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INTRODUCTION

International trade theorists recently have rediscovered a
possibility, pointed out many years ago by F. Y. Edgeworth [4], that
growth in an open economy might actually decrease the welfare of the
community. First J. Bhagwati [2] developed a model in which he showed
that if growth was sufficiently pro-trade biased (i.e. increased the
excess demand for imports and/or increased the excess supply of exports
at the initial terms of trade), the effect of the subsequent deter-
ioration in the terms of trade might outweigh the effects of the
increased productive capacity so that the country would be worse off
as a result of this growth. Then Harry Johnson [6] indicated that
within the context of a two commodity, two factor model, a country
facing fixed terms of trade might experience immiserizing growth
under a tariff if the expansion were sufficiently biased towards the
import-competing sector. Such a result has serious implications from
the point of view of a country considering the use of tariff induced
growth aimed at import substitution. Bhagwati [3] then suggested
some general conclusions concerning the phenomenon of immiserizing
growth, contending in particular that in models of the type mentioned
above immiserization could occur as the result of growth only in the
presence of non-optimal policies. In the first case, where the terms

of trade could be affected by the actions of the home country the appropriate policy would be to impose an optimum tariff[1] at all points in time and in the second case, where the country was so small that it could not affect the terms of trade, the optimal policy would be free trade.

In a recent paper Trent Bertrand and I [1] examined a particular case of Johnson's model in order to determine precise conditions under which growth due to capital accumulation in a country facing fixed terms of trade would lead to a fall in real national income, measured at world prices. The purpose of this paper is to shed further light on the phenomenon of immiserizing growth by generalizing these results in two important respects. The first is to obtain necessary and sufficient conditions for growth due to the accumulation of both factors of production in a country facing fixed terms of trade. The importance of this generalization should be obvious: to make policy recommendations to a less developed country based on the results of a model which ignores the effects of population growth must be considered somewhat naive to say the least. The second extension takes account of a consideration which arises only when population growth is allowed for: if population is growing, then an increase in aggregate income is not sufficient to guarantee an expansion of per capita income. Since the latter is probably a better index of welfare than the former, it would be useful to discover the conditions

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1. See Harry G. Johnson [5], Ch. II for a lucid exposition of the theory of the optimum tariff.

under which growth would lead to a fall in per capita real income.
This paper derives such conditions and demonstrates that Bhagwati's
conclusion [3] that immiserization will occur only in the presence
of non optimal policies (tariffs in this case) is not valid when
immiserization is interpreted in terms of per capita incomes.

The organization of the paper is to derive conditions for
the occurence of immiserization due to factor accumulation, first
in the sense of declining aggregate real incomes (section 1) and
second in the sense of declining per capita incomes (section 2).

1.   AGGREGATE REAL INCOME

1.i   The Model and Some Necessary Conditions

The essential features of the economy are represented
in Figure 1. Y, the capital intensive good, is represented on the
vertical axis, and X, labor intensive, on the horizontal axis; the
fixed terms of trade are represented by II, and the pre-growth produc-
tion point is P. If Y were the import good, this would mean that con-
sumption was occurring at some point on II above P, while if X were
imported, consumption would occur below P along II. Now if Y were the
import good and capital accumulation were to take place, the production
point P would move along the negatively sloped Rybczynski line[2] for

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2. The Rybczynski line is defined as the output expansion
locus as the supply of a factor (or both factors in a fixed propor-
tion) is increased at constant prices. As Rybczynski [8] showed,
for the increase of one factor the Rybczynski line is negatively
sloped, i.e. the production of one good expands and the other dimin-
ishes; linearity follows from the assumption of linear homogeneous
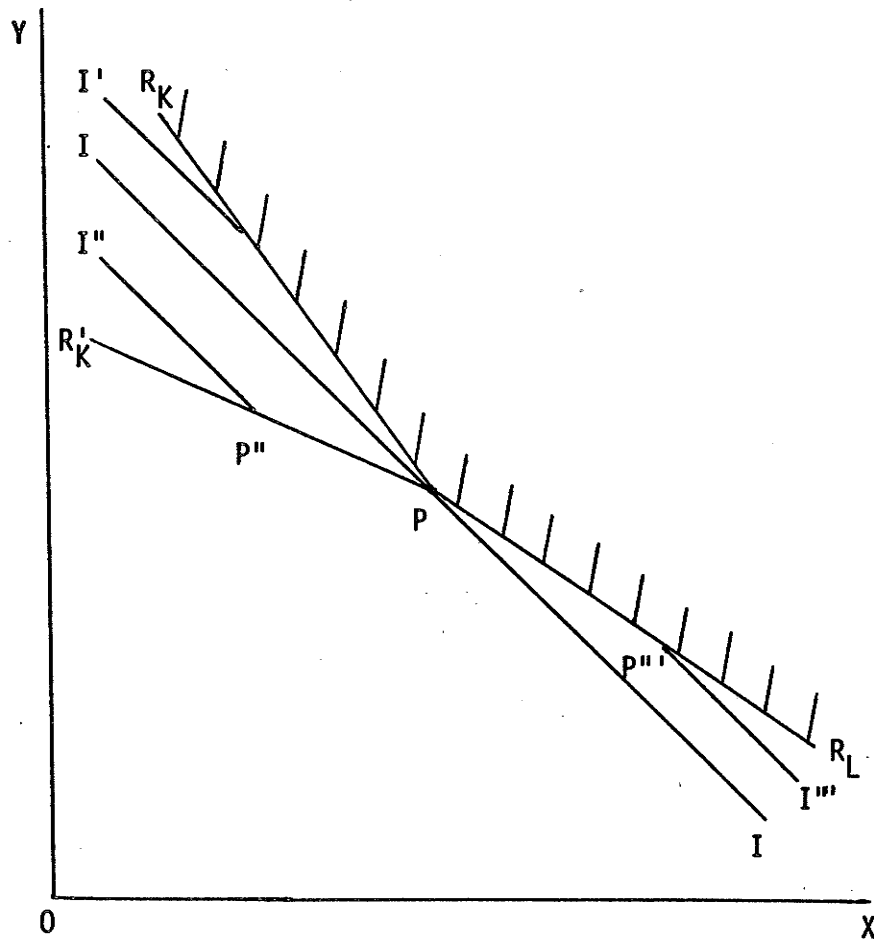production functions.

