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Long-term information, short-lived derivative securities

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Abstract

This paper characterizes how the short-lived nature of derivative securities, such as option and futures contracts, affects trading behavior and pricing. Specifically, the paper explores strategic trade in short-lived derivative securities by agents that possess long-term information about an underlying asset. In contrast to trading equity, where an informed agent will ultimately benefit from his trades, trading short-lived securities is profitable only if the private information is impounded in the price before expiry. A consequence is that a risk neutral informed agent's holdings of the short-lived security affects his trading behavior: Past informed trading leads to greater future informed trading. The shorter horizon in which information must be impounded for a short-lived security to pay off makes an informed agent more reluctant to trade at earlier dates. By characterizing the conditions under which liquidity traders choose to incur extra costs to roll over their short-term positions rather than trade in distant contracts, this paper provides a possible explanation for why most markets for longer-term derivative securities have little liquidity and large bid-ask spreads. The paper also shows how fixed trading costs and round lot restrictions can lead to multiple equilibria.

This paper explores strategic trading by agents who have long-term information about an underlying asset but trade short-lived derivative products. To illustrate the issues involved, consider the following hypothetical example. Suppose that in 1995, an individual becomes aware that most computer databases cannot properly recognize the year 2000 (the "Y2K computer bug"). The general public has not yet become aware of the magnitude of the problem and, as such, this information has not been reflected in the stock price of a database management company, BigRed. The individual, who has limited funds, would like to trade on his "private" information.

Although the information will be revealed with certainty on January 1, 2000, the company may make a public announcement prior to this date. The individual, the "informed agent", can engage in at least two possible trading strategies. One strategy is to shortsell BigRed. An advantage of shortselling is that as long as the informed agent can maintain his position, his information will eventually be incorporated into the share price. A possible disadvantage is that a temporary rise in the share price might expose the informed agent to large losses and force him to close his position. An alternative is to purchase put options for BigRed. Put options allow the informed agent to leverage his position and limit downside risk. Unfortunately for the informed agent, his private information may not be revealed publicly by the time the option expires, and hence may not be impounded in the price, in which case the option will not pay off. Since the informed agent will have the opportunity to purchase newly-issued options with later expiration dates as time passes, the question becomes when, and how frequently, should the informed agent purchase options? This paper addresses these issues.

When the informed agent purchases a derivative security, he conveys information to the market, *even* if the contract does not pay off, thereby adversely affecting the price of future contracts. We show that these strategic information costs are higher for derivative securities than for equity, and more generally for shorter-term contracts than longer-term contracts. This is because, in contrast to trading equity, where an informed agent will ultimately benefit from his trades, trading short-term derivative securities is only profitable if the private information is impounded in the price before expiry. Ignoring leverage concerns, this makes an informed agent more reluctant to trade immediately in short-term derivative securities than in equity. Indeed, an informed agent may not act immediately on his private information even if it is profitable in expectation to do so, preferring to defer in the hope of obtaining better prices in the future. The attendant risk with this strategy is that the information may be revealed publicly before he can exploit his information.

In further contrast to equity, a risk neutral informed agent's holding of the short-lived security affects his trading behavior: Higher past informed trading leads to greater future informed trading. This is because the larger is the informed agent's holding of the short-lived security, the less he minds if his additional trading reveals his information, since he will profit on his existing stake. In contrast, with equity, as long as the informed agent does not have to close his position, he will eventually profit on his existing stake, so his holdings do not affect his trading strategy. So, too, the time-to-expiry of the short-lived derivative affects trading intensities. We provide innocuous sufficient conditions for the informed's trading intensity to rise as the time-to-expiry approaches, so that prices become more sensitive to order flow.

Our model also provides insights into a puzzling empirical feature of derivative security markets — despite the existence of a widespread desire to hedge against long-term risk, longer-term option and futures contracts have little liquidity and large spreads. For example, Fleming and Sarkar (1999) found that 90% of total trading volume in Treasury Futures in 1993 occurred in the *nearby* contract (the contract with expiration month closest to the trading date), and that distant contracts generally had far *lower trading volumes* and *larger realized spreads*. The following quote from an energy industry newsletter describes the issue:

> The main dilemma is that many students of commodity and financial derivatives are so busy trying to confect econometric masterpieces that they have completely disregarded the basic mechanics of futures trading. I have always told my students that beyond 9 or 10 months, the derivatives market in most commodities generally narrows to the swaps market, with activity in futures and options being reduced to a comparatively low level. This has become common knowledge in New York and Chicago; but at the Washington meeting, we were grandly informed that futures contracts for crude oil now exist with maturities up to 7 years. When I asked one of the gentlemen active in this market if this were true, his reply was that if contracts of this maturity are what producers and consumers of oil naively desire, then The Market will make them available.
>
> What he did not say, however, was that liquidity in such long duration 'paper' markets was almost non-existent, and if at some point late in those 7 years a transactor wanted to close a position, then he or she might have to accept a resounding loss. The advantages presented by copious liquidity (i.e., always being able to trade at or near the last quoted price) are why large traders elect to employ short-dated contracts, although 'rolling over' these contracts poses dangers of its own. ("An Outlook on the Supply of Oil," Ferdinand E. Banks, International Association for Energy Economics, Winter 1998 Newsletter.)

Our paper details conditions under which liquidity traders chose to incur extra costs to roll over their short-term positions, "rolling the hedge", rather than trade distant option and futures contracts. The intuition is that informed agents place a greater value on distant contracts because their information is more likely to be revealed before the contract expires, while liquidity traders value only the reduced roll-over costs. As long as fixed trading costs are slight, the greater adverse selection costs in distant contracts more than offset the greater fixed trading costs, so that long-term liquidity traders prefer to trade nearby contracts. If all liquidity traders prefer the nearby contract, informed agents must also trade it.

Our exposition considers an agent attempting to leverage up his capital by trading in options. Our model captures a number of other situations where informed agents, with trading horizons extending beyond the date when their information will become public knowledge, trade in markets where they have to realize their position before this date. For example, our model also captures futures trading in the commodities market, where an informed agent cannot readily take an 'equity' position by purchasing the commodity (e.g. soy beans) and where futures contracts with longer time horizons either do not exist or have poor liquidity. So, too, our analysis is relevant for an informed agent who wants to short-sell and faces margin requirements. If the informed agent has limited resources, and the price moves against him, he may have to close his position before his information is incorporated into the price.

Our model incorporates features of the model proposed in Dow and Gorton (1994) in order to consider an informed agent contemplating purchases of a series of call options competitively priced by a market maker who also receives orders from random liquidity traders. We show that the informed agent's trading decisions and profit depend subtly on both the likelihood of liquidity trade and how far the price is expected to diverge from its 'fundamental' value. We then extend the model to consider how the informed agent's strategy depends on his accumulated position and the availability of options of different durations.

The paper is organized as follows. We next detail how this paper relates to previous literature on informed trading in equity. Section 1 sets out the basic model. Section 2 characterizes the informed agent's equilibrium trading strategy and details how outcomes vary with the parameters describing the economy. Section 3 characterizes how the likelihood of liquidity trade affects equilibrium outcomes. It shows that multiple trading equilibria can arise even in a simple two period context. If the informed agent's private information is slight, then he prefers to defer from trading, hoping, instead to manipulate favorably market maker beliefs. In this case, he values a high likelihood of liquidity trade. In contrast, if the informed agent's private information diverges more from expectations, then he prefers that liquidity trade be less likely. Consequently, expected trading profits need not rise with the volatility of liquidity trade. Section 4 explores how the informed agent's accumulated position affects his trading strategy. An informed agent with an accumulated position takes into consideration that submitting an order this period increases the probability that his information will be revealed and that his previously purchased option will then be exercised at a profit. Section 5 extends the analysis to consider: (i) longer holding periods; longer-lived liquidity traders; and (ii) the choice between trading derivative securities with different holding periods. We characterize how the time until expiry affects trading behavior, and show that when agents can choose whether to trade long- or short-term derivatives, then for reasonable parameterizations of the economy, only shorter-term derivatives are traded. Section 6 concludes.

Related Literature: Our model shares features of the overlapping generations model proposed by Dow and Gorton (1994). Dow and Gorton endogenize the choice to act on private information made by informed agents with short-horizons and *one* trading opportunity. In their model, information is only revealed through trade. Hence, an informed agent trades only if he believes it sufficiently likely that a future informed agent will trade before he has to realize his position, so that prices will reflect his information. Therefore, an 'arbitrage chain' only commences so far in advance of the event.

Because Dow and Gorton allow an informed agent only one opportunity to trade, their model does not capture the essence of repeatedly trading short-lived derivative securities written on a longer-lived asset about which informed agents have private information. In particular, their informed agents do not internalize the effects of current trade on future trading opportunities, so that immediate trade is more attractive and private information becomes public sooner.

In another model with short trading horizons, De Long *et al.* (1990) offer a rationale for why risk averse arbitrageurs may have a limited willingness to bet against noise traders: a concern that noise trader beliefs may not revert to the true value for a long time, but may instead deviate further away. Even if there is no 'fundamental' risk associated with purchasing 'under-priced' stocks, 'arbitrage' may be imperfect because of this 'noise trader' risk.

Treynor and Ferguson (1985) highlight the issues explored in our paper. They discuss an investor who receives what he believes to be nonpublic information about a security. They observe that to use this information effectively, the investor must be concerned with: (i) the value of the information in terms of its price impact; (ii) the process by which information is transmitted to the market; (iii) the probability the market will receive the information at a particular time; and (iv) a portfolio strategy which permits capitalizing on the information. These issues are central for an investor who must determine how best to exploit his long-term information when trading short-lived securities, and underlie our key modeling assumptions.

Models of strategic informed trade in options markets include Back (1993), Biais and Hillion (1994), and John *et al.* (2000). Back integrates a long-lived call option into a continuous time Kyle model. Biais and Hillion consider a static model in which a single trader (either informed or a rational, risk averse liquidity trader) chooses whether to trade the stock or the option. Adding the opportunity to trade options may reduce informed profits because of the effects on liquidity trading strategies. Also in a static model, John *et al.* explore how margin requirements affect the choice by informed agents of which asset to trade.

Our model also contrasts with models of informed trading in equity (*e.g.*, Kyle (1985), Back *et al.* (1999), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996)) where informed agents have multiple opportunities to trade a long-lived asset, but do not need to realize positions before their information becomes public. As in our setting, informed agents care about both current and future prices, but because they do not have to unwind positions there is no penalty to investing early; eventually stock prices reflect all private information.

1 The Basic Model

Consider a multi-period economy with two primitive assets: a safe asset and a risky asset. The risky asset may pay a one-time dividend at some distant date, T. At date T, the value of the underlying risky asset is $D_0 \in \{B, G\}$, where $D_0 = B$ in the bad state, $D_0 = G$ in the good state, and $G > B$. The *ex ante* probability that the good state occurs is κ. Let $i = T - t$ denote the number of periods *remaining* until date T: Period i corresponds to the date with i periods remaining until date T.

We consider a trading environment with three types of market participants: (1) a risk neutral, competitive market maker; (2) short-lived liquidity traders who arrive randomly; and (3) an informed agent. With probability δ, an informed agent has private information about the risky asset: he knows whether the realization was G or B. The true future state may be revealed to the public in period i either by the equilibrium trading order flow (in a manner detailed below), or if the information leaks out. The probability the information leaks out to the public exogenously in period i is λ_i. As date T approaches, the asset's value is more likely to be revealed to the public, $\lambda_i \leq \lambda_{i-1}$. Let $a_i \in \{b, n, g\}$ reflect whether bad news (b), no news (n), or good news (g) leaked out at period i.

In addition to the risky asset, our economy has two other types of assets. The first asset type is riskless with return $r = 0$.The second asset type is a derivative based on the underlying risky asset. Each derivative security is short term, existing only for one period. This assumption permits us to capture the essence of trading short-lived derivative securities written on longer-lived assets, while circumventing the need to model trade in multiple markets at each date. Section 4 extends the model to consider longer-term derivative securities.

Underlying risky asset: Each period, the market maker sets the prices of the underlying risky asset and the derivative security. We first assume that informed agents choose not to trade the underlying risky asset directly. This decision can be motivated by the informed agent's desire to maximize their leverage or by practical considerations associated with delivery of commodities. Since order flow in the underlying asset does not provide the market maker information, the model permits us to focus on trade in the derivative asset market.

Competitive pricing ensures that the current price of the underlying risky asset is equal to the market maker's expectation of its value at date T (period 0). In other words, the basis between the current spot price and the expected future price of the underlying asset is assumed to be zero. This assumption simplifies the analysis by allowing us to focus on adverse selection considerations without complications arising from basis risk. At the beginning of period i, given current and past order flows, the market maker assigns a probability β_i^o to the good state occurring at date T. The price of the underlying risky asset at the beginning of period i is then set at $D_i^o = \beta_i^o G + (1 - \beta_i^o)B$. At the end of period i, the market maker updates her belief about the good state to β_i^c to reflect the possible leak of the private information during the period. At the end of period i, the price of the underlying risky asset is $D_i^c = \beta_i^c G + (1 - \beta_i^c)B$.

All agents have a sufficient endowment of money, $M \geq 1$, that they are not wealth constrained in equilibrium. Agents incur a transaction cost of $c \geq 0$ when buying or selling the derivative asset. This fixed trading cost represents brokerage fees plus time costs, etc. We assume that agents can only submit orders in round (*i.e.* integer) lots; this assumption never impacts on equilibrium outcomes, and hence is without loss of generality.

This environment can be used to investigate a variety of derivative types. For clarity, we consider derivatives with simple payoff structures that ease algebraic derivations. The derivative can be interpreted as a European binary call option or as a futures contract:

European Binary Call Option: The "derivative" asset can be interpreted as a one-period European binary call option written on the value of the underlying risky asset. The binary option is available for purchase at the beginning of each period i at price p_i^o and expires at the end of period i for a payoff of p_i^c. The payoff at expiration to a European binary call option that matures at the end of period i with strike price K is:

$$p_i^c(D_i^c; K) = \begin{cases} 1 & \text{if } D_i^c \geq K \\ 0 & \text{if } D_i^c < K \end{cases}$$

The strike price for the binary call option is selected such that the option is in-the-money if and only if the good state is revealed. Specifically, the strike price is selected such that $K = G > \kappa G + (1 - \kappa)B > B,\ \forall \kappa \in (0, 1)$. We focus on binary options for the same reason that Dow and Gorton (1994) consider assets that pay off zero or one — they reduce algebra and the qualitative predictions extend to more general options.

At the beginning of period i, the market maker assigns a probability β_i^o to the good state occurring at date T given the current and past order flows that she observes. She then prices the option sold at the beginning of period i at

$$p_i^o(D_i^o; K) = \text{prob}[D_i^c \geq K | D_i^o] = \begin{cases} 1 & \text{if } \beta_i^o = 1 \\ \beta_i^o \lambda_i & \text{if } \beta_i^o < 1 \end{cases} . \tag{1}$$

Futures contract: The derivative security can also be interpreted as a futures contract to purchase the underlying risky asset. We first consider futures contracts with a time-to-maturity of one period. Let F_i^o be the price of a futures contract written at the beginning of period i that matures at the end of period i: $F_i^o(D_i^o) = E[D_i^c | D_i^o] = D_i^o$. At the end of period i, the offsetting position is taken at a cost of $-F_i^c$, where $F_i^c = D_i^c$, resulting in a profit of $D_i^c - F_i^o$. New futures contracts are available for purchase at the beginning of each period.

The analysis using futures contracts is equivalent to the analysis using one period binary options (most transparently so when $G = 1$ and $B = 0$). For the most part, however, we will refer to the derivative security using option terminology.

