# Population Genetics and Economic Growth*

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**Abstract**

This paper builds an age-structured model of human population genetics in which agents are endowed with a high-dimensional genome that determines their cognitive and physical characteristics. Young adults optimally search for a marriage partner, work for firms, consume goods, save for old age and, if married, decide how many children to have. Applying the fundamental genetic operators, children receive genetic material from their parents. An agent's human capital (productivity) is an aggregate of the received genetic endowment and environmental influences. Thus, the population of agents and the economy co-evolve. The model examines the impact of social and economic institutions on economic performance, including inequality in income and genetic attributes, the transition to an information economy, population bottlenecks, matchmaking, and love. We find that institutional factors significantly impact economic performance by affecting marriage, family size, and the intergenerational transmission of genes.

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*For its implications throughout biology and the social sciences, no subject is intellectually more interesting [than the co-evolution of genes and environment].*

Edward O. Wilson, *Consilience,* 1998

# 1  INTRODUCTION

IT IS WELL ACCEPTED that human capital is the source of long-run growth, as human capital engenders technological innovation (Lucas, 1988; Romer, 1990). This paper provides a biological foundation for the production of human capital by modeling the production of humans. Behavioral geneticists have shown that one's genetic endowment and early childhood experiences are the fundamental factors that beget adult cognitive skills (McClearn *et al*, 1997; Plomin & Petrill, 1994; Plomin, 1994). As a result, the way that individuals meet and mate, and the post-mating household structure, fundamentally determine adult productivity. Put more simply, we follow Becker (1993) by modeling the nature *and* nurture constituents of human capital.

In this paper we seek to take nature and nurture seriously. We do this by endowing each agent in the model with a high-dimensional genome which is derived from the primary genetic operators acting on his or her parents' genes: recombination, linkage, and mutation. An agent's genome codes for baseline cognitive ability, physical appearance, and gender. To characterize where an individual's genetic material comes from, adults make two-sided marriage decisions, and, if married, choose how many children to have. Natural selection occurs as some individuals are unable to find a mate with whom to reproduce.

Because the model includes the search for a marriage partner, we use findings from evolutionary psychology to identify the factors that lead individuals to marry,

including a spouse's human capital, physical attractiveness, the ability to effect reproduction, and the value of companionship. In this way, the model seeks to integrate the biology and psychology of human mating and reproduction with the economic and social environments that individuals finds themselves in. The model is therefore an exercise in population genetics (Takahata & Clark, 1993; Little & Haas, 1989; Keyfitz, 1984; Emlen, 1984; Ginzberg, 1983), but departs from the biological literature by modeling the (boundedly) rational decision-making unique to humans. Such a research program has been advocated by E. O. Wilson (1984), who stated during a conference in 1981 that "The optimization technique is, I think, very valuable for micro-evolutionary analyses, particularly when one [has] a thorough knowledge of the species and of its adaptive repertoires...Behavior and genetic distribution are then optimized over a small portion of the conceivable adaptive landscape."

In terms of economic choices, viz. consumption and production, an agent's genetic endowment acts as a constraint on individual choices. For example, education raises an agent's human capital and labor income, but does so only given one's genetic endowment. As a result, genetics constrains achievable consumption, savings, and one's ability to find a marriage partner. These in turn affect the economic landscape in the following generation. The population of agents and the economy therefore co-evolve.

Considering genetics and family structure as the building blocks of human capital stands in contrast with the literature that models human capital as purely education or experience with or without time or financing constraints (Lucas, 1988, 1993; Stokey, 1996; Galor & Zeira, 1993; Galor & Tsiddon, 1997a,b). Rather, the model in this paper is grounded in behavioral genetics (see Zak, 2000), extending the analysis of human capital to the level of the genome. The model is driven by marriage and fertility, and thus is related to the seminal analysis of fertility decisions by Becker & Tomes (1976), and subsequent work by Becker & Barro (1988), Becker, Murphy &

Tamura (1990), and Tamura (1996), as well as the literature on marriage matching (Becker, 1973, 1974; Lam, 1988; Burdett & Coles, 1997, Weiss, 1997). The primary point of departure of this paper is the inclusion of explicit biological factors for mating and reproduction that underpin the creation of human capital.

The model is solved numerically and calibrated to match U.S. demographic data. Once the model is calibrated, we examine a number of environmental and institutional changes that impact marriage and reproduction, including inequality in income and genetic attributes, the transition to an information economy, population bottlenecks, matchmaking, and love. We show that marriage and fertility decisions affect the economy's growth rate via the transmission of human capital from parents to children, and that the environment and institutions that impact these choices cause variations in economic performance. Indeed, inequality in income and pulchritude have persistent negative effects on fertility and output. Diseases that reduce population size also negatively effect the economy, with economic recovery times depending on the immunity pattern of the population. Endogamy through matchmaking also has pernicious effects on the economy, and can lead to population and economic extinction. Finally, including love in the model promotes genetic diversity while maintaining some of the rational aspects of marriage decisions. The primary finding of this paper is that genetic diversity is necessary for sustained economic growth, but too much diversity negatively impacts growth by reducing marriage opportunities.

# 2    THEORY

In this section we present a behavioral theory of economic growth in which individuals are heterogeneous over many dimensions: age, the expression of genes, marital status, gender, income, and the number of their siblings. Let the index $i \in \mathbb{R}^+$ identify individuals by all traits in which they vary from one another, with each agent having

a unique index. The model has a large number of agents who live three periods in overlapping generations.[1] The first period of life (period 0) is childhood, the second period is young adulthood (period 1), and the third period is old age (period 2). Following the standard economic approach, agents in this model maximize the lifetime utility derived from a variety of activities–consuming goods, being married, and having children–rather than maximizing, for example, reproductive fitness.[2] We show below that biological and psychological factors directly impact utility maximizing decisions so that fitness and utility are closely related.

Because our purpose is to endogenize the production of human capital by focusing on biological and environmental influences impacting marriage and fertility decisions, in the following subsections we characterize marriage and reproduction choices separately before presenting the full model that, in addition to these, includes choices for family size, consumption, and savings.

## 2.1   FINDING A MARRIAGE PARTNER

The first task that agents execute upon passing from childhood to young adulthood is to search for a marriage partner. Agents have a limited period of time to find a spouse, with matching occurring before any other decisions are made. In each matching round, two individuals meet and, if they are of the opposite sex, evaluate each other regarding suitability for marriage. That is, this is a two-sided matching

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[1]The overlapping generations framework was introduced in the ecological model of Leslie (1945), and independently developed and applied to economic exchange by Samuelson (1958). The model was significantly extended to include production by Diamond (1965), and subsequently many others; see Azariadis (1993).

[2]See Graefen (1998, 2000) for models of fertility in which agents explicitly maximize reproductive fitness.

problem.[3]  Note that the marriage market is competitive.  Buss (1994, p144) notes that "People with higher desirability have more resources to offer and so can attract a mate with a higher value. Those with low value must settle for less." To wit, when one marries one foregoes unions with other potential marriage partners.

In the marriage market, an agent knows his or her own attributes, with a potential mate's attributes observable upon meeting.  As each round of the marriage market opens, agents know the distribution of types of remaining potential spouses.  Agents who marry as young adults stay married through old age; agents do not die young, and divorce is not part of the model.[4]  Because siblings who are raised together almost never copulate due to a biologically-based sexual aversion to each other known as the Westermarck effect, we rule out marriage between siblings who meet during the matching game.[5]

Marriage confers several benefits to individuals.  First, as young adults of both sexes work in this model, marriage raises household income, though we make no distinction between market and nonmarket work as in Becker (1973).[6]  Because labor

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[3]Same-sex meetings impose no decisions for agents as friendship is not valued and homosexual unions are not part of the model.

[4]Marriage matching under uncertainty, divorce, and remarriage are examined in a companion paper, Zak & Park (2000); see also Greenwood, Guner & Knowles (1999) and Geddes & Zak (2000). On optimal separation, see Burdett & Coles (1999).

[5]Finnish anthropologist Edvard Westermarck (1891) first identified the sexual aversion of siblings. Subsequent genetic analysis identified the biological basis for this behavior as the presence of lethal genes that all humans carry.  On average, every individual has two lethal genes; if a particular location on the genome has a pair of lethal genes (i.e. one each from the father and mother), the fetus dies in utero or upon birth.  Mating by siblings raises the probability of homozygous lethal genes in an offspring to 1 in 4, from 1 in 50,000 in the general population.  This is known as "inbreeding depression," and is also associated with semi-lethal genetic diseases such as hemophilia.  Inbreeding depression is the presumed basis for incest taboos (Wolf, 1995).

[6]On market versus nonmarket labor supply choices for men and women, see Grosbard-Schechtman (1984), Grosbard-Schechtman & Neuman (1988), Weinhold, Zak & Morhenn (2000), and the survey

income increases in human capital, a spouse with more human capital is desirable for the income he or she brings into the household. We ignore intrahousehold bargaining as in Lundberg & Pollak (1993, 1996) and Greenwood, Guner & Knowles (1999) as the spousal division of income does not affect the model's results. Instead, we model household income and costs as being equally shared by both partners, using a variant of the Samuelson (1956) household consensus model. Equal sharing means that a married individual's consumption of goods can be higher or lower relative to what he or she would consume if single. Marriage still occurs between individuals of different human capital levels because there are several other utility flows from marriage besides the consumption of goods.

In addition to bringing in income, a marriage partner also enables reproduction. All agents in the model have preferences for children which captures the genetically programmed desire to reproduce. We ignore organic infertility in the model, though some individuals are infertile because they are unable to find a suitable spouse. Further, reproduction only occurs within marriage.[7] Married individuals also receive a pure utility flow from the act of being married, which we call the "joy of marriage." The joy of marriage can be understood as the utility from companionship, sex, or any of the other benefits that individuals receive from being married (Regan & Berscheid, 1997).

The literature in evolutionary psychology identifies two primary factors that consistently determine a mate's desirability across time and cultures: earnings ability and physical attractiveness (Buss, 1994, Buss, et al, 1990; Buss, 1989; Buss & Barnes, 1986). Following Burdett & Coles (1997), we will call an individual's desirability on the marriage market his or her *pizzazz*. Because women invest more biological

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in Weiss (1997).

[7]Throughout this paper we use "marriage" in its broad sense to denote a pair of agents who form a union and reproduce.

resources in their offspring than men, e.g. pregnancy, lactation, etc., over millennia women evolved to be highly sensitive to the availability of resources when choosing a mate (Trivers, 1972; Seger & Trivers, 1986). Historically, if a woman became pregnant by a man with few resources, both she and her child were unlikely to survive the rigors of pregnancy and post-partum childcare. Men, on the other hand, invest few resources in procreation. As a result, over evolutionary time genetically successfully men have focused on cues of a woman's health as an indicator of her desirability, i.e. the ability to bear and raise children to reproductive maturity. Standards of beauty have evolved to correspond to indicators of health and youthfulness, including smooth skin, large eyes, body symmetry, small waist to hip ratio, full lips, and shiny hair. (Johnston & Franklin, 1993; Buss, 1994.) We will use the short-hand *beauty* to denote physical appearance.

