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The Stock of Intangible Capital in Canada: Evidence from the Aggregate Value of Securities

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Abstract

This paper measures the size of the stock of intangible capital in Canada using newly released data on the market value of all securities in the economy. The approach taken relies on a quantitative application of the q-theory of investment to generate the quantity of capital owned by firms. I find that the intangible capital stock accounted for approximately 30% of overall capital since 1994. Of this, the R&D reported by national accounts makes up only 23%. These results imply that official Canadian statistics failed to account for 26% of the value of the capital stock in their 2005 quarterly data collection. In addition, I find that the magnitude of the intangible capital stock is comparable to that reported using a cost approach, confirming the size of the findings and ascertaining the need to include intangibles in empirical models and in investment data.

Keywords: q-Theory, Unrecorded Capital, Intangible Capital.

JEL classifications: E22, E44, G12, G31, O16.

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1 Introduction

Many firms devote resources to the production of capital goods that are not intended for commercialization but instead are kept in-house. Such capital goods include research and development expenditures, training expenses, brand equity and organizational change and development. For reasons explained below, this capital formation is not recorded in national income accounts. As a consequence, little is known about its overall size. Using newly released Canadian data, this paper is a first attempt to uncover the magnitude of this unrecorded capital stock in Canada.

The methodology followed is a quantitative application of the q-theory of investment. The market value of all firms in the economy is given by the aggregate value of their net financial liabilities. Assuming that investors price securities rationally, the value of these liabilities can then be used to infer the overall stock of installed capital inside firms. From this inferred capital, I subtract that part which is recorded in national income accounts and back out the unrecorded capital stock. Baily (1981) pioneered this approach by empirically linking the value of securities to the stock of capital and Hall (2001) applied it using a q-theory of investment to infer the stock of capital in the U.S. economy. This paper applies Hall's approach to Canadian data.

What is this unrecorded capital made of? First, it is important to be clear about certain concepts. Capital formation is defined as the expenditure on inputs that will not be consumed by firms in the accounting period. Consumption by firms is the act of using up goods and services in the current period (United Nations (1998)). These "consumed" goods are known as intermediate inputs. Capital is then a produced good "that is used repeatedly or continuously in production over several accounting periods (more than one year)" (United Nations (1998)). A business expenditure that aims to acquire a capital good will be recorded by national income accounts as capital formation only if it is *identifiable* and if it involves the *acquisition* of a capital good from the market instead of being produced in-house.¹ The requirement of identification is met whenever national income accountants can classify the expenditure on the item under a well-defined category of products. On the other hand, the

¹Software expenditures are an exception. Since 2001, even when produced in-house, software is treated as capital expenditure (see Statistics Canada (2001)).

necessity to observe that the item was acquired from the marketplace ensures the existence of an accurate valuation of the good which is captured by the market price.

Given these two requirements and the definition of capital formation, all capital expenditures by firms which are either non-identifiable or are intended to produce a capital good in-house do not end up being recorded as capital formation. The convention in national income accounts is to treat this spending as intermediate consumption expenditure. Consequently, this practice lowers the value added of final produced output and understates the existing stock of capital in the economy.

Research and experimental development (R&D) expenditures offer a good illustration of the consequence of this convention. Even though national income accounts incorporate data on R&D spending, this expenditure is treated as an expense rather than an investment mainly because of the lack of a market price on the output of R&D activities². Training expenses constitute a different example where no data is systematically collected by national income accounts since it is a difficult good to identify or classify.

Accountants distinguish two categories of capital goods: tangible and intangible. Tangible capital goods comprise a list of items that have a physical embodiment such as machines, tools and equipment. On the other hand, intangible capital goods are associated with items that have a knowledge or informational component such as patents, copyrights and brands. Since most intangible capital is created in-house with the two characteristics of being difficult to identify and generally not acquired or sold on the market, most unrecorded capital is often assimilated to, or completely identified with, intangible capital. This needs not be the case but it points to natural candidates that would compose the stock of the unrecorded capital that this chapter identifies. Indeed, the unrecorded capital could, for example, include mis-measurement of the tangible capital stock. Therefore, equating the unrecorded capital stock with intangible capital is only an approximation. However, for the sake of exposition, the terms “unrecorded capital” and “intangible capital” will be used interchangeably throughout this text.

²Some R&D spending leads to the creation of a patent which will carry a price if commercialized. However, the market for patents is extremely thin: very few patents change hands. For example, Serrano (2006) documents that only about 20% of all U.S. patents issued to small innovators (*i.e.*, firms that were issued no more than five patents in a given year) are traded once or more.

Why care about these unrecorded intangible assets? In the last few years, there has been a growing perception among academics and policy-makers that a significant and increasing part of total business investment is directed towards intangible investment. Intangible investment is the expenditure on items which have a knowledge component, such as research and development, training, organizational change, marketing and software. To some researchers, this phenomenon is “what put the new in the new economy” (Nakamura, 1999), while others acknowledge that “although investment in intangible capital is not counted as capital investment in the national income and product accounts, they appear to be quantitatively important.” (Bernanke, 2005)

Unfortunately, the lack of systematic statistical information on intangible investment makes it difficult to directly substantiate this phenomenon, monitor its progress and assess its importance for growth. Moreover, the difficulty of defining intangibles, given their impalpable nature, contributes to an opacity of language and, consequently, to a lack of agreement on their precise size. Finally, researchers’ various goals in measuring and using intangibles have led to diverse approaches in evaluating their magnitude, sometimes with conflicting results.

The current state of omission and mismeasurement of intangible capital has several implications. First, because spending on intangibles is not treated as investment, aggregate savings and investment may be significantly understated in official statistics. Monetary policymakers could be misled by such an imprecise picture of the economy in setting interest rates. Second, resource allocation and investment decisions within firms and across firms in a given industry become more difficult. Third, fiscal policy can be affected in various ways such as in the design of a fair tax system. Finally, the lack of good information on intangibles will lead to opaqueness and volatility in capital markets given the increased difficulty of estimating the future cash flows that some investments will generate.

Notwithstanding, preliminary estimates point to an average investment level in intangibles in the U.S. of 6% of GDP in the early 1990s. By the late 1990s, investments in intangible capital by U.S. businesses were argued to have been as large as investment in traditional, tangible capital (Corrado *et al.*, 2005). The picture in Canada is less clear. Baldwin *et al.* (2005) note that there are no reliable data in Canada that would give a complete account of

expenditures on intangible capital.

As detailed below, such a state of affairs can be remedied by taking the indirect approach of Hall (2001) to measuring the stock of capital. I find that the size of the intangible stock has been increasing from 1994 to 2001 and averaged 29% of the overall capital from 1994 to the middle of 2006. The nature of this stock is shown to consist of about 23% R&D capital with the rest made up of other intangible capital goods. This finding implies that official Canadian statistics failed to account for \$380 billion worth of capital stock in their 2005 quarterly data collection or about 26% of the inferred capital stock. The results obtained mirror qualitatively the findings of similar approaches conducted for the U.S. and the U.K. which document a substantial rise in the size of the intangible capital stock up to 2001. The findings are also in line with the results of Belhocine (2008) who uses a direct expenditure approach to measure intangible investment in Canada. This cross-method checking of the findings offers even more assurance about the size of intangibles reported in this paper.