For simplicity, the basic model only considers the "buy" side of the market.[1] As a result, the subgame equilibrium when the bad state occurs just has the informed agent declining to place an order. We later discuss how incorporating a non-trivial "sell" side of the market affects outcomes. Until then, without loss of generality, we focus on the case where the good state occurs so that the informed agent may trade.

Short-lived liquidity traders: Each period i, a short-lived liquidity trader enters the market with probability η to place a buy order of size one for the derivative security. The liquidity

[1] Our modeling assumptions can be motivated by Easley *et al.* (1997). They document empirically that (i) option markets are a key venue for information-based trading; and (ii) it is important to distinguish between 'positive' and 'negative' news option volumes.

trader is uninformed and trades only once. With probability $1 - \eta$, no short-lived liquidity trader trades in period i. Let $N_i^S \in \{0, 1\}$ represent the order of the short-lived liquidity trader at period i for the short-lived derivative security. This stark contrast between high noise trading (one order) and low noise trading (no orders) allows the model to capture as simply as possible the feature that with positive probability the market maker will detect the informed agent's presence in the market. While we do not offer a rationale for the noise traders, one can endogenize the behavior of the noise traders along the lines of Dow and Gorton (1994). They assume that the risky asset pays a stream of serially uncorrelated dividends that are negatively correlated with an outside wage received in the second period of an uninformed agent's two period life so that the stock represents a means of hedging.

Informed agent: There is at most one privately informed agent in the market. The informed agent can trade the derivative security as often as he wishes. Of course, given positive transactions costs, he will not want to trade once his information becomes public. The informed agent and the possible liquidity trader submit their orders simultaneously. Trade occurs through a risk neutral market maker who observes the aggregate order flow and sets a price equal to the conditional expected value of the derivative security given her information about the history of order flow and the history of public announcements.

Sequence of Events: The sequence of events during period i is as follows:

1. The market maker enters with a prior β_{i+1}^c that reflects past trade and announcements.

2. The informed agent, having observed past order flows and public announcements, selects a trading probability. With probability η, a liquidity trader also submits an order.

3. The market maker observes total order flow, updates her beliefs to β_i^o, and sets an opening option price of p_i^o.

4. During the period, the informed agent's private information may or may not be revealed. This information is revealed publicly with probability λ_i.

5. At period's end, the market maker updates her prior (β_i^c) to reflect whether the private information was revealed publicly.

6. Outstanding options either are exercised or expire. If the good state was revealed, the informed agent can exercise the options (and receive 1 for each option) or sell them at their closing price, $p_i^c = 1$. If the good state was not revealed, the options expire worthlessly, $p_i^c = 0$.

Equilibrium: Let $S_i = Y_i^S + N_i^S$ represent the total order flow for the short-lived derivative security at period i. Let $\mathbf{H}_i^S = \{S_T, S_{T-1}, ..., S_i\}$ and $\mathbf{A}_i = \{a_T, a_{T-1}, ..., a_i\}$ denote respectively, the histories through period i of past order flows and past public announcements.

The order submission function, $\gamma_i \left(Y_i^S = x_i^S | D_0, \mathbf{H}_{i+1}^S, \mathbf{A}_{i+1} \right)$, is a period strategy for the informed agent, mapping the date T asset value, D_0, and history of order flows and announcements into a probability distribution over the set of feasible individual orders for the short-lived derivative security, $\mathbf{X}^S$, for each period i. A period strategy for the market maker is a pair of pricing functions, $p_i^o(\mathbf{H}_i^S, \mathbf{A}_{i+1})$ and $p_i^c(\mathbf{H}_i^S, \mathbf{A}_i)$, for the open and close of trading respectively,[2] that map the order flow and public announcement histories into prices.

In a sequentially rational (perfect Bayesian) equilibrium: (i) the order submission strategy of the informed agent maximizes (recursively) lifetime expected profits given correct beliefs about pricing functions; and (ii) the pricing function is consistent with the behavior of the informed and earns the market maker zero expected profits conditional on the order flow.

We solve for the equilibrium recursively. If the total order flow in the market exceeds one or is a non-integer quantity, the market maker knows that the informed agent was present. Hence, if the informed agent submits an order, he will try to conceal his trade from the market maker by placing an order of size one. Therefore, in equilibrium, $Y_i^S \in \{0, 1\}$. For simplicity, let γ_i denote the probability that the informed agent submits an order of size one at period i, so that $1 - \gamma_i$ is the probability that the informed agent defers from trading at period i.

Given competitive pricing, in equilibrium, the history of order flow and public announcements through period i can be summarized by the market maker's belief at the end of period i that the good state will occur (β_i^c). Since the informed agent only trades when $D_0 = G$, the informed agent's strategy can be summarized by $\gamma_i(\beta_{i+1}^c)$. In equilibrium, the market maker's pricing strategy at period i can also be summarized by $p_i^o(\beta_{i+1}^c, S_i)$ and $p_i^c(\beta_{i+1}^c, S_i, a_i)$.

We now develop the economy formally. We first derive the beliefs the market maker forms about the probability that the good state will occur. At the beginning of period T, market maker beliefs correspond to the *ex ante* probability that the good state occurs: $\beta_{T+1}^c = \kappa$. At any period $i \leq T$, the market maker will receive either zero, one or two orders. In equilibrium, market maker beliefs will reflect the *equilibrium* trading probabilities of the informed agent.

[2]The pricing functions for futures contracts are denoted by F_i^o and F_i^c.

If there are no orders, the market maker assigns equilibrium probability,

$$\beta_i^o(\beta_{i+1}^c, S_i = 0) = \frac{[1 - \gamma_i(\beta_{i+1}^c)\delta]\beta_{i+1}^c}{1 - \gamma_i(\beta_{i+1}^c)\delta\beta_{i+1}^c} \tag{2}$$

to the good state. This reflects that there are three possible situations in which no orders will be submitted: (i) with probability, $\delta(1-\beta_{i+1}^c)(1-\eta)$, no liquidity trader enters and the informed sees bad news; (ii) with probability, $\delta\beta_{i+1}^c[1 - \gamma_i(\beta_{i+1}^c)](1 - \eta)$, no liquidity trader enters and the informed agent receives good news but refrains from trading; and (iii) with probability $(1 - \delta)(1 - \eta)$, no liquidity trader enters and there is no informed agent.

If there is one order, the market maker assigns equilibrium probability

$$\beta_i^o(\beta_{i+1}^c, S_i = 1) = \frac{(1 - 2\eta)\delta\beta_{i+1}^c\gamma_i(\beta_{i+1}^c) + \beta_{i+1}^c\eta}{(1 - 2\eta)\delta\beta_{i+1}^c\gamma_i(\beta_{i+1}^c) + \eta} \tag{3}$$

to the good state. This reflects that there are four possible situations in which one order will be submitted: (i) with probability $\eta\delta(1-\beta_{i+1}^c)$, a liquidity trader enters, but the informed agent sees bad news; (ii) with probability $\eta\delta\beta_{i+1}^c[1 - \gamma_i(\beta_{i+1}^c)]$, a liquidity trader enters, and the informed agent has good news but refrains from trading; (iii) with probability $\eta(1-\delta)$ a liquidity trader enters and there is no informed agent; and (iv) with probability $(1 - \eta)\delta\beta_{i+1}^c\gamma_i(\beta_{i+1}^c)$, no liquidity traders enter,and the informed agent sees good news and places an order.

Finally, if the market maker observes two orders, she realizes that the liquidity trader entered the market and the informed agent has good news and placed an order, so that

$$\beta_i^o(\beta_{i+1}^c, S_i = 2) = 1. \tag{4}$$

It is important to emphasize that these updating rules reflect the *equilibrium* outcome resulting from the consistency of market maker beliefs and the informed agent's actions. The probability $\beta_i^o(\beta_{i+1}^c, S_i)$ determines the price at which the option is purchased at the beginning of period i. The closing price will reflect whether the true state was revealed publicly. If the information is not revealed, $\beta_i^c = \beta_i^o$. Otherwise, $\beta_i^c = 1$ if the good state is revealed and $\beta_i^c = 0$ if the bad state is revealed. To reduce notation, in what follows we denote β_i^c as simply β_i.

The extreme revision of market maker beliefs in response to an order flow of two is a consequence of restricting attention to only the "buy" side of the market. If market participants can trade on both sides of the market, then the informed agent may sometimes want to trade against his information in order to manipulate market maker beliefs. If so, the market maker revises her beliefs less dramatically in response to an order flow of two. The model's qualitative predictions are otherwise unchanged if market participants can trade on both sides. Hence, in the interest of analytical clarity, we only consider the "buy" side of the market.

2 Informed Agent's Problem

One Opportunity to Trade: To begin, consider an informed agent with just one opportunity to trade. We use this to illustrate the equivalence of the analysis using one-period binary options and futures contracts. If the derivative security is a one-period call option, the expected payoff from submitting an order at the beginning of period i is

$$E[p_i^c|\beta_{i+1}, Y_i = 1] - E[p_i^o|\beta_{i+1}, Y_i = 1] - c = (1-\eta)[\lambda_i - p_i^o(\beta_{i+1}, S_i = 1)] - c. \tag{5}$$

If the derivative security is a futures contract, the expected one-period payoff from submitting an order at the beginning of period i is

$$E[F_i^c|\beta_{i+1}, Y_i = 1] - E[F_i^o|\beta_{i+1}, Y_i = 1] - c = (1-\eta)\lambda_i[1 - F_i^o(\beta_{i+1}, S_i = 1)] - c. \tag{6}$$

Since $F_i^o(\beta_{i+1}, S_i = 1) = \beta_i(\beta_{i+1}, S_i = 1)$ in the futures contract case[3] and $p_i^o(\beta_{i+1}, S_i = 1) = \beta_i(\beta_{i+1}, S_i = 1)\lambda_i$ in the options case, we observe that expressions (5) and (6) are identical: the analysis for the two types of financial assets is equivalent. For simplicity, the remainder of the paper refers to the derivative security as a binary option.

Substituting for $\beta_i(\beta_{i+1}, S_i = 1)$ yields the expected trading profits of an informed agent with one opportunity to trade:

$$(1-\eta)\left[\lambda_i - \frac{(1-2\eta)\delta\beta_{i+1}\gamma_i(\beta_{i+1}) + \beta_{i+1}\eta}{(1-2\eta)\delta\beta_{i+1}\gamma_i(\beta_{i+1}) + \eta}\lambda_i\right] - c = \frac{(1-\eta)\lambda_i\eta(1-\beta_{i+1})}{(1-2\eta)\delta\beta_{i+1}\gamma_i(\beta_{i+1}) + \eta} - c. \tag{7}$$

Multiple Trading Opportunities: Now consider an informed agent with additional trading opportunities. The structure of the economy is such that equilibrium outcomes can be characterized by solving the appropriate specification of the informed agent's dynamic programming problem characterizing his decision in each period of whether or not to submit an order, when we substitute in consistent beliefs of the market maker. The analysis exploits the fact that trading is *less* attractive (prices are higher) when the market maker believes the informed agent is *more* likely to trade. The functional equation can be written as:

$$V_i(\beta_{i+1}) = \max_{Y_i \in \{0,1\}} E\left[\pi_i(\beta_{i+1}, S_i(Y_i)) + V_{i-1}(\beta_i(\beta_{i+1}, S_i(Y_i)))\right],$$

where π_i are period payoffs and $V_0(\cdot) = 0$. In equilibrium, the informed agent's mixed trading strategy (γ_i) will correspond to the probability that the market maker assigns to an informed investor with good news trading this period. If equilibrium is characterized by a mixed trading

[3] Assuming that $G = 1$ and $B = 0$.

strategy at period i, then we can solve for the equilibrium probability by finding the market maker's beliefs about the trading probability implicit in the updating rule, $\beta_i(\beta_{i+1}, S_i)$, that leave the informed agent indifferent between trading and not.

3 Equilibrium outcomes: $\eta = 0.5$

We first characterize equilibrium outcomes analytically when, as in Dow and Gorton (1994), the probability of a liquidity trade is one-half ($\eta = 0.5$). For other liquidity trading probabilities, we characterize outcomes numerically, because analytical (closed-form) solutions for the equilibrium do not obtain. When the probability of liquidity trade is one-half, market maker beliefs about the probability of the good state do not change following an order flow of one: $\beta_i(\beta_{i+1}, S_i = 1; \eta = 0.5) = \beta_{i+1}$.

Period 1: The value of the informed agent's private information with one period remaining until his private information is sure to be revealed publicly is:

$$V_1(\beta_2) = \max\Big[0.5\lambda_1(1-\beta_2) - c, \;\; 0\Big]. \tag{8}$$

The value function is independent of γ_1, so it is easy to compute the informed agent's equilibrium trading probabilities: $\gamma_1(\beta_2) = 0$ if $\beta_2 > 1 - \frac{2c}{\lambda_1}$, and $\gamma_1(\beta_2) = 1$ if $\beta_2 < 1 - \frac{2c}{\lambda_1}$.

Period 2: The value function with two periods remaining is

$$V_2(\beta_3) = 0.5(1-\lambda_2)V_1(\beta_3) + 0.5\max\Big[\lambda_2(1-\beta_3) - 2c, \;\; (1-\lambda_2)V_1(\beta_2(\beta_3, S_2 = 0))\Big]. \tag{9}$$

In equilibrium, the market maker's beliefs must be consistent with the informed agent's (either pure or mixed) strategies. Consistency of the market maker's posterior beliefs with informed trading may require that the informed agent use a mixed trading strategy. This reflects the fact that $\beta_2(\beta_3, S_2 = 0)$ decreases in γ_2. As a result, the market maker's belief about the probability of the good state falls more in response to an order flow of zero when the informed agent has a pure strategy to buy than when he has a pure strategy to sell. The equilibrium is characterized by a non-degenerate mixed trading strategy when:

$$(1-\lambda_2)V_1(\beta_2(\beta_3, S_2 = 0, \gamma_2 = 1)) > \lambda_2(1-\beta_3) - 2c > (1-\lambda_2)V_1(\beta_2(\beta_3, S_2 = 0, \gamma_2 = 0)).$$

If the informed agent mixes in equilibrium, he must be indifferent between trading and not:

$$\lambda_2(1-\beta_3) - 2c = (1-\lambda_2)V_1(\beta_2(\beta_3, S_2 = 0)). \tag{10}$$

Our first result is that the more likely the market maker believes the good state is, the less likely the informed agent is to trade on his (good) private information:

Lemma 1 *The probability that the informed agent with good news trades with two periods remaining, $\gamma_2(\beta_3)$, decreases in β_3; strictly decreasing with β_3 if it is interior, $0 < \gamma_2(\beta_3) < 1$.*

Period i: The value function with i periods remaining is:

$$V_i(\beta_{i+1}) = 0.5(1-\lambda_i)V_{i-1}(\beta_{i+1}) + 0.5\max\left[\lambda_i(1-\beta_{i+1}) - 2c, (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, S_i = 0))\right].$$

If the informed agent adopts a non-trivial mixed strategy with i periods remaining, then:

$$\lambda_i(1-\beta_{i+1}) - 2c = (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, S_i = 0)). \quad (11)$$

Before extending lemma 1 to the general period i case, it is useful to characterize how informed trading intensities vary as time passes. One might conjecture that the informed agent would trade more aggressively as the number of remaining trading opportunities falls. The following example illustrates that the analysis is more subtle. Without imposing conditions on the likelihood with which the informed agent's information is publicly revealed, it may be that for a given market maker prior, $\bar{\beta}$, the informed agent trades *less* aggressively as i falls. To see this, suppose that $\bar{\beta} = 0.5$, $\delta = 0.4$, $c = 0.1$, $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, and $\lambda_3 = 0.49$. Then, $\gamma_3(\bar{\beta}) = 0.897 > 0 = \gamma_2(\bar{\beta})$. Intuitively, because information is far more likely to become public in period 1 than period 2, (*i.e.* $\lambda_1 >> \lambda_2$), the gain from deferring in period 2 in hopes of profiting on trade in period 1 is high. Information, however, is only marginally more likely to be revealed at the end of period 2 than period 3, so in period 3 the choice boils down to a choice between trading in period 3 or waiting until period 1. But since his information is sufficiently likely to become public before period 1, it does not pay to defer in period 3.

