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Group Consumption, Free Riding, and Informal Reciprocity Agreements

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Abstract

We examine conditions under which group consumption is likely to involve informal and tacit reciprocity agreements rather than formal contracts and a price system. Our model shows that informal reciprocity agreements are more likely to be used when transactions costs of formal agreements are high, the good is relatively inexpensive, each consumer's demand is not too responsive to price changes, the group is likely to continue to interact over time, the consumers are patient, the time between interactions is short, and the group is small and homogeneous. Further, the results suggest that informal sharing agreements are more likely to involve goods that are consumed along with other group benefits, such as conversation and companionship. We conclude by analyzing investments in social capital and discussing the effects of deeper social interactions constrained by norm structures on our results.

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1. Introduction

"Economics cannot go far enough without sociological facts and theory." Vilfredo Pareto, reported remarks paraphrased at his jubilee celebration, University of Lausanne (July 6, 1917).

Economic literature has long recognized two "truths": Generally speaking, the most efficient method of allocating private consumption goods is via private property and the market; the most efficient method of allocating goods with public consumption characteristics is through collective action, such as clubs, non-profit firms or governments, which serves to coordinate aggregate purchases.[1] These "truths" are at odds, however, with many and varied real world observations. Valuable private goods are frequently and voluntarily put into the commons by consent, while many goods with collective consumption characteristics are privately and near spontaneously provided with no formal attempts to coordinate their aggregate purchases to achieve putatively superior allocations.

Two examples illustrate this point. Colleagues often meet for lunch or drinks after work and it is not unusual for one person to offer to pick up the final tab - an act which converts essentially private consumption into common property consumption. Alternately, the bill is split equally, without regard to the exact share that individual tabs would have yielded. In the case of goods with joint consumption characteristics, neighbors often chauffeur each other's children to community events and even share in neighborhood activities such as local park maintenance, all without formally organizing into clubs.

While one might be tempted to regard these instances as simple cases of charity, such need not be the case. Actions such as these frequently imply informal and implicit reciprocity agreements. The colleague who picks up the tab this time usually expects to be treated for the next round of drinks or the next lunch. Likewise the neighbor who chauffeurs another's child today is likely to ask that the favor be returned next week. Although these are examples of sequential sharing, sometimes the sharing is contemporaneous and certain. The family who contributes a casserole to a pot-luck supper is the immediate recipient of dishes prepared

[1] Samuelson (1954) recommends government provision of pure public goods, while Buchanan (1965) shows that private clubs can efficiently provide congestible (and excludable) public goods.

by other group members. Such simultaneity also occurs in public good-type settings when neighbors gather on a particular day to help clean up the local park.

Contrary to textbook wisdom, this sort of consumption in the commons hardly implies allocative inefficiency. Admittedly, surplus losses may well be realized, since such implicit reciprocity agreements are rarely exact compared to a formal contract with fully internalized prices. While group members share in the consumption of goods, they do not formally reimburse the individual purchasers for benefits received. These deficiencies, however, must be weighted against the transaction costs of their elimination. While the lunch/drink tab could be divided up exactly based on individual consumption, a transactions cost would be incurred in calculating individual consumption, taxes, tips and, of course, the annoyance of interrupting a social occasion with petty accounting details. Similarly, neighbors could set up club arrangements in which certain goods are shared or joint efforts coordinated, but such more structured arrangements would obviously impose substantial transactions costs as well. If these transactions costs are greater than the efficiency losses incurred by group consumption under informal, hence, imperfect, reciprocity agreements, then it is more efficient to maintain the informal agreements.

Our main contribution in this paper is a formal model of informal reciprocity agreements which allows us to examine particular demand characteristics of goods that recommend themselves to be supplied through informal reciprocity agreements, i.e., attributes that imply that the loss of surplus from inefficient choices is small relative to the transactions costs of organizing and coordinating a superior allocation through the negotiation, monitoring and enforcement of "complex contracts".[2] In the next section, we develop a model that focuses on an individual's behavior, interacting with identical persons, but with transactions costs, prices, and the size of the interacting group as variable and important constraints. Our model focuses on the case in which the good is private, but the expense can be shared.[3] Our analysis makes use of the theory of infinitely repeated games.[4] We use our model to

[2] Borcherding (1978, 1983) uses this term, but it was originally suggested to him by the various works of Harold Demsetz. Much is made of complexity as a reaction to interdependence by Demsetz, and his intellectual contribution to modern understanding of this question cannot be overstressed.

[3] We leave the public goods case for future work, but we believe that many of our conclusions will apply to that case as well. We discuss this conjecture further in the conclusion, Section 4.

[4] In some respects, our approach is similar to that taken by Calvert (1995), who uses the theory of

determine conditions under which a good will be shared under informal or tacit reciprocity agreements. We then present a brief extension of our model that considers heterogeneous agents.

Our efforts yield the following results: Informal reciprocity agreements are more likely to be used when transactions costs are high, the unit cost of the good is small, each consumer's demand is not too responsive to price changes, the group is likely to continue to interact, the consumers are patient, the time between interactions is short, and the group is small and homogeneous. Further, our results suggest that informal sharing agreements are more likely to involve goods that are consumed along with other indivisible group benefits, such as conversation, company, and friendship, hereafter referred to as companionship.

Following the formal results in Section 2, we discuss in Section 3 the possibility of eliminating surplus losses through deeper social interactions, neither the formal ones of private property and markets nor those of formal collective choice institutions. We call these arrangements social reciprocity agreements, in contrast to the more spontaneous reciprocities explored in the previous sections. We use the reciprocity model to analyze investments in reciprocity networks and norms, two important components of social capital. Conclusions and additional conjectures are presented in Section 4.

2. A Reciprocity Model of Consumer Choice in the Commons

"I am saying that the economic approach provides a valuable unified framework for understanding *all* human behavior..." Gary S. Becker, "The Economic Approach to Human Behavior." (1976).

The model has n consumers who dine together once each period. For now, assume all are identical. There are two possible payment schemes. In the first, each consumer figures out what he owes and then pays for his own meal. In the second scheme, the consumers use an informal reciprocity agreement. They take turns paying the bill for the whole group - so agent i pays only every n periods, but pays for everyone's meal when he pays. We refer to

infinitely repeated games to compare different institutions, which in his setting are different arrangements for overcoming prisoner's dilemma problems.

this second scheme as "bill sharing".[5]

In order to fix ideas, we focus on the dining example throughout the paper. However, we believe that the basic features of the model apply to many situations in which individuals take turns putting themselves at risk of being taken advantage of in return for the reward of reciprocity in the future.

Consider first the case in which consumer i pays for his own meal. There are two subcases. In the first, consumer i is part of the group, and in the second, he is not. Assume that there are some intrinsic benefits associated with interacting with the group, companionship, that differ in the two subcases. Denote the level of intrinsic benefits inside and outside the group by y and y_o, respectively, where $y \geq y_o$. Denote the optimal food choices in the two subcases by x^* and x_o^* respectively.