Figure 1.

capital, R_K or R'_K , to some new point, P' or P". If the Rynczynski line were steeper than II (R_K), the new set of consumption possibilities (I'P') would completely dominate the pre-growth situation, and capital accumulation would lead to an unambiguous increase in aggregate (and per capita) real incomes. However, if R_K were less steep than II (say R'_K), the new production point would be one such as P", and consumption possibilities (I"P") would be everywhere worse than in the pre-growth situation. Similarly, if X were the import good

and the labor force alone were to grow, production would expand to the south east along the negatively sloped Rybczynski line for labor (R_L). In this case aggregate income would fall if and only if R_L were steeper than II. Finally, if the capital stock and the labor force were both to grow, production would expand along a "composite" Rybczynski line whose slope would depend on the relative growth rates of capital and labor. If R_K and R_L were the Rybczynski lines for capital and labor, then the composite Rybczynski line would be somewhere in the shaded area between R_K and R_L.[3] Thus if immiserization were not possible under either labor force growth or capital accumulation alone, it is clear that it could not occur with some combination of the two. On the other hand, if immiserization could occur with either capital accumulation or labor force growth alone, then it might also occur for at least certain combinations of growth rates of the two.

To obtain a more rigorous and precise statement of these conditions it is first necessary to derive expressions for the slope of Rybczynski line(s). Production conditions are represented by two linear homogeneous production functions:

$$X = L_X F^X(k_X)$$

$$Y = L_Y F^Y(k_Y)$$

(1)

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3. If the capital stock and labor force both expand in a given proportion a point on the composite Rybczynski line for this proportion is obtained geometrically by moving from P along R_K and R_L respectively in the given proportion and then completing the parallelogram three of whose vertices are P and the points reached along R_K and R_L. The composite Rybczynski line is then obtained by joining P, the pre-growth production point, and the fourth vertex of the parallelogram.

where L_X and L_Y are the amounts of labor employed in the X and Y industries, F^X and F^Y represent output per worker of X and Y and k_X and k_Y are the capital-labor ratios $(\frac{K}{L})$ in the two industries. It is assumed that $K_X + K_Y = K$ and $L_X + L_Y = L$ are both constant. By differentiating and simplifying the two equations in (X.1) under the assumption that domestic prices and hence k_X and k_Y remain constant the following expressions are obtained:

$$\frac{dX}{dK} = \frac{F^X}{k_X-k_Y} \text{,} \qquad\qquad \frac{dY}{dK} = \frac{F^Y}{k_Y-k_X} \qquad\qquad (2)$$

$$\frac{dX}{dL} = \frac{F^X k_Y}{k_Y-k_X} \text{,} \qquad\qquad \frac{dY}{dL} = \frac{F^Y k_X}{k_X-k_Y} \qquad\qquad (3)$$

These then can be used to obtain expressions for the slopes of the Rybczynski lines:

$$\left.\frac{dY}{dX}\right|_{dL=0} = - \frac{F^Y}{F^X} \qquad\qquad (4)$$

$$\left.\frac{dY}{dX}\right|_{dK=0} = - \frac{F^Y k_X}{F^X k_Y} = - \frac{G^Y}{G^X} \qquad\qquad (5)$$

where G^X and G^Y represent output per unit of capital in the X and Y industries. Now suppose that capital and labor both grow in such a manner that $dK = \rho dL$ where ρ is the incremental capital-labor endowment ratio. Then from (2) and (3) the following expressions are obtained:

$$dX = \frac{F^X(\rho - k_Y)}{k_X - k_Y} dL$$

$$dY = \frac{F^Y(\rho - k_X)}{k_Y - k_X} dL$$

(6)

so that the slope of the composite Rybczynski line is:

$$\left.\frac{dY}{dX}\right|_{dK=\rho dL} = - \frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)}$$

(7)