The paper is organized as follows: the next section reviews the related literature, section 3 introduces the model, section 4 describes the data, section 5 presents the empirical results, section 6 outlines the sensitivity analysis conducted to assess the robustness of the findings and section 7 concludes the paper.

2 Relationship to related literature

The literature review will focus on those papers that look at the economy as a whole, as opposed to only one segment, and that attempt to account for the sum of all types of intangible capital goods³.

At the root of most investigations into the level of intangible investment lies a dissatisfaction with the practice of national income accountants in treating expenses on intangibles as

³There is a large literature in the field of Industrial Organization which aims at uncovering the size or the value of some of the components of the intangible capital stock. These papers typically use panel or survey data which cover short periods of time or just some portions of the economy. The focus of this research strand is on industry dynamics as exemplified by their focus on firms' entry, exit, mergers and the dynamic of life and growth of population of firms or plants. Two indicative studies are the papers of Atkeson and Kehoe (2006) for the U.S. and Baldwin and Gelatly (2006) for Canada. The first paper focuses on the measurement of organizational capital for a panel of plants in the late 1980s and covers two years. The second paper investigates the expenditures of a set of Canadian firms on organizational change using survey data.

operating costs. Given that intangibles are assets, they should be capitalized because they are not entirely used up in the production of final output. In this way, they ought to be treated as investment instead of being expensed as intermediate consumption goods (Nakamura, 2003a and Corrado *et al.* 2005.). Nakamura (1999) and Corrado *et al.* (2005) attempt to calculate the size of intangible capital investment using a similar approach. Corrado *et al.* (2005) identify a list of intangible items and investigate different data sources to inform the investment of U.S. firms on intangible capital goods. They show that by the end of the 1990s, the size of the investment in intangible capital was as big as the size of the investment in physical capital.

Hall (2001) and McGrattan and Prescott (2005a) rely on the unmeasured levels of intangible capital to rationalize the rise in the stock market in the late 90s in the U.S. and in the U.K. Hall (2001) shows that the rise in the stock market coincides with an ever increasing accumulation of intangible capital. McGrattan and Prescott (2005a) are able to rationalize the size of intangible investment found in Corrado *et al.* (2005) while using the change in tax regulations to explain the different performance of the U.K. and the U.S. stock markets. McGrattan and Prescott (2005b) show that by explicitly accounting for intangible capital, one can explain the productivity paradox. In particular, they argue that GDP in national income accounts is undervalued because of the expensing of intangible investment which ultimately created a downward bias in the estimates of productivity in the early 90s.

The paper of Eliades and Weeken (2004) applies Hall's methodology to the U.K. These authors find no trace of intangible capital for the U.K. before 1990 but reach the same qualitative results as Hall (2001) for the late 90s.

To the best of our knowledge, no studies to date have attempted to give a macroeconomic account of the overall stock of intangible capital in Canada.

3 Methodology

3.1 A quantitative approach to the q-theory

The model is a standard neoclassical model of investment as developed in Hayashi (1982). It ultimately relates the value of securities to the value of installed capital which then allows

to back out the unobserved quantity of installed capital.⁴

There is perfect competition in input and output markets. The production function is homogeneous of degree one in capital, k , and labor, l , and is denoted by $F(k, l)$. Firms buy capital from the market or produce it in-house. The problem of the firm is to choose the optimal level of labor and investment so as to maximize the net present value of future profits subject to the technology of investment accumulation, the starting level of capital and the non-feasibility of Ponzi schemes:

$$\begin{aligned} \max_{\{x_s, l_s\}} \quad & v_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{F(k_{s-1}, l_s) - w_s l_s - x_s - C(x_s, k_{s-1})\} \\ \text{s.t.} \quad & \end{aligned} \quad (1)$$

$$k_{s-1} \quad (2)$$

$$k_s = (1 - \delta)k_{s-1} + x_s \quad (3)$$

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T v_{s+T} = 0 \quad (4)$$

where k stands for capital stock, x for investment, δ for the depreciation rate, l for labor, r for the real interest rate, w for the real wage rate and $C(\cdot)$ for the adjustment cost function. The value function v_t is the net present value at time t of future payout to securities' holders. Indeed, after the firm pays inputs their due, the left over income is paid to owners. Their ownership materializes through the possession of titles in the form of securities. Hence, v_t is also the value of the firm.

It is assumed that there is perfect substitutability between the recorded investment by national income accounts and the unrecorded investment *i.e.*,

$$x_s = x_s^{\text{Recorded}} + x_s^{\text{Unrecorded}}. \quad (5)$$

Note that the unrecorded investment consists of the sum of both in-house produced capital goods (for example, training expenses that goes into producing human capital inside the firm) and the externally acquired capital goods. The latter are nevertheless expensed because of

⁴The empirical performance of the q-theory of investment appears to be decent but not more (see Caballero (1999) for a survey). The belief here is that past tests of the theory suffered from specification problems by not taking into account the investment of firms in intangibles (Hall (2004) pp.914-915 provides a related discussion.) Moreover, the exercise in this paper is not intended to test the theory but instead to explore its quantitative implications.

the convention in national accounts of expensing all intangible capital goods (the purchase of a patent for example). Notice also that this approach is not intended to explain the reasons for the firm's choice to not commercialize the in-house produced capital good. As such, there is no inherent difference between a capital good bought from the market and a capital good produced in-house; perfect competition will ensure that they both have the same price.

The Lagrangian \mathcal{L} at time t is given by

$$\mathcal{L}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{F(k_{s-1}, l_s) - w_s l_s - x_s - C(x_s, k_{s-1}) - q_s [k_s - (1-\delta)k_{s-1} - x_s]\} \quad (6)$$

where q is the Lagrangian multiplier or the shadow price of an additional unit of capital.

The first order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial x_s} : \quad q_s = 1 + C_x(x_s, k_{s-1}) \quad (7)$$

$$\frac{\partial \mathcal{L}_t}{\partial l_s} : \quad w_s = F_l(k_{s-1}, l_s) \quad (8)$$

$$\frac{\partial \mathcal{L}_t}{\partial k_s} : \quad q_s(1+r) = F_k(k_s, l_{s+1}) - C_k(x_{s+1}, k_s) + (1-\delta)q_{s+1} \quad (9)$$

$$\frac{\partial \mathcal{L}_t}{\partial q_s} : \quad k_s = (1-\delta)k_{s-1} + x_s. \quad (10)$$

Note that bubbles in the shadow price of capital are ruled out, *i.e.* $\lim_{T \rightarrow \infty} (1+r)^{-T} q_{s+T} = 0$. Equation 7 illustrates the equality of the lifetime return to increasing capital by one unit with its marginal cost given by the price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. Equation 8 states the usual equilibrium condition for the labor market whereby the real wage is equal to the marginal product of labor. Equation 9 shows the dynamic equilibrium equation of the q with its continuation value. Finally, Equation 10 recasts the investment technology constraint.