Consider the first subcase.[6] Each meal, consumer i chooses the amount of food to consume, x^*, by solving the following net surplus maximization problem:

$$\max_x V(x, y) - px - c, \tag{2.1}$$

where x represents the amount of food, $V(x, y)$ represents the gross surplus (in dollars) the consumer obtains from consuming x and y, p is the price of units of food, and c is a transactions cost associated with figuring out what one owes.[7] As mentioned in the introduction, c is composed of a variety of computational, inconvenience, and other transactions costs.[8] Assume that $V(x, y)$ is increasing and strictly concave in both x and y. Then consumer $i's$ choice of x^*, his consumption, must satisfy the following first order condition:

$$V_x(x^*, y) - p = 0, \tag{2.2}$$

[5] We could also have modelled "pot sharing", in which individuals take turns providing the meal itself, and the food is split amongst the group equally. The putative inefficiency in pot sharing is in the direction of too little instead of too much, but the results in terms of deadweight losses and the factors affecting them are exactly the same. Formal proofs for this case are available from the authors on request.

[6] The second subcase is identical, just replace y with y_0 and x^* with x_o^* in what follows.

[7] Clearly, x can be interpreted as efficiency units of food, and p can be interpreted as the price of efficiency units. In other words, more x can mean either more food, better food, or some combination of the two.

[8] We treat the transactions cost associated with figuring out what one owes as an exogenous constant. In reality, it may depend on the number of members in the group, the size of the transaction, and other variables considered here. We discuss this complication below in fn. 12.

where V_x represents the first derivative of $V(x, y)$ with respect to x.

Now consider the case in which consumer i is part of the bill-sharing agreement, and pays every nth time. In the period that consumer i pays the bill, he consumes x^*, as above. Assume that in the periods in which he does not pay the bill, he free rides and orders, hence consumes, x', where $x' \geq x^*$.[9] The assumption that $x' \geq x^*$ nests two extreme cases. In the first case, $x' = x^*$. In this case, consumer i does not consume any extra food on the days he does not pay. In the second case, the quantity x' represents the amount of food consumer i demands at a zero price. Most informal reciprocity arrangements likely involve consumption in between these two extremes. In part, x' depends on how responsive consumer i is to price changes, since on days he does not pay, his effective price drops from p to 0. In part, x' also depends on the complexities of the social interactions that we discuss further below in Section 3. For now, take x' as given.

Consumer i's net surplus from paying for everyone's meal at the beginning of a cycle and then attending $n - 1$ free meals is

$$V(x^*, y) - px^* - px'(n-1) - c + \frac{\delta - \delta^n}{1 - \delta} V(x', y), \tag{2.3}$$

where δ is a discount factor. The discount factor represents the combination of several variables. First, it reflects consumer patience - more patient consumers have higher discount factors (they discount future payoffs at lower rates: $\delta = \frac{1}{1+r}$, where r is the discount rate). Second, it reflects the consumers' shared belief about the likelihood of sharing meals in the future - the higher is the likelihood of dining again, the higher is the discount factor. Third, it reflects the time between shared meals - the shorter is the time between shared meals, the higher is the discount factor (because discounting occurs at lower rates over shorter time intervals). The parameter δ is bounded between 0 and 1.

It is clear that there are benefits and costs from the sharing arrangement. The benefit to consumer i is that, on the days he does not have to pay, he consumes x' instead of x^*,

[9] We will not discuss the case of $x' < x^*$, but it is clear that in some sharing agreements, this may be the case. If companionship benefits or transactions costs savings are sufficiently high, individuals may be willing to forego some consumption of private goods in order to maintain the agreement. We discuss the determination of x' further in Section 3.

and incurs no transactions costs of determining what he owes. Further, he receives group benefits of y, which are higher than the non-sharing benefits y_o. The cost to consumer i is that, every n periods, he has to pay $px'(n-1)$, the cost of everyone else's meal.

2.1. Reciprocity.

Group members may have an incentive to cheat in a bill-sharing agreement in two ways. In the first, when consumer i does not have to pay, he might consume a quantity beyond that recognized as "acceptable" by the group. In the second, when it is consumer i's turn to pay, he can refuse to pay for everyone else's meal, and by doing so can save himself $px'(n-1)$. In our formal discussion, we focus on the second way of cheating, and treat x' as an agreement-specific exogenous variable, but the basic arguments apply to either type of deviation from the informal agreement.[10]

There are several ways that the group might punish consumer i for refusing to pay. Below, we consider two possibilities. First, we analyze a renegotiation-proof equilibrium. In a renegotiation-proof equilibrium, when consumer i cheats, the bill-sharing agreement does not break down, but instead is renegotiated. Second, we analyze a trigger strategy equilibrium, in which as soon as one consumer cheats, bill sharing breaks down and is never restored. As discussed further below, we obtain similar results in both cases.

First consider a renegotiation-proof equilibrium. We derive conditions under which the following strategy is a renegotiation-proof equilibrium strategy: Each consumer pays the bill if it is his turn to pay, consumes x^* if he is paying, and consumes the free-riding level x' if he is not paying (as in the above subsection). If consumer i does not pay for everyone else's meal when it is his turn, then the agreement is renegotiated, and consumer i agrees to pay for everyone's meal the next time. If he does not pay for the next meal then it remains his turn to pay until he pays. Once he pays, the consumers resume taking turns paying. Clearly,

[10] The assumption that x' is fixed is equivalent to an assumption that group-specific norms conditioned by price responsiveness put an upper bound on an acceptable choice of x. If group members cannot consume more than x', then since more is better, they will consume x'. It is not difficult to imagine enforceable norms that bound x. It may be possible to punish excess consumption beyond the level acceptable to the group instantly, and therefore ensure that such cheaters receive no benefits. For example, if consumer i consumes too much, the consumer whose turn it is to pay may refuse to pay for consumer $i's$ meal, and may be supported by the group for refusing to pay instead of punished. A snide comment aimed at influencing the excessive consumer may also be effective. We discuss such methods in Section 3.

in order for the agreement to be sustained, each consumer must prefer to pay as soon as he is first required to pay.

If each consumer follows the above strategy, then each receives the bill-sharing payoff in expression (2.3) forever. Denote each consumer's value of this discounted stream of payoffs by W_s, where

$$W_s = \frac{1}{1-\delta^n}\left\{V(x^*,y) - px^* - px'(n-1) - c + \frac{\delta - \delta^n}{1-\delta}V(x',y)\right\}. \tag{2.4}$$

Taking consumption choices as given, consumer i's only choice in this game is to either pay for everyone's meal when it is his turn or not. In order for consumer i to prefer paying for everyone else's meal to refusing to pay and then renegotiating, the following inequality must hold:

$$W_s \geq V(x^*,y) - px^* - c + \delta W_s. \tag{2.5}$$

That is, consumer i's payoff from paying and maintaining the agreement, W_s, must be at least as large as the payoff associated with paying only for his own meal today, $V(x^*,y) - px^* - c$, and then renegotiating to restore the sharing agreement the next time, δW_s. Note that this inequality applies whenever it is $i's$ turn to pay, whether or not i is on the equilibrium path. The inequality can be rearranged and expressed as

$$W_s \geq V(x^*,y) - px^* - c + \frac{\delta}{1-\delta}[V(x^*,y) - px^* - c] \tag{2.6}$$

A second condition must also hold. In order for consumer i to prefer paying for everyone else's meal to refusing to pay and then abandoning the group, the following inequality must hold:

$$W_s \geq V(x^*,y) - px^* - c + \frac{\delta}{1-\delta}[V(x_o^*,y_o) - px_o^* - c]. \tag{2.7}$$

That is, consumer $i's$ payoff from maintaining the sharing agreement, W_s, must be at least as large as the payoff associated with paying only for his own meal today, $V(x^*,y) - px^* - c$, and then abandoning the group forever, $\frac{\delta}{1-\delta}[V(x_o^*,y_o) - px_o^* - c]$.

