Ordinal cheap talk in common value auctions*

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Abstract

Sellers benefit on average from revealing information about their goods to buyers, but the incentive to exaggerate undermines the credibility of seller statements. When multiple goods are being auctioned, we show that *ordinal cheap talk*, which reveals a complete or partial ordering of the different goods by value, can be credible. Ordinal statements are not susceptible to exaggeration because they simultaneously reveal favorable information about some goods and unfavorable information about other goods. Any informative ordering increases revenues in accordance with the linkage principle, and the complete ordering is asymptotically revenue-equivalent to full revelation as the number of goods becomes large. These results provide a new explanation in addition to bundling, versioning, and complementarities for how a seller benefits from the sale of multiple goods.

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1 Introduction

When can a seller credibly reveal information to buyers? The linkage principle (Milgrom and Weber, 1982) shows that such revelation strengthens competition and, on average, increases seller revenues by narrowing information differences among buyers. But the importance of the linkage principle would seem limited by the seller's incentive to only reveal good information or even lie about bad information. It is usually assumed that the credibility of seller information, and the applicability of the linkage principle, depends on the seller's incentive to maintain a trustworthy reputation over time.[1]

We show that the credibility problem is less severe than normally supposed when a seller has multiple goods. Although the seller has an incentive to lie about the value of each individual good, cheap talk about the comparative values of the goods is often credible. For instance, an auction house can credibly rank the likely values of different goods even if absolute estimates are not credible. Comparative statements can be part of an equilibrium strategy because they simultaneously reveal favorable information about one good and unfavorable information about another good. The incentive to lie is thereby diminished, and in many situations is completely eliminated.

To investigate this issue formally we consider simultaneous, common value auctions of stochastically equivalent and independently distributed goods by an informed seller. For each of the goods there is a set of different buyers who each have a private signal about the value of the good and are interested only in that good. Before the auctions the seller publicly makes a cheap talk statement to all the buyers about the comparative values of the different goods. This statement may disclose a complete ordering of the goods according to her own private signal, or a partial ordering in which multiple goods are grouped into the same categories.

When the seller's information is a complement to the buyers' signals in determining buyer valuations, the seller has an incentive to sell a better good precisely when the buyers are expecting a better good. We show that this simple condition is necessary and sufficient to make ordinal cheap talk by the seller credible. For a sufficiently large number of goods we find that an ordinal cheap talk equilibrium involving a partial ordering always exists under standard conditions. We then provide a simple and natural class of auctions where revealing

[1]Of course, legal restrictions or contractual obligations may also provide an incentive for truthfulness, but the private nature of seller information makes verification inherently difficult. Moreover, common law has long protected "puffery" – the right of sellers to boast about their goods.

a complete ordering of the goods is always an equilibrium for any number of goods.

We find that the revenue gains from comparative cheap talk can be substantial and that, consistent with the linkage principle, finer comparisons imply higher revenue. Moreover, as the number of goods increases, revealing the seller's complete ordering is asymptotically equivalent to revealing all of the seller's private information. In a common values auction with a perfectly informed seller, buyer information rents therefore go to zero. This limit result is distinct from other recent results in the auction literature. Bali and Jackson (2002) find that as the number of bidders for a good increases, buyer information rents disappear in the limit for all standard auction formats. Pesendorfer and Swinkels (2000) find that for a uniform price auction with identical goods, the auction is efficient in the limit as the number of goods and buyers goes to infinity in a bounded ratio with more buyers than goods. In both these models the competition between buyers is the driving force in eliminating buyer information rents. In our model there is a fixed number of buyers who bid for each good. Buyer information rents fall purely because of credible revelation of the seller's information.

Crawford and Sobel (1982) show that limited cheap talk statements are often credible when sender and receiver interests are neither directly opposed nor directly aligned. While the sender still has an incentive to lie about the absolute value of an unknown parameter, the sender can reveal to the other party an interval in which the unknown parameter lies, provided sender and receiver interests are sufficiently aligned. For instance, the seller can state that the parameter is above or below some level. In our model the buyers for each good and the seller have directly opposing interests so the seller always has an incentive to exaggerate the value of the good and such interval cheap talk is not credible.[2]

While interval cheap talk in the sense of Crawford and Sobel is not possible, there still exist informative equilibria that are partitional in the space of the common values of the goods, with different elements of the partition differing only in the ordinal nature of the information they convey. These equilibria are distinct from those in Crawford and Sobel because they are limited to ordinal information and because each element of a partition must contain at least one good. For instance, a seller cannot state that two goods are both above average but can state that one good is better than another good.

The ability to make credible comparisons provides a new explanation in addition to bundling,

[2] Farrell and Gibbons (1989) show that cheap talk is possible with otherwise opposing interests when there are costs to trading. For instance a potential buyer might reveal a strong interest in a good to a seller so as to persuade the seller that it is worth the trouble of bargaining.

complementarities, and versioning for how a single seller benefits from the sale of multiple goods. The bundling literature shows that selling multiple goods as a package can be used to reduce variation in buyer demands (McAffee, McMillan, and Whinston, 1989) or restrict entry by sellers of single products (Nalebuff, 2000). The combinatorial auctions literature shows that simultaneous auctions can be designed to take advantage of complementarities across goods (Rassenti, Smith, and Bulfin, 1982). These factors are not present in our model because each buyer demands only one good. More relatedly, the versioning literature shows how selling multiple goods of varying quality allows a monopolist to discriminate among different buyers (Varian, 1989). In our model there is no such advantage since buyers do not vary in their taste for quality. The gains we identify are therefore entirely due to reductions in buyer information rents via the linkage principle.

The seller in our model could be the actual owner of multiple goods, a separate evaluator of the goods whose earnings are dependent on overall sales, or an auction house that sells goods on behalf of different owners. In the last case the auction house's private information could be in the form of background information about the owners. For instance, on-line auction houses have feedback mechanisms that accumulate information about the performance of sellers, but the design of these mechanisms appears to induce overly positive reports (Resnick and Zeckhauser, 2001). This paper shows that the relative performance scores of different sellers can be credible even when the absolute scores are not.

In Section 2 we set up the model and in Section 3 we consider ordinal cheap talk equilibria. In Section 3.1 we provide a result on the existence of one informative equilibria, while in Section 3.2 we characterize different ordinal cheap talk strategies in terms of their revenues. In Section 4 we provide two simple special cases of our general model. Section 5 concludes while the Appendix contains some of the proofs.

2 The Model

A seller has $N \geq 2$ different goods indexed by $k \in \{1, ..., N\}$. For each good k the seller observes the value of the good, $V_k \in [0, 1]$. Let $V \in [0, 1]^N$ represent the vector of values for the goods. Let $V_{k:N}$ denote the k–th lowest value (i.e., the kth order statistic) with $V_{1:N} \leq ... \leq V_{N:N}$.

We suppose that for each good k there are $n \geq 2$ buyers, indexed by $i_k \in \{1, ..., n\}$. The utility for buyer i_k from obtaining good k at a price p is equal to $V_k - p$ and the sets of buyers for any two goods are disjoint. Each buyer i_k has a private signal about good k, $X_{i_k} \in [0, 1]$

for $i_k \in \{1, ..., n\}$. Denote by $X_k = (X_{i_k}, ..., X_{n_k})$ the vector of buyer signals for good k.

We suppose that the random variables (X_k, V_k) are independently and identically distributed across $k \in \{1, ..., N\}$. Let F denote the joint distribution of (X_k, V_k). Following Milgrom and Weber (1982), hereafter MW, we assume that F is symmetric in its first n arguments and that it displays affiliation. Affiliation implies if one player (including the seller) observes a high private signal of the value of a good, other players are also more likely to observe high private signals of the value of that good. When F has a density f, affiliation implies that f is log–supermodular in its arguments. In the appendix we provide a general definition of affiliation.