The differential investments by men and women in reproduction causes the evaluation of a mate's pizzazz to be asymmetric. The male-female asymmetry for mate selection preferences has been borne out in an exhaustive study of 37 cultures in 33 countries by Buss, et al (1989). Following directly from these evolutionarily-driven findings, the pizzazz of agent $i$, $p^i$, as evaluated by a potential spouse is a weighted sum of his or her human capital $h^i$, and beauty, $\Upsilon^i$,

$$p^i = \eta_q h^i + (1 - \eta_q)\Upsilon^i, \tag{1}$$

where the weight $\eta_q$ placed on each attribute varies across sexes $q = m, f$. The utility flow from the joy of marriage is increasing in one's mate's pizzazz as in Burdett & Coles (1997) since a higher pizzazz mate is more highly valued for both biological and economic reasons.[8]

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[8]Hammermesh & Biddle (1994) show that people identified as beautiful earn higher incomes. Because their sample is not representative, their findings may not generalize, so we ignore this issue in the present study. Nevertheless, it is easily included by permitting an agent's labor income to depend on pizzazz.

In sum, there are four factors that affect the marriage matching decision: random meetings, beauty, human capital, and reproduction. These factors are included in the model for two reasons. First, the available evidence indicates that these factors significantly and universally affect individuals' marriage decisions.[9] Second, the degree of assortative mating of marriage partners with respect to human capital alone is low (Regalia & Rios-Rull, 1999). Including psychological mating criteria in the model generates asymmetries in mating, though positive assortative mating vis-à-vis pizzazz continues to obtain. Further, searching for the best available marriage partner provides one's children with the best possible genes when they enter the marriage market; that is, this strategy is akin to maximizing reproductive fitness.

## 2.2    Genetics and Human Capital

The primary motivation for endowing each agent with a genome follows from recent work in behavioral genetics showing that a significant proportion of one's cognitive ability is inherited. Genetic factors that contribute to cognitive ability include short- and long-term memory, the ability to learn new information, awareness of social cues, decision-making aptitude, and language skills. McClearn *et al* (1997) estimate, by studying adult twins, that 62% of cognitive ability is due to genetics. Plomin & Petrill (1997) and Hamer & Copeland (1998, Ch. 6) survey a large number of studies of twins, siblings, and adopted children that use a variety of analytical techniques and report that the heritable proportion of intelligence is estimated between 48% and 75%.[10] In related work, Behrman & Taubman (1989) find that 81% of educational

[9]Other factors affecting mate choice also matter, including social and cultural factors, age of mate, chastity, religion, ambition, and love. These are all outside the model except for love, which we examine in Section 3.7.

[10]These studies are consistent with biological research that has begun to identify the specific genes responsible for intelligence. See Plomin, McClearn & Rutter (1997) and Brody (1992) on the genetic basis for intelligence.

attainment is due to genetics.

These studies indicate that formal education only accounts for about one-quarter of one's abilities. This occurs because highly educated members of society self-select to obtain more education as they are endowed with the ability to do so (Behrman, Rosenzweig & Taubman, 1994; Ashenfelter & Rouse, 1998; Rubinstein & Tsiddon, 1999). Self-selection by innate ability also occurs when workers choose particular types of jobs (Bartel & Sicherman, 1999). Plomin & Bergeman (1991) call self-selection of particular environments directed by one's innate abilities "the nature of nurture." Thus, human capital has a significant innate component.

In addition to genetics, family environment plays an important role in the development of human capital. Parents transmit social and cultural information, called "memes" by Dawkins (1976), to their children. Indeed, humans are distinct among primates in the length of their childhood, which is related to the time required for parents to teach children complex skills (Weiss & Mann, 1985). Household environment accounts for between 11% and 25% of the variance of cognitive ability in estimates by McClearn *et al* (1997), and Plomin & Petrill (1997), respectively. Because per child parental nurturing is affected by the number of children in a family, as family size increases beyond two each child's average educational attainment falls (Behrman & Taubman, 1989) as do grades in school (Downey, 1995). This effect obtains even after controlling for parents' income, education, and other family attributes. Interestingly, children's outcomes are better when there are two children in a family, rather than only one. Children without siblings receive maximal parental nurturing, but less child-to-child nurturing, and as a result their acquisition of human capital is lower than that for children in two-child families. Thus, parents have two effects on their children's human capital, the genetic endowment they provide them with at conception, and the familial nurturing that stimulates or inhibits the expression of genetic abilities.

An individual's genome is represented by a 52 dimensional vector, $\mathcal{A}$, with the first fifty dimensions corresponding to the genes for cognitive ability, the 51st determining gender, and the 52nd coding for beauty. The on-off nature of genes is captured representing the genes that code for cognitive ability and gender as taking values of either one or zero. Further, the genes for cognitive ability are ordered so that the most important factor, $a_1$, is first, and the second most important, $a_2$, is second, etc. Though this ignores the importance of combinations of genetic factors in the generation of innate ability, it will be a useful construct when comparing ability across individuals. Lastly, the genetic factors that produce beauty are compressed into a scalar, $b^i \in [0, 1]$. Using 52 genes in agents' genomes allows genetic operators (discussed below) to produce sufficient heterogeneity among individuals while at the same time ignoring DNA that does not appear to code for proteins or is redundant. Such "junk DNA" makes up 97% of the human genome.[11]

When the economy begins, the genetic endowment for the initial population of Adams and Eves is generated by a random draw from a uniform distribution at each location on the genome. After the initial generation, when individuals meet, marry, and reproduce, the genome of each child is determined by the fundamental genetic operators, *recombination, linkage*, and *mutation*, acting on his or her parents' genomes. Recombination (also called crossover) is the primary method through which a child's genetic material is produced from his or her parents' DNA. The contribution of the mother's and father's genes produce one of two possible outcomes. Suppose at a particular location, the mother has the allele $Aa$, where as is standard, $A$ denotes a dominant trait and $a$ a recessive trait, and the father is an $aa$. That is, the mother is heterozygous and the father is homozygous in this gene. Then, the child has a one in two chance of being $Aa$, and an equal chance of being $aa$ following the rules of

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[11]More precisely, *introns* which act as punctuation for genes are about 10% of the human genome, while *intergenic sequences* that have no known function are about 87% of the genome.

Mendelian inheritance.

Because of the difficulty in assigning dominance and recessiveness to traits as complicated as those that code for cognition, we adopt a simplified method for recombination that simply identifies the *expression* of each trait as "on" or "off." When genetic material is represented by zeros and ones, irrespective of dominance or recessiveness, if an child receives $(0, 0)$ from his or her parents, the expression of this trait will be "off," while if he or she receives $(1, 1)$, the trait is "on." The only difficulty occurs for heterozygosity, $(0, 1)$ or $(1, 0)$. In this case, we simply randomize to determine which trait is expressed, i.e. either a 1 or 0.[12] By doing this, we can reduce an agent's genome to a single vector of zeros and ones that represent the expression of each trait. A generic genetic code for an individual is

$$[0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0.456].$$

In this example, the first fifty columns of the genome vector code for the expression of genes that determine innate ability, the $51^{\text{st}}$ column is one's sex (0 denotes female; 1 denotes male), and the $52^{\text{nd}}$ codes for the agent's beauty, in this case, 0.456.

The second genetic operator is linkage. Linkage binds the alleles of adjacent genes together so that they always occur as a pair. Because recombination is the primary way that genetic material is distributed across generations, linkage has a small effect on human genetics (Strickberger, 1985). Further, the extent of linkage for any two adjacent genes is, at the present time, an open question. As a result, we include this operator by linking genes 10 and 11, which would imply that roughly 2,000 genes are linked for an average human being. By linking these two genes, we can examine how pairs of traits evolve over time.

The final genetic operator is mutation. For each gene, after recombination and

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[12]We ignore a host of other effects that impact recombination, such as maternal age, cell temperature, nutrition, the presence of chemicals or radiation, as well as genotype and chromosome structure as these are second-order effects.

linkage specify the potential genetic draw, there is a small chance that a type 0 will mutate to a type 1 or vice-versa. In humans, the mutation rate is one to two for a genome of 100,000 genes (Strickberger, 1985). We calibrate this rate to our smaller genome and apply it to each gene independently as the final genetic operator.[13]

The single gene for beauty is used because beauty does not affect human capital directly, but does so only indirectly via marriage matching. For this reason, we telescope the many factors that produce physical characteristics into a single index. Because beauty is a scalar, its inheritance dynamics are simpler than that for cognitive characteristics. In particular, a child receives the average of his or her parents' beauty, plus or minus a stochastic disturbance. In this way, beauty runs in families, consistent with evidence that a wide variety of physical characteristics such as height and weight do (Mueller, 1976). Further, this construct biases beauty to be mean-reverting absent assortative mating, but permits some agents to be more or less beautiful than their parents. To accomplish the latter, we use a random draw for noise that is uniformly distributed on $[-.10, .10]$. That is, the maximum difference between a child's beauty and the average beauty of his or her parents is 20%.

We now construct an agent's human capital as an aggregate of genetic factors, familial nurturing, and education/experience. In this model, education, $e$, which we define to include experience, is taken to be a unit mean random variable, capturing formal education and learning-by-doing, and is essentially the earnings shock in Becker & Tomes (1976). Then, human capital of agent $i$ is

$$h^i = \omega \Sigma_{j=1}^{50} \alpha_j a_j^i - \theta \mid b^{ip} - 2 \mid + \aleph e^i, \tag{2}$$

where $a_j$ is the expression of gene $j$ (i.e. a zero or one) in the genome $\mathcal{A}$ which is weighted by the monotonically declining sequence $\alpha_1 > \alpha_2 > ... \alpha_{50} > 0$, $\theta \in (0,1)$ is the dilution effect of familial nurturing from having either a single child or more

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[13]If either location 10 or 11 mutates, the new gene pair become linked.

than two children in a family, where $b^{ip}$ is the total number of children agent $i$'s parents have, and $\aleph \in (0, 1)$ is the impact of education on human capital. Equation (2) is designed so that genetic factors and education are positively weighted, with $\Sigma_{j=1}^{50} \alpha_j + \aleph = 1$, while parental dilution of nurturing is either zero or negative. The declining series of weights in $\Sigma_{j=1}^{50} \alpha_j$ corresponds to the ordering of genes by their importance in the generation of human capital. Traits that are "off," i.e. are 0, receive no weight in producing one's innate ability. As a result, education does not produce much human capital absent innate ability, consistent with the empirical analysis of education in Hendricks (2000).

The final parameter in (2), $\omega \geq 1$, captures the "Flynn effect" after Flynn (1987) in which IQ in developing countries has been shown to increase between ten and twenty-five percent per generation. Wills (1998) argues that the evolution of intelligence is accelerating because of rapid cultural changes, including the ubiquity of visual media, and improvements in child health due to better nutrition and vaccines. The parameter $\omega$ is included to capture the environment–genetics feedback in a non-Lamarckian manner.[14]

## 2.3   THE MODEL

We now formalize the marriage matching and reproduction aspects described above, as well as specify agents' consumption/savings decisions. As is standard, all economic variables are written relative to an agent's human capital so that human capital enters the model tractably. This model has a single good that can be used for consumption

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[14]Lamarck (1809) argued that learned traits entered the germline and were passed down to children, a theory for which little evidence exists today. Rather, the genetics-environment feedback occurs through environmentally mediated "regulator genes" that turn off and on "structural genes" that code for proteins, affecting childhood development without passing through to the germline. Edelman (1992) and Gazzaniga (1992) apply the regulator genes model to explain intelligence and consciousness, though the molecular basis for this mechanism is just beginning to be discovered.

or investment, with the mathematical structure being quite similar to a standard overlapping generations model (Zak, 2000). During childhood one's consumption is chosen and funded by one's parents. For this reason, children do not derive utility during childhood, though their parents receive utility $\gamma \ln(b)$ from the total number of births, $b$, in the family for $\gamma > 0$. The cost of raising a child is $db$, where $d$ is the per-child cost. Agent $i$ receives utility from consuming $c_1^i$ as a young adult, and $c_2^i$ during old-age when he or she is retired. Young adults work, earning labor income $wh^i$ which is the economy-wide average wage $w$ times agent $i$'s human capital $h^i$.

