



Queen's Economics Department Working Paper No. 82

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IN SOME AGGREGATE MODELS OF
WAGE-DETERMINATION

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5-1972

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Discussion Paper No. 82

May, 1972

In a recent paper [1], we discussed the possible availability of prior information with respect to the source of the moving-average components of mixed moving-average autoregressive processes in the context of empirical investigations of wage-determination. Our earlier discussion of this generalization of the Yule-Slutsky effect ignores the possible presence of lagged endogenous variables, which may be due to explicit temporal spillovers between different wage-bargains in the labour market. This omission is remedied here and several final equations for models with temporal spillovers are given below. These can be contrasted with the final equation for a simple model in which the spillover feedback is restricted to autoregressive processes for the stochastic errors of theoretical relations. Notation from the earlier discussion is retained. The different specifications can be associated with a collection of linear hypotheses for which conventional least-squares statistics provide suitable test-statistics if samples of data are sufficiently large. Knowledge of the weights for the moving-average components is an essential framework for the procedures which are outlined.

Model

The temporal pattern of wage-changes for a particular bargaining group in the labour market is discontinuous. These changes form a step sequence in quarterly data and they occur at different points in time for different bargaining groups. Many models of wage-determination have sought to overcome this difficult feature of the market by use of prior adjustments of data; see, for example, the accounts of Dicks-Mireaux and Dow [2], Perry [3], and our own contributions [4, 5]. However, the reconciliation of these adjustments with the problem of temporal spillovers has never been explicitly

raised even though both are concerned with similar features of the labour market. The adjustments attempt to take account of the different times of adjustment for different bargaining groups and temporal spillover is the recognition that a bargaining group can take into account bargains achieved in earlier periods by itself and by other groups.

Suppose the supply of labour can be classified into four distinct groups according to the quarter in which members of each group negotiate and obtain their annual wage-bargains. Assume that each group can consider two distinct collections of factors when negotiating their individual bargains; namely, factors which affect the general well-being of all groups and factors which reflect the relative wage-standards of each group as compared with those of other groups. If the first collection is the same for each group and if we can focus attention solely on temporal influences at the expense of spatial ones, then the determination of wage-rates can be described by linear equations of the form

$$(1) \begin{bmatrix} y_{4t+4} \\ \dots \\ y_{4t+1} \end{bmatrix} = \sum_{s=1}^4 \begin{bmatrix} d_{4s} & & 0 \\ & \ddots & \\ 0 & & d_{1s} \end{bmatrix} \begin{bmatrix} y_{4t+4-s} \\ \dots \\ y_{4t+1-s} \end{bmatrix} + \sum_{j=1}^k \begin{bmatrix} a_{4j} & & 0 \\ & \ddots & \\ 0 & & a_{1j} \end{bmatrix} \begin{bmatrix} x_{4t+4} \\ \dots \\ x_{4t+1} \end{bmatrix}_j + \begin{bmatrix} e_{4t+4} \\ \dots \\ e_{4t+1} \end{bmatrix}$$

for $t = 0, 1, 2, \dots, n_0$ and $n = 4(n_0 + 1)$,

where y_{4t+i} represents the average wage-rate arranged by members of the i -th group in the period indexed by $(4t+i)$, $[x_{4t+i}]_j$ is the level of the

j -th explanatory variable considered by the i -th group in obtaining its bargain during period $(4t+i)$. k and n are the number of explanatory variables (apart from lagged endogenous variables) and observations respectively. The positivity of the coefficients $\{d_{js}$ for $j = 1,2,3,4$ and $s = 1,2,3,4\}$ is indicative of explicit spillover and this may be classified into two distinct types. "Inter-group spillover" and "historical within-group spillover" are represented by $\{d_{js}$ for $j = 1,2,3,4$ and $s = 1,2,3\}$ and $\{d_{js}$ for $j = 1,2,3,4$ and $s = 4\}$, respectively, and groups may be described as "temporally myopic" to the extent that some or all of these coefficients are zero. If different quarterly groups can be identified with either "key" sectors or "follower" sectors of the labour market, then the coefficients $\{d_{js}\}$ will vary with index j for all given values of index s . In particular, d_{js} will be relatively small, perhaps zero, if j represents a key sector and relatively large if j represents a follower sector. The errors $\{e_{4t+i}$ for $i = 1,2,3,4$ and $t = 1,2, \dots, n_0\}$ are assumed to be generated by an autoregressive process in the form