Let F_V denote the marginal distribution of V_k. We suppose that either F_V admits a positive density f_V with compact support $\mathbf{V} = \{v|f_V(v) > 0\}$ or that F_V is a step function, i.e., V_k takes finitely many values and $f_V(v) = \Pr[V_k = v]$. For each $v \in \mathbf{V}$, let $F_{X|V}(\cdot|v)$ denote the distribution of X_k conditional on $V_k = v$. We suppose that either $F_{X|V}(\cdot|v)$ admits a positive density $f_{X|V}(.|v)$ for each v or that $F_{X|V}(\cdot|v)$ is a step function for each v. In the latter case, X_k takes a finite number of values with $f_{X|V}(x|v) = \Pr[X_k = x|V_k = v]$. For each $v \in \mathbf{V}$, let $\mathbf{X}(v) = \{x|f_{X|V}(x|v) > 0\}$ denote the support of $f_{X|V}(\cdot|v)$ (that may depend on v). Finally, assume that $f_{X|V}(\cdot|v)$ is a bounded function of v with at most a finite number of discontinuities.

The seller sells the goods in the form of N simultaneous "English" or continuous ascending clock auctions to the N different groups of buyers.[3] Formally, such an auction consists of a price $p \in [0, 1]$ rising continuously from 0 to 1. At any price p, each buyer has to decide whether to remain active or drop out after observing the number of previously active bidders and when other bidders have dropped out. Drop outs are final. Let $\iota(p) \in \{1, ..., n\}$ be the number of bidders who are active at p. Let p_{i_k} be the price at which i_k drops out (with p_{i_k} set equal to 1 if i_k never drops out). The winner of the auction is the bidder with the maximum p_{i_k}, with ties being decided uniformly. The price P *that the winner pays* is equal to $\inf\{p|\iota(p) \leq 1\}$ if it exists, and is equal to 1 otherwise.

Since the auctions are simultaneously held, none of the buyers in any auction observe any of the proceedings of any other auction. The only possible information transmission between auctions takes the form of a public announcement sent by the seller before the auctions start. The seller's announcement strategy is represented as a function $m(V)$ choosing a message (or

[3] We do not consider the optimality of the English auction nor other mechanism design issues such as reserve prices and entry fees.

a probability distribution over messages) from a finite set M (with at least $N!$ elements), as a function of her private information V. We ignore reputational considerations, so that the seller's announcement is pure cheap talk and the seller is only interested in maximizing her total revenues from the N auctions.

Recall that we assume separate sets of buyers for each good. In practice, there will often be some overlap between buyers in which case neither our assumption of disjoint buyer sets nor the more common assumption of identical buyer sets will hold. Separation is most likely when the goods are of different types, e.g., a government privatizes a number of firms in different industries and buyers are industry-specific, or an on-line auction house sells a range of different goods and buyers are interested in a specific good. Use of disjoint buyer sets greatly simplifies the analysis of equilibrium bidding behavior and also highlights the fact that the gains we identify from selling multiple goods are not due to increased competition from a corresponding increase in buyers for each good, nor to strategies such as bundling that depend on each buyer having an interest in multiple goods, but are exclusively due to cheap talk by the seller.

3 Ordinal Cheap Talk

3.1 Existence of Equilibrium

An equilibrium for our cheap talk and bidding game consists of an announcement strategy $m(V)$ for the seller and bidding strategies for each buyer of each good such that: given the message m, the bidding strategies constitute a symmetric Bayesian Nash equilibrium of the auction for each good k; and, given the bidding strategies, the seller's announcement strategy maximizes her expected revenues for each possible realization of V.

A full characterization of the symmetric Bayesian Nash equilibria for English auctions is found in MW.[4] In brief, the bidding proceeds in two stages. In stage 1, the $n-2$ bidders with the lowest signals successively drop out at points p_{i_k} that depend on their private signals, enabling the two remaining bidders to infer their signals. In stage 2, each of the two remaining bidders drop out at the point p reaches the expected value of the good conditional on the (inferred) values of the lowest $n-2$ signals, the message m sent by the seller, and on the fact

[4] While the analysis in MW is carried out for the case where the buyer signals X_{i_k} are continuous random variables admitting a density (so that ties are zero probability events), it is straightforward to check that their analysis carries over to the case of discrete buyer signals when the seller employs an English auction in the sense defined above.

that the buyer is *tied* for the highest signal. Notice that when $n = 2$ this is equivalent to equilibrium bidding behavior in a second price auction.

For $i_k = 1, ..., n$, let Y_{i_k} be the i–th highest signal among the n signals of the buyers of good k and let $Z_{i_k} = (Y_{i_k}, ..., Y_{n_k})$. When the value of the second highest signal is y_2 and the value of the lowest $n-2$ signals is equal to (the vector) z_3, the bidder with the second highest signal will drop out at the point $b_m(y_2, z_3)$ defined by:

$$b_m(y_2, z_3) = E[V_k | Y_{1_k} = Y_{2_k} = y_2, Z_{3_k} = z_3, m]. \tag{1}$$

It may seem that the expectation in (1) may not be well–defined if the seller's message m is inconsistent with the buyer signal(s) X_k.[5] However, for the set of cheap talk equilibria that we focus on, where the seller only discloses ordinal information, such a possibility never arises, as the distribution of V_k given any message m always enjoys the same support as the prior distribution of V_k. Furthermore, for each ordinal message m, our assumptions imply that the function b_m is non–decreasing in each argument. Let $P_{m,k}$ be the price that the seller receives for good k given a message m. Since $Z_{2_k} = (Y_{2_k}, Z_{3_k})$, we can write

$$P_{m,k} = b_m(Z_{2_k}). \tag{2}$$

We now turn to considering the seller's announcement strategies $m(V)$ that may constitute a cheap talk equilibrium. As is usual in cheap talk games, there always exists one uninformative or babbling equilibrium where the buyers ascribe no meaning to the seller's announcement, so that the seller does not send any informative message. For the babbling case we will denote by $b_u(\cdot)$ the function in (1) that defines the price at which the bidder with the second highest signal drops out, regardless of the seller's message m. Since b_u does not depend on the seller's message,

$$b_u(y_2, z_3) = E[V_k | Y_{1_k} = Y_{2_k} = y_2, Z_{3_k} = z_3]. \tag{3}$$

Denote by $P_{u,k} = b_u(Z_{2,k})$ the price that the seller obtains for good k in a babbling equilibrium.

Regarding informative equilibria, note that since buyer and seller interests are directly opposed on each good, there is no room for cheap talk that refers to the value of each good independently of the value of other goods. We are interested in the possibility of cheap talk equilibria where the seller's message consists of disclosing a partial or complete order of the

[5]This may occur when, for example, the seller has announced that the value of the good is equal to 1 and the buyer knows from his signal that the value of the good is less than 1 with probability 1.

values $V_1, ..., V_N$ of his N goods. Such a message contains information about each good that is *not* independent of the information it contains about other goods. We call such strategies ordinal cheap talk strategies and such equilibria, if they exist, ordinal cheap talk equilibria.

Formally, let $\mathbf{C}_N = (c_1, ..., c_J)$ denote an ordering of the N goods into $J \in \{1, ..., N\}$ elements or categories such that category $j = 1, ..., J$ contains $c_j \in \{1, ..., N\}$ goods with $\sum_j c_j = N$. For $j = 1, ..., J$, let $\psi_j = \sum_{j'=1}^{j} c_{j'}$ be the number of goods in or below category j, with $\psi_0 = 0$.

The ordinal cheap talk strategy that corresponds to the ordering $\mathbf{C}_N$ is described as follows. For each realization of V, the seller announces that the c_1 goods with the lowest values belong to category 1, the next c_2 goods belong to category 2, and so on. If there are ties between some of the V_k's, the seller uniformly randomizes when she sorts those goods into different categories. Consequently, buyers know that goods in higher categories have a weakly higher value and cannot distinguish between goods within a category based on the seller's message.