Each agent, whether single or married, makes choices for himself or herself, though married agents' choice sets and constraints differ from that of singles. Success or a lack of it in the marriage market determines which group an agent ends up in since every agent searches for a spouse as a young adult. If agent $i$ marries agent $j$ who has pizzazz $p^j$, then $i$ receives the joy of marriage utility $\pi p^j$, for $\pi > 0$ as in Becker (1973, 1974). After marriage, $i$ and $j$ are able to reproduce. Couples choose family size by comparing each parent's share of household labor income $\frac{w(h^i+h^j)}{2}$ with the per parent opportunity cost of children, $\frac{d^{ij}b^{ij}}{2}$ as shown below. Note that by equally sharing income and expenses, there is no disagreement within a couple over how many children to have in contrast to Galor & Weil (1996).

When agents' preferences over goods are logarithmic, an agent of type $i$ born at time $t-1$ and potentially married to a spouse $j$ solves

$$Max_{c_1^i, c_2^i, \zeta b^{ij}} (1-\beta)\ln(c_{1,t}^i) + \beta\ln(c_{2,t+1}^i) + \zeta\gamma\ln(b_t^{ij}) + \zeta\pi p^j \tag{3}$$

s.t.

$$c_{1,t}^i = \frac{w_t(h_t^i + \zeta h_t^j)}{1 + 1\zeta} - \zeta\frac{d_t^{ij}b_t^{ij}}{2} - a_{t+1}^i \tag{4}$$

$$c_{2,t+1}^i = R_{t+1}a_{t+1}^i \tag{5}$$

$$b_t^{ij} \geq 2. \tag{6}$$

Because each individual makes marriage, consumption, and savings decisions on his or her own, the budget constraint for agent $i$ when he or she is a young adult, (4), who is married ($\zeta = 1$) to an agent with human capital $h^j$ equally shares household income and expenses, and saves assets $a^i$ for old-age consumption. The parameter $\beta \in (0,1)$ specifies the weight placed on old-age vs. young consumption, i.e. $\beta$ measures patience. If $i$ is single ($\zeta = 0$), the consumption/savings decision is based on his or her own labor income, absent child-rearing costs. Consumption when $i$ is old, (5), comes from the principle and interest on savings as a young adult, with $R$ being one plus the interest rate. The last constraint (6) limits the number of children that married couples have to be two or greater in order to match U.S. data showing that the average number of children produced by married couples is two (U.S. Census Bureau, 1998, Table B).

Because the primary cost of children is the time required to raise them (Birdsall, 1988), the cost relation $d^{ij}$ is parameterized as a function of labor income,

$$
d_t^{ij} = \begin{cases} D\left(\frac{w(h_t^i + h_t^j)}{2}\right)^2 & \text{for } \frac{w(h_t^i + h_t^j)}{2} < \kappa \\ D_1 \frac{w(h_t^i + h_t^j)}{2} & \text{for } \frac{w(h_t^i + h_t^j)}{2} \geq \kappa \end{cases}
\tag{7}
$$

for the constants $0 < D < \frac{1}{w(h_t^i + h_t^j)/2}$ $\forall i, t$, and $D_1 = \frac{\gamma}{2(1+\gamma)}$, where $\kappa \equiv \frac{\gamma}{D(1+\gamma)}$. This bifurcated cost function arises because of the lower bound on the number of children, with the change occurring at $b^{ij} = 2$. The function $d^{ij}$ is designed so that agents' economic choices vary in a consistent manner with the number of children in a family, i.e. are identical whether $b^{ij} > 2$ or $b^{ij} = 2$.

It is convenient to present the solution to the agent's lifetime utility maximization problem in the reverse order in which it is executed. To wit, the optimal solution to (3) for the amount to save from period $t$ to $t+1$ when agent $i$ is married to agent $j$ ($\zeta = 1$) or unmarried ($\zeta = 0$), is

$$
a_{t+1}^{i\star} = \left[\frac{\beta}{1 + \zeta\gamma}\right] \frac{w_t(h_t^i + \zeta h_t^j)}{1 + 1\zeta}.
\tag{8}
$$

Optimal consumption during young adulthood and old age are found by substituting optimal savings (8) into the budget constraint for each period (4) and (5); let us call these consumption optima $c_{1,t}^{i\star}$ and $c_{2,t+1}^{i\star}$. Relation (8) shows that savings is proportional to $i$'s share of household income, rises as patience $\beta$ rises, and falls as the value placed on children $\gamma$ increases (since children have a cost).

If agent $i$ is married to $j$, the household must also decide how many children to have. The solution for the desired number of children from (3) is

$$b_t^{ij\star} = Max\{\frac{\gamma}{D(1+\gamma)\frac{w_t(h_t^i+h_t^j)}{2}}, 2\}.$$                    (9)

The desired number of children is strictly decreasing in household income due to the opportunity cost of child-rearing, and increases as the preference for children increases. It is straightforward to show that the desired number of children is (left) continuous at it minimum, two. The decrease in the desired number of children as income rises is consistent with the data (Feng, Kugler & Zak, 2000).

Before agents execute their consumption/savings, and if married, family size decisions, they search a marriage partner. Recall that there are three benefits the accrue to agent $i$ if she marries agent $j$: $i)$ savings and consumption increase if $h^j > h^i$ by (8), otherwise, consumption falls; $ii)$ $i$ receives the utility from children $\gamma \ln(b^{ij})$ by (9); and $iii)$ $i$ receives the joy of marriage utility flow proportional to $j$'s pizzazz, $\pi p^j$. Note that agent $j$ receives symmetric benefits if he marries $i$. The costs of marriage from $i$'s point of view include a potential reduction in consumption (if $h^j < h^i$), bearing half the cost of raising children $\frac{d^{ij}b^{ij}}{2}$, and, if $i$ and $j$ decide to marry before the final marriage matching round, the inability to meet and marry a different person. Thus, marriage is a relatively standard two-sided matching problem.

Upon meeting, agents simultaneously decide whether or not to marry by comparing indirect utility when married vs. single, taking into account the lost opportunity to marry another agent if the matching market is not in the final round. This choice problem can be clearly seen by examining the final matching round. In the last

matching period, agent $i$ is indifferent between marriage to agent $j$ and staying single when

$$(1 - \beta) \ln(c_t^{i\star} |_{\zeta=1}) + \beta \ln(c_{t+1}^{i\star} |_{\zeta=1}) + \gamma \ln(b_t^{ij\star}) + \pi p^j$$
$$= (1 - \beta) \ln(c_t^{i\star} |_{\zeta=0}) + \beta \ln(c_{t+1}^{i\star} |_{\zeta=0}). \tag{10}$$

The left-hand side of equation (10) is the lifetime utility of $i$ when married ($\zeta = 1$), while the right-hand side is the utility if $i$ remains single ($\zeta = 0$), where both are evaluated using optimality condition (8) to generate optimal consumption levels, and (9) for the number of children $i$ and $j$ would choose to have. Equation (10) indicates that agent $i$ who meets agent $j$ in her last round of the marriage market will choose to marry if the right-hand side exceeds the left-hand side (at equality we will assume agents marry).

Before the marriage market opens, that is, when the distribution of types is known but before agent $i$ has actually encountered a potential mate, equation (10) evaluated at its expected value can be used to find the minimum pizzazz $i$'s mate must have to induce her to marry. Let us denote the minimum pizzazz that $i$'s spouse must have as $\overline{p}^{ij}$. Note that this minimum is agent-specific since it depends on $i$'s pizzazz, and is also sex-dependent since males and females differentially value the components of pizzazz (human capital and beauty). Further, a mate's minimum pizzazz also depends on the expectation of meeting a member of the opposite sex–that is, on the sex ratio. For example, if there is a shortage of women relative to men, than a male agent is less likely to meet a woman during any matching round (all the women may be "taken"). This will cause men to optimally be less "picky" when determining a mate's minimum required pizzazz. Thus, as the data show, when there is a shortage of women in the marriage market, women will be more choosy about a mate's attributes and marry later in life (Buss, 1994). The symmetric result obtains when there is a

relative surplus of women.[15]

The analysis above determines a mate's minimum pizzazz in the final round of matching. For the penultimate matching round, taking into account the opportunity cost of meeting a different spouse during the additional matching period, a mate's minimum required pizzazz is higher than the lifetime minimum. Repeating this logic, an agent's minimum pizzazz for a mate is higher than the lifetime minimum for each earlier matching round. In order to reduce the computations in an already computationally heavy model, we use an extrapolation from the lifetime minimum required spouse's pizzazz to determine each agent's minimum spouse's pizzazz in each round of the marriage market.[16] Agents marry if both the man and woman are above the other's minimum pizzazz for the round they are in; otherwise they remain single for that round. Specifically, agent $i$ who meets agent $j$ in round $r$ of the matching game desires to marry $j$ if

$$p^{ij} \geq \overline{p}^{ij} \exp(\frac{\rho - r}{\rho}), \tag{11}$$

where $\rho$ is the number of matching rounds. The marriage occurs if $j$'s analog of (11) is also satisfied for $i$. Once the marriage market closes, unmatched young agents remain single for the remainder of their lives.

## 2.4   PRODUCTION AND PRICES

To close the model, we specify how prices are determined and output is produced. Output $Y$ is produced with physical capital, $K$, and aggregate human capital $H_t \equiv$

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[15]The sex of a couple's children is a 50-50 draw in the model so that there is no reason to expect a balanced sex ratio at any point in time.

[16]The approximation of the marriage matching rule is consistent with agents having limited information processing capacities, as has been shown by Herbert Simon (1997). Simon argues that humans use rules of thumb to approximate optima in complex problems; see also the discussion in Cosmides & Tooby (1995).

$\int_0^\infty h_t^i d\mu_t$, where $\mu$ is an appropriately defined measure over young adults, $\int_0^\infty d\mu_t = N_t$. To keep the model's focus on marriage and genetics, we consider this economy to be part of a global system where goods and physical capital flow freely, while human capital (that is, humans) remain in their country of origin.[17]

By assumption, the country under analysis is a sufficiently small part of the global economy that the return on savings, $R$, and the economy-wide wage $w$ are set in international markets and are constant over time. Put differently, physical capital that flows in to or out of the country under analysis equalizes factor prices internationally, maintaining a constant ratio of human to physical capital each period. Let the production function be Cobb-Douglas, $Y_t = K_t^{1-\alpha} H_t^\alpha$. Then, the ratio of physical to aggregate human capital, $\frac{K_t}{H_t}$, is constant at each time period $t$ due to international flows of physical capital. As a result, output is linear in aggregate human capital,[18]

$$Y_t = AH_t, \tag{12}$$

and output per worker is $y_t \equiv \frac{Y_t}{N_t} = Ah_t$, where $h_t \equiv \frac{H_t}{N_t}$ is average human capital.