Our assumption that $y \geq y_o$ implies that if inequality (2.6) holds, then inequality (2.7) must hold as well. Therefore, we can focus on inequality (2.6). Substituting in for W_s from expression (2.4) and rearranging, inequality (2.6) can be expressed as

$$-px' + \frac{\delta - \delta^n}{(n-1)(1-\delta)} \left[V(x', y) - V(x^*, y) + px^* + c \right] \geq 0. \tag{2.8}$$

When consumers use renegotiation-proof strategies, bill-sharing can be sustained if inequality (2.8) is satisfied. Since inequality (2.8) has both positive and negative components, it is satisfied for some values of the parameters, but not for others. Under which parameter values is inequality (2.8) likely to be satisfied? In Appendix A, we prove that the expression on the left-hand side in (2.8) is increasing in transactions costs c and the discount factor δ, but decreasing in the group size n, the price p, and the free-riding level of consumption x'. We can say nothing in general about the effect of y on whether or not bill-sharing is used. The sign of the effect of y on the left-hand side of inequality (2.8) depends on the sign of

$$V_y(x', y) - V_y(x^*, y), \tag{2.9}$$

and thus depends on whether companionship is a complement of a good meal or a substitute for one!

Now consider the case in which consumers use trigger strategies. In a trigger strategy equilibrium, the punishment for failing to pay is permanent ostracism. Consider the following trigger strategy: Each consumer pays the bill if it is his turn to pay, consumes x^* if he is paying, and consumes the free-riding level x' if he is not paying (as above). If consumer i does not pay for everyone else's meal when it is his turn, then the agreement breaks down and consumer i must leave the group and not return. In this case, the only relevant inequality is inequality (2.7). In Appendix B, we obtain results that are similar to the renegotiation-proof case. We show that inequality (2.7) is more likely to hold when transactions costs c and the discount factor δ are high, and when n and x' are low. Further, as long as x_o^* is not "too large", inequality (2.7) is more likely to hold when the price p is low.[11] Further, the higher

[11]See Appendix B. The required condition is $-(n-1)x' - \delta^n x^* + (\delta + \delta^2 + ...\delta^n)x_o^* \leq 0$. This condition always holds if $x_o^* \leq x^*$, which occurs if consumption and companionship are complements. It also holds as

the value of y, and the lower the value of y_o, the greater the incentive each consumer has to maintain the sharing arrangement. This implies that when ostracism is a possible reaction to cheating, bill sharing is more likely if intrinsic benefits are high within the sharing agreement and low outside it. This suggests that informal sharing agreements are more likely to involve goods that are consumed along with other group benefits, such as companionship.

In summary, our results suggest that informal sharing agreements are more likely as transactions costs, patience, the probability of meeting again, and companionship within the group rise, and as the time between meetings, group size, unit price, the (absolute) price elasticity, and companionship outside the group fall. These results are quite intuitive.[12] One might expect to see informal reciprocity agreements when sharing a good does little to alter the consumption level of the original purchaser. Likewise, the unenforceable, but customary ritual of taking turns buying drinks works well among small groups, but poorly for large ones. And, while several neighbors may trade household items back and forth, it is unlikely that entire neighborhoods will partake in such exchanges. Even among small groups, we should not expect individuals to share in goods with highly price-elastic demands. Finally, our analysis provides an insight as to why informal reciprocity agreements are more common for relatively "small ticket" items that are frequently purchased, like drinks and lunches, rather than more expensive items such as dinners and concerts. These results also imply that certain public-type goods which seem to be provided with the least obvious forms of enforcement, e.g., manners and common courtesies, may involve services whose demands are reflexively established, i.e., almost costless, are highly price inelastic, and work out best when group size is small.

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long as x' is sufficiently greater than x_o^*.

[12] All of our results are derived under the assumption that the transactions cost c is exogenous. Although this assumption yields clear, intuitive results, it is admittedly extreme. In reality, c may depend on the number of members in the group, the size of the transaction, and the other variables considered here. This relaxation would lead to additional terms in the partial derivatives used to obtain our results. The nature of the dependence of c on our other variables varies from context to context, and although we can make reasonable conjectures about the directions of effects (for example, increasing group size likely increases transactions costs) we cannot determine the magnitude of the effects (an empirical question). We can conclude that if transactions-cost changes are small relative to the size of the other terms in the partial derivatives, then our results continue to hold. However, it is possible that in some settings, changes in transactions costs could dominate the effects we have described, and our conclusions would be modified or even reversed.

2.2. Consumer Heterogeneity.

In this subsection we extend the above model to examine consumer heterogeneity, and make the argument that bill sharing is more likely to be sustained between similar consumers than different ones. We focus on a renegotiation-proof equilibrium. To make our argument in the simplest possible way, we consider only two consumers. Suppose that consumer i consumes x_i^* when he pays, x_i' when he does not pay; obtains y_i in companionship per meal; and incurs transactions costs c_i when he pays the bill, where $i = 1, 2$. We derive conditions under which bill sharing can be sustained that are similar to the conditions derived above.

Following our model presented above, consumer i prefers to continue to cooperate if the following inequality, a modified version of inequality (2.5), holds:

$$
\begin{aligned}
& \frac{1}{1-\delta^2}\left[V_i(x_i^*, y_i) - px_i^* - px_j' - c_i + \delta V_i(x_i', y_i)\right] \\
\geq \;& V_i(x_i^*, y_i) - px_i^* - c_i + \frac{\delta}{1-\delta^2}\left[V_i(x_i^*, y_i) - px_i^* - px_j' - c_i + \delta V_i(x_i', y_i)\right],
\end{aligned} \tag{2.10}
$$

where $i = 1, 2$, $j = 1, 2$, and $i \neq j$. Rearranging, the above inequality simplifies to

$$
-px_j' + \delta\left[V_i(x_i', y_i) - V_i(x_i^*, y_i) + px_i^* + c_i\right] \geq 0, \tag{2.11}
$$

It is possible that inequality (2.11) is satisfied for one consumer and not for the other. If this is the case, then the expression on the left-hand side of inequality (2.11) will differ in magnitude depending on whether $i = 1$ and $j = 2$ or $i = 2$ and $j = 1$. The difference between the two left-hand-side expressions is given by the following expression (subtract the expression when $i = 1$ and $j = 2$ from the expression when $i = 2$ and $j = 1$):

$$
p(x_2' - x_1') + \delta\left\{[p(x_2^* - x_1^*) + c_2 - c_1] + [V_2(x_2', y_2) - V_2(x_2^*, y_2)] - [V_1(x_1', y_1) - V_1(x_1^*, y_1)]\right\}. \tag{2.12}
$$

It is clear from expression (2.12) that the differences between the two left-hand-side expressions depend on the differences between x_2' and x_1', x_2^* and x_1^*, c_2 and c_1, and $V_2(x_2', y_2) -$

$V_2(x_2^*, y_2)$ and $V_1(x_1', y_1) - V_1(x_1^*, y_1)$ (the last terms measure the extra happiness that consumer i obtains from consuming x_i' instead of x_i^*).