In the rest of this paper we will denote an ordinal cheap talk strategy by the corresponding ordering $\mathbf{C}_N$. Note that such a strategy consists of a partition of $\mathbf{V}^N$. The partition has $N!/\prod_j (c_j!)$ elements, each element corresponding to one way of sorting the N goods into the categories specified by $\mathbf{C}_N$. We emphasize that the ordering $\mathbf{C}_N$ itself is fixed and does not depend on the realization of V. When buyers believe that the seller's announcement strategy is given by $\mathbf{C}_N$, we assume that to *each* message $m \in M$ they ascribe a unique meaning corresponding to one element of the partition of $\mathbf{V}^N$ that is generated by $\mathbf{C}_N$, thus ruling out the possibility of out–of–equilibrium messages.

Given N, we will say that an ordering $\mathbf{C}_N$ is finer than an ordering $\mathbf{C}'_N$ if it is a finer partition of $\mathbf{V}^N$ in the usual sense of partitions. The finest possible ordering, denoted by $\mathbf{C}^*_N$, is when the seller completely orders her N goods, i.e., $J = N$ and $c_j = 1$ for all j. On the other hand, the coarsest possible ordering, denoted by $\mathbf{C}^u_N$, has $J = 1$ and corresponds to the uninformative babbling equilibrium discussed above. Note that every ordinal cheap talk strategy $\mathbf{C}_N \neq \mathbf{C}^u_N$ is informative about V_k for all k.

For any ordering $\mathbf{C}_N$, when the seller announces that good k belongs to category j, the buyers of good k know that the value of good k is equally likely to be one of the c_j order statistics $\{V_{\psi_{j-1}+1:N}, ..., V_{\psi_j:N}\}$. Let $F_{j:\mathbf{C}_N}$ be the distribution of good k when the seller announces that good k belongs to category j. Due to our symmetry and independence assumptions $F_{j:\mathbf{C}_N}$ does not depend on k. We will denote by $V_{j:\mathbf{C}_N}$ the random variable that has distribution $F_{j:\mathbf{C}_N}$, with $\overline{V}_{j:\mathbf{C}_N} = E[V_{j:\mathbf{C}_N}]$. Note from the definition of $\mathbf{C}_N$ that $\overline{V}_{j:\mathbf{C}_N}$ is increasing in j.

Similarly, using (1), (2) and our symmetry assumptions, we let $P_{j:\mathbf{C}_N} = b_{j:\mathbf{C}_N}(Z_{2,k})$ denote the price that the seller obtains when her message m implies that good k belongs to category j. By affiliation and the definition of $\mathbf{C}_N$, $E[P_{j:\mathbf{C}_N}|V_k = v]$ is non–decreasing in v and j. The following lemma provides a necessary and sufficient condition for an ordinal cheap talk strategy characterized by an ordering $\mathbf{C}_N$ to be an equilibrium.

Lemma 1 *The ordering $\mathbf{C}_N = (c_1, ..., c_J)$ is an equilibrium if and only if for all v, v' with $v > v'$*

$$E[P_{j:\mathbf{C}_N}|V_k = v] - E[P_{j:\mathbf{C}_N}|V_k = v'] \text{ is non–decreasing in } j \in \{1, ..., J\}. \tag{4}$$

Proof. Necessity is immediate: if there existed j, j' with $j > j'$ such that

$$E[P_{j:\mathbf{C}_N}|V_k = v] - E[P_{j:\mathbf{C}_N}|V_k = v'] < \ E[P_{j':\mathbf{C}_N}|V_k = v] - E[P_{j':\mathbf{C}_N}|V_k = v'] \tag{5}$$

for some v, v' with $v > v'$, then for a realization of V such that $V_k = v > V_{k'} = v'$ and such that good k should be in category j and good k' in category j', the seller would do better to announce that k is in category j' and k' in j, keeping the rest of her announcement unchanged.

To show sufficiency, consider any subset of $L \leq N$ goods, indexed by k_l, $l = 1, ..., L$, such that $V_{k_1} \leq ... \leq V_{k_L}$. Fixing the categories announced for the other $N - L$ goods, suppose that $\{j_l\}_{l=1}^{L}$ are the categories the seller has available to announce for the L remaining goods, with $j_1 \leq ... \leq j_L$. We show by induction on L that she would want to announce the highest possible category j_L for good k_L, and so on, announcing category j_1 for good k_1. For $L = 2$, this is identical to (4).

Suppose as the inductive hypothesis that the claim is true for $L - 1$ and observe that the expected revenues from the L goods, from announcing a category other than $j_{l^*} \neq j_L$ for good k and category j_L announced for good $k_{l'}$, is equal to

$$\begin{aligned} & E[P_{j_{l^*}:\mathbf{C}_N}|V_{k_L}] + E[P_{j_L:\mathbf{C}_N}|V_{k_{l'}}] + \sum_{\substack{j_l \neq j_{l^*}, j_L \\ k_l \neq k_L, k_{l'}}} E[P_{j_l:\mathbf{C}_N}|V_{k_l}] \\ \leq \quad & E[P_{j_{l^*}:\mathbf{C}_N}|V_{k_{l'}}] + E[P_{j_L:\mathbf{C}_N}|V_{k_L}] + \sum_{\substack{j_l \neq j_{l^*}, j_L \\ k_l \neq k_L, k_{l'}}} E[P_{j_l:\mathbf{C}_N}|V_{k_l}] \end{aligned}$$

by (4). Thus, it is weakly optimal for the seller to announce category j_L for good k_L. By the inductive hypothesis, it is weakly optimal to put good k_{L-1} in category j_{L-1}, and so on, so that the sufficiency of (4) follows. ■

The 'increasing difference' condition in Lemma 1 for the existence of an ordinal cheap talk equilibrium captures an information complementarity between buyer valuations and the seller's information. It says that the gain in expected revenue from selling a higher-valued good in any auction must be non-decreasing in how optimistic the buyers are in that auction, given the seller's message. Such a condition is necessary and sufficient for the seller to announce higher categories for her better goods and lower categories for her worse goods.

In general, whether or not an informative ordinal cheap talk equilibrium exists depends on the structure of information held by the buyers and the seller.[6] The next result shows that one such equilibrium always exists for a large enough number of goods provided that the monotone likelihood ratio property implied by affiliation holds strictly rather than just weakly, that the support of the buyer signals is the same for all values of the goods, and that either the set of possible buyer signals or the set of possible values of the goods has a finite number of elements.

Theorem 1 *Suppose that for all $v, v' \in \mathbf{V}$ with $v > v'$, $\mathbf{X}(v) = \mathbf{X}(v') = \mathbf{X}$ and further $\frac{f_{X|V}(x|v)}{f_{X|V}(x|v')}$ is strictly increasing in $x \in \mathbf{X}$. Then if either $\mathbf{V}$ is finite or $\mathbf{X}$ is finite, there exists $\overline{N}$ such that for each $N > \overline{N}$ the set of informative equilibrium orderings is non–empty.*

Proof. See the Appendix. ■

The proof of Theorem 1 is constructive and proceeds by considering, for each N, an ordering $\mathbf{C}_N = (1, N-1)$ so that the goods are divided into two categories with 1 good in the lower category and $N-1$ goods in the higher category. For such an ordering, as N becomes large, buyers become more and more certain that the good in the lower category is likely to be of the lowest possible value and bid accordingly, regardless of their signals and the actual value of the good. However, as N becomes large, buyers do not have such strong beliefs about a good in the higher category since almost all of the goods are in that category and the probability distribution for the value of a good in the higher category therefore approaches the prior distribution. As a result, buyers pay more attention to their own private signals and on average bid more for a good that is better. Since the impact on prices from selling a better good is greater for the higher category, the increasing difference condition (4) is satisfied.[7]

[6] It also depends on other factors such as the auction format and on the number of bidders for each auction, but the necessary and sufficient condition (4) remains unchanged.