From the individual's point of view, he or she is able to transfer resources over time to smooth consumption via the global market for physical capital. Knowing one's human capital and international prices $w$ and $R$ is sufficient to solve the consumption-savings-family size decision problem (3)-(6) as well as choose a marriage partner via (10)-(11) taking the expectation over all possible values of a partner's pizzazz $p^j$.

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[17]For a related model with immigration, see Zak, Feng & Kugler (2000).

[18]I thank Lutz Hendricks for suggesting this representation of output.

# 3    Responses to Institutional and Environmental Changes

In the following sections, we discuss the numerical calibration of agents' optimal decision rules for marriage, family size, consumption, and savings. We then examine a variety of environmental and institutional changes that impact marriage and fertility, and subsequently economic performance over time. To reiterate: the impact on economic performance occurs as institutional factors affect the rate and types of marriages that are formed and therefore the production of humans and of human capital, with these effects cumulating over generations.

## 3.1    Base Model Calibration

*What is truly revolutionary about molecular biology in the post-Watson–Crick era is that it has become digital...the machine code of the genes is uncannily computer-like.*

Richard Dawkins, *River Out of Eden,* 1995

Because of the high degree of heterogeneity in the model, we construct the sequence of equilibria numerically using equations (8), (9), and (11), along with the law of motion for human capital (2) and the definition of pizzazz (1). In equilibrium, all agents maximize and prices equate supply and demand in the labor, and capital (savings) markets. Note that because of the limited number of marriage matching rounds as well as minimum pizzazz thresholds, the marriage market does not clear in each generation (i.e. (11) is not satisfied for all agents), leaving some agents single and unable to reproduce. This means that the "fertility" market is also not in equilibrium. Put differently, because the model is evolutionary, purely economic markets are

in equilibrium at each point in time but there is not a bio-economic equilibrium–some agents who wish to marry and have children are unable to do so.[19]

The model is calibrated to replicate U.S. demographic data. This is done not because we seek to replicate U.S. economic data with the model; rather, it is because U.S. demographic data are high quality, available, and reasonably representative of developed-country numbers. The calibration procedure involves first setting parameter values which are well-estimated in the literature. The remaining parameters are chosen so that the model replicates moments in the data.

We first set the marriage and genetics parameters. Buss (1989) reports results from surveys of individuals in the U.S. on mate preference which permits a determination of the relative weights men and women put on human capital versus beauty when choosing a mate, $\eta_m$ and $\eta_f$, in the pizzazz equation (1). Isolating the effects on matching of earning ability and attractiveness, the following values arise: $\eta_m = .34$, and $\eta_f = .54$. Thus, when evaluating a prospective marriage partner, men value beauty almost twice as much as human capital (66% vs. 34%), while women value human capital and beauty nearly equally (54% to 46%). This is consistent with the discussion above of the biologically-driven differences parental investment in offspring. The number of marriage matching rounds is set to twelve.

Next, we draw on the behavioral genetics literature to identify the parameters in the intergenerational human capital transmission function (2). The weighting parameters $\alpha_j$ for $j = 1, 2, ..., 50$ for the genetic constituents of human capital are chosen as a declining geometric sequence satisfying $\Sigma_{j=1}^{50}\alpha_j + \aleph = 1$. Plomin (1994) estimates the weight $\aleph$ on education to be .25. The education variable is specified so that $e \sim N(1, .25^2)$, indicating that the sequence $\{\alpha_j\}$ is chosen with $\Sigma_{j=1}^{50}\alpha_j = .75$. Lastly, the dilution effect on parental and child-to-child nurturing in families that do

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[19]Evolutionary epistemology and its relationship to equilibrium are discussed in Zak & Denzau (2000).

not have two children, $\theta = .02$, according to Behrman, Hrubec, Taubman & Wales (1980, Tables 6.7 and 6.8) who estimate earnings regressions similar to equation (2).

Flynn (1987) shows that IQ scores have increased between 10% to 25% over the last two generations in developed countries. Taking the mean increase to be the average of .10 and .25 and dividing by two to find the increase per generation indicates that gene-culture co-evolution causes average human capital to increase 9% per generation. The parameter $\omega$ is chosen to match this effect each generation when fertility is at its replacement rate. Thus, $\omega$ is set so that $\omega E[\Sigma_{j=1}^{50} \alpha_j a_j - \theta(3.3 - 2) + \aleph e] = 1.09$ which is the expectation of human capital equation (2) evaluated at population replacement fertility.[20] The resulting value for $\omega = 2.31$.[21]

The patience parameter $\beta$ can be calculated from values used in the real business cycle literature. A standard value for the subjective discount rate is 1% per year (Cooley, 1995). Considering a generation (a "period" in our model) to be twenty years, the equivalent discount factor is $.99^{20} = .818 = \beta$. The remaining parameters are either used to calibrate the model or set to innocuous values if there is insufficient data to evaluate them. For the latter reason, we set the cost of children scale factor $D = .3$, the production function constant $A = 1$, and the joy of marriage parameter, $\pi = .1$. Lastly, we choose the parameter for the utility of children, $\gamma$, and the constant average wage $w$ so that the model replicates the 60% marriage rate in the data, and produces an endogenous fertility rate that leads to a roughly constant population for

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[20]The Flynn effect essentially internalizes aggregate technological change as in Romer (1986, 1990) and Weitzman (1998).

[21]The calibration of $\omega$ proceeds as follows: For any individual, the genetic draws $a_j$, and education $e$ are random variables, with $E\{a_j\} = \frac{1}{2} \; \forall j$, and $E\{e\} = 1$. Thus, $E\{\Sigma_{j=1}^{50} \alpha_j a_j\} = \frac{75}{2}$. When 60% of the population marries (U.S. Census Bureau, 1999), the replacement fertility rate among married households is $b = 3.3$, which produces a total fertility rate (the average number of children born to women during their reproductive years) of 2.0 which is the population replacement rate absent childhood deaths.

twenty generations.[22] We found this combination of $\gamma$ and $w$ via a grid search that produced the closest match to these two statistics. A constant population is important for the base model so that the effect on the economy of institutional changes that impact marriage and fertility in the experiments that follow can be clearly discerned. The resulting values are $\gamma = .0125$ and $w = .218$.

To begin the simulations, we need to generate initial data for the economy. The initial distribution of genetic material $a_j$ at each location $j$ is generated via a 50-50 draw for zero or one. The exception to this rule is the linked genes 10 and 11. When 10 is generated, 11 inherits this value. We randomly generate an initial distribution of genetic material in this way in the absence of information suggesting a better biological fit. The initial population size is 300 agents. The distribution of beauty, $\Upsilon$ is uniform on [0,1], producing an initial mean beauty of 0.5. For each institutional change reported below, we generate the identical set of random variables so that, though the environment in which agents make decisions is stochastic, realizations of stochastic variables do not drive the results.

Running the base model for 40 generations, we find output grows over time as aggregate human capital accumulates, with a geometric output growth rate of 1.6% per generation. Figure 1 shows that output growth is not monotone. The dips in output shown in the figure are caused by variations in the marriage matching rate, which reduces aggregate fertility and the size of the labor force in the following generation. For example, in generation 17 the marriage rate falls to 62% from 71% in generation 16. As a result, aggregate births fall 10% and aggregate output declines by 5%. At the same time, the difference in spouses' human capital and pizzazz fall when the marriage rate falls, as agents are more "desperate" to marry.

This simulation exhibits positive assortative mating since the standard deviation of human capital falls by 27% during the forty generations, and average beauty rises by

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[22]As mentioned above, we are ignoring immigration in the model.

one-third. As a result, average pizzazz increases 45%, though this is primarily driven by the growth in human capital. Mean human capital, which is per worker output, strictly increases throughout this simulation, growing steadily at 1% per generation. Indeed, because human capital is unbounded while beauty is bounded above by unity, as living standards increase, human capital causes pizzazz to grow, while beauty is bounded from above. Agents still desire beautiful spouses, but a growing economy is the result of the growth of "brains" not beauty, and this is reflected in mating behavior.

In the following subsections, we examine the impact of environmental and institutional changes and compare the resulting economic performance to the base model.

## 3.2   INEQUALITY

*For how shall we know the source of inequality between men, if we do not begin by knowing mankind? And how shall man hope to see himself as nature made him, across all the changes which the succession of place and time must have produced in his original constitution?*

Jean Jacques Rousseau, *What is the Origin of Inequality among Men,*

*and is it Authorized by Natural Law?*, 1754

The first policy experiment we undertake is to vary the initial distribution of human capital so that income inequality increases. The relationship between inequality and economic performance is a current area of controversy with rising inequality generally reducing growth (Perotti, 1996), and it is therefore worth examining. We perform this exercise by changing the initial distribution of genetic material. Rather than generate agents' genomes in the first generation by via a 50-50 draw, we split the sample so that half the agents have a 60-40 draw for a 1 at any particular lo-

cation, and the other half have a 40-60 draw for a 1. This change causes the initial distribution of human capital to skew relative to the base model; though the means are the same, the variance of initial human capital for this experiment is 65% higher. All other aspects of the economy are unchanged.

Following this economy for 70 generations, aggregate output exceeds the base model until generation 8 as shown in Figure 2. At this point, the downward pressure on fertility from rising incomes is not matched by an increasing marriage rate as in the base model, and population and aggregate output begin to fall. Income per capita continues to rise, even as aggregate output falls, further decreasing the fertility rate and leading aggregate income to be 63% lower than the base model by generation 49. The decline in output is driven by a shrinking workforce; by generation 49, the number of young adults has fallen 80%. In this simulation, the increased heterogeneity in human capital results in a lower marriage rate, leading to an epoch of contraction. Nevertheless, by generation 61, output recovers its initial level as the marriage rate rises rapidly, and with it aggregate fertility and the size of the labor force.

The growth slowdown occurs because of a decrease in marriage rates and subsequent reproduction due to inequality–there are simply fewer agents who meet the marriage criterion. This changes when the number of agents shrinks–agents optimally become less picky when the population is sufficiently small. For example, after the economy bottoms-out in generation 49, the marriage rate for the remaining generations is very high, ranging between 80-91%, with a corresponding increase in the average difference between spouses' human capital. Interestingly, the variance of the distribution of human capital at generation 40 is identical to the variance for this generation in the base model. Nevertheless, the long period of economic decline is the result of the cumulative impact of low marriage rates.

A second experiment uses the male-female asymmetry in the evaluation of pizzazz to examine the effects of inequality in beauty. In this experiment, we skew the

initial distribution of beauty to produce a society with a proportionally large number of very beautiful and very plain agents of both sexes. To do this, we change the initial distribution that generates beauty from uniform to bimodal. A simple way to generate a bimodal distribution is by taking a uniform distribution and applying a mean-preserving spread. We generate an agent's beauty in this way as $.5 + .05 \times \Delta$, where $\Delta$ is a random variable that takes values -1 or 1 with a 50% probability. The initial distribution of human capital for this simulation matches that in the base model, so that the only difference in this economy is the initial distribution of beauty.