What appears to lead to this non-monotonicity appears is that the probability information is revealed publicly evolves in a convex pattern. If the probability that the event is revealed publicly, λ_i, varies over time, we can verify in the three period environment that:

Result 1 *A sufficient condition for $\gamma_2(\bar{\beta}) \geq \gamma_3(\bar{\beta})$ is $\frac{\lambda_3}{\lambda_2} \leq \frac{\lambda_2}{\lambda_1}$.*

More generally, we conjecture that

Conjecture 1 *If $\frac{\lambda_{i-1}}{\lambda_i} \leq \frac{\lambda_i}{\lambda_{i+1}}, \forall i < T$, then $\gamma_i(\bar{\beta}) \geq \gamma_{i+1}(\bar{\beta})$.*

The intuition underlying this conjecture is that this condition ensures that the probability of information release does not jump in a non-convex way so that fixing beliefs the expected future trading profits as the number of remaining trading opportunities falls.

What we can prove is that if the probability that the information leaks out does not vary over time, then the informed agent's trading intensity rises as the date T is approached:

Proposition 1 *Let $\lambda_i = \lambda,\ \forall i$. Then in any period i, the informed agent trades with positive probability if and only if the price is not too close to its fundamental value, i.e., $\bar{\beta} < 1 - \frac{2c}{\lambda}$. If $\bar{\beta} < 1 - \frac{2c}{\lambda}$, then the informed agent is more likely to submit an order as date T approaches: $\gamma_i(\bar{\beta}) \geq \gamma_{i+1}(\bar{\beta})$, and $\gamma_i(\bar{\beta}) > \gamma_{i+1}(\bar{\beta})$ if $\gamma_{i+1}(\bar{\beta}) < 1$. Hence, as date T approaches, equilibrium prices become more sensitive to order flow.*

Underlying this result is a lemma that we prove in the appendix. Lemma 5 details that if the probability of information leakage does not vary over time, then the informed agent's expected profits rise with the number of periods available for him to trade on his information: $V_i(\beta) \leq V_{i-1}(\beta)\ \forall \beta \in [0,1]$.Using this result, the proposition can be interpreted as follows. When $\lambda_i = \lambda,\ \forall i$, the expected return from submitting an order, $0.5\lambda(1-\bar{\beta}) - c$, does not vary over time. Accordingly, to keep the informed agent indifferent between trading and not as time passes, continuation profits from deferring must not fall even though fewer opportunities remain in which to benefit from the potential price improvement gained from deferring. Hence, as time passes, the price must fall by more following an order of zero: market maker beliefs must be revised downward by more following an order of zero. In turn, this requires that the informed agent trade more aggressively on his information as date T approaches.

We now return our attention to extending lemma 1 to the general period i case. When the probability of information leakage jumps in a non-convex way, it is difficult to rule out the possibility of multiple solutions to (11) and hence the possibility that the informed agent's trading intensity *rises* with an increase in the market maker's prior.

To preclude this possibility, the remainder of this section makes the following assumption:

Assumption (A1): $\gamma_i(\beta) > 0$ implies that $\gamma_j(\beta) > 0$, $\forall j < i$.

This assumption (which Proposition 1 ensures is satisfied if $\lambda_i = \lambda, \forall i$) is sufficient to rule out multiple solutions to (11). Given this assumption, it is now possible to extend lemma 1 and characterize the informed agent's equilibrium trading strategy for the general i period case. The next proposition details that the more likely the market maker believes the good state is, the less likely the informed agent is to trade:

Proposition 2 *The equilibrium probability with which the informed agent submits an order, $\gamma_i(\beta_{i+1})$, declines monotonically in β_{i+1}; strictly decreasing where it is interior.*

This result is subtle. The more likely the market maker believes the good state is to occur, the smaller are period trading profits. However, future market maker beliefs about the good state will also be higher (if the information is not revealed), so future trading profits are also lower. The key is to show that current profits fall more rapidly with β than do continuation profits. The assumption that $\gamma_i(\beta) > 0$ implies that $\gamma_j(\beta) > 0, j < i$ ensures that continuation payoffs following an order flow of zero are a concave function of β, while trading profits are a linear function of β. Then, *ceteris paribus*, increasing β reduces the value the informed agent places on trading relative to manipulating market maker beliefs. To keep the informed agent indifferent, market maker beliefs must fall by less following an order flow of zero — so the informed agent must be less likely to trade.

Proposition 2 implies that if the informed agent defers from trading at period i when the market maker's prior belief about the probability of the good state is zero, the informed agent will always defer from trading at period i. Corollary 1 offers sufficient conditions for beliefs to exist such that the informed agent will submit an order with positive probability in period i.

Corollary 1 *A sufficient condition for the existence of a range of $\beta_{i+1} \in [0,1]$ such that the informed agent trades with positive probability at period i is:*

$$\lambda_i - 2c > \sum_{j=1}^{i-1} \left((0.5)^{i-j} (\lambda_j - 2c) \left[\prod_{k=j+1}^{i} (1 - \lambda_k) \right] \right)$$

and $\lambda_1 > 2c$. If $\lambda_i = \lambda$, $\forall i$, then the sufficient condition reduces to $\lambda > 2c$.

We next characterize how the parameters describing the economy affect informed trading.

Proposition 3 *When $\eta = 0.5$, the probability with which the informed agent submits an order at period i, γ_i, is characterized by the following:*

- *The more likely the informed agent's information is to be revealed publicly, the more aggressively he trades: γ_i is weakly increasing in λ_i; strictly increasing for interior values of γ_i. Also, when $\lambda_i = \lambda\ \forall i$, γ_i is weakly increasing in λ.*

- *The greater are the fixed trading costs, the less aggressively the informed agent trades: γ_i is weakly decreasing in c; strictly decreasing for interior values of γ_i.*

- *The more likely there is to be an informed trader, the less aggressively the informed agent trades: γ_i is weakly decreasing in δ; strictly decreasing for interior values of γ_i.*

These findings are quite intuitive. As λ_i rises, the informed agent's information is more likely to be impounded into the closing period i price, raising the expected return from an option purchased at period i. Further, an increase in λ_i reduces the expected gain from deferring from trade because the informed agent is less likely to have private information to trade on in the next period. Less obviously, this relationship still holds when the probability that the private information is revealed is constant across periods. There are three direct effects: First, an increase in λ increases the informed agent's expected return from submitting an order this period. Second, an increase in λ also reduces the expected return from deferring since it increases the probability that information will be revealed before next period. The third, potentially offsetting effect, is that, if the information is not revealed this period, the expected return of submitting an order next period increases. We show, however, that the first two effects dominate the third.

The fixed trading cost (c) reduces the expected return from submitting an order each period. When c is high, the benefit from trading is low unless the price deviates substantially from the true value. As c rises, the informed agent is more likely to defer from trading, hoping to obtain possibly lower prices next period, which would raise the net profit margin.

As δ increases, the market maker believes that an informed agent is more likely, so her beliefs about the likelihood of a good state fall more sharply following an order flow of zero. Hence, as δ increases, it becomes more profitable for the informed agent to defer from trading.

3.1 Equilibrium Characterizations: $\eta \in [0, 1]$

Analytical results obtain only when the probability of liquidity trade is one-half. One consequence of $\eta = 0.5$ was that market maker beliefs did not change following an order flow of size one. $\eta = .5$ also maximizes the variance of liquidity trade. In Kyle (1985), the more volatile is liquidity trade, the easier it is for the informed agent to "hide" his trades and hence, the larger is his profit. Here, however, informed profits are typically not maximized by $\eta = 0.5$, because as η rises, the market maker is more likely to receive two orders when the informed agent trades, and this would reveal his information. The probability of liquidity trade (η) affects in subtle ways how the market maker updates her beliefs in response to observing one order:

Proposition 4 *When one order has been received, the price of an option at period i rises with γ_i when $\eta < 0.5$, but falls as a function of γ_i when $\eta > 0.5$.*

That is, if the probability of a liquidity trader is high, then *raising* the probability that the informed agent places an order *reduces* the price charged for an option, *in response to observing one order*. The intuition is as follows. If η and γ_i are high, the market market expects *two* orders. If the market maker only receives *one* order, she views this as a probable indication that the informed agent did not trade. Since a higher value of γ_i suggests that the informed agent would almost certainly trade if he had good news, the market maker's prior changes to reflect a lower belief that the good news event will occur. As a result, the option price falls.

Result 2 *If* $\eta \geq 0.5$*, multiple equilibria may exist.*

Figure 1 illustrates this possibility. The figure plots equilibrium trading probabilities, γ_1, for different values of liquidity trade, $\eta \in (0,1)$, when $c = .1$, $\delta = .4$, $\lambda_1 = .8$ and $\beta_2 = .5$.[4] For values of $\eta \in (0.75, 0.7864208074)$, there are three possible consistent equilibrium trading probability levels (and associated pricing). For η in this range, in one equilibrium, the informed agent always trades when he has good news; and in another he always defers. For other values of η, the equilibrium is unique. If the probability of liquidity trade is low (*i.e.* η is small), the market maker tends to attribute order flow to the informed agent, reducing informed trading incentives, so that γ_1 is small. If, instead, η is high, the informed agent will not place an order, since doing so is too likely to reveal his information to the market maker.

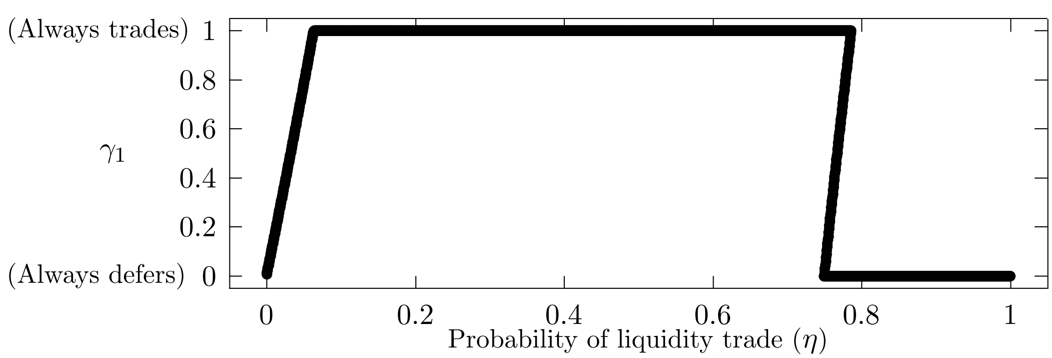

Figure 1: Informed agent's period 1 trading strategy when $c = .1$, $\delta = .4$, $\lambda_1 = .8, \beta_2 = .5$.

The more likely the market maker believes the good state, the less likely the informed agent is to trade. To illustrate, we modify the example illustrated in figure 1, by increasing the market maker's prior to $\beta_2 = .8$, while keeping the other parameters the same. Figure 2 plots

[4] We chose a relatively high value for the fixed transaction cost in order to provide sufficient contrast in the figures to illustrate the main points of interest.

equilibrium values of γ_1 for different values of η. Increasing η has two (potentially) opposing effects on the payoff from submitting an order. First, if liquidity trade is more likely, the informed agent is more likely to be revealed by an order flow of two. Obviously, this reduces the informed agent's expected payoff. Second, an increase in the probability of liquidity trade affects how the market maker adjusts her beliefs in response to an order flow of one. For $\eta \leq .5$, an increase in η reduces $\beta_1(S_1 = 1, \beta_2)$. For $\eta < .25$, the first effect dominates — the decrease in prices more than offsets the increase in the probability of getting "caught". For $\eta > .25$, the second effect dominates — the increase in the probability of getting "caught" is not sufficiently compensated for by lower prices. Since the informed agent mixes between submitting and not in equilibrium, his expected payoff from submitting an order at period 1 is always zero. For $\eta > .5$, the second effect goes in the wrong direction — an increase in η raises $\beta_1(S_1 = 1, \beta_2)$, so the informed agent does not trade.

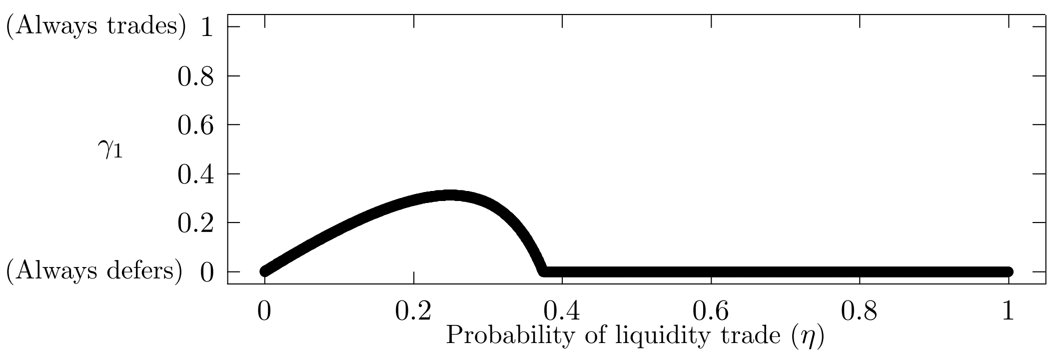

Figure 2: Informed agent's period 1 trading strategy when $c = .1$, $\delta = .4$, $\lambda_1 = .8, \beta_2 = .8$.

The analysis can be extended to two periods. In contrast to a one period trading horizon, where the payoff from deferring was zero, the period 2 payoff from deferring may be positive. Figure 3 plots equilibrium values of γ_2 for $\eta \in (0, 1)$ when $c = .1$, $\delta = .4$, $\lambda_1 = .8$, $\lambda_2 = .6$, and $\beta_3 = .5$. The kink resulting around $\eta = .08$, corresponds to $\gamma_1(\beta_2(S_2 = 1, \beta_3)) = 1$. For $\eta < .08$, $\gamma_1(\beta_2(S_2 = 1, \beta_3)) < 1$ so that $V_1(\beta_2(S_2 = 1, \beta_3)) = 0$. There is no kink corresponding to when $\gamma_1(\beta_2(S_2 = 0, \beta_3)) = 1$ because $\beta_2(S_2 = 0, \beta_3)$ is independent of the probability of liquidity trade (if $S_2 = 0$, the market maker knows that there is no liquidity trade).

The remainder of this paper uses the two period model to investigate two key features of trading in derivative securities. Section 4 details how the informed agent's accumulated position affects his trading strategy. Section 5 provides a possible explanation for why markets for shorter-term derivatives are so much more liquid than those for longer-term derivatives.

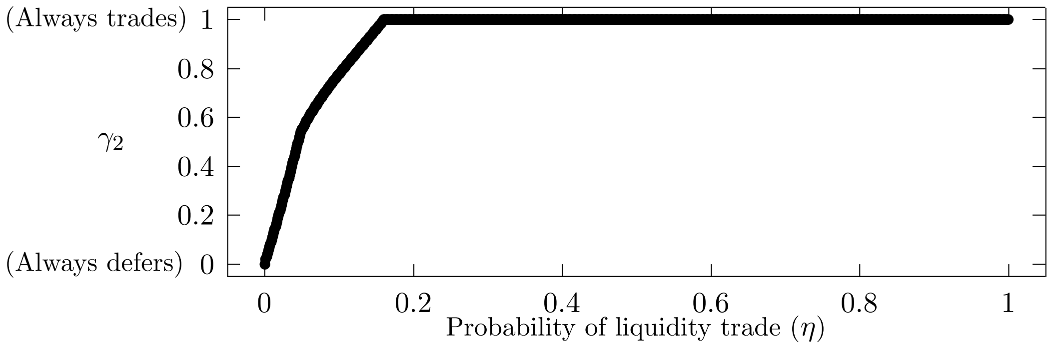

Figure 3: Period 2 informed trading strategy when $c = .1$, $\delta = .4$, $\lambda_1 = .8$, $\lambda_2 = .6, \beta_3 = .5$.