From (7) it can be confirmed that the slope of the composite R-line will always be between that of the R-line for capital alone (R_K) and that of labor alone (R_L). With $\rho > k_Y > k_X$, that is with the incremental capital-labor endowment ratio greater than that used in the capital intensive industry and hence, the capital stock growing faster than the labor force, the composite R-line is negatively sloped and steeper than R_K. With $k_Y > \rho > k_X$, that is with the incremental capital-labor endowment ratio intermediate between those employed in the capital and labor intensive sectors, the composite R-line is positively sloped. A special case here is that in which ρ is equal to the initial capital-labor endowment ratio, that is, in which the labor force and capital stock are growing at the same rate. In this case aggregate real income grows at the same rate as the factors are accumulating.[4] Finally

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4. This result follows immediately from the assumptions of fixed terms of trade and linear homogeneity of the production functions.

with $k_Y > k_X > \rho$, that is with the incremental capital-labor endowment ratio less than that in the labor intensive sector and hence with the labor force growing faster than the capital stock, the composite R-line is negatively sloped but less steep than R_L. Since immiserization cannot occur unless the R-line is negatively sloped, these facts establish as a necessary condition for immiserization that either $\rho > k_Y > k_X$ or $k_Y > k_X > \rho$, i.e. that the incremental capital-labor endowment ratio lie outside the bounds set by the capital-labor ratios employed in the two sectors.

The next step is to prove Bhagwati's contention [3] that immiserization cannot occur under free trade in this model. This is an important result in its own right and it is also useful in strengthening the necessary conditions for immiserization that have just been stated. The proof is accomplished by establishing two propositions concerning the relationship between the slope of the R-line and that of the domestic price line. First, for $\rho > k_Y > k_X$ the R-line is always steeper than the domestic price line. Suppose that this were not true - that is:

$$\frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)} \lessgtr \frac{P_X^D}{P_Y^D} \tag{8}$$

or

$$\frac{P_X^D F^X}{P_Y^D F^Y} \gtrless \frac{(\rho - k_X)}{(\rho - k_Y)} \tag{9}$$

By assumption, the right side of (9) is greater than unity; further, since profit maximization by producers equates the value of marginal

products in both sectors ($P_X^D MPL_X = P_Y^D MPL_Y$), the left side of (9)

is equal to ϕ_L^Y / ϕ_L^X , the ratio of the share of labor in the Y industry

to that in the X industry, which by the factor intensity assumption

is less than unity. Thus equation (9) contradicts the assumptions

and it is proved that for $\rho > k_Y > k_X$ the R-line must be steeper than the

domestic price line. The second proposition is that for $k_Y > k_X > \rho$ the

R-line is less steep than the domestic price line. Suppose that the

converse were true:

$$\frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)} = \frac{k_Y G^Y(\rho - k_X)}{k_X G^X(\rho - k_Y)} = \frac{P_X^D}{P_Y^D} \tag{10}$$

or

$$\frac{\left(\frac{\rho}{k_X}\right) - 1}{\left(\frac{\rho}{k_Y}\right) - 1} \geqq \frac{G^X P_X^D}{D^Y P_Y^D} \tag{11}$$

By assumption, the left side of (11) is less than unity; and the

right side is equal to ϕ_K^Y / ϕ_K^X which is greater than unity by the

factor intensity assumption. Thus (11) is contradicted by the

initial assumptions and the second proposition is proved: for

$k_Y > k_X > \rho$ the R-line is less steep than the domestic price line. This

means that for increases in the endowments of either or both factors

the relevant part of the R-line can never lie between the origin and

the domestic price line; and since under free trade domestic prices

are equal to world prices, immiserization can never occur under free

trade.[5]

Under a tariff, however, the domestic price ratio diverges from the world price ratio and thus the capital-labor ratios and average products of labor and capital will in general be different from their free trade values. Hence the slope of the R-line will also tend to be different from its free trade value; this is why immiserization might occur under a tariff. Two cases, however, can be ruled out immediately. First, as has just been shown, for $k_Y > k_X > \rho$ the R-line is always less steep than the domestic price line. Since the effect of a tariff on Y (the capital intensive good) is to make the domestic line _less_ steep than the world price line, it is impossible for a tariff on Y to make the R-line _more_ steep than the world price line and hence to cause immiserization to occur under these circumstances (see Figure 2a). Similarly, for $\rho > k_Y > k_X$ the R-line is always steeper than the domestic price line, and since a tariff on X makes the domestic price line _steeper_ than the world price line, it is impossible for a tariff on X to make the R-line _less steep_ than the world price line and hence to cause immiserization to occur in this situation (see Figure 2b). Thus it is a necessary condition for immiserization to occur that either $k_Y > k_X > \rho$ and X be protected, or $\rho > k_Y > k_X$ and Y be protected. This extends the previously stated conditions.