Running this economy for 70 generations we find that aggregate output initially exceeds the base model, but begins to fall in generation 5 as a lower marriage rate coupled with the negative impact on fertility of income growth for young adults causes the population to shrink. Indeed, after 37 generations the labor force falls 92% and aggregate output is 86% below is initial value. By generation 40, the population of young adults is so small that agents optimally widen their criteria for an acceptable spouse. This causes the marriage rate to increase, raising aggregate fertility as in the previous case. By generation 50, the trend is strongly upward, and aggregate output and population grow apace. By generation 64, the economy has recovered the initial level of output. The plot of aggregate output for this experiment qualitatively matches the graph in Figure 2, though output is less than in the previous case throughout this simulation showing the powerful effect that beauty has on marriage matching, fertility, and output.

In both these experiments we find that in aggregate, increased inequality is bad for output–causing an extended period of decline–even while income on a per capita basis generally rises throughout.

## 3.3 The Change from an Industrial to an Information Economy

*Times are changing. These nerds are a threat to our way of life.*

*Revenge of the Nerds,* Twentieth Century Fox, 1984

This simulation examines the effect on economic performance when the components that produce human capital for one economic environment are differently valued as the marketplace changes. Consider an economy that has run for 20 generations as an industrial power where a certain type of worker (e.g. manufacturing) is highly valued. (We run the economy for 20 generations before making a change so that initial conditions do not affect the results.) Then, the information revolution occurs so that "nerds" rather than manufacturing workers are highly valued in the labor market. This corresponds to a change in the weights placed on the genetic constituents of human capital, $\alpha_j$. That is, this is not a genetic change, but a change in the market valuation of various genetic factors that produce human capital ("brains vs. brawn").

In this simulation, the weighting in the production of human capital for children in generation 20 changes from $\alpha_1 > \alpha_2 > ...\alpha_{50}$ to $\alpha_{50} > \alpha_{49} > ...\alpha_1$. That is, society's valuation of innate attributes reverses in period 20. Presumably, the change in values for various attributes during the information revolution in the twentieth century occurred slowly, but we undertake this radical experiment to generate stark results. Intuitively, this change fundamentally affects marriage and fertility decisions–now "nerds" rather than craftsmen are the hot properties on the marriage market, inverting the previous pattern of assortative mating. In every other way, this economy is unchanged from the base model.

Figure 3 plots aggregate output for 40 generations for this economy. It shows that there is an immediate and long-run increase in aggregate output–well beyond the base

model–while per capita income drops but quickly recovers. In this case, the increased genetic diversity raises output growth, leaving marriage and fertility rates virtually unchanged. The information revolution engenders an economic renewal, raising both population and aggregate output.

This result is sensitive, though, to the weights placed on the genetic components of the new high-human capital agents. In the experiment discussed above, weights were inverted, but unchanged in value location-by-location. If, on the other hand, the weights on the previous low values are raised after the inversion (i.e. "nerds" are more highly valued than were industrial workers, with the weights continuing to sum to 0.75), after an initial growth spurt following generation 20, the new elites inbreed excessively, raising inequality and causing output to sharply decline. As genetic diversity falls, marriage and reproduction lag, and aggregate output follows. Even for a 15% increase in the weights on the first 10 genetic factors, aggregate output declines and does not begin to grow until population and output have fallen, respectively, 92% and 96%. At this point, the marriage rate rises and with it population and output. The initial level of output is exceeded only in generation 61. This is consistent with the effects of inequality–sufficiently large differences among groups in society reduce genetic diversity through inbreeding, adversely affecting output.

## 3.4   POPULATION BOTTLENECKS

*The power of population is so superior to the power in the earth to produce subsistence for man, that premature death must in some shape or other visit the human race.*

Thomas Malthus, *An Essay on the Principle of Population,* 1798

When an extreme environmental event, such as a pandemic, kills off a large proportion of a species so that the remaining group inbreeds, genetic adaptations and

anomalies cumulate.[23] This type of environmental selection pressure, known as the *founder effect*, causes a twist in this branch of the genetic tree. Examples include Tay-Sachs disease in Ashkenazi Jews, and colorblindness in Pingelap Islanders. We investigate such a twist by considering an environmental change that within a generation kills a large proportion of the population in several particular ways.

These variants of the model examine the effects of events like the Black Plague that killed 25 million in fourteenth century Europe, where people ate "lunch with their friends and dinner with their ancestors in paradise" (Boccaccio, 1351). The first scenario we consider is a plague that randomly kills one-half of the young population before they are able to marry and reproduce in the twenty-first generation, after twenty generations of plague-free evolution. In this case, we find that aggregate output falls 23% in generation 21 and continues downward (with slight upticks) until generation 31 when an upward trend is re-established (Figure 4). By generation 45, output recovers the level it was at in generation 20. The slow recovery of the economy occurs because income per capita continues to rise, and following the plague, marriage rates plummet and remain low until after generation 30, with both effects keeping fertility low.

In the second variation, rather than having the plague kill agents randomly, disease kills agents who do not possess a congenital immunity.[24] We model this by positing that agents who have 19 or more "on" genes in *even locations* on the genome are immune to the pandemic.[25] The value of 19 was chosen so that roughly half the population would be killed in the plague, matching the first population bottleneck.

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[23]Over long time intervals, subgroup inbreeding can lead to speciation where interbreeding between subgroups is no longer possible.

[24]On the genetic resistance to diseases, see the survey by Qureshi, Skamene & Malo (1999).

[25]Since the 52nd gene determines an agent's beauty, we use a standard rounding rule to include this factor in the immunity calculation. This is consistent with the constituents of beauty, especially body symmetry infectious, signaling the ability to resist disease.

As above, we run the simulation for 20 generations and then let the plague hit young adults before they enter the marriage market. Figure 5 shows that output initially falls 45%, recovers and falls again, beginning an upward trend in generation 30. In generation 43, this economy recovers the generation 20 level of aggregate output. Interestingly, in generation 40, the distribution of human capital for the post-plague economy has the same mean and variance as the base economy in generation 40, though the former is slightly skewed leftward. Thus, the plague has little effect on the composition of human capital, with its primary effect being a reduction in the size of the labor force.

The final bottleneck we examine is gender specific. In this experiment, the disease only infects men, barring a high degree of immunity. For example, there is evidence that common viruses such as Epstein-Barr (which causes mononucleosis) may be fatal to boys with an X-linked immunodeficiency (Purtilo, Cassel, Yang & Harper, 1975). In this experiment, only young males who have less than 19 "on" genes in even locations die; females are completely immune. As before, the disease-free economy runs for 20 generations, and the plague hits young males in generation 21 prior to reproduction. Two generations after the plague hits, output falls 33% as Figure 6 depicts, and recovers its pre-plague level by generation 34. Immediately after the plague, an asymmetry appears in marriage matching rates. The 72% marriage rate for males and females in generation 20 changes to a 59% rate for males, and 40% for females. Within two generations, marriage rates recover to a symmetric 70% and continue to rise, even as fertility slowly declines as per capita income rises.

In all three population bottlenecks examined, output falls by less than does the population, keeping per capita incomes high, even as nearly half of the population is killed. Although the aggregate effects are substantial, living standards for survivors are relatively unaffected.

## 3.5   MATCHMAKERS

*Forever, Fortune, wilt thou prove*
*An unrelenting foe to love*

James Thomson, *The Castle of Indolence,* 1748

In this section we examine the aggregate impact on fertility and output of matchmakers. By "matchmakers" we mean any matching technology that changes the matching pattern from globally random to one which is limited to a certain subset of the population. This includes traditional matchmakers who arrange marriage by family ties and compatibility, to dating services that permit individuals to sort themselves and prospective mates into identified categories. For example, the most popular internet dating site, Match.com, permits individuals to search for potential mates by a variety of characteristics including age, location, education, and a desire for (or not for) children. Match.com enables individuals to anonymously email each other to determine compatibility, leading in some cases to phone calls and eventually tête-à-tête meetings. In the year 2000, this site had over 925,000 active members and six million visitors. Since its inception in 1995, Match.com has facilitated 160,000 "meaningful relationships" and more than 900 marriages. A similar internet dating site that specializes in matching alumni and faculty from the Ivy League is GoodGenes.com.

The "sorting" aspect of matchmaking is built into the model by limiting the random meeting of agents to only those similar to themselves. Once a couple meets, both choose whether to marry using criterion (11); couples are never compelled by outside forces to marry. This is therefore a model of endogamy (marrying within one's own group). For simplicity, we ignore any costs associated with matchmaking.[26]

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[26]The assumption of costless matching of potential spouses is consistent with individuals who sort themselves regarding where to live, work, and play by pizzazz.

Figure 7 plots aggregate output when there are three subgroups of agents such that one can only meet a potential a marriage partner from the group one is in. The figure shows that output initially peaks in generation 2, then declines steadily until generation 34, after which the marriage matching rate rapidly rises, raising the fertility rate even as fertility per household falls steadily over time. By generation 52, output recovers its initial level and population and output continue to grow. This simulation shows that matchmaking increases inequality among individuals, and reduces the marriage rate over many generations.

The experiments with matchmaking, inequality, and population bottlenecks suggest that there is a threshold such that when population shrinks over time to something near 40 agents, the marriage rate shoots up as does the average difference between spouses' human capital. Agents optimally loosen the thresholds for an acceptable marriage partner when prospects are scarce. To investigate this issue further, we reran the experiment with matchmakers but increased the number of subgroups from which one can meet a mate from three to five. This puts an additional friction on matching opportunities with an extreme effect on economic performance. After 29 generations, this society becomes extinct; there are no marriages and therefore no reproduction. The higher degree of endogamy in this case disallows agents in a small society from marrying those who remain, leading to its demise.

## 3.6  LOVE

*Who ever loved that loved not at first sight?*

Christopher Marlowe, *Hero and Leander,* 1598

In this section we examine the effect of love on marriage in a manner similar to Becker (1974), and characterize its impact on economic performance. Even though the basic model includes both beauty and the joy of marriage, these are not "love"

because they are part of a rational decision process. The outstanding feature of love is that, though it has many dimensions, one is either in it or not. Love is a "biological imperative" (Ackerman, 1995) because it is the mechanism that guarantees the perpetuation of genes. Buss (1994) writes that "Love is one of the most important cues to commitment" (p42), and therefore signals to a potential mate whether one will remain in the relationship–a critical cue when choosing whether to marry and have children.

By including love in the model we seek to take into account the impact of the emotions on mate choice. Following Becker (1974), we expect love to decrease the degree of positive assortative mating. That is, the average difference in human capital between married men and women is higher with love than without. Thus, this is a model of (partial) exogamy.