Following Hayashi (1982), the adjustment cost function is quadratic and displays constant returns to scale:

$$C(x_t, k_{t-1}) = \frac{\alpha}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1}. \quad (11)$$

where α is the adjustment cost parameter. Its exact meaning will be explained below. Assuming $s = t$ and substituting this cost function into the first order condition that described

the equality of the q with the marginal cost of augmenting capital by one unit, we obtain the following equation:

$$x_t = \frac{1}{\alpha}(q_t - 1)k_{t-1}. \quad (12)$$

This is known as the investment equation since it relates the behavior of investment to the shadow price of capital q_t . Investment is positive when the lifetime return to increasing capital by one unit exceeds its price. This equation has limited empirical use since q_t is by definition a shadow price and therefore, it is unobservable.

Hayashi (1982) showed that by combining all the first order conditions with the no-Ponzi scheme constraint, the value of the firm v_t would be equal to the value of the installed capital $k_t q_t$. This finding combined with the first order condition of the equality of the q with the marginal cost of increasing capital by one unit results in a recursive system of the form:

$$\begin{cases} v_t = k_t q_t \\ q_t = 1 + C_x(x_t, k_{t-1}) \Leftrightarrow x_t = \frac{1}{\alpha}(q_t - 1)k_{t-1} \end{cases} \quad (13)$$

This approach is introduced in Hall (2001). The system of two equations can be solved for the pair (k_t, q_t) at each t given v_t and k_{t-1} . To obtain an explicit solution, rewrite the investment equation as

$$x_t = \frac{1}{\alpha} \left(\frac{v_t}{k_t} - 1 \right) k_{t-1}. \quad (14)$$

Substituting the investment term x_t by the capital accumulation expression and rearranging, we obtain the following quadratic equation:

$$\alpha k_t^2 + [1 - \alpha(1 - \delta)]k_{t-1}k_t - v_t k_{t-1} = 0 \quad (15)$$

Hall (2001) shows that a unique solution exists for a general convex cost function with constant returns to scale. This equilibrium is stable and is therefore not sensitive to initial conditions in the long-run. The positive root expresses the law of motion of the capital stock⁵:

$$k_t = \frac{1}{2\alpha} \left([\alpha(1 - \delta) - 1]k_{t-1} + \sqrt{([\alpha(1 - \delta) - 1]k_{t-1})^2 + 4\alpha v_t k_{t-1}} \right). \quad (16)$$

All variables are observable and v_t is a sufficient statistic to back out the stock of capital in the economy. k_t is therefore the endogenous variable to be calculated at each point in

⁵Since a capital stock is a positive quantity, the negative root is meaningless in this context.

time. Notice here that at every t , $k_t = k_t^{Recorded} + k_t^{Unrecorded}$. $k_t^{Recorded}$ is the observed capital stock constructed from national income accounts by cumulating the recorded investment $x_t^{Recorded}$ overtime.

3.2 Empirical strategy

The parameters in the law of motion of the capital stock need to be specified. For the sake of comparison, the same parameters as those in Hall (2001) are used. These are also used by Eliades and Weeken (2004). Section 6 analyzes the impact of specifying different ranges of parameters on the implied stock of capital k_t and further discusses the rationale for the choice of certain parameter values.

In order to account for irreversibility in investment, it is assumed that the cost function is piece-wise quadratic :

$$C(x_t, k_{t-1}) = \begin{cases} \frac{\alpha^+}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t > 0 \\ \frac{\alpha^-}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t < 0 \end{cases} \quad (17)$$

where the adjustment-cost parameter α^+ (α^-) represents the time it takes for the capital stock to double (halve) when q doubles (halves). To see this, rewrite the investment equation as

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha}(q_t - 1). \quad (18)$$

If q doubles permanently, say from one to two, it will initially cause the investment-capital ratio to increase by $\frac{1}{\alpha}$. For the investment-capital ratio to double, the increase in $\frac{1}{\alpha}$ must be repeated for α periods. By allowing the downward adjustment-cost parameters to be higher than the upward adjustment-cost parameter, this asymmetry in the investment decision will reflect irreversibility of investment.

Hall (2001) cites the work of Shapiro (1986) to justify the choice of a doubling time parameter of 8 quarters. He also sets the downward adjustment-cost parameter to ten times higher than the upward adjustment-cost parameter. The depreciation rate of 2.6% per quarter is used by national income accounts for physical capital. Finally, to start the iteration on the law of motion of capital, the value of the initial capital stock k_{t-1} needs to be set. We will assume that the economy is in a steady-state equilibrium at the pre-initial

Table 1: Parameter values

Name	Parameter	Value	Rationale
Upward adjustment-cost	α^+	8	Shapiro (1986)
Downward adjustment-cost	α^-	80	Hall (2001)
Depreciation rate	δ	0.026	Hall (2001)
Initial capital stock	k_{t-1}	v_{t-1}	Assuming $q_{t-1} = 1$ at $s = t - 1$

quarter, *i.e.* q_{t-1} takes its equilibrium value of 1. Since investment will be nil at this pre-initial quarter, the relationship $v_t = k_t q_t$ implies that $k_{t-1} = v_{t-1}$. Because the recursion was shown to be insensitive to initial condition, this equilibrium assumption is not going to affect the behavior of the system in the long-run. In fact, the derivative of the capital stock in 1994 with respect to the initial condition is only 0.1 and dies to 0 soon after. It will be shown in the sensitivity analysis section that this assumption is inconsequential after the recursion runs for some quarters⁶.

Table 1 summarizes the parameter values used and the rationale for the choice of each value.

4 Data Description

The Canadian system of national accounts is made of three main accounts: the national income and expenditure account (NIEA), the financial flow accounts (FFA) and the national balance sheet accounts (NBSA). The FFA reports the flow of assets and liabilities that occur during a period while the NBSA reports the evolution of the stock of assets and liabilities overtime. The NBSA can be viewed as an aggregate account statement that merges all balance sheets of firms, for all sectors of the economy. The financial liabilities that are reported can be divided into three major categories: shares, bonds and other liabilities (loans and mortgages, short-term paper, trade payables, life insurance and pensions).

Until recently, the data on the aggregate value of financial instruments given by the NBSA was available only at book value. Since June 2004, Statistics Canada produces this data at market value. The data coverage starts in 1990 at quarterly frequency. This represents a

⁶The focus throughout the text is on the period post 1994 given that the recursion is shown to be invariant to the initial capital level after 4 years. The results from 1990 and 1994 are therefore not as precise (see Figure 8 in the sensitivity analysis section.)