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5. It should also be noted that equations (9) and (11) can be interpreted, if P_X^D / P_Y^D is replaced by the world price ratio, as necessary and sufficient conditions for immiserization to occur for the cases in which $\rho > k_Y > k_X$ and $k_Y > k_X > \rho$ respectively.
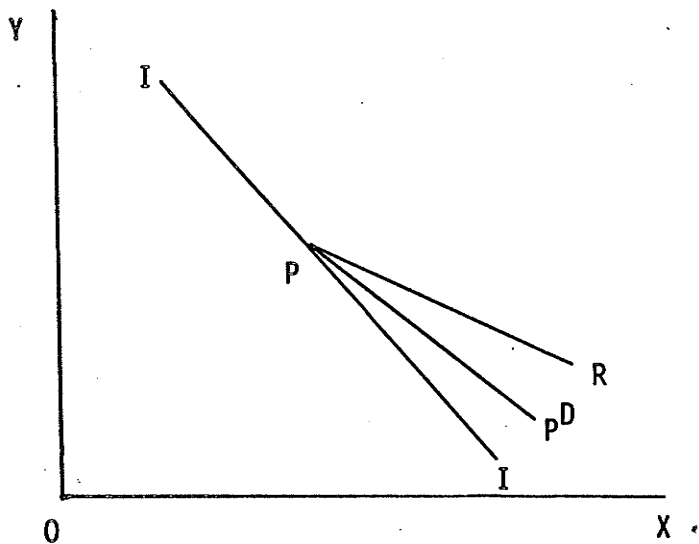
(a)
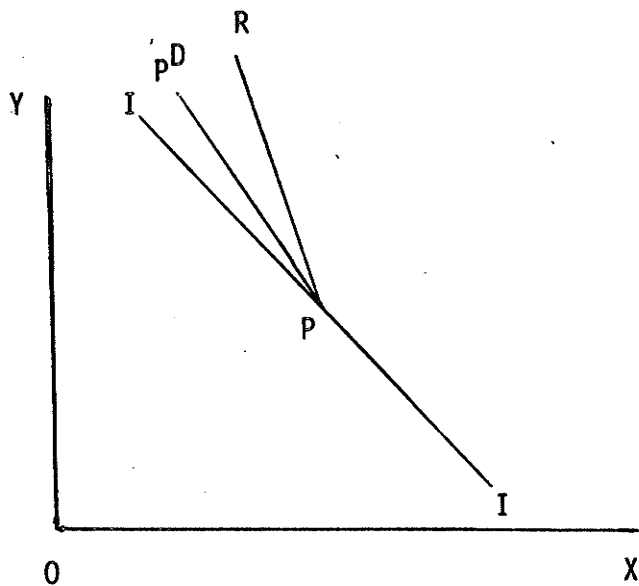
(b)

Figure 2.

1.ii   <u>Necessary and Sufficient Conditions</u>

In order to discover exactly under what conditions immiserization will occur it is necessary to discover how the slope of the R-line under a tariff diverges from its free trade value. From an initial position of free trade the imposition of a tariff will result in a change in the domestic price ratio which will in turn raise or lower the domestic wage-rental ratio. In order to derive a more precise necessary condition for immiserization the relationship between the change in the slope of the R-line and changes in the wage-rental ratio is derived. Then a necessary and sufficient condition is established by linking changes in the wage-rental ratio with the magnitude of the tariff.

By differentiating equation (7) and making the appropriate simplifications the following expression is derived, showing the relative change in the slope of the R-line resulting from a given relative change in the domestic wage-rental ratio:[6]

$$\left(\left.\frac{dY}{dX}\right|_{dK=\rho\,dL}\right)^* = \left(-\left.\frac{dY}{dX}\right|_{dK=\rho\,dL}\right)^* = \left(\frac{F^Y(\rho-k_X)}{F^X(\rho-k_Y)}\right)^*$$

$$= \left[\sigma_Y\left(\phi_K^Y + \frac{k_Y}{\rho-k_Y}\right) - \sigma_X\left(\phi_K^X + \frac{k_X}{\rho-k_X}\right)\right]\left(\frac{w}{r}\right)^* \qquad (12)$$

where σ_X and σ_Y represent the elasticities of factor substitution in

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6. In this derivation the following two relationships are used: $(F^Y)^* = \phi_K^Y(k_Y)^*$, and $(k_Y)^* = \sigma_Y(\frac{w}{r})^*$. The same relationships are true for X.

the X and Y industries, $\frac{w}{r}$ represents the wage-rental ratio, and an asterisk above a variable signifies the relative change in that variable. This expression can then be used to derive a necessary condition for immiserization to occur.

First consider the case in which Y is the protected industry. Since it has already been established that immiserization can occur in this case only if $\rho > k_Y > k_X$, this means for immiserization to take place it is necessary that the imposition of a tariff cause the R-line to rotate in a counter-clockwise direction from its free trade position. This requires that the relative change in the slope of the R-line be negative. Since Y is the capital-intensive industry the imposition of a tariff on it makes $(\frac{w}{r})^*$ negative. Thus an examination of (12) shows that a necessary condition for immiserization to occur is that the term in square brackets in (12) be positive. Similar reasoning will establish that if X is the protected industry it is necessary in order to observe immiserization that the imposition of a tariff cause the R-line to rotate in a clockwise direction--that is, that the relative change in its slope be positive. Since the imposition of a tariff on X causes $(\frac{w}{r})^*$ to be positive, this means once again that the term in square brackets in (12) must be positive. Thus in either case, whether Y is protected and $\rho > k_Y > k_X$, or X is protected and $k_y > k_X > \rho$, a necessary condition for the occurrence of immiserization is that:

$$\frac{\sigma_Y \left(\phi_X^Y + \frac{k_Y}{\rho - k_Y} \right)}{\sigma_X \left(\phi_K^X + \frac{k_X}{\rho - k_X} \right)} > 1 \tag{13}$$