To make our point that heterogeneity makes it less likely that the consumers will engage in informal reciprocity, we consider an extreme, but still plausible, case. Suppose that consumer 2 consumes more than consumer 1, both when he has to pay and when he does not have to pay (so $x_2^* > x_1^*$ and $x_2' > x_1'$). Further, suppose that consumer $2's$ additional pleasure from obtaining a free meal exceeds consumer $1's$ additional pleasure (so $V_2(x_2', y_2) - V_2(x_2^*, y_2) > V_1(x_1', y_1) - V_1(x_1^*, y_1)$). Finally, suppose that the transactions costs are the same for the two consumers. Then it follows that expression (2.12) is strictly positive, which implies that consumer 2 is willing to engage in an informal reciprocity agreement for a much larger range of parameter values than consumer 1. Consumer 1 is less likely to approve of the agreement because consumer 1 has to pay for consumer $2's$ expensive meals but does not consume expensive meals himself, and does not obtain sufficient extra pleasure from having 2 pay the bill every second time out to compensate him for his loss.[13] We discuss heterogeneity further in the following section.

3. Social Capital, Tacit Agreements, and Transactions Costs

"[Social] structures must not be conceptualized as simply placing constraints on human agency, but as enabling [it]..." Anthony Giddens, *The Constitution of Society* (1984).

As previously mentioned, surplus losses are created by the inexact nature of informal reciprocity agreements. The magnitude of these surplus losses depends on whether and to what extent reciprocity exists. The reciprocity model presented above allows for the fact that consumers are unlikely to internalize price externalities at the margin (since x' can exceed x^*). In the extreme case in which x' represents the consumer's unconstrained free-riding point, where $V_x(x', y) = 0$, there is no marginal reciprocity - on days that consumer i does not have to pay, consumer i acts as though the meal is completely free, and does not take into consideration that one of his companions has to pay the bill. If the true social costs

[13] Of course, the notion of likes preferring to interact with likes has long been known to hold for clubs and local governments supplying local public goods (Cornes and Sandler, 1986).

and benefits were internalized in the interaction, however, all consumers would consume x^* whether or not they were paying. Further, it is easy to show that such consumer restraint is Pareto improving - the transactions costs of dividing up the bill can be avoided, but there are no losses from excess consumption. The derivative of the per-cycle payoff associated with the informal reciprocity agreement presented above, expression (2.3), with respect to x' is $-p(n-1) + \frac{\delta - \delta^n}{1-\delta} V_x(x', y)$. Since $\frac{\delta - \delta^n}{1-\delta} \leq n-1$ and $V_x(x', y) \geq 0$, this derivative is less than or equal to $(n-1)[-p + V_x(x', y)]$. Since x^* internalizes all price effects (it satisfies the first order condition $V_x(x^*, y) - p = 0$) and $V(x, y)$ is strictly concave in x, $V_x(x', y) < p$ for all x' greater than x^*. This implies that expression (2.3) is decreasing in x' as long as $x' > x^*$. This implies that $x' = x^*$ is socially optimal, which is another way of saying that consuming x^* every period is Paretian.

In this section, we explore price internalizations generated by social reciprocity agreements, and investments in social capital. We explore how socially constructed reciprocities, but not black-letter formal mechanisms, might arise which dominate informally uncoordinated Nash-Cournot reactive ones. We might call these Coasian social interactions, where much if not all of the price externalities are internalized through tacit reciprocities and social arrangements.[14]

For tacit reciprocity to take place, two things must be in place. First, norms or mores must inform members of the interacting group how to behave, and second, conditions must permit

[14] Note that alternative mechanisms that do not involve reciprocity may also discourage excess consumption while reducing some transactions costs. For example, consider the following "Chinese restaurant" case. Suppose that, every meal, each consumer orders x_c^*, and only consumes $\frac{1}{n}$ of what he orders, but also consumes $\frac{1}{n}$ of what everyone else orders. At the end of the evening, the consumers each pay $\frac{1}{n}$ of the total bill. Consumer i takes the choices of the other consumers as given, and chooses x_c^* to maximize his net surplus:

$$\max_{x_i} V\left(\frac{x_i}{n} + \frac{\sum_{j \neq i} x_j}{n}, y\right) - p\left(\frac{x_i}{n} + \frac{\sum_{j \neq i} x_j}{n}\right) - c_c,$$

where c_c is the transactions cost associated with negotiating, monitoring, and enforcing this agreement. In a symmetric equilibrium, the resulting first-order condition is

$$V_x(x_c^*, y)\frac{1}{n} = \frac{p}{n},$$

which implies that consumer i chooses $x_c^* = x^*$. If c_c is sufficiently low, and if the supply technology allows for such a division in consumption, then this arrangement may be preferred to a bill sharing agreement of the type presented above.

mechanisms that operate to enforce these norms or mores. These issues have been discussed in depth by Coleman (1987, 1990) and Frank (1992), using Coleman's now well-known notion of "social capital." The latter represents a set of understandings that prescribes or proscribes desirable and undesirable behavior in a given social context as well as mechanisms for rewards and sanctions that enforce said dicta.

Although neither Coleman nor Frank derive the supply function of social capital in any formal way, they speak to its operations under various social conditions. We view norms and mores as capital too, but we believe they can be analyzed as we would durable assets using the theory of investment and capital accumulation over time. Such social investments yield benefits to the group, but without means to enforce each individual's compliance by either the reward of social acceptance or the punishment of excludable benefits of sodality, social capital will be deficient in quantity, and informal, hence uncoordinated, agreements will not be converted to socially constructed ones via tacit understandings that internalize the aforementioned price externalities. Besides appealing to the theory of investment, the other concept necessary to develop a positive notion of social capital is that of transaction costs. Returns must be favorable to the group, but individuals must be motivated to keep these tacit agreements. Thus, monitoring and enforcement efforts modulate the production of useful social constraints.

Sociologists have long labored over exactly this question, and we will attempt to translate their thoughts into those theoretically coherent to economists and rational choice scholars.[15] For example, Granovetter(1985) discusses the notion of "embeddedness," the networks of trust and norm enforcement that through approbation and ostracism foster cooperation and censure free riding.[16] Hechter(1987) devotes a book to these relationships with empirical

[15] Granovetter (1990) traces the severance of the discipline of sociology from economics since the *Methodenstreit* and offers some thoughts explaining this breakup. He also speaks to the issue of social awareness and constraint, the basis of Coleman's view of social capital, and offers a methodological critique of economists disinterest in the concept until comparatively recently.

