreciation.[44]

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[44] This conclusion does not contradict the notion that the mix of capital may eventually morph into faster depreciating assets, like computers instead of abacuses. But for a given asset type, comparative advantage indicates that broader trading zones allow more specialization, and improved product quality should be a byproduct.

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