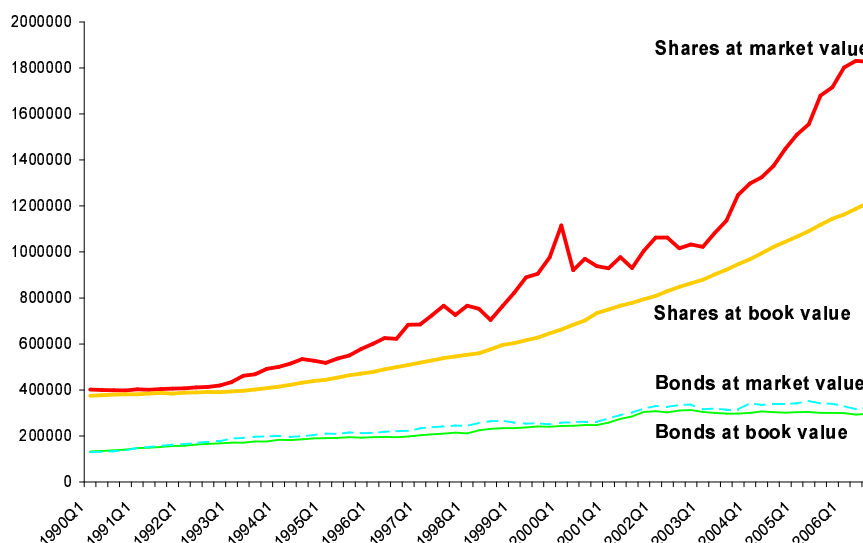


Figure 1: Shares and Bonds in Book and Market Value (in millions)

major improvement in the reporting given that the discrepancy between the historical price of an asset and its current market value can be substantial, especially for equities and long-term financial assets. Figure 1 illustrates the difference in the size of shares and bonds when measured at book value versus at market value.

It is interesting to note that the balance sheet for the U.S. economy reports only equity at market value and leaves bonds at book value. Hall (2001) manipulates the stock of bonds at book value to obtain a series at market value. In the end, the two bonds' series evolve in the same way as those shown for Canada in Figure 1. This cross-checking with an official statistical agency is reassuring and highlights that most of the variation in the market value of firms in the U.S. and in Canada is associated with equity.⁷

The market value of net financial claims (financial liabilities minus financial assets) is used as the measure of v_t since the value of the ownership claims is a reflection of the installed capital inside the firm. Indeed, v_t was defined as the present value of payouts to securities' holders. Assuming that investors are rational, it follows that the present value of payouts v_t will equal the value of securities on the market. Since for all t , $v_t = q_t k_t$, then the value of securities equals the value of the installed capital stock.

⁷The financial instruments that constitute "other liabilities" were not displayed on the graph because their market value is reported to be similar to their book value. This is most likely the case given the majority of these financial instruments are not traded (for example, loans and mortgages).

Table 2: Accounting Balance Sheet

Assets	Liabilities
Current Assets	Current Liabilities
Cash	Accounts payable
Accounts receivable	Notes payable
Marketable securities	Other payables
Inventory	
Long-Term Assets	Long-Term Liabilities
Land	Notes payable
Plant and Equipment	Bonds payable
Other Assets	Owner's Equity
Goodwill	Equity
Intangible assets	Retained earnings
=Total Assets	=Total Liabilities & owner's equity

Notice that v_t includes all financial claims towards firms net of financial assets that firms hold against others. This represents a departure from the way the literature in the q-theory of investment interpreted v_t as covering only equity values or as consisting of equity plus bonds. This departure is mainly due to the availability of data.

Another way of understanding the equality of v_t with net financial claims is to rearrange the usual accounting balance sheet of a firm, like the one illustrated in Table 2⁸. A firm's balance sheet stresses the distinction between assets and liabilities.

The modified accounting framework shown in Table 3 uses the equality of assets and liabilities to recast the balance sheet into financial assets versus non-financial assets with the result that the two must be equal. As Table 3 indicates, net financial liabilities serve as an estimate to the value of the firm's productive assets v_t , for all t . Note that this balance sheet approach is an identity (accounting convention) while the equality between assets and liabilities under the q-theory of investment is an equilibrium condition.

When conducting the data analysis, the focus will be on the non-farm, non-financial corporate sector. This sector is chosen because it is the most amenable to fit the perfectly competitive framework of this paper. The removal of the farming sector aims to control for

⁸This exposition is borrowed from Hall (2001).

Table 3: Modified Accounting Framework

Financial claims	Nonfinancial assets
Equity outstanding	Plant and equipment
Debt outstanding	Land
Value of payables and other financial obligations	Inventories
	Intangibles
<i>Less</i> equity, debt, receivables, cash and other financial claims on others	<i>Less</i> nonfinancial assets claimed on others
=Net financial claims outstanding	=Net value of nonfinancial assets

the presence in the overall capital stock of land, a capital input in fixed supply, which therefore earns rents. The choice of the corporate sector ensures that securities are continually priced to reflect accurately new information regarding the value of the capital stock. This would not be true for the installed capital of unincorporated businesses. Another reason to focus on this sector is dictated by the fact that the farming sector, the non-corporate sector and the financial sector suffer from data quality problems. The use of the non-farm, non-financial corporate sector is not restrictive given that this sector owns around 90% of the non-residential fixed capital stock in the economy.

In terms of the needs of this study, the NBSA data suffers from two limitations. First, the data starts in the first quarter of 1990, limiting the determination of the capital stock to the period post-1990. Statistics Canada plans to publish NBSA tables at market value starting in 1970 at yearly frequency. Their release was planned for September 2006 but has been postponed *sine die*.

The second limitation relates to the composition of the equity data. The valuation of equity at market value in the NBSA is made difficult by the existence of two types of shares: listed (quoted) and unlisted (unquoted). Only listed shares have a market value while unlisted shares are evaluated at book value (Statistics Canada (2004)). As a result, the reported value of corporate shares in the NBSA does not price all categories of equity present in the balance sheet of corporations at market value. I report in the Appendix the

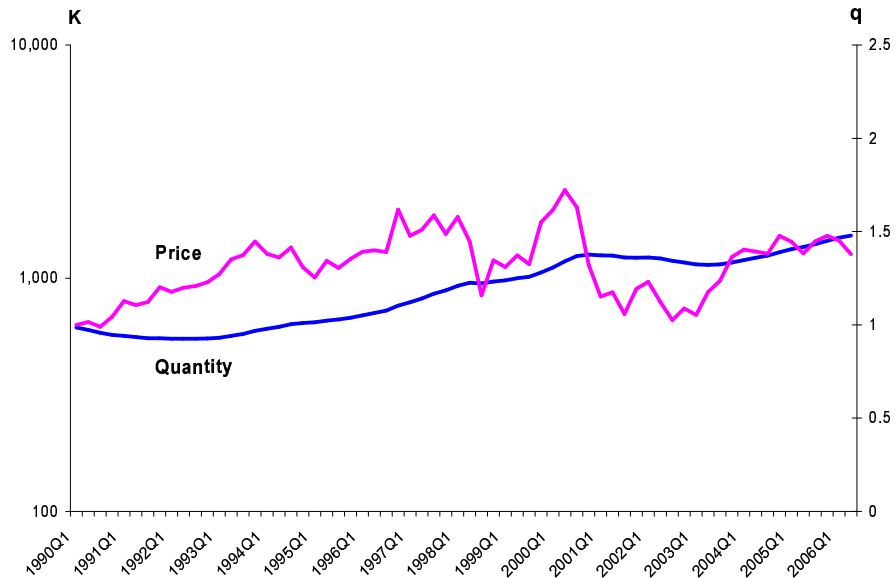


Figure 2: Quantity and Shadow Price of the Inferred Capital Stock

ensuing data series and I explain how I convert the book value data into market value. A full description of data sources and data manipulations can also be found in the Appendix.