In order now to obtain a necessary and sufficient condition for immiserization the precise relationship between a tariff and changes in the wage-rental ratio must be obtained and substituted into equation (12). The necessary and sufficient condition will be derived for two cases, the first being that in which Y (the capital-intensive industry) is protected. In this case immiserization will occur if and only if under a tariff the R-line rotates in a counterclockwise direction until it becomes at least as flat as the world price line, that is, if and only if the absolute value of the percentage difference between the free trade slope of the R-line and its slope under the tariff is no less than the absolute value of the percentage difference between its free trade slope and that of the world price line:

$$\left(-\frac{dY}{dX} \bigg|_{dK=\rho dL} \right)^* = -\left[\sigma_Y \left(\phi_K^Y + \frac{k_Y}{\rho - k_Y} \right) - \sigma_X \left(\phi_K^X + \frac{k_X}{\rho - k_X} \right) \right] \left(\frac{w}{r} \right)^*$$

$$\geq -\left(\frac{P_X}{P_Y} - \frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)} \right) \left| \frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)} \right. \tag{14}$$

Noting that: (i) from the condition of equality of the values of the marginal product of labor in each sector, the right side of (14) can

be rewritten as $\left(1 - \dfrac{\phi_L^Y(\rho-k_Y)}{\phi_L^X(\rho-k_X)}\right)$; (ii) the relative change in the

wage-rental ratio can be shown to be $(\frac{w}{r})^* = \left(\dfrac{P_X^D}{P_Y^D}\right)^* \Bigg/ (\phi_K^Y\phi_L^X - \phi_K^X\phi_L^Y);$[7]

(iii) the relative change in the domestic price ratio is simply -t

where t is the ad valorem tariff rate on imports of Y, (14) can be

simplified and rewritten:

$$\left[\sigma_Y(\phi_K^Y + \frac{k_Y}{\rho-k_Y}) - \sigma_X(\phi_K^X + \frac{k_X}{\rho-k_X})\right] \gtreqqless \left(1 - \frac{\phi_L^Y(\rho-k_Y)}{\phi_L^X(\rho-k_X)}\right) \frac{(\phi_K^Y\phi_L^X - \phi_K^X\phi_L^Y)}{t} \quad (15)$$

Thus (15) is a necessary and sufficient condition for immiserization

when Y is the protected industry. As long as the necessary condition

for immiserization is fulfilled (equation   (13) is satisfied) the left

side of (15) is positive and the equation can be manipulated slightly

to obtain an explicit statement of the minimum tariff required to pro-

duce immiserization:

$$t \gtreqqless \left(1 - \frac{\phi_L^Y(\rho-k_Y)}{\phi_L^X(\rho-k_X)}\right) \frac{(\phi_K^Y\phi_L^X - \phi_L^Y\phi_K^X)}{\left[\sigma_Y(\phi_K^Y + \frac{k_Y}{\rho-k_Y}) - \sigma_X(\phi_K^X + \frac{k_X}{\rho-k_X})\right]} \quad (16)$$

In a similar fashion the necessary and sufficient condition

for the occurrence of immiserization when the labor-intensive sector

is protected can be derived. (Recall that for this to produce immis-

erization it must be true that $k_Y>k_X>\rho$.) In this case immiserization

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7. See for instance R. W. Jones [7], equations (3.2) and
(4.2) on p. 560.

will occur if and only if under a tariff the R-line rotates in a
clockwise direction enough that it becomes at least as steep as the
world price line - that is if and only if the absolute value of the
percentage difference between the free trade slope of the R-line and
its slope under a tariff is no less than the absolute value of the
percentage difference between its free trade slope and that of the
world price line:

$$
\left(\left.\frac{dY}{dX}\right|_{dK=\,dL}\right)^* = \left[\sigma_Y(\phi_K^Y + \frac{k_Y}{\rho-k_Y} - \sigma_X(\phi_K^X + \frac{k_X}{\rho-k_X})\right](\frac{w}{r})^* \gtreqqless \left(\frac{P_X}{P_Y} - \frac{F^Y(\rho-k_X)}{F^X(\rho-k_Y)}\right) \Bigg/ \frac{F^Y(\rho-k_X)}{F^X(\rho-k_Y)} \tag{17}
$$