[16] The bibliographies and the discussion in Coleman (1990) and Hechter (1987) provide an enormous aid to the economist unfamiliar with the sociological literature. We would have been hard pressed to speak with any confidence of the social capital concept without this tutorship. Recently we have been directed by our colleague Arthur Denzau to an important study by anthropologist Jean Emsinger (1992), whose excellent first chapter reviews the intersection of anthropology, economics, and sociology that, among other matters, speaks to this issue. Political scientist Robert Putnam's book (1993) has an entire chapter, six, devoted to social capital and its complementarity with political institutions. Margaret Levi (1988) considers the

case studies of their effects on group norms, their production and maintenance, and resultant behavior. Coleman (1987, 1990) has been mentioned, and his very theoretical writings on social capital cover much the same ground. Frank (1988), an economist, devotes much of his book to explaining how groups can neutralize free riding.

These and other key works suggest that the benefits of stocks of social capital are likely high when groups are quite homogenous, when individual members are linked by multiple and overlapping networks, and when individual behavior is not overly sensitive to private prices, i.e., price inelastic. The solidarity of the group, its traditions, and its durability over time as an entity give individuals longer views, thus discounting is at lower rates, again conditions which add value to social capital.

The presence of high group value to tacit rules of behavior is a surely a necessary but hardly a sufficient condition for social capital to be formed. Individuals must not feel inclined to chisel on the tacit agreement and its dictates. Free-riders must be punished for defecting, while those supporting the social agreement must be rewarded. This we think is a function of social sympathies. Since Adam Smith's *Theory of Moral Sentiments* (1759) scholars of human behavior have noticed the intense desire of individuals to be accepted and to crave the approbation of individuals in their group. Whether this is biological or socially constructed, we will not inquire, but take said motivation as given.[17] It is the relevant group that parcels out these rewards and punishments, much like the owners and managers of a firm contract for inputs and distribute the surplus. The size and scope of this social group is determined by transaction costs, just as Coase, Alchian and Demsetz, Klein, Williamson et al. of the New Institutional Economics tradition suggested for the firm (Furboton and Richter (1991)). If other complementary social networks exist, even those that span private markets, formal clubs, governmental relationships, or other legally enforceable interactions, sanctions can be

issue of tax compliance in new institutional economics terms, but with much attention to "norm generating structures," an infelicitous term Coleman used before he coined social capital. North (1990) devotes a chapter to "Informal Constraints" in his book on the development of institutions and their effect on economic growth. The various books of social theorist Jan Elster, too numerous to cite, are very readable and instructive. We found *Nuts and Bolts* (1988) particularly useful. Recently, a volume of essays by scholars in various disciplines (Hedstrom and Swedberg (1998)) explores the microfoundations of social mechanisms supporting norms, mores, and social capital using rational choice analysis.

[17] Current wisdom in sociobiology, particularly evolutionary psychology, suggests that there is at least some genetic component to norms (Ridley (1997), J.Q. Wilson (1993), E.O. Wilson (1998), and Wright (1994)).

magnified by removal of individuals from one or several of this rent-generating networks. This "disembedding" attenuates opportunism. The latter transaction costs must be reduced to some economic level for the group to flourish and social capital investment to take hold.

Generally speaking, conditions that lower transaction costs of social capital formation very much parallel those that favor high values of social capital. Solidarity, homogeneity, durability of relationships, multiplicity of overlapping trust, felicity, and mutual-help networks, and the like make enforcement of tacit agreements easier.

3.1. Investments in Social Capital which Encourage Reciprocity

Our reciprocity model can be used to analyze investments in a reciprocity norm and a reciprocity network, two important components of social capital. Here, we provide a simple example. Suppose that there are two individuals who currently do not have an informal reciprocity agreement. Perhaps they have just met. In Subsection 2.2, individuals differed in a variety of ways. Here, for simplicity, assume that individuals differ only by their x' values. Assume that each individual is drawn at random from the same population, and that proportion λ in the population has $x' = x'_l$, and proportion $1 - \lambda$ has $x' = x'_h$, where $x'_h > x'_l$. Assume that each individual knows his own value of x', but is uncertain about others. In other words, when Joe meets Mary, Joe thinks that Mary has x'_l with probability λ and x'_h with probability $1 - \lambda$.

Suppose that Joe has x'_l. To make the problem interesting, assume that if Mary has x'_l then it is worthwhile to have an informal reciprocity agreement with her, but if Mary has x'_h it is not. Formally, this implies that

$$V(x^*, y) - px^* - px'_l - c + \delta W_{s|x'_l} \geq W_o \tag{3.1}$$

and

$$V(x^*, y) - px^* - px'_h - c + \delta W_{s|x'_h} < W_o, \tag{3.2}$$

where

$$W_{s|x_l'} = \frac{1}{1-\delta^2}\left\{V(x_l', y) + \delta\left[V(x^*, y) - px^* - px_l' - c\right]\right\}, \tag{3.3}$$

$$W_{s|x_h'} = \frac{1}{1-\delta^2}\left\{V(x_h', y) + \delta\left[V(x^*, y) - px^* - px_h' - c\right]\right\}, \tag{3.4}$$

and $W_o = \frac{1}{1-\delta}\left[V(x_o^*, y_o) - px_o^* - c\right]$. $W_{s|x_l'}$ and $W_{s|x_h'}$ represent the present values of reciprocity agreements with types x_l' and x_h', respectively. W_o represents the present value of having no agreement.

Attempting to initiate an informal reciprocity agreement is risky because of the uncertainty involved. Suppose that Joe can attempt to initiate an agreement by inviting Mary to lunch and offering to pay. In doing so, Joe may be taken advantage of if Mary is type x_h'. Joe finds it worthwhile to invite Mary as long as

$$V(x^*, y) - px^* - c + \lambda\left[-px_l' + \delta W_{s|x_l'}\right] + (1-\lambda)\left[-px_h' + \delta W_o\right] \geq W_o. \tag{3.5}$$

The left-hand side of inequality (3.5) is increasing in λ. Therefore, investments in informal reciprocity agreements are more likely to occur when λ is high. When is λ likely to be high? It is likely to be high when there is existing social capital that leads people to practice reciprocity with each other. If there is a social norm that causes people to restrict their temptation to take advantage of someone who makes a generous offer, then the more widespread the norm, the higher the proportion of people with x_l'.

What other conditions lead to investment in reciprocity agreements? The intuition we obtained analyzing the model in Section 2 continues to hold here. In Appendix C we prove that investment is more likely to occur, the higher are the transactions costs c, the lower are the free-riding levels of consumption x_l' and x_h', the higher is the discount factor, the higher is companionship within the agreement y, and the lower is companionship outside the agreement y_o. Further, as long as x_o^* is not "too large", investment is more likely to occur, the lower is the price p.

Note that there are two offsetting effects of existing social capital on investment in new social capital. First, as already noted, when existing reciprocity norms are strong, we expect that λ will be higher, and, other things equal, more agreements will emerge. However, when existing social capital is high, individuals may receive high social benefits outside the agreement, and y_o may be quite high. In that case, investments in additional social capital are less necessary, hence are discouraged.