$$(2) \quad \begin{bmatrix} e_{4t+4} \\ \dots \\ e_{4t+1} \end{bmatrix} = \sum_{i=1}^4 \begin{bmatrix} & & & 0 \\ b_{4i} & & & \\ & & & \\ 0 & & & b_{1i} \end{bmatrix} \begin{bmatrix} e_{4t+4-i} \\ \dots \\ e_{4t+1-i} \end{bmatrix} + \begin{bmatrix} u_{4t+4} \\ \dots \\ u_{4t+1} \end{bmatrix}$$

for $t = 0, 1, 2, \dots, n_0$, where $\{u_{4t+i}\}$ is white noise with constant variance. The positivity of the coefficients $\{b_{ji}\}$ indicates implicit spillover due to omitted factors and, again, two types of spillover may be distinguished by the value of the second subscript.

These specifications can be written in an alternative and more convenient matrix form. Equations (1) become

$$(3) \quad y = \sum_{s=1}^4 [I_n \otimes D_s] y_{-s} + \sum_{j=1}^k [I_n \otimes A_j] x_{0j} + e,$$

where $\{A_j\}$ are diagonal matrices $\{dg(a_{4j}, a_{3j}, a_{2j}, a_{1j})\}$ for $j = 1, 2, \dots, k\}$, $\{D_s\}$ are diagonal matrices $\{dg(d_{4s}, d_{3s}, d_{2s}, d_{1s})\}$ for $s = 1, \dots, 4\}$, I_n is an identity matrix of order $(n_0 + 1)$, and y, y_{-s}, x_{0j} and e are column vectors of order n .

$$y \equiv \begin{bmatrix} y_n \\ \dots \\ y_1 \end{bmatrix}, \quad y_{-s} \equiv \begin{bmatrix} y_{n-s} \\ \dots \\ y_{1-s} \end{bmatrix}, \quad x_{0j} \equiv \begin{bmatrix} x_n \\ \dots \\ x_1 \end{bmatrix}_j \quad \text{and} \quad e \equiv \begin{bmatrix} e_n \\ \dots \\ e_1 \end{bmatrix}$$

$[I_n \otimes A_j]$ is the right direct product of I_n and A_j (see MacDuffee [6]); namely, a quasidiagonal matrix with $(n_0 + 1)$ diagonal blocks formed by A_j .

Similarly, equations (2) can be represented by

$$(4) \quad e = \sum_{i=1}^4 [I_n \otimes B_i] e_{-i} + u,$$

where $\{B_i\}$ are diagonal matrices $\{dg(b_{4i}, b_{3i}, b_{2i}, b_{1i})\}$ for $i = 1, 2, 3, 4\}$,

$$e_{-i} \equiv \begin{bmatrix} e_{n-i} \\ \dots \\ e_{1-i} \end{bmatrix} \quad \text{and} \quad u \equiv \begin{bmatrix} u_n \\ \dots \\ u_1 \end{bmatrix}.$$

In summary, the two autoregressive specifications represent distinct spillover effects. The presence of lagged dependent variables in (1) and (3) indicates the influences of explicit spillovers between the bargains of different groups, which may be asymmetric in the sense associated with key and follower sectors, and the autoregressive specification for the generating process of the errors in (2) and (4) indicates implicit spillover. Clearly,

adjustment is used for all variables in the model and that the two sets of coefficients $\{a_{ij}\}$ and $\{d_{is}\}$ are both free to vary over values of the first subscript i . Hence the model is free from the problems of "linear aggregation bias" which were pointed out by Theil [7] in their general form and by Gupta [8] in the specific context of labour markets.