5 Results

Figure 2 shows the solution to the recursive system with the breakdown of the value of the installed capital v_t into a shadow price q_t , represented on the right axis, and an inferred stock of capital k_t , represented on the left axis in log scale. The figure shows a smoothly increasing inferred capital stock in the economy from 1990 to 2006. The shadow price of capital is constantly above one, a finding that is representative of many calculations of the q in the literature ⁹.

Figure 3 shows the breakdown of the components of the aggregate value of firms. The difference between the value of securities v and the inferred quantity of capital is reflects the price q . The shadow price is the variable that absorbs all the volatility in securities values. The figure also uncovers the size of the unrecorded capital stock (intangibles) by subtracting the recorded capital stock in national income accounts from the inferred quantity of capital. Note that there is a smooth pattern of increase of the stock of intangibles from 1992 to 2001.

⁹See Laitner and Stolyarov (2003) and the references therein.

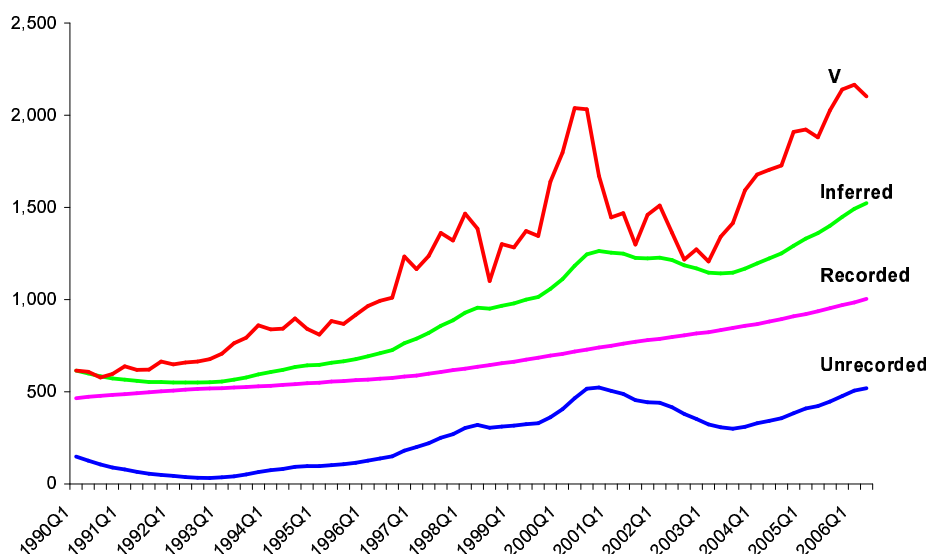


Figure 3: Decomposition of the Aggregate Value of Firms (in billions of 1997 dollars)

After 2001, this stock falls to a level comparable to the one in 1998 and increases back after 2003.

The ratio of intangibles to the inferred capital stock averages 29% from 1994 onwards with a standard deviation of 7.9%. The behavior of this ratio is depicted in Figure 4. The relative size of intangible capital increases with the rise of the value of securities in the late 90s. It grows from a proportion of 12% in 1994 to a proportion of 41% at the peak of the value of securities in the last quarter of 2000. With the fall in the value of securities in 2001, the relative stock of intangible capital falls to reach a proportion of 33% by the end of 2003. The recent rise in the value of securities is once again accompanied by a rise of the stock of intangible capital.

The coincidence in the rise of Canadian securities values with the accumulation of intangible capital reflects the slower pace at which the accumulation of the recorded capital proceeds relative to the pace at which the value of securities rise. The same process works in opposite direction in the case of a fall in the value of securities. The relative rise of intangible capital in the overall capital stock reflects the increasing reliance of companies on knowledge capital. This is viewed as a consequence of the information-technology revolution that started in the 70s (Greenwood and Jovanovic (1999), Hobijn and Jovanovic (2001)).

The proportion of intangibles in the overall capital stock is smaller than the one found

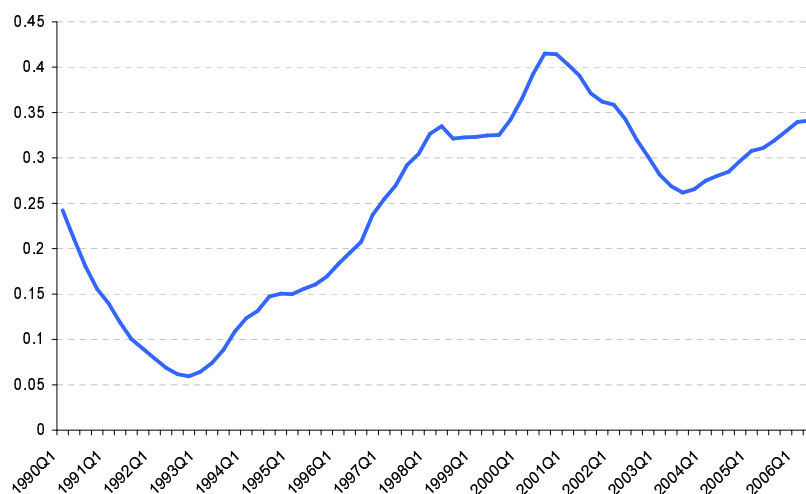


Figure 4: Ratio of Intangible Capital Stock to the Inferred Capital Stock

in Hall (2001) for the U.S. and bigger than the one found in Eliades and Wecken (2004) for the U.K. Both papers document a sharp rise in the proportion of intangibles accompanying a run up in the value of securities in the late 90s. Although Figure 3 shows a comparable rise for Canada, the sharpness of this increase is actually about half as large as that of the U.S. The share of intangible capital in the U.S. by the end of 2000 constituted half of the inferred stock of capital. It is important to note that an extension of the findings of Hall (2001) to the year 2005 shows a collapse in the relative size of intangibles to levels slightly below the ones found for Canada in 2005¹⁰. Both countries experienced the same qualitative behavior and are seemingly heading to a similar steady-state ratio.

Next, the nature of the unrecorded capital stock is explored. This information will also help estimate the size of the capital stock for which national income accounts do not collect any data.

As mentioned in the introduction, there are no available data from national income accounts on the investment of firms on different types of intangible capital goods. Statistics Canada collects only data on R&D expenditures. The stock of R&D is calculated using the perpetual inventory method and is compared with the size of the unrecorded capital. The procedure is detailed in the Appendix and the results are shown in Figure 5. This figure depicts the evolution of the R&D stock together with the evolution of the unrecorded capital

¹⁰These results can be obtained from the author upon request. No such exercise was conducted for the U.K.