4 Accumulated position

An implication of Proposition 3 is that the longer is the holding period, the more aggressively the informed agent trades. That is, an increase in λ captures the effects of a longer holding period, ignoring the impact of an informed agent's accumulated position on his trading behavior. We now show that an informed agent's accumulated position further increases the informed agent's trading intensity. An informed agent with an accumulated position takes into consideration that submitting an order for an option this period increases the probability that his information will be revealed, in which case his previously purchased options will then be exercised at a profit. This situation does not arise in equity because, as long as an informed agent can hold his stake, his information will eventually be incorporated into the price of a previously-accumulated equity position. In contrast, with options there is a risk that the option contract may expire before the information is revealed publicly. In this context, increasing the probability that the information will be revealed has a positive value, a value which rises with the size of the informed agent's accumulated position.

We document this in the simplest possible context. We consider the last two periods remaining before date T of an environment with a long-lived option that is written at the beginning of period 3 and expires at the end of period 1. We assume that $\lambda_1 < 1$, so that the informed agent's accumulated position could expire unexercised. Each period, market participants can purchase options that expire at the end of period 1. Each period, with probability one-half, a liquidity trader places an order.

Let $W_i = \sum_{\iota=i}^{T} Y_\iota$ denote the informed agent's accumulated position at the end of period i. For simplicity, we do not model trading activity at period 3, but rather assume that β_4 is within the range where the informed agent adopts a mixed trading strategy, submitting an order of size one with probability $\gamma_3 \in (0,1)$. Therefore, the informed agent enters period 2 with an accumulated position, $W_3 \in \{0,1\}$. The informed agent's period i trading strategy, $\gamma_i (Y_i = x | D, \mathbf{H}_{i+1}, \mathbf{A}_{i+1}, W_{i+1})$ is a probability distribution over feasible order sizes, $x \in \mathbf{X}$. Note that the informed agent's strategy depends on his accumulated position.

In equilibrium, any informed order of size $Y_i \notin \{0,1\}$ reveals the informed agent's information to the market maker, in which case the price reflects his information. Hence, without loss of generality, we need only consider the following informed period trading strategies: (1) submit an order of size two that reveals his information; (2) submit an order of size one; (3) defer from trading. The associated expected period i payoffs are:

$$E\left[\pi_i\left(Y_i|\mathbf{H}_{i+1}, W_{i+1}\right)\right] = \begin{cases} W_{i+1} - c & \text{if } Y_i = 2 \\ 0.5\left(\lambda_i - p_i\left(S_i = 1|\mathbf{H}_{i+1}\right)\right) - c + \left(0.5 + 0.5\lambda_i\right) W_{i+1} & \text{if } Y_i = 1 \\ \lambda_i W_{i+1} & \text{if } Y_i = 0 \end{cases}$$

The period 1 value function is therefore:

$$V_1\left(\mathbf{H}_2, W_2\right) = \max_{Y_1 \in \{0,1,2\}} E\left[\pi_1\left(Y_1|\mathbf{H}_2, W_2\right)\right].$$

Proposition 5 *For any given market maker beliefs, the expected size of the informed agent's order in period 1 rises with the size of his accumulated position.*

The period 2 value function is:

$$V_2\left(\mathbf{H}_3, W_3\right) = \max_{Y_2 \in \{0,1,2\}} E\left[\pi_2\left(Y_2|\mathbf{H}_3, W_3\right) + (1-\lambda_2) V_1\left(\mathbf{H}_2(Y_2), W_3 + Y_2\right)\right].$$

Lemma 2 *In equilibrium, the informed agent never places a period 2 order of size two.*

In a more general multi-period context, the informed agent would never submit an order of size two until the option is about to expire.[5] The intuition is that the informed agent can always submit an order of size two at the last period if his private information has not been revealed. Rather than submit an order of size two at period $i > 1$, he can defer from trading. If he defers, his information may be revealed through a public announcement prior to period 1 so that he would not incur the trading cost c; if, instead, the information is not revealed, he can always place an order of size two at period 1, thereby revealing the information.

We now extend proposition 5 to show that a greater accumulated position at date 2 increases the probability that the informed agent submits an order at period 2:

[5]This result depends on the assumption that $r = 0$. If $r > 0$, the opportunity cost of potentially waiting until the last period may make it worthwhile for the informed agent to attempt to reveal his information earlier.

Proposition 6 *Given market maker beliefs, β_3, an informed agent is more likely to submit an order at period 2 if he has an accumulated position than if he does not.*

Propositions 5 and 6 reveal that holding both the informed agent's information and total past (informed plus liquidity) trade the same, future expected informed trade and hence future expected volume will be greater when the informed agent has a greater share of past trade. That is, the market maker updates prices in the same manner independently of who submitted the orders, but the informed agent trades more aggressively in the future if he traded more aggressively in the past and hence acquired a greater stake.

5 Time-to-maturity and the liquidity of derivative markets

This section investigates why option and futures contracts that are relatively close to maturity are far more liquid than similar contracts with more distant expiration dates. This empirical regularity is especially puzzling given the numerous reasons why agents might want to use longer-term derivative securities to hedge against long-term risk.

To provide context, consider the pattern of open interest and daily trading volume for the Chicago Mercantile Exchange's Live Cattle December 1999 Futures Contract, illustrated in figure 4. The expiry cycle is such that contracts for delivery in the upcoming months of February, April, June, August, October and December are always available. The timeline is divided into six sections to represent the last trading day of each contract.

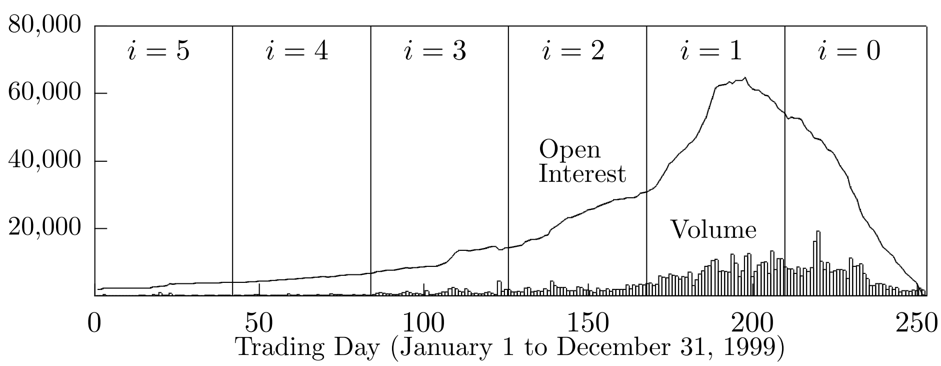

Figure 4: **CME December 1999 Live Cattle Futures Contract:** Daily trading volume (indicated by bars) and daily open interest (indicated by solid line).

In general, cattle farmers and food producers know well in advance that they will be selling or buying cattle on a regular basis. Suppose that in January a farmer knew that he was going to sell cattle in November. The farmer has three possible strategies: (a) sell the cattle at the harvest-time cash price; (b) use a December futures contract to guarantee (approximately) a price for his cattle today; (c) sequentially roll over futures contracts — i.e. every two months roll over into the nearest to expiration contract.

Heuristic industry evidence suggests that the strategy of sequentially rolling over nearby futures contracts is quite common. Rolling over shorter-term positions, often referred to as "rolling the hedge", has two additional costs over buying the longer-term futures contract:

1. Transaction costs (commissions, bid-ask spread) incurred each time a position is rolled over.
2. Agents must pay for the "spread" between two futures contracts with different expiration dates. This cost may be significant if there are systematic differences in the basis between the local cash price and the price of futures contracts with different durations.

To focus on issues related to adverse selection, we do not address this second cost associated with rolling over futures contracts. Specifically, our model has no rollover risk because the basis between the cash price and futures contract price is always zero, for all maturities. Neuberger (1999) considers the implications of rollover risk. He derives optimal long-term hedging strategies using multiple short-term futures contracts to minimize this risk.

Within the context of the widespread use of rolling the hedge, figure 4 can be interpreted as follows. In general, agents prefer not to hold a *commodity-based* futures contract into the delivery month unless actually delivering against the contract. This provides a buffer against holding a contract on the delivery date and either not being able to supply the commodity or not having to take delivery (this is less of a concern if the underlying asset is a financial product.) Agents also normally select a futures contract that expires after their planned cash sale. This eliminates the need to buy back the hedge before the commodity is sold. These two factors explain the significant drop in open interest and daily volume in period $i = 0$ as agents close out their positions prior to the delivery month (December).

As a result, period $i = 1$ most accurately captures the period in which the December futures contract has the shortest *effective* time-to-maturity. During this period, there is a significant increase in both open interest and daily volume that would partially correspond to agents rolling over their positions from October contracts. Also consistent with our interpretation, the contract's liquidity, as measured by open interest and volume, falls as time-to-maturity increases. These trading patterns characterize other derivative security contracts, (*e.g.* longer-term options such as LEAPs (**L**ong-term **E**quity **A**ntici**P**ation **S**ecurities)).

To investigate why markets for nearby options and futures contracts are so much more liquid than those for distant contracts we now extend the two-period model to

- Allow for the possible arrival of a second, *long-lived* liquidity trader who can trade in both periods. The long-lived and short-lived liquidity agents each trade with probability one-half, and these probabilities are independently distributed.

To simplify the exposition we also assume that

- Information is revealed with certainty at period 1 ($\lambda_1 = 1$). Consequently, the informed agent does not have to submit a large order in period 1 to reveal his information.
- Fixed trading costs are sufficiently small that it is always profitable for the informed agent to trade in the last period if his information has not been revealed through trading or a public announcement. Lemma 6 in the appendix details that a sufficient condition for the informed agent to trade in the last period if his information has not been revealed is that the ex ante probability that the good state will occur is sufficiently small.

Until now, we have considered only short-lived options that expire after one period. In practice, options exist for various lengths of time. As an option's time until expiry increases, the model tends to that of a long-lived trader in equity (Kyle 1985), with agents able to hold on to positions for as long as it takes the market maker to learn the asset's true value. In contrast to Kyle, in our model, agents may not commence trading once they receive private (good) information; if the price of the asset is too close to its fundamental value an informed agent prefers to strategically defer in the hope of manipulating the market. This result is a consequence of fact that our model allows for non-convexities — the fixed trading costs and round lot restrictions. Kyle's normality assumptions preclude consideration of these features, so that submitting a sufficiently small order dominates the strategy of delaying trade.

To investigate how time-to-expiry matters, we introduce an additional market for 2–period (or *long-lived*) options. The market maker observes total order flows in *both* markets. Let S_i denote total orders received by the market maker for the short-lived option at period i. Let L_i denote total orders received by the market maker for the long-lived option at period i. At period 1, orders received for the short- and long-lived options are equivalent.

Let W_i denote the informed agent's accumulated position at the end of period i of long-lived options which expire in period $i-1$. The set of feasible *individual* order sizes for the short- and long-lived option, are respectively denoted by $\mathbf{X}^S$ and $\mathbf{X}^L$, and $\tilde{\mathbf{X}}^S$ and $\tilde{\mathbf{X}}^L$ represent the corresponding sets of feasible *aggregate* order flows for the short- and long-lived option. Let Y_i^S and Y_i^L denote the size of the informed agent's order for the short- and long-lived option, respectively, at period i. The informed agent's strategy at period i is a joint probability distribution, $\gamma_i\left(Y_i^S = x^S, Y_i^L = x^L \middle| D_0, W_{i+1}, \mathbf{H}_{i+1}^S, \mathbf{H}_{i+1}^L, \mathbf{A}_{i+1}\right)$, over feasible orders in each market. The market maker selects a set of pricing functions for the short-lived option at the open and close, $P_{io}^S(\mathbf{H}_i^S, \mathbf{H}_i^L, \mathbf{A}_{i+1})$ $P_{ic}^S(\mathbf{H}_i^S, \mathbf{H}_i^L, \mathbf{A}_i)$, and a set of pricing functions for the long-lived option at the open and close, $P_{io}^L(\mathbf{H}_i^S, \mathbf{H}_i^L, \mathbf{A}_{i+1})$, $P_{ic}^L(\mathbf{H}_i^S, \mathbf{H}_i^L, \mathbf{A}_i)$. Note that the pricing functions depend on order flow in *both* markets.

Given the additional long-lived option market, the long-lived liquidity trader can choose between two possible strategies that hedge him against the possible arrival of good news in both periods. One hedging strategy is to purchase the short-lived option at period 2 and, if the information is not revealed through trading or a public announcement, purchase the short-lived option at period 1. The other hedging strategy is to purchase the long-lived option at period 2. The liquidity trader will select the strategy with the lowest expected cost.

5.1 Long-lived liquidity trader always trades in short-lived option market

We first consider the possibility that the long-lived liquidity trader purchases the short-lived option each period, as long as the private information is not revealed through trading or a public announcement. If the long-lived liquidity trader never submits an order for the long-lived option, then any order from the informed agent for the long-lived option would reveal his existence to the market maker. Therefore, the informed agent only submits orders to the short-lived option market, so liquidity in the long-lived option market dries up.

The possibility of two liquidity traders provides the opportunity for the informed agent to submit an order of size two without being revealed with certainty to the market maker. The following result, however, reveals that this strategy is never optimal:

Lemma 3 *It is never profit-maximizing for the informed agent to submit an order of size two.*

Essentially, the probability that a short-lived liquidity trader places a trade is high enough to discourage the informed agent from submitting a larger order. This result holds in period 1 *even* though the informed agent may know from period 2 order flow net of his trade that there is no long-lived liquidity trader (so that the maximum liquidity trade is one), whereas, the market maker is unable to make such a distinction. That is, if the market maker observes one order at period 2, the market maker cannot determine which market participant submitted the order, but if the informed agent submitted that order he knows that there is no long-lived liquidity trader. Consequently, the informed agent has better information than the market maker in period one about both the value of the asset *and* the likely level of liquidity trade.

Market Maker Beliefs: At the beginning of period 2, the market maker's beliefs about the likelihood of the good state correspond to the *ex ante* prior, κ. If the market maker observed an order for the long-lived option she would update her conditional belief that the good state will occur to one. Since equilibrium order flow in the long-lived option is therefore zero, to reduce notation, we suppress the order flow in the long-lived option in this section. At period 2, the market maker updates her beliefs, $\beta_2(S_2)$, in response to the order flow, S_2, in the short-lived option market, where $S_2 \in \{0, 1, 2, 3\}$ along the equilibrium path.

The market maker updates her period 1 beliefs, $\beta_1(S_1, S_2)$, based on order flows in the short-lived option market in periods 1 and 2. Now, the history of order flow and public announcements up to the end of period 2 cannot be summarized just by β_2. This is because the order flow in period 2 reveals information to the market maker about the existence of the long-lived liquidity trader. If no orders are submitted in period 2, the market maker knows that there is no long-lived liquidity trader, so that an order flow of 2 or greater in period 1 reveals the informed agent. If one order is submitted at period 2, then the market maker cannot rule out the possibility of either an informed agent or a long-lived liquidity trader. In this situation, a period one order flow of 2 or less does not reveal the informed agent. Finally, if two orders are received at period 2, the market maker knows that a long-lived trader (either liquidity or informed) exists. The market maker's updating rules are outlined in the appendix.