Using the same simplifications as in deriving (2.15) from (2.14),
except that in this case the relative change in the domestic price
ratio, $(P_X^D/P_Y^D)^*$, is t, the ad valorem tariff rate applied to imports
of X, and noting further that

$$
\left(\frac{\phi_L^Y}{k_Y}\right) \Bigg/ \left(\frac{\phi_L^X}{k_X}\right) = \frac{\phi_K^Y}{\phi_K^X} , \qquad \text{(17) becomes:}
$$

$$
\left[\sigma_Y(\phi_K^Y + \frac{k_Y}{\rho-k_Y} - \sigma_X(\phi_K^X + \frac{k_X}{\rho-k_X})\right] \gtreqqless \left(\frac{\phi_K^Y(\frac{\rho}{k_Y} - 1)}{\phi_K^X(\frac{\rho}{k_X} - 1)} - 1\right)\frac{(\phi_K^Y\phi_L^X - \phi_K^X\phi_L^Y)}{t} \tag{18}
$$

which is the necessary and sufficient condition for the occurrence of
immiserization when the labor-intensive industry is protected. Simi-
larly (18) can be manipulated to obtain a statement of the minimum

tariff required to produce immiserization:

$$
t \gtreqqless \left(\frac{\phi_K^Y(\frac{\rho}{k_Y} - 1)}{\phi_K^X(\frac{\rho}{k_X} - 1)} - 1 \right) \frac{(\phi_K^Y \phi_L^X - \phi_K^X \phi_L^Y)}{\left[\sigma_Y(\phi_K^Y + \frac{k_Y}{\rho - k_Y}) - \sigma_X(\phi_K^X + \frac{k_X}{\rho - k_X}) \right]} \tag{19}
$$

1.iii Factors Affecting the Likelihood of Immiserization

Equations (15) and (18) can now be studied to discover what factors will tend to affect the chance of immiserization. First the case in which the capital intensive industry is protected is examined. Since both bracketed terms on the left side of (15) are positive, the value of the left side will increase the greater is σ_Y relative to σ_X. Since both $\phi_K^Y - \phi_K^X$ and $(\frac{k_Y}{\rho - k_Y}) - (\frac{k_X}{\rho - k_X})$

increase as k_Y increases or k_X decreases, the value of the left side will increase with differences in factor intensities between the two sectors. And since $(\frac{k_Y}{\rho - k_Y}) - (\frac{k_X}{\rho - k_X})$ varies inversely with ρ, the value of the left side will also decrease as ρ increases. Similarly, both terms on the right side of (15) tend to increase with differences in factor intensities between the two sectors. And since $(\rho - k_Y)$ / $(\rho - k_X)$ increases with ρ, the value of the right side also will tend to vary inversely with ρ. Finally, the value of the right side varies inversely with the size of the tariff imposed on imports of Y. Thus immiserization will be more likely the greater is the elasticity of factor substitution in the capital intensive sector relative to that in the labor intensive sector--the greater is the ratio between the

two, the faster will the capital-labor ratio and output per head in the capital intensive sector fall relative to the same variables in the labor intensive sector for any given tariff on imports of the capital intensive good.[8] The effect of differences in factor intensities on the likelihood of immiserization is ambiguous: the greater are the differences the faster will the R-line rotate for any given decrease in the wage-rental ratio (see the term on the left side); on the other hand, the greater are these differences the greater will be the tariff required to produce any given change in the wage-rental ratio (the second term on the right side) and the greater will be the difference between the free trade slope of the R-line and that of the world price line (the first term on the right side). The first effect tends to increase the chance of immiserization, while the latter two tend to decrease its likelihood. Similarly, the effect of increases in ρ (the incremental capital-labor endowment ratio) is also ambiguous: from the left side of (15) it is seen that increases in ρ decrease the rate at which the R-line rotates for any given decrease in

---

8. The absolute value of the slope of the R-line (from equation (7) is

$$\frac{dY}{dX}\bigg|_{dK=\rho dL} = \frac{F^Y(\rho - k_X)}{F^X(\rho - k_Y)}$$

As the result of a decrease in the wage rental ratio output per head and the capital-labor ratio in each sector fall. The greater is σ_Y relative to σ_X the greater will be the rate of decrease of F^Y and k_X relative to F^X and k_X and thus the smaller will be the absolute value of the slope of the R-line.

the wage-rental ratio, while from the right side it is clear that

increases in ρ decrease the difference between the free trade slope

of the R-line and that of the world price line. Finally, the possi-

bility of immiserization will increase with the size of the tariff

on imports of the capital intensive good--the greater is t the greater

will be the change in the wage-rental ratio and thus the greater will

be the rotation of the R-line from its free trade position.[9]

Next the case in which the labor intensive sector is protected

is examined. Since for $k_Y>k_X>\rho$ it is true that $\dfrac{k_Y}{\rho-k_Y} < \dfrac{k_X}{\rho-k_X} < -1$, and

$\phi_K^X<\phi_K^Y<1$, each of the bracketed terms on the left side of (2.18) is

negative. Thus the value of the left side will be greater the larger

is ρ_X and the smaller is ρ_Y. Similarly, the value of the left side

can be shown to increase as differences in factor intensities between

the two sectors increase and to decrease as ρ increases. From the

factor intensity assumption and from the falsity of equation   (16)

both terms on the right side of (18) are positive. Further, both

terms can be shown to increase as differences in factor intensities

between the sectors increase, and to decrease as ρ decreases and as

t increases. From these conclusions it follows that the greater is

the elasticity of factor substitution in the labor intensive sector

relative to that in the capital intensive sector the greater will be

9. Of course, the greater is the tariff, the greater is the
chance that it will be prohibitive. Under such circumstances immis-
erization would be impossible - the economy would proceed in autarky
with real income at domestic prices growing with factor accumulation.