3.2. Some Thoughts about "Connectedness"

Our reciprocity model provides us with intuition about the features of goods that are most likely to be shared using informal reciprocity agreements, and the model is extendable to analyze investments in social capital. We believe that, although it is beyond the scope of this paper, the model could be extended to analyze overlapping networks as well, giving a formal theoretical basis for Granovetter's "strength of weak ties" (Granovetter (1973)). In the remaining paragraphs in this section, let us discuss some of the likely impacts of overlapping social networks on our results.

First, we make the following conjectures about the evolution of networks and norms. Suppose that, in the environment described in the above subsection, there is initially a large number of consumers who belong to no networks, and a continuum of possible x' values. Early on in the evolution of the society, as long as investments in social capital are worthwhile, each consumer gets invited to several lunches. Once learning about types begins, given that consumers have limited time and resources, it seems likely that consumers invest more in networks that have low x' values. Consumers that belong to networks that have high x' values continue to search for better networks. Over time, low-x' networks flourish, and high-x' networks cease to exist. Early on in the evolution of the system of networks, a consumer with a relatively high x' value might belong to several networks, but over time, absent a change in the consumer's behavior, the consumer is ostracized. As the society evolves, more and more members either acquire high social capital or are left out. Initially, reciprocity involves extremely small groups with low price sensitivities consuming cheap goods with high transactions costs. As the networks with lower x' values develop, group size rises, and more expensive and price-elastic goods are shared. The overlapping networks provide stronger threats to defectors, and greater rewards to those who continue to share.

The effect of group size is one element of our analysis that troubles us. We showed above, other things equal, that as group size rises it is less likely that informal reciprocity agreements can be sustained. However, transaction costs also are liable to rise as group size grows, again other conditions given, so little can be said on this margin without a great deal more thought. Still, one can speculate that as the affinity of individual members to one another increases, as is the case for certain ethnic and religious groups embedded in larger societies, scope and scale economies of solidarity may emerge making even large groups behave as we might otherwise suspect only small groups would.[18]

Before closing this section let us give this discussion a little empirical-cum-anecdotal thought based on the section's discussion. Consider our previous example of close friends who take turns picking up the bill. An additional reason why arrangements of this sort are sustainable is that each consumer restricts himself from over-consuming. This occurs because the private price at the margin is taxed by the realization that fellow diners, friends and long-time acquaintances would think poorly of such "maximizers," and tend to shun them in other situations where trust and propriety are required.[19] In addition, spending a great deal of time parsing out the bill exactly as to who had what would suggest that such individuals place a low opportunity cost on their time or, perhaps worse, that they are "cheap" and too narrowly concerned with self.

To take another common example, if one is driving one's own child to the Little League game and asks other neighboring parents if they wish their child to ride along, one feels a certainty that next week it will be that other parent's turn. Acceptance of such an offer implicitly binds that parent to this unspoken compact. Those who treat these obligations casually and free ride will be frozen out in subtle ways, i.e., it may induce negative spillovers into other aspects of the chiseling individual's social or business life which will be internalized by others, to the detriment of the free rider.

Finally, consider academic colleagues who attend seminars and read papers expecting same, but who never voice this expectation. They also spend hours on peer refereeing and

[18]Rodney Stark's interesting book on pre-Constantinian Christians is an extended study of free-riding control (Stark (1996)). Posner's discussion of primitive societies (Posner (1980)) explores Coasian social internalizations in stateless contexts.

[19]Japanese salarymen groupings after working hours come especially to mind as Fukuyama (1995) points out, but stable groups of close acquaintances and friends are just as apt examples.

mentoring younger colleagues, sometimes in areas not exactly their own. Why do they do it? They don't know, exactly, but as Tevye in "Fiddler on the Roof" says, "it's tradition." But we (the authors) think we do know why they do so. Being considered a "team player" has value in the academy, and those who break these rules of reciprocity, or those who are more self-absorbed than is considered acceptable, must be differentially more productive as scholars to overcome their reputations for excessive self interest and indifference to larger institutional and collegial concerns.

Thus, the aforementioned relationships of boosters, fellow workers, friends, parents, and teams insure a higher degree of cooperation, and a lower rate of free ridership than an unconsidered estimate might otherwise suggest. Nevertheless, we should not be surprised that such social understandings vary depending on the returns they generate to individuals in the relevant group, and the cost of their creation and long-run maintenance. Of course, if both informal agreements and social capital fail to generate sufficient returns to individual participants, more formal solutions using common law and statutes, through market co-operatives or polities, may become attractive. One cannot assume a priori this will be the case, especially given the weight of everyday observation to the contrary.

4. Conclusions

"The performance of all social institutions... depends on how these [free riding] problems are resolved." Robert D. Putnam, *Making Democracy Work* (1993).

Probably little of this analysis has escaped casual observation and conversation of economists, public choice scholars, and sociologists, yet to our knowledge the literature has not produced a simple, generalized discussion of the relationship of non-formally coordinated givings and takings in private goods settings. In any case, abstracting from the question of originality, one can make positive predictions using this simple model; to wit: only when unit price is low, demands are very price-inelastic, and group size is small and homogeneous and individuals in the group interact frequently will one likely find little institutional complexity where activities involve commons sharing practices of private goods.

Although the examples presented in the introduction demonstrate that public goods are also often shared using informal reciprocity agreements, we have left the public goods case for future work. We believe, however, that the basic conditions that apply to private goods also apply to public goods. Still, public goods have additional sharing economies and congestion effects that we have not analyzed here. We believe that in the public goods case, control of group size would have to be analyzed with care because of these additional effects, but that otherwise our results would continue to hold.[20]

That we observe great numbers of shared goods with little or no formal contracting indicates to us that the surplus loss in these "inefficient markets" is small relative to the transactions costs of developing formal trading mechanisms. Informal reciprocity arrangements take place between consumers, as we have described, but also happen between firms that transact frequently, agents within organizations that interact with each other, politicians that campaign for each other, etc. Yet, informal sharing agreements have been neglected by the current theoretical and empirical literature of the New Institutional Economics. Markets with formal contracts disciplined by common law, reputational or goodwill capital, and formal statutes, on the other hand, are analyzed to the smallest detail. In the public sector, the theory of special and general interest legislation, and voting and agency relationships, are now part-and-parcel of the general economist's vocabulary.

What is interesting to us is to note the usual pairings of comparisons in the New Institutional Economics literature - market contract A with market contract B, market contract A with government mechanism A, and government mechanism A with market mechanism B. Somehow, the cultural and sociological possibilities governing exchange have not been studied nearly as carefully by rational choice analysts as they deserve. We hope our attempt

[20] Consider two sorts of public goods entering our analysis, one pertaining to companionship y and the other pertaining to consumption x. First, suppose that, over some range, as group size n grows, so does y, but then as n continues to increase y eventually falls. There exists an n^* which is optimal. With simple interactions and a modicum of social capital, n^* greater than 1 is called for, but it is likely not large. For example, tables at restaurants generally seat four to six. Chinese restaurant round tables are often larger, but fn. 14 suggests that price perception constraints assure optimality under conditions encountered in these contexts.