Given the market maker's updating rules, the long-lived liquidity trader's expected cost from submitting an order for the short-lived option each period is:

$$C^S = 0.5\Big[\delta\kappa\rho_S + \beta_2(2)\lambda_2 + (1-\delta\kappa\rho_S)\beta_2(1)\lambda_2\Big] + c + (1-\lambda_2)(1-0.5\delta\kappa\rho_S)c$$

$$+0.25(1-\lambda_2)\Big[2\delta\kappa - \delta\kappa\rho_S + \beta_1(2,2) + [1-\delta\kappa\rho_S]\beta_1(2,1) + [1-\delta\kappa]\beta_1(1,2) + [1-\delta\kappa]\beta_1(1,1)\Big]$$

Section 5.2 compares these costs with the long-lived liquidity trader's expected cost from submitting an order for the long-lived option. First, it it is useful to provide intuition about the market maker's equilibrium pricing strategies in this environment.

The market maker prices so that her expected losses to informed agents are offset by her profits from uninformed liquidity traders. In the appendix, we derive the market maker's expected losses to the informed agent ($E[\Psi_I]$), the period 2 short-lived liquidity trader ($E[\Psi_{S2}]$), the period 1 short-lived liquidity trader ($E[\Psi_{S1}]$), and the long-lived liquidity trader ($E[\Psi_L]$). In equilibrium, $E[\Psi_I] + E[\Psi_{S2}] + E[\Psi_{S1}] + E[\Psi_L] = 0$. Figure 5 illustrates the market maker's expected loss from each type of market participant when $\delta = 0.4$, $\kappa = 0.2$, and $c = 0.005$. Observe that the market maker expects to lose money to the informed agent for all values of λ_2. To offset these losses, the market maker expects to earn non-negative profits from each of the short-lived liquidity traders and strictly positive profits from the long-lived liquidity trader.

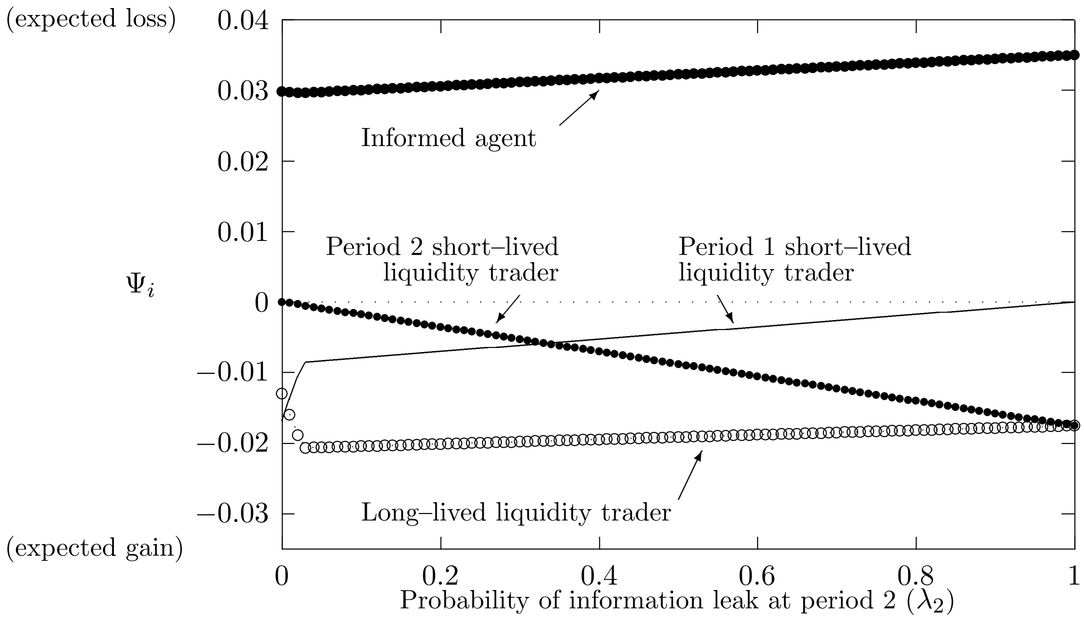

Figure 5: *Ex ante* expected market maker losses from each market participant if the long-lived liquidity trader always trades the short-lived option: $\delta = .4$, $\kappa = .2$, $c = .005$, and $\lambda_1 = 1$.

The period 2 short-lived liquidity trader benefits from a decrease in λ_2 since the market maker reduces prices to reflect a reduced likelihood of informed trade. In contrast, the period 1 short-lived liquidity trader benefits from an increase in λ_2 since it reduces the probability that he will have to enter an unprofitable trade. When $\lambda_2 = 1$, the market maker expects the same profit from the long-lived liquidity trader and the period 2 short-lived liquidity trader.

Note that when $\lambda_2 = 0$, the market maker does *not* expect the same profit from the long-lived liquidity trader and the period 1 short-lived liquidity trader: $E[\Psi_L] > E[\Psi_{S1}]$ when $\lambda_2 = 0$. The reason for this result is as follows. If there is a long-lived liquidity trader, he will place an order in period 2 regardless of the value of λ_2. The informed agent also submits an order at period 2 with positive probability even if $\lambda_2 = 0$. This is because it is advantageous for the informed agent to have the market maker believe that there could be a long-lived liquidity trader; if no orders are received in period 2, the market maker knows that there is no long-lived liquidity trader so that an order flow of two at period 1 would reveal the informed agent. As a result, the informed agent's period 2 trades may reveal his presence.

5.2 Long-lived liquidity trader always trades in long-lived option market

Suppose now that the long-lived liquidity trader always trades in the long-lived option market. Because order flow at period 2 can provide the market maker with information regarding the existence of the long-lived liquidity trader, the market maker's updating rule at period 1 depends on both her original prior and observed past order flow. Given that κ is sufficiently small, the informed agent's value function at the last period is

$$V_1(S_2, L_2) = L_2 + 0.5[1 - \beta_1(S_1 = 1, S_2, L_2)] - c.$$

The informed agent has four possible period 2 trading strategies: (1) buy the long-lived option; (2) buy the short-lived option; (3) buy both the long-lived option and the short-lived option; or (4) defer from trading. The associated period 2 value function is:

$$V_2 = \max_{Y_2^S, Y_2^L} E\left[\pi_2\left(Y_2^S, Y_2^L\right) + (1 - \lambda_2)V_1\left(S_2(Y_2^S), L_2(Y_2^L)\right)\right]$$

where

$$E\left[\pi_2\left(Y_2^S, Y_2^L\right)\right] = 0.25\begin{cases} 2\lambda_2 - \beta_2(0,1) - \beta_2(1,1) - 4c & \text{if } Y_2^S = 0, Y_2^L = 1 \\ 2\lambda_2 - \beta_2(1,0)\lambda_2 - \beta_2(1,1)\lambda_2 - 4c & \text{if } Y_2^S = 1, Y_2^L = 0 \\ 2\lambda_2 - \beta_2(1,1)\lambda_2 - \beta_2(1,1) - 8c & \text{if } Y_2^S = 1, Y_2^L = 1 \\ 0 & \text{if } Y_2^S = 0, Y_2^L = 0 \end{cases}$$

The next lemma shows that we can rule out the third trading strategy:

Lemma 4 *The informed agent never buys both options in equilibrium:* $\gamma_2(Y_2^S = 1, Y_2^L = 1) = 0$.

In the appendix, we use lemma 4 to describe how the market maker updates her beliefs in response to different period order flows. At this point, it is convenient to denote the probability that the informed agent submits an order to the long-lived option market at period 2 by ρ_L where $\rho_L = \gamma_2(Y_2^S = 0, Y_2^L = 1)$ and to denote the probability that the informed agent submits an order to the short-lived option market at period 2 by ρ_S where $\rho_S = \gamma_2(Y_2^S = 1, Y_2^L = 0)$. The following result characterizing the informed agent's period 2 strategy obtains:

Proposition 7 *The informed agent is more likely to trade the longer-term option than the shorter term option: If $\lambda_2 < 1$, then $\rho_L > \rho_S$.*

The intuition for why the informed agent finds the longer-term option more attractive is the following: If the informed agent's information becomes public in the second period, he profits from trade in either market. If, instead, his information does not become public in period 2, then he still profits from trading the longer-lived option, but not from trading the shorter one. Independently of which option the informed agent traded in the second period, he will trade again in the first period if his information was not revealed.

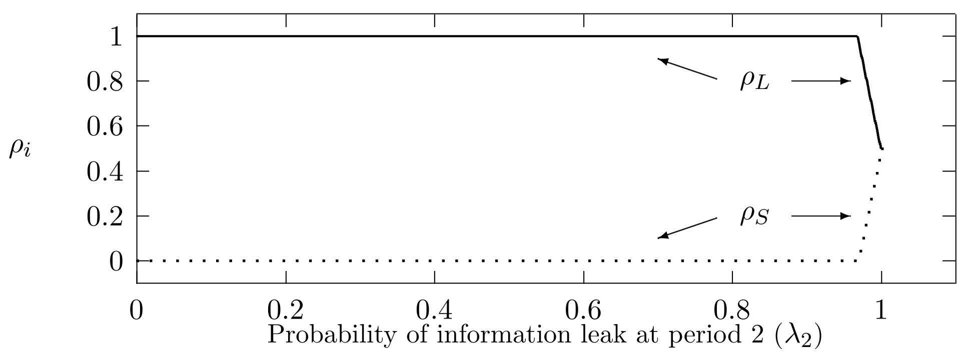

Figure 6: Informed agent's period 2 strategy when $\delta = .4$, $\kappa = .2$, $c = .005$, and $\lambda_1 = 1$. The informed agent submits an order in the long-lived option market with probability ρ_L, and with probability $\rho_S = 1 - \rho_L$ he submits an order to the short-lived option market.

Figure 6 illustrates the informed agent's period 2 strategy when $\delta = .6$, $\kappa = .2$, $c = .005$, and $\lambda_1 = 1$. For $\lambda_2 < 1$, the informed agent prefers to trade in the long-lived option market. As $\lambda_2 \to 1$, the long- and short-lived option markets become equivalent and the informed agent begins to mix his order flow between the two markets. Paradoxically, it is the informed agent's preference for the long-lived option market that contributes to its poor liquidity. The long-lived liquidity trader faces higher adverse selection costs from trading the long-lived option since a higher proportion of the order flow there is attributed to the informed agent.

The long-lived liquidity trader's expected cost of buying the long-lived option is:

$$C^L = \delta\kappa\rho_L + 0.5(1 - \delta\kappa(\rho_S + \rho_L))(\beta_2(0,1) + \beta_2(1,1)) + 0.5\delta\kappa\rho_S(1 + \beta_2(1,1)) + c.$$

Even if the market maker believes the long-lived liquidity trader will always buy the long-lived option (the best possible scenario), it is still not always optimal for the the long-lived liquidity trader to do so. The expected cost of submitting an order to the short-lived option market at period 2 and rolling over the position in the period 1 short-lived option market if the information is not revealed publicly is:

$$\hat{C}^S = 0.5 + 0.5\delta\kappa\rho_S + 0.5[\rho_N\kappa\delta + 1 - \delta\kappa]\beta_2(1,0)\lambda_2 + 0.5\rho_L\kappa\delta\beta_2(1,1)\lambda_2 + c$$

$$+(1-\lambda_2)(1 - 0.5 - 0.5\delta\kappa\rho_S)c + 0.25(1-\lambda_2)\left[(1-\delta\kappa)\beta_2(1,0) + 2\rho_L\kappa\delta + 2\rho_N\kappa\delta + 1 - \delta\kappa\right]$$

When $C^L < \hat{C}^S$, both long- and short-lived option markets can potentially exist in equilibrium. This condition will hold for most reasonable parameter ranges.

In summary, our model suggests that there are two possible equilibrium outcomes: (1) pooling equilibrium (only trade in the short-lived option market); (2) separating equilibrium (trade in both the short- and long-lived option markets).[6] Empirically, some derivative security markets seem to have a greater concentration of trade in nearby contracts than others. A relevant question is then: What characteristics determine which equilibrium outcome will prevail? Our model offers some intuition about how transaction costs, the percentage of informed trade, the rate of information dispersion, and the distribution of time horizons of liquidity traders impact the equilibrium outcome.

Suppose that the equilibrium outcome which prevails is the equilibrium that minimizes the long-lived liquidity trader's expected costs. In equilibrium, the long-lived liquidity trader will elect to trade in the short-lived option market if and only if $C^L > C^S$. Under these conditions, the liquidity in the long-lived option market will vanish. We now consider how the difference $C^L - C^S$ changes in response to changes in our parameters of interest.

Transaction costs: The long-lived liquidity trader's relative cost of rolling over his position in the short-term option market rise as c increases. As figure 7 illustrates using representative parameter values, the pooling equilibrium will occur when the transaction cost is sufficiently small that the increased transaction costs associated with rolling over his position are more than compensated for by lower adverse selection costs.

The advent of electronic trading and discount brokerages have reduced transactions costs. *Ceteris paribus*, this would reduce the cost of sequentially trading short-lived options and tend to concentrate trade in nearby contracts. Also, our model predicts that markets for derivative securities with smaller fixed transaction costs should have relatively less liquidity in more distant contracts.

[6] The possibility for stable pooling and separating equilibrium outcomes frequently arises in market microstructure models with asymmetric information. For example, Seppi (1990) characterizes possible equilibrium outcomes when informed institutional trade can be executed either as a large block trade or as a sequence of smaller trades.

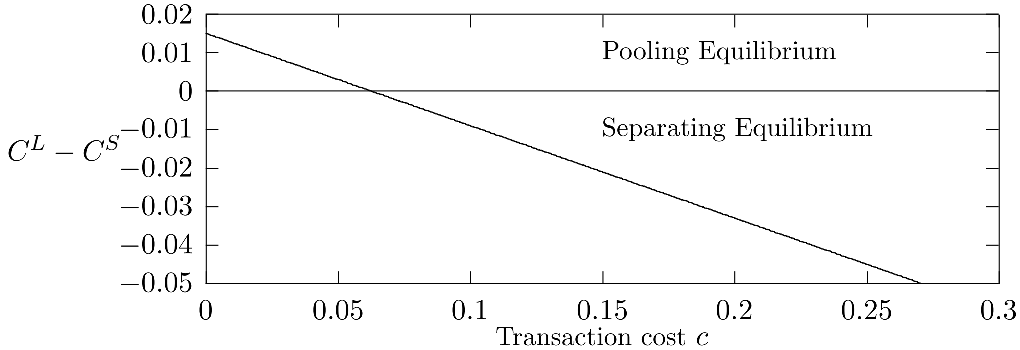

Figure 7: Effect of transaction cost, c, on the difference between the long-lived liquidity trader's expected cost from trading in the long- and short-lived option markets when $\delta = .4$, $\kappa = .2$, $\lambda_2 = .7$, and $\lambda_1 = 1$.

Percentage of informed trade: The parameter δ represents the probability that an agent has private information. As δ increases, the importance of adverse selection costs rise, raising the relative costs of the long-lived option market for the long-lived liquidity trader. Figure 8 illustrates this result for representative parameter values. Thus, *ceteris paribus*, derivative markets with more informed trade should have relatively greater concentrations of trade in nearby contracts.

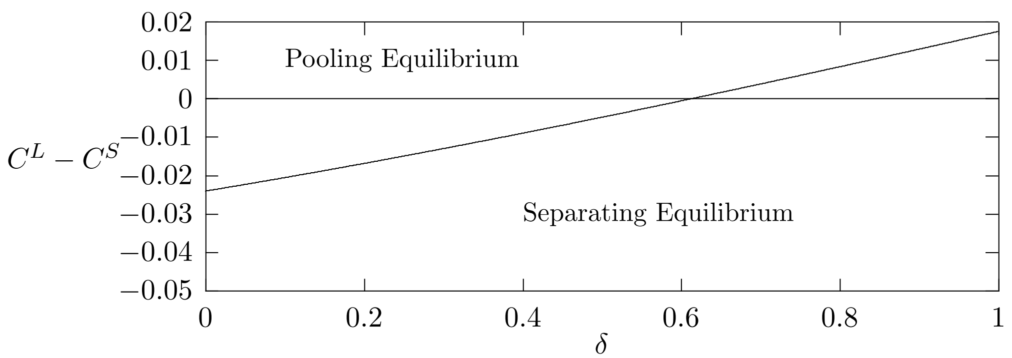

Figure 8: Effect of the probability that the informed agent has private information, δ, on the difference between the long-lived liquidity trader's expected cost trading in the long- and short-lived option markets when $\kappa = .2$, $\lambda_2 = .7$, $c = .1$ and $\lambda_1 = 1$.