the chance of immiserization. This, of course, is opposite to the conclusion reached in the case where the capital intensive sector is protected, for in the present case the R-line becomes more steep the more rapidly the labor-capital ratio and output per head in the capital intensive sector fall relative to the same variables in the labor intensive sector as the result of any given increase in the wage-rental ratio.[10] In a manner exactly analogous to the previous case it can be shown that the effect of differences in factor intensities between the two sectors and of decreases in ρ (increasing growth of the labor force relative to that of the capital stock) on the likelihood of immiserization are ambiguous, and increases in the tariff rate applied to imports of the labor intensive good raise the chance of its occurrence.[11]

2. PER CAPITA REAL INCOME

Thus far necessary and sufficient conditions have been obtained

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10. By dividing the numerator and denominator of (7) by $k_X k_Y$, the absolute value of the slope of the R-line can be shown to be:

$$- \frac{dY}{dX}\bigg|_{dK=\rho dL} = \frac{G^Y(1 - \frac{\rho}{k_Y})}{G^X(1 - \frac{\rho}{k_Y})}$$

As the result of an increase in the wage-rental ratio the labor-capital ratio (1/k) and output per unit of capital (G) fall in each sector. The greater is σ_X relative to σ_Y the greater will be the fall of G^X and $1/k_X$ relative to G^Y and $1/k_Y$ and hence the greater will be the absolute value of the R-line's slope.

11. Once again the proviso must be made that the tariff not become sufficiently large to become prohibitive.

for factor accumulation to lead to a fall in <u>aggregate</u> real incomes. However, it is still possible, insofar as factor accumulation results from increases in population, that even if <u>aggregate</u> real incomes do not fall, <u>per capita</u> incomes might fail to rise. In what follows, necessary and sufficient conditions for the occurrence of this latter eventuality will be derived under the assumption that the labor force is a fixed proportion of the population and the capital stock is independent of population size.

The first obvious point is that if only the capital stock expands, the condition for immiserization in the aggregate sense will be the same as that for immiserization in the per capita sense. Thus it is necessary and sufficient for capital accumulation alone to fail to increase real per capita incomes that the capital intensive industry be protected and that:

$$\sigma_Y \, \phi_K^Y - \sigma_X \phi_K^X \; \gtreqless \; \left(1 - \frac{\phi_L^Y}{\phi_L^X} \right) \frac{(\phi_K^Y \phi_L^X - \phi_K^X \phi_L^Y)}{t} \qquad (20)$$

where (20) is simply what (15) reduces to as ρ approaches infinity.[12] It was shown in Bertrand and Flatters [1] that in this case immiserization will be more likely the greater is σ_Y, the smaller is σ_X, the larger is the tariff, and the smaller are the differences in factor intensities between the two sectors.

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12. This equation represents the necessary and sufficient condition for immiserization in the case examined in Bertrand and Flatters [1].

If, however, the population is expanding also, there are two cases to be considered: that in which the import competing industry is capital intensive and that in which the import competing industry is labor intensive. Consider first the case in which the capital intensive industry is protected and suppose initially that the capital stock is growing faster than the population. The factor accumulation can be divided into two parts: first a proportional increase in the supply of both factors, and then a further increase in the capital stock alone. After the proportional increase in both factors, per capita income remains unchanged from its initial position.[13] Now, if capital accumulation alone leads to an increase in aggregate income it is clear that per capita income will also rise, and the total effect of factor accumulation in this case will be to raise per capita income. On the other hand if capital accumulation alone leads to a fall in aggregate real income, the total effect of factor accumulation will be to lower per capita real income. Thus, if the capital intensive industry is protected, and if the capital stock grows faster than population, immiserization in the per capita income sense will occur if and only if capital accumulation alone is immiserizing--that is, if equation (20) is satisfied. The same type of reasoning is used to examine the case in which population is expanding more rapidly than the capital stock, except that in this case the second step in

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13. Recall that this result was presented above in the discussion of equation (7).

the process is to reduce, rather than increase the capital stock.
If it is true that capital accumulation alone is immiserizing, then
it is a direct corollary that capital decumulation will lead to an
increase in real income (both aggregate and per capita).[14] Similarly,
if capital accumulation alone is not immiserizing, capital decumula-
tion alone will be immiserizing. Thus it follows immediately that
if population is growing faster than the capital stock, and the
capital intensive industry is protected, immiserization in the per
capita income sense will occur if and only if capital accumulation
alone is not immiserizing--that is if equation (20) is satisfied
with the inequality reversed.

Suppose now that the labor intensive sector is protected.
It will be recalled that it already has been established that the
R-line will be steeper than the domestic price line whenever it is
true that $\rho > k_Y > k_X$, and so this will be true in particular when the
capital stock is growing with the labor force constant. Further,
with a tariff on imports of the labor intensive good (X), the world
price line will necessarily be less steep than the domestic price
line. Thus, considering the case where the capital stock is accumu-
lating faster than population, both factors are first assumed to
increase proportionately so that production takes place at P (Figure
3) where per capita income remains unchanged from its initial level.