We include love in the model, while maintaining some rationality in mating choices by making two assumptions. First, being in love does not necessarily lead to marriage. That is, one can meet and fall in love with someone, but not find them a compatible long-term mate. The second assumption is the partial converse of the first: No matter how high is a potential mate's pizzazz, marriage does not occur unless both the man and woman are in love. We operationalize this approach by having each agent in the model determine the minimum pizzazz a mate must have in the final matching round, as in the base model, but this minimum is not scaled up for earlier rounds of the marriage market. Instead, we introduce a new variable $\heartsuit^{ij} \in \{0, 1\}$ that indicates whether agent $i$ is in love with agent $j$ ($\heartsuit^{ij} = 1$), or is not in love ($\heartsuit^{ij} = 0$). Upon meeting, agents $i$ and $j$ marry in any matching round if they are both above the other's final round minimum mate's pizzazz ($p^{ij} > \overline{p}^{ij}$ and $p^{ji} > \overline{p}^{ji}$), and both agents are in love with each other $\heartsuit^{ij} = 1 = \heartsuit^{ji}$. In the absence of a theory of love (rather than attraction, which is pizzazz), the variable $\heartsuit^{ij}$ is generated for each agent in each matching round with a 40% chance of being a 1. We use a 40% value as we are unable

to assign a probability distribution to this variable from a review of the literature, though below we discuss the sensitivity of our results as this probability changes.

Figure 8 plots aggregate output for the model with love. The figure shows that the inclusion of love causes per capita income to grow spectacularly. After 40 generations, this economy more than quadruples in size, with a growth rate of 3.7% per generation. Moreover, the growth rate accelerates to 5.6% during the generations 25-40. This by far exceeds output growth in any other of the experiments we consider. This dramatic result occurs as love generates positive assortative mating while maintaining sufficient genetic diversity among couples (and their children). A measure of diversity is the average difference between spouses' human capital; this measure is 25% larger over 40 generations for the model with love relative to the base model. Love balances an increase in genetic diversity (which also occurred in the first experiment) with a strong desire to marry assortatively. The result of this balance is rapid output growth.

If the probability of falling in love with a member of the opposite sex rises above 40%, the number of marriages–as well as population and output–increase even more rapidly than in the previous case. As the probability of falling in love falls, the number of marriages, population, and output growth also fall. For example, if the probability that $\heartsuit^{ij} = 1$ is below 17%, then output falls for nearly 40 generations before slowly recovering. Thus, an economy in which agents are unlikely to fall in love suffers a sustained contraction in output. These simulations therefore demonstrate that love is important to economic performance.

# 4    Conclusion

We have set out a model of population genetics in order to characterize the effects that marriage and reproduction have on economic performance. One of the contributions of this paper is the framework linking genetics, and more generally the bio-economics

of the family, to economic growth. The model shows that in every environment, positive assortative mating obtains to various degrees, but the marriage rate may be so low that population and aggregate output fall for extended periods of time. Inequality, including that induced by matchmaking, is shown to induce long periods of economic contraction, as do population bottlenecks. Absent these, marriage and fertility lead to nonmonotone, though generally positive output growth. The strongest growth effect occurs for the combination of positive assortative mating with sufficient genetic diversity which obtains when the model includes love; output growth in this case is very rapid and nearly always positive over subintervals.

The bio-economic approach developed here can be extended both in its biological foundations (e.g. see Banzhaf & Eeckman, 1995), in the environmental factors examined, and in the way agents are identified. This should not affect, though, the underlying message of the paper: genetic diversity is critical to sustained economic growth. More generally, we have shown that the social-economic-institutional environment matters because it affects decisions by the human animal–decisions driven by our evolutionary makeup–leading to a genetics-environment co-evolutionary cycle.