[7] In the proof of Theorem 1, the finiteness of $\mathbf{V}$ is used to guarantee a lower bound on the left–hand side and an upper bound on the right–hand side of (4) that are both independent of v and v', guaranteeing that for N large enough (4) holds for all v, v'. When $\mathbf{V}$ is a continuous random variable, one can achieve the same end if $\mathbf{X}$ is finite.

The argument underlying Theorem 1 also applies when the lower category has any fixed number c_1 of goods and N becomes large. More generally, the existence of an equilibrium does not require that there be only two categories or that N is large. In Section 4 we consider two examples of our general model where the set of informative ordinal cheap talk equilibria is quite large even for small N. But first we investigate the expected revenues from informative ordinal cheap talk strategies.

3.2 Revenues

Informative cheap talk equilibria are especially interesting because of their beneficial effect on ex–ante expected revenues via the linkage principle. Consider an ordering $\mathbf{C}_N = (c_1, ..., c_J)$ and let $R(\mathbf{C}_N)$ be the seller's per–good ex–ante expected revenue when she uses the ordering $\mathbf{C}_N$ and when the buyers believe that the seller is doing so. Then,

$$R(\mathbf{C}_N) = \frac{1}{N} \sum_{j=1}^{J} c_j E[P_{j:\mathbf{C}_N}]. \tag{6}$$

Note that when $\mathbf{C}_N$ is an equilibrium ordering, $R(\mathbf{C}_N)$ is the seller's equilibrium expected revenue.

The next result states that finer orderings lead to higher expected revenues. It implies that the expected revenues from any informative ordinal cheap talk equilibrium is higher than $R(\mathbf{C}_N^u)$, the expected revenue from the uninformative babbling equilibrium. The result follows from a direct application of Theorem 13 in MW.[8]

Theorem 2 *If $\mathbf{C}_N$ is finer than $\mathbf{C}'_N$ then $R(\mathbf{C}_N) \geq R(\mathbf{C}'_N)$.*

We now consider the complete ordering $\mathbf{C}_N^*$ and show that as the number of goods N becomes large, the per–good expected revenues converge to $\overline{V}$, the ex–ante expected value of each good. In other words, in the limit, the seller obtains the same revenue as she would from being able to fully disclose her information.

Theorem 3 *For the complete ordering $\mathbf{C}_N^*$, $\lim_{N\to\infty} R(\mathbf{C}_N^*) = \overline{V}$.*

Proof. See the Appendix. ■

[8] See also footnote 4.

For example, suppose that V_k is uniformly distributed in $[0,1]$. When buyers know the complete ordering of the goods, for a large number of goods it is almost sure that the value of the highest ranked good is close to 1, and that the value of the lowest ranked good is close to 0. Because of this increased certainty, buyers will be willing to bid close to 1 and 0 respectively for the two goods, regardless of their private signals, so that the prices will also be close to 1 and 0, respectively. Theorem 3 uses the Glivenko–Cantelli Theorem and shows that this same logic applies along the entire distribution— for any $p \in (0,1)$, if there are N goods, then the value of the pN–th good is likely to be very close to p as N becomes large, and buyers will pay close to that value. As the number of goods increases, buyers become more and more certain that the ranking of the good narrowly constrains the good's likely value so that per–good information rents converge to zero.

Our last result of this section considers *partial* orderings $\mathbf{C}_N$ that asymptotically yield expected revenue equal to $\overline{V}$ as the number of goods becomes large when $\mathbf{V}$ is finite, i.e.,

$$V_k \in \{v_1, ..., v_H\} \text{ with } \Pr[V_k = v_h] = \lambda_h \in (0,1) \text{ for } h = 1, ..., H. \tag{7}$$

For each N consider the ordering $\mathbf{C}_N = (c_1^N, ..., c_H^N)$ with the asymptotic property that

$$\lim_{N \to \infty} \frac{c_h^N}{N} = \lambda_h \text{ for } h = 1, ..., H. \tag{8}$$

In other words, $\mathbf{C}_N$ orders the goods into H categories, with the number of goods in each category $h \geq 1$ being in proportion (asymptotically) to the probability that V_k take its hth value. As N becomes large, the probability that a good in the h–th category takes the value v_h becomes arbitrarily close to 1. This implies that information rents vanish for each good and asymptotic revenues equal $\overline{V}$.

Theorem 4 *Assume (7). For any sequence of orderings* $\{\mathbf{C}_N\}_N$ *satisfying (8),* $\lim_{N \to \infty} R(\mathbf{C}_N) = \overline{V}$.

Proof. See the Appendix. ■

4 Two Special Cases

In view of Theorems 3 and 4, it is of interest to identify models where the set of ordinal cheap talk equilibria is large and includes, in particular, the complete ordering $\mathbf{C}_N^*$ or the asymptotically revenue-equivalent partial orderings characterized by (8). In this section, we develop two simple special cases of our general model above with these properties. In the first case, we impose restrictions on the nature of signals that the buyers for each good can possess while keeping the seller's information unrestricted. In the second case, we do the opposite.

4.1 Informed/Uninformed Buyers

Let F_V be as before but suppose that each buyer i_k knows V_k with probability $\beta \in (0,1)$ and has no information with probability $1-\beta$. The probability that any buyer is informed is independent across buyers. It is straightforward to check that such a structure satisfies all the assumptions of the model.

For any candidate equilibrium ordering $\mathbf{C}_N$, recall that $\overline{V}_{j:\mathbf{C}_N}$ is the expected value of a good that belongs to category j. The following bidding behavior constitutes a symmetric Bayesian Nash Equilibrium of the auction for any good k that belongs to category j according to the seller's announcement, as can be easily checked.

Any informed bidder i_k who knows $V_k = v$ drops out with probability 1 when the price $p \geq v$ and remains active with probability 1 for all $p < v$ regardless of how many other bidders are or have been active. Any bidder i_k who is uninformed drops out with probability 1 whenever the number of active bidders $\iota(p)$ is less than n, or whenever the price $p \geq \overline{V}_{j:\mathbf{C}_N}$, and stays active with probability 1 otherwise.

As a result, when $V_k \geq \overline{V}_{j:\mathbf{C}_N}$ the seller obtains a price equal to $\overline{V}_{j:\mathbf{C}_N}$ when at most 1 out of n bidders are informed, and obtains a price equal to V_k otherwise. On the other hand, when $V_k < \overline{V}_{j:\mathbf{C}_N}$, the seller obtains a price equal to $\overline{V}_{j:\mathbf{C}_N}$ when no bidder is informed, and obtains a price equal to V_k otherwise. Thus,

$$E[P_{j:\mathbf{C}_N}|V_k] = \begin{cases} (1-\Pi)V_k + \Pi\overline{V}_{j:\mathbf{C}_N} & \text{if } V_k \geq \overline{V}_{j:\mathbf{C}_N} \\ (1-\pi)V_k + \pi\overline{V}_{j:\mathbf{C}_N} & \text{if } V_k < \overline{V}_{j:\mathbf{C}_N} \end{cases} \tag{9}$$

where $\pi = (1-\beta)^n$ is the probability that none of the n bidders is informed and $\Pi = \pi + n\beta(1-\beta)^{n-1}$ is the probability that at most 1 of the n bidders is informed.