Rate of information dispersion: The parameter λ_2 represents the probability that the informed agent's private information is revealed publicly at period 2. Using representative parameter values, figure 9 illustrates how changing λ_2 affects the difference between the long-lived liquidity trader's expected costs from trading in the short-lived option market (when only the short-lived option market exists) and his expected costs from trading in the long-lived option market (when both markets exist). As long as λ_2 is not too low, the long-lived liquidity trader incurs lower expected costs by selecting the short-lived option market.

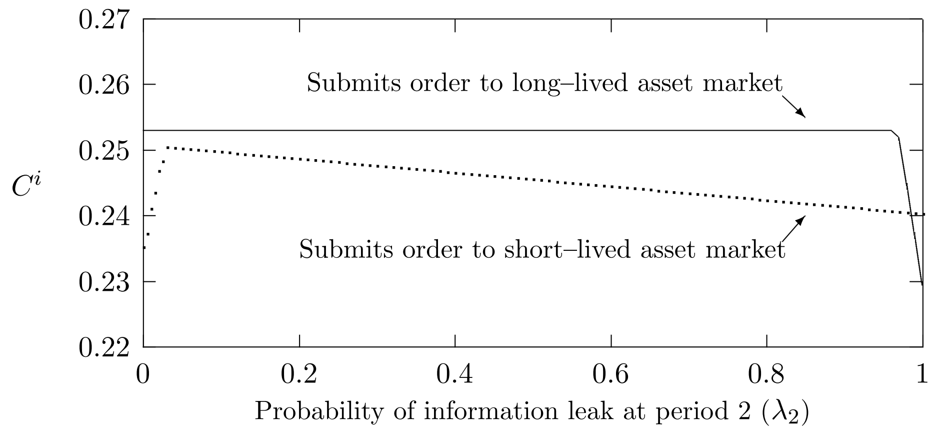

Figure 9: Long-lived liquidity trader's expected costs from always trading in the short-lived option market vs always trading in the long-lived option market, when $\delta = .4$, $\kappa = .2$, $c = .005$, and $\lambda_1 = 1$.

For most values of λ_2, the expected cost of submitting an order to the long-lived option market is constant. Only as $\lambda_2 \to 1$, so the long-lived option market and the short-lived option market become similar, does the informed agent begin to split order flow between the two markets, instead on sending all orders to the long-lived option market. As ρ_L falls, the adverse selection costs of the long-lived liquidity trader fall, reducing expected costs.

The expected cost of purchasing the short-lived option first rise as the informed agent increases the probability of submitting an order in period 2 (*i.e.* γ_2 increases). Once γ_2 reaches one, the expected costs of submitting an order to the short-lived option market begin to fall with λ_2. This decrease in costs results because the period 2 short-lived liquidity trader bears a disproportionate amount of the increased level of adverse selection.

λ_2 is a proxy for the rate at which private information is revealed to the marketplace. The development of the Internet, 24 hour business news television stations, etc. have increased the rate at which private information is revealed and incorporated into asset prices. Over most (intermediate) parameter values, our model predicts that these trends will reduce adverse selection costs for liquidity traders and increase the relative attractiveness of hedging long-term risk using longer-lived derivative securities. Our model, however, implies that these predictions could be reversed for extremely high or low values of λ_2.

Distribution of liquidity trader time horizons: Our previous analysis assumed that long-lived and short-lived liquidity trade are equally likely. We now enrich the analysis, and allow these two probabilities to differ. Let η_L denote the probability that a long-lived liquidity trader places an order and let η_S denote the probability that a short-lived liquidity trader places an order. The expressions for cost functions, expected payoffs, and pricing functions (available upon request from the authors) are now far more complex and are therefore not reproduced here.

Holding $\eta_S = 0.5$ fixed, we vary η_L over the range $(0, 0.5]$ to determine how the relative proportion of long-lived liquidity traders affects equilibrium outcomes. As figure 10 illustrates, an increase in η_L reduces the long-lived liquidity trader's expected costs in both possible equilibrium outcomes. An increase in η_L reduces adverse selection costs and has an impact similar to that of a decrease in δ. Lower adverse selection costs reduce the long-lived liquidity trader's expected cost of always submitting an order to the long-lived option market faster than the expected cost of always submitting an order to the short-lived option market.

These results suggest that we should observe increased liquidity levels in longer-lived derivative securities based on underlying assets that are more likely to be hedged over longer time horizons. It seems reasonable to conjecture that the time horizon of liquidity traders in commodity-based derivatives is shorter than the time horizon of liquidity traders in financial instruments such as interest rate and exchange rate futures contracts. So, too, the relatively poor liquidity of long-lived equity options, such as LEAPs, can be partially explained by the typical relatively short holding periods of the underlying securities by liquidity traders.

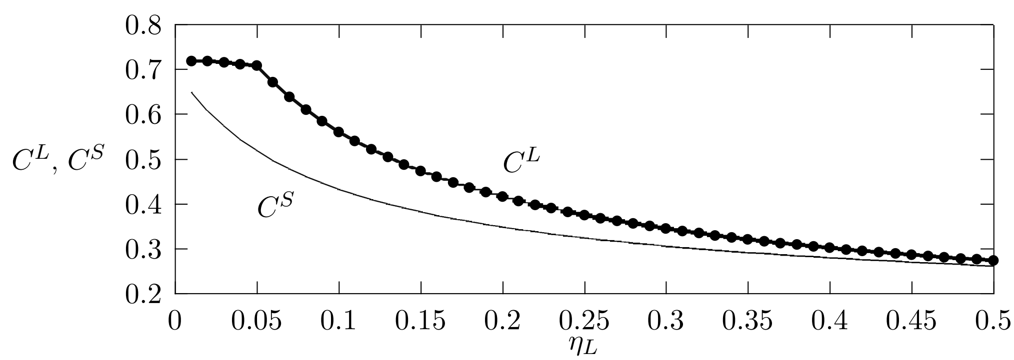

Figure 10: Long-lived liquidity trader's expected cost from always submitting an order to the short-lived option market vs. long-lived option market for different levels of η_L when $\delta = .4$, $\kappa = .2$, $c = .01$, $\lambda_2 = .7$, $\lambda_1 = 1$ and $\eta_S = .5$.

6 Conclusion

Strategic trading of short-lived derivative securities, such as option or futures contracts, has largely been ignored by the academic literature. This paper documents that there are important differences exist between the strategic trading of short-lived derivative securities and the strategic trading of equity. We find that the short-lived nature of the securities affects trading behavior and pricing in the following ways:

1. With short-lived securities, the greater a risk neutral informed agent's holdings of the short-lived security, *ceteris paribus*, the more aggressively he trades in the future. This behavior contrasts with that found in equity markets where an informed agent's accumulated position does not affect his trading behavior.

2. The shorter horizon in which information must be impounded for a short-lived security to pay off makes an informed agent more reluctant to trade, especially when the informed's information is longer-term in nature. Given innocuous technical conditions, relatively, as the event about which the informed has private information approaches, informed trading intensities rise so that prices become more sensitive to order flow.

3. For reasonable parameter ranges, liquidity traders prefer to incur extra costs to roll over their short-term positions rather than trade in distant contracts. We provide a theoretical rationale for the empirical finding that markets for longer-term derivative securities have little liquidity and large spreads.

Finally, we contribute to the existing literature by investigating how non-convexities, introduced by fixed trading costs and round lot trading, can lead to multiple equilibria.

A Appendix

A.1 Derivations for Section 5

To clarify the exposition in section 5, we omitted many of the details regarding the market maker's updating rules. For completeness, we present those updating rules here.

Long-lived liquidity trader always submits to short-lived option market: Let $\rho_S = \gamma_2(Y_2^S = 1, Y_2^L = 0)$ denote the probability that the informed agent submits an order to the short-lived derivative market at period 2. The market maker's updated beliefs become

$$\beta_2(S_2 = 0) = \frac{\kappa(1 - \delta\rho_S)}{1 - \delta\kappa\rho_S}; \;\; \beta_2(S_2 = 1) = \frac{\kappa(2 - \delta\rho_S)}{2 - \delta\kappa\rho_S}; \;\; \beta_2(S_2 = 2) = \frac{\kappa(1 + \delta\rho_S)}{1 + \delta\kappa\rho_S}; \;\; \beta_2(S_2 \geq 3) = 1.$$

The market maker's up-dating rule in the last period reflects the fact that the period 2 order flow provides information about the existence of the long-lived liquidity trader:

Case 1 ($S_2 = 0$): If no orders are submitted in period 2 there cannot be a long-lived liquidity trader. Consequently, the market maker updates her beliefs at period 1 as follows:

$$\beta_1(S_1 = 0|S_2 = 0) = \frac{\kappa(1 - \delta)}{1 - \delta\kappa}; \;\; \beta_1(S_1 = 1|S_2 = 0) = \frac{\kappa(1 - \delta\rho_S)}{1 - \delta\kappa\rho_S}; \;\; \beta_1(S_1 = 2|S_2 = 0) = 1.$$

Case 2 ($S_2 = 1$): At period 1, the market maker updates her beliefs as follows:

$$\beta_1(S_1 = 0|S_2 = 1) = \frac{\kappa(1 - \delta)}{1 - \delta\kappa}; \;\; \beta_1(S_1 = 1|S_2 = 1) = \frac{\kappa(2 - \delta)}{2 - \delta\kappa};$$

$$\beta_1(S_1 = 2|S_2 = 1) = \frac{\kappa(1 + \delta - \delta\rho_S)}{1 + \delta\kappa - \delta\rho_S\kappa}; \;\; \beta_1(S_1 = 3|S_2 = 1) = 1.$$

Case 3 ($S_2 = 2$): An order flow of two at period 2 implies the existence of a long-lived trader (either liquidity or informed). At period 1, the market maker updates her beliefs as follows:

$$\beta_1(S_1 = 1|S_2 = 2) = \frac{\kappa(1 - \delta + \delta\rho_S)}{1 - \delta\kappa + \delta\kappa\rho_S}; \;\; \beta_1(S_1 = 2|S_2 = 2) = \frac{\kappa(1 + \delta\rho_S)}{1 + \delta\kappa\rho_S}; \;\; \beta_1(S_1 = 3|S_2 = 2) = 1.$$

The market maker's expected loss from an informed agent in the last two periods is

$$E[\Psi_I] = 0.5\kappa\delta\rho_S\Big[\lambda_2(1 - \beta_2(2)) + 0.5\lambda_2(1 - \beta_2(1))$$
$$+0.25(1 - \lambda_2)\Big(5 - 2\beta_1(2,2) - \beta_1(1,2) - \beta_1(2,1) - \beta_1(1,1)\Big)\Big] +$$
$$0.25\kappa\delta(1 - \rho_S)\Big[(1 - \lambda_2)\Big(6 - \beta_1(2,2) - \beta_1(1,1) - 2\beta_1(2,1) - \beta_1(2,0) - \beta_1(1,0)\Big)\Big].$$

The market maker's expected loss from a short-lived liquidity trader in period 2 is:

$$E[\Psi_{S2}] = 0.25\lambda_2\Big[\kappa\Big((1-\beta_2(2)) + (1-\delta\rho_S)(1-\beta_2(1))\Big) - (1-\kappa)\Big(\beta_2(2)+\beta_2(1)\Big)\Big].$$

The market maker's expected loss from a short-lived liquidity trader at period 1 is:

$$E[\Psi_{S1}] = 0.5(1-\lambda_2)\Big[0.25\kappa\Big(\delta\rho_S[2-\beta_1(2,2)-\beta_1(2,1)]$$

$$+\delta(1-\rho_S)[2-\beta_1(2,1)-\beta_1(2,0)] + (1-\delta)[4-\beta_1(2,2)-\beta_1(2,1)-\beta_1(1,1)-\beta_1(1,0)]\Big)$$

$$-0.25(1-\kappa)\Big(\beta_1(2,2)+\beta_1(2,1)+\beta_1(1,1)+\beta_1(1,0)\Big)\Big].$$

The market maker's expected loss from a long-lived liquidity trader in the last two periods is:

$$E[\Psi_L] = 0.25\Big[\kappa\Big(\lambda_2(1-\beta_2(2)) + (1-\delta\rho_S)\lambda_2(1-\beta_2(1))$$

$$+0.5(1-\lambda_2)(1-\beta_1(2,2) + (1-\delta\rho_S)(1-\beta_1(2,1)) + (1-\delta)[2-\beta_1(1,2)-\beta_1(1,1)])\Big)$$

$$-(1-\kappa)\Big(\lambda_2(\beta_2(2)+\beta_2(1)) + 0.5(1-\lambda_2)(\beta_1(2,2)+\beta_1(1,2)+\beta_1(2,1)+\beta_1(1,1))\Big)\Big].$$

Long-lived liquidity trader always submits to long-lived option market: Given Lemma 4, the market maker's updated period 2 beliefs are:

$$\beta_2(S_2=0,L_2=0) = \frac{\kappa[1-\delta(1-\rho_N)]}{1-\kappa\delta(1-\rho_N)};\quad \beta_2(S_2=0,L_2=1) = \frac{\kappa(1-\delta\rho_S)}{1-\kappa\delta\rho_S};$$

$$\beta_2(S_2=1,L_2=0) = \frac{\kappa(1-\delta\rho_L)}{1-\kappa\delta\rho_L};\quad \beta_2(S_2=1,L_2=1)=\kappa;\quad \beta_2(S_2=2,\cdot) = \beta_2(\cdot,L_2=2) = 1$$

At period 1, the market maker's updated beliefs are

$$\beta_1(S_1=0,S_2,L_2) = \frac{\kappa(1-\delta)}{1-\delta\kappa};\quad \beta_1(S_1=1,S_2,L_2) = \beta_2(S_2,L_2);\quad \beta_1(S_1=2,S_2,L_2)=1.$$

A.2 Proofs

Proof of Lemma 1: Let LHS and RHS be the values of the left hand side and right hand side, respectively, of equation (10). Clearly, $LHS < 0$ for $\beta_3 > 1-\frac{2c}{\lambda_2}$. Since $RHS \geq 0$, any solution to (10), if one exists, must occur for $\beta_3 < 1-\frac{2c}{\lambda_2}$. Intuitively, the informed agent will only submit an order with positive probability if the expected one-period return from submitting an order this period is positive.