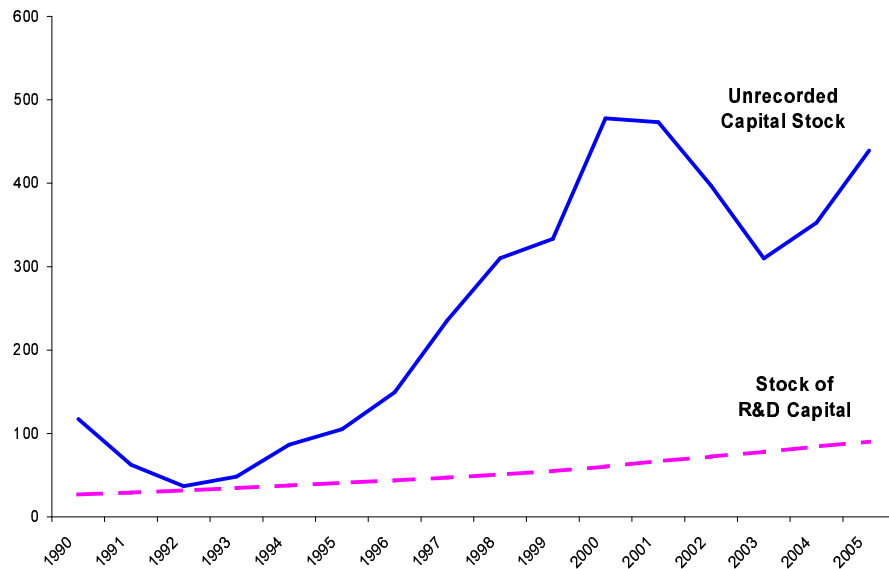


Figure 5: The Part of R&D in the Unrecorded Intangible Capital Stock (in billions)

stock. The ratio of the R&D stock to the unrecorded capital stock falls from a level of 43% in 1994 to a level of 12% by 2000. This proportion grows afterwards to reach an average of 20% between 2001 and 2005. This trend shows that the composition of the intangible capital stock shifted towards less R&D capital goods.

About 23% of the size of the unrecorded capital is made of R&D capital since 1994 on average. The rest would be made of the accumulated stock of capital that resulted from expenditures on training, organizational change, advertising and any expenditure which is intended to increase future production. Since only 23% of the 29% of unrecorded capital can be explained, we conclude that Statistics Canada misses about 26% of the overall productive capital stock in its data collection. This represented about \$380 billion in the last quarter of 2005.

It is possible to compare the results of this approach with the findings in Belhocine (2008) who uses a direct expenditure approach to measure intangible investment in Canada. Belhocine (2008) found that the growth of the investment of firms in intangibles between 1998 and 2005 averaged 6.6% per year. On the other hand, backing out the investment in intangibles from the inferred stock of intangible capital documented above shows that the similar growth rate is equal to 7.15%. The absolute numbers are hard to compare because of the extreme volatility which characterizes the investment in intangibles that is backed out

from the value of securities. However, the average yearly stock of intangible capital implied by the results of Belhocine (2008) for the period considered is about \$365 billions¹¹. This number is comparable to the average inferred stock of intangibles per year of \$387 billions found in this paper. This cross-checking of the findings is reassuring.

6 Sensitivity Analysis

Three checks for robustness are conducted in this section. It is found that for a range of depreciation rates, starting values and adjustment costs parameters, the qualitative results pertaining to the behavior and to the relative size of intangible capital stock are unchanged. The size of the stock of intangibles does naturally vary depending on the range of investment adjustment cost and depreciation rates considered.

6.1 Allowing for Different Adjustment Cost Parameters

First, the experiment of removing the asymmetry assumption in the adjustment cost parameters by allowing $\alpha^- = \alpha^+ = 8$ is conducted. The quantity of intangibles and the corresponding q in the case of symmetry are superimposed on the values obtained with the baseline model. This result reflects the absence of negative net investment *i.e.*, gross investment has always, at the very least, kept up with depreciation. In other words, no instances of decline in the firms' value have ever provoked discarding of capital. Therefore, the assumption on the relationship between the upward and the downward adjustment cost parameters is innocuous.

No studies were found for Canada that attempted to estimate the adjustment-cost parameter involved in increasing the amount of installed capital. In the baseline calibration exercise, the upward adjustment cost parameter was set to match the finding of Shapiro (1986). Hall (2004) followed essentially the procedure in Shapiro (1986), elaborating on the econometrics and using annual manufacturing industry data rather than aggregate data. He found even smaller adjustment cost parameters, in the range reported by Cooper and

¹¹The procedure to calculate the stock of intangibles is based on the perpetual inventory method. This approach is similar to the one used to calculate the stock of R&D capital described in the Appendix. The depreciation rate used is also the same.

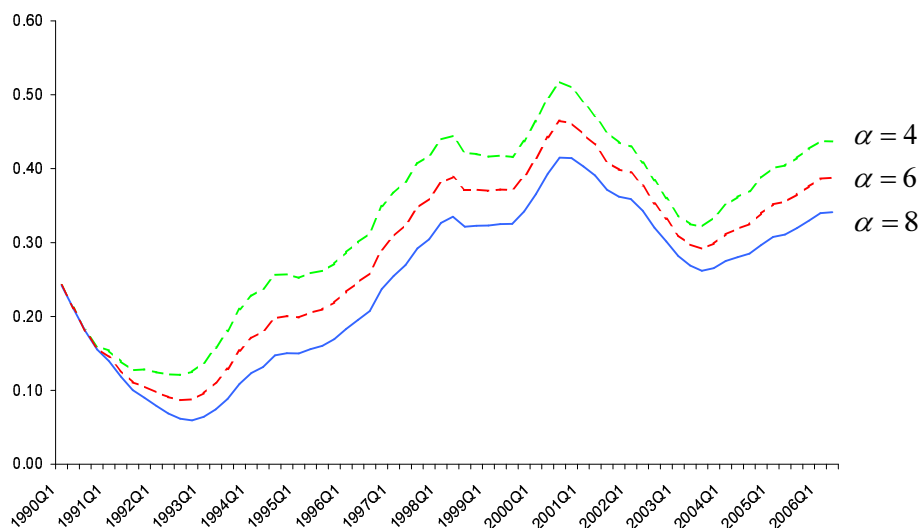


Figure 6: Sensitivity to Various Adjustment Costs of the Ratio of Intangible Capital to the Inferred Capital Stock

Haltiwanger (2006) for plant level data. Groth and Khan (2007) confirm the results of Hall (2004) when allowing for investment adjustment costs on top of the capital adjustment costs. Following these findings, lower adjustment costs were considered in the sensitivity analysis: adjustment costs of $\alpha^+ = 6$ and $\alpha^+ = 4$ corresponding respectively to an adjustment period of capital following a shock of a year and half and a year. The implications on the implied ratio of intangibles to tangible capital levels are shown in Figure 6.

As the adjustment cost parameter decreases, the implied capital stock will increase given that the firm can now install capital at a lower cost. In particular, halving the adjustment time will increase the ratio of intangible capital stock to an average of 38% for the period considered. The high estimate that was assumed in setting $\alpha^+ = 8$ is viewed appropriate at this stage of the evidence and research on intangibles. Indeed, little is known about the exact size of the stock of intangible capital and its relative size in the overall capital stock, which warrants a conservative approach. Further research will provide more guidance on this matter.