Proposition 1 *In the informed/uninformed buyer model, for each N, any ordering $\mathbf{C}_N$ is an equilibrium.*

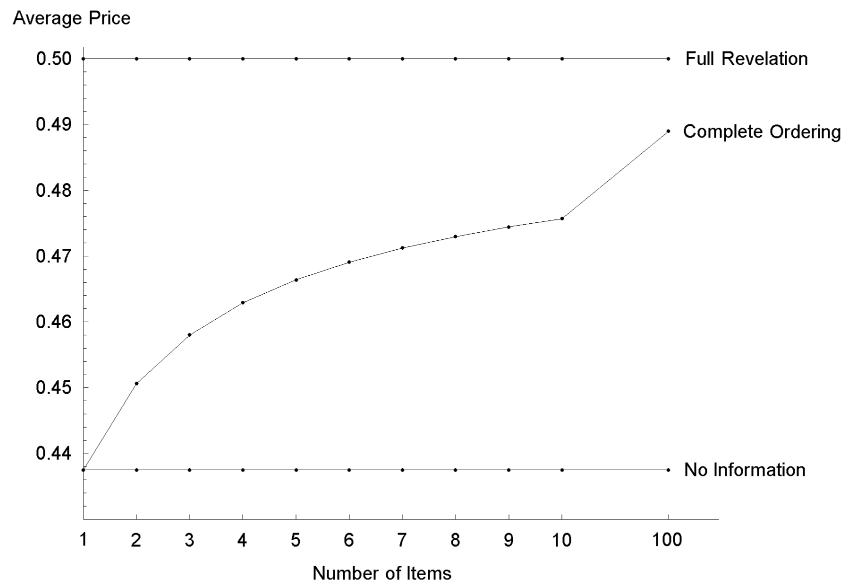

Figure 1: Informed/uninformed buyer model, $n = 2$, $V_k \sim U[0,1]$, $\beta = 1/2$.

Proof. Pick an ordering $\mathbf{C}_N$ and categories j, j' with $j > j'$. Since $\overline{V}_{j:\mathbf{C}_N} > \overline{V}_{j':\mathbf{C}_N}$, we observe from (9) that for any v,

$$E[P_{j:\mathbf{C}_N} - P_{j':\mathbf{C}_N} | V_k = v] = \begin{cases} \pi(\overline{V}_{j:\mathbf{C}_N} - \overline{V}_{j':\mathbf{C}_N}) & \text{if } v < \overline{V}_{j':\mathbf{C}_N} \\ (\Pi - \pi)v + \pi\overline{V}_{j:\mathbf{C}_N} - \Pi\overline{V}_{j':\mathbf{C}_N} & \text{if } \overline{V}_{j':\mathbf{C}_N} \leq v < \overline{V}_{j:\mathbf{C}_N} \\ \Pi(\overline{V}_{j:\mathbf{C}_N} - \overline{V}_{j':\mathbf{C}_N}) & \text{if } \overline{V}_{j:\mathbf{C}_N} \leq v \end{cases} \quad (10)$$

Since $\Pi > \pi$ the expression above is non–decreasing in v. But this is equivalent to (4) so that $\mathbf{C}_N$ is an equilibrium ordering. ■

Proposition 1 implies that the full ordering $\mathbf{C}_N^*$ is an equilibrium for every N. Thus the asymptotic revenue result from Theorem 3 is relevant. Figure 1 shows the per–good ex–ante expected revenues as a function of the number of goods N when there are two bidders for each good ($n = 2$), F_V is the uniform distribution on $[0, 1]$, and the probability β that a bidder is informed is equal to $\frac{1}{2}$. For the uniform distribution, $\overline{V}_{k:\mathbf{C}_N^*} = \frac{k}{N+1}$ for $k = 1, ..., N$. The average price under full revelation for any number of goods is the expected value of the good, $\frac{1}{2}$. Of course, such revelation is not credible in this example nor more generally. If the seller does not make any credible statements then the per–good expected revenue is just .4375 for any number of goods. Under ordinal cheap talk with a complete ordering the price rises to .4506 for 2 goods and continues to rise with the number of goods. Buyer information rents, as represented by the difference in the expected value and the expected price, fall by over 50% for six goods and by over 80% for 100 goods.

4.2 Binary Seller Information

Suppose that $V_k \in \{0,1\}$ where $\Pr[V_k = 1] = \lambda \in (0,1)$ for $k \in \{1, ..., N\}$. Suppose also that $\mathbf{X}(v) = \mathbf{X}$ for all $v \in \{0,1\}$ and that the likelihood ratio of buyer signals conditional on V_k is bounded:

$$\bar{l} = \sup_{x \in \mathbf{X}} \frac{f_{X|V}(x|1)}{f_{X|V}(x|0)} < \infty. \tag{11}$$

Consider the ordering

$$\mathbf{C}_N = (c_1, c_2) \text{ such that } c_2 \geq \lambda N. \tag{12}$$

Let $\lambda_{j:\mathbf{C}_N} = \Pr[V_{j:\mathbf{C}_N} = 1]$ for $j = 1, 2$. Note that $\lambda_{1:\mathbf{C}_N} = E[\max\{0, \frac{Y - c_2}{c_1}\}]$ and $\lambda_{2:\mathbf{C}_N} = E[\min\{1, \frac{Y}{c_2}\}]$ where Y is a binomial random variable with parameters λ and N. We have the following result.

Proposition 2 *Consider the binary seller information model with (11) and suppose that $\lambda \leq \frac{1}{1+\bar{l}^2}$. Then for all N such that λN is an integer, any ordering $\mathbf{C}_N$ satisfying (12) is an equilibrium. Further, there exists $\overline{N}$ such that for all $N > \overline{N}$ any ordering $\mathbf{C}_N$ satisfying (12) is an equilibrium.*

Proof. See the Appendix. ■

The orderings given by (12) contain those covered by Theorem 4, e.g., the ordering with $c_2 = \lceil \lambda N \rceil$. Consequently, in the binary signal model with (11) and λ low enough, there exists an ordinal cheap talk equilibrium with the property that per–good expected revenues are approximately equal to $\overline{V}$, the ex–ante expected value, when the seller has a large number of goods.

In the rest of this section we consider an example of this model where we can explicitly solve for equilibrium bids and where we can strengthen the conclusions of Proposition 2. Suppose that there are two buyers for each good, and each buyer gets a binary signal $X_{i_k} \in \{0,1\}$, with

$$\Pr[X_{i_k} = 1 | V_k = 1] = \Pr[X_{i_k} = 0 | V_k = 0] = \beta \in (\frac{1}{2}, 1). \tag{13}$$

The buyers' signals are independent conditional on the value of the good. Note that condition (11) holds for this signal structure with $\bar{l} = \frac{\beta}{1-\beta} > \frac{1}{2}$.

With two bidders, in the symmetric equilibrium of the English (equivalently, second price) auction, each bidder bids the probability that the good has value 1 conditional on his own signal $X_{ik} = x$, on the other bidder having the same signal, and on the announced category for the good. Consider an ordering $\mathbf{C}_N = (c_1, c_2)$ that divides the goods into two categories with $\lambda_{j:\mathbf{C}_N}$ being the probability that $V_k = 1$ given that it is in category $j \in \{1, 2\}$. Let $b_{j:\mathbf{C}_N}(x) = \Pr[V_k = 1|X_{1k} = x = X_{2k}]$ be the equilibrium bid of a buyer with signal $x \in \{0, 1\}$,when the seller has announced that the good is in category j :

$$b_{j:\mathbf{C}_N}(1) = \frac{\beta^2 \lambda_{j:\mathbf{C}_N}}{\beta^2 \lambda_{j:\mathbf{C}_N} + (1-\beta)^2 (1 - \lambda_{j:\mathbf{C}_N})} \tag{14}$$

and

$$b_{j:\mathbf{C}_N}(0) = \frac{(1-\beta)^2 \lambda_{j:\mathbf{C}_N}}{(1-\beta)^2 \lambda_{j:\mathbf{C}_N} + \beta^2 (1 - \lambda_{j:\mathbf{C}_N})} \tag{15}$$

Note that the high bid is received only when both buyers have a high signal. Therefore,

$$\begin{aligned} E[P_{j:\mathbf{C}_N}|V_k &= 1] = \beta^2 b_{j:\mathbf{C}_N}(1) + (1-\beta^2) b_{j:\mathbf{C}_N}(0) \\ E[P_{j:\mathbf{C}_N}|V_k &= 0] = (1-\beta)^2 b_{j:\mathbf{C}_N}(1) + (1-(1-\beta)^2) b_{j:\mathbf{C}_N}(0). \end{aligned}$$

It is straightforward to check that in this model the necessary and sufficient condition (4) for the existence of equilibrium reduces to the simple condition

$$\lambda_{1:\mathbf{C}_N} + \lambda_{2:\mathbf{C}_N} \leq 1 \tag{16}$$

that is independent of β. Furthermore, since both $\lambda_{1:\mathbf{C}_N}$ and $\lambda_{2:\mathbf{C}_N}$ are decreasing in c_2, there exists a cutoff value $\bar{c}_2$ (depending on λ and N) such that the ordering $\mathbf{C}_N = (c_1, c_2)$ is an equilibrium if and only if $c_2 \geq \bar{c}_2$. The following result shows that for this example Proposition 2 can be considerably strengthened.