We know: (a) $\beta_2(\beta_3, S_2 = 0) < \beta_3$; (b) $\lambda_1 \geq \lambda_2$. Hence, for $\beta_3 \in \left(0, 1 - \frac{2c}{\lambda_2}\right)$, it must be that $0.5\lambda_1(1 - \beta_2(\beta_3, S_2 = 0)) - c > 0$ and the RHS can be expanded as:

$$(1 - \lambda_2)[0.5\lambda_1(1 - \beta_2(\beta_3, S_2 = 0)) - c]. \tag{12}$$

Holding γ_2 constant, the first derivative of (12) with respect to β_3 is

$$-0.5\lambda_1(1 - \lambda_2)(1 - \gamma_2\delta)(1 - \gamma_2\delta\beta_3)^{-2} < 0$$

and the second derivative of (12) with respect to β_3 is

$$-\lambda_1(1 - \lambda_2)\gamma_2\delta(1 - \gamma_2\delta)\,(1 - \gamma_2\delta\beta_3)^{-3} < 0.$$

Thus, RHS is strictly concave for $\beta_3 \in \left(0, 1 - \frac{2c}{\lambda_2}\right)$. The derivative of LHS with respect to β_3 is $-\lambda_2 < 0$. Hence, there is at most one solution to (10) for $\beta_3 \in [0, 1]$. Holding γ_2 "fixed" at a solution to (10) at period 2, it follows that LHS must fall more quickly with an increase in β_3 than RHS. Therefore, to preserve equality, since LHS is independent of γ_2 and RHS rises with γ_2, the mixing probability γ_2 must fall with β_3, falling strictly for $\gamma_2 \in (0, 1)$. ■

Proof of Result 1: This result is shown numerically using an extensive grid search over the space $\{c, \delta, \lambda_1, \epsilon, \bar{\beta}\} \in [0, 1]^5$ where $\lambda_2 = \epsilon\lambda_1$. A finite grid search is sufficient to prove the result since payoffs are a continuous function of the parameters, with bounded derivatives. ■

Proof of Proposition 1: The mixing probability $\gamma_i(\bar{\beta})$ solves

$$\lambda(1 - \bar{\beta}) - 2c = (1 - \lambda)V_{i-1}(\beta_i(\bar{\beta}, 0)) \tag{13}$$

The proposition has two parts (A and B):

Part A: We show that $\bar{\beta} < 1 - \frac{2c}{\lambda}$ is *necessary* and *sufficient* for $\gamma_i(\bar{\beta}) > 0$.

Necessity: If $\bar{\beta} \geq 1 - \frac{2c}{\lambda}$, then $\lambda(1 - \bar{\beta}) - 2c < 0$. Since $V_{i-1}(\cdot) \geq 0\ \forall i$, the informed agent defers.

Sufficiency: Proof by induction.
Period 1: If $\bar{\beta} < 1 - \frac{2c}{\lambda}$, then $0.5\lambda(1 - \bar{\beta}) - c > 0$ and the informed agent submits an order.
Period 2: If $\bar{\beta} < 1 - \frac{2c}{\lambda}$ and $\gamma_1(\bar{\beta}) = 1$, then

$$\lambda(1 - \bar{\beta}) - 2c > (1 - \lambda)V_1(\bar{\beta}) = (1 - \lambda)0.5[\lambda(1 - \bar{\beta}) - 2c].$$

That is, if the market maker *believes* that the informed agent will defer, then the informed agent's expected return from submitting an order exceeds his expected continuation payoff.

Therefore, in equilibrium, $\gamma_2(\bar{\beta}) > 0$.

Arbitrary period i: If $\bar{\beta} < 1 - \frac{2c}{\lambda}$ and $\gamma_j(\bar{\beta}) > 0 \; \forall j < i$, then

$$\lambda(1-\bar{\beta}) - 2c > (1-\lambda)V_{i-1}(\bar{\beta}) = [\lambda(1-\bar{\beta}) - 2c]\sum_{p=1}^{i-1}[0.5(1-\lambda)]^p,$$

since $1 > \sum_{p=1}^{i-1}[0.5(1-\lambda)]^p$. Hence, $\gamma_i(\bar{\beta}) > 0$. Before proceeding to the second part of proposition 1, we develop the following lemma:

Lemma 5 *If $\lambda_i = \lambda \; \forall i$, then $V_i(\beta) \geq V_{i-1}(\beta) \; \forall \beta \in [0,1], \; \forall i$.*

Proof of Lemma 5: Consider the following two cases: (1) If $\beta \geq 1 - \frac{2c}{\lambda}$, then $V_i(\beta) = (1-\lambda)^{i-1}V_1(\beta) = 0$. This implies that $V_i(\beta) = V_{i-1}(\beta)$. (2) If $\beta < 1 - \frac{2c}{\lambda}$, then $V_i(\beta) = 0.5(1-\lambda)V_{i-1}(\beta) + 0.5[\lambda(1-\bar{\beta}) - 2c]$ and $V_{i-1}(\beta) = 0.5(1-\lambda)V_{i-2}(\beta) + 0.5[\lambda(1-\bar{\beta}) - 2c]$. It follows that $V_i(\beta) > V_{i-1}(\beta)$ if and only if $V_{i-1}(\beta) > V_{i-2}(\beta)$. Since $V_1 = 0.5[\lambda(1-\bar{\beta}) - 2c] > V_0 = 0$, the result follows from induction. ■

Part B: Consider the following three cases:

Case 1: If $\gamma_i(\bar{\beta}) = 1$, then it follows immediately that $\gamma_i(\bar{\beta}) \geq \gamma_{i+1}(\bar{\beta})$.

Case 2: If $\gamma_i(\bar{\beta}) = 0$, then from part A it follows that $\lambda(1-\bar{\beta}) - 2c < 0$ and $\gamma_{i+1}(\bar{\beta}) = 0$.

Case 3: If $\gamma_i(\bar{\beta}) \in (0,1)$ then either $\gamma_{i+1}(\bar{\beta}) \in (0,1]$ or $\gamma_{i+1}(\bar{\beta}) = 0$, then

$$(1-\lambda)V_{i+1}(\beta_i(\bar{\beta},0)) = (1-\lambda)V_i(\beta_{i+1}(\bar{\beta},0)) = \lambda(1-\bar{\beta}) - 2c$$

Hence, $V_{i+1}(\beta_i(\bar{\beta},0)) = V_i(\beta_{i+1}(\bar{\beta},0))$. From lemma 5, this occurs only if $\beta_i(\bar{\beta},0) < \beta_{i+1}(\bar{\beta},0)$. Since $\frac{\partial \beta_i(\bar{\beta},0)}{\partial \gamma_i} \leq 0$, it follows that $\gamma_i(\bar{\beta}) > \gamma_{i+1}(\bar{\beta})$. ■

Proof of Proposition 2: Interior values for period i trading probabilities $\gamma_i(\beta_{i+1})$ must solve:

$$\lambda_i(1-\beta_{i+1}) - 2c = (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, S_i = 0)). \tag{14}$$

Let $LHS(i)$ and $RHS(i)$ be the period i values of the left and right hand sides, respectively, of equation (14). Clearly, $LHS(i) < 0$ for $\beta_{i+1} > 1 - \frac{2c}{\lambda_i}$. Since $RHS(i) \geq 0$, any solution to (14) must occur for $\beta_{i+1} < 1 - \frac{2c}{\lambda_i}$. We know: (a) market maker beliefs are non-increasing in response to aggregate order flows of 0 or 1 ($\beta_i(\beta_{i+1}, S_i \in \{0,1\}) < \beta_{i+1} \; \forall i$); and (b) the probability of information leakage is non-decreasing, ($\lambda_i \geq \lambda_{i+1}, \; \forall i$). Hence, for $\beta_{i+1} \in \left(0, 1 - \frac{2c}{\lambda_i}\right)$, it must be that

$$0.5\lambda_i\left(1 - \beta_j\left(\beta_{i+1}, \{S_j, S_{j+1}, ..., S_i\}\right)\right) - c > 0, \; \forall j < i. \tag{15}$$

Using an induction argument and (A1), we now show that if γ_j declines with β_{j+1}, $\forall j < i$, then γ_i falls with β_{i+1}.

Periods 1 and 2: Immediate. γ_1 must fall with β_2 since

$$\gamma_1(\beta_2)\begin{cases} = 1 & \text{for } \beta_2 < 1 - \frac{2c}{\lambda_1} \\ \in [0,1] & \text{for } \beta_2 = 1 - \frac{2c}{\lambda_1} \\ = 0 & \text{for } \beta_2 > 1 - \frac{2c}{\lambda_1} \end{cases}$$

We also showed in lemma 1 that γ_2 fell with β_3.

Period 3: For $\beta_4 \in \left(0, 1 - \frac{2c}{\lambda_3}\right)$, expand $RHS(i = 3)$ as:

$$.5(1-\lambda_3)\Big[(1-\lambda_2)\left(.5\lambda_1(1-\beta_3(\beta_4,0)) - c\right) +$$

$$\max\left\{\lambda_2(1-\beta_3(\beta_4,0)) - 2c,\ (1-\lambda_2)(.5\lambda_1(1-\beta_2(\beta_3(\beta_4,0),0)) - c)\right\}\Big], \tag{16}$$

using (15). Let $\beta_4 \in (0, \beta_4^*)$ denote the range of β_4 such that $\gamma_3(\beta_4) > 0$. From assumption (A1), if $\gamma_3(\beta_4) > 0$, then $\gamma_2(\beta_4) > 0$. Further $\gamma_3(\beta_4) > 0$ implies $\gamma_2(\beta_3(\beta_4, 0)) > 0$ because: (a) $\beta_3(\beta_4, 0) \leq \beta_4$; and (b) $\gamma_2(\beta)$ is falling in β. Hence, we need only consider (16) for β_4 corresponding to $\gamma_2(\beta_3(\beta_4, 0)) > 0$.

Since $LHS(i = 3) > RHS(i = 3)$ for $\beta_4 \geq 1 - \frac{2c}{\lambda_3}$, (A1) implies that if $\gamma_3(\beta_4) > 0$ then $LHS(i = 3) > RHS(i = 3)$ for $\beta_4 > \beta_4^*$. Given these observations and the fact that the derivative of $LHS(i = 3)$ with respect to β_4 is constant and equal to $-\lambda_3 < 0$, a sufficient condition to ensure at most one solution exists to (14) evaluated at period 3 is that

$$(1-\lambda_3)\left[0.5(1-\lambda_2)(0.5\lambda_1(1-\beta_3(\beta_4,0)) - c) + 0.5\lambda_2(1-\beta_3(\beta_4,0)) - c\right] \tag{17}$$

be strictly concave. Expression (17) corresponds to $RHS(i = 3)$ evaluated over $\beta_4 \in (0, \beta_4^*)$. Holding γ_3 constant, the first derivative of (17) with respect to β_4 is

$$-(1-\lambda_3)[0.25(1-\lambda_2)\lambda_1 + 0.5\lambda_2]\frac{\partial\beta_3(\beta_4,0)}{\partial\beta_4} < 0$$

and the second derivative is

$$-(1-\lambda_3)[0.25(1-\lambda_2)\lambda_1 + 0.5\lambda_2]\frac{\partial^2\beta_3(\beta_4,0)}{\partial\beta_4^2} < 0.$$

Thus $RHS(i = 3)$ is strictly concave for $\beta_4 \in (0, \beta_4^*)$ and there is at most one solution to (14) at period 3. Holding γ_3 "fixed" at a solution to (14) at period 3, it follows that $LHS(i = 3)$ falls more quickly with an increase in β_4 than $RHS(i = 3)$. Therefore, to preserve equality, since $LHS(i = 3)$ is independent of γ_3 and $RHS(i = 3)$ rises with γ_3, the mixing probability γ_3 must decline with β_4, strictly falling if $\gamma_3 \in (0, 1)$.

Period i: Using an induction argument, we extend the analysis to an arbitrary period i. Suppose the informed's mixing probability for periods $j < i$, falls with β_{j+1}. Using the same logic as for period 3, we show that if $\gamma_i(\beta_{i+1}) > 0$, then $\gamma_j(\beta) > 0,\ \forall \beta \leq \beta_{i+1},\ \forall j < i$. Let β^*_{i+1} be the maximum β_{i+1} such that $\gamma_i(\beta_{i+1}) > 0$. Hence, possible solutions for the period i analog of equation (14) are in some range $\beta_{i+1} \in (0, \beta^*_{i+1})$. Expanding $RHS(i)$ yields,

$$\sum_{p=1}^{i-1}\left((0.5)^{i-p}(\lambda_p(1-\beta_i(\beta_{i+1},0))-2c)\left[\prod_{k=p+1}^{i}(1-\lambda_k)\right]\right). \tag{18}$$

The first derivative of (18) with respect to β_{i+1} is given by

$$-\sum_{p=1}^{i-1}\left((0.5)^{i-p}\left(\lambda_p\frac{\partial\beta_i(\beta_{i+1},0)}{\partial\beta_{i+1}}\right)\left[\prod_{k=p+1}^{i}(1-\lambda_k)\right]\right) < 0.$$

The second derivative of (18) with respect to β_{i+1} is given by

$$-\sum_{p=1}^{i-1}\left((0.5)^{i-p}\left(\lambda_p\frac{\partial^2\beta_i(\beta_{i+1},0)}{\partial\beta_{i+1}^2}\right)\left[\prod_{k=p+1}^{i}(1-\lambda_k)\right]\right) < 0.$$

Therefore, (18) is strictly concave over $\beta_{i+1} \in (0, \beta^*_{i+1})$. Since the derivative of $LHS(i)$ with respect to β_{i+1} is constant and equal to $-\lambda_i < 0$ and since, by assumption, $LHS(i) > RHS(i)$ if $\gamma_j(\beta) = 0$ for $j < i$ and $\beta < \beta_{i+1}$, there can be no more than one solution to (14) in period i. Holding γ_i "fixed" at a solution to (14) at period 3, it follows that $LHS(i)$ falls more quickly with an increase in β_{i+1} than $RHS(i)$. Therefore, to preserve equality, since $LHS(i)$ is independent of γ_i and $RHS(i)$ rises with γ_i, the mixing probability γ_i must decrease with β_{i+1}, strictly decreasing for $\gamma_i \in (0, 1)$. ■

Proof of Corollary 1: Clearly, the $RHS(i)$ and $LHS(i)$ of (14) are continuous and decreasing in β_{i+1}, and $LHS(i) > RHS(i)$ for $\beta_{i+1} \geq 1 - \frac{2c}{\lambda_i}$. Hence, a sufficient condition for the informed agent to trade with positive probability at period i is that $RHS(i) > LHS(i)$ evaluated at $\beta_{i+1} = 0$:

$$\lambda_i - 2c > \sum_{p=1}^{i-1}\left((0.5)^{i-p}(\lambda_p - 2c)\left[\prod_{k=p+1}^{i}(1-\lambda_k)\right]\right) \tag{19}$$

for $i > 1$ and corresponds to $\lambda_i > 2c$ for $i = 1$. Given (A1), if (19) holds at period i, the informed agent will submit an order each period j, $j \leq i$.

When $\lambda_i = \lambda\ \forall i$, (19) becomes

$$\lambda - 2c > (\lambda - 2c)\sum_{p=1}^{i-1}[0.5(1-\lambda)]^p. \tag{20}$$

If $\lambda - 2c > 0$, this is always satisfied. ■

Proof of Proposition 3 (Comparative Statics):

1a. Change in λ_i: Interior values for the period i trading probabilities $\gamma_i(\beta_{i+1})$ must solve (14). When λ_i decreases, $LHS(i)$ decreases and $(1-\lambda_i)$ increases. In order to maintain equality, it follows that $V_{i-1}(\beta_i(\beta_{i+1},0))$ decreases and therefore $\gamma_i(\beta_{i+1})$ decreases.

1b. Change in λ: The period i analog to (14) for the case where $\lambda_i = \lambda$ is

$$\lambda(1-\beta_{i+1}) - 2c = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1},0)). \tag{21}$$

Let $LHS^*(i)$ and $RHS^*(i)$ be the period i values of the left hand side and the right hand side, respectively, of (21). Part A of proposition 1 implies that: (a) a solution to (21) can occur only in the range $\beta_{i+1} \in \left(0, 1-\frac{2c}{\lambda}\right)$; and (b) $\gamma_j(\beta) > 0 \ \forall j \forall \beta < 1-\frac{2c}{\lambda}$. Hence, for interior values of $\gamma_i(\beta_{i+1})$ it follows that

$$V_{i-1}(\beta_i(\beta_{i+1},0)) = 0.5(1-\lambda)V_{i-2}(\beta_i(\beta_{i+1},0)) + 0.5\lambda(1-\beta_i(\beta_{i+1},0)) - c.$$

Proof proceeds as follows:

Period 1: Recall that $\gamma_1(\beta_2) = 1$ if $\beta_2 < 1-\frac{2c}{\lambda}$ and $\gamma_1(\beta_2) = 0$ if $\beta_2 > 1-\frac{2c}{\lambda}$. Since $\frac{\partial}{\partial\lambda}\left[1-\frac{2c}{\lambda}\right] = \frac{2c}{\lambda^2} > 0$, it follows that $\gamma_1(\beta_2)$ is weakly increasing in λ.