6.2 Allowing for Different Depreciation Rates

There is no information on the depreciation rate of intangible capital in general. However, estimates of the depreciation rate for R&D capital are available. The Bureau of Economic

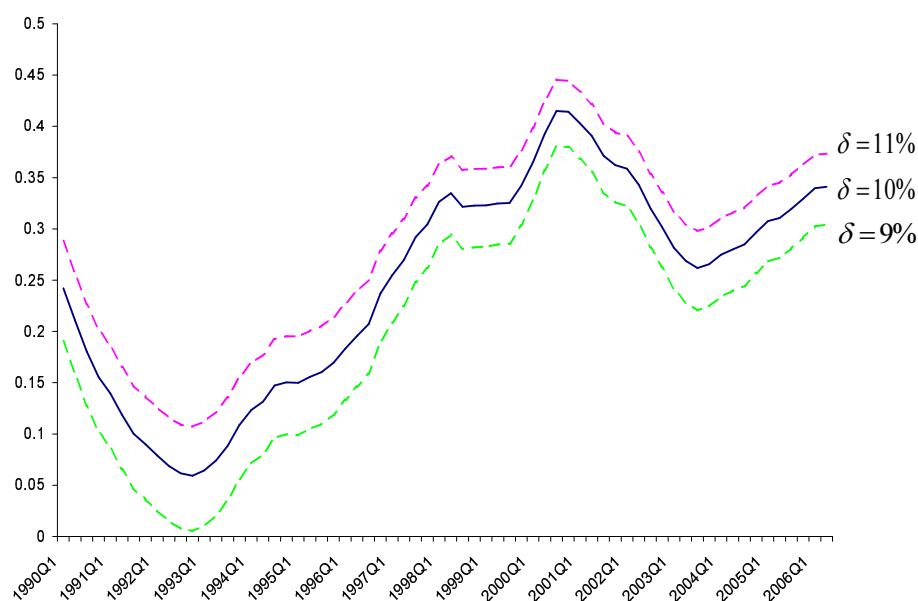


Figure 7: Sensitivity of the Ratio of Intangible Capital to the Overall Capital Stock to Various Depreciation Rates

Analysis (1994) estimates the depreciation rate of R&D to be around 11% while the Bureau of Labor Statistics (1989) uses a rate of 10%. Adams (1990) calculated an annual depreciation rate for basic research of between 9% to 13%, while Nadiri and Prucha (1996) estimated a depreciation rate of 12% for industrial R&D. No similar work was found documenting such rates for Canada. Given that R&D was found to compose only 23% of the overall intangible capital stock, the depreciation rate to use for the overall intangible capital is not obvious.

Nevertheless, the depreciation rate was allowed to vary between 11% and 9% in accordance with the only evidence available that was just cited. With a depreciation rate of 11%, the ratio of intangible capital stock to the overall level of capital increases by 3 percentage points to a level of 32%. Increasing the depreciation rate lowers the inferred capital stock but lowers even more the physical capital stock which nets out to an increase in the intangible capital stock. With a depreciation rate of 9%, the intangible capital stock averages 25% of the overall capital stock. These bounds are shown in Figure 7.

The form of the theoretical law of motion of the capital stock from the model does not allow the use of a different depreciation rate for the physical capital stock and the intangible capital. In light of what the aforementioned statistical agencies use, the physical depreciation rate of 10% applied implicitly on intangibles is ultimately a reasonable level to consider.

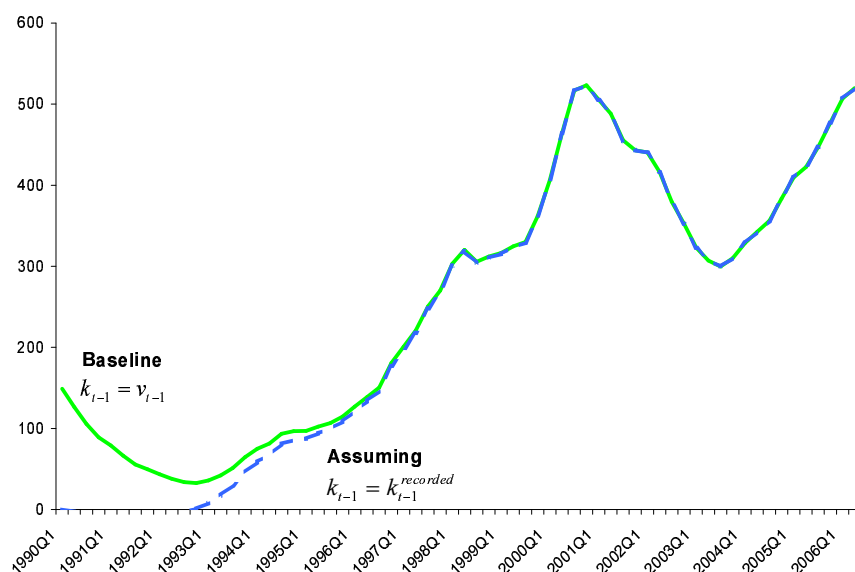


Figure 8: Sensitivity of the Intangible Capital Stock to Various Initial Values

6.3 Allowing for Different Initial Values

In the baseline calibration, the initial capital stock k_{t-1} was set to equal the level that prevails when the system is in equilibrium at $t - 1$. This corresponds to v_{t-1} or a level equal to \$614 billion. Here, the initial capital stock is set to equal the recorded capital stock. In other words, we assume that at $t - 1$, the stock of intangibles is zero. The level of the recorded capital stock is now initially equal to \$466 billion. Hence, the initial starting value is lower than in the baseline model. Figure 8 shows the implied intangible level of capital for both scenarios. We can see that despite the sizeable difference in starting values, the implied level of intangibles converges to a common value by 1996, with a similar and almost equal values starting in 1994. Hence, the pre-1994 downward trend in intangible levels is not accurate and the estimates are afterwards robust to the choice of a starting value.

7 Conclusion

In this paper, newly released Canadian data on the aggregate market value of all securities in the economy was used to calculate the size of the intangible capital stock. It was found that the stock of unrecorded capital was about 29% of the overall capital since 1994. The accumulation of intangible capital played a bigger role in the rise of the capital stock in the late 90s than the accumulation of physical capital. This relative rise coincided with the increase in the value of securities. Similar studies conducted for the U.S. and the U.K. reached similar conclusions on the increasing prevalence of intangible capital. The nature of the intangible capital stock was shown to consist of 23% R&D capital. The composition of intangible capital shifted towards less R&D capital goods overtime. Since Statistics Canada collects data only on R&D capital, it misses about 26% of the overall capital stock in its data collection. This chapter provided a sense of how much national income accounts underestimated the stock of capital in times of a shift in the form of capital owned by businesses.

There are many promising avenues for future research. For example, it would be desirable to extend the model to account for differences that are inherent in the two capital goods. For example, the model could be enriched by explicitly accounting for the non-rival nature of most intangible goods and allow for the existence of spillovers. The perfectly competitive structure and the non-existence of externalities that the model assumed did not allow for such possibilities. Another avenue that needs further exploration is the impact of the presence of rents once imperfect competition is introduced. Part of the value of the firm will then include a portion that is made up of rents. If this portion is large then the baseline model could be confusing some of the capital accumulation for rent accumulation with the consequence of underestimating the shadow price of capital. An analogous implication would be the overestimation of the quantity of capital. I plan to address this issue by extending the framework of this paper to accommodate for the existence of rents and measure their economic importance. Finally, the future extension of the data for the period pre-1990 by Statistics Canada will be very valuable as it will open the door to computing a longer series of intangibles and explore the consequence of the existence and behavior of intangible capital on the measures of productivity performance of the Canadian economy.