Proposition 3 *Consider the binary seller information model with $n = 2$ and buyer signals satisfying (13). There exists an equilibrium sequence of orderings $\mathbf{C}_N = (c_1^N, c_2^N)$ such that $\lim_{N\to\infty} \frac{c_2^N}{N} = \lambda$.*

Proof. For this case $\lambda_{1:\mathbf{C}_N} + \lambda_{2:\mathbf{C}_N} \leq 1$ (equivalently, $\frac{1-\lambda_{2:\mathbf{C}_N}}{\lambda_{1:\mathbf{C}_N}} \geq 1$) is necessary and sufficient for an ordering $\mathbf{C}_N = (c_1^N, c_2^N)$ to be an equilibrium. For $\lambda \leq \frac{1}{2}$, let $c_2^N = \lceil \lambda N \rceil$ (i.e., the smallest integer at least as high as λN) and note that since $\lambda_{1:\mathbf{C}_N} \leq \lambda_{2:\mathbf{C}_N}$, by symmetry and the definition of conditional probabilities

$$\lambda = \frac{c_1^N}{N}\lambda_{1:\mathbf{C}_N} + \frac{c_2^N}{N}\lambda_{2:\mathbf{C}_N} = \frac{N - \lceil \lambda N \rceil}{N}\lambda_{1:\mathbf{C}_N} + \frac{\lceil \lambda N \rceil}{N}\lambda_{2:\mathbf{C}_N} \geq (1-\lambda)\lambda_{1:\mathbf{C}_N} + \lambda\lambda_{2:\mathbf{C}_N} \quad (17)$$

so that

$$\frac{1-\lambda_{2:\mathbf{C}_N}}{\lambda_{1:\mathbf{C}_N}} \geq \frac{1-\lambda}{\lambda} \geq 1. \quad (18)$$

This proves the result for $\lambda \leq \frac{1}{2}$.

Now consider $\lambda > \frac{1}{2}$. From Theorem 1, we are guaranteed the existence of a two category informative ordering $\mathbf{C}_N = (1, N-1)$ for N large enough. For each such N, let $\overline{\mathbf{C}}_N = (\overline{c}_1^N, \overline{c}_2^N)$ be the informative ordering with the lowest value of c_2^N. Using arguments similar to those used for establishing (18), we see that $\overline{c}_2^N > \lambda N$. We want to show that $\lim_{N\to\infty} \frac{\overline{c}_2^N}{N} = \lambda$.

Suppose not. Since $\frac{\overline{c}_2^N}{N} \in [0,1]$ for each N, there exists $\varepsilon > 0$ and a convergent subsequence $\{N_r\}$ such that $\lim_{r\to\infty} \frac{\overline{c}_2^{N_r}}{N_r} > \lambda+\varepsilon$. For each N_r, consider the ordering $\widetilde{\mathbf{C}}_{N_r} = (\overline{c}_1^{N_r}+1, \overline{c}_2^{N_r}-1)$. Since $\lim_{r\to\infty} \frac{\overline{c}_1^{N_r}+1}{N_r} < 1-\lambda$, by the Law of Large Numbers it follows that $\lim_{r\to\infty} \lambda_{1:\widetilde{\mathbf{C}}_{N_r}} = 0$. Furthermore, since for each N_r, $\widetilde{\mathbf{C}}_{N_r}$ is not an equilibrium ordering we must have

$$\lambda_{1:\widetilde{\mathbf{C}}_{N_r}} + \lambda_{2:\widetilde{\mathbf{C}}_{N_r}} > 1 \quad (19)$$

so that $\lim_{r\to\infty} \lambda_{2:\widetilde{\mathbf{C}}_{N_r}} = 1$. Since for each N_r,

$$\lambda = \frac{\widetilde{c}_1^{N_r}}{N_r}\lambda_{1:\mathbf{C}_{N_r}} + \frac{\widetilde{c}_2^{N_r}}{N_r}\lambda_{2:\mathbf{C}_{N_r}} \quad (20)$$

it follows that $\lim_{r\to\infty} \frac{\widetilde{c}_2^{N_r}}{N_r} = \lambda$. But since $\lim_{r\to\infty} \frac{\overline{c}_2^{N_r}}{N_r} = \lim_{r\to\infty} \frac{\widetilde{c}_2^{N_r}}{N_r}$, this establishes a contradiction, completing the proof. ■

Proposition 3 shows that we are guaranteed the existence of a sequence of equilibrium orderings for which Theorem 4 applies. Figure 2 plots the expected per–good revenue from the ordering $\mathbf{C}_N = (N - \lceil \lambda N \rceil, \lceil \lambda N \rceil)$ when $\lambda = \frac{1}{2}$ and $\beta = \frac{3}{4}$ as N varies. In the no information or "babbling" case buyers are very unsure whether a good is high or low value, and so each bidder reduces their bid out of fear of the winner's curse. The expected price of .35 is therefore substantially below the expected value of $\lambda = \frac{1}{2}$. Categorizing the goods based on their relative values increases revenues. As the number of goods increases, the probability that a good in the low category has value 0 rises as does the probability that a good in the high category has value 1. Buyers are therefore more and more confident of the likely value of each good, so competition intensifies and buyer information rents fall.

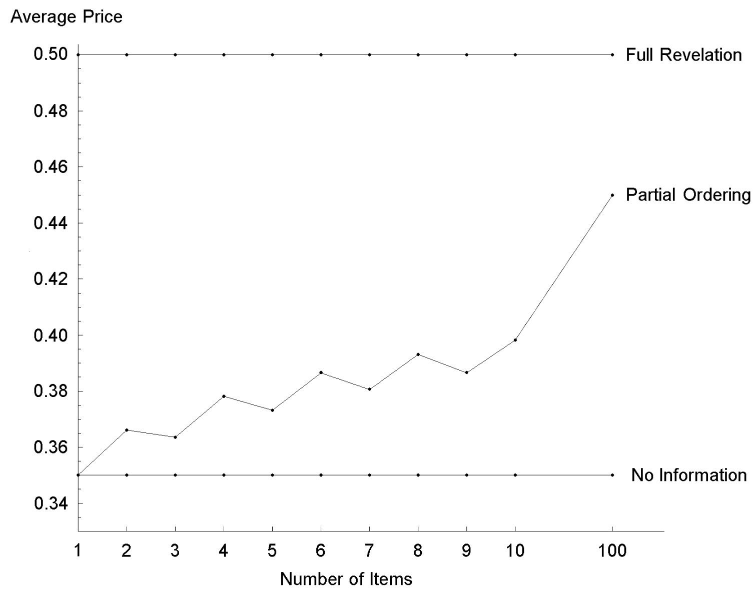

Figure 2: Binary seller information model, $n = 2$, $V_k \in \{0, 1\}$, $\lambda = 1/2$, $\beta = 1/2$.