Period 2: Observe that
$\lambda\frac{\partial LHS^*(i=2)}{\partial\lambda} - (1-\lambda)c = \lambda(1-\beta_3) - (1-\lambda)c > \lambda(1-\beta_3) - 2c = (1-\lambda)V_1(\beta_2(\beta_3,0))$,
and
$\lambda\frac{\partial RHS^*(i=2)}{\partial\lambda} - (1-\lambda)c = \lambda\left[(1-\lambda)\frac{\partial V_1(\beta_2(\beta_3,0))}{\partial\lambda} - V_1(\beta_2(\beta_3,0))\right] - (1-\lambda)c$
$< \lambda(1-\lambda)\frac{\partial V_1}{\partial\lambda} - (1-\lambda)c = (1-\lambda)\left[0.5\lambda(1-\beta_2(\beta_3,0)) - c\right] = (1-\lambda)V_1(\beta_2(\beta_3,0))$.
It follows that $\frac{\partial LHS^*(i=2)}{\partial\lambda} > \frac{\partial RHS^*(i=2)}{\partial\lambda}$. To maintain the equality, $\gamma_2(\beta_3)$ must increase.

Period i: The argument used for period 2 is extended to any period i. Define

$$a_i = \sum_{k=1}^{i-1} 0.5^{k-1}(1-\lambda)^k < 2 \qquad i = 2,3,...,T.$$

Observe that
$\lambda\frac{\partial LHS^*(i)}{\partial\lambda} - a_i c = \lambda(1-\beta_{i+1}) - a_i c > \lambda(1-\beta_{i+1}) - 2c = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1},0))$,
and
$\lambda\frac{\partial RHS^*(i)}{\partial\lambda} - a_i c = (1-\lambda)\left[\sum_{p=1}^{i-1}[.5(1-\lambda)]^{p-1}[.5\lambda(1-\beta_i(\beta_{i+1},0)) - V_{i-1-p} - c]\right]$

$< (1-\lambda)\left[\sum_{p=1}^{i-1}[0.5(1-\lambda)]^{p-1}[0.5\lambda(1-\beta_i(\beta_{i+1},0))-c]\right] = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1},0)).$

It follows that $\frac{\partial LHS^*(i)}{\partial \lambda} > \frac{\partial RHS^*(i)}{\partial \lambda}$. To maintain the equality, $\gamma_i(\beta_{i+1})$ must increase. Therefore, $\gamma_i(\beta_{i+1})$ is weakly increasing in λ.

2. Change in c: Interior values of the mixing probability γ_i are defined by (14). Differentiating $LHS(i)$ and $RHS(i)$ of (14) with respect to c:

$$\frac{\partial LHS(i)}{\partial c} = \frac{-2}{1-\lambda_i}; \quad \frac{\partial RHS(i)}{\partial c}\Big|_{\gamma_i} = \frac{\partial V_{i-1}}{\partial c}\Big|_{\gamma_i} \geq -1.$$

Since $\frac{\partial LHS(i)}{\partial c} < \frac{\partial RHS(i)}{\partial c}\Big|_{\gamma_i}$. To restore equilibrium, $RHS(i)$ must decrease. Hence, γ_i must fall.

3. Change in δ: For a given γ_i, an increase in δ causes the market maker's belief about the probability of the good state to fall more rapidly in response to observing an aggregate order flow of zero $\left(\frac{\partial \beta_i(\beta_{i+1},0)}{\partial \delta}\Big|_{\gamma_i} \leq 0\right)$. As δ increases, $\beta_i(\beta_{i+1},0)$ falls and $V_{i-1}(\beta_i(\beta_{i+1},0))$ increases. Hence, $RHS(i)$ of (14) rises with δ. Since $LHS(i)$ does not vary with δ, γ_i must fall to restore the mixing equilibrium condition given by (14). ■

Proof of Proposition 4: When one order has been received, the price of an option at period i is

$$p_i = \beta_i(\beta_{i+1}, S_i = 1)\lambda_i = \frac{(1-2\eta)\delta\beta_{i+1}\gamma_i + \beta_{i+1}\eta}{(1-2\eta)\delta\beta_{i+1}\gamma_i + \eta}\lambda_i.$$

The sign of the derivative of the price with respect to γ_i is

$$\text{sign}\left[\frac{\partial p_i}{\partial \gamma_i}\right] = \text{sign}\left[(1-2\eta)\delta\beta_{i+1}\eta(1-\beta_{i+1})\lambda_i\right].$$

Since $\delta > 0$, $\beta_{i+1} > 0$, $\lambda_i > 0$ and $(1-\beta_{i+1}) \geq 0$, it follows that $\frac{\partial p_i}{\partial \gamma_i} > 0$ when $\eta < 0.5$ and $\frac{\partial p_i}{\partial \gamma_i} < 0$ when $\eta > 0.5$. ■

Proof of Lemma 3: Consistent market maker beliefs imply $\beta_1(S_1 = 2|S_2 = 1, \kappa) \geq \beta_1(S_1 = 1|S_2 = 1, \kappa)$ and $\beta_1(S_1 = 2|S_2 = 2, \kappa) \geq \beta_1(S_1 = 1|S_2 = 2, \kappa)$. We now show that the informed agent does not submit an order of size two at period 1. There are six possible scenarios:

Case 1: $S_2 = 0$. This reveals to the market maker that there is no long-lived liquidity trader. Thus, an order flow of 2 at period 1 reveals the informed agent.

Case 2: $S_2 = 1$ and $Y_2 = 1$. This order flow reveals to the informed agent that there is no long-lived liquidity trader. The informed agent's expected period 1 payoffs from submitting one and two orders are:

$$E[\pi_1(Y_1 = 1|S_2 = 1, Y_2 = 1, \kappa)] = 1 - 0.5\beta_1(2,1) - 0.5\beta_1(1,1) - c \quad (22)$$

$$E[\pi_1(Y_1 = 2|S_2 = 1, Y_2 = 1, \kappa)] = 1 - \beta_1(2,1) - 2c \tag{23}$$

Since $\beta_1(2,1) + 2c > \beta_1(1,1)$, it follows that (22) $>$ (23).

Case 3: $S_2 = 1$ and $Y_2 = 0$. The corresponding expected payoffs are:

$$E[\pi_1(Y_1 = 1|S_2 = 1, Y_2 = 0, \kappa)] = 1 - 0.5\beta_1(2,1) - 0.25\beta_1(1,1) - 0.25(1) - c \tag{24}$$

$$E[\pi_1(Y_1 = 2|S_2 = 1, Y_2 = 0, \kappa)] = 2(1 - 0.75(1) - 0.25\beta_1(2,1) - c) \tag{25}$$

It follows that (24) $>$ (25) since $1 + 4c > \beta_1(1,1)$.

Case 4: $S_2 = 2$ and $Y_2 = 2$. This reveals to the informed agent that there is no long-lived liquidity trader. Therefore, the corresponding expected payoffs are:

$$E[\pi_1(Y_1 = 1|S_2 = 2, Y_2 = 2, \kappa)] = 1 - 0.5\beta_1(2,2) - 0.5\beta_1(1,2) - c \tag{26}$$

$$E[\pi_1(Y_1 = 2|S_2 = 2, Y_2 = 2, \kappa)] = 2(1 - 0.5(1) - 0.5\beta_1(2,2) - c) = 1 - \beta_1(2,2) - 2c \tag{27}$$

Since $\beta_1(2,2) + 2c > \beta_1(1,2)$, it follows that (26) $>$ (27).

Case 5: $S_2 = 2$ and $Y_2 = 1$. The corresponding expected payoffs are:

$$E[\pi_1(Y_1 = 1|S_2 = 2, Y_2 = 1, \kappa)] = 1 - 0.5\beta_1(2,2) - 0.25\beta_1(1,2) - 0.25(1) - c \tag{28}$$

$$E[\pi_1(Y_1 = 2|S_2 = 2, Y_2 = 1, \kappa)] = 2(1 - 0.75(1) - 0.25\beta_1(2,2) - c) \tag{29}$$

Since $1 + 4c > \beta_1(1,2)$, it follows that (28) $>$ (29).

Case 6: $S_2 = 2$ and $Y_2 = 0$. This reveals to the informed agent the existence of a long-lived liquidity trader. He does not submit two orders because doing so would reveal his information.

Accordingly, in period 2, since the informed does not submit an order of two in period 1, the informed's expected payoff from submitting one order at period 2 is:

$$E[\pi_2(Y_2 = 1, \beta_3)] = 0.25\lambda_2[1 - \beta_2(1)] + 0.5\lambda_2[1 - \beta_2(2)] - c$$

$$+0.25(1 - \lambda_2)E[\pi_1(Y_1 = 1|S_2 = 1, Y_2 = 1, \beta_3)] + 0.5(1 - \lambda_2)E[\pi_1(Y_1 = 1|S_2 = 2, Y_2 = 1, \beta_3)].$$

His expected payoff from submitting two orders at period 2 is:

$$E[\pi_2(Y_2 = 2, \beta_3)] = 0.5\lambda_2[1 - \beta_2(2)] - 2c + 0.5(1 - \lambda_2)E[\pi_1(Y_1 = 1|S_2 = 2, Y_2 = 2, \beta_3)].$$

Hence, $E[\pi_2(Y_2 = 1, \beta_3)] > E[\pi_2(Y_2 = 2, \beta_3)]$. ■

Lemma 6 *A sufficient condition for the informed agent to trade in the last period if the information has not been revealed through trading or through a public announcement is that the ex ante probability that the good state will occur is sufficiently small. If there is trade only in the nearby contract, the sufficient condition is $\kappa < \frac{1-4c}{1+4c\delta}$. If there is trade in the distant contract, the sufficient condition is $\kappa < 1 - 2c$.*

Proof of Lemma 6: **Trade in nearby contract only:** If the information has not been revealed, the largest possible value of β_1, and hence the lowest expected profit, occurs when $\gamma_2 = 1$, $S_1 = 2$, and $S_2 = 2$. In this situation, the informed agent submits an order at period 1 if

$$0.25\left(1 - \frac{\kappa(1+\delta)}{1+\delta\kappa}\right) - c > 0 \Leftrightarrow \kappa < \frac{1-4c}{1+4c\delta}.$$

Trade in distant contracts: If the information has not been revealed, the lowest expected profit occurs when $\rho_L = 0$. Then, the informed agent submits an order at period 1 if $0.5[1 - \kappa] - c > 0 \Leftrightarrow \kappa < 1 - 2c$. ■

Proof of Proposition 7: a) $\lambda_2 = 1$: Consistent market maker beliefs imply that

$$E[\pi_2(0,1) - \pi_2(1,0)] = 0.25(\beta_2(1,0) - \beta_2(0,1)) = 0 \Leftrightarrow \beta_2(1,0) = \beta_2(0,1) \Leftrightarrow \rho_L(\kappa) = \rho_S(\kappa).$$

b) $\lambda_2 < 1$: By contradiction. Suppose $\rho_L \leq \rho_S$. It follows that $\beta_2(0,1) \leq \beta_2(1,0)$, which implies $1 - \beta_2(0,1) > \lambda_2(1 - \beta_2(1,0))$ and $1 - \beta_2(1,1) > \lambda_2(1 - \beta_2(1,1))$. Also, $V_1(\beta_2(0,1)) \geq V_1(\beta_2(1,0))$ because $\frac{\partial V_1(\beta)}{\partial \beta} \leq 0$. These observations imply that the informed agent's expected return from trading the distant contract exceeds his expected return from trading the nearby contract. But then his optimal trading strategy is not consistent with market maker's beliefs. ■

Proof of Lemma 4: The difference

$$E\left[\pi_2(0,1) + (1-\lambda_2)V_1(S_2(0), L_2(1))\right] - E\left[\pi_2(1,1) + (1-\lambda_2)V_1(S_2(1), L_2(1))\right]$$

$$= 0.25\left[1 - \beta_2(0,1)\right] - 0.25\lambda_2\left[1 - \beta_2(1,1)\right] + c + 0.25(1-\lambda_2)V_1(\beta_2(0,1))$$

is always positive since consistent market maker beliefs imply that $\beta_2(0,1) \leq \beta_2(1,1)$. ■

Proof of Proposition 5: Observe that

$$\frac{\partial E[\pi_1(Y_1|\mathbf{H}_2, W_2)]}{\partial W_2} = \begin{cases} 1 & \text{if } Y_1 = 2 \\ 0.5 + 0.5\lambda_1 & \text{if } Y_1 = 1 \\ \lambda_1 & \text{if } Y_1 = 0 \end{cases} \tag{30}$$

Since
$$\frac{\partial E[\pi_1(Y_1=2|\mathbf{H}_2,W_2)]}{\partial W_2} \geq \frac{\partial E[\pi_1(Y_1=1|\mathbf{H}_2,W_2)]}{\partial W_2} \geq \frac{\partial E[\pi_1(Y_1=0|\mathbf{H}_2,W_2)]}{\partial W_2}$$
(strict when $\lambda_1 < 1$), it follows that the informed agent is more likely to submit a larger order at period 1 as W_2 increases. ■

Proof of Lemma 2: The payoff from submitting an order of size two at period 2 is $W_3 - c$. The informed agent can realize a higher expected return by deferring at period 2 and, if his information has not yet been revealed, submit an order of size two at period 1. This alternative strategy has expected return $W_3 - c(1-\lambda_2) > W_3 - c,\ \lambda_2 > 0$. ■

Proof of Proposition 6: Lemma 2 ensures that the informed agent never submits an order of size two at period 2. Conditional on W_3, the informed's expected return from submitting a one unit order at period 2 is

$$submit(W_3) = 0.5(1-\lambda_2)V_1([S_2=1,\mathbf{H}_3],W_3+1)+0.5(\lambda_2-p_2(S_2=1,\mathbf{H}_3))-c+0.5(1+\lambda_2)W_3, \quad (31)$$

and the expected return from deferring at period 2 is

$$defer(W_3) = 0.5(1-\lambda_2)\left[V_1([S_2=1,\mathbf{H}_3],W_3) + V_1([S_2=0,\mathbf{H}_3],W_3)\right] + \lambda_2 W_3. \quad (32)$$

If $submit(1) - defer(1) > submit(0) - defer(0)$, the relative value of submitting an order is larger when the informed agent has an accumulated position. Rewriting this condition yields

$$1 + [V_1(1,2) - V_1(1,1)] - [V_1(0,1) - V_1(0,0)] - [V_1(1,1) - V_1(1,0)] > 0, \quad (33)$$

where we suppress the order flow history, $\mathbf{H}_3$, and only report S_2 to conserve space.

Equation (30) shows that the magnitude of the increase in the value of the informed's information at period 1, due to an increase in W_2, rises with Y_1. Lemma 5 ensures that for any given market maker beliefs, the informed agent is more likely to submit a larger order at period 1 as W_2 increases. These two observations imply that $V_1(1,2) - V_1(1,1) \geq V_1(1,1) - V_2(1,0)$.

Clearly, the informed agent never submits an order of 2 at period 1 when $W_2 = 0$. Therefore, it follows from (30) that $V_1(0,1) - V_1(0,0) < 1$. These observations ensure that condition (33) is satisfied, and hence, $\gamma_2(Y_2 = 1|W_3 = 1, \cdot) \geq \gamma_2(Y_2 = 1|W_3 = 0, \cdot)$. ■

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