Another sort of public good is in x where imperfect consumption indivisibilities cause price-sharing economies to dominate over congestion costs for some group size up to n^* where these margins equate. Again, casual empiricism about neighborhood clean-ups suggest n is not so small, though free riding by some is ubiquitous. Participants, however, enjoy great companionship economies. Refereeing and mentorship efforts may also be of this sort, and the "good colleague" reputational returns are probably important here, since, again, n is rather large.

herein is a start in this direction. Of course, one reason for this scholarly inattention by economists is the extreme difficulty of measuring constructs like social capital. Primitives like ideology, moral force, networks, power, trust, etc. are not so easy to define and harder yet to measure. They are "latent variables", to use the terms that sociologists coined decades ago, and proxies for them are searched for and their changes traced by these scholars to observable behavior, a point Max Weber (1922/1947) noted seventy-five years ago.

Economists, since the *Methodenstreit*, have instinctively shied away from such indistinct and refractory concepts, since sociological economic-based theories are difficult to establish and quantitative measures are often considered unobtainable. We economists are, perhaps, materialists, more comfortable with the theory of production, than with demand theory based on preference assumptions.[21] However, there is every reason to believe that since social constraints and spontaneous informal coordinating is ubiquitous, economists will find the will and wit to make some sense of it, as they have of contract and law, market and polity.

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Appendix A.

Here, we prove that the left-hand side of inequality (2.8) is increasing in c and δ, and decreasing in n, p, and x'. All of the steps use partial derivatives or discrete changes of the left-hand side of inequality (2.8) with respect to the relevant variable, and then sign the effect.

A1. c: The derivative of the left-hand side of inequality (2.8) with respect to c is $\frac{\delta-\delta^n}{(n-1)(1-\delta)}$, which is positive.

A2. δ: First, note that $V(x',y) - V(x^*,y) + px^* + c > 0$, because $x' \geq x^*$. Second, note that $\frac{\delta-\delta^n}{1-\delta} = \delta + \delta^2 + ...\delta^{n-1}$, which is increasing in δ. These two results imply that the left-hand side of (2.8) is increasing in δ.

A3. n: Consider an increase in n to $n+1$. The change in the left-hand side of inequality (2.8) is

$$\left[\frac{\delta-\delta^{n+1}}{n(1-\delta)} - \frac{\delta-\delta^n}{(n-1)(1-\delta)}\right]\left[V(x',y) - V(x^*,y) + px^* + c\right].$$

Since $x' \geq x^*$, the second term is positive. The first term is negative by the following argument: $\left[\frac{\delta-\delta^{n+1}}{n(1-\delta)} - \frac{\delta-\delta^n}{(n-1)(1-\delta)}\right] = \frac{\delta}{(1-\delta)n(n-1)}\left[-1-(n-1)\delta^n + n\delta^{n+1}\right]$. The sign of this expression depends on the sign of $-1-(n-1)\delta^n + n\delta^{n+1}$, which is strictly increasing in δ. As $\delta \to 1$, $\left[-1-(n-1)\delta^n + n\delta^{n+1}\right] \to 0$. Therefore, for all $\delta < 1$, the expression is negative.

A4. p: The derivative of the left-hand side of inequality (2.8) with respect to p is $-x' + \frac{\delta-\delta^n}{(n-1)(1-\delta)}\left[-V_x(x^*,y)\frac{\partial x^*}{\partial p} + p\frac{\partial x^*}{\partial p} + x^*\right]$. From the first-order condition (2.2), $-V_x(x^*,y)\frac{\partial x^*}{\partial p} + p\frac{\partial x^*}{\partial p} = 0$. The remaining expression is less than or equal to $-x' + x^*$, because $\frac{\delta-\delta^n}{1-\delta} = \delta + \delta^2 + ...\delta^{n-1}$ and $\delta + \delta^2 + ...\delta^{n-1} \leq n-1$. Since $x' \geq x^*$, this implies that the derivative is negative.

A5. x': The derivative of the left-hand side of inequality (2.8) with respect to x' is $-p + \frac{\delta-\delta^n}{(n-1)(1-\delta)}V_x(x',y)$, which is less than or equal to $-p + V_x(x',y)$, because $\frac{\delta-\delta^n}{1-\delta} = \delta + \delta^2 + ...\delta^{n-1}$ and $\delta + \delta^2 + ...\delta^{n-1} \leq n-1$. The following argument shows that $-p + V_x(x',y) \leq 0$. Since $x' \geq x^*$ and $V(x,y)$ is strictly concave in x, $V_x(x',y) \leq V_x(x^*,y)$. From the first-order condition (2.2), $V_x(x^*,y) = p$. Combining these results yields $V_x(x',y) \leq p$.

Appendix B.

Here, we prove that inequality (2.7) is more likely to hold when c, δ, and y are high, and when n, p, x', and y_o are low. Inequality (2.7) can be simplified, and holds if the following expression is non-negative:

$$-px' + \frac{\delta - \delta^n}{(n-1)(1-\delta)} \left[V(x', y) - V(x^*, y) + px^* + c\right] \quad (4.1)$$
$$+\frac{\delta - \delta^{n+1}}{(n-1)(1-\delta)} \left[V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^*\right].$$

In what follows, we prove that expression (4.1) is increasing in c, δ, and y, and decreasing in n, x', and y_o. We prove that expression (4.1) is decreasing in p as long as x_o^* is not "too large" in a sense made precise below. As in Appendix A, all of the steps use partial derivatives or discrete changes of expression (4.1) with respect to the relevant variable, and then sign the effect.

B1. c: The derivative of expression (4.1) with respect to c is $\frac{\delta-\delta^n}{(n-1)(1-\delta)}$, which is positive.

B2. δ: We showed in Appendix A that $-px' + \frac{\delta-\delta^n}{(n-1)(1-\delta)} \left[V(x', y) - V(x^*, y) + px^* + c\right]$ is increasing in δ. Therefore, it suffices to show that $\frac{\delta-\delta^{n+1}}{(n-1)(1-\delta)} \left[V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^*\right]$ is increasing in δ. First, note that $V(x^*, y) - px^* > V(x_o^*, y_o) - px_0^*$ because $y \geq y_o$. This implies that $V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^* > 0$. Second, note that $\frac{\delta-\delta^{n+1}}{1-\delta} = \delta + \delta^2 + ...\delta^n$, which is increasing in δ. These two results imply that expression (4.1) is increasing in δ.

B3. n: We showed in Appendix A that $-px' + \frac{\delta-\delta^n}{(n-1)(1-\delta)} \left[V(x', y) - V(x^*, y) + px^* + c\right]$ is decreasing in n. Therefore, it suffices to show that $\frac{\delta-\delta^{n+1}}{(n-1)(1-\delta)} \left[V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^*\right]$ is decreasing in n. Consider an increase in n to $n + 1$. The change in this term is

$$\left[\frac{\delta - \delta^{n+2}}{n(1-\delta)} - \frac{\delta - \delta^{n+1}}{(n-1)(1-\delta)}\right] \left[V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^*\right].$$

We showed in Step B2 that $V(x^*, y) - px^* - V(x_o^*, y_o) + px_0^* > 0$. The first term is negative by the following argument: $\left[\frac{\delta-\delta^{n+2}}{n(1-\delta)} - \frac{\delta-\delta^1}{(n-1)(1-\delta)}\right] = \frac{\delta}{(1-\delta)n(n-1)} \left[-1 - (n-1)\delta^{n+1} + n\delta^n\right]$. The sign of this expression depends on the sign of $-1 - (n-1)\delta^{n+1} + n\delta^n$, which is strictly increasing in δ. As $\delta \to 1$, $\left[-1 - (n-1)\delta^{n+1} + n\delta^n\right] \to 0$. Therefore, for all $\delta < 1$, the expression is negative.