5 Conclusion

Sellers often make comparative statements about the values of their goods. In a multi-object auction, we show that such statements can be credible even though full revelation of the seller's information is not. As a result, buyer information rents fall and seller revenues rise in accordance with the linkage principle. Moreover, seller revenues asymptotically approach revenues under full revelation as the number of goods increases.

Ordinal cheap talk also has applications to related non-auction selling environments. For instance, a salesperson might have information regarding what good is better for a particular buyer. The buyer might be suspicious of the salesperson's claims about the value of each good, but might still believe claims that one good is likely to be preferred over another. Similarly, in financial markets, an analyst's claims about the likely returns to a stock might not be credible, but the statement that one stock is a better bet than another might be.

6 Appendix

Definition of Affiliation

A subset S of $\mathbb{R}^n$ is a sub–lattice if, whenever $x, x' \in S$, so are their component–wise maximum (meet) and component–wise minimum (join). The indicator function $1_A(x)$ of any subset A of $\mathbb{R}^n$ is defined to be equal to 1 if $x \in A$ and equal to 0 otherwise. A set A is increasing if its indicator function is non–decreasing. Let $X = (X_1, ..., X_n)$ be a random n–vector with probability distribution P (i.e., $P(A) = \Pr[X \in A]$).

Definition 1 *$X_1, ..., X_n$ are affiliated if for all increasing sets A and B and every sub–lattice S, $P[A \cap B|S] \geq P[A|S]P[B|S]$.*

Proof of Theorem 1

For each N, consider the ordering $\mathbf{C}_N = (1, N-1)$. Let $v_{\min} = \min \mathbf{V}$. Note that as N becomes large $V_{1:\mathbf{C}_N}$ converges almost surely to $v_{\min}$ and $V_{2:\mathbf{C}_N}$ converges in distribution to V_k. Given that $\mathbf{X}(v) = \mathbf{X}$ for each $v \in \mathbf{V}$, as well as the assumed properties of F_V and $F_{X|V}$, it follows that for *each* $z_2 = (y_2, z_3)$, $b_{1:\mathbf{C}_N}(z_2)$ converges to $v_{\min}$ and $b_{2:\mathbf{C}_N}(z_2)$ converges to $b_u(z_2)$, where $b_u(z_2)$ is defined in (3). Furthermore, the strict monotone likelihood ratio condition assumed in the statement of the theorem implies that $b_u(z_2)$ is strictly increasing in its arguments and that $E[b_u(Z_{2,k})|V_k = v]$ is strictly increasing in v.

Suppose first that $\mathbf{V}$ is finite. Since $P_{j:\mathbf{C}_N} = b_{j:\mathbf{C}_N}(Z_{2,k})$ for $j = 1, 2$ it follows that there exists $\varepsilon \geq 0$ and $\overline{N}$ such that for all $N > \overline{N}$,

$$\max_{v>v'}\{E[P_{1:\mathbf{C}_N}|V_k = v] - E[P_{1:\mathbf{C}_N}|V_k = v']\} \leq \min_{v>v'}\{E[b_u(Z_{2,k})|V_k = v] - E[b_u(Z_{2,k})|V_k = v']\} - \varepsilon \tag{21}$$

and

$$\min_{v>v'}\{E[b_u(Z_{2,k})|V_k = v] - E[b_u(Z_{2,k})|V_k = v']\} - \varepsilon \leq \min_{v>v'}\{E[P_{2:\mathbf{C}_N}|V_k = v] - E[P_{2:\mathbf{C}_N}|V_k = v']\}. \tag{22}$$

But then (4) holds.

Suppose next that $\mathbf{X}$ is finite. Since b_u is strictly increasing, there exists $\varepsilon > 0$ such that[9]

$$\min_{z_2 > z_2'}\{b_u(z_2) - b_u(z_2')\} > \varepsilon. \tag{23}$$

[9]Following usual convention, the inequality $z_2 > z_2'$ allows the vectors z_2 and z_2' to be identical in some but not all components.

Observe that since $P_{j:\mathbf{C}_N} = b_{j:\mathbf{C}_N}(Z_{2,k})$ for $j = 1, 2$, condition (4) can be rewritten as

$$E[b_{2:\mathbf{C}_N}(Z_{2,k}) - b_{1:\mathbf{C}_N}(Z_{2,k})|V_k = v] \text{ is non–decreasing in } v. \tag{24}$$

To establish (4), by affiliation of V_k and X_k it suffices to show that there exists $\overline{N}$ such that for all $N > \overline{N}$, the function $b_{2:\mathbf{C}_N}(z_2) - b_{1:\mathbf{C}_N}(z_2)$ is non–decreasing in its arguments. But since $b_{1:\mathbf{C}_N}(z_2)$ converges to $v_{\min}$ and $b_{2:\mathbf{C}_N}(z_2)$ converges to $b_u(z_2)$ for each z_2, there exists $\overline{N}$ such that for all $N > \overline{N}$,

$$\max_{z_2 > z_2'}\{b_{1:\mathbf{C}_N}(z_2) - b_{1:\mathbf{C}_N}(z_2')\} < \min_{z_2 > z_2'}\{b_u(z_2) - b_u(z_2')\} - \varepsilon \tag{25}$$

and

$$\min_{z_2 > z_2'}\{b_u(z_2) - b_u(z_2')\} - \varepsilon \leq \min_{z_2 > z_2'}\{b_{2:\mathbf{C}_N}(z_2) - b_{2:\mathbf{C}_N}(z_2')\}. \tag{26}$$

But this implies that for $N > \overline{N}$, $b_{2:\mathbf{C}_N}(z_2) - b_{1:\mathbf{C}_N}(z_2)$ is non–decreasing in its arguments, establishing (24). ■

Proof of Theorem 3

Observe first that when F_V is a step function (so that V_k takes a finite number of values), the result follows from Theorem 4, via Theorem 2, as $\mathbf{C}_N^*$ is the finest possible ordering. Accordingly, we provide here a proof for the case where F_V has a positive density f_V, so that F_V is invertible.

Note that $\int_0^1 F_V^{-1}(q)dq = \overline{V}$ and let $\lceil x \rceil$ denote the smallest integer at least as large as x. By the Glivenko–Cantelli Theorem, for each $q \in (0, 1)$,

$$\lim_{N\to\infty} V_{\lceil qN \rceil :N} = F_V^{-1}(q) \text{ a.s.} \tag{27}$$

Since V_k is bounded in $[0, 1]$, it follows from (1) and (2), via the dominated convergence theorem for conditional expectations, that for each such q:

$$\lim_{N\to\infty} E[P_{\lceil qN \rceil :\mathbf{C}_N^*}] = F_V^{-1}(q). \tag{28}$$

Pick $\varepsilon > 0$ and let $\{q_l\}_{l=0}^{L}$ be a collection such that $0 = q_0 < .... < q_L = 1$, for all $l = 1, ..., L$, and, furthermore,

$$\sum_{l=0}^{L-1}(q_{l+1} - q_l)F_V^{-1}(q_l) > \int_0^1 F_V^{-1}(q)dq - \varepsilon = \overline{V} - \varepsilon. \tag{29}$$

Since we can write

$$R(\mathbf{C}_N^*) = \frac{1}{N}\sum_{k=1}^{N} E[P_{k:\mathbf{C}_N^*}], \tag{30}$$

and, by affiliation, for k, l such that $q_l N \leq k < q_{l+1}N \quad E[P_{k:\mathbf{C}_N^*}] \geq E[P_{\lceil q_l N\rceil:\mathbf{C}_N^*}]$, it follows that

$$\begin{aligned}
\lim_{N\to\infty} R(\mathbf{C}_N^*) &= \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} E[P_{k:\mathbf{C}_N^*}] \\
&\geq \lim_{N\to\infty}\frac{1}{N}\sum_{l=0}^{L-1}(\lceil q_{l+1}N\rceil - \lceil q_l N\rceil)E[P_{\lceil q_l N\rceil:\mathbf{C}_N^*}] \\
&\geq \lim_{N\to\infty}\sum_{l=0}^{L-1}(q_{l+1}-q_l)E[P_{\lceil q_l N\rceil:\mathbf{C}_N^*}] - \lim_{N\to\infty}\frac{L}{N} \\
&= \sum_{l=0}^{L-1}(q_{l+1}-q_l)F_V^{-1}(q_l) > \overline{V} - \varepsilon.
\end{aligned}$$