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14. To put it slightly differently, the handmaiden of
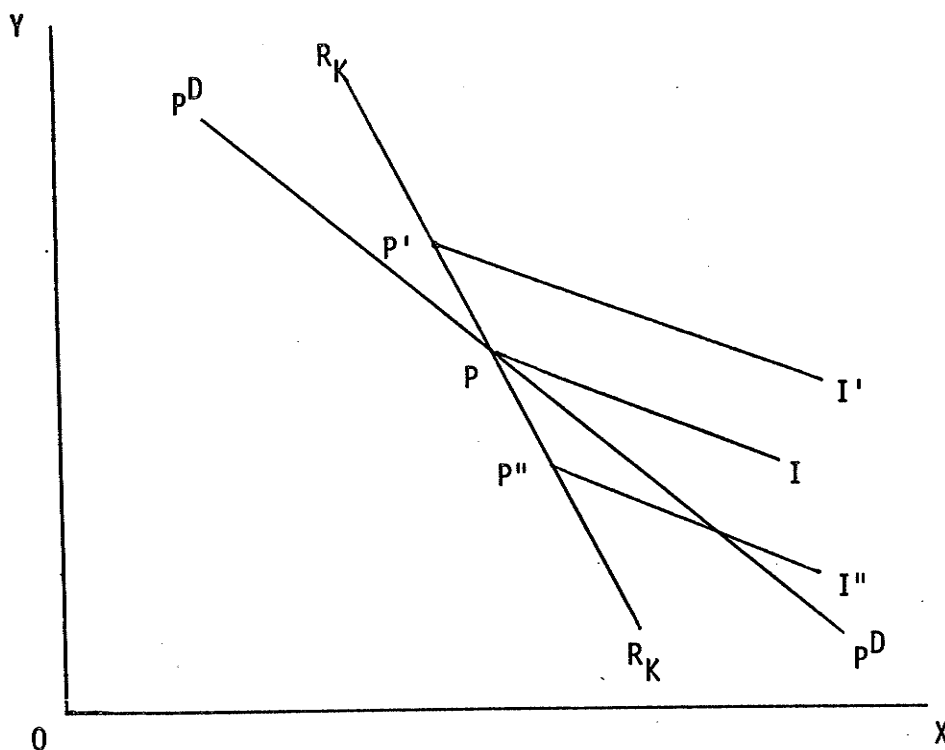immiserizing growth is beneficial shrinkage.

Figure 3.

Then the capital stock alone is assumed to increase so that production takes place above P along R_K, where it is necessarily true that aggregate, and thus per capita income has increased from its level at P (P'I' > PI). If on the other hand the capital stock were to decline from P, production would move down along R_K, to a point such as P", where aggregate and per capita incomes have necessarily fallen. It should be noted, however, that these results are independent of the degree of protection given the labor intensive sector. In particular, even if there were no protection and thus the world price line coin-

cided with the domestic price line it can be verified simply from an examination of figure 3 that the conclusions just stated would continue to hold. Thus, as long as the import competing sector is labor intensive, it is necessary and sufficient for immiserization in the per capita income sense to occur that population expand more rapidly than the capital stock.

This result is obvious from a consideration of the necessary conditions for immiserization in the sense of declining aggregate real income derived in section 1.i. It was shown there that when the import competing industry was labor intensive, capital accumulation alone ($\rho > k_Y > k_X$) could never lead to immiserization and, as a corollary, capital decumulation must always be immiserizing. Since the question of immiserization of per capita incomes can be reduced to a question of aggregate income immiserization under capital accumulation alone, the result that the occurrence of per capita income immiserization depends only on the relative growth rates of the capital stock and the labor force and is independent of the level of protection is not surprising. It is only when the import competing industry is capital intensive that the level of protection is important.

A final conclusion that follows from the case in which the import competing industry is labor intensive is that Bhagwati's [3] contention that immiserizing growth occurs only as a result of non optimal policies does not generalize to a consideration of changes in per capita real incomes.

3. <u>SUMMARY</u>

Necessary and sufficient conditions have been derived for the occurrence of declining aggregate and per capita incomes as a result of factor accumulation in an open economy facing fixed terms of trade. These conditions are summarized for the various possible cases in Table 1 on the following page.

Will Immiserization Occur

Relative Growth Rates of K and L	(a) in the aggregate income sense?	(b) in the per capita income sense?
L* = K*	No	Per capita income constant
L* > K*	No	If and only if (20) is satisfied, with inequality reversed
K* > L*	If and only if (15) is satisfied	If and only if (20) is satisfied

Table 1a.

Possibility of Immiserization when
Import-Competing Industry
is Capital Intensive

L* = K*	No	Per capita income constant
L* > K*	If and only if (17) is satisfied	Yes, necessarily
K* > L*	No	No

Table 1b.

Possibility of Immiserization when
Import-Competing Industry
is Labor Intensive

TABLE 1.

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