B4. p: Since x^* and x_o^* are both chosen to satisfy first-order conditions, we can ignore the effects of changes in p on x^* and x_o^* when computing marginal effects. The derivative of expression (4.1) with respect to p is $-x' + \frac{\delta-\delta^n}{(n-1)(1-\delta)}x^* + \frac{\delta-\delta^{n+1}}{(n-1)(1-\delta)}[-x^* + x_o^*]$. This can be simplified and expressed as

$$\frac{1}{n-1}\left[-(n-1)x' - \delta^n x^* + (\delta + \delta^2 + ...\delta^n)x_o^*\right],$$

which is non-positive as long as $x_o^* \leq \frac{(n-1)x'+\delta^n x^*}{(\delta+\delta^2+...\delta^n)}$. This inequality always holds if $x_o^* \leq x^*$, and is more likely to hold when δ and x_o^* are small, and when x' and x^* are large.

B5. x': The derivative of expression (4.1) with respect to x' is $-p + \frac{\delta-\delta^n}{(n-1)(1-\delta)}V_x(x', y)$, which was shown to be negative in Step A5.

B6. y: Since x^* is chosen to satisfy a first-order condition, we can ignore the effects of changes in y on x^* when computing marginal effects. Therefore, the derivative of expression (4.1) with respect to y, after simplifying, is $\frac{\delta-\delta^n}{(n-1)(1-\delta)}V_y(x', y) + \frac{\delta^n}{n-1}V_y(x^*, y)$, which is positive.

B7. y_o: Since x_o^* is chosen to satisfy a first-order condition, we can ignore the effects of changes in y_o on x_o^* when computing marginal effects. Therefore, the derivative of expression (4.1) with respect to y_o is $-\frac{\delta-\delta^n}{(n-1)(1-\delta)}V_y(x_o^*, y_o)$, which is negative.

Appendix C.

Here, we prove that inequality (3.5) is more likely to hold when c, δ, and y are high, and when p, x'_l, x'_h, and y_o are low. Inequality (3.5) can be simplified, and holds if the following expression is non-negative:

$$\left(1+\frac{\lambda\delta^2}{1-\delta^2}\right)[V(x^*,y)-px^*-V(x^*_o,y_o)+px^*_o]-px'_l\frac{\lambda}{1-\delta^2}-px'_h(1-\lambda) \quad (4.2)$$
$$+\frac{\lambda\delta}{1-\delta^2}[V(x'_l,y)+c-V(x^*_o,y_o)+px^*_0].$$

In what follows, we prove that expression (4.2) is increasing in c, δ, and y, and decreasing in x'_l, x'_h, and y_o. We prove that expression (4.2) is decreasing in p as long as x^*_o is not "too large" in a sense made precise below. All of the steps use partial derivatives of expression (4.2) with respect to the relevant variable, and then sign the effect.

C1. c: The derivative of expression (4.2) with respect to c is $\frac{\lambda\delta}{1-\delta^2}$, which is positive.

C2. δ: This proof makes use of inequality (3.1). Substituting in for $W_{s|x'_l}$ and W_o and simplifying, inequality (3.1) can be expressed as

$$V(x^*,y)-px^*-px'_l-V(x^*_o,y_o)+px^*_o+\delta[V(x'_l,y)-V(x^*_o,y_o)+px^*_o+c]\geq 0. \quad (4.3)$$

We will demonstrate that this condition (which must hold in order for reciprocity to be desirable) implies that expression (4.2) is increasing in δ. The derivative of expression (4.2) with respect to δ is

$$\frac{2\lambda\delta}{(1-\delta^2)^2}[V(x^*,y)-px^*-V(x^*_o,y_o)+px^*_o]-px'_l\frac{2\lambda\delta}{(1-\delta^2)^2}$$
$$+\frac{\lambda+\lambda\delta^2}{(1-\delta^2)^2}[V(x'_l,y)+c-V(x^*_o,y_o)+px^*_0].$$

Multiply this expression by $\frac{(1-\delta^2)^2}{2\lambda\delta}$ to see that its sign depends on the sign of

$$V(x^*, y) - px^* - px'_l - V(x^*_o, y_o) + px^*_o + \frac{(1+\delta^2)}{2\delta}\left[V(x'_l, y) - V(x^*_o, y_o) + px^*_o + c\right].$$

Since $V(x'_l, y) - V(x^*_o, y_o) > 0$, and since $\frac{(1+\delta^2)}{2\delta} \geq \delta$, this expression is greater than or equal to the left-hand side of inequality (4.3), and thus is non-negative.

C3. p: Since x^* and x^*_o are both chosen to satisfy first-order conditions, we can ignore the effects of changes in p on x^* and x^*_o when computing marginal effects. The derivative of expression (4.2) with respect to p is $\left(1 + \frac{\lambda\delta^2}{1-\delta^2}\right)[x^*_o - x^*] - x'_l \frac{\lambda}{1-\delta^2} - x'_h(1-\lambda) + \frac{\lambda\delta}{1-\delta^2}x^*_o$. This derivative is non-positive as long as

$$x^*_o \leq \frac{(1-\delta^2+\lambda\delta^2)x^* + \lambda x'_l + (1-\lambda)x'_h}{1-\delta^2+\lambda\delta(1+\delta)}.$$

This inequality always holds when $x^*_o \leq x^*$, and is more likely to hold, the lower is x^*_o, and the higher are x^*, x'_l, and x'_h.

C4. x'_l: The derivative of expression (4.2) with respect to x'_l is $-p\frac{\lambda}{1-\delta^2} + \frac{\lambda\delta}{1-\delta^2}V_x(x'_l, y)$, which is less than or equal to $\frac{\lambda}{1-\delta^2}\left[-p + V_x(x'_l, y)\right]$, which is negative, by an argument similar to that of step A5 above.

C5. x'_h: The derivative of expression (4.2) with respect to x'_l is $-p(1-\lambda)$, which is negative.

C6. y: Since x^* is chosen to satisfy a first-order condition, we can ignore the effects of changes in y on x^* when computing marginal effects. Therefore, the derivative of expression (4.2) with respect to y is $\left(1 + \frac{\lambda\delta^2}{1-\delta^2}\right)V_y(x^*, y) + \frac{\lambda\delta}{1-\delta^2}V_y(x'_l, y)$, which is positive.

C7. y_o: Since x^*_o is chosen to satisfy a first-order condition, we can ignore the effects of changes in y_o on x^*_o when computing marginal effects. Therefore, the derivative of expression (4.2) with respect to y_o is $-\left(1 + \frac{\lambda\delta^2}{1-\delta^2}\right)V_y(x^*_o, y_o) - \frac{\lambda\delta}{1-\delta^2}V_y(x^*_o, y_o)$, which is negative.