With any choice for G , the macro-equations are obtained from (3) and (4).

$$(6) \quad Gy = \sum_{s=1}^4 G[I_n \otimes D_s] y_{-s} + \sum_{j=1}^k G[I_n \otimes A_j] x_{0j} + Ge$$

$$(7) \quad Ge = \sum_{i=1}^4 G[I_n \otimes B_i] e_{-i} + Gu$$

In order to write these equations in terms of the "macro-data" $\{Gy, (Gy)_{-s}, Gx_{0j}, Ge$ and $(Ge)_{-i}\}$, matrices N_j, R_s and M_i must be found such that

$$(8) \quad G[I_n \otimes A_j] \equiv N_j G \quad \text{for } j=1, 2, \dots, k$$

$$(9) \quad G[I_n \otimes D_s] \equiv R_s G \quad \text{for } s=1, 2, 3, 4$$

$$(10) \quad G[I_n \otimes B_i] \equiv M_i G \quad \text{for } i=1, 2, 3, 4.$$

Given the particular specification (5) for G , this matrix has full row rank so that its Moore-Penrose inverse, denoted G^- , is $G'(GG')^{-1}$ and GG^- is an identity matrix of order $(n-q)$. Hence, postmultiplication by the Moore-Penrose inverse in (8), (9) and (10) yields explicit expressions for $N_j, R_s, \text{ and } M_i$.

$$(11) \quad N_j = G[I_n \otimes A_j]G^- \quad \text{for } j=1, 2, \dots, k$$

$$(12) \quad R_s = G[I_n \otimes D_s]G^{-1} \quad \text{for } s=1, 2, 3, 4.$$

$$(13) \quad M_i = G[I_n \otimes B_i]G^{-1} \quad \text{for } i=1, 2, 3, 4.$$

$$(14) \quad (Gy) = \sum_{s=1}^4 R_s(Gy)_{-s} + \sum_{j=1}^k N_j(Gx_{0j}) + (Ge)$$

$$(15) \quad (Ge) = \sum_{i=1}^4 M_i(Ge)_{-i} + (Gu)$$

The errors of the macro-equation (14) are generated by a mixed moving-average autoregressive process for which the weights of the moving-average component are known. This form provides the only basis for estimation when the macro-data are the sole source of information.

Notice that the definitions of N_j , R_s and M_i indicate

$$(16) \quad G[I_n \otimes A_j] = G[I_n \otimes A_j]H \quad \text{for all } j,$$

$$(17) \quad G[I_n \otimes R_s] = G[I_n \otimes R_s]H \quad \text{for all } s,$$

and

$$(18) \quad G[I_n \otimes B_i] = G[I_n \otimes B_i]H \quad \text{for all } i$$

if H denotes the product $G^{-1}G$. This product is a symmetric idempotent matrix and has an important role in the estimation procedure which is indicated below. Equations (16), (17) and (18) permit the macro-equation (14) to be arranged in a convenient form after it has been subjected to two more adjustments.

Estimation and the Macro-Equations

Let θ represent the lag operator such that $\theta^s z_t$ is equal to z_{t-s} for an arbitrary vector of observations z_t . Then $L(\theta)$ can be used to denote a matrix function of the lag operator such that the equation for the

macro-error (15) can be written as

$$L(\theta) \cdot (Ge) = Gu$$

where

$$L(\theta) \equiv I - \sum_{i=1}^4 M_i \theta^i$$

Application of this autoregressive transformation to (14) yields

$$(19) \quad Gy = \sum_{i=1}^4 (M_i + R_i)(Gy)_{-i} - \sum_{i=1}^4 \sum_{s=1}^4 M_i R_s (Gy)_{-i-s} \\ + \sum_{j=1}^k N_j (Gx_{0j}) - \sum_{i=1}^4 \sum_{j=1}^k M_i N_j (Gx_{0j})_{-i} + (Gu) .$$

Let this equation be pre-multiplied by the Moore-Penrose inverse G^- to obtain an alternative equation which is more convenient to use.