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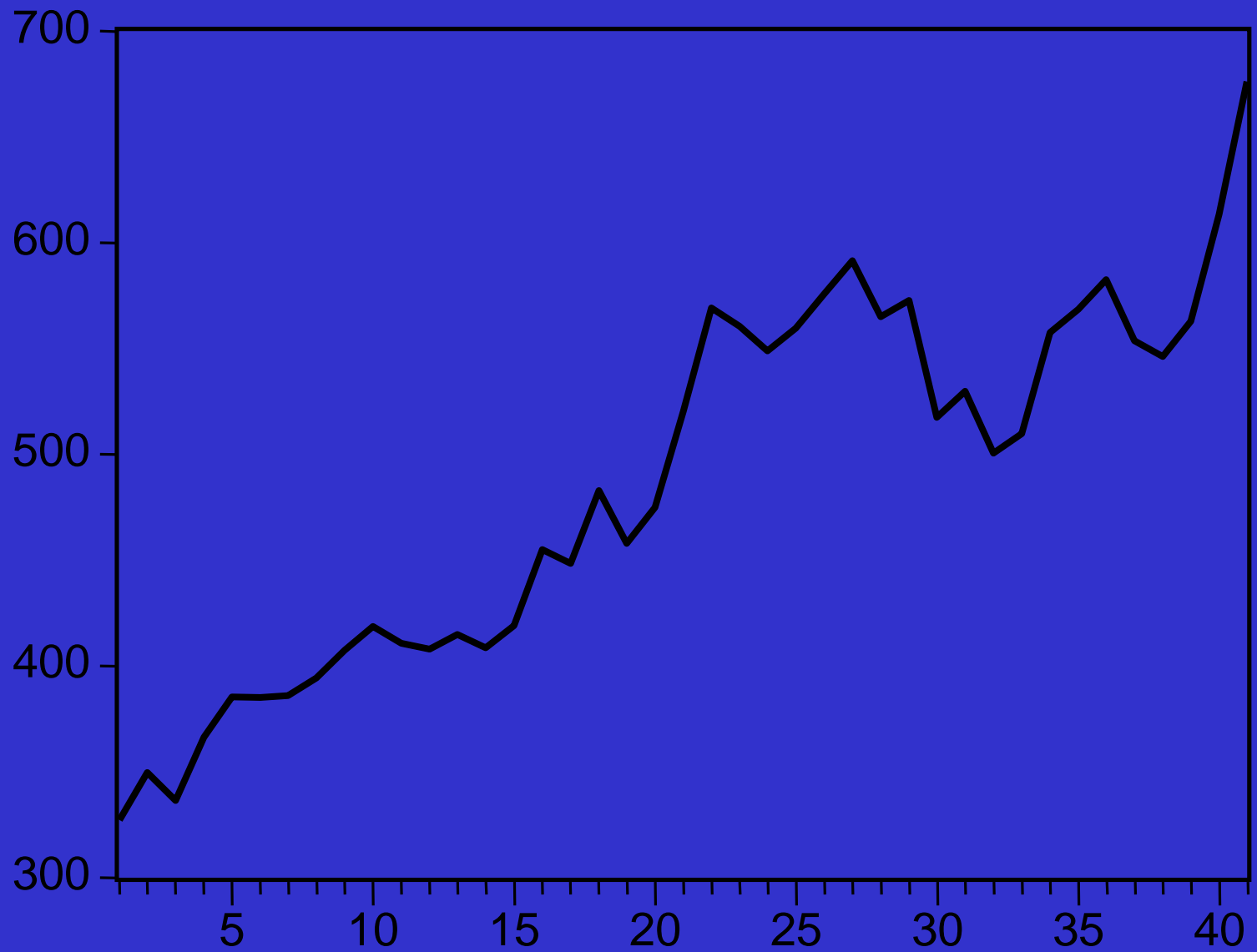
Fig.1    Aggregate Output of Basic Model
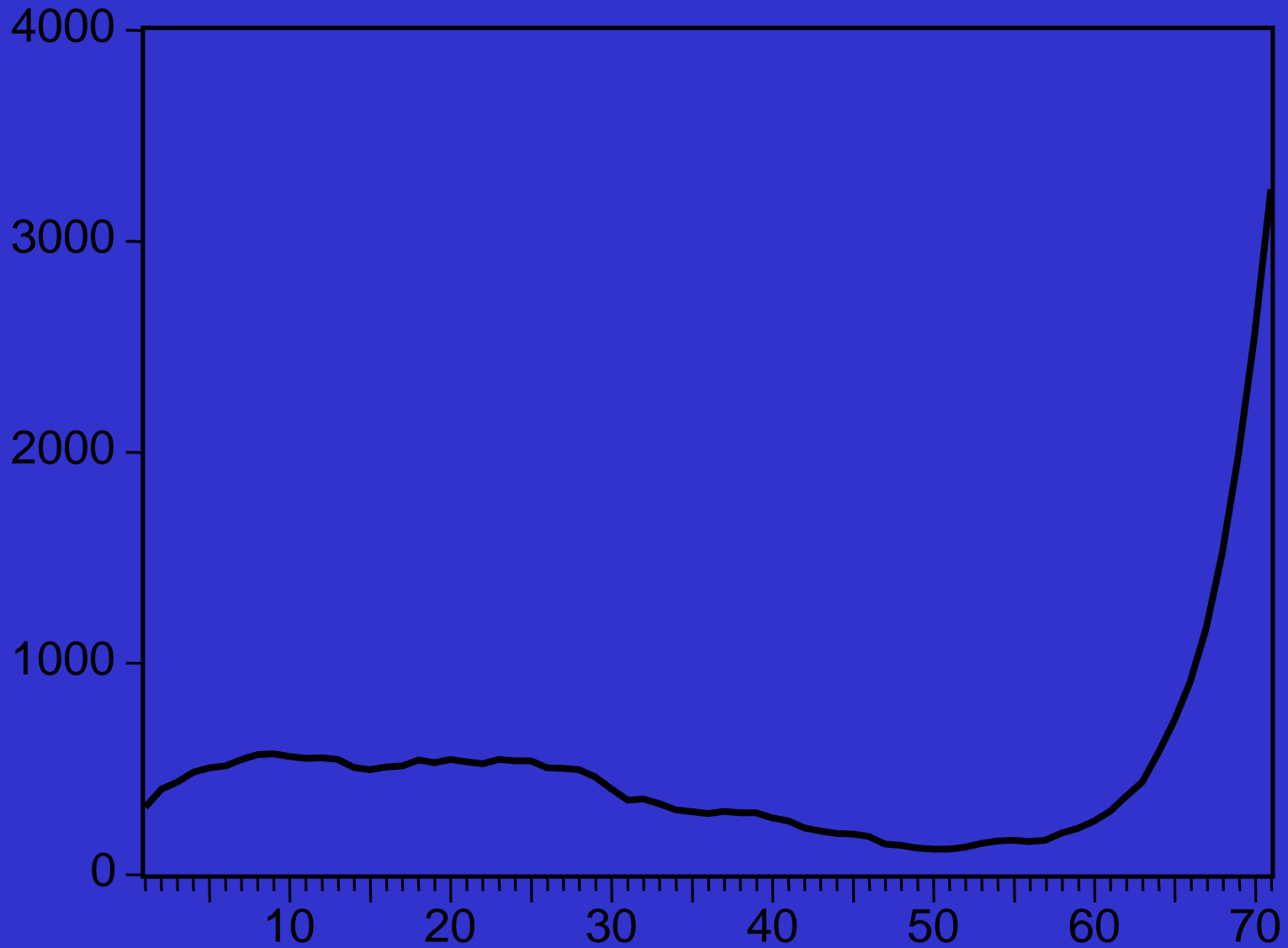
Fig.2   Aggregate Output with Inequality
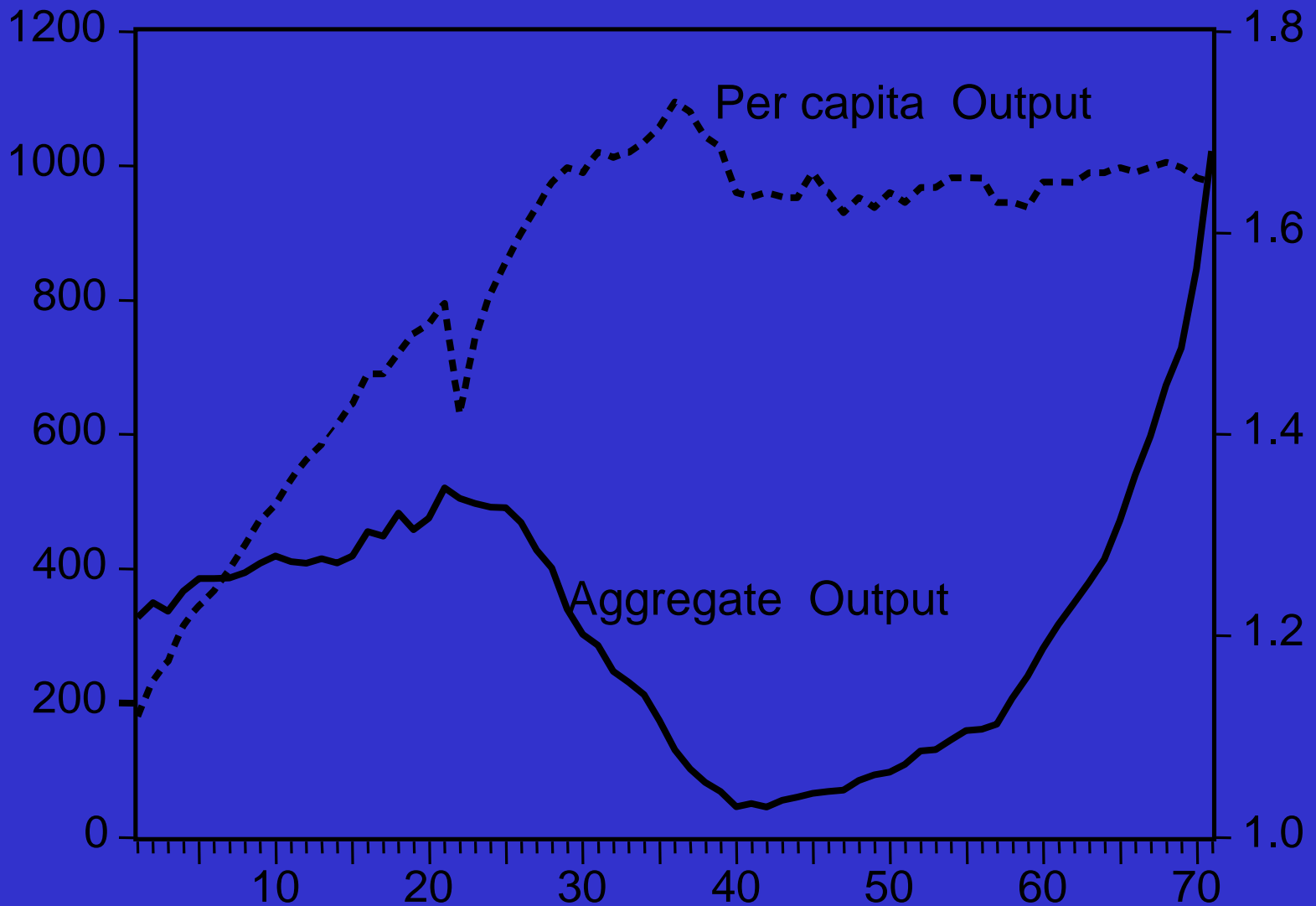
Fig.3   Output of Information Economy
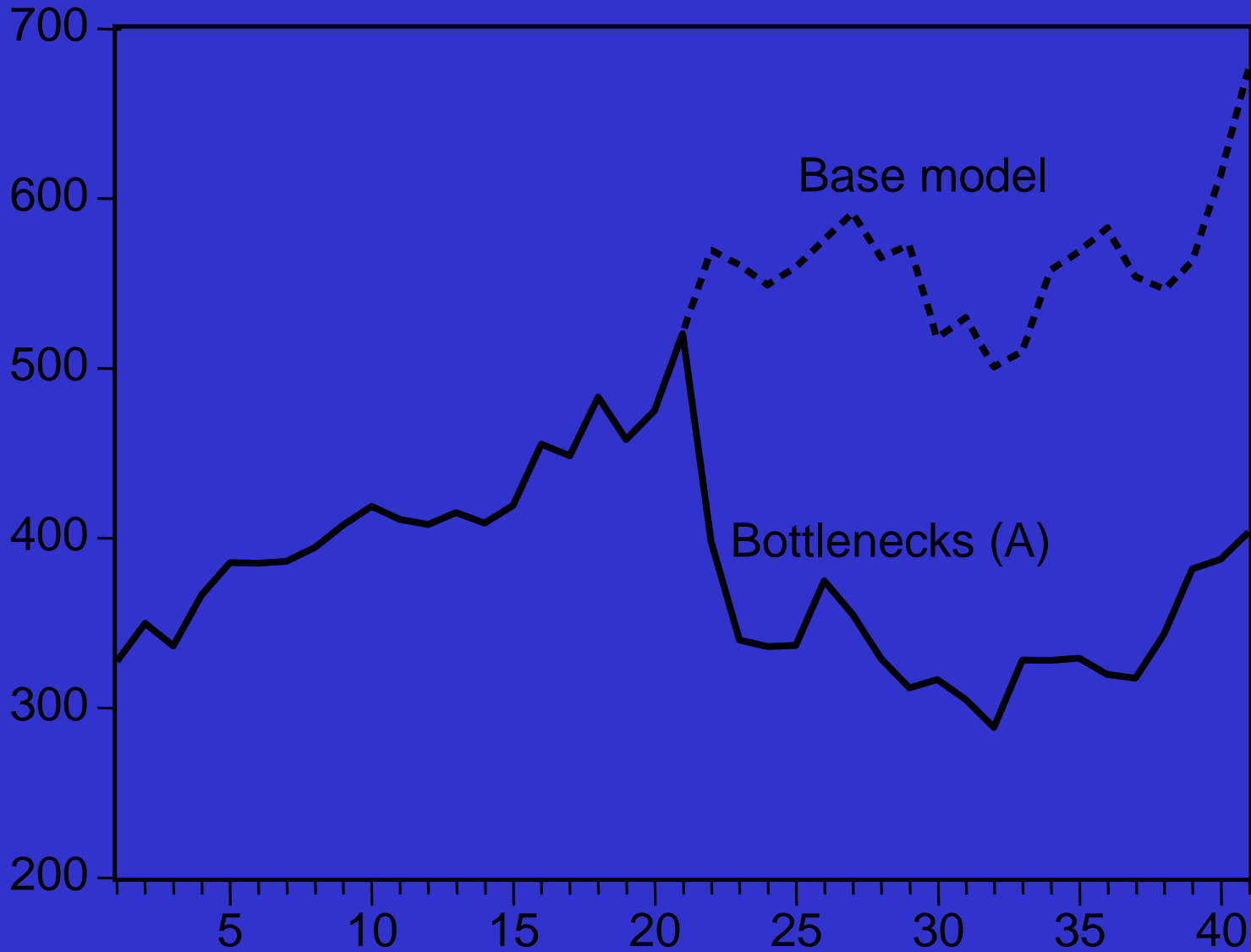