Since ε was arbitrary, this completes the proof. ■

Proof of Theorem 4

For the ordering $\mathbf{C}_N$ defined by (8) let $\psi_h^N = \sum_{h'=1}^{h} c_h^N$ with $\psi_0^N = 0$. For any $h = 1, ..., H$, notice that the value of a good given that it is in category h is equally likely to be one of the order statistics $\{V_{\psi_{h-1}^N+1:N}, ..., V_{\psi_h^N:N}\}$, so that

$$\Pr[V_{h:\mathbf{C}_N} = v_h] = \frac{1}{c_h^N}\sum_{k^N=1}^{c_h^N}\Pr[V_{\psi_{h-1}^N+k^N:N} = v_h]. \tag{31}$$

Fix h and pick an $\varepsilon \in (0, \lambda_h)$. Pick $\delta \in (0, \frac{\varepsilon\lambda_h}{2})$ and let N be large enough so that $\frac{2\lceil\delta N\rceil}{N} < \frac{c_h^N}{N}$. Observe that

$$\Pr[V_{h:\mathbf{C}_N} = v_h] \geq \frac{1}{c_h^N}\sum_{k^N=\lceil\delta N\rceil}^{c_h^N-\lceil\delta N\rceil}\Pr[V_{\psi_{h-1}^N+k^N:N} = v_h]. \tag{32}$$

Now, for each $k^N \in \{\lceil\delta N\rceil, ..., c_h^N - \lceil\delta N\rceil\}$,

$$\sum_{h'=1}^{h-1}\lambda_{h'} < \lim_{N\to\infty}\frac{\psi_{h-1}^N + k^N}{N} < \sum_{h'=1}^{h}\lambda_{h'} \tag{33}$$

so that by the Law of Large Numbers, $\lim_{N\to\infty}\Pr[V_{\psi_{h-1}^N+k^N:N} = v_h] = 1$. Thus,

$$\lim_{N\to\infty}\Pr[V_{h:\mathbf{C}_N} = v_h] \geq \lim_{N\to\infty}\frac{c_h^N - 2\lceil\delta N\rceil}{c_h^N} = 1 - \frac{2\delta}{\lambda_h} > 1 - \varepsilon \tag{34}$$

and we conclude that $V_{h:\mathbf{C}_N}$ converges to v_h in probability for all $h = 1, ..., H$.

Consequently, from (1), for all y_2, z_3 such that $f_{X|V}(y_2, y_2, z_3|v_h) > 0$, the winning bid $b_{h:\mathbf{C}_N}(y_2, z_3)$ must converge to v_h, so that $\lim_{N\to\infty} E[P_{h:\mathbf{C}_N}|V_{h:\mathbf{C}_N} = v_h] = v_h$. Furthermore, as $V_k \in [0, 1]$, we must have

$$\Pr[V_{h:\mathbf{C}_N} = v_h]E[P_{h:\mathbf{C}_N}|V_{h:\mathbf{C}_N} = v_h] + (1 - \Pr[V_{h:\mathbf{C}_N} = v_h]) \geq E[P_{h:\mathbf{C}_N}] \tag{35}$$

and

$$E[P_{h:\mathbf{C}_N}] \geq \Pr[V_{h:\mathbf{C}_N} = v_h]E[P_{h:\mathbf{C}_N}|V_{h:\mathbf{C}_N} = v_h]. \tag{36}$$

Taking limits on both sides of (35) and (36) we conclude that $\lim_{N\to\infty} E[P_{h:\mathbf{C}_N}] = v_h$ for all $h = 1, ..., H$ and the result follows. ■

Proof of Proposition 2

Recall from (24), that it is sufficient to prove that $b_{2:\mathbf{C}_N}(z_2) - b_{1:\mathbf{C}_N}(z_2)$ is non–decreasing in each argument. With $z_2 = (y_2, z_3)$, let $l(z_2) = \frac{f_{X|V}(y_2,y_2,z_3|1)}{f_{X|V}(y_2,y_2,z_3|0)}$ and observe that this is non–decreasing in each argument, by affiliation. From (1) observe also that

$$b_{2:\mathbf{C}_N}(z_2) - b_{1:\mathbf{C}_N}(z_2) = \frac{l(z_2)\lambda_{2:\mathbf{C}_N}}{l(z_2)\lambda_{2:\mathbf{C}_N} + (1 - \lambda_{2:\mathbf{C}_N})} - \frac{l(z_2)\lambda_{1:\mathbf{C}_N}}{l(z_2)\lambda_{1:\mathbf{C}_N} + (1 - \lambda_{1:\mathbf{C}_N})} \tag{37}$$

which is non–decreasing in z_2 iff

$$\bar{l}^2 \leq \frac{1 - \lambda_{2:\mathbf{C}_N}}{\lambda_{1:\mathbf{C}_N}} \frac{1 - \lambda_{1:\mathbf{C}_N}}{\lambda_{2:\mathbf{C}_N}}. \tag{38}$$

Since $\lambda_{2:\mathbf{C}_N} > \lambda_{1:\mathbf{C}_N}$, for the ordering with $c_2 \geq \lambda N$ we must have

$$\lambda = \frac{c_1}{N}\lambda_{1:\mathbf{C}_N} + \frac{c_2}{N}\lambda_{2:\mathbf{C}_N} \geq (1 - \lambda)\lambda_{1:\mathbf{C}_N} + \lambda\lambda_{2:\mathbf{C}_N} \tag{39}$$

so that $\frac{1-\lambda_{2:\mathbf{C}_N}}{\lambda_{1:\mathbf{C}_N}} \geq \frac{1-\lambda}{\lambda}$. Furthermore, note that for $\lambda < \frac{1}{2}$ and any ordering with $c_2 = \lceil \lambda N \rceil$ we have $c_1 \geq c_2$, at least for N large. Since $\lambda_{1:\mathbf{C}_N} = E[\max\{0, \frac{Y - c_2}{c_1}\}]$ and $\lambda_{2:\mathbf{C}_N} = E[\min\{1, \frac{Y}{c_2}\}]$, where $E[Y] = \lambda N$, by Jensen's inequality we obtain

$$\begin{aligned}
\lambda_{1:\mathbf{C}_N} + \lambda_{2:\mathbf{C}_N} &= E[\max\{0, \frac{Y - c_2}{c_1}\} + \min\{1, \frac{Y}{c_2}\}] \\
&\leq \max\{0, \frac{\lambda N - c_2}{c_1}\} + \min\{1, \frac{\lambda N}{c_2}\} \\
&\leq 1
\end{aligned}$$

so that $\frac{1-\lambda_{1:\mathbf{C}_N}}{\lambda_{2:\mathbf{C}_N}} \geq 1$. Since $\lambda_{1:\mathbf{C}_N}$ and $\lambda_{2:\mathbf{C}_N}$ are both decreasing in c_2, we obtain the same inequality for $c_2 > \lceil \lambda N \rceil$.

We conclude that for all orderings satisfying (12), the right–hand side of (38) is greater than or equal to $\frac{1-\lambda}{\lambda}$ for $\lambda \leq \frac{1}{2}$, for all N such that λN is an integer, and for N large enough otherwise. Thus, for $\lambda \leq \min[\frac{1}{2}, \frac{1}{1+\bar{l}^2}] = \frac{1}{1+\bar{l}^2}$, (38) holds and the result follows. ■

7 Bibliography

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