$$(20) \quad Hy = \sum_{i=1}^4 (G^-M_i + G^-R_i)(Gy)_{-i} - \sum_{i=1}^4 \sum_{s=1}^4 G^-M_i R_s (Gy)_{-i-s} \\ + \sum_{j=1}^k G^-N_j (Gx_{0j}) - \sum_{i=1}^4 \sum_{j=1}^k G^-M_i N_j (Gx_{0j})_{-i} + Hu .$$

$$\text{But } G^-M_i (Gy)_{-i} = H[I_n \otimes B_i] (Hy)_{-i} \\ = [I_n \otimes B_i](Hy)_{-i} \quad \text{using (18).}$$

$$G^-R_s (Gy)_{-s} = [I_n \otimes D_s](Hy)_{-s} \quad \text{using (17).}$$

$$G^-N_j (Gx_{0j}) = [I_n \otimes A_j](Hx_{0j}) \quad \text{using (16).}$$

$$G^-M_i N_j (Gx_{0j})_{-i} = H[I_n \otimes B_i] H [I_n \otimes A_j](Hx_{0j})_{-i} \\ = [I_n \otimes B_i][I_n \otimes A_j](Hx_{0j})_{-i} \\ = [I_n \otimes B_i A_j](Hx_{0j})_{-i}$$

$$G^{-1}M_i R_s(Gy)_{-i-s} = [I_n \otimes B_i D_s](Hy)_{-i-s}$$

$$(21) \quad (Hy) = \sum_{i=1}^4 [I_n \otimes (B_i + D_i)](Hy)_{-i} - \sum_{i=1}^4 \sum_{s=1}^4 [I_n \otimes B_i D_s](Hy)_{-i-s} \\ + \sum_{j=1}^k [I_n \otimes A_j](Hx_{0j}) - \sum_{i=1}^4 \sum_{j=1}^k [I_n \otimes B_i A_j](Hx_{0j})_{-i} + (Hu).$$

Further notation is necessary if this equation is to be arranged in a more convenient form. Let a bar associated with an arbitrary column vector z , of order n , represent the operation which takes the elements of the vector and forms a matrix, of order n by 4,

$$\bar{z} \equiv \begin{bmatrix} dg(z_n, z_{n-1}, z_{n-2}, z_{n-3}) \\ \dots & \dots \\ dg(z_8, z_7, z_6, z_5) \\ dg(z_4, z_3, z_2, z_1) \end{bmatrix} \quad \text{if } z \equiv \begin{bmatrix} z_n \\ \dots \\ z_2 \\ z_1 \end{bmatrix}.$$

$$\text{Let } b_i \equiv \begin{bmatrix} b_{4i} \\ b_{3i} \\ b_{2i} \\ b_{1i} \end{bmatrix}, \quad d_s \equiv \begin{bmatrix} d_{4s} \\ d_{3s} \\ d_{2s} \\ d_{1s} \end{bmatrix}, \quad a_j \equiv \begin{bmatrix} a_{4j} \\ a_{3j} \\ a_{2j} \\ a_{1j} \end{bmatrix},$$

$$b_i * a_j \equiv \begin{bmatrix} b_{4i} & a_{4j} \\ b_{3i} & a_{3j} \\ b_{2i} & a_{2j} \\ b_{1i} & a_{1j} \end{bmatrix} \equiv -c_{ij}^{ab}, \quad \text{and}$$

$$b_i * d_s \equiv \begin{bmatrix} b_{4i} & d_{4s} \\ b_{3i} & d_{3s} \\ b_{2i} & d_{2s} \\ b_{1i} & d_{1s} \end{bmatrix} \equiv -c_{is}^{bd},$$

the Schur products of (b_i, a_j) and (b_i, d_s) , respectively, for all i, j and s .

$$(22) \quad (Hy) = \sum_{i=1}^4 \overline{(Hy)}_{-i} (b_i + d_i) + \sum_{i=1}^4 \sum_{s=1}^4 \overline{(Hy)}_{-i-s} c_{is}^{bd} \\ + \sum_{j=1}^k \overline{(Hx_{0j})} a_j + \sum_{i=1}^4 \sum_{j=1}^k \overline{(Hx_{0j})}_{-i} c_{ij}^{ab} + (Hu)$$