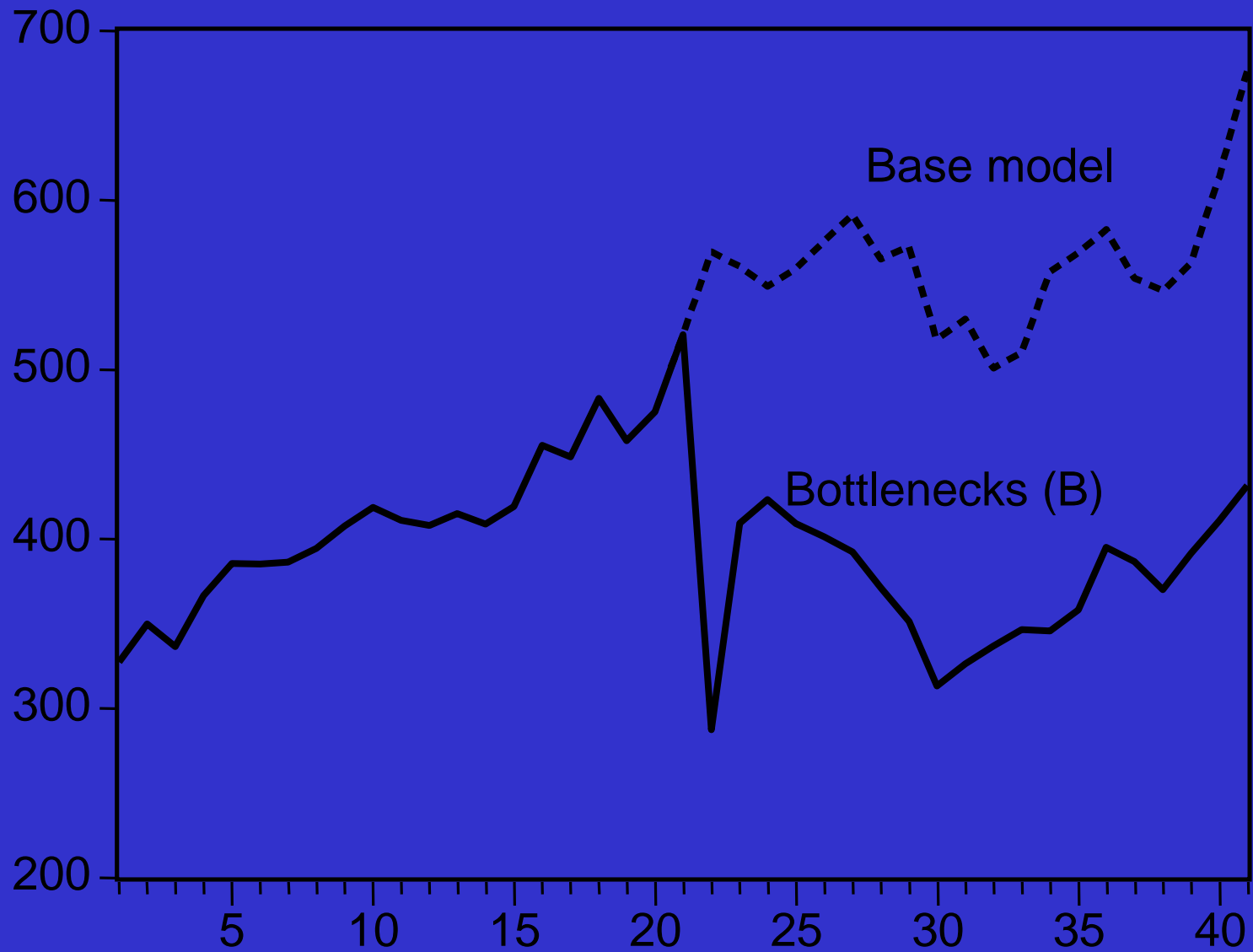Fig.4   Aggregate Output  (case A)
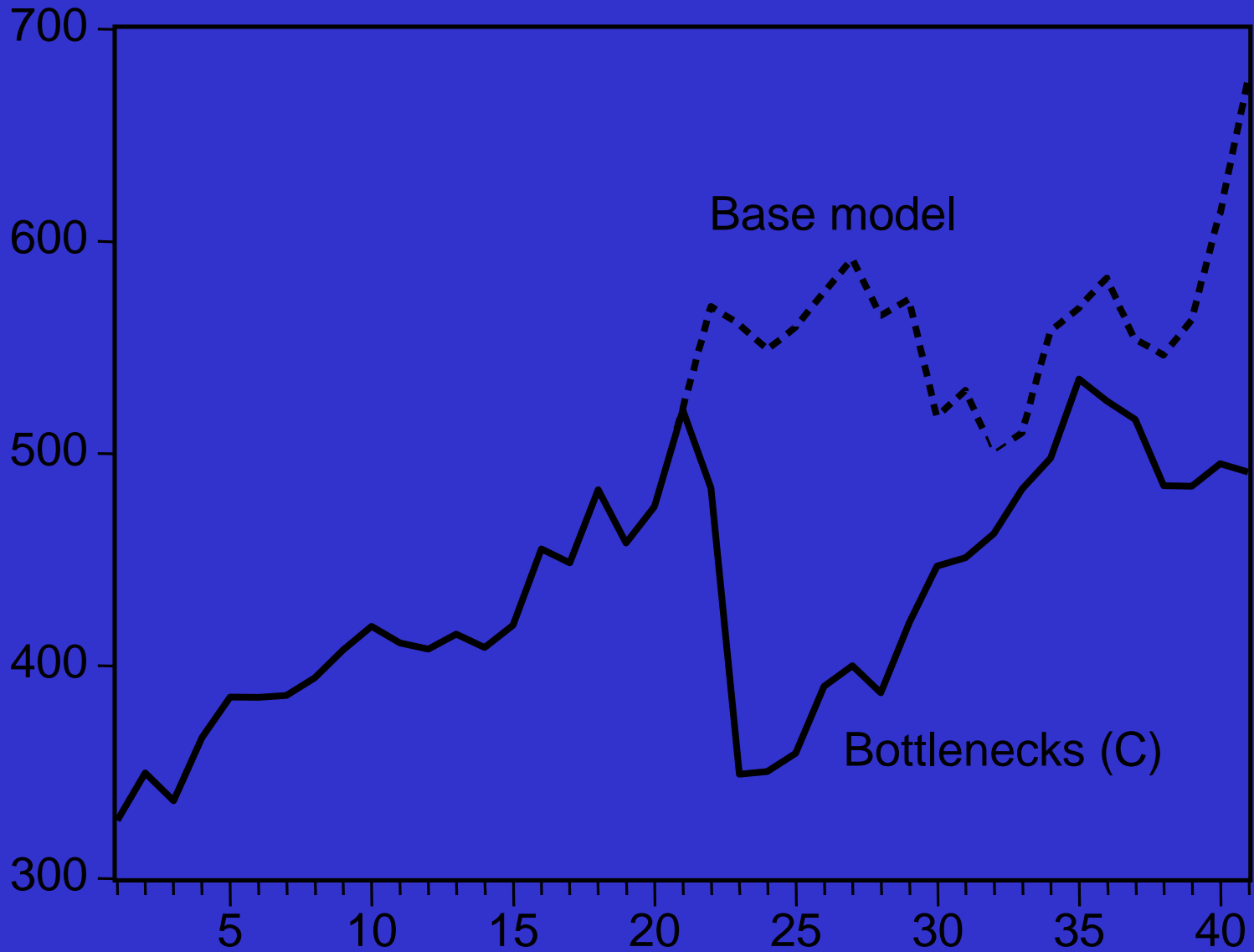
Fig.5   Aggregate Output  (case B)
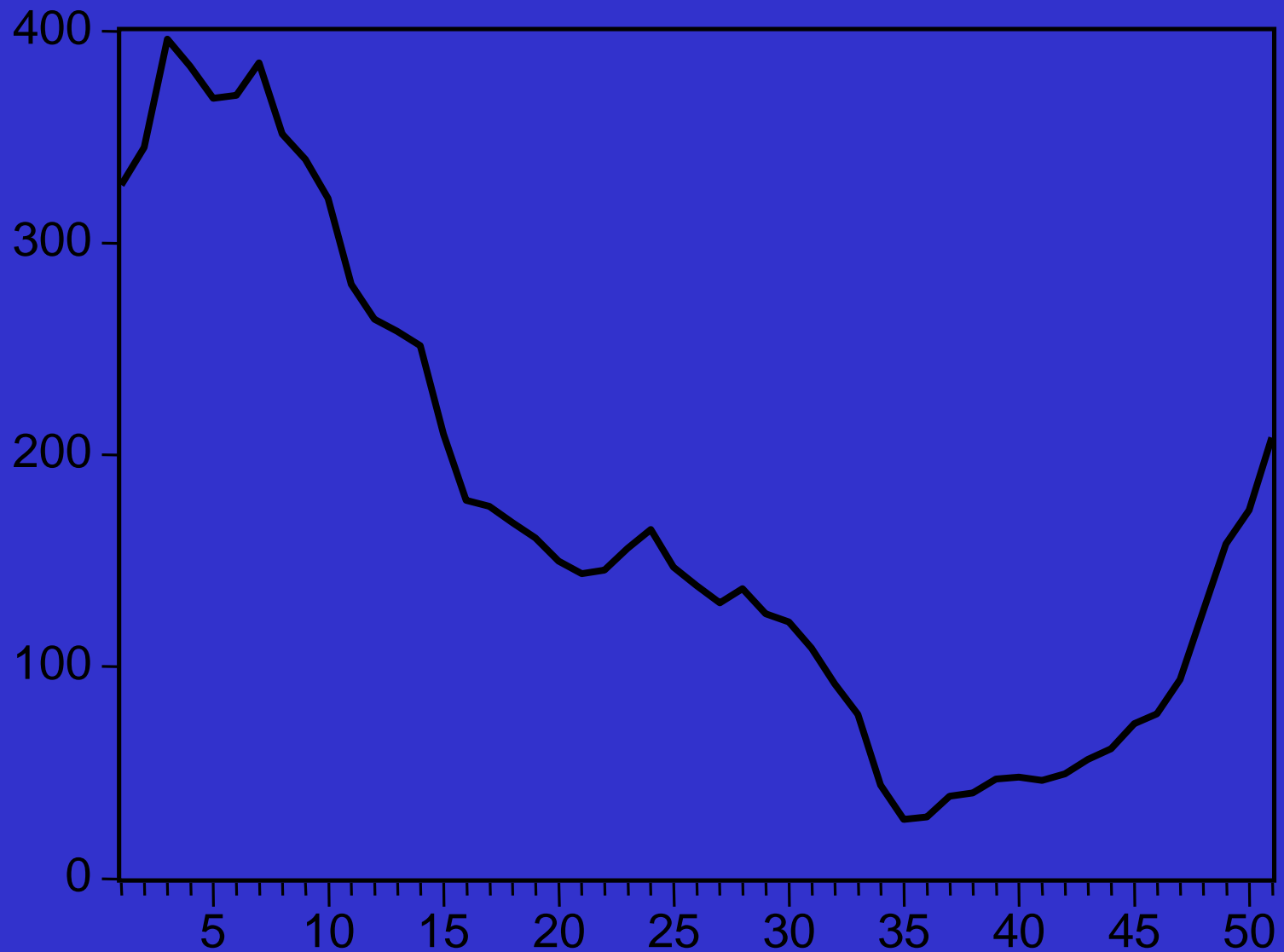
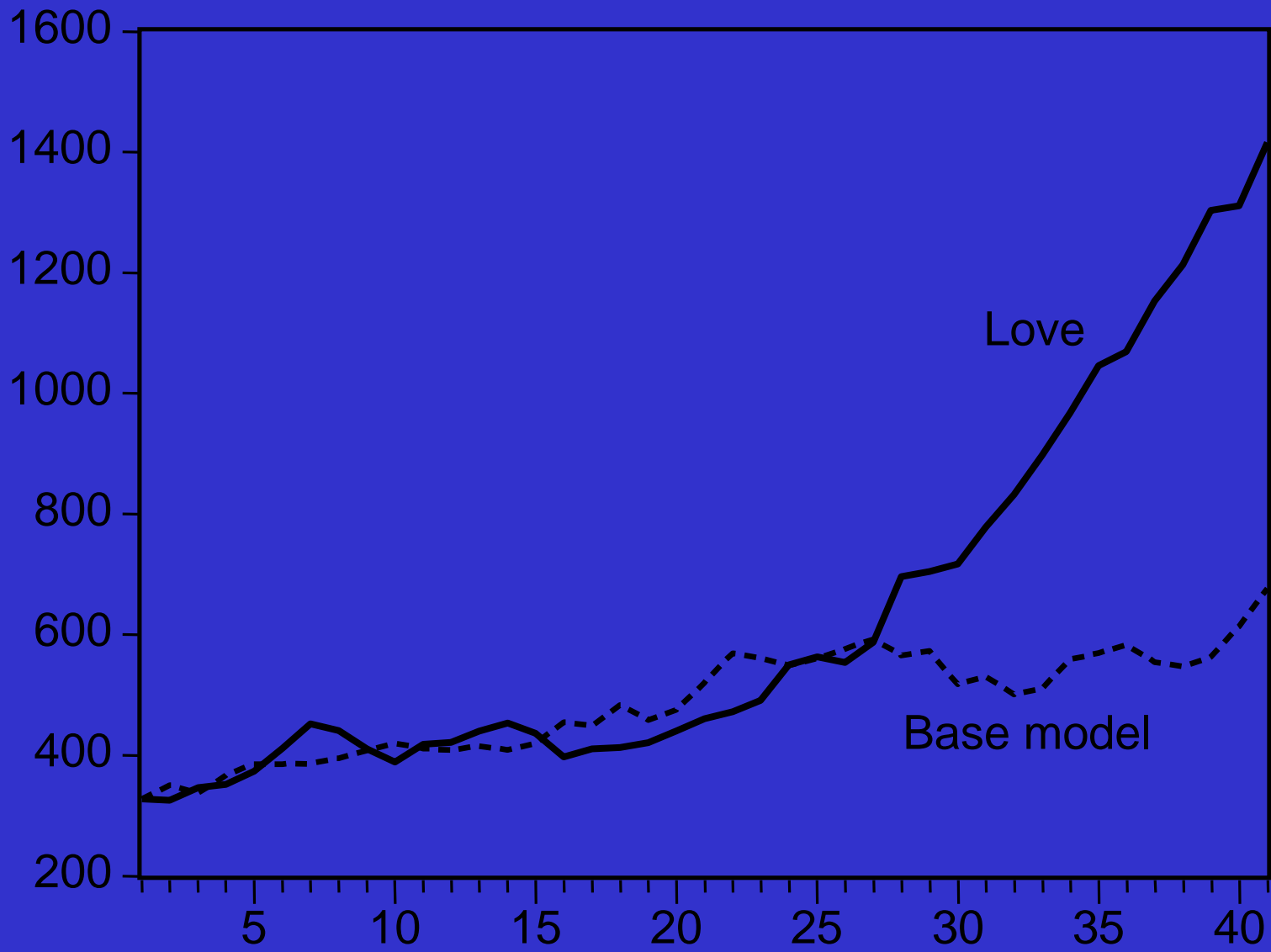Fig.6   Aggregate Output  (case C)

Fig.7   Aggregate Output  with Matchmaker

Fig.8   Aggregate Output with Love