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Appendix: Data sources and manipulations

The procedure to insure that all shares of the non-farm non-financial corporate sector are in market value is as follows. First, I obtain the market value of all listed shares from the World Federations of Exchanges website (<http://www.world-exchanges.org/WFE/>). This series represents the market capitalization of the TSX and is available starting January 1995 under the heading “statistics/monthly”. I extend the series backward to 1990 using the S&P/TSX composite index obtained from CANSIM (series label V122620). Second, I take the level of shares at book value and market value of all corporations from the NBSA (respectively the labels V20682659 and V28368658). I obtain the value of unlisted shares at book value by subtracting the market capitalization of the TSX from the level of shares of all corporations at market value. I then obtain the value of listed shares at book value by subtracting from the shares of all corporations at book value the unlisted shares portion just calculated. Finally, I construct a price index by dividing the listed shares at market value by the listed shares at book value. I then use this price index to inflate the book value series of the non-farm non-financial corporate sector. I use it on both the book value series of liabilities and assets of this sector (given respectively by the labels V20682692 and V20682673). These two data series are reported in Table 4.

The nominal net market value of securities is obtained by subtracting the value of liabilities series from the value of assets. This amount is deflated by the investment deflator which is obtained by dividing the gross nominal private investment series (V498927) by the gross real private investment (V1992271). The resulting series is the variable v_t .

The series for the recorded capital stock is calculated by cumulating overtime the quarterly investments in fixed capital by the non-farm, non-financial corporate sector taken from the FFA (V34914) while removing at the same time the depreciated part at each quarter assuming a depreciation rate of 10%. This investment in fixed capital is deflated each quarter by a deflator obtained from dividing the nominal value of non-residential and equipment investment series (V498929) by the real value of non-residential and equipment investment (V1992273) both taken from the NIEA. The initial level of the capital stock is taken to be the one given in the last quarter of 1961 made of the the summation of the stock of machinery and equipment (V33216), the stock of non-residential capital (V33215), all taken from the

Quarters	Market value of liabilities	Market value of financial assets	Quarters	Market value of liabilities	Market value of financial assets
1990Q1	1,069,347	461,118	1998Q2	2,761,116	1,356,233
1990Q2	1,057,895	452,767	1998Q3	2,224,033	1,104,446
1990Q3	1,006,034	432,616	1998Q4	2,633,052	1,311,218
1990Q4	1,019,384	435,703	1999Q1	2,601,037	1,305,822
1991Q1	1,062,623	462,979	1999Q2	2,809,568	1,427,221
1991Q2	1,060,513	460,003	1999Q3	2,770,789	1,413,242
1991Q3	1,060,993	458,014	1999Q4	3,343,751	1,703,599
1991Q4	1,101,445	472,715	2000Q1	3,691,009	1,891,020
1992Q1	1,091,813	479,557	2000Q2	4,256,924	2,181,759
1992Q2	1,114,047	491,099	2000Q3	4,220,281	2,161,963
1992Q3	1,117,999	493,858	2000Q4	3,504,809	1,807,549
1992Q4	1,157,615	513,798	2001Q1	3,031,529	1,560,048
1993Q1	1,227,042	548,535	2001Q2	3,167,348	1,640,818
1993Q2	1,323,151	587,364	2001Q3	2,781,554	1,439,411
1993Q3	1,392,012	628,004	2001Q4	3,151,825	1,646,010
1993Q4	1,531,017	693,552	2002Q1	3,286,813	1,705,871
1994Q1	1,528,409	699,204	2002Q2	3,030,176	1,591,654
1994Q2	1,546,083	705,558	2002Q3	2,701,386	1,419,164
1994Q3	1,656,469	764,850	2002Q4	2,831,656	1,497,154
1994Q4	1,572,813	736,239	2003Q1	2,648,691	1,392,792
1995Q1	1,520,312	707,575	2003Q2	2,940,932	1,545,696
1995Q2	1,672,590	787,231	2003Q3	3,120,966	1,654,865
1995Q3	1,640,113	776,379	2003Q4	3,563,604	1,935,844
1995Q4	1,735,100	828,036	2004Q1	3,823,860	2,102,784
1996Q1	1,814,899	867,267	2004Q2	4,126,673	2,300,326
1996Q2	1,914,776	923,999	2004Q3	4,153,015	2,317,611
1996Q3	1,961,333	954,634	2004Q4	4,450,058	2,487,784
1996Q4	2,347,897	1,132,167	2005Q1	4,574,016	2,597,895
1997Q1	2,265,592	1,111,234	2005Q2	4,697,921	2,687,315
1997Q2	2,463,546	1,221,725	2005Q3	5,053,895	2,910,910
1997Q3	2,730,379	1,367,555	2005Q4	5,254,254	3,042,614
1997Q4	2,582,517	1,258,609	2006Q1	5,342,077	3,109,073
1998Q1	2,876,877	1,398,559	2006Q2	5,341,878	3,104,318

Table 4: Financial Assets and Liabilities at Market Value

NBSA of the non-farm, non-financial corporate sector.

Statistics Canada does not collect data on the R&D expenditure incurred by the non-farm, non-financial corporate sector. The closest data available for the needs of this study is the nominal R&D business enterprise expenditure (V617324) collected at yearly frequency. To calculate the R&D investment of the non-farm, non-financial corporate sector, the relative investment amounts of this sector in the overall business investment was computed and used to scale the R&D expenditures data accordingly. The relative investment of the non-farm, non-financial sector at each quarter was calculated by dividing the investment in fixed capital of the non-farm, non-financial corporate sector taken from the FFA (V34914) by the investment in fixed capital by all businesses (V498929) taken from the NIEA. Once the R&D portion of this sector was obtained, it was deflated by the same deflator used for the stock of recorded capital, at yearly basis. Finally, the stock of R&D capital was calculated using the perpetual inventory method¹²:

$$R_t = I_t^{R\&D} + (1 - \delta)R_{t-1}$$

where R_t is the stock of R&D at time t , $I_t^{R\&D}$ is the investment flow in R&D at time t and δ is the depreciation rate set equal to 10%. The initial stock of R&D in 1963 was calculated using the formula obtained by backward recursive substitution of the above equation:

$$R_{1963} = \left(\frac{1 + g}{\delta + g} \right) I_{1963}^{R\&D}$$

where g is the historical average growth rate of R&D expenditures. It is assumed that preceding the initial observation, there was a long period of constant investment growth in R&D of g which is set equal to the average growth rate for the period 1963-2005. In any case, the stock of R&D obtained in 1990 is not sensitive to these initial starting point assumptions given the small magnitude of investment in R&D in the early 60s.

¹²There is a long tradition, prominently described in Griliches (1979), of calculating the stock of R&D capital as a weighted sum of past expenditures in R&D.