If the initial explanatory variables (x_{0j}) , associated with the collection of general factors, are non-stochastic and if all redundant variables are omitted from equation (22), then the principle of least-squares can be applied to this equation, even though the final errors (Hu) are not spherical, to derive consistent estimators $b_i, d_s, a_j, c_{ij}^{bd}$ and c_{ij}^{ab} under fairly general conditions. These estimators will, of course, not take account of the nonlinear constraints on the parametric vectors and they are conditional upon the availability of sufficient observations to avoid the problem of multicollinearity. An approximation to the principle of generalized least-squares would involve the calculation of a matrix N , of full row rank, such that NHN' is a scalar matrix. Equation (22) would be pre-multiplied by N and the least-squares technique applied to the new equation. The resultant estimators of the parametric vectors would be identical with those obtained without the use of the further transformation N . This equivalence follows from the equality of $H'H$ and $H'N'NH$ if both N and G have full row rank and NHN' is the identity matrix of order $(n-q)$.

A simple change in notation permits (22) to be written in a more convenient form:

$$(23) \quad (Hy) = \sum_{p=1}^8 \overline{(Hy)}_{-p} \delta_{1p} + \sum_{j=1}^k \sum_{i=0}^4 \overline{(Hx_{0j})}_{-i} \delta_{2p}^j + (Hu),$$

where the vectors of macro-coefficients $\{\delta_{1p}, \delta_{2p}^j\}$ are identified in terms of the micro-parameters by elements of Tables One and Two below.

TABLE ONE. MACRO-COEFFICIENTS OF LAGGED DEPENDENT VARIABLES

p	δ_{1p}
1	$b_1 + d_1$
2	$b_2 + d_2 - (b_1 * d_1)$
3	$b_3 + d_3 - (b_1 * d_2) - (b_2 * d_1)$
4	$b_4 + d_3 - (b_1 * d_3) - (b_2 * d_2) - (b_3 * d_1)$
5	$-(b_1 * d_4) - (b_2 * d_3) - (b_3 * d_2) - (b_4 * d_1)$
6	$-(b_2 * d_4) - (b_3 * d_3) - (b_4 * d_2)$
7	$-(b_3 * d_4) - (b_4 * d_3)$
8	$-(b_4 * d_4)$

TABLE TWO. MACRO-COEFFICIENTS OF OTHER EXPLANATORY VARIABLES
(for $j = 1, 2, 3, 4, \dots, k$)

p	δ_{2p}^j
0	a_j
1	$-(a_j * b_1)$
2	$-(a_j * b_2)$
3	$-(a_j * b_3)$
4	$-(a_j * b_4)$

The four forms of spillover can be associated with specific vectors of micro-coefficients as shown in Table Three and, hence, with zero restrictions on the vectors of macro-coefficients as indicated by the zero entries in Table Four. Equation (23) contains all four forms of spillover and the omission of variables from this equation can be identified with the absence of one or more of the forms of spillover. Notice that if implicit historical spillover is absent within groups, then we cannot discriminate between no further absence of spillover, absence of explicit inter-group spillover and absence of explicit historical within-group spillover. (See rows 3, 7 and 9 of Table Four.) Similarly, we cannot discriminate between the absence of only explicit inter-group spillover and no absences of any form. However these alternatives can be distinguished if either explicit historical spillover within-groups (rows 1, 2 and 5) or explicit inter-group spillover (rows 4 and 10) is known to be absent.

The roles of key or follower sectors are illustrated by differences within the vectors of parameters. If they are wholly absent, the bars over the variables in equation (23) can be omitted and the vectors of macro-coefficients reduce to single elements. Key sectors might be expected to correspond to zero or small elements in the parametric vectors $\{d_s\}$ whereas follower sectors might be associated with larger values. An index of relative bargaining strength might be based upon the relative entries in the vectors $\{a_j\}$ with bargaining strength being positively correlated with the size of the entries.

Conclusion

If sufficient data are available, the influence of temporal spillover upon movements in the aggregate wage-index can be investigated in four distinct forms. Linear hypotheses of a simple type can be identified with the absence of one or more of these forms of spillover and approximate test statistics for these hypotheses can be obtained either by application of the least-squares approach to a doubly-transformed equation or by application of Aitken's generalization of the least-squares approach to an equation which has been subjected to a simple autoregressive transformation of fourth-order.

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